Cost Reducing Investment, Competition and Industry Dynamics*

Emmanuel Petrakis
Departmento de Economia, Universidad Carlos III de Madrid, Madrid, Spain,

and

Santanu Roy
Department of Economics, Erasmus University, Rotterdam, The Netherlands.

First Version: September, 1996, Revised Version: June, 1997.

ABSTRACT: We demonstrate the possibility of *shake-out* of firms and emergence of inter-firm heterogeneity along the (socially optimal) dynamic equilibrium path of a competitive industry with free entry and exit, even when there is no uncertainty and all firms are *ex ante* identical with perfect foresight. Atomistic firms with upward sloping marginal cost curves undertake investment in *firm-specific* cost reduction. They earn negative net profit in early periods, compensated later by strictly positive net profits; no entry occurs after the initial time period. Some firms may exit before others even while other firms earn positive net profit.

JEL Classification: D41, D92, L11, O31
Key-words: Cost Reducing Investment, Industry Dynamics, Learning, Competitive Equilibrium, Shake Out.

Author Addresses:
Emmanuel Petrakis, Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe (Madrid), Spain.
[E-mail: *petrakis@eco.uc3m.es.*, Tel: (+34-1)-6249652, Fax:(+34-1)-6249875]
Santanu Roy, Department of Economics, Erasmus University H7-28, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands.
[E-mail: *roy@few.eur.nl.*, Tel: (+31)-10-4081420, Fax:(+31)-10-2121724]

---

* We thank the Tinbergen Institute at Rotterdam and the Universidad Carlos III de Madrid for supporting short-term research visits during the writing of this paper. We gratefully acknowledge helpful comments made by participants at the meetings of the Society for Economic Dynamics and Control (1995, Barcelona), the Econometric Society (1997, New Orleans), the Royal Economic Society (1997, Stoke-on-Trent) and seminar audience at Arizona State University. The current version is forthcoming in the *International Economic Review* and has considerably benefitted from the suggestions made by two anonymous referees of that journal. Financial support from DGICYT grant PB92-0245 is acknowledged.

1. Introduction.

One of the most important factors behind intertemporal variations in market structure, as well as prices, output and profitability of firms in an industry, is productivity and technology improvements occurring at firm-level. Activities such as research and development (R&D) and innovation which lead to the emergence and adoption of new technologies are crucial factors behind such changes. These activities are potentially beneficial to all firms and even if direct "free-riding" is prevented by patents, there are widespread indirect spill-over effects. There is, however, another class of activities which includes firm-specific learning[1], organizational innovation, and other (firm-specific) efficiency enhancing activities which also play a significant role in productivity improvement and cost reduction by firms. Their common feature is firm-specificity - they are determined almost entirely through internal investment and accumulation of experience within the firm with little, if any, potential benefit to other firms. This paper focuses on the latter class of activities and analyzes the incentives for cost reduction offered by a competitive market as well as their consequences on the dynamics of industry structure and market variables. In particular, we characterize the dynamic equilibrium path of a competitive industry with a

[1]The applied literature on the "learning curve" in industries identifies a complex set of factors behind cost reduction in firms many of which can be clubbed under "indirect" labour learning i.e., increases in labour productivity arising from conscious management effort, improvements in "architectural aspects" of modern manufacturing organizations such as inventory and communication systems, interdepartmental coordination, degree of job specialization and task rotation, decentralization of responsibilities, process quality control, methods of motivating and training workers etc. - all of which require investment and experience. See, Mookherjee and Ray (1992) for further details.

continuum of price-taking firms, free entry and exit, where individual firms undertake investment over time in order to reduce their future production cost.[2]

We consider production technologies which exhibit decreasing returns; in each time period, the current marginal cost of production for a firm is strictly increasing in output[3]. This reflects the fact that in each period, the organizational capital and stock of knowledge of a firm is given by past history. This allows price-taking firms to earn strictly positive profit as compensation for past investment without violating market equilibrium. Individual firms, by making investments, are able to reduce their total as well marginal cost of production over time in a deterministic way. A firm incurs a strictly positive "fixed cost" every period it stays in the industry, *even if no output is produced*. Thus, the average cost curve of a firm in any time period is U-shaped. Investment can reduce the fixed cost incurred by active firms. We impose restrictions

---

[2]Petrakis, Rasmusen and Roy (1997) analyze a two period model of a competitive industry with atomistic firms who engage in "learning by doing"; future production cost is a decreasing function of cumulative past *output* (rather than direct investment). As a result, firms overproduce (relative to what is suggested by current price and marginal cost) in order to reduce future cost. A number of empirical studies have pointed out that cumulative investment (as considered in the present paper) is a better explanatory variable for firm performance compared to cumulative output (see Dutton and Thomas 1984).

[3] If the firm-level technology exhibits constant returns at each point of time, then the dynamic scale economies associated with the possibility of unit cost reduction through investment leads to a breakdown of perfect competition. The market cannot compensate an individual firm for past investment because whenever price is greater than the current marginal cost, price-taking firms produce indefinitely large output.

on the cost-reduction technology so that the dynamic scale economies for a firm are eventually bounded. A dynamic competitive equilibrium exists for the industry. As there are no externalities or inter-firm spill-overs, the equilibrium path is socially optimal. The competitive market provides just the right incentives for investment in cost reduction.[4]

Firms with perfect foresight, take the price path as given and make their entry, exit, output and investment decisions over time, so as to maximize their intertemporal profit (net of investment cost). In equilibrium, each firm that enters the industry earns exactly zero intertemporal profit over its period of stay; no firm can earn strictly positive intertemporal profit by changing its entry and exit decisions. Firms which stay for more than one period undertake strictly positive investment in cost reduction. Equilibrium prices are (weakly) decreasing over time and lie below the static competitive price i.e., the minimum average cost for a new entrant. Firms undertake investment and earn negative net profits in their earlier periods of stay, which is compensated by positive future profits. Price is greater than the minimum average cost of mature firms and they typically produce above their minimum efficient scale.

Recent empirical studies indicate that a distinctive characteristic of industry evolution is the high degree of heterogeneity encountered: high variance of growth rates across firms, high dispersion in size and significant rates of turnovers of firms[5]. Since the early eighties, models of

---

[4] Competitive equilibrium theory of industry dynamics with atomistic firms was first developed by Lucas and Prescott (1971). Hopenhayn (1990) establishes results on existence and social optimality of dynamic industry equilibrium with entry and exit, for a very general class of intertemporal technology and stochastic shocks, both aggregate and firm specific.

[5] For example, Dunne, Roberts and Samuelson (1989) study a sample of U.S. manufacturing industries over a period of 5 years and report rates of entry ranging from 30.7%

stochastic evolution and selection in competitive industries have been used to explain these empirical regularities. Jovanovic (1982) analyzes the dynamics of a competitive industry where atomistic firms, uncertain about their productivity, acquire noisy information about how efficient they are; incumbents that are afflicted by unfavourable signals conclude that they are inefficient and exit the market to be replaced by new entrants. Pakes and Ericson (1990) discuss the implications of a more general version of this model and compare it with those of a stochastic model of their own, where firms actively undertake investment in order to influence the conditional distribution of future shocks (see also, Lippman and Rummelt 1982). In a fairly general model with firm level exogenous technology shocks, Hopenhayn (1992 a,b) shows the possibility of entry and exit as part of the limiting behaviour of an industry. In a similar model, Hopenhayn (1993) relates the observed patterns of entry and exit to stochastic demand expansion and technological change. Jovanovic and MacDonald (1994) discuss a competitive model where innovational opportunities fuel entry and failure to innovate, whose chances are exogenously specified, fuels exit. The unifying feature of this class of models is the role of firm-level uncertainty in creating heterogeneity among firms. The process of market selection then leads to exit of firms afflicted by unfavourable shocks, often opening up room for entrants with more favourable initial characteristics (who are therefore more optimistic about future profitability than firms that exit)[6]. In contrast, our model is fully *deterministic*. The dynamics of the industry are

---

to 42.7% and an equally dramatic exit rate ranging from 30.8% to 39% across industries.

[6] In a somewhat different exercise, Lambson (1991) analyzes a dynamic competitive model where firms make investments which entail sunk cost and whose relative profitability is influenced by exogenous stochastic shocks over time; the equilibrium path can exhibit high turnover of plants. See, also, Dixit (1989).

determined exclusively by deterministic shifts in the cost structure of firms resulting from deliberate investment under perfect foresight. Further, all firms are *ex ante* identical and market demand is stationary over time. Even so, our model explains many of the stylized empirical observations relating to industry dynamics.

We show that on the industry equilibrium path, some firms may exit the market before others. This *shake-out* of firms is not a result of predatory behaviour (even though it occurs simultaneously with expansion of the size of incumbent firms through investment); nor does it result from firms being subject to unfavourable shocks. The reason behind shake-out in our model is that investment in cost reduction may lead to expansion of firms' optimal scale. Given that market demand is stationary, the industry may no longer be able to sustain all the existing firms without excessive price reductions which can harm the incentives to invest. Thus, it is in order to give appropriate incentives for cost reductions that the market requires some firms to exit. The role of exiting firms, from the point of social efficiency, is to reduce the industry-wide cost of production in initial periods when the marginal cost curve could be very steep. We establish a simple sufficient condition for shake-out which is satisfied if the minimum efficient scale expands fast enough with cost-reducing investment and the market demand is relatively inelastic. We construct an example where exit occurs only in the first period if the demand is not too inelastic and if demand is sufficiently inelastic, exit occurs every period. If investment reduces only the fixed cost of being active in the industry, firms' supply curves stay unchanged over time and therefore, shake-out does not occur.

Shake-out is a widely observed empirical phenomenon. In a study of the evolution of 46 new products, Gort and Klepper (1982) find an average rate of shake-out of firms - measured by the number of firms after the decrease, relative to the peak, - of roughly 40%. The rate of exit is higher among relatively smaller firms and the average size of incumbent firms in the industry increases with maturity (see, among others, Dunne, Roberts and Samuelson 1989, Davis and Haltiwanger 1992). In our model, exiting firms make typically less cumulative investment than staying firms. Thus, the supply curve of firms which stay on in the industry is to the right of that of exiting firms. Exiting firms are smaller in size. No entry occurs in the industry after period 1; this apparent barrier to entry co-exists with mature incumbents earning positive net profits. This too is not due to any anti-competitive activity of incumbent firms, but is part of the socially efficient competitive outcome. The absence of late entry is intuitive as there is no randomness governing the fortunes of firms, nor does market demand change over time. All initial entrants realize their planned intertemporal return from current investment with certainty; the level of entry in the initial period is such that any possibility of a gain through late entry is precluded. As there is no late entry and exiting firms have typically lower cumulative investment compared to firms that stay on, firm numbers decrease and the size of incumbent firms increase as the industry matures. Even though firms are *ex ante* identical, heterogeneity emerges in the output, investment, production cost and profit profile of active firms because of differences in their length of stay in the industry.

We also illustrate some other interesting possibilities through numerical examples. An increase in opportunities for cost reduction may lead to an increase in the number of firms active in the industry (lower "concentration"); investment as a proportion of revenue may increase even while the number of active firms increase. These contrast sharply with standard "Schumpeterian" notions as well as results obtained in oligopolistic models of R&D.[7]

---

[8]See, for example, the static models of oligopolistic R&D contained in Dasgupta and Stiglitz (1980) and Dasgupta (1986). Dynamic non-cooperative games where firms make cost reducing investments of the kind considered in this paper, have been analyzed, among others, by

Section 2 describes the model formally and states the definitions as well as basic results related to the existence of dynamic industry equilibrium and its social optimality. The main results of the paper relating to characterization of the equilibrium path in terms of prices, output, entry and exit of firms are contained in Section 3. Section 4 discusses the phenomenon of exit or shake-out in our model and establishes a sufficient condition for exit. Section 5 contains some numerical examples which highlight certain interesting effects of change in demand elasticity and the effectiveness of investment in cost reduction on industry variables. Section 6 concludes. The proof of proposition 5 is contained in the appendix.

2. The Model and Preliminary Results

Consider a homogenous good industry which lasts for T periods, $1 < T < \infty$. The market demand in period t depends only on the current price and is stationary over time. Denote by D(p) the market demand function and by P(Q), the inverse demand function. There is a continuum of *ex ante* identical firms (of indefinitely large measure) which can enter the industry in any period and there is no sunk cost of entry. Each firm is of measure zero and is indexed by $i \in R_+$. In each period t, an active firm i produces output $q_t(i) \geq 0$ and makes an investment $x_t(i) \geq 0$. Let $z_t(i) \in R_+$ be the cumulative investment of firm i at the beginning of period t. The latter summarizes firm i's stock of firm-specific learning, organizational capital and other efficiency enhancing attributes which we may collectively refer to as its *stock of knowledge* in period t. The initial stock of knowledge for an entering firm is normalized to be equal to zero. If firm i enters in period $\hat{o}$, then for $t > \hat{o}$

$$z_t(i) = x_{\hat{o}}(i) + x_{\hat{o}+1}(i) + ... + x_{t-1}(i),\ z_{\hat{o}}(i) = 0$$

Flaherty (1980), Spence (1984), Ericson and Pakes (1995), and Lach and Rob (1996).

We shall use the letters q, x and z without time or firm specific indices to indicate respectively the current output, (flow) investment and the stock of knowledge of a generic firm.

A firm's production cost in any period depends on its current output as well as its stock of knowledge. Specifically, firm i's cost of producing output q in period t is given by $C(q,z_t(i))$. On the other hand, firm i's cost of making investment $x_t(i)$ in any period t is given by ã(x_t(i)). Given prices $(p_1,...p_T)$, a firm i which enters in period ô $\geq$ 1 and exits in period ô+k $\leq$ T, earns intertemporal payoff equal to total discounted sum of profit, net of investment cost, that is,

$$\sum_{t=\text{ô},..\text{ô}+k} \text{ä}^{t-\text{ô}}[p_t q_t(i) - C(q_t(i),z_t(i)) - \text{ã}(x_t(i))],$$

where ä is the discount factor lying in (0,1]. We follow the convention of discounting profit streams to the period of entry. We will also follow the convention that a firm which enters does so at the beginning of a period and exits, if it so wishes, at the end of a period after production has taken place. The firm is free to decide its period of entry and exit. Once a firm exits, it loses all its acquired technological knowledge (embedded in its employed factors) and can re-enter the industry only as a fresh entrant with z = 0. As there is an indefinitely large measure of identical potential entrants, we assume without loss of generality that a firm which exits does not re-enter.

We make the following assumptions on the inverse demand function P(Q) and the cost functions C(q,z), ã(x):

(A1) $P:R_+ \rightarrow R_+$ is continuous, strictly decreasing and integrable on any bounded interval.

(A2) $C:R^2_+ \rightarrow R_+$ is continuously differentiable; for any given $z \geq 0$, $C_q(q,z)$ is strictly increasing in q on R_+; $C(0,z) > 0$ for all $z \geq 0$.

(A3) $C_z(q,z) \leq 0$; for any $q \geq 0$, $z^1 > z^2$ implies that $C_q(q,z^1) \leq C_q(q,z^2)$.

(A4) There exists $h > 0$ such that $[C(q,z)/q] \geq h$ for all $q,z \geq 0$.

(A5) ã:$R_+ \rightarrow R_+$ is continuously differentiable, ã(0) = 0, ã$'$(x) > 0 for x > 0.

(A6) For any q > 0, there exists some z > 0 such that

$$ã(0) + äC(q,0) < ã(z) + äC(q,z).$$

Let $p_m(0)$ denote the current minimum average cost for a new entrant with zero stock of knowledge:

$$p_m(0) = \min\{C(q,0)/q: q \geq 0\}$$

(A7) $\lim_{Q\downarrow 0}P(Q) > p_m(0)$.

For any vector of outputs $(q_1,.....q_T) \geq 0$, let $G(q_1,q_2,....q_T)$ denote the minimum cost of production after taking into account the investment in cost reduction:

$$G(q_1,q_2,....q_T) = \quad \text{Min} \sum_{t=1,..T} ä^{t-1}[C(q_t,z_t) + ã(x_t)]$$
$$x_t \geq 0,\ t = 1,...T$$
$$z_t = \sum_{ô = 1,....,t-1} x_{ô},\ t > 1,\ z_1 = 0.$$

It can be checked that there exists a solution to the minimization problem. One can think of G as the "net" cost of producing an output vector. Our next assumption imposes a restriction on the behaviour of the net cost function for large output levels:

(A8) There exists K > 0, such that for any $(q_1,q_2,....q_T) \geq 0$, $q_{ô} > K$ implies

$$G(q_1,..,q_t,..q_T) > G(y_1,..,y_t,...y_T) + ä^{ô\text{-}1}(q_{ô} - K)p_m(0)$$

where $y_t = q_t$ for $t \neq ô$, $y_{ô} = K$.

Assumption (A1) states that the market demand function in any period depends only on the current price, is stationary over time and is strictly decreasing. Further, it is integrable on any bounded interval so that the net social surplus is well defined. Assumption (A2) says that the cost function is smooth and that the marginal cost of production is strictly increasing in output, no matter what the stock of knowledge is for the firm. Thus the current production technology in any period exhibits decreasing returns to scale. Further, production cost for an active firm is strictly positive *even if no output is produced*, no matter how large the firm's cumulative investment. This can be interpreted as a "fixed" cost that must be paid by the firm every period it stays in the industry. It can be reduced by investing, for instance, in organizational capital. Among other things, assumption (A2) implies that, in any time period, the current average cost curve is U-shaped and the current minimum efficient scale of an active firm is always bounded away from zero. This ensures that only a finite measure of firms enter the industry in equilibrium. In the absence of this assumption and given a downwards sloping demand curve, it is possible that equilibrium involves indefinitely large number of firms in the industry, each producing infinitesimal output in each period.

Assumption (A3) states that the total as well as the marginal costs of production are (weakly) decreasing in the amount of accumulated investment. Note that we do not assume that C is convex in both arguments. Thus, the marginal return to investment in cost reduction may be initially increasing in the level of investment made by the firm. Assumption (A4) says that the average cost of production for any firm is uniformly bounded below by some $h > 0$. In other words, no matter how large the accumulated stock of knowledge for a firm, the minimum average cost of production can never be made arbitrarily close to zero. This is a technical assumption which ensures that the equilibrium prices are bounded away from zero so that the total output and "size" of the industry is uniformly bounded over all time periods, which in turn is used to ensure the existence of dynamic industry equilibrium. Assumption (A5) describes the properties of the cost of investment in any period. The marginal cost of making positive investment is strictly positive. Typically, one would expect the function $\tilde{a}(x)$ to be convex though we do not make this assumption. Assumption (A6) ensures that the cost reduction technology is sufficiently "productive"; there exists some investment level which is worth undertaking for a firm that has made no past investment and plans to produce a strictly positive output next period. This assumption ensures that all firms except those which stay for only one period, find it worthwhile to make strictly positive investment in their initial period of stay. Note that this assumption allows for the existence of a "threshold level" below which investment in cost reduction is not worthwhile. Assumption (A7) is a restriction which is necessary to ensure that the market is non-trivial and market equilibrium exists for $T = 1$.

The crucial assumption in the model is (A8); it limits the dynamic scale economies implied by the possibility of cost reduction. It ensures that the firm's technology exhibits "asymptotic decreasing returns" even after taking into account the possibility of cost reduction through investment. More particularly, as we allow for free entry and exit, equilibrium prices are bounded above by $p_m(0)$, the minimum average cost of a new entrant. Assumption (A8) states that, if we restrict attention to price vectors which lie in $[0,p_m]^T$, no firm will produce output in excess of bound K; the net marginal cost (taking into account investment cost) exceeds $p_m(0)$ if output exceeds K. From the standpoint of social efficiency, instead of having one firm produce $q_{ô} > K$, it is better to reduce its output to K and have the excess, viz. $(q_{ô} - K)$ produced by a measure of fresh entrants which stay in the market for one period and produce at their minimum efficient scale. Note that this assumption allows for environments where the current marginal cost of production $C_q(q,z)$ falls to zero as z becomes very large, provided the marginal cost of investment, $ã'(x)$, increases sharply enough so that the overall effect is to restrict firm size and prevent any natural monopoly outcome[8]. However, even when the marginal cost of investment is constant, (A8) is satisfied if the infimum of the set of possible marginal production cost that a firm can attain through investment exceeds $p_m(0)$, when its output is larger than K[9].

Let $\underline{ô}(i)$ and $ô(i)$ denote, respectively, the periods of entry and exit for firm i, $1 \leq \underline{ô}(i) \leq ô(i) \leq T$. If firm i never enters, we simply use the convention that $\underline{ô}(i) = ô(i) = T+1$. Given time periods $\underline{ô}$ and $ô$, where $1 \leq \underline{ô} \leq ô \leq T$, let $S(\underline{ô},ô)$ be the set of firms which enter in period $\underline{ô}$ and exit in period $ô$. Further, let $n(\underline{ô},ô)$ denote the (Lebesgue) measure of the set $S(\underline{ô},ô)$, that is the "*number of firms*" which enter in period $\underline{ô}$ and exit in $ô$. Given price vector $\mathbf{p} = (p_1,...,p_T)$ and time periods $\underline{ô}$, $ô$ where $1 \leq \underline{ô} \leq ô \leq T$, we denote by $Ð(\mathbf{p},\underline{ô},ô)$ the maximum discounted sum of profit (net of investment cost) that a firm which enters in period $\underline{ô}$ and exits in period $ô$ can achieve. Thus:

$$(1) \qquad Ð(\mathbf{p},\underline{ô},ô) = \max \sum_{t=\underline{ô}}^{ô} ä^{t-\underline{ô}} \{p_t q_t - C(q_t,z_t) - ã(x_t)\}$$

$$\text{with respect to } (q_t,x_t) \geq 0,\ t = \underline{ô},...ô$$

$$\text{where } z_t = \sum^{t-1} x_{ô},\ t > \underline{ô},\ z_{\underline{ô}} = 0.$$

---

[8]For example, $C(q,z) = e^{q-kz} + F$, with $F, k > 0$ and $ã(x) = ax^2$, $a > 0$.

[9]This is satisfied by cost functions in the family $C(q,z) = G(q)H(z)+F$ where $F > 0$, G is strictly convex and strictly increasing, H is decreasing in z and $\lim_{z\to\infty}H(z) > 0$; for instance, $C(q,z) = q^2(1 + e^{-z}) + F$.

$\hat{o}=\underline{\hat{o}}$

Let Ø($\mathbf{p}$,$\underline{\hat{o}}$,$\hat{o}$) be the set of solutions to the maximization problem on the right hand side of (1). As C(q,z) is not necessarily convex, there can be multiple solutions to the profit maximization problem.

Let S_t denote the set of all firms which are active in period t = 1,2,...T. Obviously, S_t is the union of all sets S($\underline{\hat{o}}$,$\hat{o}$) where $\underline{\hat{o}} \leq t \leq \hat{o}$. We are now ready to define industry equilibrium:

Definition of Industry Equilibrium: An industry equilibrium consists of:

(i) measurable sets S($\underline{\hat{o}}$,$\hat{o}$) of firms that enter in period $\underline{\hat{o}}$ and exit in period $\hat{o}$, $1 \leq \underline{\hat{o}} \leq \hat{o} \leq T$.

(ii) output and investment profile $\{(q_t(i),x_t(i)), t = \underline{\hat{o}},..,\hat{o}\}$ for all $i \in S(\underline{\hat{o}},\hat{o})$, $1 \leq \underline{\hat{o}} \leq \hat{o} \leq T$; $q_t(i)$, $x_t(i)$ integrable on S($\underline{\hat{o}}$,$\hat{o}$).

(iii) price vector $\mathbf{p} = (p_1,....p_T)$

such that:

(a) $D(p_t) = Q_t$, where $Q_t = \int_{S_t} q_t(i)di$

and, for all ($\underline{\hat{o}}$,$\hat{o}$) such that $1 \leq \underline{\hat{o}} \leq \hat{o} \leq T$

(b) if $n(\underline{\hat{o}},\hat{o}) > 0$, then for all $i \in S(\underline{\hat{o}},\hat{o})$, $\{(q_t(i),x_t(i)),t = \underline{\hat{o}},..,\hat{o}\} \in$ Ø($\mathbf{p}$,$\underline{\hat{o}}$,$\hat{o}$)

(c) Ð($\mathbf{p}$,$\underline{\hat{o}}$,$\hat{o}$) $= 0$, if $n(\underline{\hat{o}},\hat{o}) > 0$,

≤ 0, otherwise.

Condition (a) says that the market should clear every period. Condition (b) says that, given the equilibrium price vector $\mathbf{p}$, the output-investment profile of each active firm over its period of stay should maximize its net discounted sum of profits. Condition (c) ensures that all active firms, no matter when they enter and exit the industry, earn exactly zero net intertemporal profit over their periods of stay. In addition, there is no incentive for further entry and no active firm can make a strict gain by changing its entry-exit decision.

Under assumptions (A1)-(A8), there exists an industry equilibrium. The fact that there is a continuum of firms makes the aggregate technology for the industry a closed convex cone. Our assumptions limit the size of individual firms as well as the industry. It can be shown that the set of equilibrium allocations are equivalent to those which solve the social planner's problem of maximizing the discounted sum of net social surplus in the industry. Ensuring existence of solution to the planner's problem guarantees existence of an industry equilibrium. This also proves that the industry equilibrium is socially optimal. In fact, the equilibrium is unique in prices and total output produced.

At this stage, we should define precisely the social planner's problem. The social planner chooses:

(i) sets $S(\underline{\hat{o}},\hat{o})$, $1 \leq \underline{\hat{o}} \leq \hat{o} \leq T$, of firms that enter in period $\underline{\hat{o}}$ and exit in period $\hat{o}$ (which also defines the sets S_t of firms which are active in period t);

(ii) output and investment profile $\{(q_t(i),x_t(i)),t = \underline{\hat{o}},..,\hat{o}\}$ for all $i \in S(\underline{\hat{o}},\hat{o})$, $1 \leq \underline{\hat{o}} \leq \hat{o} \leq T$; $q_t(i),x_t(i) \geq 0$ and integrable on $S(\underline{\hat{o}},\hat{o})$.

so as to maximize

$$\sum_{t=1}^{T} \ddot{a}^{t-1} \left\{ \int_{0}^{Q_t} P(y)dy \; - \int_{S_t} [C(q_t(i),z_t(i)) + \tilde{a}(x_t(i))]di \right\}$$

subject to:

$$Q_t = \int_{S_t} q_t(i)di$$

$$z_t(i) = \sum_{\hat{o}=\underline{\hat{o}}}^{t-1} x_{\hat{o}}(i),\ t > \underline{\hat{o}},\ z_{\underline{\hat{o}}}(i) = 0.$$

Proposition 1. *There exists an industry equilibrium. The allocation pertaining to an industry equilibrium always solves the social planner's problem (SPP) and every solution to SPP can be sustained as an industry equilibrium (with appropriately defined prices).*

Proof: Hopenhayn (1990) contains a proof of existence and social optimality of industry equilibrium in a very general stochastic model. Under assumption (A8), we can restrict, without loss of generality, the output produced by a firm in any period to lie in [0,K]. As the total investment in any period cannot exceed maximum total intertemporal revenue and the prices can, without loss of generality, be restricted to lie in $[h,p_m(0)]$,[10] (where h is the lower bound on average cost as defined in assumption (A4)), we can construct a modified model with compact aggregate technology (technology of firm consists of feasible vectors of outputs, investments and their intertemporal cost) and compact state space (state = stock of knowledge). Further, as the average cost of production is always bounded below by $h > 0$ and $P(Q) \downarrow 0$ as $Q \to \infty$, the net social surplus goes to $-\infty$ (in the modified model) if total industry output is infinitely large. These are sufficient to ensure all the assumptions made in that paper. Therefore, there exists an industry equilibrium and every such equilibrium path is socially optimal. It can be checked that the modifications made on the action and state spaces do not matter for industry equilibrium as well as social planner's problem. We now give the outline of a direct proof which is based on Jovanovic (1982) and virtually identical to that contained in Petrakis, Rasmusen and Roy (1994).

First, consider the social planner's problem. Suppose the planner wishes to produce some vector of total output $(y_{\text{ô}},....y_{\text{ô}+k})$ using firms which enter in period t and exit in period (t+k). The number (measure) of such firms n and their optimal path $(q_t(i), x_t(i))$, $t = \text{ô},...\text{ô}+k$, must solve the social cost minimization problem

$$\text{(P1)} \qquad \text{Min} \quad \sum_{t=\text{ô}}^{\text{ô}+k} \text{ä}^{t-\text{ô}} \int_0^n [C(q_t(i), z_t(i)) + \text{ã}(x_t(i))]di$$

subject to:

$$\text{(*.1)} \qquad y_t \leq \int_0^n q_t(i)di$$

$$z_t(i) = \sum_{j=\text{ô}}^{t-1} x_j(i), \; t > \text{ô}, \; z_{\text{ô}}(i) = 0.$$

$(q_t(i), x_t(i)) \geq 0$ and integrable.

Let $R_{\text{ô},\text{ô}+k}(y_{\text{ô}},....,y_{\text{ô}+k})$ be the value of this minimization problem. Under our assumptions, particularly (A8), there exists a solution to this minimization problem (for example, using

[10]We show later (Proposition 2) that equilibrium prices can never fall below h or exceed $p_m(0)$ so the modification of the problem by restricting prices to lie $[h,p_m(0)]$ does not matter.

Theorem 6.1 in Aumann and Perles (1965)). As we have a continuum of firms, a direct application of the Lyapunov-Richter theorem (see L.1.3 in MasColell (1985)) shows that $R_{\hat{o},\hat{o}+k}$ is a convex function. Taking variations around the optimal solution, it can be established that $R_{\hat{o},\hat{o}+k}$ is differentiable and that the partial derivative $\partial R_{\hat{o},\hat{o}+k}/\partial y_t = \mu_t$ where μ_t is the optimal value of the dual variable corresponding to constraint (*.1). Further, the first order conditions imply that:

(*.2) $\quad \mu_t = C_q(q_t(i), z_t(i))$, if $q_t(i) > 0$,

(*.3) $\quad \tilde{a}'(x_t(i)) + \sum_{j=t}^{\hat{o}+k} \ddot{a}^{j-t} C_z(q_j(i), z_j(i)) = 0$, if $x_t(i) > 0$

Let $y_t(\hat{o},\hat{o}')$ denote the total output produced in period t by firms who enter in period ô and exit in period ô'; $R_{\hat{o},\hat{o}'}(y_{\hat{o}}(\hat{o},\hat{o}'),...,y_{\hat{o}'}(\hat{o},\hat{o}'))$ the minimum social cost of producing a total output vector $(y_{\hat{o}}(\hat{o},\hat{o}'),...,y_{\hat{o}'}(\hat{o},\hat{o}'))$ by using such firms. Note that $R(0,0,..0) = 0$.

Now, rewrite the social planner's intertemporal net surplus maximization problem as:

(P2) $\quad \max \quad \sum_{t=1}^{T} \ddot{a}^{t-1} \left[\int_0^{Q_t} P(x)dx\right] - \sum_{1\leq\hat{o}\leq\hat{o}'\leq T} R_{\hat{o},\hat{o}'}(y_{\hat{o}}(\hat{o},\hat{o}'),...,y_{\hat{o}'}(\hat{o},\hat{o}'))$

subject to:

$$Q_t = \sum_{\hat{o}\leq t\leq\hat{o}'} y_t(\hat{o},\hat{o}')$$

with respect to $y_t(\hat{o},\hat{o}') \geq 0$, $1 \leq \hat{o} \leq \hat{o}' \leq T$.

As $P(Q) \to 0$ as $Q \to +\infty$, (A4) implies that there exists a finite solution to the social planner's problem. As the maximand is strictly concave, there is a unique solution in total output and $Q_t > 0$ for t = 1,2,...T. From the first order conditions of this maximization we have:

(*.4) $\quad P(Q_t) = \partial R_{\hat{o},\hat{o}'}/\partial y_t$ if $\hat{o} \leq t \leq \hat{o}'$ and $y_t(\hat{o},\hat{o}') > 0$

Set $p_t = P(Q_t)$. If $\hat{o} \leq t \leq \hat{o}'$ and $y_t(\hat{o},\hat{o}') > 0$, then, $p_t = \mu_t$, the dual variable corresponding to constraint (*.1) in the social cost minimization problem for output vector $(y_{\hat{o}}(\hat{o},\hat{o}'),...,y_{\hat{o}}(\hat{o},\hat{o}'))$. From standard duality results, it follows that for each firm i that enters in period ô and exits in period ô', $(x_t(i),q_t(i))$, $t = \hat{o},...\hat{o}'$, maximizes intertemporal profit $\sum_{t=\hat{o},..\hat{o}'} \ddot{a}^{t-\hat{o}}[p_t q_t - C(q_t,z_t) - \tilde{a}(x_t)]$ and that this maximum is exactly equal to zero. Thus, each firm maximizes profit, gets zero net intertemporal profit and markets clear every period at prices $\{p_t\}$. It can also be shown that if no firm enters in period ô and exits in period ô', then $p_t \leq \lim_{y\downarrow 0}(\partial R_{ô,ô'}/\partial y_t)$ where the partial derivative is evaluated at output vectors with zero output in all periods and output y in period t. These together can be used to show that no firm can earn strictly positive intertemporal profit at prices $\{p_t\}$. Thus, the social planner's solution can be implemented as an industry equilibrium with prices $\{p_t\}$.

Next, consider any industry equilibrium. If $n(ô,ô') > 0$ and $(y_{ô},...,y_{ô'})$ is the total output produced by firms in $S(ô,ô')$, then the total social cost of producing $(y_{ô},...,y_{ô'})$ is minimized by $(q_t(i),x_t(i))$, $t = ô,..,ô'$, $i \in S(ô,ô')$. If this is not true, then there is some other way of producing this total output vector where the total intertemporal cost across all firms would be lower; however, in the alternative solution, at prices $\{p_t\}$, the total intertemporal revenue of all firms together in the industry would still be the same, viz. $\sum_{t=ô,..ô'} ä^{t-ô} p_t y_t$. This implies at least one firm can earn strictly positive intertemporal profit at prices $\{p_t\}$, contradicting the definition of industry equilibrium. Using the first order conditions of profit maximization it can be shown that p_t is equal to the optimal value of the dual variable in the social cost minimization problem (P2) corresponding to $(y_{ô},...,y_{ô'})$. Now, considering (P2) which is a convex problem, it can be checked that the first order (equality and inequality) conditions of maximization are indeed satisfied by the industry equilibrium allocation. Thus, every industry equilibrium path maximizes social welfare. As the social planner's problem has a unique solution in terms of total industry output, industry equilibrium is unique in prices.//

For firm $i \in S(\underline{ô},ô)$, $1 \leq \underline{ô} \leq ô \leq T$, the equilibrium path of output, investment and stock of knowledge $\{(q_t(i), x_t(i), z_t(i)),\ t = \underline{ô},..,ô\}$ satisfies the following first order conditions:

(2) $$p_t = C_q(q_t(i), z_t(i)),\ \text{if } q_t(i) > 0,$$

(3) $$ã'(x_t(i)) + \sum_{ô=t+1}^{ô} ä^{ô-t} C_z(q_{ô}(i), z_{ô}(i)) = 0,\ \text{if } x_t(i) > 0$$

(2) simply says that a firm will equate price to its current marginal cost every period it produces positive output; in each period, the marginal cost curve of the firm is its individual supply curve.

As the stock of knowledge increases over time, the supply curve shifts to the right. It is easy to see that a firm will produce $q_t(i) > 0$ if and only if $p_t > C_q(0,z_t(i))$. (3) says that a firm will undertake investment up to the point at which the current marginal cost of investment is equal to the discounted sum of future marginal returns on such investment i.e., the discounted sum of marginal decrease in future costs of production at the planned output levels. Obviously, $x_ô(i) = 0$ if ô = ô i.e., no firm invests in its last period in the market.

Given a price vector **p**, the profit maximization problem (1) for firm i need not have a unique solution as we have not assumed C(q,z) to be convex. However, as $C_q(.,z)$ is strictly increasing for any z, there is a unique optimal output path associated with any vector of investment decisions over time. All firms start with the same supply curve in their period of entry, i.e. the marginal cost curve corresponding to z = 0. Over time firms may follow different investment paths and therefore exhibit heterogeneity in supply decisions. Heterogeneity in output decisions may occur in our model even if C and ã are strictly convex in both arguments (and thus, there is a unique solution to the maximization problem in (1)). Identical firms may follow different investment-output paths because their planned lengths of stay in the industry differ.

Observe that, a firm might choose to stay in the industry and incur the fixed cost rather than exit, if it expects to make sufficient profit in the future. Exiting would make the firm lose its accumulated knowledge.

3. Properties of Industry Equilibrium Path.

In this section, we establish a set of interesting properties which characterize the equilibrium path of the industry. The first result characterizes the equilibrium prices. We show that on the equilibrium path, prices are (weakly) decreasing over time. As firms accumulate investment in cost reduction, their marginal costs of production fall and this is passed on to consumers in the form of lower prices. Further, the prices always lie below $p_m(0)$, the minimum average cost for a new entrant. Note that $p_m(0)$ is the unique equilibrium price in a static model with free entry (the so-called "long run" equilibrium in textbooks). Let $q_m(0)$ denote the (unique) minimum efficient scale for a new entrant, i.e. $p_m(0) = [C(q_m(0),0)/q_m(0)]$.

Proposition 2. *The industry equilibrium price path* $\mathbf{p}=(p_1,...p_T)$ *satisfies* $p_t \geq p_{t+1}$ *for all* $t = 1,2,..,T-1$. *Further,* $p_1 \leq p_m(0)$ *and* $p_t < p_m(0)$ *for all* $t > 1$. *Also,* $p_t \geq h$ *for* $t = 1,2,...T$.

Proof. To see that equilibrium prices are non-increasing, suppose to the contrary that $p_t < p_{t+1}$ for some $t < T$. First, suppose that no exit occurs at the end of period t. Then $S_{t+1} \geq S_t$ and by assumption (A3), $z_t(i) \leq z_{t+1}(i)$ for each active in periods t and t+1 firm i, so that $C_q(q,z_t(i)) \geq C_q(q,z_{t+1}(i))$ for all $q \geq 0$. As $p_t < p_{t+1}$, it follows from (2) that $q_t(i) \leq q_{t+1}(i)$ for all these firms, and thus $Q_t \leq Q_{t+1}$ which violates condition (a) of industry equilibrium (as D(p) is strictly decreasing). Next, suppose that exit occurs at the end of period t. Let i be a firm exiting at the end of period t. Then $x_t(i) = 0$ and $[p_t q_t(i) - C(q_t(i),z_t(i))] \geq 0$. This implies $q_t(i) > 0$. If firm i stays on till period (t+1) and makes no further investment, then as $p_{t+1} > p_t$, $[p_{t+1}q_t(i)-C(q_t(i),z_t(i)] > [p_t q_t(i)-C(q_t(i),z_t(i))] \geq 0$; hence, it can make a strictly positive profit in period (t+1), so condition (c) of industry equilibrium is violated, a contradiction.

Next, if $p_t > p_m(0)$ for some t, then $Đ(\mathbf{p},t,t) > 0$ which contradicts condition (c) of industry equilibrium. Further, if $p_t = p_m(0)$ for any $t > 1$, then $p_{ô} = p_m(0)$ for all ô = 1,.,t-1. Then assumption (A6) implies that $Đ(\mathbf{p},1,2) > 0$ since this firm makes positive investment. Finally, if $p_t < h$ for some t, then $p_{ô} < h$ for all ô = t+1,.,T; as the average cost of production always exceeds h, no firm would stay in the market beyond period (t-1) which would violate market clearing at such prices. The proof is complete. //

Our next result states that the process of cost reduction through investment by initial entrants in an industry is such that no entry occurs after period 1. The equilibrium path could allow room for entry only if industry output expands over time which would require strict decrease in prices. The latter, however, implies that late entrants could always make greater intertemporal profit by entering earlier. As the model is fully deterministic, firms have perfect foresight and all firms *ex ante* have equal chance of entering the industry in period 1, all possibilities for profitable late entry are eliminated. Expansion of industry output and price reduction over time takes place only as a consequence of outward shifts in the supply curves of incumbent firms. In our model, the lack of late entry co-exists with mature incumbent firms earning strictly positive current profit (net of fixed and investment costs) - often supposed by regulatory agencies to be indicative of the existence of anti-competitive barriers to entry.

Proposition 3. *In the industry equilibrium path, no firm enters after period 1 that is, $n(\underline{\hat{o}},\hat{o}) = 0$ for $\underline{\hat{o}} > 1$.*

Proof. Suppose not. Let $\hat{o} > 1$ be the first period in which entry occurs after period 1. In particular, $n(\hat{o},\hat{o} + k) > 0$ for some $k \geq 0$.

First, we show that $p_t = p_{t-1}$ for all $t \leq \hat{o}$. If this were not true, then by Proposition 2, $p_t < p_{t-1}$ for some $t \leq \hat{o}$, which implies that $p_1 > p_{\hat{o}}$. Suppose a late entrant stays for $k > 1$ periods (it cannot break even by staying for only one period as $p_{\hat{o}} < p_m(0)$). Further, as $p_{\hat{o}+j} \leq p_{1+j}$ for $j = 1,..k$, if the late entrant had entered in period 1 and exited in period k+1, it could have made strictly positive profits, i.e. $Đ(\mathbf{p},1,k+1) > Đ(\mathbf{p},\hat{o},\hat{o}+k) = 0$, a contradiction. Thus, if late entry occurs in any period $\hat{o} > 1$, prices must be constant till period $\hat{o}$. Let us denote this price by p^*. Obviously, $p^* < p_m(0)$ by Proposition 2.

Next, we claim that exit cannot occur at the end of period $(\hat{o}$-1). If a firm exits at the end of period $(\hat{o}$-1), then it must have stayed in the market for at least two periods in order to break-even (as $p^* < p_m(0)$). By assumption (A6), the exiting firm has invested in its period of entry which means that, in order to break even, p^* must be strictly greater than its minimum average cost in period $(\hat{o}$-1). But then such a firm could always make strictly positive profit by staying on till period $\hat{o}$ as $p_{\hat{o}} = p^*$, a contradiction. Therefore, $S_{\hat{o}} > S_{\hat{o}-1}$. Observe also that for strictly positive output to be produced in period 1, $p^* > C_q(0,0) \geq C_q(0,z)$ for all $z \geq 0$; thus all active firms in periods $(\hat{o}$-1) and $\hat{o}$ produce strictly positive output. Given assumptions (A2) and (A3) and $p_{\hat{o}} = p_{\hat{o}-1} = p^*$, total output in period $\hat{o}$ is strictly greater than in period $(\hat{o}$-1). For the market to clear in both periods $(\hat{o}$-1) and $\hat{o}$, it should be the case that $p_{\hat{o}} < p_{\hat{o}-1}$, a contradiction. The proof is complete. //

As no new firms enter the market after period 1, the set of active firms in any period $t > 1$ is a subset of the set of active firms in period (t-1), the difference between the two sets being the set of firms which exit at the end of period (t-1) (i.e., the *shake-out*). However, as prices are non-increasing over time, the total output is non-decreasing over time. This is consistent with shake-out, as firms which stay in the industry undertake greater investment than exiting firms and as a result, their marginal cost curves (their individual supply curves) shift out to the right. It stands to reason, therefore, that exiting firms at their point of exit are typically "smaller in size" and have lower stock of knowledge compared to staying firms. We establish this result under certain restrictions on the cost function.

Let $p_m(z)$ be the minimum average cost for a firm with accumulated stock $z \geq 0$ of knowledge and $q_m(z)$ the corresponding (unique) minimum efficient scale:

$$p_m(z) = \min \{[C(q,z)/q]: q \geq 0\} = [C(q_m(z),z)/q_m(z)]$$

Proposition 4. *Suppose that at least one of the following holds:*

(a) $p_m(z)$ is strictly decreasing in z on R_+

(b) C and ã are convex functions and at least one of them is strictly convex

Then, if firm i exits at the end of period $\hat{o} < T$, and firm j stays on till a later period, that is, $i \in S(1,\hat{o})$ and $j \in S(1,\hat{o}')$ where $\hat{o} < \hat{o}'$, then $z_{\hat{o}}(i) \leq z_{\hat{o}}(j)$ and $q_{\hat{o}}(i) \leq q_{\hat{o}}(j)$ i.e., the size of a firm prior to exit increases with its time of exit.

The proof of Proposition 4 is contained in the appendix. Observe that part (a) of the hypothesis of Proposition 4 is satisfied if for all $q > 0$, the total cost of producing q is always strictly decreasing in the level of accumulated investment. Part (b) is a restriction which would ensure that for any given time horizon, there is a unique output-investment path which solves a firm's intertemporal profit maximization problem. The hypothesis of proposition 4 requires that at least one of parts (a) or (b) be satisfied; it does not cover environments where costs are non-convex and, in addition, the marginal effect of investment on production cost becomes zero after a certain point.

4. On the Possibility of Shake-Out

The phenomenon of shake-out in this model is crucially linked to the way in which both marginal as well fixed costs of production decrease with investment. In addition, for shake-out to occur, the demand curve has to be sufficiently inelastic so that the industry output expands in a relatively conservative fashion. A competitive equilibrium with exit is socially efficient because most of the investment is then concentrated in some firms whose marginal costs decrease rather sharply. These firms then undertake future production, while others exit the industry. Exiting firms remain active in the industry in earlier periods only because their marginal cost curve is steep when the stock of knowledge is small which makes it too costly for a social planner to concentrate all production in a few firms.

Consider first an industry where investment only reduces the fixed cost of production; the marginal cost curve does not depend on the stock of knowledge:

$$C(q,z) = C_v(q) + F(z)$$

where $C_v'(q) > 0$, $C_v''(q) > 0$, $C_v(0) = 0$, $F(z) > 0$ for all $z \geq 0$, $F'(z) < 0$. Then the supply curves of individual firms remain unchanged over time. If $p_t > p_{t+1}$ for some t, then all active firms produce lower output in period (t+1) than in period t; as late entry is ruled out on the equilibrium path (Proposition 3), the total output in period (t+1) is smaller than in period t, which is inconsistent with market clearing. Thus, prices are constant over time and so is the output of each firm. This implies that there is *no exit* on the equilibrium path; if firms did exit, the total output would fall which would again violate market clearing (given constant prices over time). There is no heterogeneity in the industry. The market price lies strictly below $p_m(0)$ and firm output is initially below minimum efficient scale; firms earn negative *gross* profit in addition to incurring investment cost. As the stock of knowledge increases and F decreases, the minimum average cost falls below the constant price and the minimum efficient scale falls below the constant output level; firms earn positive profits that compensate negative profits and investment cost incurred in earlier periods.

The absence of shake-out in the industry considered above depends critically on the fact that the marginal cost curve does not shift outwards as investment occurs, and thus the minimum efficient scale of firms decreases over time. If, on the contrary, the stock of knowledge increases the supply curve of firms, then exit is likely to occur, especially (but not necessarily) if the optimal scale of firms expands over time. In fact, we can give a fairly simple condition on the demand and cost functions which is *sufficient* to ensure that shake-out occurs on the equilibrium path. Recall that $p_m(z)$ and $q_m(z)$ are, respectively, the minimum average cost and minimum efficient scale when the stock of knowledge is z.

Proposition 5: *If*

(4) $$[D(p_m(z))/q_m(z)] < [D(p_m(0))/q_m(0)] \text{ for all } z > 0,$$

then exit must occur in some period ô < T.

Proof: To see this, observe that if no exit occurs prior to period T, then the measure of firms active in the industry is constant over time. Let us denote this measure by n. All firms produce identical output q_1 in period 1. As $p_1 \leq p_m(0)$, $q_1 \leq q_m(0)$ so that

(5) $$n = D(p_1)/q_1 \geq D(p_m(0))/q_m(0)$$

However, $p_T \geq p_m(z_T(i))$ for all active firms i. Note that $z_T = \inf\{z_T(i): i \in S(1,T)\} > 0$; if zT = 0, then pT ≥ pM(zT(i)) for all i which would imply that pT ≥ pM(0) which in turn would contradict Proposition 1. Therefore, $p_T \geq p_m(z_T)$. This implies that $q_T(i) \geq q_m(z_T)$ for all active firms i. Thus,

(6) $$n \leq D(p_T)/q_m(z_T) \leq D(p_m(z_T))/q_m(z_T)$$

(5) and (6) together contradict (4) //.

The hypothesis of Proposition 5 requires the function $[D(p_m(z))/q_m(z)]$ to attain its maximum value at a unique point viz., z = 0. A sufficient condition for this to hold is that $[D(p_m(z))/q_m(z)]$ be strictly decreasing in z. One can think of $[q_m(z)/D(p_m(z))]$ as a measure of the size of a competitive firm relative to the industry size, when the firm operates at its minimum efficient scale. If the minimum efficient scale $q_m(z)$ increases rapidly with z, relative to the increase in industry demand at price equal to minimum average cost, then there is less and less room for firms; though the industry size expands, the optimal firm size expands faster.

Let $N(z) = [D(p_m(z))/q_m(z)]$. Further, suppose that D(p), $p_m(z)$ and $q_m(z)$ are differentiable functions and that $p_m'(z) < 0$. Then, one can check that

(7) $$-[N'(z)/N(z)] = [ç_D(p_m(z))][p_m'(z)/p_m(z)] + [q_m'(z)/q_m(z)]$$

where $ç_D$ is the elasticity of demand. Therefore, $N'(z) < 0$ (that is, $[D(p_m(z))/q_m(z)]$ is strictly decreasing in z) if and only if, $q_m'(z) > 0$ and, further,

$$ç_D(p_m(z)) < |d(\log q_m(z))/d(\log p_m(z))|$$

Thus, $D(p_m(z))/q_m(z)$ is decreasing in z if the elasticity of demand is low and for any marginal increase in the stock of knowledge, the rate of expansion of the minimum efficient scale is large enough relative to the rate of decline in the minimum average cost.

We present below a numerical example of an industry where investment reduces the marginal cost curve sharply and shake out occurs *in every period*. This example also illustrates the features of the equilibrium path as characterized in the previous section.

Example 1[11]: T = 3, D(p) = 100 - p, $C(q,z) = e^{q-z} + 10$, ã(x) = $0.5x^2$, ä = 0.5.

Equilibrium prices:

$p_1 = p_m(0) = 8.64403$, $p_2 = 5.16861$, $p_3 = 2.75447$

Measure of active firms in period 1: 42.3558

Measure of active firms in period 2: 19.6378

Measure of active firms in period 3: 17.1709

Output-investment path followed by firms:

$q_1(i) = q_m(0) = 2.15687$ for all $i \in S_1$.

For $i \in S(1,1)$, $x_1(i) = 0$.

For $i \in S(1,2)$, $x_1(i) = 2.5843$, $q_2(i) = 4.22691$, $x_2(i) = 0$.

For $i \in S(1,3)$, $x_1(i) = 3.27292$, $q_2(i) = 4.91553$, $x_2(i) = 1.37724$, $q_3(i) = 5.66338$, $x_3(i) = 0$.

Firms which stay for three periods invest in both period 1 and 2. Shake-out of firms occurs at the end of both period 1 and period 2. The biggest shake-out occurs at the end of period 1. All firms earn exactly zero profit in period 1, gross of investment cost. In period 2, the profit of firms which exit at the end of that period is strictly positive and just compensates (all appropriately discounted) the investment cost made in period 1. As for firms which stay for all three periods, they earn strictly positive gross profit in both periods 2 and 3 (even while exit occurs!), prices are strictly greater than their minimum average cost in these periods and their output is greater than the minimum efficient scale; net of investment cost, they earn negative profit in period 2 which

[11]In examples 1, 2 and 3, all magnitudes are numerical approximations obtained by solving nonlinear equations defining the equilibrium conditions, using *Mathematica*.

is compensated by the profit in period 3. Exiting firms make much less investment than staying firms (if at all) and their marginal cost curve is far to the left of the marginal cost curve for mature staying firms (smaller efficient scale).

5. Numerical Examples

In this section, we present some numerical examples which highlight certain interesting features that may be observed in industry equilibrium.

Example 2: $T = 3$, $D(p) = [10/p]$, $C(q,z) = e^{q-kz} + 1$, ã(x) = $0.5x^2$, ä = 0.9.

Here, $k \geq 0$ is a parameter that captures the "innovational opportunities" in the industry. As k increases, the marginal effectiveness of current investment in reducing total as well as marginal costs of production increases. For this industry, there is no exit on the equilibrium path. All firms produce identical output q_t and make identical investment x_t in each period. Let n denote the measure of firms active in the industry. Observe that demand is unit elastic everywhere.

At $k = 1$, the equilibrium is given by:

$p_1 = 3.51727$, $p_2 = 1.56248$, $p_3 = 1.208191$, $q_1 = 1.25769$, $q_2 = 2.83115$, $q_3 = 3.66136$, $x_1 = 2.38487$, $x_2 = 1.08737$, $n = 2.26059$.

At $k = 1.5$, the equilibrium is given by:

$p_1 = 3.42012$, $p_2 = 1.10245$, $p_3 = 0.81471$, $q_1 = 1.22968$, $q_2 = 3.81481$, $q_3 = 5.16214$, $x_1 = 2.47818$, $x_2 = 1.09986$, $n = 2.37776$.

The interesting feature that is illustrated by this example is that *an increase in innovational opportunities in the industry (the effectiveness of investment in cost reduction) may lead to an increase in the number of firms active in the industry*[12]. This contrasts with both, the general

[12]As we have a continuum of firms and each firm is of measure zero, strictly speaking we cannot apply the usual indices of "market concentration". However, if an industry has a finite number of symmetric firms all producing identical output, then any decrease in the number of firms leads to an increase in industry concentration (using any reasonable measure of

"Schumpeterian" notion that higher innovational opportunities are associated with more "concentrated" industries and, in particular, the theoretical prediction in strategic static R&D models (see, for example, Dasgupta and Stiglitz 1980; Dasgupta 1986).

Another prediction of the latter class of models is that higher *research intensity* (expenditure on R&D as a ratio of total revenue) is associated with smaller number of firms. In the context of our dynamic model we may define "research intensity" as:

$$R = [(x_1 + \text{ä}x_2)/(p_1q_1 + \text{ä}p_2q_2 + \text{ä}^2p_3q_3)]$$

For k = 1, R = 0.280572, while for k = 1.5, R = 0.304287. As noted above, an increase in k also leads to an increase in the active firm numbers. Therefore, this example indicates that, contrary to the received doctrine, *higher research intensity may be associated with larger active firm numbers*.

Example 3: T = 3, $D(p) = [100/p^b]$, $C(q,z) = e^{q-z} + 10$, $\text{ã}(x) = 0.5x^2$, ä = 0.5

If the elasticity of demand is low enough, i.e. for values of $b \leq 0.5$ then it can be checked that $[D(p_m(z))/q_m(z)]$ is strictly decreasing in z. For such low values of b, demand elasticity is lower than the ratio of the rate of expansion of the firm's minimum efficient scale over the rate of decline in its minimum average cost (see (7)). From our earlier discussion, it follows that exit occurs in at least one period. We describe the market equilibrium corresponding to four different values of b viz., 0.5, 0.3, 0.2 and 0.1. When b = 0.5 or 0.3, the equilibrium is one where exit occurs only at the end of period 1. This indicates, among other things, that $[D(p_m(z))/q_m(z)]$ being strictly decreasing in z is not sufficient for exit to occur in every period or even in multiple periods. However, if demand elasticity is extremely low - in this example, for b = 0.2 or 0.1, exit occurs at the end of both periods 1 and 2.

---

concentration). Extending this to the case of continuum of firms and noting that in this example, all firms produce identical output, we can interpret an increase in the active firm numbers as a "decrease in concentration". In this spirit, our example shows that *an increase in innovational opportunities in the industry may lead to a decrease in concentration*.

Let n_t denote the measure of firms active in the market in period t. The equilibrium configurations corresponding to b = 0.5 and 0.3 are described below:

b = 0.5: p_1 = 8.64403, p_2 = 4.90811, p_3 = 3.19; For i $\in$ S(1,3), q_1(i) = 2.15687, q_2(i) = 4.84244, q_3(i) = 6.00657, x_1(i) = 3.25155, x_2(i) = 1.595; For j $\in$ S(1,1), q_1(j) = 2.15687; n_1 = 15.7695, n_2 = n_3 = 9.32134; *Measure of exiting firms = 6.44816*.

b = 0.3: p_1 = 8.64403, p_2 = 5.08082, p_3 = 2.90487; For i $\in$ S(1,3), q_1(i) = 2.15687, q_2(i) = 4.8921, q_3(i) = 5.78546, x_1(i) = 3.26663, x_2(i) = 1.45244; For j $\in$ S(1,1), q_1(j) = 2.15687; n_1 = 24.2751, n_2 = n_3 = 12.5523; *Measure of exiting firms = 11.7228*.

These two equilibrium configurations illustrate the fact that *as demand elasticity decreases, the rate of exit increases*. One should interpret this in the context of equation (7), which says that the rate of decrease of [D(p_m(z))/q_m(z)] is higher if the elasticity of demand is lower.

Now, let us consider the equilibrium configurations corresponding to b = 0.2 and 0.1. Here, exit occurs in both periods 1 and 2. Therefore, the prices are determined exclusively by the zero intertemporal profit conditions for firms which stay in the market for one, two and three periods; these also determines the optimal output and investment decisions of firms. As the cost functions C and ã as well as the values of T and ä are exactly the same as in Example 1 (where too, exit occurs in both periods 1 and 2), the equilibrium prices, outputs and investments of firms are exactly the same as in Example 1. The only difference is in the measure of active firms in the market:

b = 0.2: n_1 = 30.1185, n_2 = 14.6845, n_3 = 14.4184; *measure of firms exiting at the end of period 1 = 15.434, measure of firms exiting at the end of period 2 = 0.2661*.

b = 0.1: n_1 = 37.3684, n_2 = 17.47487, n_3 = 15.9559; *measure of firms exiting at the end of period 1 = 19.89353, measure of firms exiting at the end of period 2 = 1.51897*.

Observe that the decline in demand elasticity increases the rate of exit at the end of *both* periods 1 and 2. Lastly, comparison of rates of exit at the end of period 1 across all four values of demand elasticity shows that a decrease in demand elasticity always increases the rate of exit at the end of period 1.

6. Conclusion.

We have characterized the dynamic equilibrium path of a competitive industry with free entry and exit, where atomistic firms undertake investment over time in order to reduce their future production costs by increasing their firm-specific stock of knowledge. Cost reduction is deterministic and there are no inter-firm spill-overs. The instantaneous marginal cost function is upward sloping and the average cost curve is U-shaped; investment reduces the marginal as well as average cost curves over time. However, the associated dynamic scale economies are eventually bounded so that a competitive industry equilibrium exists and is socially optimal. Equilibrium prices are (weakly) decreasing over time. Firms invest in cost reduction and earn negative net profit when they are young. In later periods, they face prices above their minimum average cost, produce beyond their minimum efficient scale and earn strictly positive net profit. The industry equilibrium path allows for no new entry after the initial time period. Though all firms are *ex ante* identical with perfect foresight and the model is fully deterministic, some firms may exit before others (*shake-out*). Exit may occur even while incumbent firms earn strictly positive current profits. Exiting firms have relatively "small size" compared to incumbents; as the industry matures, firm numbers decrease and the average size of incumbent firms increase. Heterogeneity in behaviour and size of firms emerges endogenously through differences in their length of stay in the industry and in the absence of any random shocks. These results offer an alternative explanation for a number of empirical regularities.

Appendix

<u>Proof of Proposition 4</u>:

It is sufficient to show that $z_{ô}(i) \le z_{ô}(j)$. Suppose, to the contrary, that $z_{ô}(i) > z_{ô}(j)$. From definition of equilibrium,

(8) $$Ð(\mathbf{p},1,ô) = \sum_{t=1,..ô} ä^{t-1}[p_t q_t(i) - C(q_t(i),z_t(i)) - ã(x_t(i))] = 0$$

(9) $$\sum_{t=1,..ô} ä^{t-1}[p_t q_t(j) - C(q_t(j),z_t(j)) - ã(x_t(j))] \le 0$$

There are two possibilities: (i) $x_t(j) = 0$, for all $t = ô,......,ô'$, and (ii) $x_t(j) > 0$ for some $t \ge ô$.

First, consider case (i). Here $z_t(j) = z_{ô}(j)$ for $t = ô,..ô'$. Note that as firm i exits in period ô even though $z_{ô}(i) > z_{ô}(j)$, it must be true that

(10) $$p_{ô+1} \le p_m(z_{ô}(i)) \le p_m(z_{ô}(j))$$

for otherwise firm i could make strictly positive profit by staying one period more.

On the other hand, as firm j stays on till period $ô'$,

(11) $$p_{ô'} \ge p_m(z_{ô}(j))$$

as $z_{ô'}(j) = z_{ô}(j)$. Since prices are decreasing over time, (10) and (11) imply

(12) $p_t = p_m(z_{ô}(j)) = p_m(z_{ô}(i))$ for $t = ô+1,....,ô'$

Suppose condition (a) of the proposition holds so that $p_m(z)$ is strictly decreasing in z. Then, $p_m(z_{ô}(i)) < p_m(z_{ô}(j)) = p_{ô+1}$, which contradicts (12). Now, consider the situation where condition (b) of the proposition holds so that C and ã are convex and at least one of them is strictly convex. Then, there is a unique solution to the intertemporal profit maximization problem for a firm which enters in period 1 and exists in $ô'$. Observe from (11) that firm j earns exactly zero net profit from period $ô+1$ till $ô'$. It is easy to check that the price-investment path (q_t,x_t), $t=1,...ô'$ where $q_t = q_t(i)$, $t= 1,..ô$, $x_t = x_t(i)$, $t=1,...ô-1$, $x_t = x_t(j)$, $t = ô,...ô'$, $q_t = q_t(j)$ for $t = ô+1,....ô'$ yields non-negative intertemporal profit and hence, by definition of equilibrium, must yield exactly zero intertemporal profit. Thus (q_t,x_t), $t=1,...ô'$ is another profit maximizing solution for a firm which stays from period 1 till period $ô'$ which is distinct from $(q_t(j),x_t(j))$, $t=1,...ô'$, a contradiction.

Next, consider case (ii). Let $t^* = \min\{t \ge ô: x_t(j)>0\}$. Choose å such that $0 < å < \min\{x_{t*}(j), z_{ô}(i)-z_{ô}(j)\}$. Consider the output-investment path $(q_t\text{^},x_t\text{^})$, $t = 1,....,ô'$,: $q_t\text{^} = q_t(i)$, $t = 1,..., ô$; $x_t\text{^} = x_t(i)$,

$t = 1,...,ô\text{-}1$; $x_t{}^\wedge = x_t(j)$, $t \neq t^*$ and $ô \leq t \leq ô'$; $x_{t^*}{}^\wedge = x_{t^*}(j) - å$; $q_t{}^\wedge = q_t(j)$, $t = ô+1,...,ô'$. Check that, under our supposition and by construction, $z_t{}^\wedge \geq z_t(j)$ for all $t =1,...,ô'$. Then,

$Ð(\mathbf{p},1,ô') \geq \sum_{t=1,..ô'} ä^{t-1}[p_t q_t{}^\wedge - C(q_t{}^\wedge,z_t{}^\wedge) - ã(x_t{}^\wedge)]$

$= \sum_{t=1,..ô-1} ä^{t-1}\{p_t q_t(i) - C(q_t(i),z_t(i)) - ã(x_t(i))\}$

$+ ä^{ô-1}\{p_ô q_ô(i) - C(q_ô(i),z_ô(i)) - ã(x_ô{}^\wedge)\} + [\sum_{t=ô+1,..ô'} ä^{t-1}\{p_t q_t(j) - C(q_t(j),z_t{}^\wedge) - ã(x_t{}^\wedge)\}]$

$= Ð(\mathbf{p},1,ô) - ä^{ô-1}ã(x_ô{}^\wedge) + [\sum_{t=ô+1,..ô'} ä^{t-1}\{p_t q_t(j) - C(q_t(j),z_t{}^\wedge) - ã(x_t{}^\wedge)\}]$

$> -ä^{ô-1}ã(x_ô(j)) + [\sum_{t=ô+1,..ô'} ä^{t-1}\{p_t q_t(j) - C(q_t(j),z_t(j)) - ã(x_t(j))\}]$

as $Ð(\mathbf{p},1,ô) = 0$, $z_t{}^\wedge \geq z_t(j)$ for all $t = ô+1,......,ô'$, $x_t{}^\wedge = x_t(j)$ for $t \neq t^*$, $ô \leq t \leq ô'$, $x_{t^*}{}^\wedge < x_{t^*}(j)$ and ã(.) is strictly increasing. Lastly, we claim that

$-ä^{ô-1}ã(x_ô(j)) + [\sum_{t=ô+1,..ô'} ä^{t-1}\{p_t q_t(j) - C(q_t(j),z_t(j)) - ã(x_t(j))\}] \geq 0,$

which, in turn, implies that $Ð(\mathbf{p},1,ô') > 0$, contradicting the definition of equilibrium. To prove this last claim note that if

$$-ä^{ô-1}ã(x_ô(j)) + [\sum_{t=ô+1,..ô'} ä^{t-1}\{p_t q_t(j) - C(q_t(j),z_t(j)) - ã(x_t(j))\}] < 0$$

then firm j would earn zero intertemporal net profit only if

$$\sum_{t=1,..ô-1} ä^{t-1}[p_t q_t(j) - C(q_t(j),z_t(j)) - ã(x_t(j))] + ä^{ô-1}\{p_ô q_ô(j) - C(q_ô(j),z_ô(j))\} > 0$$

which would imply $Ð(\mathbf{p},1,ô) > 0$, a contradiction to the definition of equilibrium. The proof is now complete. //

References

Aumann, R.J. and M. Perles, "A Variational Problem Arising in Economics", Journal of Mathematical Analysis and Applications 29 (1965), 152-173.

Davis, S. and J. Haltiwanger, "Gross Job-Creation, Gross Job Destruction, and Employment Reallocation", Quarterly Journal of Economics 107 (1992), 819-863.

Dasgupta, P., "The Theory of Technological Competition" in J. Mathewson and J.E. Stiglitz, eds., Modern Approaches to Industrial Economics (London: Macmillan, 1986), 519-47.

Dasgupta, P. and J.E. Stiglitz, "Industrial Structure and the Nature of Innovative Activity", Economic Journal 90 (1980), 266-93.

Dixit, A., "Entry and Exit Decisions under Uncertainty", Journal of Political Economy 97 (1989), 620-638.

Dunne T., M. Roberts and L. Samuelson, "The Growth and Failure of U.S. Manufacturing Plants", Quarterly Journal of Economics 104 (1989), 671-98.

Dutton, J. and A. Thomas, "Treating Progress Functions as a Managerial Technology", Academy of Management Review 9 (1984), 235 - 247.

Ericson, R. and A. Pakes, "Markov Perfect Industry Dynamics: A Framework for Empirical Work", Review of Economic Studies 62 (1995), 53-82.

Flaherty, M.T., "Industry Structure and Cost Reducing Investment", Econometrica 48 (1980), 1187-1209.

Gort, M. and S. Klepper, "Time Paths in the Diffusion of Product Innovations", Economic Journal 92 (1982), 630 - 653.

Hopenhayn, H., "Industry Equilibrium Dynamics: A General Competitive Theory", mimeo, Stanford University, 1990.

Hopenhayn, H., "Entry, Exit and Firm Dynamics in the Long Run Equilibrium", Discussion Paper # 1180, Graduate School of Business, Stanford University, December 1991.

Hopenhayn, H., "Entry, exit and Firm Dynamics in Long Run Equilibrium", Econometrica 60 (1992a), 1127-50.

Hopenhayn, H., "Exit, Selection and the Value of Firms", Journal of Economic Dynamics and Control 16 (1992b), 621-53.

Hopenhayn, H., "The Shakeout", Working Paper # 33, Department of Economics, University of Pompeu Fabra, 1993.

Jovanovic, B., "Selection and the Evolution of Industry", Econometrica 50 (1982), 649-70.

Jovanovic, B. and G. MacDonald, "The Life Cycle of a Competitive Industry", Journal of Political Economy 102 (1994), 322-47.

Lach, S. and R. Rob, "R&D, Investment, and Industry Dynamics", Journal of Economics and Management Strategy 5 (1996), 217-249.

Lambson, V.E., "Industry Evolution with Sunk Costs and Uncertain Market Conditions", International Journal of Industrial Organization 9 (1991), 171 - 196.

Lippman, S.A. and R.P. Rummelt, "Uncertain Imitability: An Analysis of Interfirm Differences in Efficiency under Competition", Bell Journal of Economics 13 (1982), 418-438.

Lucas, R. and E.C. Prescott, "Investment under Uncertainty", Econometrica 39 (1971), 659-681.

Mas-Colell, A., The Theory of General Economic Equilibrium: A Differentiable Approach, (Cambridge, England: Cambridge University Press, 1985).

Mookherjee, D. and D. Ray, "Learning by Doing and Industrial Structure: An Overview" in B. Dutta, S. Gangopadhyaya, D. Mookherjee and D. Ray, eds., Theoretical Issues in Development Economics (New Delhi: Oxford University Press, 1992).

Pakes, A. and R. Ericson, "Empirical Implications of Alternative Models of Firm Dynamics", Working Paper, Yale University, 1990.

Pertakis, E. and S. Roy, "Cost Reducing Investment, Competition and Industry Dynamics", Working Paper #96-62, Departmento de Economia, Universidad Carlos III de Madrid, 1996.

Petrakis E., E. Rasmusen and S. Roy, "The Learning Curve in a Competitive Industry", Rand Journal of Economics 28 (1997), 248-268.

Spence, M., "Cost Reduction, Competition and Industry Performance", Econometrica 52 (1984), 101-121.