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What to Do if a Dollar is Not a Dollar? The Impact of Inflation Risk on Production and Risk Management*

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What to Do if a Dollar is Not a Dollar? The Impact of Inflation Risk on Production and Risk Management

An entrepreneur faces two types of risk, one from income generation, one from income spending. His income from firm profits is risky due to output price fluctuations and other risks. As a consumer, he is also exposed to inflation risk since he maximizes expected utility of real income. This paper focuses on optimal production and risk management decisions of a risk-averse entrepreneur jointly facing tradable output price risk and untradable inflation risk. Inflation risk applies multiplicatively to the entrepreneur's entire nominal income. Relative risk aversion and the risks' joint distribution determine the effect of introducing a futures market on production. For dependent risks, this effect may be negative if relative risk aversion is above one. Relative risk aversion and the joint distribution also determine optimal risk management with futures contracts where speculation on a real risk premium and cross hedging may be conflicting objectives.

JEL classification: D81, G11, D21

Keywords: inflation risk, output price risk, production, futures market, relative risk aversion, risk management

INTRODUCTION

Consider a farmer growing corn. Since the price of corn is uncertain, he faces output price risk. High corn prices may seem to be excellent news for him because they imply a high dollar income. However, as a consumer, the farmer is not primarily interested in dollars per se but in consumption. Since the price of his consumption bundle is uncertain as well, he might face a scenario in which corn prices are up only slightly but consumer prices are up even more. In this case, the farmer is worse off compared to another scenario where corn prices and consumer prices remain unchanged. This demonstrates that the farmer is affected by both output price risk and inflation risk. When making the decisions on production and, if possible, on hedging output price risk in a commodity futures market, the farmer will anticipate the possibility of changes in the inflation rate and its co-movement

with corn prices. For example, if the farmer's dollar income and the inflation rate move in the same direction, his income is higher when consumption bundles are expensive and lower when they are cheap. Then, real income and, hence, consumption vary less than in the absence of inflation risk due to a natural hedge in his real income. It is intuitively clear that under these circumstances the optimal futures position is smaller. It is also clear that the introduction of a commodity futures market does not necessarily stimulate production.

The purpose of this paper is to examine the decision problem of an entrepreneur facing joint exposure to potentially tradable commodity price risk from his output and untradable inflation risk. Since inflation risk applies multiplicatively to the entire nominal income, including the random commodity price, the paper is based on a multiplicative combination of risks. Using the farmer's problem as an example, the paper focuses on three questions: What is the optimal production decision? How does inflation risk affect the reaction to the introduction of an unbiased futures market? What is the optimal futures position? These questions are analyzed in a single period framework where the farmer maximizes expected utility of real income which is linked to nominal income by the purchasing power index for his consumption bundle. The main objective of this paper is to answer the first two questions. The third question has recently been addressed in a closely related paper by Adam-Müller (2000).

The main assumption of the paper is that the farmer operates in an incomplete financial market in which inflation risk cannot be traded. For obvious reasons, there are no contracts on an individual farmer's consumption bundle. But even contracts on a general price index or an inflation rate are unavailable in most countries. Besides the undiversified farmer's problem, it could be argued in the light of the Fisher hypothesis that well-diversified investors are able to form stock portfolios that provide a hedge against inflation risk. However, a number of papers have shown that empirical evidence in support of this argument, if any, is quite weak, see Lee (1992), Ely and Robinson (1997) and Hess and Lee (1999). Thus, assuming untradable inflation risk seems to be reasonable for the problem considered here.

Holthausen (1979) and others analyze the classical production and risk management problem in which there is only tradable output price risk. Re-

cent papers analyze the effects of additional risks which are untradable. Briys et al. (1993) incorporate independent additive background risk. Cross hedging additive risks is discussed by Broll et al. (1995), Lence (1995), Broll et al. (1999), Broll and Wong (1999) and others. Moschini and Lapan (1995) and Viaene and Zilcha (1998) analyze production risk. Adam-Müller (1997) and Giaccotto, Hedge and McDermott (2001) discuss the impact of additional revenue risk. In these papers, utility is defined over nominal income.

In the classical problem (i.e. under deterministic inflation), the existence of a futures market has the following three well-known consequences: (1) Given a futures market, the optimal production decision is separable from the distribution of the random output price and the farmer's attitude towards risk. (2) The introduction of an unbiased futures market leads to an increase in production. (3) Full hedging is optimal if and only if the futures market is unbiased.

This paper shows that some of these results change significantly while others remain valid if inflation is risky: For any joint distribution of output price risk and inflation risk, the separation theorem is robust to the existence of inflation risk despite the fact that it is untradable. The effect of introducing a nominally unbiased futures market and the decision on the optimal futures position are not robust. If untradable inflation risk is a monotone function of output price risk plus noise, the effect of introducing a futures market may be negative, depending on the level of relative risk aversion (RRA) and the real risk premium in the futures market. In addition, full hedging in an unbiased futures market is not necessarily optimal.

Briys and Schlesinger (1993) were the first to analyze a related problem in the presence of untradable inflation risk. They use a state-dependent preference model with two possible realizations of the inflation rate and show that inflation risk does not affect the sign of a speculative futures position if marginal utility is state-independent. In a related paper, Adam-Müller (2000) analyzes the hedging problem using state-independent preferences but allowing for any probability distribution of the inflation rate. His framework is similar to the one employed in this paper, hence, his results are shortly restated here. In contrast to Briys and Schlesinger (1993) and Adam-Müller (2000), this paper endogenously derives the exposure to inflation risk since the production decision is incorporated.

The paper is organized as follows. First, the model is presented. Then,

the optimal production decision in the presence of inflation risk and the effect of introducing a futures market are examined, followed by a numerical example for a negative effect. A brief analysis of the optimal risk management decision follows. All proofs are given in the Appendix.

THE MODEL

The farmer's optimal production and risk management decisions are analyzed in a two-date framework where decisions are made at date 0 and uncertainty is resolved at date 1. At date 0, the farmer purchases inputs at prevailing nominal prices. Inputs are used to produce a homogeneous output x, e.g. corn. The nominal value of the inputs at date 1 is given by the cost function $c(x)$. The cost function is increasing and strictly convex.

At date 1, production is completed and output is entirely sold on a competitive market at the stochastic nominal output price $\tilde{p}$ with $p > 0$.[1] The $\tilde{p}$-risk is called *output price risk*. In addition, there is a futures contract on the $\tilde{p}$-risk such that output price risk is tradable. p_0 denotes the competitive date 0 futures price for delivery of one unit of output at date 1; H is the quantity of output sold in the futures market. In addition, there is a fixed component of nominal income, denoted $\bar{\Pi}^n$, which summarizes the farmer's deterministic income from other sources. All payments are made at date 1. The farmer's nominal income at date 1 is

$$\tilde{\Pi}^n = \tilde{p}x - c(x) + H(p_0 - \tilde{p}) + \bar{\Pi}^n \,. \tag{1}$$

Inflation risk is modeled via a stochastic purchasing power index $\tilde{y}$ with $y > 0$. Without loss of generality, assume $\mathrm{E}\tilde{y} = 1$ such that $(\tilde{y}-1)$ represents the surprise in the purchasing power change. The randomness of $\tilde{y}$ gives rise to *inflation risk*. It is assumed that inflation risk cannot be traded. Then, $\tilde{y}$ is a multiplicative background risk that applies to the farmer's entire nominal income. The multiplicative combination of output price risk and inflation risk is an important characteristic of the model. The farmer's real income $\tilde{\Pi}$ at date 1 is given by

$$\tilde{\Pi} \;=\; \tilde{y}\,\tilde{\Pi}^n \;= \tilde{y}\,\left(p_0\,x - c(x) + (H - x)(p_0 - \tilde{p}) + \bar{\Pi}^n\right). \tag{2}$$

His preferences are summarized in a von Neumann-Morgenstern utility function $U(\cdot)$ which is defined over real income Π at date 1.[2] $U(\Pi)$ is at least

twice continuously differentiable and strictly concave such that the farmer is risk-averse. Based on the assessment of the joint distribution of $\tilde{p}$ and $\tilde{y}$, the optimization problem is to

$$\max_{x,H} \mathrm{E}\Big[U(\tilde{\Pi})\Big] \tag{3}$$

where $\tilde{\Pi}$ is defined in equation (2). Since $\mathrm{E}[U(\tilde{\Pi})]$ is strictly concave in x and H, the optimal values x^* and H^* are the unique solution of the first-order conditions

$$\mathrm{E}\Big[U'(\tilde{\Pi}^*)\ \tilde{y}\ (\tilde{p} - c'(x^*))\Big] = 0\,, \tag{4}$$

$$\mathrm{E}\Big[U'(\tilde{\Pi}^*)\ \tilde{y}\ (p_0 - \tilde{p})\Big] = 0\,. \tag{5}$$

The following assumptions are made to avoid lengthy discussions of corner solutions for x^* and for H^*: Optimal output x^* is positive, disregard of whether there is a futures market or not. Furthermore, let $\bar{\Pi}^n$ be sufficiently large to ensure positive nominal income for any optimal output x^*, that is $p_{min}x^* - c(x^*) + \bar{\Pi}^n > 0$ where p_{min} denotes the smallest possible realization of $\tilde{p}$. In addition, assume $U'(\Pi) \to \infty$ for $\Pi \to 0$ in order to preclude optimal futures positions that lead to $\Pi^* \leq 0$. Hence, $\Pi^* > 0$ in each state which is equivalent to $\Pi^{n*} > 0$ since $y > 0$.

Before analyzing optimal decisions, it is helpful to review some commonly used definitions concerning the optimal futures position and the futures market. Full hedging is defined by $H = x$. It eliminates all $\tilde{p}$-risk and makes nominal income Π^n risk-free. Overhedging [underhedging] is defined by $H >$ $[<]\, x$. Nominal unbiasedness [contango] [backwardation] characterizes the situation where the nominal risk premium in the futures market, $\mathrm{E}[p_0 - \tilde{p}]$, is zero [positive] [negative]. The real risk premium of a futures contract is given by $\mathrm{E}[\tilde{y}(p_0 - \tilde{p})] = \mathrm{E}[p_0 - \tilde{p}] - \mathrm{cov}(\tilde{p}, \tilde{y})$. It is important that the nominal and the real risk premium differ by $-\mathrm{cov}(\tilde{p}, \tilde{y})$.

OPTIMAL PRODUCTION

This section derives the optimal production decision and its reaction to the introduction of a futures market. The optimal production decision in the presence of a futures market is analyzed first. Surprisingly, inflation risk does not affect optimal output if there is a futures market for output price risk. This is formally stated in Theorem 1.

Theorem 1 *Suppose that output price risk is tradable with futures contracts. Disregard of whether there is untradable inflation risk or not, the optimal production decision is given by* $c'(x^*) = p_0$.

Theorem 1 states that the optimal production decision is independent of the existence of inflation risk provided there is a futures market. Hence the separation property follows: x^* is independent of the degree of risk aversion and the (joint) distribution of risk(s) involved. The intuition behind Theorem 1 is that any violation of the optimality condition $c'(x^*) = p_0$ allows the farmer to earn riskless nominal income. Since higher nominal income is desirable for any realization of the inflation rate, optimal output is independent of the existence and the nature of inflation risk.

In the literature, various separation results have been derived given the existence of financial contracts that allow for complete elimination of risk(s). Typically, separation results cannot be derived if there are untradable risk(s) as follows from, for example, Moschini and Lapan (1995) and Adam-Müller (1997). Theorem 1 is in sharp contrast to previous results since separation remains valid even if inflation risk is untradable. It is sufficient for the separation result to have a contract on $\tilde{p}$ only, despite the fact that the farmer cannot achieve a riskless position with this contract. This surprising result is due to the fact that untradable inflation risk applies multiplicatively to the entire nominal income which is not the case for the untradable risk(s) in the contributions mentioned above.

One might ask whether separation can also be derived in case of a futures market for the *joint* risk $\tilde{y}\tilde{p}$. It is easy to show that this is not the case because the inflation risk arising from nominal cost and deterministic nominal income cannot be hedged with contracts on $\tilde{y}\tilde{p}$. Thus, a contract on $\tilde{y}\tilde{p}$ alone does not allow for complete elimination of risks. The same applies to a contract on $\tilde{y}$ alone. If a riskless real income is to be achieved, contracts on both $\tilde{p}$ and $\tilde{y}\tilde{p}$ are required. But, as Theorem 1 shows, a contract on $\tilde{p}$ alone is sufficient to achieve separation despite the fact that complete elimination of risks is impossible.

Next, the effect of introducing a futures market for output price risk on the optimal production decision is analyzed. Consider the optimal production decision in the absence of a futures market where the farmer's real income is denoted by $\tilde{\Pi}_a = \tilde{y}\,\tilde{\Pi}_a^n = \tilde{y}\,(\tilde{p}x_a - c(x_a) + \bar{\Pi}^n)$. Analogously, x_a^* denotes the optimal production decision in this case. x_a^* is solely determined

by (4) which can be rewritten as

$$\mathrm{E}\tilde{p} - c'(x_a^*) = -\frac{\mathrm{cov}\left(\tilde{p}, \tilde{y}\,U'(\tilde{\Pi}_a^*)\right)}{\mathrm{E}\left[\tilde{y}\,U'(\tilde{\Pi}_a^*)\right]}\,. \tag{6}$$

The effect of introducing a nominally unbiased futures market becomes apparent if x_a^* and x^* are compared. According to Theorem 1, x^* is solely determined by $p_0 = c'(x^*)$. In contrast, (6) clearly indicates that x_a^* is affected by the shape of $U(\cdot)$ and the joint distribution of $\tilde{p}$ and $\tilde{y}$. The sign of $\mathrm{E}\tilde{p} - c'(x_a^*) = p_0 - c'(x_a^*)$ depends on the sign of $\mathrm{cov}(\tilde{p}, \tilde{y}\,U'(\tilde{\Pi}_a^*))$ since $yU'(\cdot) > 0$. This covariance cannot be signed without imposing additional assumptions on the shape of $U(\cdot)$ and/or on the joint distribution of $\tilde{p}$ and $\tilde{y}$.[3] Hence, introducing a futures market does not necessarily increase output.

The covariance in (6) and, thus, the comparison between x^* and x_a^* under inflation risk is analyzed for three scenarios. For deterministic inflation, it is well-known that the introduction of an unbiased futures market stimulates output, see Holthausen (1979), Eldor and Zilcha (1987) and others. Now, it will be shown that this result is robust to the introduction of inflation risk in the first two scenarios, but not necessarily so in the third scenario.

In the first scenario, the utility function is logarithmic such that RRA equals unity. As is well-known from Adler and Dumas (1983), inflation risk is completely ignored under logarithmic utility. However, logarithmic utility does not seem to be a reasonable assumption since inflation risk will be ignored even if it is perfectly correlated with output price risk, for example.

In the second scenario, output price risk is conditionally independent of inflation risk, $\mathrm{E}[\tilde{p}|y] = \mathrm{E}\tilde{p}\,\forall\, y$.[4] Since conditional independence implies uncorrelatedness, the real risk premium in the futures market coincides with the nominal risk premium. Hence, it is straightforward that introducing a nominally unbiased futures market increases production for any risk-averse utility function since output price risk can be sold at zero nominal and real cost.[5]

However, it is unlikely that the price of the farmer's output and the inflation rate move independently. Therefore, the third scenario focuses on a joint distribution where output price risk and inflation risk are stochastically dependent. The analysis concentrates on the following type of dependence: $\tilde{y}$ is the sum of a monotone function $a(\tilde{p})$, a fixed term $\bar{y}$ and a noise term $\tilde{\epsilon}$ with $\mathrm{E}\tilde{\epsilon} = 0$, $\tilde{y} = a(\tilde{p}) + \bar{y} + \tilde{\epsilon}$. $a(\tilde{p})$ is a deterministic, differentiable and monotone function. $\tilde{p}$ is conditionally independent of $\tilde{\epsilon}$, $\mathrm{E}[\tilde{p}|\epsilon] = \mathrm{E}\tilde{p}\,\forall\,\epsilon$.

The economic intuition behind this assumption is the following: To some extent, the purchasing power index $\tilde{y}$ depends on $\tilde{p}$, either directly or indirectly. This is captured by the function $a(\tilde{p})$. If the price of the farmer's output directly enters the calculation of the purchasing power index $\tilde{y}$, there is a direct effect. Then, $a(\tilde{p}) = \alpha/\tilde{p}$ for some positive weight α such that $a'(\tilde{p}) < 0$.[6] If, alternatively, the farmer's output is used as an input in the production of consumption goods, there is an indirect effect of $\tilde{p}$ on $\tilde{y}$ requiring a more sophisticated specification of $a(\tilde{p})$. Since higher input prices generally lead to higher consumer prices, the indirect effect implies $a'(\tilde{p}) < 0$ as well. Thus, it seems realistic to assume $a'(\tilde{p}) < 0$.[7] In addition to the effect of $\tilde{p}$ on $\tilde{y}$, there are other sources of expected and unexpected changes in the purchasing power index $\tilde{y}$. They are represented by $(\bar{y} + \tilde{\epsilon})$ where $\bar{y}$ covers the expectation and $\tilde{\epsilon}$ the expectation error.

For notational simplicity, $\bar{y}$ and $a(\tilde{p})$ are summarized in the function $b(\tilde{p}) = \bar{y} + a(\tilde{p})$. $b(\tilde{p})$ is simply a linear transformation of $a(\tilde{p})$. Hence $b'(\tilde{p}) < 0$. Conditional independence of $\tilde{p}$ from $\tilde{\epsilon}$ implies uncorrelatedness of $\tilde{p}$ and $\tilde{\epsilon}$ such that $\text{cov}(\tilde{p}, \tilde{y}) = \text{cov}(\tilde{p}, a(\tilde{p})) = \text{cov}(\tilde{p}, b(\tilde{p})) < 0$ for $a'(\tilde{p}), b'(\tilde{p}) < 0$. If the nominal risk premium in the futures market is zero, $\text{E}[p_0 - \tilde{p}] = 0$, the real risk premium amounts to $\text{E}[\tilde{y}(p_0 - \tilde{p})] = -\text{cov}(\tilde{p}, b(\tilde{p}))$ which is positive.

Given this type of dependence, the effect of introducing a nominally unbiased futures market for output price risk is no longer unequivocal but depends on the level of RRA, as claimed in the next theorem.

Theorem 2 *Suppose there is output price risk and inflation risk. Suppose further that $\tilde{y} = b(\tilde{p}) + \tilde{\epsilon}$ where $\tilde{p}$ is conditionally independent of $\tilde{\epsilon}$ and $b'(\tilde{p}) < 0$. The introduction of a nominally unbiased futures market for output price risk*
a) increases production, $x^* > x_a^*$, if relative risk aversion is below one in all states in the absence of a futures market,
b) may decrease production, $x^* < x_a^*$, if relative risk aversion is above one at least in some states in the absence of a futures market,
c) decreases production if relative risk aversion tends to infinity.

Stochastic dependence between $\tilde{p}$ and $\tilde{y}$ has two opposing effects on the level of x_a^* as compared to x^*. The first is a cost effect, the second is a risk reduction effect. Both effects are absent if there is a futures market.

Begin with the *cost effect*. Expected real income is given by $\mathrm{E}\tilde{\Pi}_a = \mathrm{E}\tilde{\Pi}_a^n + x_a \operatorname{cov}(\tilde{p}, b(\tilde{p}))$ where the covariance is negative since $b'(\tilde{p}) < 0$. Hence, the covariance can be interpreted as an additional per unit cost of production since it negatively enters expected real income. This decreases production for any level of risk aversion and establishes the cost effect.

The cost effect can be isolated from the risk reduction effect if a risk-neutral farmer is considered. By definition, the risk-neutral farmer ignores risk and, thus, any risk reduction effect. It is straightforward to show that his optimal output, x_a^{*rn}, satisfies $c'(x_a^{*rn}) = \mathrm{E}\tilde{p} + \operatorname{cov}(\tilde{p}, b(\tilde{p})) < \mathrm{E}\tilde{p} = c'(x^*)$. Hence, a risk-neutral farmer produces less compared to the case in which there is an unbiased futures market, $x_a^{*rn} < x^*$. This is solely attributable to the cost effect.

Next, turn to the *risk reduction effect*. Since $b'(\tilde{p}) < 0$ implies negatively correlated $\tilde{p}$ and $\tilde{y}$, there is a natural hedge in the farmer's real income. Taken in isolation, this risk reduction effect increases production. A risk-averse farmer takes both the cost effect and the risk reduction effect into account. The higher his risk aversion, the more relative weight he attaches to the risk reduction effect.

Now, Theorem 2 can be interpreted using the cost effect and the risk reduction effect. For low risk aversion as represented by RRA below one, the risk reduction effect is given only a small weight. Consequently, the cost effect always dominates the risk reduction effect in the sense that $x_a^* < x^*$. This is the scenario described in Theorem 2a).[8] However, for RRA above unity, the risk reduction effect is given a higher weight and may even dominate the cost effect. In other words, if $x_a^* > x^*$, risk aversion must be high as represented by RRA above one (Theorem 2b)). A numerical example with RRA $= 2$ is provided in the next section. As Theorem 2c) states, the risk reduction effect always dominates if RRA approaches infinity. Taken together, parts b) and c) of Theorem 2 indicate that for any joint distribution there is a critical level of RRA at which the risk reduction effect dominates the cost effect such that $x_a^* > x^*$.

Existing empirical evidence indicates that RRA well above unity is the realistic case. Thus, Theorem 2b) is relatively bad news from a policy maker's point of view since it is not automatically ensured that introducing a futures market has a positive effect on production. This is in sharp contrast to almost all previous contributions.[9]

AN EXAMPLE

This section presents an example in which the introduction of a futures market for output price risk reduces optimal production. The purpose of the example is to illustrate Theorem 2b). It is based on the same assumptions except for the (slightly) stronger assumption of constant RRA given by power utility $\check{U}(\Pi) = \Pi^{\gamma}/\gamma$ where constant RRA equals $(1-\gamma)$. Let $\gamma = -1$ and $\tilde{y} = \bar{y} + 3/\tilde{p}^3 + \tilde{\epsilon}$ such that RRA $= 2$ and $b'(\tilde{p}) < 0$. The joint probability distribution of $\tilde{p}$ and $\tilde{\epsilon}$ and the resulting distribution of $\tilde{y}$ is given in Table 1. There are four states of nature s_i, $i = 1, ...\, 4$, which are equally probable. Setting $\bar{y} = .01194$ yields $\mathrm{E}\tilde{y} = 1$. $\tilde{\epsilon}$ and $\tilde{p}$ are stochastically independent. The correlation coefficient for $\tilde{p}$ and $\tilde{y}$ is -0.96665 such that there is a strong natural hedge and the potential for a large risk reduction effect.

Table 1: Probability distributions of $\tilde{p}$, $\tilde{\epsilon}$ and $\tilde{y}$

state of nature	s_1	s_2	s_3	s_4
prob s_i	.25	.25	.25	.25
p_i	1.30	1.30	1.70	1.70
ϵ_i	.10	-.10	.10	-.10
y_i	1.47744	1.27744	.72256	.52256

The farmer's income in the absence of a futures market is given by $\tilde{\Pi}_a = \tilde{y}(\tilde{p}x_a - c(x_a) + \bar{\Pi}^n)$. The cost function is specified as $c(x) = x + x^2/8$. In addition, $\bar{\Pi}^n = 2.50$. Then, the optimal production decision in the absence of a futures market is $x_a^* = 2.10$.

If there is a nominally unbiased futures market, the futures price is $p_0 = 1.50$. The nominal risk premium is zero, but the real risk premium is not since $\mathrm{E}[\tilde{y}(p_0 - \tilde{p})] = .07549 > 0$. The optimal production decision is given by $c'(x^*) = p_0$ according to Theorem 1. In the present case, $c'(x^*) = 1 + x^*/4$ or, equivalently, $x^* = 2.00$. Thus, optimal output in the presence of a nominally unbiased futures market, $x^* = 2.00$, is smaller than in the absence of such a market where $x_a^* = 2.10$.

OPTIMAL RISK MANAGEMENT

This section presents the optimal risk management decision. Due to the separation stated in Theorem 1, optimal output is fixed at x^*. Thus, the

optimal futures position can be derived using (5) in isolation. This section restricts itself to briefly summarizing the main result of Adam-Müller (2000) who has recently analyzed this condition for the same three scenarios used in the previous section.

In the first scenario, i.e. under logarithmic utility, inflation risk is completely ignored. Therefore, the optimal futures position is as follows: Underhedging [full hedging] [overhedging] is optimal if and only if the futures market exhibits nominal backwardation [unbiasedness] [contango], see Holthausen (1979). For the second scenario where $\tilde{p}$ is conditionally independent of $\tilde{y}$, the same qualitative statement holds for any risk-averse utility function.[10]

In the third scenario, inflation risk is a monotone function of output price risk plus noise, $\tilde{y} = b(\tilde{p}) + \tilde{\epsilon}$, with $\tilde{p}$ conditionally independent of $\tilde{\epsilon}$ and $b'(\tilde{p}) < 0 \, \forall \, p$ as above. In this case, there are two effects determining the optimal futures position. The first effect is cross hedging, the second is speculation on a real risk premium. Begin with *cross hedging*: Since output price risk and inflation risk are correlated, the farmer is able to cross hedge the otherwise untradable inflation risk. The second effect is *speculation*. The real risk premium is given by $\mathrm{E}[\tilde{y}(p_0 - \tilde{p})] = \mathrm{E}[p_0 - \tilde{p}] - \mathrm{cov}(\tilde{p}, b(\tilde{p}))$. Thus, a nominally unbiased futures market exhibits a positive real risk premium since $b'(\tilde{p}) < 0$. Then, any futures position changes expected real income while leaving expected nominal income unaffected. The existence of a real risk premium is an incentive for speculation. Consequently, the farmer will assume a speculative position. Given these two effects, the following theorem characterizes the optimal futures positions in a nominally unbiased futures market.

Theorem 3 *Suppose $\tilde{y} = b(\tilde{p}) + \tilde{\epsilon}$ where $\tilde{p}$ is conditionally independent of $\tilde{\epsilon}$ and $b'(\tilde{p}) < 0$. Suppose further that the futures market is nominally unbiased.*
a) Underhedging is optimal if relative risk aversion is above one for all possible levels of real income.
b) Overhedging is optimal if relative risk aversion is below one for all possible levels of real income.

The optimal futures position can be easily interpreted if it is split into three components as $H^* = H^d + H^c + H^s$. H^d is the hedging component aimed at directly hedging tradable output price risk without taking the

risk premium and untradable inflation risk into account. It follows that $H^d = x^*$. H^c denotes the cross hedging component which is aimed at reducing real income risk by acquiring nominal income for states with low purchasing power against nominal income in states with high purchasing power. Thus, negatively correlated $\tilde{p}$ and $\tilde{y}$ imply a negative cross hedging component, $H^c < 0$, since this provides higher nominal income in states with low purchasing power. H^s is a speculative component that is due to the existence of the real risk premium $-\text{cov}(\tilde{p}, b(\tilde{p})) > 0$. Expected real income can be raised by taking a positive speculative position $H^s > 0$.

To sum up, the cross hedging component and the speculative component have opposite signs. Theorem 3 states that their relative size is determined by the level of RRA. If RRA is above one, the cross hedging component dominates the speculative component, $|H^c| > |H^s|$. Then, underhedging is optimal, $H^* - H^d = H^* - x^* = H^s + H^c < 0$. In other words, if risk aversion is high, reducing the variability of real income (by cross hedging otherwise untradable inflation risk) is more attractive than increasing expected real income via speculation. Analogously, if RRA is below one, increasing expected real income is more important such that the speculative component dominates the cross hedging component and overhedging is optimal.

Theorem 3 deals with a nominally unbiased futures market. An extension to nominally biased futures markets as well as a numerical example can be found in Adam-Müller (2000).

CONCLUSION

This paper analyzes the optimal production and risk management decisions of a risk-averse entrepreneur in the presence of tradable output price risk and untradable inflation risk. The latter applies multiplicatively to the entrepreneur's entire nominal income.

In the presence of a futures market, the optimal production decision is separable from the entrepreneur's attitude towards risk as well as the joint distribution of output price risk and inflation risk despite the fact that a riskless position cannot be achieved.

The effect of introducing a nominally unbiased futures market depends on the joint distribution and the level of RRA. If the output price is positively correlated with the inflation rate, introducing a futures market

decreases production if RRA is above one and sufficiently high. This is in contrast to almost all previous contributions and has interesting policy implications: Introducing an unbiased futures market does not necessarily stimulate output for realistic levels of RRA.

For the same type of dependence, the level of RRA is crucial for the optimal futures position as well. Cross hedging and speculating on a real risk premium are conflicting objectives. Cross hedging is more important for high RRA whereas speculating incentives are predominant if RRA is low. In either case, full hedging is not optimal in a nominally unbiased futures market.

APPENDIX

The proofs are based on the uniqueness of the optimal solution. Lemma 1 is used in the proofs of Theorems 2 and 3.

Lemma 1 *Let $\tilde{v}$ and $\tilde{w}$ be two random variables with $v, w > 0$ and $f(\tilde{v}, \tilde{w})$ a deterministic function that satisfies $f(v, w) > 0 \,\forall\, v, w$. Then, under mild regularity conditions,*

$$\operatorname{cov}\Big(\tilde{v}, f(\tilde{v}, \tilde{w})\Big) = \mathrm{E}\Big[\operatorname{cov}\Big(\tilde{v}, f(\tilde{v}, w)\Big|w\Big)\Big] + \operatorname{cov}\Big(\mathrm{E}[\tilde{v}|w], \mathrm{E}[f(\tilde{v}, w)|w]\Big)\,. \quad (7)$$

Proof: Apply the Law of Iterated Expectations. □

Conditional independence of $\tilde{v}$ from $\tilde{w}$ implies $\mathrm{E}[\tilde{v}|w] = \mathrm{E}\tilde{v} \,\forall\, w$ which is a constant. In this case, the second summand in (7) is zero.

Proof of Theorem 1

Rewriting (4) and (5) and combining yields

$$\mathrm{E}\Big[\tilde{p}\,\tilde{y}\,U'(\tilde{\Pi}^*)\Big] = c'(x^*)\,\mathrm{E}\Big[\tilde{y}\,U'(\tilde{\Pi}^*)\Big] = p_0\,\mathrm{E}\Big[\tilde{y}\,U'(\tilde{\Pi}^*)\Big]\,. \quad (8)$$

Dividing by $\mathrm{E}[\tilde{y}\,U'(\tilde{\Pi}^*)]$ proves that $c'(x^*) = p_0$ is optimal. Deterministic inflation implies $y = 1$ for all states but leaves the result unchanged. □

Proof of Theorem 2

In the absence of a futures market, $\tilde{y} = b(\tilde{p}) + \tilde{\epsilon}$ implies $\tilde{\Pi}_a = [b(\tilde{p}) + \tilde{\epsilon}](\tilde{p}x_a - c(x_a) + \bar{\Pi}^n)$. Now, the optimality condition for x_a^* as given in (4) is evaluated

at x^* where $\mathrm{E}\tilde{p} = c'(x^*)$. Using Lemma 1 and conditional independence of $\tilde{p}$ from $\tilde{\epsilon}$, this evaluation results in

$$\begin{aligned} \mathrm{E}\Big[U'(\tilde{\Pi}_a)\,\tilde{y}\,(\tilde{p}-\mathrm{E}\tilde{p})\Big] &= \mathrm{cov}\Big(\tilde{p},[b(\tilde{p})+\tilde{\epsilon}]\,U'(\tilde{\Pi}_a)\Big) \\ &= \mathrm{E}\Big[\mathrm{cov}\Big(\tilde{p},[b(\tilde{p})+\epsilon]\,U'(\tilde{\Pi}_a)\Big|\epsilon\Big)\Big]\,. \end{aligned} \tag{9}$$

It is straightforward to show that

$$\frac{\mathrm{d}yU'(\Pi_a)}{\mathrm{d}p} = \Big(1-\mathrm{RRA}(\Pi_a)\Big)\,b'(p)\,U'(\cdot) + [b(p)+\epsilon]^2\,x_a\,U''(\cdot)\,. \tag{10}$$

The second summand is always negative since $x_a > 0$, $[b(p)+\epsilon] = y > 0\,\forall\, p,\epsilon$ and $U''(\cdot) < 0$. For $b'(p) < 0$ combined with $\mathrm{RRA} < 1\,\forall\,\Pi_a$, the first summand is negative. Then, the conditional covariance in (9) is negative for all p and ϵ. Thus, the whole expression in (9) is negative. Given the strict concavity of the problem, this implies $x_a^* < x^*$. This proves part a).

For $x_a^* > x^*$, (9) indicates the necessity of $\mathrm{cov}(\tilde{p},[b(\tilde{p})+\epsilon]\,U'(\tilde{\Pi}_a)|\epsilon) > 0$ for at least some ϵ. By (10), this requires $\mathrm{RRA} > 1$ for these values of ϵ. This proves part b).

For $\mathrm{RRA} \to \infty\,\forall\,\Pi_a$ and $b'(p) < 0$, the first summand in (10) is positive and exceeds the second summand. Hence, the expressions in (9) and (10) are positive. $x_a^* > x^*$ follows directly. This proves part c). □

Proof of Theorem 3

Let $\xi = p_0x^* - c(x^*) + \bar{\Pi}^n > 0$ and $U'(y\xi)y = \theta(y)$. $U'(\cdot) > 0$ and $y > 0$ imply $\mathrm{E}[\theta(\tilde{y})] > 0$. At $H = x^*$, real income simplifies to $\tilde{\Pi} = \tilde{y}\xi$. Evaluating (5) at $H = x^*$ yields

$$\mathrm{E}\Big[\theta(\tilde{y})(p_0-\tilde{p})\Big] = \mathrm{E}\Big[\theta(\tilde{y})\Big]\,\mathrm{E}[p_0-\tilde{p}] - \mathrm{cov}\Big(\tilde{p},\theta(\tilde{y})\Big)\,. \tag{11}$$

The first summand in (11) is zero due to nominal unbiasedness. It remains to sign the covariance in (11). $\tilde{y} = b(\tilde{p}) + \tilde{\epsilon}$ implies

$$\begin{aligned} \frac{\mathrm{d}\theta(y(p))}{\mathrm{d}p} = \theta'(y)b'(p) &= [U''(y\xi)y\xi + U'(y\xi)]\,b'(p) \\ &= [1-\mathrm{RRA}(y\xi)]\;U'(y\xi)\,b'(p) \quad \forall\,\epsilon\,. \end{aligned} \tag{12}$$

Combining $\mathrm{RRA} < [>]\,1$ with $b'(p) < 0$ implies $\mathrm{d}\theta(y(p))/\mathrm{d}p < [>]\,0\,\forall\,\epsilon$. Since $\tilde{p}$ is conditionally independent of $\tilde{\epsilon}$, $\mathrm{cov}(\tilde{p},\theta(\tilde{y})) < [>]\,0$ follows by Lemma 1. Theorem 3 directly follows from the strict concavity of the problem. □

BIBLIOGRAPHY

Adam-Müller, A. F. A. (1997). Export and hedging decisions under revenue and exchange rate risk: a note. European Economic Review, 41, 1421-1426.

Adam-Müller, A. F. A. (2000). Hedging price risk when real wealth matters. Journal of International Money and Finance, 19, 549-560.

Adler, M., & Dumas, B. (1983). International portfolio choice and corporation finance: a synthesis. Journal of Finance, 38, 925-988.

Briys, E., Crouhy, M., & Schlesinger, H. (1993). Optimal hedging in a futures market with background noise and basis risk. European Economic Review, 37, 949-960.

Briys, E., & Schlesinger, H. (1993). Optimal hedging when preferences are state dependent. Journal of Futures Markets, 13, 441-451.

Broll, U., Wahl, J. E., & Zilcha, I. (1995). Indirect hedging of exchange rate risk. Journal of International Money and Finance, 14, 667-678.

Broll, U., & Wong, K. P. (1999). Hedging with mismatched currencies. Journal of Futures Markets, 19, 859-875.

Broll, U., Wong, K. P., & Zilcha, I. (1999). Multiple currencies and hedging. Economica, 66, 421-432.

Eldor, R., & Zilcha, I. (1987). Discriminating monopoly, forward markets and international trade. International Economic Review, 28, 459-468.

Ely, D. P., & Robinson, K. J. (1997). Are stocks a hedge against inflation? International evidence using a long-run approach. Journal of International Money and Finance, 16, 141-167.

Giaccotto, C., Hedge, S. P., & McDermott, J.B. (2001). Hedging multiple price and quantity exposures. Journal of Futures Markets, 21, 145-172.

Hess, P. J., & Lee, B.-S. (1999). Stock returns and inflation with supply and demand disturbances. Review of Financial Studies, 12, 1203-1218.

Holthausen, D. M. (1979). Hedging and the competitive firm under price uncertainty. American Economic Review, 69, 989-995.

Lee, B.-S. (1992). Causal relations among stock returns, interest rates, real activity, and inflation. Journal of Finance, 47, 1591-1603.

Lence, S. H. (1995). On the optimal hedge under unbiased futures prices. Economics Letters, 47, 385-388.

Moschini, G., & Lapan, H. (1995). The hedging role of options and futures under joint price, basis, and production risk. International Economic Review, 36, 1025-1049.

Sandmo, A. (1971). On the theory of the competitive firm under price uncertainty. American Economic Review, 61, 65-73.

Viaene, J.-M., & Zilcha, I. (1998). The behavior of competitive exporting firms under multiple uncertainty. International Economic Review, 39, 591-609.

Notes

[1]Random variables are given a tilde, their realizations are not.

[2]If the payments for inputs are due at date 0, a more complicated intertemporal expected utility framework with preferences over payments at dates 0 and 1 is required. Alternatively, $c(x)$ can be interpreted as the date 1 repayment (including interest payments) of a loan raised at date 0 to pay for the inputs.

[3]It is well-known from Sandmo (1971) that $\mathrm{E}\tilde{p} > c'(x_a^*)$ characterizes the optimal production decision if there is neither inflation risk nor a futures market for output price risk.

[4]Conditional independence is slightly weaker than independence.

[5]To see why, set $b'(p) = 0$ in the proof of Theorem 2a). This is equivalent to conditional independence of $\tilde{p}$ from $\tilde{y}$. For $b'(p) = 0$, the first summand in (10) vanishes.

[6]For additively combined risks, a linear specification of $a(\tilde{p})$ has been used in the cross hedging problem analyzed by Briys et al. (1993).

[7]All results of the paper related to dependent $\tilde{p}$ and $\tilde{y}$ can be also derived for $a'(\tilde{p}) > 0$, with some obvious modifications.

[8]For logarithmic utility where RRA = 1, the cost effect is still dominant.

[9]Broll et al. (1995) is the exception.

[10]To justify this statement, set $b'(p) = 0 \,\forall\, p$ in the proof of Theorem 3. This implies $\mathrm{cov}(\tilde{p}, \theta(\tilde{y})) = 0$ such that the covariance term in (11) vanishes. Then, (11) and the strict concavity of the problem imply $\operatorname{sgn} \mathrm{E}[p_0 - \tilde{p}] = \operatorname{sgn}(H^* - x^*)$.