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Efficiency in Auctions with Private and Common Values: An Experimental Study

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Efficiency in Auctions with Private and Common Values:
An Experimental Study

Jacob K. Goeree and Theo Offerman*

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ABSTRACT

Auctions generally do not lead to efficient outcomes when the expected value of the object for sale depends on both private and common value information. We report a series of first-price auction experiments to test three key predictions of auctions with private and common values: (i) inefficiencies grow with the uncertainty about the common value while revenues fall, (ii) increased competition results in more efficient outcomes and higher revenues, and (iii) revenues and efficiency are higher when information about the common value is publicly released. We compare the predictions of several bidding models, including Nash, when examining these issues. A model in which a fraction of the bidders falls prey to a winner’s curse and decision-making is noisy, best describes bidding behavior. We find that revenues and efficiency are positively affected by increased competition and a reduction in uncertainty about the common value. The public release of high-quality information about the common value also has positive effects on efficiency, although less so than predicted by Nash equilibrium bidding.

JEL Classification: C72, D44.
Keywords: Auctions, experiments, winner’s curse, efficiency, information disclosure, competition.

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1. Introduction

Auctions are typically classified as either "private value" or "common value." In private value auctions, bidders know their own value for the commodity for sure but are unaware of others’ valuations (e.g. the sale of a painting). In contrast, in common value auctions, each bidder receives a noisy signal about the commodity’s value, which is the same for all (e.g. firms competing for the rights to drill for oil). While this dichotomy is convenient from a theoretical point of view, most real-world auctions exhibit both private and common value elements. In the recent spectrum auctions conducted by the FCC, for example, the different cost structures of the bidding firms constituted a private value element, while the uncertain demand for the final consumer product added a common value part. Alternatively, in takeover battles, bidders’ valuations are determined by private synergistic gains in addition to the target’s common market value.\(^1\)

By focusing on the "extreme" cases, the literature has inadvertently spread the belief that auctions generally lead to efficient allocations. In (symmetric) private value auctions, optimal bids are increasing in bidders’ values so the object is awarded to whom it is worth the most, and in common value auctions, any allocation is trivially efficient. When both private and common value elements play a role, however, inefficiencies should be expected. The simple intuition for this result is that a bidder with an inferior private value but an overly optimistic conjecture about the common value may outbid a rival with a superior private value. The possibility of inefficiencies in multi-signal auctions was first discussed by Maskin (1992) and further explored by Dasgupta and Maskin (1999), Jehiel and Moldovanu (1999), Pesendorfer and Swinkels (1999), and Goeree and Offerman (1999).

This paper reports a series of first-price auction experiments in which bidders receive a private and a common value signal.\(^2\) To determine the optimal bid, the two pieces of

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\(^1\) Even the standard examples of purely private or purely common value auctions are not convincing. When a painting is auctioned, for instance, it may be resold in the future and the resale price will be the same for all bidders, which adds a common value element. And in the oil drilling example, a firm’s cost of exploiting the tract adds a private value element.

\(^2\) Kirchkamp and Moldovanu (2000) report an experiment that compares the efficiency properties of the English auction and the second-price auction when bidders’ valuations are interdependent.
information have to be combined and the relative weights bidders assign to each signal determines the efficiency of the resulting allocation. For instance, if bidders completely ignore their common value signal, the auction turns into a fully efficient private value auction. In contrast, when bidders ignore their private value information, the auction is no more efficient than a random allocation rule. Rational bidders react to both pieces of information, resulting in some intermediate degree of inefficiency. One goal of this paper is to measure the extent of inefficiencies that result with financially motivated human bidders.

Observed bids will differ from rational ones when subjects do not (sufficiently) incorporate the negative information conveyed by winning into their bids. Indeed, there exists substantial experimental and empirical evidence that in purely common value auctions, bidders often ignore this adverse selection effect and forgo some profits as a result: the winner’s curse.³ In auctions with both private and common values such naive bidders may cause inefficiencies when they put too much weight on their own common value signal. To separate efficiency from winner’s curse issues, the common value in our experiment is the average of bidders’ signals. This design allows bidders to fall prey to a winner’s curse, by replacing others’ common value signals by their unconditional expected value, without affecting efficiency. As long as bidders assign the same weight to their own common value signal as rational bidders would, efficiency remains unchanged. Of course, the amount paid by a naive winner will be higher (and may even exceed the object’s value), but this monetary transfer from the winning bidder to the seller does not affect efficiency.

A second goal of this paper is to systematically explore factors that affect efficiency and to test the effectiveness of policies aimed at reducing inefficiencies. For example, when licenses to operate in a market are auctioned, interested firms will have to estimate the uncertain (but common) demand for the consumer product they will sell. There will be more uncertainty associated with licenses for new markets (e.g. wireless local loop frequencies for

multi-media applications) than with licenses for well-established markets (e.g. vendor locations at fairs). Intuitively, an increase in uncertainty about the common value makes the private values less important and causes efficiency to fall. In addition, with more uncertainty about the common value, bids have to be more cautious to avoid a winner’s curse, resulting in higher profits for the bidders and less revenue for the seller.

Another important determinant for efficiency of market outcomes is (perfect) competition (e.g. Stigler, 1987). In purely common value auctions, perfect competition also leads to "information aggregation," i.e. the winning bid converges to the actual value of the commodity for sale (Wilson, 1977). In auctions with private and common values, perfect competition leads to both information aggregation and full efficiency. These positive effects of competition are not limited to the case of an infinite number of bidders, however: even a moderate increase in the number of bidders (e.g. from three to six) is predicted to have a substantial effect on efficiency and revenues. The reason for this "small numbers" effect is that bidders weigh their common value information substantially less even if there is only a moderate increase in competition. Hence, the private value differences become more important and an efficient outcome more likely. In fact, the introduction of new bidders makes the auction more efficient even when these bidders themselves have no chance of winning (see Goeree and Offerman, 1999).

Finally, the seller often possesses information about the object for sale and the public disclosure of such information affects bidding behavior. Milgrom and Weber (1982) have shown that the public release of information is in the auctioneer’s best interest because it raises revenues. Elsewhere we have shown that public information release also raises efficiency. The resulting increase in total surplus generated by the auction benefits the seller, not the winning bidder, and the predicted effects on efficiency and revenues are stronger the higher the quality of the public information (Goeree and Offerman, 1999).

This paper investigates the empirical validity of these theoretical predictions, i.e. the effects of uncertainty about the common value, competition, and information disclosure on efficiency and revenues. The order of topics is as follows: the design of the auctions is explained in section 2. The Nash bidding functions and other theoretical benchmark models are derived in section 3, together with some comparative statics results. In section 4 we
investigate the effects of the winner’s curse in auctions with private and common values. In sections 5 and 6 we focus on how bidders process the private and common value signal, and report the realized level of efficiency. The effects of uncertainty about the common value, the public release of information, and increased competition are dealt with in section 7. Section 8 concludes. The Appendices contain instructions, the Nash equilibrium bids for one of the treatments, and figures that show actual and predicted bids in each treatment.

2. Design of the Auctions

Bidder $i$’s valuation for the object for sale consists of a private value, $t_i$, and the common value, $V$. The realization of $V$ is unknown at the time of bidding, but is revealed after the bidding phase. When bidder $i$ wins the auction with a bid $b_i$, she receives a net amount of $V + t_i - b_i$, where the common value is the average of bidders’ signals:

$$V = \frac{1}{n} \sum_{i=1}^{n} v_i. \quad (1)$$

This formulation for the common value has previously been used in both theoretical and experimental work. An advantage of (1) is that it is easier to explain and understand than the "traditional" formulation of the common value, where $V$ has some known prior distribution and bidders’ signals are draws conditional on the particular realization of $V$ (Wilson, 1970). While being simpler, (1) captures the two main features of the traditional formulation: (i) the value of the object for sale is the same for all bidders, and (ii) in order not to fall prey to a winner’s curse, bidders should take into account the information conveyed in winning. Holt and Sherman (2000), for instance, report clear evidence of a winner’s curse in a two-person first-price auction experiment using the formulation in (1), and Avery and Kagel (1997) find similar evidence in a second-price auction.

The (completely computerized) experiment consisted of two parts; subjects received the instructions for part 2 only after all twenty periods of part 1 were finished (see Appendix 4). Theoretical papers using this formulation include Bikchandani and Riley (1991), Albers and Harstad (1991), Krishna and Morgan (1997), Klemperer (1998), Bulow and Klemperer (1999) and Bulow, Huang and Klemperer (1999).
A for a translation of the instructions). In the experiment, subjects earned points, which were exchanged into guilders at the end of the experiment at a rate of 4 points = 1 guilder (≈ $0.50). In total we conducted seven treatments: low-3, high-3, high-6, low-3+, high-3+, high-6+, high-3++. The labels "low" and "high" indicate whether the variance of the common value distribution was small or large, the number indicates group size, and the "+" (or "++") sign indicates that subjects were once (twice) experienced. For statistical reasons, group-composition was kept constant during the whole experiment, although subjects did not know this, to avoid repeated-game effects.

**Part 1. The Basic Setup**

The first part of the experiment lasted for 20 periods. Bidders were given a starting capital of 120 points, which they did not have to pay back. In each period, subjects’ private values, $t_i$, were uniformly distributed between 75 and 125, i.e. $t_i \sim U[75,125]$. Common value signals were $U[0,200]$ distributed in the high-3 and high-6 treatments, and were $U[75,125]$ distributed in the low-3 treatments. Both private and common value signals were i.i.d. across subjects and periods, and the procedure for generating the signals was common knowledge. Bids were restricted to lie between the lowest and highest possible valuation for the commodity: in treatments low-3 and low-3+, subjects had to enter integer bids between 150 and 250 points, while in the other treatments bids had to be between 75 and 325 points. At the end of a period subjects were told the bids in their group (ordered from high to low), the common value, and whether or not they won the auction. In case of a tie, the winner was selected at random from the highest bidders. They only received information about others’ bids, not about others’ private or common value signals. Finally, the winner’s profit was communicated only to the winner.

**Part 2. Public Information Disclosure**

The second part lasted from periods 21 to 30, and was designed to evaluate the effects of public information release on efficiency, revenues, and profits. While the effects of

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5 The experiment was programmed using the Rat-Image Toolbox, see Abbink and Sadrieh (1995).

6 In case of a tie, the winner was selected at random from the highest bidders.
increased competition and increased uncertainty about the common value were investigated using a *between* subject design, we used a *within* subject design to determine the effects of public information release. In each period, subjects made two decisions, the first decision similar to that of part 1. After all subjects made their first decision, they received an additional signal about the value of the object for sale, the *auctioneer’s signal*, and were asked to bid again. The auctioneer’s signal was an independent draw from the same distribution as the common value signals of the bidders. Everyone in the group received the same auctioneer’s signal, and subjects’ private and common value signals for the second decision were the same as for the first. The common value in part 2 was:

\[ V = \frac{1}{n+1} \sum_{i=0}^{n} v_i, \]

(2)

where \( v_0 \) represents the auctioneer’s signal.

The implicit assumption underlying (2) is that the auctioneer’s signal has the same precision as that of the bidders. One way to model a more precise auctioneer’s signal is to give it more weight than the bidders’ signals. This was the case in treatment high-3++, for which the common value in part 2 was calculated as:

\[ V = \frac{1}{10} \left( 7v_0 + \sum_{i=1}^{3} v_i \right). \]

(3)

Decisions with and without an auctioneer signal had an equal chance of being selected for payment, and subjects learned which decision was chosen only after everyone had made both decisions. They only received information pertaining to the decision that was selected, and the information provided was analogous to that in part 1.

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7 This procedure to evaluate the effects of an auctioneer report does not affect subjects’ choices in an undesired way (Kagel and Levin, 1986, and Kagel, Harstad, and Levin, 1987).

8 This can be motivated as follows: suppose the variance of the bidders’ signals is \( \sigma^2_b \) (the same for all bidders) and the variance of the auctioneer’s signal is \( \sigma^2_a \). The "best" (i.e. unbiased and smallest variance) estimator of the commodity’s value is then given by \( V = (\lambda v_0 + \sum_{i=1}^{n} v_i)/(n+\lambda) \) where \( \lambda = \sigma^2_b/\sigma^2_a \) is a measure of the (relative) quality of the auctioneer’s information. The formulation in (2) corresponds to \( \lambda = 1 \) or \( \sigma^2_a = \sigma^2_b \), and (3) corresponds to \( \lambda = 7 \) or \( \sigma^2_a = \sigma^2_b/7 \).

9 A subject’s starting capital in period 21 equaled the total amount earned in part 1 plus a 60-point bonus.
Subjects and Bankruptcy

We conducted sessions with both inexperienced and experienced subjects, who were recruited at the University of Amsterdam. The experiment was finished within two hours and subjects made 61.25 guilders (≈ $30.60) on average. Their starting capital of 120 points provided some buffer against bankruptcy.\footnote{Even when everyone plays according to the Nash strategy, there is some chance that the winner loses money.} A subject went bankrupt when her cash balance became negative, in which case she was asked to leave the experiment without receiving any money. If a subject went bankrupt in a treatment with 6 bidders per group, the computer bid 0 for this subject for the remainder of the experiment. The other bidders in the group then proceeded as before, now with one less opponent.\footnote{Of course, if one of six bidders enters a zero bid, the theoretical predictions are affected (Nash and alternative models). We take this into account when analyzing the data.} If a subject went bankrupt in a treatment with 3 bidders per group, the computer submitted Nash bids for this person for the remainder of the experiment. (We did not use 0 bids in this case because we feared it would make collusion too easy.) The other two bidders proceeded as before, now facing one "human" and one "computerized Nash" opponent. The periods after a bankruptcy in a group of three were played out only to give the remaining two bidders a chance to make some money: the data from these periods are discarded.

Subjects who did not go bankrupt could voluntarily subscribe for one of the experienced sessions. (Subjects who went bankrupt were not given this opportunity.) The majority of subjects in experienced sessions participated in the same treatment as in their inexperienced session. Treatment high-3++ was conducted two months after the other treatments, and subjects in this treatment had participated in two earlier sessions. Table 1 summarizes the different treatments and the number of participants.

3. Theoretical Background

While few people would dispute that most real-world auctions exhibit both private and common value features, surprisingly little is known about their equilibrium properties. The difficulty with multiple signals is how to combine the different pieces of information into a
single statistic that can be mapped into a bid (Milgrom and Weber, 1982, p.1097). This is not a problem, however, for the linear formulation in (1): it is routine to verify that the summary statistic is given by the "surplus" $s = v/n + t$. The optimal bids then follow from the work of Milgrom and Weber (1982) who characterize the equilibrium for standard auctions when bids are based on a univariate statistic.

First, let us fix the notation. Due to symmetry we can, without loss of generality, focus on bidder 1 whose surplus is given by $s_1 = v_1/n + t_1$. Lower case letters are used to denote the highest surplus of the $n - 1$ others, e.g. $y_1 = \max_{j=2, \ldots, n} (v_j/n + t_j)$, and capital letters indicate order statistics when they pertain to all $n$ bidders, e.g. $Y_1$ ($Y_2$) is the maximum (second highest) of $n$ surplus draws. To keep the notation simple we only use one expectation symbol, e.g. the expected private value of the winner $E(t_{\text{winner}}) = E_{Y_1}(E(t|s = Y_1))$ is simply written as $E(t|s = Y_1)$.

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12 A bidder’s expected payoff is: $\pi^e = (\text{expected gain} - \text{expected payment}) \times \text{probability of winning}$. The expected payment and the probability of winning are independent of a bidder’s private and common value signals (but will depend on her bid and others’ bidding strategies). Moreover, for the average formulation of the common value, bidder $i$’s expected gain equals her "surplus," $s_i = v/n + t_i$, plus terms that are independent of her signals. The first-order conditions for profit maximization therefore determine optimal bids in terms of her surplus, $s_i$. 

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Table 1. The Different Treatments

<table>
<thead>
<tr>
<th>treatment</th>
<th>experience</th>
<th>#subjects</th>
<th>#bidders per group</th>
<th>private value</th>
<th>common value</th>
<th>weight auctioneer’s signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-3</td>
<td>none</td>
<td>30</td>
<td>3</td>
<td>$U[75, 125]$</td>
<td>$U[75, 125]$</td>
<td>1</td>
</tr>
<tr>
<td>low-3+</td>
<td>once</td>
<td>18</td>
<td>3</td>
<td>$U[75, 125]$</td>
<td>$U[75, 125]$</td>
<td>1</td>
</tr>
<tr>
<td>high-3</td>
<td>none</td>
<td>30</td>
<td>3</td>
<td>$U[75, 125]$</td>
<td>$U[0, 200]$</td>
<td>1</td>
</tr>
<tr>
<td>high-3+</td>
<td>once</td>
<td>18</td>
<td>3</td>
<td>$U[75, 125]$</td>
<td>$U[0, 200]$</td>
<td>1</td>
</tr>
<tr>
<td>high-6</td>
<td>none</td>
<td>54</td>
<td>6</td>
<td>$U[75, 125]$</td>
<td>$U[0, 200]$</td>
<td>1</td>
</tr>
<tr>
<td>high-6+</td>
<td>once</td>
<td>18</td>
<td>6</td>
<td>$U[75, 125]$</td>
<td>$U[0, 200]$</td>
<td>1</td>
</tr>
<tr>
<td>high-3++</td>
<td>twice</td>
<td>21</td>
<td>3</td>
<td>$U[75, 125]$</td>
<td>$U[0, 200]$</td>
<td>7</td>
</tr>
</tbody>
</table>
Proposition 1. The \( n \)-tuple of strategies \((B(\cdot), \ldots, B(\cdot))\), where

\[
B(x) = E(V + t_1 \mid s_1 = x, Y_1 = x) - E(Y_1 - y_1 \mid s_1 = x, Y_1 = x),
\]

(4)
is an equilibrium of the first-price auction. The winner’s expected profit is \( \pi_{\text{winner}} = E(Y_1) - E(Y_2) \) and the auctioneer’s expected revenue is \( R = E(V) + E(t_{\text{winner}}) - \pi_{\text{winner}}. \)

The intuition behind (4) is as follows: the first term on the right side represents what the commodity is worth (on average) to a bidder assuming that her surplus, \( x \), is the highest and the second term shows how much she "shades" her bid to make a profit.

When the auctioneer reveals information about the object for sale in the form of an extra signal, \( v_0 \), the optimal bids in (4) will have to be adjusted to incorporate this information. Consider, for instance, the case when the auctioneer’s signal is of the same quality as the bidders’ signals as per (2). The new optimal bids will then be functions of the summary statistic \( s_i^A = v_i/(n+1) + t_i \). Likewise, if the auctioneer’s signal is of a higher quality, as in (3), the summary statistic changes to \( s_i^A = v_i/(n+7) + t_i \). With this redefinition of the surplus variable and the corresponding order statistics, the Nash equilibrium bids again follow from Proposition 1. In other words, the functional form in (4) remains valid.\(^\text{13}\)

Since expected efficiency depends only on the weight bidders place on their common value signals, any factor that reduces this weight will positively affect efficiency. This intuition underlies the following comparative statics results:

**Proposition 2.** Expected efficiency in a Nash equilibrium rises when (i) more bidders enter the auction, (ii) information is publicly released, and (iii) the variance of the common value signals is reduced.

The proofs of Propositions 1 and 2 can be found in Goeree and Offerman (1999). The comparative statics predictions of Proposition 2 are tested in section 6.

\(^{13}\) The Nash bid is: \( B(x) = E(V + t_1 \mid v_0, s_1^A = x, Y_1^A = x) - E(Y_1^A - y_1^A \mid s_1^A = x, Y_1^A = x) \), with the common value, \( V \), given by (2). The winner’s profit is \( \pi_{\text{winner}} = E(Y_1^A) - E(Y_2^A) \) and the seller’s revenue is \( R = E(V) + E(t_{\text{winner}}) - \pi_{\text{winner}}. \)
Finally, we discuss two models of bidding behavior that include rational bidding as a special case. First, we allow subjects to weigh their private and common value signals differently. In particular, when the summary statistic is $s^\alpha = \alpha v + t$, optimal bids become

$$B^\alpha(x) = E(V + t|s_{1}^\alpha = x, Y_{1}^\alpha = x) - E(Y_{1}^\alpha - y_{1}^\alpha | s_{1}^\alpha = x, Y_{1}^\alpha = x),$$

which reduces to the Nash bidding function (4) iff $\alpha = 1/n$. Second, previous experiments based on purely common value auctions have demonstrated that subjects often fail to take into account the information conveyed in winning. Therefore we also consider a model of naive bidding, i.e. when bidders replace others’ common value signals by their (unconditional) expected value:

$$B_{\text{curse}}^\alpha(x) = E(V + t|s_{1}^\alpha = x) - E(Y_{1}^\alpha - y_{1}^\alpha | s_{1}^\alpha = x, Y_{1}^\alpha = x).$$

We will refer to (6) for $\alpha = 1/n$ as the "Naive" benchmark. Even though $B_{\text{curse}}^\alpha(x) > B^\alpha(x)$ for all $x$, the models predict the same winner because both are functions of $s_{1}^\alpha$. Hence, there is no efficiency loss due to the winner’s curse. Efficiency depends only on the relative weight bidders place on their common value signal: the higher is $\alpha$, the lower is expected efficiency. In section 5 we estimate $\alpha$ from the data, using both the winner’s curse model (6) and the "curse free" model (5).

4. Experimental Results: the Winner’s Curse

The winner’s curse is well documented for purely common value auctions, and is also clearly present in auctions with private and common values. Tables 2 and 3 give a first impression of the severity of the winner’s curse in the different treatments. A winner’s curse is absent only in treatments low-3 and low-3+, which is not surprising since the adverse selection problem is relatively small in these treatments. In treatments high-3 and high-6, however, more than half the bids are above $E_{\text{Nash}}$, the object’s expected value given that rivals bid according to Nash (Table 2), resulting in an average bid that differs significantly from the Nash prediction (Table 3). Of course, when others systematically bid too high, even a
A rational bidder will depart from Nash and the fourth column of Table 2 may thus overstate the prevalence of a winner’s curse. We have also included the percentage of bids exceeding the object’s expected value given the empirical distribution of bids and common value signals (column five in Table 2). Although this fraction is generally lower, it is still substantial. The final column shows that subjects rarely bid above $E_{\text{naive}}$, the object’s expected value predicted by the Naive model.\textsuperscript{14} This result is mirrored in Table 3, which shows that the average bid predicted by the Naive model is significantly higher than the actual average.\textsuperscript{15}

The bottom panel of Table 3 indicates that overbidding in the inexperienced treatments high-3 and high-6 is costly. In high-3, winners only realize about half the available Nash profits and in high-6, winners even lose money on average. Despite the fact that there is not much overbidding in low-3, winners make less profit than predicted by either the Naive or

\textsuperscript{14} Given a bidder’s surplus, $s$, this expected value is easily calculated as: $E_{\text{naive}} = s + (n-1)/n \ E(v)$.

\textsuperscript{15} Bids show no systematic time trend within a treatment. Nevertheless, some aspects in the data are consistent with learning direction theory (Selten and Buchta, 1994). Most notably, when winning the auction results in a loss, subjects increase their bid factor $(s + (n - 1)/n \ E(v) - \text{bid})$ by 22.9 (10.3) in the inexperienced (experienced) treatment.

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**Table 2. Winner’s Curse Percentages**

<table>
<thead>
<tr>
<th></th>
<th>% auctions with positive profits</th>
<th>% auctions won by highest surplus</th>
<th>% bids (winning bids) &gt; $E_{\text{Nash}}$</th>
<th>% bids (winning bids) &gt; $E_{\text{emp}}$</th>
<th>% bids (winning bids) &gt; $E_{\text{naive}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-3</td>
<td>75</td>
<td>79</td>
<td>17 (12)</td>
<td>19 (14)</td>
<td>7 (10)</td>
</tr>
<tr>
<td>low-3+</td>
<td>81</td>
<td>76</td>
<td>13 (7)</td>
<td>12 (8)</td>
<td>3 (2)</td>
</tr>
<tr>
<td>high-3</td>
<td>62</td>
<td>65</td>
<td>50 (43)</td>
<td>45 (21)</td>
<td>6 (10)</td>
</tr>
<tr>
<td>high-3+</td>
<td>61</td>
<td>73</td>
<td>58 (41)</td>
<td>54 (33)</td>
<td>7 (9)</td>
</tr>
<tr>
<td>high-6</td>
<td>48</td>
<td>51</td>
<td>58 (57)</td>
<td>57 (49)</td>
<td>17 (9)</td>
</tr>
<tr>
<td>high-6+</td>
<td>52</td>
<td>64</td>
<td>55 (47)</td>
<td>38 (30)</td>
<td>9 (9)</td>
</tr>
<tr>
<td>high-3++</td>
<td>64</td>
<td>72</td>
<td>53 (37)</td>
<td>37 (24)</td>
<td>3 (4)</td>
</tr>
</tbody>
</table>

\textit{Notes}: $E_{\text{Nash}}$ is the object’s expected value given Nash rivals, $E_{\text{naive}}$ is the object’s expected value assuming a winner’s curse, and $E_{\text{emp}}$ is the object’s expected value given the empirical distribution of bids and common value signals.
Nash benchmark. This is because the auction is not always won by the bidder with the highest surplus (see Table 2), as predicted by Nash/Naive bidding. One possible explanation for why a bidder with an inferior surplus wins the auction is that subjects put too much weight on their common value signal. An alternative explanation is that bidding is more erratic or "noisy" (see section 6).

Overall, subjects’ performance is somewhat better in the experienced sessions than in the inexperienced sessions. First, in the inexperienced sessions 7 subjects (6%) went bankrupt, while none of the experienced subjects went bankrupt. Second, earnings are higher in the experienced sessions. This improved performance may be either the result of learning, selection, or both. Subjects that subscribed for an experienced session earned, on average, 1.62 points per period in the inexperienced sessions, while those that did not subscribe earned 1.03 points. This supports the idea that selection plays a role, although the difference between the earnings is far from significant (a Mann-Whitney test with subjects as the unit of observation yields \( p = 0.77 \)). The 45 subjects that participated twice in the same treatment

<table>
<thead>
<tr>
<th>Table 3. Bids and the Winner’s Curse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Panel: Bids, Lower Panel: Winner’s Profit Per Period</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>low-3</th>
<th>high-3</th>
<th>high-6</th>
<th>low-3+</th>
<th>high-3+</th>
<th>high-6+</th>
<th>high-3++</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>189.6</td>
<td>172.6</td>
<td>182.4</td>
<td>189.5</td>
<td>179.5</td>
<td>176.7</td>
<td>175.6</td>
</tr>
<tr>
<td>Naive</td>
<td>191.5</td>
<td>186.7</td>
<td>194.3</td>
<td>191.5</td>
<td>187.0</td>
<td>194.6</td>
<td>187.0</td>
</tr>
<tr>
<td>Nash</td>
<td>188.3</td>
<td>159.9</td>
<td>171.1</td>
<td>188.3</td>
<td>160.4</td>
<td>169.3</td>
<td>160.7</td>
</tr>
<tr>
<td>E_{Nash}</td>
<td>197.1</td>
<td>173.9</td>
<td>176.4</td>
<td>197.1</td>
<td>174.5</td>
<td>174.3</td>
<td>174.8</td>
</tr>
</tbody>
</table>

| Actual   | 7.27  | 11.88  | -2.75  | 9.62   | 10.55   | 5.34    | 12.44    |
| Naive    | 12.02 | 8.54   | 2.67   | 11.72  | 8.83    | 2.23    | 9.31     |
| Nash     | 13.34 | 21.77  | 10.01  | 13.05  | 21.96   | 9.83    | 22.45    |

Notes: The \( p \)-value of a Wilcoxon rank test comparing predictions of the benchmark models with the actual bid are displayed in italics. Groups are the unit of observation. Test results are based on only three pair-wise observations in high-6+. \( E_{Nash} \) is the object’s expected value given Nash rivals.
earned somewhat higher profits and deviated slightly less from Nash (in an absolute sense) in the experienced session. Thus, there are also some (weak) signs for learning, although learning mainly occurs within the inexperienced session, and not between the inexperienced and experienced sessions.

Despite the improved performance in the experienced sessions, subjects still fall prey to the winner’s curse and in the high-3+, high-3++, and high-6+ sessions, they systematically overbid at a considerable cost. In high-6+, bids are now cautious enough to result in a small profit, but in high-3+, bids are even somewhat more aggressive than in the inexperienced session high-3.\textsuperscript{16} Figures 1-3 in Appendix C show the actual bidding data together with predictions of Nash (bottom line) and the Naive benchmark (top line). Note that observed bids tend to increase in surplus and that the winner’s curse tends to be more serious when a subject’s surplus is smaller. This is intuitive: winning the auction is more informative about others’ common value signals when own surplus is small, so neglecting this information leads to a bigger bias.

One obvious question is whether the first-price auction institution is capable of mitigating the effects of the winner’s curse. Indeed, it is often argued that economic institutions correct individual biases. In market settings, for instance, "biased" traders can learn from "unbiased" traders via signals provided by market prices. Or, vigorous trading of rational traders may to some degree neutralize the effects produced by noise traders. There is some experimental evidence that markets may alleviate the effects of judgmental biases (\textit{e.g.} Camerer, 1987; Camerer, Loewenstein, and Weber, 1989; Anderson and Sunder, 1995; Ganguly, Kagel, and Moser, 1998). Interestingly, the selection process in auctions may aggravate individual biases, since the bidder with the strongest curse (i.e. the bidder that deviates more from Nash than the average bidder with the same signals), tends to win the auction. The data confirm this intuition. A logistic regression with the probability to win the auction as the dependent variable and "surplus" and "curse" (= actual bid - Nash bid) as

\textsuperscript{16} One possible explanation for why bidding in high-6 becomes more cautious while bidding in high-3 becomes more aggressive, is that in high-3 losing subjects more frequently experience regret when they find out that their value for the object is higher than the winning bid (46.3% in high-3 versus 36.4% in high-6). Furthermore, when they experience regret, it is stronger: in high-3 an average of 39 points was left on the table versus 21 points in high-6. Thus, regret may have caused a stronger upward pressure on bids in high-3.
independent variables, shows that the estimated parameter for surplus is 0.10 (s.d. 0.003), the estimated parameter for curse is 0.06 (s.d. 0.003) and the estimated parameter for the constant is -14.45 (s.d. 0.443). Hence, subjects with a stronger curse have a higher probability to determine the price for the commodity.17

5. Experimental Results: Efficiency

Recall from section 3 that the relative weight bidders place on their common value signal determines the efficiency level of the auction. For example, if bidders would ignore their common value information, bids are ranked according to the private value signals, and 100% efficiency is attained. At the other extreme case where bidders neglect their private information, the auction is no more efficient than a random allocation rule. For Nash and Naive bidders, the weight of the common value signal is \( \frac{1}{n} \), which results in an intermediate level of inefficiency.

Table 4 shows the efficiency levels realized in the experiment, per block of ten periods, and the predictions of several benchmark models are added for comparison. The efficiency levels are determined as follows. Let \( t_{\text{winner}} \) denote the private value of the winner and \( t_{\text{min}} \) (\( t_{\text{max}} \)) the minimal (maximal) private value in the group, then

\[
\text{realized efficiency} = \frac{t_{\text{winner}} - t_{\text{min}}}{t_{\text{max}} - t_{\text{min}}} \times 100\%.
\]  

(7)

The efficiency level predicted by a benchmark is obtained by replacing \( t_{\text{winner}} \) with the private value of the bidder predicted to win by the model. The predicted efficiencies of the Nash and Naive benchmarks are the same since both models predict the same winner.

Note that actual efficiency levels are substantially below the Nash/Naive predictions in the first ten periods of the inexperienced sessions, although this difference becomes smaller in later periods. The efficiency levels of the experienced treatments are roughly constant and in the same range as those in the last twenty periods of the inexperienced sessions. In the next

---

17 This selection force may weaken in the long run, however, as bidders with more severe curses go bankrupt and disappear from the auctions.
section we compare the benchmarks of section 3 to the individual data.

6. Analysis of the Individual Bidding Data

The Nash and Naive benchmark both make point predictions, and one has to make an assumption about how players err to evaluate these models. We invoke a commonly made assumption: for each of the benchmarks a random error term is added to the predicted bid. The error terms are drawn from a truncated Normal distribution with mean 0 and variance $\sigma^2$, and are identically and independently distributed across subjects and periods.\footnote{The distribution is truncated to ensure that bids stay between the lower and upper limit on bids.} This method of transforming deterministic models into stochastic models can easily be criticized on theoretical grounds, but there is no \textit{a priori} reason why one model will be favored over another. So this procedure seems adequate to compare the "goodness-of-fit" of different benchmark models. Since the results of the previous section suggest a difference between

\begin{table}[h]
\centering
\caption{Observed Efficiencies By Blocks of Ten Periods}
\begin{tabular}{cccc|cccc|cccc}
\hline
 & \multicolumn{3}{c}{inexperienced} & \multicolumn{3}{c}{once experienced} & \multicolumn{3}{c}{twice experienced} \\
 & 1-10 & 11-20 & 21-30 & 1-10 & 11-20 & 21-30 & 1-10 & 11-20 & 21-30 \\
\hline
\textbf{High-3} & \textbf{High-3+} & \textbf{High-3++} \\
Actual & 54 & 68 & 68 & 73 & 62 & 72 & 71 & 69 & 71 \\
Nash/Naive & 71 & 73 & 75 & 72 & 72 & 75 & 75 & 73 & 76 & 0.01 & 0.07 & 0.07 & 0.92 & 0.05 & 0.12 & 0.46 & 0.31 & 0.18 \\
\hline
\textbf{High-6} & \textbf{High-6+} \\
Actual & 72 & 81 & 85 & 93 & 89 & 90 \\
Nash/Naive & 94 & 86 & 91 & 92 & 88 & 91 & 0.01 & 0.05 & 0.07 & 1.00 & 1.00 & 1.00 \\
\hline
\textbf{Low-3} & \textbf{Low-3+} \\
Actual & 79 & 87 & 89 & 90 & 91 & 86 \\
Nash/Naive & 96 & 93 & 98 & 97 & 92 & 97 & 0.01 & 0.11 & 0.01 & 0.03 & 0.92 & 0.04 \\
\hline
\end{tabular}
\end{table}

Notes: The $p$-value of a Wilcoxon rank test comparing a model’s efficiency with realized efficiency is displayed in italics. Groups are the unit of observation.
bidding behavior in the first ten and later periods (see Table 4), we provide separate tables for periods 1-10 and periods 11-30.

Table 5A reports the estimation results for the final twenty periods. The top panel pertains to the Nash and Naive benchmarks. Based on the loglikelihoods there is no obvious ranking of the two models: Nash performs better in low-3, Naive in high-3, and the results are mixed for high-6. Glancing at Figures 1-3, it seems plausible that there is some heterogeneity among subjects, with some bidders suffering from the winner’s curse while others don’t. This is tested in the Nash-Naive combined model, which allows subjects to bid

<table>
<thead>
<tr>
<th></th>
<th>low-3 ( n=600 )</th>
<th>high-3 ( n=519 )</th>
<th>high-6 ( n=981 )</th>
<th>low-3+ ( n=360 )</th>
<th>high-3+ ( n=360 )</th>
<th>high-6+ ( n=360 )</th>
<th>high-3++ ( n=420 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>( \sigma )</td>
<td>8.4</td>
<td>25.4</td>
<td>27.9</td>
<td>6.4</td>
<td>28.1</td>
<td>31.0</td>
</tr>
<tr>
<td></td>
<td>-logL</td>
<td>3.53</td>
<td>4.60</td>
<td>4.71</td>
<td>3.26</td>
<td>4.70</td>
<td>4.79</td>
</tr>
<tr>
<td>Naive</td>
<td>( \sigma )</td>
<td>8.8</td>
<td>21.9</td>
<td>26.2</td>
<td>7.0</td>
<td>18.5</td>
<td>34.3</td>
</tr>
<tr>
<td></td>
<td>-logL</td>
<td>3.59</td>
<td>4.50</td>
<td>4.68</td>
<td>3.37</td>
<td>4.34</td>
<td>4.95</td>
</tr>
<tr>
<td>Nash - Naive Combined</td>
<td>( \sigma_1 )</td>
<td>6.0</td>
<td>21.4</td>
<td>27.6</td>
<td>4.9</td>
<td>27.3</td>
<td>30.6</td>
</tr>
<tr>
<td></td>
<td>( \sigma_2 )</td>
<td>15.5</td>
<td>15.8</td>
<td>12.5</td>
<td>10.4</td>
<td>12.9</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>( p )</td>
<td>0.83</td>
<td>0.45</td>
<td>0.44</td>
<td>0.77</td>
<td>0.22</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>-logL</td>
<td>3.38</td>
<td>4.32</td>
<td>4.31</td>
<td>3.20</td>
<td>4.15</td>
<td>4.49</td>
</tr>
<tr>
<td>B( \alpha ) - B( \alpha_{\text{curse}} ) Combined</td>
<td>( \sigma_1 )</td>
<td>5.6</td>
<td>21.9</td>
<td>27.6</td>
<td>4.8</td>
<td>20.3</td>
<td>30.7</td>
</tr>
<tr>
<td></td>
<td>( \sigma_2 )</td>
<td>13.4</td>
<td>15.9</td>
<td>12.6</td>
<td>10.6</td>
<td>13.8</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>( p )</td>
<td>0.75</td>
<td>0.32</td>
<td>0.44</td>
<td>0.78</td>
<td>0.11</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>( \alpha )</td>
<td>0.40</td>
<td>0.47</td>
<td>0.17</td>
<td>0.28</td>
<td>0.43</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>-logL</td>
<td>3.37</td>
<td>4.30</td>
<td>4.31</td>
<td>3.19</td>
<td>4.10</td>
<td>4.49</td>
</tr>
<tr>
<td>Random</td>
<td>-logL</td>
<td>4.62</td>
<td>5.53</td>
<td>5.53</td>
<td>4.62</td>
<td>5.53</td>
<td>5.53</td>
</tr>
</tbody>
</table>

Notes: Loglikelihood per choice is displayed.
according to either the Nash or Naive benchmark. This combined model results in a much higher likelihood than either of the two individual models.

The Nash-Naive combined model should be compared to the model in the bottom panel of Table 5A, which is based on the bidding functions (5) and (6). This model also allows (a fraction of the) bidders to fall prey to the winner’s curse, and to weigh their common value signal differently than Nash bidders (i.e. the relative weight of the common value signal is not necessarily 1/n). The inclusion of the weight $\alpha$ results in a small, albeit significant increase in likelihood when there are three bidders, but adds nothing when group size is six. The maximum likelihood results suggest that while a significant fraction of the subjects (e.g. more than 50 percent) falls prey to a winner’s curse, bidders correctly weigh their private and common value information. This conclusion, which is based on the individual bidding data, is further corroborated by the aggregate results of Table 4, which shows that actual efficiency levels are close to predicted levels in the final part. To summarize, the data seem best described by a model in which bidders weigh their information in roughly the same manner as rational bidders would, while a fraction of the bidders falls prey to the winner’s curse.

Estimates for the initial ten periods of the different treatments are reported in Table 5B. For the inexperienced treatments, all models result in much lower likelihoods than those reported for the final twenty periods (Table 5A). This is partly due to the higher weight bidders assign to their common value signal, which causes the "wrong" bidder to win.

---

19 To be precise, the unconditional likelihood $L(x_{i,11,:,:30})$ of a player $i$’s choices $x$ in periods 11-30 is:

$$L(x_{i,11,:,:30}) = \prod_{t=11}^{30} (p L(x_{i,t}|\text{Nash}) + (1-p) L(x_{i,t}|\text{Naive})),$$

where $L(x_{i,t}|\text{Nash})$ represents the conditional probability of $x_{i,t}$ predicted by the Nash model, $L(x_{i,t}|\text{Naive})$ represents the conditional probability of $x_{i,t}$ given the Naive model, and $p$ is the probability that a subject plays according to the Nash equilibrium. The Nash and Naive benchmarks are nested as special cases (i.e. $p = 1$ or $p = 0$).

20 We also estimated a model in which bidders make a "logit" best response to the empirical distribution of bids (with or without a winner’s curse). This model generally resulted in a much worse description of the data (i.e. a 10-20 percent reduction in the loglikelihood per observation). Finally, we estimated a "discount" model in which bids are determined as a fraction of the (rational or naive) expected value of the object. This model yielded similar loglikelihoods as the ones in Table 5. We prefer the benchmark models of section 3, however, as they have a more sound theoretical foundation.
Table 5B. Maximum Likelihood Results for Periods 1-10

<table>
<thead>
<tr>
<th></th>
<th>low-3 $n=300$</th>
<th>high-3 $n=300$</th>
<th>high-6 $n=527$</th>
<th>low-3+ $n=180$</th>
<th>high-3+ $n=180$</th>
<th>high-6+ $n=180$</th>
<th>high-3++ $n=210$</th>
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<tbody>
<tr>
<td>Nash $\sigma$</td>
<td>11.6</td>
<td>29.7</td>
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<td>6.2</td>
<td>29.4</td>
<td>29.5</td>
<td>27.6</td>
</tr>
<tr>
<td>-logL</td>
<td>3.82</td>
<td>4.74</td>
<td>4.87</td>
<td>3.22</td>
<td>4.74</td>
<td>4.76</td>
<td>4.68</td>
</tr>
<tr>
<td>Naive $\sigma$</td>
<td>11.7</td>
<td>27.9</td>
<td>29.5</td>
<td>6.6</td>
<td>16.9</td>
<td>28.0</td>
<td>18.3</td>
</tr>
<tr>
<td>-logL</td>
<td>3.86</td>
<td>4.75</td>
<td>4.80</td>
<td>3.31</td>
<td>4.24</td>
<td>4.75</td>
<td>4.33</td>
</tr>
<tr>
<td>Nash - Naive Combined $\sigma_1$</td>
<td>8.2</td>
<td>26.4</td>
<td>36.0</td>
<td>3.9</td>
<td>20.2</td>
<td>28.1</td>
<td>22.5</td>
</tr>
<tr>
<td></td>
<td>15.5</td>
<td>20.0</td>
<td>13.9</td>
<td>7.5</td>
<td>13.9</td>
<td>7.5</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>0.63</td>
<td>0.59</td>
<td>0.53</td>
<td>0.60</td>
<td>0.12</td>
<td>0.62</td>
<td>0.34</td>
</tr>
<tr>
<td>-logL</td>
<td>3.77</td>
<td>4.60</td>
<td>4.57</td>
<td>3.10</td>
<td>4.08</td>
<td>4.30</td>
<td>4.08</td>
</tr>
<tr>
<td>B$^\alpha$ - B$^\alpha_{\text{curse}}$ Combined $\sigma_1$</td>
<td>8.1</td>
<td>27.4</td>
<td>36.6</td>
<td>3.8</td>
<td>20.0</td>
<td>26.8</td>
<td>23.5</td>
</tr>
<tr>
<td></td>
<td>14.8</td>
<td>20.0</td>
<td>14.0</td>
<td>7.5</td>
<td>12.9</td>
<td>6.5</td>
<td>11.5</td>
</tr>
<tr>
<td></td>
<td>0.61</td>
<td>0.52</td>
<td>0.51</td>
<td>0.59</td>
<td>0.11</td>
<td>0.67</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.46</td>
<td>0.42</td>
<td>0.25</td>
<td>0.37</td>
<td>0.39</td>
<td>0.13</td>
<td>0.42</td>
</tr>
<tr>
<td>-logL</td>
<td>3.76</td>
<td>4.60</td>
<td>4.56</td>
<td>3.01</td>
<td>4.06</td>
<td>4.28</td>
<td>4.06</td>
</tr>
<tr>
<td>Random $-\log L$</td>
<td>4.62</td>
<td>5.53</td>
<td>5.53</td>
<td>4.62</td>
<td>5.53</td>
<td>5.53</td>
<td>5.53</td>
</tr>
</tbody>
</table>

Notes: Loglikelihood per choice is displayed.

However, Table 5B shows that the inclusion of $\alpha$ leads to only a small improvement in the likelihood. The main reason for the worse fit of the data is that behavior is more erratic in the first ten periods, as can be seen from higher standard deviations. Notice that experienced bidders show no difference between initial and final-period behavior.21

7. Experimental Results: Comparative Statics Predictions

In this section we test the comparative statics predictions of Proposition 2. We start with the effects of a reduction in the uncertainty about the common value (see Table 6). According to both benchmark models, efficiency should increase when uncertainty decreases.

---

21 In fact, in the experienced treatments, the likelihoods and standard deviations are somewhat lower in the first ten periods.
Table 6. Effect of Uncertainty about the Common Value
Top Panel: Efficiency, Middle Panel: Winner’s Profit, Bottom Panel: Revenues

<table>
<thead>
<tr>
<th></th>
<th>Inexperienced</th>
<th></th>
<th></th>
<th>Experienced</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>high-3</td>
<td>low-3</td>
<td>p-value</td>
<td>high-3+</td>
<td>low-3+</td>
<td>p-value</td>
</tr>
<tr>
<td>Actual</td>
<td>62</td>
<td>85</td>
<td>0.00</td>
<td>69</td>
<td>89</td>
<td>0.00</td>
</tr>
<tr>
<td>Nash/Naive</td>
<td>73</td>
<td>96</td>
<td>0.00</td>
<td>73</td>
<td>95</td>
<td>0.00</td>
</tr>
<tr>
<td>Actual</td>
<td>11.88</td>
<td>7.27</td>
<td>0.27</td>
<td>10.55</td>
<td>9.62</td>
<td>0.38</td>
</tr>
<tr>
<td>Nash</td>
<td>21.77</td>
<td>13.34</td>
<td>0.00</td>
<td>21.96</td>
<td>13.05</td>
<td>0.00</td>
</tr>
<tr>
<td>Naive</td>
<td>8.54</td>
<td>12.02</td>
<td>0.10</td>
<td>8.83</td>
<td>11.72</td>
<td>0.52</td>
</tr>
<tr>
<td>Actual</td>
<td>194.1</td>
<td>202.4</td>
<td>0.03</td>
<td>198.2</td>
<td>200.8</td>
<td>0.20</td>
</tr>
<tr>
<td>Nash</td>
<td>187.0</td>
<td>198.7</td>
<td>0.00</td>
<td>187.2</td>
<td>198.5</td>
<td>0.00</td>
</tr>
<tr>
<td>Naive</td>
<td>200.2</td>
<td>200.1</td>
<td>0.94</td>
<td>200.4</td>
<td>199.9</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Notes: The third and the sixth column report p-values for a Mann-Whitney rank test results comparing high-3 with low-3 and low-3+ and high-6+ respectively. Groups are the unit of observation.

The intuition is that with less variation in the common value signal, \( v_i \), private value differences are exemplified, making it more likely that the bidder with the highest private value wins. This predicted effect of a decrease in uncertainty is borne out by the data: the realized efficiency level is substantially and significantly higher in treatment low-3 than in high-3, both for inexperienced and experienced subjects.

The effect of increased uncertainty on winner’s profits and revenues depends on the benchmark. Nash predicts that with more uncertainty, bids are less aggressive because of the increased winner’s curse and profits are higher as a result. In contrast, naive bidders neglect the fact that winning is informative and hence are insensitive to the increased risk of a winner’s curse. In fact, they will bid higher when there is more uncertainty, because the maximum surplus, \( v/n + t \), is higher when the common value signals are drawn from \( U[0,200] \) than when they are drawn from \( U[75,125] \). Table 6 shows that actual profits are lower with less uncertainty, although less so than predicted by Nash (both for inexperienced and experienced bidders). Revenue results are the opposite: Nash predicts that revenues will decrease when the uncertainty about the common value increases, while the naive benchmark
Table 7. Effect of Increased Competition
Top Panel: Efficiency, Middle Panel: Winner’s Profit, Bottom Panel: Revenues

<table>
<thead>
<tr>
<th></th>
<th>Inexperienced</th>
<th></th>
<th>Experienced</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>high-3</td>
<td>high-6</td>
<td>p-value</td>
<td>high-3+</td>
</tr>
<tr>
<td>Actual</td>
<td>62</td>
<td>79</td>
<td>0.00</td>
<td>69</td>
</tr>
<tr>
<td>Nash/Naive</td>
<td>73</td>
<td>90</td>
<td>0.00</td>
<td>73</td>
</tr>
<tr>
<td>Actual</td>
<td>11.88</td>
<td>-2.75</td>
<td>0.01</td>
<td>10.55</td>
</tr>
<tr>
<td>Nash</td>
<td>21.77</td>
<td>10.01</td>
<td>0.00</td>
<td>21.96</td>
</tr>
<tr>
<td>Naive</td>
<td>8.54</td>
<td>2.67</td>
<td>0.00</td>
<td>8.83</td>
</tr>
<tr>
<td>Actual</td>
<td>194.1</td>
<td>211.8</td>
<td>0.00</td>
<td>198.2</td>
</tr>
<tr>
<td>Nash</td>
<td>187.0</td>
<td>202.8</td>
<td>0.00</td>
<td>187.2</td>
</tr>
<tr>
<td>Naive</td>
<td>200.2</td>
<td>210.1</td>
<td>0.00</td>
<td>200.4</td>
</tr>
</tbody>
</table>

Notes: The third and the sixth column display p-values for a Mann-Whitney rank test comparing high-3 and high-6, high-3+, and high-6+ respectively. Groups are the unit of observation.

predicts that revenues will increase. Again, Nash correctly predicts the shift in observed revenues and the actual change is close to the predicted change for inexperienced bidders, but it is too small for experienced bidders.

Next, we consider the effects of competition. Both Nash and the Naive benchmark predict that efficiency levels will increase with the number of bidders. To see this, note that both models predict that bids depend on surplus \( v/n + t \). So when \( n \) increases, the private value signal becomes more important. In addition, with more bidders, the maximum of the private values increases on average, making it more likely that the highest surplus coincides with a higher private value. The data accord with this prediction: for both inexperienced and experienced bidders there is a sharp increase in the actual efficiency level when the number of bidders is increased from 3 to 6 (see Table 7). In contrast, if bidders would have weighed their private value and common value signal equally, say, a substantially smaller effect of

---

22 Note that the predicted change in revenues is not equal to the change in profits, because the total surplus is higher in low-3 than in high-3.
increasing competition would be expected (from 63% to 65% in high-6 and from 62% to 66% in high-6+).

Both benchmarks yield identical predictions for the effects of an increase in the number of bidders on profits and revenues. Revenues increase with the number of bidders for two reasons. First, with more competition, bidders are forced to bid closer to their estimate of the value of the commodity. Second, with a higher number of bidders the total surplus to be divided between seller and bidders is higher. The opposite effect is expected for the winner’s profits, which declines with more bidders according to both benchmarks. Table 7 shows that for both inexperienced and experienced bidders, the profit of the winning bidder decreases as the number of bidders increases.

Finally, consider the case where the auctioneer has an independent estimate of the common value of the object for sale. By revealing this information, the auctioneer decreases bidders’ uncertainty about the common value, which results in a more efficient allocation. The decrease in uncertainty reduces the winner’s curse, forcing bidders to be more aggressive, resulting in lower winner’s profits and higher revenues. The higher the quality of the auctioneer’s information, the stronger the predicted effects.

Table 8 shows that, by and large, bidders change their bid in the direction predicted by Nash, both when the quality of the auctioneer’s signal is the same as that of the bidders and when its quality is higher. Interestingly, the most common deviation from the Nash prediction is that a bidder does not increase her bid when Nash bidders would. This occurs when bidders neglect the fact that the extra information mitigates the winner’s curse and that, as a consequence, more aggressive bidding is warranted.

Table 9 shows that the public disclosure of the auctioneer’s signal, which is of the same quality as bidders’ signals, has no effects on efficiency, revenues, and profits (i.e. the

---

23 This argument is flawed when bidders neglect the fact that winning is informative. The disclosure of the auctioneer’s information leads to an improvement in bidders’ estimates of the value, resulting in a more efficient allocation. However, when bidders neglect the fact that winning is informative both with and without the auctioneer’s report, their bids will be equally aggressive. Due to the improved estimate of the value, higher profits for the winner are expected though. The effect of information disclosure on revenue is ambiguous, since the positive effect of a higher winner’s curse without information may be offset by a negative effect on the total surplus to be divided between the seller and the bidders.
effects are economically small, not systematic, and most often statistically insignificant). Note, however, that this lack of effect is conform the predictions of the different benchmark models. In contrast, the predicted effects of a high-quality auctioneer’s signal (treatment high-3++) are substantial. There is a significant effect on realized efficiency, although smaller than expected. The effect on actual profits and revenues is rather small and not significant. The observed changes accord more or less with naive bidding, and sharply contrast the large drop in profits (or large rise in revenues) predicted by Nash.

The data suggest the following explanation: without an auctioneer’s report there is a substantial winner’s curse in periods 21-30 (subjects bid 171.9 on average, while Nash predicts 159.8). The auctioneer’s report helps subjects to form a better estimate of the common value, thereby alleviating the winner’s curse. After the auctioneer’s report has been revealed subjects bid on average somewhat higher (178.2 versus 171.9), but now the winner’s

---

Table 8. Qualitative Effect Auctioneer’s Report on Bids

<table>
<thead>
<tr>
<th>Auctioneer’s signal weight 1</th>
<th>( b_1 &gt; b_2 )</th>
<th>( b_1 = b_2 )</th>
<th>( b_1 &lt; b_2 )</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_1 &gt; N_2 )</td>
<td>334</td>
<td>71</td>
<td>39</td>
<td>28.3%</td>
</tr>
<tr>
<td>( N_1 = N_2 )</td>
<td>38</td>
<td>11</td>
<td>19</td>
<td>4.3%</td>
</tr>
<tr>
<td>( N_1 &lt; N_2 )</td>
<td>133</td>
<td>196</td>
<td>729</td>
<td>67.4%</td>
</tr>
<tr>
<td>total</td>
<td>32.3%</td>
<td>17.7%</td>
<td>50.1%</td>
<td>( n = 1570 )</td>
</tr>
</tbody>
</table>

Notes: \( N_1 (N_2) \) represents the Nash bid without (with) auctioneer’s signal, and \( b_1 (b_2) \) represents the actual bid without (with) auctioneer’s signal. When weight=1: \( \chi^2 \) Pearson 637.83 (p=0.00). When weight=7: \( \chi^2 \) Pearson 111.77 (p=0.00).

---

24 These results are also consistent with the results reported in Kagel and Levin (1986). They investigate the effect of an auctioneer’s report in a purely common value auction and find that revenues increase less than predicted by Nash. They find that an auctioneer’s signal raises revenue when bidders do not fall prey to a winner’s curse, while revenue decreases when they do. Note that in our treatment high-3+, the treatment in which the winner’s curse is most prevalent (Table 3), the decrease in revenue is largest.
Table 9. Effects of the Public Disclosure of the Auctioneer’s Signal

<table>
<thead>
<tr>
<th></th>
<th>Efficiency</th>
<th>Profit Winner</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Nash/Naive</td>
<td>Actual</td>
</tr>
<tr>
<td>low-3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without</td>
<td>89</td>
<td>98</td>
<td>6.58</td>
</tr>
<tr>
<td>with</td>
<td>90</td>
<td>98</td>
<td>6.65</td>
</tr>
<tr>
<td></td>
<td>0.62</td>
<td>0.18</td>
<td>0.76</td>
</tr>
<tr>
<td>low-3+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without</td>
<td>86</td>
<td>97</td>
<td>6.92</td>
</tr>
<tr>
<td>with</td>
<td>90</td>
<td>98</td>
<td>8.00</td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>0.32</td>
<td>0.21</td>
</tr>
<tr>
<td>high-3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without</td>
<td>68</td>
<td>75</td>
<td>15.71</td>
</tr>
<tr>
<td>with</td>
<td>66</td>
<td>82</td>
<td>11.33</td>
</tr>
<tr>
<td></td>
<td>0.67</td>
<td>0.04</td>
<td>0.21</td>
</tr>
<tr>
<td>high-3+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without</td>
<td>72</td>
<td>75</td>
<td>10.12</td>
</tr>
<tr>
<td>with</td>
<td>76</td>
<td>82</td>
<td>12.42</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.07</td>
<td>0.25</td>
</tr>
<tr>
<td>high-6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without</td>
<td>85</td>
<td>91</td>
<td>0.01</td>
</tr>
<tr>
<td>with</td>
<td>82</td>
<td>93</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>0.11</td>
<td>0.95</td>
</tr>
<tr>
<td>high-6+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without</td>
<td>90</td>
<td>91</td>
<td>3.60</td>
</tr>
<tr>
<td>with</td>
<td>90</td>
<td>93</td>
<td>6.13</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>0.32</td>
<td>0.11</td>
</tr>
<tr>
<td>high-3++</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without</td>
<td>73</td>
<td>76</td>
<td>11.89</td>
</tr>
<tr>
<td>with</td>
<td>82</td>
<td>94</td>
<td>13.14</td>
</tr>
<tr>
<td></td>
<td>0.09</td>
<td>0.02</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Notes: Each third row reports p-values for a Wilcoxon rank tests comparing the entry with and without auctioneer’s report. Groups are the unit of observation.

curse has disappeared. In fact, they bid somewhat less than the Nash benchmark predicts (184.6). Thus, subjects bid more aggressively after the auctioneer’s report has been made public, but less so than predicted by Nash.
8. Conclusion

The majority of the theoretical and empirical literature on auctions pertains to either private or common value auctions. A remarkable feature of these polar cases is that both yield fully efficient allocations (in a Nash equilibrium). Most real-world auctions, however, exhibit both private and common value elements and inefficiencies should be expected, even in a Nash equilibrium. This paper reports a series of first-price auction experiments in which bidders receive a private value signal and an independent common value signal. We investigate the extent of inefficiency that occurs with (financially motivated) human bidders. In addition, we test several policies aimed at reducing inefficiencies.

As expected, a fraction of the bidders falls prey to the winner’s curse and this curse is more severe when winning is more informative. While there is systematical overbidding in most treatments, bidders aggregate their private and common value information in roughly the same manner as rational bidders would. As a result, realized efficiencies are of the same magnitude as predicted by Nash. Large differences occur only in the first ten periods of the inexperienced sessions, and seem mostly due to initially more erratic behavior. These findings are further corroborated by an analysis of the individual bidding data.

An increase in uncertainty about the common value leads to a substantial decrease in efficiency, accompanied by a slight increase in winner’s profits and a slight decrease in the seller’s revenues. These results are in line with Nash predictions, although the effects on profits and revenues are smaller than predicted because the increase in uncertainty aggravates the winner’s curse. The public disclosure of the auctioneer’s information about the common value has no systematic effects on efficiency, profits, or revenues when the information provided is of the same quality as that of the bidders. Furthermore, the public release of high-quality information positively affects efficiency, although less so than predicted by Nash.

Finally, our results indicate that more competition is a robust way to enhance efficiency, reduce winner’s profits, and raise the seller’s revenues. The reasons for these positive effects are partly "statistical": with more bidders, the winner will on average have better information (i.e. higher signals). More importantly, however, an increase in competition induces bidders to weigh their own common value signal significantly less, which makes their private value information more important and an efficient outcome more likely.


Selten, Reinhard and Joachim Buchta (1994) "Experimental Sealed Bid First-Price Auctions with Directly Observed Bid Functions," Discussion paper B-270, University of Bonn.


Appendix A: (Computerized) Instructions for Treatment High-3

Welcome to the experiment! You can make money in this experiment. Your choices will determine how much money you will make. Read the instructions carefully. There is paper, a pen, and a calculator on your table. You can use these during the experiment. Before the experiment starts, we will hand out a summary of the instructions and there will be one practice period.

The experiment
The experiment consists of two parts. You will earn points in both parts of the experiment. At the end of the experiment your points will be exchanged in guilders. Each point will yield 25 cent. First you will receive the instructions for part 1. When part 1 of the experiment is finished, you will receive the instructions for part 2.

Instructions part 1
You will start part 1 with a starting capital of 120 points. Part 1 consists of 20 periods. Each period you will be allocated to a group of 3 persons. A product will be sold in your group in each period. The person with the highest bid in the group will buy the product. If more than one person choose the same highest bid, the computer will determine by lottery which participant will buy the product.

The buyer will not literally receive a product. He or she will receive an amount equal to the value of the product minus the costs of the product (in points).

Value and costs of the product
The participant who buys the product will pay a price equal to the own bid. The costs of the product are thus equal to the own bid.

The value of the product is determined as follows. For each participant the value is equal to a “private value” plus a “common value”. For each participant the private value will lie between 75 and 125 points, and each number between 75 and 125 is equally likely. The private value of one participant is independent of the private value of the other participants. Your private value therefore (very) probably differs from those of the others. At the start of a period you will get to know your private value. You will not know the private value of other participants, just like other participants will not know your private value.

The common value of the product is determined as follows. Each participant in the group will receive a common component. A common component lies between 0 and 200 points. The common component of the one participant is independent of the common components of the other participants. At the start of a period each participant will only get to know her or his own common component. The COMMON VALUE equals the AVERAGE of the common components of the participants in a group. If the average is not an integer, it will be rounded to the nearest integer. For each participant, all values between 0 and 200 are equally likely to be the common component.

Notice that the private value differs for each participant, while the common value is exactly the same for all members of the group. You know your private value at the time of bidding. You will only know your own common component at the time of bidding.
not know the common value for sure, because you don't know others’ common value components. Thus at the time of bidding you only partially know the value of the product.

**Profit**

Only the person with the highest bid earns an amount, which equals his or her private value plus the common value minus his or her own bid. Notice that this amount may be negative. If the highest bidder bids higher than the value of the product, (s)he will make a loss. Just like a profit is automatically added to the amount earned up to that period, a loss will automatically be subtracted.

Note also that in the most unfavorable case the value of the product will equal 75 points, when the common value equals 0 points and your private value equals 75 points. In the most favorable case the value of the product will equal 325 points, when the common value equals 200 points and your private value equals 325 points. This is the reason that your bid will have to lie between 75 and 325 points. *Note: In low-3 and low-3+ bids were restricted to [150, 250].*

**Information**

When all bidders have entered their bid in a period, the results of the period will be communicated. You will get to know all bids in your group, ordered from high to low. You will be told whether you did or did not buy the product. You will get to know the common value of the product and your profit.

Then a new period will be started. In the new period again a product will be sold. For each participant the product will have a different value. Each participant receives a new private value for the product and a new common component. Your value for the product in a period will thus not depend on your value for the product in any other period.

Now you are asked to answer some questions about the instructions.

**Question about the common value**

Assume that your common component equals 112 points and you don't know the common components of the other participants. Assume that for the other participants it holds that:

<table>
<thead>
<tr>
<th>Participant</th>
<th>Common component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>132</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
</tr>
</tbody>
</table>

How large is the common value in this example?

*Note: Subjects could only pass a question by giving the right answer. After a wrong answer the relevant part in the instructions was explained anew, and the subject had to try again.*

**Question about the profit**

Assume that your private value for the product equals 115 points. The common value of the product in your group equals 87 points. You have bid 170 points (this bid is arbitrarily
chosen by us, and does not contain any information about how you should bid). Assume that your bid is the highest. How large is your profit in this case?

End
You have reached the end of the instructions. If you want to read some parts of the instructions again, push the button BACK.

Otherwise you push the button READY. When all participants have pushed READY, the experiment will start. When the experiment has started, you will not be able to return to these instructions. Before the experiment is started, a summary of the instructions will be handed out and a practice period will be carried out. Your profit or loss during the practice period will NOT be added to your earnings.

If you have a question, please raise your hand!

Instructions part 2
Here follow the instructions for part 2. Before part 2 actually starts, you will receive a summary of these instructions. An amount of 60 points will be added to the total that you earned until now. This is resulting amount is your starting capital for part 2. Part 2 lasts from period 21 to 30. Each period you will be allocated to a group of 3 persons, and one product will be sold to each group.

Two decisions per period
From now on you will make two decisions per period. Each period consists of part A and part B. In each part you make a decision. Only one of the two decisions will be actually paid. At the end of the period the computer determine by lottery which decision is paid out. The probability that decision A will be paid out is 50%. If decision A is not paid out, decision B will be paid out.

The decision that you will make in part A is exactly the same as the decision in the first part of the experiment.

Decision part B
When all participants have made their decision for part A, part B of a period is started. In part B each participant will again bid for the product. For a large part the situation is the same as for the decision in part A. Again each participant is allocated to a group of 3 persons. Again the product will be sold to the person with the highest bid. With more than one highest bid, the buyer will be determined by lottery. The buyer pays a price equal to the own bid.

The value of the product in part B equals a private value plus a common value. For each participant the private value equals the private value in part A. The common component is also for each participant exactly as the common component in part A. The only difference between the decision in part A and the decision in part B is that the common value for the product is computed differently.

The so-called PUBLIC COMPONENT is important for calculating the common value in part B. The public component lies between 0 points and 200 points, where each number is equally likely. The value for the public component does not depend on the value of the
common components of the participants. EACH participant gets to know the public component of her or his group at the start of part B. Thus all participants in a group are confronted with the same public component.

In part B the common value equals the AVERAGE of the common components of the participants in a group AND the public component of the group. To be precise, all common components of the participants in the group are added. Then the public component is added to this number. The resulting number is divided by four and (when needed) rounded to the closest integer number.

**Profit**

When all participants have made their decision for part B, the computer determines whether part A or part B is used for actual payment. Only the person with the highest bid in the part that is paid out earns an amount. This amount equals her or his private value plus the common value minus her or his bid. Just like in the first part of the experiment profits (losses) are automatically added (subtracted) from the amount earned up to now.

You do not have to enter the same bid twice. Nor do you have to enter a different bid for part B than for part A. It is up to you to decide what you want to do.

**Information**

When all participants have made both decisions, the results of the period will be communicated. Only the results of the part that is used for actual payment will be communicated. You will see all bids in your group ranked from high to low. You will get to know whether or not you bought the product. You will see the common value of the product and your profit.

Then a new period will be started. In the new period again a product will be sold. For each participant the product will have a different value. Each participant receives a new private value for the product and a new common component. At the start of part B each participant will see the new public component. Your value for the product in one period does not depend on your value for the product in any other period.

Now you will be asked to answer some questions about these instructions.

**Question about the common value**

Assume that your common component equals 54 points. The public component equals 113 points. Assume that for the other participants it holds that:

<table>
<thead>
<tr>
<th>Participant</th>
<th>Common component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>127</td>
</tr>
<tr>
<td>2</td>
<td>66</td>
</tr>
</tbody>
</table>

How large is the common value in this example?

*Note: Again, subjects could only pass the question by giving the right answer. After a wrong answer the relevant part in the instructions was explained anew, and the subject had to try again.*
Question about the public component
Is the following statement right or wrong?
In part B of a period each participant in a group will receive a different public component.

End
You have reached the end of the instructions. If you want to read some parts of the instructions again, push the button BACK.

Otherwise you push the button READY. When all participants have pushed READY, the second part of the experiment will start. When the experiment has started, you will not be able to return to these instructions. Before the experiment is started, a summary of the instructions will be handed out.

If you have a question, please raise your hand!
Appendix B: Nash Equilibrium Bids for Treatment High-3

Recall from section 3 that the Nash equilibrium bids are given by

\[ B(x) = E(V + t_1 | s_1 = x, Y_1 = x) - E(Y_1 - y_1 | s_1 = x, Y_1 = x), \quad (B1) \]

where the surplus variable is defined as \( s = v/n + t \), and \( Y_1 \) is the maximum of the \( n \) surplus draws. For treatment high-3, the surplus variable is the sum of two uniformly distributed random variables: \( t \sim U[75, 125] \) and \( v \sim U[0, 200] \). It is useful to decompose the support of \( s \) into three regions: \( R_1 = [75, 125] \cup R_{II} = [125, 75+200/3] \cup R_{III} = [75+200/3, 125+200/3] \). The density of the surplus variable can then be worked out as: \( f_1(s) = 3(s - 75)/10000 \), \( f_{II}(s) = 3/200 \), and \( f_{III}(s) = 3(575/3 - s)/10000 \), i.e. the density has a "trapezoid" shape.

An alternative way to write (B1) is

\[ B(x) = \frac{n - 1}{n} E(v | s \leq x) + E(y_1 | y_1 \leq x). \quad (B2) \]

The first term on the right side of (B2) can be written as

\[ E(v | s \leq x) = \int_{75}^{x} E(v | s = y) f_s(y) dy, \quad (B3) \]

with \( f_s \) the density of the surplus variable. The second term on the right side of (B2) equals

\[ E(y_1 | y_1 \leq x) = \int_{75}^{x} y dF_{n-1}(y | y_1 \leq x) = x - \int_{75}^{x} \frac{F_{s}^{n-1}(y)}{F_{n-1}(x)} dy, \quad (B4) \]

with \( F_s \) the cumulative distribution corresponding to \( f_s \). The bidding functions on each of the three regions can now be computed from the conditional expectations \( E_i(v | s = y) = 3(y - 75)/2 \), \( E_{II}(v | s = y) = 3(y - 100) \), \( E_{III}(v | s = y) = 3(y - 175/3)/2 \), and the expressions for the density, \( f_s \), given above. The explicit formulas are:

\[ B_i(x) = \frac{22}{15} x - 35, \quad (B5) \]

for \( 75 \leq x \leq 125 \).
for $125 \leq x \leq 425/3$, and

$$B_{II}(x) = \frac{5(x^3 - 240x^2 + 18,125x - 413,125)}{3(x - 100)^2}, \quad (B6)$$

for $425/3 \leq x \leq 575/3$. The optimal bids in (B5) - (B7) are shown as the lower lines in the top and bottom panels of Figure 1 in Appendix C.
Figure 1. Bids (+) in Parts 1 of High-3 (Top Panel) and High-3+, High-3++ (Bottom Panel) Together with Nash Bids (Lower Lines) and Naive Bids (Top Lines)
Figure 2. Bids (+) in Parts 1 of High-6 (Top Panel) and High-6+ (Bottom Panel) Together with Nash Bids (Lower Lines) and Naive Bids (Top Lines)
Figure 3. Bids (+) in Parts 1 of Low-3 (Top Panel) and Low-3+ (Bottom Panel) Together with Nash Bids (Lower Lines) and Naive Bids (Top Lines)