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# DETERMINANTS OF INTER-TRADE DURATIONS AND HAZARD RATES USING PROPORTIONAL HAZARD ARMA MODELS

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**ABSTRACT.** This paper puts a focus on the hazard function of inter-trade durations to characterize the intraday trading process. It sheds light on the time varying trade intensity and, thus, on the liquidity of an asset and the information channels which propagate price signals among asymmetrically informed market participants. We show, based on an exogenous information process, that the way traders aggregate information has implications for the shape of the hazard function. We use a semiparametric proportional hazard model which is augmented by an ARMA structure very similar to the wide spread ACD model to obtain consistent estimates of the baseline survivor function and to capture well known serial dependencies in the trade intensity process.

From an inspection of conditional transaction probabilities based on Bund future transaction data of the DTB we find a decreasing hazard shape providing evidence for the use of non-trading intervals as an indication for the absence of price information among market participants. However, this information content seems to be diluted by a high liquidity base level, particularly with respect to a large inflow of potential traders from the U.S.

Furthermore, we provide evidence that past sequences of prices and volumes have a significant impact on the trading intensity in accordance with theoretical models.

## 1. INTRODUCTION

The analysis of the time between transactions is an ongoing topic in the empirical analysis of market microstructure. After the seminal work of Engle (1996)<sup>1</sup> a broad range of extensions was proposed in the literature. One string of the

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<sup>1</sup>The paper is now published as Engle (2000).

literature deals with the modelling of the well known serial dependency in the inter-trade duration process and deals with more flexible dynamics, like in the SVD model proposed by Ghysels, Gouriéroux, and Jasiak (1998) or the SCD model introduced by Bauwens and Veredas (1999). The other set of extensions emphasizes more flexible forms of the underlying distribution and, thus, the hazard function of the inter-trade durations. See e.g. Grammig and Maurer (1999) and also Bauwens and Veredas (1999).

In this paper we propose a consistent estimator of the hazard function which also accounts for the serial dependency in inter-trade durations. We use the proportional hazard ARMA (PHARMA) model <sup>2</sup> that extends the traditional semiparametric proportional hazard model by allowing for serial dependency in the durations. The main advantage of this type of duration model compared with the class of ACD models proposed by Engle and Russell (1998) or the SVD model is that it requires no parametric specification of the duration distribution. It yields a nonparametric estimate of the baseline hazard function and allows for ARMA-structures in the inter-trade duration process. Explanatory variables can be included dynamically, corresponding to an ARMAX specification, but also statically, i.e. without any lag structure.

The contribution of our paper is twofold: First, by using the PHARMA model for the inter-trade duration process we model the conditional probability to observe the next transaction as a function of the elapsed time since the last trade, conditional on explanatory variables capturing the state of the market. The shape of these conditional failure probabilities, i.e. the hazard function, allows us to quantify the duration dependence, i.e. the impact of the elapsed time since the last trade on the timing of the next transaction.

Note, that a constant (flat) hazard rate corresponds to a duration-independence between the trades, i.e. the timing of one trade has no impact on the timing of the next trade, corresponding to a Poisson process for the arrival rate of transactions. On the other hand, a negative (positive) time dependence between particular trades means that the probability of observing a further transaction decreases

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<sup>2</sup>See Gerhard and Hautsch (2000).

(increases) the longer the time span since the last transaction. This duration dependency has implications for trade-to-trade market dynamics and provides some evidence on the informational content of no-trade intervals. Assuming an exogenous information process, we show that the shape of the hazard rate provides information about the way traders aggregate information.

Second, we provide some results on the impact of market microstructure variables, indicating the state of the market. We investigate whether the probability for the occurrence of a new transaction depends not only on the elapsed time since the last trade but also on market microstructure characteristics, like past and contemporaneous price changes and volumes. We investigate different hypotheses concerning the learning of market participants from past market sequences and the strategic behaviour of traders.

By analyzing Bund future transaction data of the Deutsche Terminbörse (DTB) in Frankfurt we show that the PHARMA model does a good job capturing the serial dependency in the inter-trade duration process while it allows to semiparametrically assess the shape of the hazard rate.

We obtain a decreasing hazard function indicating evidence for the use of non-trading intervals as a proxy for the absence of price information among market participants. This may be due to trading schemes of market participants who infer from past sequences of market activities and account for the intensity of price signals.

The outline of the paper is as follows: In section 2 we derive economic implications of the hazard shape and testable hypothesis originating in market microstructure models. The econometric model is presented in section 3. Section 4 gives a description of the data set, while section 5 presents the empirical results. Section 6 concludes.

## 2. MARKET MICROSTRUCTURE

**2.1. Economic implications of the hazard rate.** One of the main targets of this study is to show that the hazard rate is a key instrument to measure the informational content of inter-trade waiting times. In the following we briefly

characterize the economic implications of different shapes of the hazard function. We show that different trading schemes, i.e. different ways traders aggregate their information, lead to different shapes of the hazard function of the resulting inter-trade durations.

We assume that the arrival of exogenous information during the trading day follows a Poisson process and is observable by the informed market participants. The information corresponds to price signals where we assume that the amount of information provided by every particular signal is equal. Within this framework we assume the existence of an uninformed, risk neutral market maker. We distinguish between three types of market participants who trade on the basis of different information aggregation schemes. Our analysis is based on the trading behaviour of one representative trader of each group.

- (a) The first type of market participants trades after every arrival of a new price signal. Hence, his transaction process corresponds to the information process, thus the resulting inter-trade durations are distributed according to an exponential distribution, i.e. the hazard function is constant and has a flat shape.
- (b) The second group of traders contains market participants who are usually described as traders who trade according to exogenous reasons, like portfolio aspects. We call such a trader 'fundamental' trader. In this context this trader's behaviour is not exogenous, but dependent on the information process. This market participant does not trade after every price signal but only after observing a certain amount of new information. He adjusts his portfolio only if the deviation from his optimal portfolio exceeds a certain threshold. Each arrival of a new signal is considered to shift the trader away from his desired position as he gains successively more information on the asset. Thus, a trader of group a) can be seen as the limiting case of a trader from group b) with a threshold of zero. Motivations for the introduction of such a threshold can easily be found in the presence of transaction costs.

An important assumption in this context is, that this type of traders does not regard the speed of information arrival, i.e. the information intensity.

Hence, the fundamental trader aggregates information independent of the timing of the particular events. This kind of information aggregation leads to a trading behaviour where the market participant trades only after observing a certain amount of price signals. The following proposition establishes the shape of the resulting hazard rate.

**Proposition 1:** *Based on a Poisson process for the arrival of information, a 'fundamental' trading strategy leads to an increasing shape of the hazard rate of the resulting inter-trade durations.*

**Proof:** See Appendix.

- (c) The third type of market participants consists on traders who are usually considered to trade according to endogenous reasons, we call these traders 'technical' traders. We assume that the technical trader learns from past sequences of price signals. He wants to exploit the informational content of sequences of price signals optimally by explicitly accounting for the information intensity, i.e. the number of signals which occur within a given time interval  $T$ .

While 'fundamental' market participants wait until they observe a certain amount of information, 'technical' traders update their beliefs continuously, i.e. in market phases with a sufficiently high information intensity they trade after every new price signal. Proposition 2 states the hazard shape of inter-trade durations based on a technical trading strategy.

**Proposition 2:** *Based on a Poisson process for the arrival of information, a 'technical' trading strategy leads to a shape of the resulting inter-trade hazard rate that decreases for duration values  $\tau \in (0, T]$  and is constant for values  $\tau \in (T, \infty)$ .*

**Proof:** See Appendix.

These results show that, based on a certain information arrival process, the way market participants aggregate their information has important implications for the shape of the hazard function. The main difference between the shapes arises by the fact that 'fundamental' traders ignore the timing of the events and

accounting for the amount of information only while 'technical' trading is based on the speed of information arrival.<sup>3</sup>

In our setting it is easy to show that the hazard shape becomes more flat the more traders with different trading behaviours enter the market. Intuitively it is clear that, in the extreme case, infinitely many different information aggregation schemes lead to transactions after every price signal. Hence, the transaction process corresponds to the information process, leading to a constant hazard function.

**2.2. Determinants of inter-trade durations.** A further scope of our paper is to provide some insight into the impact of microstructure variables on the inter-trade durations. In market microstructure theory the timing of trades plays an important role in the learning mechanism of market participants drawing inferences from the trading process. In this context inter-trade durations are regarded as means to aggregate information on price signals available to individual traders in an asymmetric information environment (see Easley and O'Hara (1992)), complementary to other information channels like the sequence of price changes (see Hellwig (1982) or Diamond and Verrecchia (1981)) and traded volume (see Blume, Easley, and O'Hara (1994)).

A central assumption is that the timing of trades is not only driven by the occurrence of information but also reflects the individual decisions of traders. This implies that agents' learning from past sequences of market activities is also reflected in the expected waiting times until the next transaction. The assumption of the informativeness of past price sequences is based on the noisy rational expectation equilibrium models of Hellwig (1982) and Diamond and Verrecchia (1981) which analyze rational expectation equilibria in a market where investors learn from past prices. If traders' preference for the immediacy increases when

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<sup>3</sup>We also extended the theoretical setting by assuming the existence of different groups of traders, i.e. market participants that trade according to one of the trading schemes described above but based on different amounts of information respectively based on different time horizons. Because in this context it is difficult to calculate the resulting inter-trade hazard rates analytically, we simulated the corresponding market activities. These simulations show that the results above also hold when the market consists of agents that have the same trading behaviour but different aggregation criteria.

past market activities provide information to them then past price sequences have also an impact on the expected inter-trade duration. The assumption that the information content of a price process is correlated with its volatility leads to the following hypothesis:

**Hypothesis H1a:** *Large absolute price changes in the past imply a decreased expected waiting time until the next trade.*

Blume, Easley, and O'Hara (1994) analyze the informational role of volume. They resolve how the statistical properties of volume relate to the behaviour of market prices and show that traders can also learn from sequences of volume. The crucial result is that volume provides information that cannot be deduced from the price statistics. In our setting we want to investigate whether this informational content of trading volumes is also reflected in inter-trade durations. Based on this theoretical setting we formulate the hypothesis H1b:

**Hypothesis H1b:** *Past sequences of volumes are informative for expected inter-trade durations even if past price sequences are accounted for.*

Furthermore, we want to get insights into the impact of the heterogeneity of information on the trade-to-trade waiting times. Lang, Litzenberger, and Madrigal (1992) show that the dispersion of private information across the agents influences the trading volume, but not the price. This divergence of beliefs arising from asymmetric information plays an important role in generating activity. According to this theoretical literature one would expect that an increase of the heterogeneity of information has a significant impact on the speed of market activities. The influence of this dispersion of private information on the inter-trade durations can be empirically tested by analyzing the trade intensity at the DTB before and after the beginning of American trading. We base this investigation on the conditional survivor function and the conditional hazard function, given past and present market activities, that indicate changes in the duration dependence of the timing of trades. We formulate the following testable hypothesis:

**Hypothesis H2:** *The inflow of additional market participants from the U.S. causes*



*changes in the duration dependence of inter-trade waiting times.*

The last hypothesis we want to test concerns strategic behaviour of market participants. The theoretical literature analyzing the strategic behaviour of agents is heavily influenced by Kyle (1985). He shows that profit-maximizing informed investors attempt to camouflage their information, e.g. by spreading trades over time. Admati and Pfleiderer (1988) assume two types of uninformed traders, "discretionary" liquidity traders, who have some choice over the time at which they transact, and "nondiscretionary" liquidity traders whose orders are assumed to arrive in a random fashion. They show that it is optimal for liquidity traders and also for insiders to trade together leading to concentrations of trading in particular time periods. While both studies ignore the choice of the trade size Barclay and Warner (1993) examine the proportion of cumulative price changes that occur in certain volume categories. Based on an empirical study they find evidence that most of the cumulative price change is due to medium-sized trades. This result is consistent with the hypothesis that informed traders tend to use medium volume sizes <sup>4</sup>. Empirical evidence on this hypothesis is provided by the impact of the contemporaneous volume per transaction on the expected waiting time until the next trade. If informed investors trade medium sizes and want to exploit their informational advantage by executing a transaction as soon as possible then one would expect that medium trading volumes have the strongest impact on the trading intensity leading to a highly nonlinear relationship between inter-trade durations and the present trading volume. These implications are summarized in the following hypothesis:

**Hypothesis H3:** *A nonlinear relationship can be observed between the contemporaneous volume and the expected time between trades.*

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<sup>4</sup>A paper with a related focus is Kempf and Korn (1999) who empirically analyze the relation between unexpected net order flow and price changes and find highly nonlinear relationships.

### 3. THE PROPORTIONAL HAZARD ARMA MODEL

A specific feature of inter-trade durations is the occurrence of clustering. In the econometric literature there exist two central approaches accounting for intertemporally correlated duration data: Engle and Russell (1995) introduce the class of Autoregressive Conditional Duration (ACD) models for serially correlated inter-trade durations, which is based on a parametric specification of the conditional expectation of the duration and employs the GARCH methodology developed to model the volatility of price processes to analyze the time between trades. Ghysels, Gouriéroux, and Jasiak (1998) propose a class of two factor models for duration data, where the first factor accommodates dynamics in the conditional mean and the second factor in the conditional variance. Because of its strong relation to stochastic volatility models they call it the Stochastic Volatility Duration (SVD) model. They show that the SVD model captures interactive dynamics of the conditional mean and variance and models clustering and persistence effects in both conditional moments.

These types of models belong to the class of accelerated failure time models, i.e. models where covariates accelerate or decelerate the time between transactions. A drawback of these models is that they do not specify an impact of covariates on the hazard function, thus, it is not possible to investigate the influence of covariates on the duration dependence. A further drawback is the requirement of restrictive parametric specifications for the assumed distribution of the durations, at least if one is indeed interested in the shape of the hazard function. If only the conditional expectation of the time between transactions is of interest, one has to account for the results of Lee and Hansen (1994) and Lumsdaine (1996) for the PML interpretation of GARCH models which were applied by Engle and Russell (1998) to ACD models. Hence, the ACD model provides consistent estimates of the conditional expectation even when the distribution of the duration process is misspecified.

Grammig and Maurer (1999) analyze the performance of different ACD specifications via Monte Carlo studies and show that standard ACD models are

highly sensitive to the distributional assumptions employed. For this reason Grammig and Maurer introduced a more flexible ACD model by assuming a Burr-distribution for the standardized durations<sup>5</sup> and provide evidence in form of Monte Carlo simulations in favour of this ML estimator.<sup>6</sup> Bauwens, Giot, Grammig, and Veredas (2000) compare the predictive performance of several ACD, Log-ACD (see Bauwens and Giot (1997)) and SVD models via density forecasts and show that most of these specifications fail to capture the true (conditional) distribution of durations. They illustrate that these models work better the more flexible the assumed distribution of the durations is.

Based on the PML property of ACD models it is possible to use the empirical distribution of the residuals to obtain a nonparametric estimate of the baseline hazard. The drawback is that the resulting baseline hazard rate is based on standardized durations, i.e. durations that are standardized by their conditional duration. Because we are interested in duration dependencies based on pure, non-transformed, inter-trade durations that allow us to interpret the hazard function directly as a natural liquidity measure, the use of ACD models in this context is not reasonable.

Because of these reasons we use an extended semiparametric proportional hazard specification which allows us to account for serial dependency in the inter-trade duration process and provides a semiparametric estimation of the baseline hazard rate. Because this model is based on an ARMA structure we call it Proportional Hazard ARMA (PHARMA) model. The estimation procedure is based on a discretization of the dependent variable, thus the model corresponds to a particular quantal response model introduced by Han and Hausman (1990). Because of its dynamic structure it can be seen as a combination of ACD type models and hazard rate models<sup>7</sup>.

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<sup>5</sup>The Burr-ACD (BACD) nests the standard Exponential-ACD and Weibull-ACD models as special cases.

<sup>6</sup>Bauwens and Veredas (1999) introduced the Stochastic Conditional Duration (SCD) model where the durations are generated by a latent stochastic factor allowing an autoregressive process. The main innovation of this class of models is to allow for a wider range of shapes of hazard functions.

<sup>7</sup>For more details see Gerhard and Hautsch (2000).

Consider the sequence of arrival times  $t_0, t_1, \dots, t_n$  with  $t_0 < t_1 < \dots < t_n$  as a stochastic process. Associated with this process for the arrival times is a process for the waiting times between the trades,  $\tau_i = t_i - t_{i-1}$ ,  $i = 1, \dots, n$ , the inter-trade durations.

In order to provide a flexible baseline hazard rate we use a semiparametric proportional hazard specification as a starting point

$$(1) \quad \lambda(\tau_t | x_t) = \lambda_0(\tau_t) \exp(-x_t' \beta), \quad t = 1, \dots, n,$$

where  $\lambda_0(\tau_t)$  is an unspecified baseline hazard,  $x_t$  a vector of covariates and  $\beta$  the corresponding vector of coefficients.

The proportional hazard model admits an interpretation as a linear regression model using the fact that

$$(2) \quad \tilde{\tau}_t = x_t' \beta + \epsilon_t,$$

where  $\tilde{\tau}_t \equiv \ln \int_0^{\tau_t} \lambda_0(s) ds$  and  $\epsilon_t$  is an identically independently extreme value distributed error term with mean  $c = E[\epsilon] = -0.57722$  and variance  $V[\epsilon] = \pi^2/6$ . See e.g. Kiefer (1988) or Han and Hausman (1990).<sup>8</sup>

By augmenting the latent model by an ARMA structure to account for serial dependency in the inter-trade duration process we obtain

$$(3) \quad \tilde{\tau}_t = x_t' \beta + u_t,$$

with

$$(4) \quad u_t = \sum_{j=1}^p \phi_j u_{t-j} + \sum_{j=1}^q \theta_j (\epsilon_{t-j} + w_{t-j}' \gamma) + w_t' \gamma + \epsilon_t.$$

A vector of exogenous explanatory variables  $w_t$  is included in the dynamic structure with  $\gamma$  as the corresponding vector of coefficients.

Note that it is sufficient that the latent variable  $\tilde{\tau}_t$  is conditional extreme value distributed, given the past and the explanatory variables, hence

$$\tilde{\tau}_t | \bar{\tau}_{t-1}, \bar{w}_{t-1}, x_t \sim \mathcal{EV}.$$

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<sup>8</sup>The model can be extended to account for unobserved heterogeneity similar to the SVD model proposed by Ghysels, Gourioux, and Jasiak (1998). Such effects are included by specifying a compounder  $\omega$  acting multiplicatively with the hazard function. By analyzing LIFFE Bund Future data, Hautsch (1999) shows that unobservable effects captured by the compounder are only very weak. For this reason we ignore the impact of such effects.

Using the standard state space form, the model can be rewritten as

$$(5) \quad \tilde{\tau}_t = H \cdot \xi_t + x_t' \beta$$

$$(6) \quad \xi_t = F \cdot \xi_{t-1} + e_1 \cdot w_t' \gamma + e_1 \cdot \epsilon_t,$$

where

$$H = \begin{bmatrix} 1 & \theta_1 & \dots & \theta_{r-1} \end{bmatrix}, \quad F = \begin{bmatrix} \phi_1 & \dots & \phi_r \\ I_{r-1} & & 0 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}'$$

and  $r$  denotes the dimension of the state space defined as  $r = \max(q+1, p)$ . Note that then  $\theta_i$  and  $\phi_i$  with  $i > q$  or  $i > p$  respectively are defined to be equal zero.

We use a conditional maximum likelihood estimation procedure proposed by Gerhard and Hautsch (2000) where the model is regarded as an ordered response model with an autoregressive structure in the latent process given by eq. (3) and (4). In this context we categorize the inter-trade durations  $\tau_t$  by using  $\mu_k$ ,  $k = 1, \dots, K-1$ , as the observable thresholds between the categories.

We observe the inter-trade duration within the category  $(\mu_{k-1}; \mu_k]$  if the latent variable  $\tilde{\tau}_t$  lies between the two thresholds  $(\tilde{\mu}_{k-1}; \tilde{\mu}_k]$ , i.e. the conditional probability for observing  $\mu_{k-1} < \tau_t \leq \mu_k$  is

$$(7) \quad \text{Prob}(\mu_{k-1} < \tau_t \leq \mu_k | \bar{\tau}_{t-1}, \bar{w}_{t-1}, x_t) = \int_{\tilde{\mu}_{k-1}-m_t}^{\tilde{\mu}_k-m_t} f_\epsilon(s) ds,$$

where  $m_t$  is defined by

$$m_t \equiv E[\tilde{\tau}_t | \bar{\tau}_{t-1}] - c$$

and  $f_\epsilon(s)$  denotes the density function of  $\epsilon_t$  given by

$$f_\epsilon(s) = \exp(s - \exp(s)).$$

Hence, in this ordered response framework the conditional log likelihood function, given  $\bar{\tau}_{t-1}, \bar{w}_{t-1}, x_t$ , takes the usual form of

$$(8) \quad \log \mathcal{L} = \sum_{t=1}^N \sum_{k=1}^K y_{tk} \ln \int_{\tilde{\mu}_{k-1}-m_t}^{\tilde{\mu}_k-m_t} f_\epsilon(s) ds,$$

where the indicator variable  $y_{tk}$  is defined by

$$y_{tk} = \begin{cases} 1 & , \text{ if } \mu_{k-1} < \tau_t \leq \mu_k \\ 0 & , \text{ else.} \end{cases}$$

The nonparametric baseline survivor function is obtained directly by the estimated thresholds. It can be calculated at the  $K - 1$  discrete points by

$$(9) \quad S_0(\mu_k) = \exp(-\exp(\tilde{\mu}_k)), \quad k = 1, \dots, K - 1.$$

Note that the function  $m_t$  is based on an ARMA structure, thus, the log likelihood (8) has to be computed using a recursion of the latent process. We calculate the conditional expectation of the latent variable given past realizations of the observable dependent variable  $E[\xi_t|\bar{\tau}_{t-1}]$  using a recursion based on the state equation (6). Following the recursive procedure proposed by Gerhard (2000) the conditional mean  $E[\xi_t|\bar{\tau}_{t-1}]$  can be computed by <sup>9</sup>

$$(10) \quad E[\xi_t|\bar{\tau}_{t-1}] = c \cdot e_1 + F(E[\xi_{t-1}|\bar{\tau}_{t-2}] + e_1 \cdot E[\epsilon_{t-1}|\bar{\tau}_{t-1}]).$$

From this,  $m_t$  is directly evaluated as

$$(11) \quad m_t = E[\tilde{\tau}_t|\bar{\tau}_{t-1}] - c = H \cdot E[\xi_t|\bar{\tau}_{t-1}] - c.$$

The recursion is initialized with the unconditional expectation of the state variable

$$(12) \quad E[\xi_1|\bar{\tau}_0] = E[\xi_1] = 0.$$

Because the dynamic structure is based on variables that are unobservable ( $\tilde{\tau}_t$  and  $\epsilon_t$ ), we use estimates of these variables to execute the recursion (10). Estimates of  $\tilde{\tau}_t$  and  $\epsilon_t$  are obtained by using the concept of generalized errors proposed by Gouriéroux, Monfort, and Trognon (1985). Hence, if  $\mu_{k-1} < \tau_t \leq \mu_k$ , the conditional expectation of the error  $\epsilon_t$  is given by

$$(13) \quad E[\epsilon_t|\mu_{k-1} < \tau_t \leq \mu_k, \bar{\tau}_{t-1}, \bar{w}_{t-1}, x_t] = \frac{1}{F_\epsilon(\tilde{\mu}_k - m_t) - F_\epsilon(\tilde{\mu}_{k-1} - m_t)} \cdot \int_{\tilde{\mu}_{k-1} - m_t}^{\tilde{\mu}_k - m_t} s f_\epsilon(s) ds,$$

where  $F_\epsilon(\cdot)$  denotes the distribution function of  $\epsilon$ . Based on these generalized errors we obtain estimates of  $\tilde{\tau}_t$  that are used in the recursion.

A necessary condition for consistency of this maximum likelihood estimator is that the moment condition  $E[\frac{\partial \log \mathcal{L}}{\partial \Theta}] = 0$  holds for the true parameter  $\Theta_0$ . Because

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<sup>9</sup>For ease of notation the regressors  $x_t$  and  $w_t$  are omitted. For more details see Gerhard and Hautsch (2000).

our approach is based on a latent model the score of the observable model is not directly applicable. By using the relationships between latent and observable models proposed by Gouriéroux, Monfort, Renault, and Trognon (1987) we verify that the expectation of the score of the observable model is zero (see in the appendix).

Under the usual regularity conditions we can show consistency and asymptotic normality for this maximum likelihood estimator (see Gerhard and Hautsch (2000)).

#### 4. THE DATA

The sample contains intra-day transaction data from the Bund Future trading at the screen based trading system of the Deutsche Terminbörse (DTB), Frankfurt, from 01/30/95 to 02/24/95, corresponding to 20 trading days. Within this period the Bund-Future was one of the most liquid futures in Europe and corresponded to a 6% German government bond of DEM 250.000 face value. The Bund Future had a maturity of 8.5 years and four contract maturities per year, March, June, September and December. In the sampling interval prices were denoted in basis points of face value, thus, one tick was equivalent to a value of DEM 25.

The data set contains time stamped prices and volumes and consists of 44810 observations, where the overnight durations are omitted. Furthermore, we refrain from using the first 10 minutes of a trading day to avoid the opening phase.

We use a categorization that ensures to derive conditional failure probabilities based on longer time intervals. Because we want to get insights in the trading behaviour and learning processes of market participants, it is not reasonable to regard extremely short inter-trade durations (e.g.  $\leq 10$  seconds). For this reason we use a categorization based on 30 second intervals (30,60,...,180) that is large enough to could be associated with learning processes of traders and reveal market dynamics up to 3 minutes. Because the distribution of the waiting times is skewed to the right (see table 1) we use additional categories for lower durations to ensure satisfactory frequencies of the observations in the categories.

Several empirical studies (Wood, McInish, and Ord (1985), Engle and Russell (1995), Engle and Russell (1997), Guillaume, Dacorogna, Dave, Müller, Olsen, and Pictet (1996) or Dacorogna, Morgenegg, Müller, Olsen, Pictet, and Schwarz (1990)) found evidence for highly significant seasonality patterns in the volatility of the return series as well as in the trade frequency of the transaction process. To account for intraday seasonalities we use the flexible Fourier form proposed by Andersen and Bollerslev (1998) based on Gallant (1981) which is given by

$$(14) \quad s(\delta, t^*, P) = \delta_1 \cdot t^* + \sum_{p=1}^P (\delta_{c,p} \cos(t^* \cdot 2\pi p) + \delta_{s,p} \sin(t^* \cdot 2\pi p)),$$

where  $p$  is identical with the order of the term,  $t^* \in [0, 1]$  is defined by

$$(15) \quad t^* = \frac{\text{seconds since 8:40 a.m.}}{\text{seconds between 8:40 a.m. and 5.15 p.m.}}$$

and  $\delta_{c,p}$ ,  $\delta_{s,p}$  and  $\delta$  denote the corresponding coefficients <sup>10</sup>.

To check hypothesis H2, concerning the impact of an increase of information heterogeneity, we define two dummy variables indicating trading after 2:30 p.m., the opening of U.S. trading, and indicating the 02/20/95, the 'President's Day', American holiday. To investigate the further market microstructure hypotheses we include log-volume and absolute price changes as explanatory variables.

## 5. EMPIRICAL RESULTS

**5.1. Persistence and Intraday-Seasonalities.** To investigate the autoregressive structures in the data we start our analysis by calculating the autocorrelation (acf) and partial autocorrelation functions (pacf) of the inter-trade durations. Table 2 shows the acf and pacf of the data that indicate highly significant autocorrelations. They depict a slowly decaying rate of dependence, which is typical for a process with high persistence, a feature of the data that is well documented in recent literature <sup>11</sup>.

To simplify the model selection we first run several ARMA models on the raw and also seasonally adjusted log durations. This amounts to estimating a log ACD

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<sup>10</sup>Within the observation period at the DTB trading took place between 8:30 a.m. and 5.15 p.m.

<sup>11</sup>See e.g. Jasiak (1999) who accounts for this high persistence by modelling a fractional integrated ACD-model.



model as suggested by Bauwens and Giot (1997). The close relationship between the acf implied by the PHARMA model employed here and the ACD specification justifies this procedure. A more detailed discussion of the relationship can be found in Gerhard and Hautsch (2000). Table 3 shows the results of four ARMA specifications based on the raw data. The high values of the ARMA parameters indicate a high persistence of the duration-process and are comparable to the results found by Engle and Russell (1998) who investigated price intensities using ACD models. Table 4 presents the corresponding results based on seasonally adjusted durations <sup>12</sup>. The fact that ARMA parameters are nearly unaffected indicates that the persistence is not reduced due to the inclusion of seasonality parameters.

The model selection is based on the Bayes information criterion (BIC) leading to an ARMA(2,2) for the raw durations respectively an ARMA(1,1) for the seasonally adjusted waiting times as the best specification. We use these results as a starting point for the model selection in the PHARMA models and obtain a PHARMA(1,2) as the best specification, again based on the BIC criterion.

Table 5 shows the estimation results of the PHARMA(1,2) model <sup>13</sup> for three different specifications. Panel A presents the results of a regression without any explanatory variables, based only on the thresholds and the ARMA coefficients, while panel B and C contain the corresponding results with included seasonal covariates <sup>14</sup>. The similarity of the ARMA parameters to the simple ARMA(1,2) regressions on the raw respectively the seasonal adjusted log durations (see tables 3 and 4) indicate the robustness of the results. Furthermore, by running several regressions with different categorizations of the durations we find evidence that the estimates of the ARMA parameters and also the coefficients of the covariates

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<sup>12</sup>In this context we regress the log-durations on the variables of an flexible Fourier form of order 4, inclusive two dummies indicating the opening of American trading at 2:30 p.m. and the American 'President's Day' at 02/20/95, by OLS. Based on these consistent (but inefficient) estimates of the covariates, we calculate the residuals and use these as seasonal adjusted durations.

<sup>13</sup>The maximum likelihood estimation of the model is performed using the BFGS algorithm with numerical derivatives in GAUSS.

<sup>14</sup>We also run regressions by including dummy variables that accounted for day-of-the-week-effects but didn't find any significance concerning such effects.

are not effected by the choice of the categories. This result is in accordance with the property of semiparametric proportional hazard models that consistent estimates of the coefficients of the explanatory variables are obtained even when the form of the baseline hazard is unknown (see Meyer (1990)).

To investigate the impact of intraday-seasonalities we calculate the influence of these variables on the probability to survive a inter-trade duration of 5 and 30 seconds. Figure 1 shows the typical intraday seasonality pattern with high market activities in the morning, a significant dip at the lunch time and the shortest inter-trade durations after the opening of the American trading at 2:30 p.m.

It is an interesting result that the inclusion of the two dummies indicating trading after 2:30 p.m. and the American 'President's Day' at 02/20/95 (see panel C) decreases the significance of most of the trigonometric terms. Hence, the main impact of the estimated intraday seasonalities seems to be captured by the 2:30-dummy. The high significance of this dummy indicates that the opening of the American trading has a strong impact on the speed of market activities and increases the liquidity according to lower trade-to-trade waiting times.

**5.2. Hazard Rates and Survivor Functions.** Figure 3 illustrates estimates of the discrete hazard rate, i.e. the conditional probability for the end of a spell in the next duration category, given the time it lasted already, conditional on trading before and after 2:30 p.m. and at the American holiday, given by <sup>15</sup>

$$(16) \quad \tilde{\lambda}_0(\mu_k) = \frac{S_0(\mu_k) - S_0(\mu_{k+1})}{S_0(\mu_k)}, \quad k = 1, \dots, K - 1.$$

These conditional failure probabilities allow us to characterize the duration dependence for longer time intervals that provide us insight into the way traders infer information from no-trade-time intervals. By virtue of the chosen categorisation it is straightforward to interpret the conditional failure probabilities depicted in figure 3. There we depict the conditional probabilities for a transaction to occur within the next 30 seconds, given that we have just observed

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<sup>15</sup>The functions are conditioned on mean values for the explanatory and the dynamic variables.

the last transaction and given that the last transaction happened before 30, 60, 90, 120, and 150 seconds. We note that the hazard rate is decreasing, i.e. the longer the last trade dates back the lower the probability for observing the next transaction.

This result supports the hypothesis that market participants tend to trade according to, in our sense, 'technical' trading schemes. Hence traders, account not only for the amount of information but also for the speed of the information arrival. This result can be seen as empirical evidence for the fact that the market is dominated by agents that tend to learn from past market sequences and not only to adjust their portfolios based on exogenous criteria. This result supports in some sense the hypothesis that traders associate no-trade-intervals with the lack of information and, thus, is quite plausible in the light of the seminal contribution of Easley and O'Hara (1992).

We find slight evidence that the shape of the hazard rate flattens out for longer durations (even with a slightly increasing pattern between 90 and 120 seconds) that may be due to a high base level of liquidity, i.e. the abundance of market participants with exogenous, in our sense 'fundamental', motivations.

The comparison of the different (discrete) hazard functions conditional on trading before and after 2:30 p.m. and at the American holiday, provides further insight into the impact of liquidity trading. One could argue that the inflow of additional traders from the U.S. has a particularly high share in noise traders as the decrease in the conditional probability increases from 7 to 11 to 14 percentage points comparing the sample after 2:30 p.m. to the sample before 2:30 p.m. and to the U.S. holiday.

The slight flattening of the hazard shape after 2:30 p.m. can be seen as weak evidence for the hypothesis that the hazard becomes more flat the more traders with different trading behaviours enter the market. Hence, an increase of the liquidity seems to lead to a flattening of the hazard shape.

Figure 2 shows the estimated baseline survivor functions conditional on trading before and after 2:30 and at the American holiday <sup>16</sup>. The graphs show that the pattern of the baseline survivor functions change significantly indicating that the American trading plays an important role for the market dynamics. While the probability for observing inter-trade time intervals longer than 30 seconds is approximately 0.37 at U.S. holiday, this probability is approximately 0.09 at 'normal' days before 2:30 p.m. and nearly zero after 2:30. These significant changes of the patterns of the survivor function and, thus, the underlying duration distribution <sup>17</sup>, may economically be attributed to an increased heterogeneity in traders' price signals caused by a major inflow of potential traders from the U.S. market that would confirm hypothesis H2. While the mean inter-trade duration is at the U.S. holiday three times as large as an 'normal days' (see Table 1), the transaction volume is nearly unaffected. Hence, liquidity effects seem to be caused by an increase of the trading intensity, not by a change in trading volumes.

**5.3. Testing the market microstructure hypotheses.** In order to check the empirical evidence of the market microstructure hypotheses proposed in section 2, we run two regressions with market microstructure covariates included (see table 6). Since the main impact of deterministic intraday seasonalities is captured by the 2:30-dummy we omit the trigonometric seasonality terms. Panel D (table 6) presents regression results including a polynomial of log-volume. While the linear term  $\log vol$  is insignificant, the significant quadratic  $\log vol^2$  and cubic term  $\log vol^3$  indicate a nonlinear impact of volume on the expected time until the next transaction. To illustrate this result we plot the aggregated impact of the volume-covariates on the probability to survive 5 respectively 30 seconds (see figure 5). The graph depicts a slightly non-monotonic function with a decreasing pattern for volumes up to approximately 60 and an almost flat shape for larger volumes. Overall, the marginal influence of volume on the latent variable is very

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<sup>16</sup>The functions are conditioned on mean values for the explanatory and the dynamic variables.

<sup>17</sup>This feature is also reflected in the descriptive statistics (see Table 1).

weak and is well approximated by a linear function. Thus, we find no empirical evidence for a strategic behaviour of informed traders.

In order to test our hypothesis H1, we include the first lags of the log volume and the absolute price change dynamically (see panel E, table 6). For both variables we find highly significant negative coefficients. Hence, the higher past volumes and the more volatile the past price sequence the lower the expected trading intensity which supports hypotheses 1a and 1b. The calculation of the influence of past volumes and past absolute price changes on the present trade frequency depends not only on the coefficient of the regressor but also on the ARMA parameters. We find a slowly decaying lag structure with the median lag at 18 for both regressors. Taking also into account that the mean time between transactions is about 14 seconds (see Table 1), we can - *cum grano salis* - invoke the intuition that the weighted volume and the weighted absolute price changes of the last 3 minutes before a transaction make up about 50% of these regressors impact. The coefficients of the lag structure are all negative. Thus, these results can be interpreted as empirical evidence for the hypothesis that investors seem to increase their preference for immediacy of further transactions if past market activities provide information to them.

## 6. CONCLUSIONS

In this study we use a proportional hazard ARMA model to estimate hazard rates of inter-trade durations of the Bund-Future trading at the DTB, Frankfurt. This analysis provides insight into the duration dependence of failure probabilities, i.e. the probability for observing the next trade conditional on the elapsed time since the last transaction. The shape of this (discrete) hazard function provides evidence on the way traders aggregate information.

We estimate the hazard rate by using a PHARMA model that accounts for serial dependency in inter-trade durations and also allows for a nonparametric estimation of the underlying baseline hazard. Overall, we find a significantly decreasing hazard rate which flattens out slightly for longer durations indicating a negative duration dependency between the timing of particular trades. This

result supports the hypothesis that traders tend to trade more 'technical' oriented, i.e. they account not only for the amount of information but also for the speed of information arrival. Furthermore, we investigate the impact of an increase of the heterogeneity of information on the trading intensity by comparing survivor and hazard functions based on market phases before and after the opening of the American trading and at a U.S. holiday. We obtain significant changes in market dynamics indicated by differences in the estimated survivor functions, and, thus the duration distribution.

The inclusion of covariates statically and dynamically allows us to shed some light on the impact of microstructure variables, indicating the state of the market, on the hazard function. We investigate whether the timing of trades reflects the decisions of traders which learn from past market activities.

By including past volumes and absolute price changes as dynamic covariates we find evidence for the fact that these variables have a significantly negative impact on the expected trading intensity. These results are in accordance with market microstructure hypotheses which imply that the informativeness of past sequences of market activities is reflected in a traders' preference for immediacy of transactions, i.e. in lower inter-trade durations.

## REFERENCES

- ADMATI, A., AND P. PFLEIDERER (1988): "A Theory of Intraday Patterns: Volume and Price Variability," *Review of Financial Studies*, 1, 3–40.
- ANDERSEN, T. G., AND T. BOLLERSLEV (1998): "Deutsche Mark-Dollar Volatility: Intraday Activity Patterns, Macroeconomic Announcements, and Longer Run Dependencies," *Journal of Finance*, 53, 219–265.
- BARCLAY, M. J., AND J. B. WARNER (1993): "Stealth Trading and Volatility," *Journal of Financial Economics*, 34, 281–305.
- BAUWENS, L., AND P. GIOT (1997): "The Logarithmic ACD Model: An Application to Market Microstructure and NASDAQ," *Working Paper of the CORE, Universite Catholique de Louvain*.
- BAUWENS, L., P. GIOT, J. GRAMMIG, AND D. VEREDAS (2000): "A Comparison of Financial Duration Models Via Density Forecasts," Discussion paper, CORE.
- BAUWENS, L., AND D. VEREDAS (1999): "The Stochastic Conditional Duration Model: A Latent Factor Model for the Analysis of Financial Durations," Discussion paper, CORE.
- BLUME, L., D. EASLY, AND M. O'HARA (1994): "Market Statistics and Technical Analysis," *Journal of Finance*, 49 (1), 153–181.
- DACOROGNA, M. M., C. MORGENEGG, U. A. MÜLLER, R. B. OLSEN, O. V. PICTET, AND M. SCHWARZ (1990): "Statistical Study of Foreign Exchange Rates, Empirical Evidence of A Price Change Scalling Law, and Intraday Analysis," *Journal of Banking and Finance*, 14, 1189–1208.
- DIAMOND, D. W., AND R. E. VERRECCHIA (1981): "Information Aggregation in a Noisy Rational Expectations Economy," *Journal of Financial Economics*, 9, 221–235.
- EASLEY, D., AND M. O'HARA (1992): "Time and Process of Security Price Adjustment," *The Journal of Finance*, 47 (2), 577–605.
- ENGLE, R. (2000): "The Econometrics of Ultra-High-Frequency Data," *Econometrica*, 68, 1, 1–22.
- ENGLE, R. F. (1996): "The Econometrics of Ultra-High Frequency Data," Discussion paper, UCSD.
- ENGLE, R. F., AND J. RUSSELL (1998): "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data," *Econometrica*, 66, 1127–1162.
- ENGLE, R. F., AND J. R. RUSSELL (1995): "Forecasting Transaction Rates: The Autoregressive Conditional Duration Model," Discussion paper, UCSD.
- (1997): "Forecasting The Frequency of Changes in Qouted Foreign Exchange Prices with Autoregressive Conditional Duration Model," *Journal of Empirical Finance*, 4, 187–212.
- GALLANT, R. A. (1981): "On the Bias in Flexible Functional Forms and an Essential Unbiased Form: The Fourier Flexible Form," *Journal of Econometrics*, 15, 211–245.
- GERHARD, F. (2000): "A Latent Dynamic in the Context of the Linear Exponential Family," Discussion paper, University of Konstanz.
- GERHARD, F., AND N. HAUTSCH (2000): "A General Framework for Dynamic Proportional Hazard Models," Discussion paper, University of Konstanz.
- GHYSELS, E., C. GOURIEROUX, AND J. JASIAK (1998): "Stochastic Volatility Duration Models," Discussion paper, CIRANO.
- GOURIEROUX, C., A. MONFORT, E. RENAULT, AND A. TROGNON (1987): "Generalised Residuals," *Journal of Econometrics*, 34, 5–32.
- GOURIEROUX, C., A. MONFORT, AND A. TROGNON (1985): "A General Approach to Serial Correlation," *Econometric Theory*, pp. 315–340.
- GRAMMIG, J., AND K.-O. MAURER (1999): "Non-Monotonic Hazard Funfunctions and the Autoregressive Conditional Duration Model," *Humboldt-Universität zu Berlin, Discussion Paper*, 50, 1–46.

- GUILLAUME, D. M., M. M. DACOROGNA, R. R. DAVE, U. A. MÜLLER, R. B. OLSEN, AND O. V. PICTET (1996): "From the Birds Eye to the Microscope: A Survey of New Stylized Facts of the Intra-Daily Foreign Exchange Markets," *Finance and Stochastics*, 1, 95–129.
- HAN, A., AND J. A. HAUSMAN (1990): "Flexible Parametric Estimation of Duration and Competing Risk Models," *Journal of Applied Econometrics*, 5, 1–28.
- HAUTSCH, N. (1999): "Analyzing the Time Between Trades with a Gamma Compounded Hazard Model. An Application to LIFFE Bund Future Transactions," Discussion Paper 99/03, CoFE, Konstanz.
- HELLWIG, M. (1982): "Rational Expectations Equilibrium with Conditioning on Past Prices: A Mean-Variance Example," *Journal of Economic Theory*, 26, 279–312.
- JASIAK, J. (1999): "Persistence in Intraday Durations," Discussion paper, York University, Montreal.
- JOHNSON, N. L., S. KOTZ, AND N. BALAKRISHNAN (1994): *Distributions in Statistics: Continuous Univariate Distributions (2. Vol.)*. Wiley, New York.
- KEMPF, A., AND O. KORN (1999): "Market Depth and Order Size," *Journal of Financial Markets*, 2, 29–48.
- KIEFER, N. M. (1988): "Economic Duration Data and Hazard Functions," *Journal of Economic Literature*, 26, 646–679.
- KYLE, A. S. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53 (6), 1315–1335.
- LANG, L. H. P., R. H. LITZENBERGER, AND V. MADRIGAL (1992): "Testing Financial Market Equilibrium under Asymmetric Information," *Journal of Political Economy*, 100, 317–348.
- LEE, S., AND B. HANSEN (1994): "Asymptotic Theory for the GARCH(1,1) Quasi-Maximum Likelihood Estimator," *Econometric Theory*, 10, 29–52.
- LUMSDAINE, R. (1996): "Consistency and Asymptotic Normality of the Quasi-Maximum Likelihood Estimator in IGARCH(1,1) and Covariance Stationary GARCH(1,1) Models," *Econometrica*, 64, 575–596.
- MEYER, B. D. (1990): "Unemployment Insurance and Unemployment Spells," *Econometrica*, 58 (4), 757–782.
- WOOD, R. A., T. H. MCINISH, AND J. K. ORD (1985): "An Investigation of Transactions Data for NYSE Stocks," *Journal of Finance*, 40, 723–741.



## 7. APPENDIX

**7.1. The expectation of the score.** For ease of exposition the following proof is illustrated for an PHARMA(1,0) process without covariates. The latent process is defined as

$$\begin{aligned}\tilde{\tau}_t &= \phi \tilde{\tau}_{t-1} + \epsilon_t \\ &= \sum_{j=1}^{\infty} \phi^j \epsilon_{t-j} + \epsilon_t,\end{aligned}$$

where  $\epsilon_t$  is extreme value distributed with density

$$f(\epsilon) = \exp(\epsilon - \exp(\epsilon)).$$

Thus, the score of the latent model is given by

$$\frac{\partial \log \mathcal{L}^*}{\partial \phi} = \sum_{t=2}^T \left[ - \sum_{j=1}^{\infty} j \phi^{j-1} \epsilon_{t-j} + \exp(\epsilon_t) \sum_{j=1}^{\infty} j \phi^{j-1} \epsilon_{t-j} \right].$$

Along the lines of the work of Gouriéroux, Monfort, Renault, and Trognon (1987) (lemma 3) the score of the observable model is given by the conditional expectation of the score of the latent model, given the observable dependent variables  $\bar{\tau}_t$ ,

$$\begin{aligned}\frac{\partial \log \mathcal{L}}{\partial \phi} &= \mathbb{E} \left[ \frac{\partial \log \mathcal{L}^*}{\partial \phi} \middle| \bar{\tau}_t \right] \\ &= \sum_{t=2}^T \left[ - \sum_{j=1}^{\infty} j \phi^{j-1} \mathbb{E}[\epsilon_{t-j} | \bar{\tau}_t] + \mathbb{E} \left[ \exp(\epsilon_t) \sum_{j=1}^{\infty} j \phi^{j-1} \epsilon_{t-j} \middle| \bar{\tau}_t \right] \right].\end{aligned}$$

Hence, the expectation of the score of the observable model is given by

$$\begin{aligned}\mathbb{E} \left[ \frac{\partial \log \mathcal{L}}{\partial \phi} \right] &= \sum_{t=2}^T \left[ - \sum_{j=1}^{\infty} j \phi^{j-1} \mathbb{E} \mathbb{E}[\epsilon_{t-j} | \bar{\tau}_t] + \mathbb{E} \mathbb{E} \left[ \exp(\epsilon_t) \sum_{j=1}^{\infty} j \phi^{j-1} \epsilon_{t-j} \middle| \bar{\tau}_t \right] \right] \\ (17) \quad &= \sum_{t=2}^T \left[ - \sum_{j=1}^{\infty} j \phi^{j-1} \mathbb{E}[\epsilon_{t-j}] + \mathbb{E} \left[ \exp(\epsilon_t) \sum_{j=1}^{\infty} j \phi^{j-1} \epsilon_{t-j} \right] \right]\end{aligned}$$

$$\begin{aligned}(18) \quad &= \sum_{t=2}^T \left[ - \sum_{j=1}^{\infty} j \phi^{j-1} \mathbb{E}[\epsilon_{t-j}] + \sum_{j=1}^{\infty} j \phi^{j-1} \mathbb{E}[\exp(\epsilon_t)] \cdot \mathbb{E}[\epsilon_{t-j}] \right] \\ &= \sum_{t=2}^T \left[ - \sum_{j=1}^{\infty} j \phi^{j-1} \mathbb{E}[\epsilon_{t-j}] (1 - \mathbb{E}[\exp(\epsilon_t)]) \right] = 0.\end{aligned}$$

Eq. (17) is based on the law of iterated expectations where (18) uses the assumption of independence between the latent error terms. It is easily to show that the random variable  $\exp(\epsilon)$  is exponential distributed, thus  $\mathbb{E}[\exp(\epsilon)] = 1$  and the result follows.

**7.2. Inter-trade hazard rates implied by different trading schemes.** In the following we show that, based on a Poisson process for the arrival of price signals, 'fundamental' and 'technical' trading schemes lead to different hazard rates of the resulting inter-trade durations.

**Proof of Proposition 1:** If the trader executes a transaction after every  $(\mu + 1)$ th price signal the resulting inter-trade durations  $\tau$  are distributed according to the sum of exponential distributed random variables that follow an Erlang  $(\mu)$ -distribution<sup>18</sup> with hazard function

$$\lambda_\mu(k) = \frac{\lambda_0(\lambda_0 k)^{\mu-1}/(\mu-1)!}{\sum_{i=1}^{\mu} (\lambda_0 k)^{i-1}/(i-1)!}$$

where  $\lambda_0$  denotes the parameter of the underlying exponential distribution. It is easy to show that  $\frac{\partial \lambda_\mu(k)}{\partial k} > 0$  for  $\mu > 1$ .

**Proof of Proposition 2:** We denote the relevant time horizon of an investor by  $T$  and the duration between the price signals in period  $t-1$  and  $t$  by  $\kappa_t$ ,  $t = 1, \dots, n$ . We assume that a market participant initiates a trade if he observes the last  $\mu + 1$  signals within a time interval  $T$ , i.e. the sum of the last  $\mu$  inter-signal waiting times has to be lower than  $T$ . Furthermore, we assume that the last trade occurred in  $t$  and denote the time between  $t$  and the next transaction by  $\tau_t$ . The proof proceeds as following:

First, we characterize the shape of the inter-trade hazard function for durations  $\tau_t \leq T$  because for these durations the hazard rate has a significantly different shape than for waiting times  $\tau_t > T$ . By considering a point of time  $t + k$ ,  $0 < k \leq T$ , after the trade in  $t$ , we have to characterize different situations.

In the first case we characterize points of time  $k$  where the next price signal after the transaction in  $t$  leads to the next transaction, i.e. the inter-trade duration corresponds to the inter-signal waiting time. The second case concerns points of time where the next price signal leads with certainty to no further transaction because in these points of time the last  $\mu + 1$  signals occurred within a time interval larger than  $T$ . In the third situation we consider points of time where not the first but only the second price signal after  $t$  leads to the next transaction, i.e. the inter-trade duration corresponds to the sum of the following two inter-signal waiting times. All other points of time  $0 < k \leq T$  are characterized in a similar way. For these different cases we calculate the corresponding conditional hazard rates.

Second, based on the particular conditional hazard rates, given a certain point of time  $k$ , we calculate the unconditional hazard rate for durations  $\tau_t \leq T$  and show that the shape of this hazard function is (non-monotonically) decreasing.

Third, we provide the shape of the hazard function for durations  $\tau_t > T$ , that is easily obtained.

By considering an arbitrarily chosen point of time  $t + k$ ,  $0 < k \leq T$  we consider the following cases  $s$ :

- (s=1) Assume a point of time  $t + k$  with  $k \leq \kappa_{t+1}$ , i.e. the next signal has not occurred yet but observing it in the next instant of time would lead to the next transaction, thus,  $\tau_t = \kappa_{t+1}$ . The conditioning information for the corresponding inter-signal

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<sup>18</sup>See Johnson, Kotz, and Balakrishnan (1994).

durations is given by

$$\mathcal{A}_1 = \left\{ \kappa_{t+2-\mu}, \dots, \kappa_t \left| \kappa_{t+1} \leq T - \sum_{i=t+2-\mu}^t \kappa_i \right. \right\}.$$

The conditional hazard rate, given situation  $s = 1$ , is

$$\lambda_1(k) = \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \text{Prob}(\kappa_{t+1} = k + \Delta | \kappa_{t+1} \geq k; \mathcal{A}_1).$$

- (s=1,a) Assume a point of time  $t+k$  equivalent to  $s = 1$  but assume that observing a price signal in the next instant of time after  $t+k$  leads with certainty to no further transaction because in this point of time the last  $\mu + 1$  signals occurred within a time horizon larger than  $T$ . The conditioning information for the inter-signal durations in this situations is given by

$$\mathcal{A}_{1,a} = \left\{ \kappa_{t+2-\mu}, \dots, \kappa_t \left| \kappa_{t+1} > T - \sum_{i=t+2-\mu}^t \kappa_i \right. \right\}.$$

In this situation the conditional hazard is

$$\lambda_{1,a}(k) = 0.$$

- (s=2) We consider a point  $t+k$  with  $\kappa_{t+1} < k \leq \kappa_{t+1} + \kappa_{t+2}$ , i.e. the next price signal after  $t$  has been observed already. We assume that this price signal has not led to a trade because in  $t + \kappa_{t+1}$  the last  $\mu + 1$  signals occurred within a time larger than  $T$ . Furthermore, it is assumed that the next signal has not occurred yet but observing it in the next instant of time leads to a further trade. Hence, the conditioning information for the inter-signal durations in these points of time is

$$\mathcal{A}_2 = \left\{ \kappa_{t+2-\mu}, \dots, \kappa_t \left| \kappa_{t+2} \leq T - \sum_{i=t+3-\mu}^{t+1} \kappa_i \cap \kappa_{t+1} > T - \sum_{i=t+2-\mu}^t \kappa_i \right. \right\}.$$

The conditional hazard rate, given this situation, is

$$\lambda_2(k) = \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \text{Prob}(\kappa_{t+2} = k - \kappa_{t+1} + \Delta | \kappa_{t+2} \geq k - \kappa_{t+1}; \mathcal{A}_2).$$

Equivalently, the conditional hazard rates for the situations  $s = i$ ;  $i = 3, \dots, M$  are obtained by continuing the induction from case  $s = i$  on  $s = i+1$ , where  $M \leq \mu$  denotes the number of price signals occurring between the points of time  $t$  and  $t+T$ . Note, that the conditional hazard rates, given that in the next instant of time no transaction will occur ( $\lambda_{i,a}$ ,  $i = 1, \dots, M$ ), are zero.

The unconditional hazard rate is obtained by

$$\lambda(k) = \sum_{i=1}^M [\lambda_i(k) \cdot \text{Prob}(\mathcal{A}_i) + \lambda_{i,a}(k) \cdot \text{Prob}(\mathcal{A}_{i,a})].$$

To show that this pattern is (non-monotonically) decreasing we consider two properties of the hazard rate:

- (i)  $\frac{\partial \lambda_i(k)}{\partial k} < 0$ ;  $i = 1, 2, \dots, M$ .

The decreasing shape of the particular conditional hazard rates is caused by the fact that a price signal only leads to a transaction if it occurs within a certain time horizon. Hence, the higher  $k$ , i.e. the more time is elapsed since the last transaction, the more restrictive is the conditioning information. In the extreme case,

i.e. if  $k = T$ , then the probability for observing inter-price signals  $\kappa_t$  satisfying the conditioning is zero, leading to a hazard rate that is also zero. Thus, while the (unconditional) hazard rate of an exponential distribution is constant, the time-dependence of the conditioning leads to a decreasing shape of the conditional hazard rate.

- (ii)  $\lambda_i(k) \cdot \text{Prob}(\mathcal{A}_i) > \lambda_{i+1}(k) \cdot \text{Prob}(\mathcal{A}_{i+1})$ ,  $i = 1, \dots, M$ .

This feature is obvious from the definition of the conditionings  $\mathcal{A}_i$ , which are more informative the more price signals (without a trade) have been observed already. Note that the situation  $s = i$  can only be observed when situation  $s = i - 1$  is survived already. Hence, equivalent to the argument in (i) the conditioning information is more restrictive the more price signals have been observed since the last trade.

Due to the fact that particular conditional hazard rates are zero, given that in the next instant of time no transaction will occur,  $\lambda_{ia}$ ,  $i = 1, \dots, M$ , the unconditional hazard  $\lambda(k)$  can have a non-monotonic (decreasing) shape.

- (b)  $\tau_t > T$

For these durations restrictions concerning the elapsed time since the last price signal are not informative, hence the resulting hazard rate corresponds to the hazard of the underlying exponential distributed inter-signal durations, leading to a constant hazard rate.

### 7.3. Empirical Results.

#### 7.3.1. Descriptive Statistics.

TABLE 1. Descriptive statistics of inter-trade durations and volume per transaction. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations.

A: After 2:30 p.m., no U.S. holiday.

B: Before 2:30 p.m., no U.S. holiday.

C: U.S. holiday, (President's Day (02/20/95)).

D: Overall observations.

	A	B	C	D
Inter-trade durations				
0.25-quantile	1	3	5	2
0.5-quantile	5	7	17	6
0.75-quantile	12	20	57	16
Mean	10.22	17.68	49.52	14.16
Std. Dev.	17.74	32.07	84.29	26.56
Volume per transaction				
0.25-quantile	5	4	3	5
0.5-quantile	11	10	10	10
0.75-quantile	23	20	20	21
Mean	19.77	18.67	16.07	19.19
Std. Dev	25.49	25.85	24.25	25.68

TABLE 2. acf and pacf functions of inter-trade durations. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations.

Column A: Raw durations.

Column B: Seasonal adjusted durations (based on flexible Fourier form of order  $p = 4$ , 2:30 p.m- and holiday-dummies).

	A		B	
	acf	pacf	acf	pacf
lag1	0.2171	0.2171	0.1539	0.1539
lag2	0.2013	0.1617	0.1368	0.1159
lag3	0.1859	0.1230	0.1203	0.0870
lag4	0.1874	0.1122	0.1216	0.0824
lag5	0.1967	0.1113	0.1317	0.0870
lag6	0.1896	0.0915	0.1240	0.0711
lag7	0.1745	0.0670	0.1076	0.0492
lag8	0.1867	0.0785	0.1209	0.0630
lag9	0.1735	0.0571	0.1065	0.0433
lag10	0.1722	0.0532	0.1052	0.0408
lag11	0.1522	0.0286	0.0836	0.0169
lag12	0.1584	0.0385	0.0904	0.0276
lag13	0.1672	0.0480	0.0999	0.0378
lag14	0.1621	0.0391	0.0944	0.0298
lag15	0.1684	0.0461	0.1016	0.0379

TABLE 3. Estimates of ARMA models for raw inter-trade log-durations. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

	ARMA(1,1)		ARMA(1,2)		ARMA(2,2)		ARMA(3,3)	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
<i>AR1</i>	0.9788	0.0000	0.9823	0.0000	1.7296	0.0000	0.8150	0.0000
<i>AR2</i>					−0.7325	0.0000	0.7754	0.0000
<i>AR3</i>							−0.5973	0.0000
<i>MA1</i>	0.9153	0.0000	0.8899	0.0000	1.6398	0.0000	0.7264	0.0000
<i>MA2</i>			0.0349	0.0000	−0.6542	0.0000	0.7650	0.0021
<i>MA3</i>							−0.5242	0.0000
<i>Mean</i>	1.3551	0.0000	1.3552	0.0000	1.3554	0.0000	1.3554	0.0000
<i>BIC</i>	−195150		−195108		−195074		−195090	
AR and MA Roots								
<i>AR1</i>	1.0215		1.0179		1.3494		1.4371	
<i>AR2</i>					1.0115		1.0121	
<i>AR3</i>							1.1509	
<i>MA1</i>	1.0925		26.5386		1.4580		1.5665	
<i>MA2</i>			1.0779		1.0482		1.0510	
<i>MA3</i>							1.1584	

TABLE 4. Estimates of ARMA models for seasonal adjusted log inter-trade durations. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

[illegible]

7.3.2. *Regression Results.*

TABLE 5. Estimates of PHARMA(1,2) models for grouped durations.  
Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to  
02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

Variable	A		B		C	
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Thresholds						
$\mu_1$ ( $\tau_t = 1$ )	-4.0915	0.0000	-3.2266	0.0000	-2.8096	0.0000
$\mu_2$ ( $\tau_t = 5$ )	-3.0255	0.0000	-2.1591	0.0000	-1.7407	0.0000
$\mu_3$ ( $\tau_t = 10$ )	-2.5136	0.0000	-1.6472	0.0000	-1.2282	0.0000
$\mu_4$ ( $\tau_t = 30$ )	-1.7431	0.0000	-0.8769	0.0000	-0.4571	0.0001
$\mu_5$ ( $\tau_t = 60$ )	-1.2342	0.0000	-0.3671	0.0016	0.0538	0.3346
$\mu_6$ ( $\tau_t = 90$ )	-0.9456	0.0000	-0.0772	0.2678	0.3455	0.0030
$\mu_7$ ( $\tau_t = 120$ )	-0.7634	0.0000	0.1062	0.1969	0.5308	0.0000
$\mu_8$ ( $\tau_t = 150$ )	-0.6038	0.0003	0.2669	0.0160	0.6940	0.0000
$\mu_9$ ( $\tau_t = 180$ )	-0.4948	0.0025	0.3766	0.0012	0.8061	0.0000
Intraday Seasonalities						
<i>trend</i>			0.2228	0.0000	1.0286	0.0000
$\delta_{s,1}$			0.1946	0.0000	0.0314	0.2759
$\delta_{s,2}$			0.0785	0.0045	0.0348	0.1387
$\delta_{s,3}$			-0.1099	0.0000	-0.0328	0.1035
$\delta_{s,4}$			0.0744	0.0009	0.0275	0.1584
$\delta_{c,1}$			-0.2183	0.0000	-0.0140	0.3886
$\delta_{c,2}$			0.1811	0.0000	0.0676	0.0169
$\delta_{c,3}$			-0.0724	0.0018	-0.0542	0.0081
$\delta_{c,4}$			0.0310	0.0988	0.0767	0.0003
2:30 and Holiday Dummy						
After 2:30					-0.8264	0.0000
U.S. Holiday					0.8684	0.0000
ARMA Parameters						
AR1	0.9877	0.0000	0.9815	0.0000	0.9779	0.0000
MA1	0.8879	0.0000	0.8836	0.0000	0.8815	0.0000
MA2	0.0443	0.0000	0.0411	0.0000	0.0402	0.0000
BIC and Mean Log Likelihood						
Mean Log Likelihood	-1.6229		-1.6225		-1.6209	
BIC	-72786.4101		-72816.6819		-72755.6961	

TABLE 6. Estimates of PHARMA(1,2) models for grouped durations and BIC. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

Variable	D		E	
	Coeff.	p-value	Coeff.	p-value
Thresholds				
$\mu_1$ ( $\tau_t = 1$ )	-2.9277	0.0000	-3.2667	0.0000
$\mu_2$ ( $\tau_t = 5$ )	-1.8577	0.0000	-2.2166	0.0000
$\mu_3$ ( $\tau_t = 10$ )	-1.3441	0.0000	-1.7034	0.0000
$\mu_4$ ( $\tau_t = 30$ )	-0.5710	0.0000	-0.9314	0.0000
$\mu_5$ ( $\tau_t = 60$ )	-0.0589	0.2397	-0.4212	0.0000
$\mu_6$ ( $\tau_t = 90$ )	0.2332	0.0025	-0.1315	0.1006
$\mu_7$ ( $\tau_t = 120$ )	0.4188	0.0000	0.0510	0.3103
$\mu_8$ ( $\tau_t = 150$ )	0.5827	0.0000	0.2099	0.0212
$\mu_9$ ( $\tau_t = 180$ )	0.6958	0.0000	0.3178	0.0011
Intraday Seasonalities				
trend	1.0347	0.0000	1.0532	0.0000
Volume				
log vol	0.0241	0.1647		
(log vol) <sup>2</sup>	-0.3863	0.0009		
(log vol) <sup>3</sup>	0.5034	0.0021		
Volume, Absolute Price Changes (dynamically)				
lvol lag1			-0.0367	0.0000
dp lag1			-0.0301	0.0005
2:30- and Holiday-Dummy				
After 2:30	-0.8858	0.0000	-0.8982	0.0000
U.S. Holiday	0.8530	0.0000	1.0171	0.0000
ARMA Parameters				
AR1	0.9781	0.0000	0.9800	0.0000
MA1	0.8820	0.0000	0.8845	0.0000
MA2	0.0373	0.0000	0.0422	0.0000
BIC and Mean Log Likelihood				
Mean Log Likelihood	-1.6191		-1.6229	
BIC	-72642.9075		-72810.0709	



7.4. Figures.

Figure 1: Impact of Intraday Seasonalities on Probabilities to Survive 5 sec. (solid) and 30 sec. (dotted)

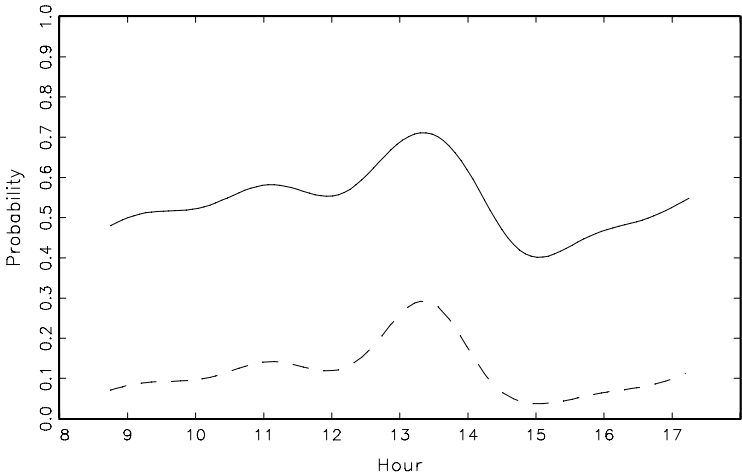


Figure 2: Survivor Function,  
(above: U.S. Holiday, 10:00; middle: 10:00; below: 15:00)

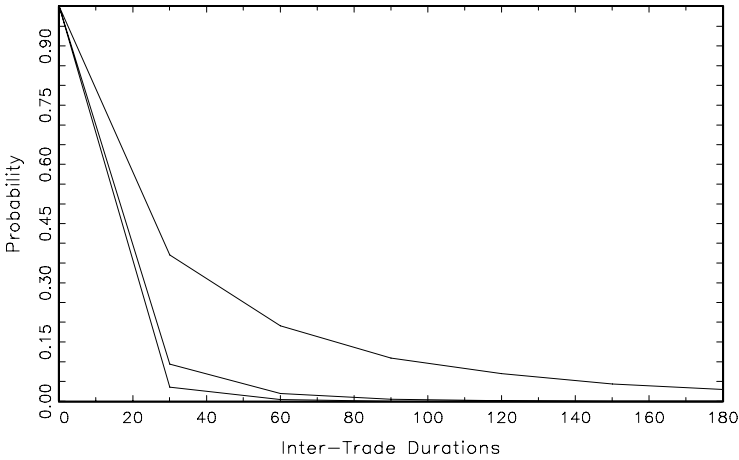


Figure 3: Conditional Failure Probabilities,  
(above: 15:00; middle: 10:00; below: U.S. Holiday, 10:00)

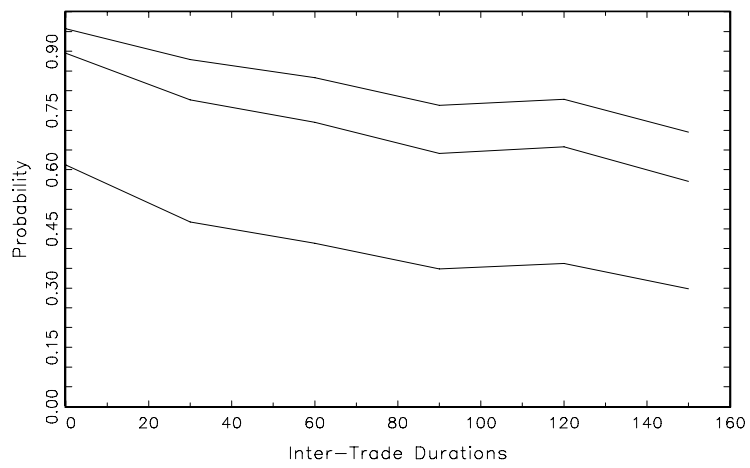


Figure 4: Impact of Volume on Probabilities to Survive  
5 sec. (solid) and 30 sec. (dotted)

