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# Evolutionary Model of the Bank Size Distribution

*Joachim Kaldasch*

**Abstract**
An evolutionary model of the bank size distribution is presented based on the exchange and expansion of deposit money. In agreement with empirical results the derived size distribution is lognormal with a power law tail. The key idea of the theory is to regard the creation of money as a slow process compared to exchange processes of deposit money. The exchange of deposits causes a preferential growth of banks with a fitness determined by the competitive advantage to attract permanent deposits. They generate the lognormal part of the size distribution. Sufficiently large banks, however, benefit from economies of scale leading to a Pareto tail. The model suggests that the liberalization of the banking system in the last decades is the origin of an increasing skewness of the bank size distribution.

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**Keywords** Evolutionary economics; bank size; money; competition; Gibrat's law

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## 1. Introduction

Extensive empirical and theoretical investigations have been carried out to understand the bank size distribution and its dynamics (Alhadeff and Alhadeff 1964, Rhoades and Yeats 1974, Yeats et al. 1975, Tschoegl 1983, Wilson and Williams 2000, Goddard et al. 2002, Goddard et al. 2004, Janicki and Prescott 2006, Benito 2008). The empirical bank size distribution has been shown to exhibit a highly skewed shape. It is explained similar to the firm size distribution by Gibrat's law of proportionate effects (Gibrat 1913). Gibrat's law suggests that bank growth is driven by unsystematic random factors such that the bank growth rates are uncorrelated. The multiplicative growth process generates a lognormal size distribution. The validity of Gibrat's law is of major interest for the size distribution (Tschoegl 1983, Enrique Benito 2008). Moreover, during the last decades the banking industry has experienced significant changes. Liberalization and deregulation processes and the technological progress declined the number of institutions in many countries, mainly by mergers and acquisitions (Berger et al. 1993, Berger et al. 1999). Despite temporal variations, however, the general shape of the bank size distribution can be always described by a lognormal distribution with a Pareto tail (Janicki and Prescott 2006, Benito 2008).

The aim of this paper is to derive the bank size distribution from the growth dynamics of banks, which is essentially determined by the exchange and expansion of deposit money. Taking advantage from a separation of the time scales the exchange of permanent deposits causes an evolutionary competition for customers. Additionally restructuring processes in the banking system induced by the exit and entrance of banks are taken into account. In particular by large banks benefit from economies of scale induced by the expansion of deposit money. Taking these effects into account the presented dynamic model establishes a bank size distribution that is in agreement with empirical studies. Consequences of the evolutionary model are discussed in the conclusion.

## 2. The Model

We want to characterize the size of a bank by its deposits. The size of the *i-th* bank at time step *t* is denoted $S_i(t)$ and corresponds to the total amount of deposit money of a bank. In a closed banking system the total amount of deposits $S(t)$ is determined by the sum over all banks:

$$S(t) = \sum_{i=1}^{n(t)} S_i(t) \tag{1}$$

where $n(t)>1$ is the total number of banks. In order to establish a continuous model, the size of a bank is scaled by a large number $M$:

$$s_i(t) = \frac{S_i(t)}{M} \tag{2}$$

such that $s_i(t)$ can be treated as a real positive number. The total amount of scaled deposit money becomes $s(t)=\Sigma s_i(t)$. We further demand that the parameter $M$ is sufficiently large that $s(t)<1$ is always fulfilled in the considered time interval.

In order to model the dynamics of bank deposits, four processes changing the amount of deposit money are taken into account:

1. Due to economic activity there is a flow of money between banks. This flow is characterized by an exchange rate $y_i$ . The size of the *i-th* bank is governed by the balance:

$$\frac{ds_i(t)}{dt} \sim y_i(t) = y_i^{\,in}(t) - y_i^{\,out}(t) \tag{3}$$

where $y_i^{in}\geq 0$ is the inflow rate and $y_i^{out}\geq 0$ the outflow rate of money. The balance relation suggests that a positive $y_i$ is related to an effective inflow and a negative to an effective outflow of money.

2. The core activity of a bank is granting loans. Banks lend out money from current deposits and generate new fiat money leaving a certain percentage as a minimum reserve $s_i^{\,0}$. The amount of money that can be created by a bank is therefore proportional to the amount of current deposits. The growth of deposit money of the *i-th* bank by the generation of fiat money has the form:

$$\frac{ds_i(t)}{dt} \sim \alpha_i(t) s_i(t)$$

(4)

The money growth rate $\alpha_i$ can be written as:

$$\alpha_i(t) = \alpha_i^{'}(t) - \alpha_i^{''}(t)$$

(5)

where $\alpha_i'$ is the generation rate of deposit money per unit time by granting loans and $\alpha_i'' s_i$ is the total backflow of money by repaying loans.

3. In order to increase the ability to lend out money, banks try to attract money for a longer time period by offering interests and advantages for their customers. The success of the migration of permanent deposits is taken into account by an additional growth term. The growth of deposit money of the *i-th* bank can be written as:

$$\frac{ds_i(t)}{dt} \sim \eta_i(t) s_i(t)$$

(6)

where the growth rate $\eta_i = \eta_i^{in} - \eta_i^{out}$ is the difference between inflow and outflow of permanent deposit money.

4. Restructuring processes by the entry and exit of banks change the number of banks *n(t)* and leads also to a shift of permanent deposits. In particular mergers and acquisitions increase the amount of permanent deposit money of the surviving bank. The growth of the *i-th* bank caused by restructuring of the banking system is taken into account by the growth term:

$$\frac{ds_i(t)}{dt} \sim \beta_i(t) s_i(t)$$

(7)

where $\beta_i$ is the corresponding growth rate.

Since these processes can be regarded as independent, the time evolution of deposit money of a bank can be approximated by:

$$\frac{ds_i(t)}{dt} = \alpha_i(t) s_i(t) + \beta_i(t) s_i(t) + \eta_i(t) s_i(t) + y_i(t)$$

(8)

The key idea of the model is to separate the processes with respect to their velocity. The exchange of money between banks due to economic activity is

regarded as the fastest process. The competition of banks for permanent deposits due to attraction and restructuring processes are considered to take place much slower. And the generation of new money is regarded to be the slowest process.Taking advantage from the relation of the growth rates:

$$\alpha_i << \beta_i, \eta_i$$

(9)

the growth rate $\alpha_i$ must be a small parameter of the order $\varepsilon$:

$$\alpha_i \sim \varepsilon$$

(10)

with $\varepsilon << 1$.

In this case the time evolution of deposit money can be studied by introducing two different time scales, a long and a short time scale. The short time scale $\tau$ is related to the long time scale by:

$$t = \varepsilon\tau$$

(11)

On the short time scale the growth of the total amount of deposit money can be neglected, because:

$$\frac{ds}{d\tau} = \varepsilon \sum_{i=1}^{n} \alpha_i s_i = \varepsilon \langle \alpha \rangle s \sim \varepsilon^2 \cong 0$$

(12)

where brackets indicate the average over deposits. For sufficiently short time periods the time evolution of the bank size is given by:

$$\frac{ds_i}{d\tau} \cong \varepsilon \left( \beta_i(\tau) s_i(\tau) + \eta_i(\tau) s_i(\tau) + y_i \right)$$

(13)

It implies that deposit money can be only exchanged between banks.[1]

*Economic Activity*

The short term flow of money due to economic activity is treated as a first approximation as random process. The exchange of deposit money between banks can be given an average exchange rate denoted *d*. The chance that money flows into respectively out of a bank is in a random process proportional to the size of a bank.

[1] This relation implies that the entrance and exit of banks is not accompanied with a considerable change of the total amount of deposit money.

Therefore the outflow of money can be approximated on the short time scale by[2]:

$$y_i^{out}(\tau) = ds_i(\tau) + \zeta_i^{out}(\tau)$$

(14)

while the inflow has the form:

$$y_i^{in}(\tau) = ds_i(\tau) + \zeta_i^{in}(\tau)$$

(15)

while $\zeta_i^{in}$ and $\zeta_i^{out}$ are fluctuating terms. We obtain for the size evolution of the *i-th* bank:

$$y_i(\tau) \cong \zeta_i(\tau)$$

(16)

where $\zeta_i = \zeta_i^{in} - \zeta_i^{out} << ds_i(\tau)$. The average exchange of money between banks due to economic activity cancels out and we obtain simply a fluctuating contribution as a result of this fast process.

*The Evolutionary Dynamics*

We want to continue by considering the evolution of deposits that vary much slower on the short time scale, called permanent deposits. These deposits represent money that is not used in fast processes of economic activity and deposits bonded to the bank by offering high interest rates. Neglecting the fluctuations due to economic activity we obtain from Eq.(13):

$$\frac{ds_i(\tau)}{d\tau} \cong \varepsilon f_i(\tau) s_i(\tau)$$

(17)

where

$$f_i(\tau) = \beta_i(\tau) + \eta_i(\tau)$$

(18)

[2] Note that money exchange implies the transformation of deposit money into cash money and vice versa. This exchange process has, however, no impact on the result of the model as long as the condition of a random money flow between banks due to economic activity is satisfied. Only in the case of a "bank run", where the outflow of deposits and hence the need for cash money is much larger than the inflow, this condition is violated. Hence a bank run in not included in the model.

The relocation of permanent deposits can be interpreted as the result of a deposit market. Banks represent the supply side and customers interested in a relocation of their deposits the demand side of this market. As derived in appendix A the parameter $\eta_i$ can be interpreted as a preference rate for the *i-th* bank.

In the evaluation of the bank size evolution, we have to take the condition that the total amount of deposit money is fixed on the short time scale into account. Eq.(12) can be satisfied by adding a free parameter to Eq.(17), such that:

$$\frac{ds_i(\tau)}{d\tau} \cong \varepsilon\left(f_i(\tau)+\xi\right)s_i(\tau)$$

(19)

Applying Eq.(12) yields for the free parameter:

$$\xi = -\langle f \rangle = -\sum_{i=1}^{n} f_i s_i$$

(20)

With this relationship the short term growth dynamics of banks is determined by a replicator equation:

$$\frac{ds_i(\tau)}{d\tau} = \varepsilon\left(f_i(\tau)-\langle f \rangle\right)s_i(\tau)$$

(21)

The presented model suggests therefore that the time evolution of banks is governed for short time periods by a preferential growth process. It expresses the competition between banks for permanent deposits. The replicator equation is determined by the parameter *f* which is usually termed as fitness. Hence, the rate $f_i$ characterizing the ability to attract permanent money can be viewed as a bank fitness. The replicator dynamics suggests that banks with a higher than the mean fitness attract a higher amount of deposit money and can increase their size in time at the expense of banks with a lower fitness. This can be done either by attracting deposits from competitors or by mergers and acquisitions.

For further use we introduce the fitness advantage of the *i-th* bank by:

$$\delta f_i(\tau) = f_i(\tau) - \langle f \rangle$$

(22)

and Eq.(21) becomes:

$$\frac{ds_i(\tau)}{d\tau} = \varepsilon\delta f_i(\tau)s_i(\tau)$$

(23)

*The Bank Size Distribution*

The size distribution of banks *P(s)* is determined by the probability to find the size of a bank $s_i$ in the interval *s* and *s+ds*. The size distribution is governed by the long term dynamics of banks summarized in Eq.(8).

The fast process of money exchange due to economic activity leads to an additive noise contribution to the evolution of a bank. A key process of bank growth is the relocation of permanent deposits, because it limits the ability to generate fiat money. This shift is determined by the ability to attract permanent deposits and merge with other banks. This ability is characterized in this model by the bank fitness advantage *δf*. In the competition process for permanent deposits banks have the tendency to increase their fitness. As a result the fitness variable alters its magnitude on the long time scale. The varying success of banks in the competition process can be taken into account by regarding the fitness advantage as a fluctuating variable.

Also the effective money growth rate *α(t)* is not constant but generally fluctuates on the long time scale under the impact of varying loan granting. The money growth rate can be written as the sum of a mean growth rate over all banks $<\alpha>$ and individual growth rate fluctuations $\delta\alpha(t)$:

$$\alpha(t) = \langle\alpha\rangle + \delta\alpha(t)$$

(24)

while we will neglect for brevity the index. The growth dynamics of a bank on the long time scale given by Eq.(8) can be rewritten as:

$$\frac{ds(t)}{dt} = \langle\alpha\rangle s(t) + \rho(t)s(t) + \zeta(t)$$

(25)

where the fluctuating variable $\rho(t)$ is characterized by money growth rate fluctuations and the result of the competition for permanent deposits:

$$\rho(t) = \delta\alpha(t) + \delta f(t)$$

(26)

Hence the time evolution of a bank is governed by a mean growth due to the expansion of deposit money and multiplicative respectively additive growth contributions. We can further regard multiplicative growth processes as dominant compared to random fluctuations and therefore neglect the additive contribution $\zeta(t)$ in the remainder of the model.

As a first approximation the fluctuating function $\rho(t)$ is treated as an independent, identical distributed (iid), random variable with mean value and time correlation:

$$\langle \rho(t) \rangle_t = 0$$

$$\langle \rho(t), \rho(t') \rangle_t = 2D\delta(t-t')$$

(27)

while $D$ is a white noise amplitude and brackets with index $t$ indicate the time average.

Taking advantage from Eq.(10) the growth process depends on the size of a bank. For small banks $s \leq \varepsilon$ the first term in Eq.(25) is of the order $\varepsilon^2$ and can be neglected. The growth dynamics becomes for small banks on the long time scale:

$$\frac{ds(t)}{dt} \cong \rho(t)s(t)$$

(28)

This relation describes a multiplicative stochastic growth process and represents Gibrat's law of proportionate effects. Note that it is in this model a direct consequence of the competition between banks for deposit money. With Eq.(27) the central limit theorem suggests that the size distribution for small banks ($s \leq \varepsilon$) is given by a lognormal probability distribution of the form (Sornette 2006):

$$P(s,t) = \frac{1}{\sqrt{2\pi t}\omega s} \exp\left( -\frac{\left(\ln(s/s') - ut\right)^2}{2\omega^2 t} \right)$$

(29)

where $u$ and $\omega$ are free parameters and $s/s$' is the bank size scaled by the size at $t=0$.

For large banks with $s > \varepsilon$, however, the growth process is given by Eq.(25). This relation can be interpreted as a generalized Langevin equation (Richmond and Solomon 2000, Kaldasch 2012). It yields after a sufficiently long time a size distribution of the form (see appendix B):

$$P(s) \sim \frac{1}{s^{\left(1+\frac{\langle \alpha \rangle}{D}\right)}}$$

(30)

The bank size distribution can be described for large banks by a power law (Pareto) distribution which can be related to Zipf's law (Saichev 2011). The evolutionary model suggests therefore that the size distribution of banks counted

in deposits is generally a lognormal distribution with a power law tail. For $<\alpha>\approx 0$, however, the Pareto tail disappears and the size distribution is dominated by the lognormal contribution.

## 3. Conclusion

The presented evolutionary model derives a bank size distribution that is in agreement with empirical results. It is derived from the fundamental processes of a financial system, the exchange and growth of money. In this theory the size distribution is on the one hand the result of the competition between banks for permanent deposit money. They can be increased either by attracting potential customers interested in a relocation of their deposits or by mergers and acquisitions. The competition for deposits can be described by a preferential growth process which is characterized by a bank fitness function. The varying success in the competition process is captured by a fluctuating bank fitness, which is the origin of Gibrat's law. It generates the lognormal contribution of the bank size distribution.

On the other hand banks growth by the creation of fiat money. This size dependent contribution to the growth process can be interpreted as a preferential attachment process (Newman 2005, Kaldasch 2012). Since large banks benefit more from the mean growth of money than smaller banks, economies of scale cause the power law tail in the bank size distribution.

The presented model suggests that the Pareto tail is governed by the Pareto exponent $1+<\alpha>/D$. An increasing exponent indicates a more evenly distributed Pareto tail (Newman 2005), which is the case when the mean money growth rate dominate deposit fluctuations $<\alpha>>D$.[3] The power law tail becomes, however, more uneven when the bank evolution is suffered from increased growth rate fluctuations, $<\alpha><D$. It indicates a more intense competition between banks for permanent deposits. In particular mergers lead to large fluctuations of permanent deposits and hence to an increasing skewness of the bank size distribution, accompanied with a concentration of banks in the Pareto tail. This effect has been found in empirical studies (Janicki and Prescott 2006, Benito 2008). The empirical fact that an increasing number of banks became "too big to fail" is therefore a direct consequence of the liberalization of the banking systems in many countries over the last decades.

[3] Note that Zipf's law is retained if the mean money growth rate and growth rate fluctuations of deposits are of the same order $<\alpha>\approx D$.

## Appendix A

*The Deposit Market*

We want to denote owners of deposits interested in a relocation of their deposits as potential customers. The total size of permanent deposit money associated with such a relocation is indicated by the variable *Z(t)*, respectively their relative size by $z(t)=Z(t)/M<1$. For simplicity we assume that potential customers occur randomly with the same rate independent of the bank. The amount of deposit money of potential customers generated per unit time in the *i-th* bank is then $g(t)s_i(t)$, where $g(t)$ is the mean generation rate of potential deposits per unit time. The amount of potential deposit money available for relocation can be obtained for the *i-th* bank from the balance relation:

$$\frac{dz_i(t)}{dt} = s_i(t)g(t) - \lambda_i^{out}(t)$$

(A1)

The amount of potential deposits increases with their generation rate $s_i g$ and decreases with the relocation to another bank with an outflow rate $\lambda_i^{out}$.

Under the condition that $z_i$ relaxes sufficiently fast to its stationary state we obtain from $dz_i/d\tau=0$:

$$\lambda_i^{out}(t) = s_i(t)g(t)$$

(A2)

This relation suggests that the outflow of deposits per unit time is proportional to the size of a bank. The total amount of potential deposits $z(t)$ can be determined by the balance:

$$\frac{dz(t)}{dt} = g(t) - \sum_{i=1}^{n} \lambda_i^{out}(t)$$

(A3)

The inflow rate of deposit money is the result of the competition in the deposit market. The market consists of a demand and a supply side. The demand side is determined by the total number of potential customers (respectively their deposits $z(t)$). The supply side on the other hand is given by the number of available banks *n-1*. When supply meets demand potential customers relocate their deposits. The inflow rate of the *i-th* bank $\lambda_i^{in}$ is zero if the total number of potential deposits $z(t)$ is zero. Under the condition that potential customers can

only shift their deposits to existing banks $\lambda_i^{in}$ is also zero if the size of a bank $s_i(t)$ is zero. Therefore the inflow rate can be regarded to be a function of potential and current deposits $\lambda_i^{in} = \lambda_i^{in}(z,s_i)$. As a first approximation the inflow rate is expanded in both variables and we obtain for the first nonzero contribution:

$$\lambda_i^{in}(t) \cong \eta'_i(t) z(t) s_i(t)$$

(A4)

The total inflow rate of permanent deposits is therefore proportional to the size of a bank and a rate $\eta'_i \geq 0$ which characterizes the chance that potential customers prefer the *i-th* bank. It expresses the impact of the marketing efforts to attract potential deposits.

The effective flow of permanent deposits becomes:

$$\lambda_i^{in}(t) - \lambda_i^{out}(t) = \left(\lambda_i^{in}(t) - \lambda_i^{out}(t)\right) s_i(t) = \left(\eta'_i(t) z(t) - g(t)\right) s_i(t)$$

(A5)

Under the condition that the number of banks remain constant in the considered time interval $(\beta_i = 0)$ the condition Eq.(12) suggests that on the short time scale:

$$\sum_{i=1}^{n} \lambda_i^{in}(\tau) - \lambda_i^{out}(\tau) = 0$$

(A6)

Applying Eq.(A5) this condition is satisfied when:

$$g = \langle \eta \rangle$$

(A7)

and

$$\eta_i(\tau) = \eta'_i(\tau) z(\tau)$$

(A8)

The parameter $\eta_i$ can be interpreted as the preference rate for the *i-th* bank. It is determined on the one hand by the chance that the bank is chosen for a shift of permanent deposits and on the other hand by the total amount of potential

deposits. If there is little interest in a relocation of bank deposits, marketing efforts are unsuitable since the preference rate for all banks will be small ($z(\tau)$ is small). Note that the average preference rate is in this model equal to the mean generation rate of potential deposits per unit time.

## Appendix B

Neglecting the additive noise contribution $\zeta(t)$, Eq.(25) represents a generalized Langevin equation (Richmond and Solomon 2000). It has the form:

$$\frac{ds}{dt} = F(s) + \rho G(s) \tag{B1}$$

with $F(s)=<\alpha>s$ and $G(s)=s$. It is a multiplicative stochastic relation that can be transformed into a relation with additive noise by introducing the functions $h(s)$ and $V(s)$ according to:

$$\frac{dh(s)}{dt} = \frac{1}{G(s)}\frac{ds}{dt} \tag{B2}$$

and

$$-\frac{dV(s)}{dh(s)} = \frac{F(s)}{G(s)} \tag{B3}$$

Inserting these relations in (B1) we obtain the usual Langevin equation:

$$\frac{dh}{dt} = -\frac{dV}{dh} + \rho \tag{B4}$$

For uncorrelated fluctuations as suggested by Eq.(27), this relation describes a random walk of $h$ in the potential $V$. For a sufficiently long time the probability distribution for $h$ becomes (Sornette 2006):

$$B(h)dh = \frac{1}{\mathrm{N'}}\exp\left(-\frac{V(H)}{D}\right)dH \tag{B5}$$

where $N'$ is a normalization constant. In terms of the original variable, we get:

$$P(s)ds = B(h)dh = \frac{1}{\mathrm{N'}}\exp\left(-\frac{1}{D}\int\frac{F(s')}{G(s')^2}ds'\right)\frac{ds}{G(s)}$$

(B6)

which yields with the corresponding functions for *G(s)* and *F(s)*:

$$P(s) \sim \frac{1}{s^{\left(1+\frac{\langle\alpha\rangle}{D}\right)}}$$

(B7)

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