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Insights from physics into development processes: Are fat tails interesting for development research?

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Uta Schultze

Insights from Physics into Development Processes:

Are Fat Tails Interesting for Development Research?

Number 23

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The **CENTER FOR DEVELOPMENT RESEARCH (ZEF)** was established in 1997 as an international, interdisciplinary research institute at the University of Bonn. Research and teaching at ZEF aims to contribute to resolving political, economic and ecological development problems. ZEF closely cooperates with national and international partners in research and development organizations. For information, see: http://www.zef.de.

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Abstract

This article explores the question of whether concepts and methods used in non-equilibrium statistical physics, already successfully applied in biology, epidemeology and finances, can also be a useful tool for research into economic development. The aim of this article is to describe one such phenomenon, the phenomenon of "fat tails", for a multi-disciplinary public and to stimulate discussion on its potential use. The method is explained and empirically tested by looking at the example of the probability distribution of per capita GNP in various countries. It is shown that this distribution does indeed have "fat tails". The significance of the appearance of such fat tails is discussed and it is stressed that they reflect a complex dynamic which cannot be captured using only mean values and variances. The article also suggests some other pertinent problems of development research where they may possibly be applied.

Zusammenfassung

Die Autorin untersucht die Frage, ob Konzepte und Methoden, die in der statistischen Physik, der Biologie, der Epidemeology und den Finanzwissenschaften bereits erfolgreich eingesetzt werden, auch in der Entwicklungsökonomie ein sinnvolles Instrument darstellen können. Das Ziel des Artikels ist, eine dieser Methoden, nämlich die der Identifikation von statistischen Mustern, sogenannten "fat tails", für die multidisziplinäre Öffentlichkeit zu beschreiben und damit eine Diskussion über deren potenziellen Nutzen in Gang zu setzen. Die Methode wird erklärt und am Beispiel der Wahrscheinlichkeitsverteilung des Bruttosozialprodukts verschiedener Länder empirisch getestet. Es zeigt sich, daß deren Verteilung tatsächlich "fat tails" aufweist. Die Signifikanz der Existenz von "fat tails" wird diskutiert und es wird betont, daß sie eine komplexe Dynamik reflektieren, die mit einfachen Mittelwerten oder Varianzen nicht erfasst werden könnte.

1 Introduction

Ideas and methods from the field of physics were already being applied to other scientific fields back in the 19th century and had a widespread influence on public perceptions of the world (concerning the topics of the "mechanistic worldview", "reductionism", "deterministic theory" see West 1985, Popper 1982, Mirowsky 1988, Georgescu-Reogen 1971, Prigogine 1980). In the last 30 years, concepts that have emerged from the study of complex systems such as self-organization (also known in physics as pattern formation), turbulence and chaos have not only penetrated economics, and especially financial analysis, but also biology (see Murray 1989), epidemeology (see Anderson 1982) and even the socio-economic sciences. An interesting example of the latter is the work of Nunes-Amaral et al. (1997), which showed that the conditional probability of an enterprise of size x_0 having a size of x_1 after one year exhibits universal properties. The dispersions of the probability distributions have a power dependence of x_0 with an exponent which is universal. That means that the exponent does not depend on the type of the enterprise (i.e. it can be a shoe factory or a crane factory).

In this paper, the applicability of some ideas that have come out of non-equilibrium statistical physics and that are already used in finance (see Bouchaud and Potter 1996) and are being applied to social phenomena like sizes of cities and populations of countries (Laherre and Sornette 1998) will be discussed by looking at the example of economic growth in a cross-sectional comparison. The concept we are interested in is related to the appearance of so-called "fat tails" in certain probability distributions. More precisely, if we look at the probability distribution in relation to a large fluctuation of a certain index of the stock exchange (e.g. the DAX) or several cities with large populations, we discover that the corresponding probabilities are high. This means that it is likely that big differences frequently occur, in contrast to a Gauss distribution, which would predict small perturbations because its negligible tail values near the average are much more likely. The existence of fat tails has not only theoretical but also practical implications, such as the modification of the celebrated Black-Scholes formula for high-risks option pricing (Matacz 1997). The appearance of fat tails in various domains of science calls for an inter-disciplinary exchange on the concept.

If a distribution has fat tails, this means that large fluctuations are important. This view calls into question the traditional perception of an equilibrium economy. The main motivation for this work was to understand how fluctuations, which are commonly neglected, can be taken into account in economic growth models. In order to do so, we have considered systems which display different degrees of freedom and can be described by stochastic laws. This allows a better interpretation of the data and a more accurate description of the system under consideration. The probability distribution functions thus obtained are not necessarily of the Gauss type but often present fat tails.

Since the purpose of this paper is to stimulate an exchange of ideas between scientists from different disciplines, it starts with a short review of physical concepts and methods used in describing complex systems. The process of understanding a complex system leads to the discovery of new levels of complexity and of new structures. Through this process the old methods and concepts find their limitations and new, richer ones, appear. One obtains in this way a deeper understanding of nature and/or society. Since this of course cannot be explained here in detail, but readers who are interested are kindly referred to the literature cited.

The paper is organized as follows: Section 2 gives a short review of macroscopic and microscopic approaches in physics, some of which are already partly used in economics. Section 3 shows the appearance of fat tails using a simple model. In contrast to the random walker described by a Gauss distribution, the Levy flier leads to fat tails. In Section 4 we look at the probability distribution of the GNP per capita for different countries. Although only limited data is available, the distributions are smooth. The domains of countries with a high p.c. GNP and with a low p.c. GNP are considered separately. Both domains show fat tails but of a different nature. We also notice that, surprisingly, there has been no obvious change in the probability distributions in the last twenty years. The conclusions from the discussion of the concept and the example tested are summarized in Section 5. The possible relevance of this work for future studies on topics related to economic development research is discussed.

2 Micro and Macroscopic Approaches in Non-Equilibrium Physics (from Newtonian Dynamics to Pattern Formation)

Newtonian physics was an attempt to make precise predictions about natural phenomena – predictions which could be verified by observations and experiments. The goal was to understand nature as a deterministic "clockwork" universe. This deterministic universe was supposed to be predictable and capable of being described by differential equation systems.

The application of probability distributions came in slowly. Early uses of probability arguments in physics focused on simple distributions parameterized by their means and variances. The paradigm was the Gaussian law of errors which was related to the uncertainty of measurements in systems with many degrees of freedom. However, when it came to Maxwell-Boltzmann and Planck distributions, the whole distribution now acquired a physical meaning. That had a conceptual advantage, since deviations from the mean were no longer an error and a large variance did not have to indicate poor measurement. In fact the whole distribution is a prediction (for detailed discussion see West 1985). Recently, distributions very different from the Gauss ones (with large values of the cumulants) have also been observed in nature (Solomon et al 1993).

In non-equilibrium statistical physics, the Gauss distribution appears in many contexts. The simplest one is the time evolution of a random walker. This type of random behavior can be seen experimentally in the Brownian motion of grains in a fluid. The theoretical breakthrough achieved by Einstein in 1905 was to look into the microscopic structure of the problem and to relate the diffusion constant to the radius of the grain and the viscosity coefficient. Moreover he was able to find the correct temperature dependence of the diffusion constant. In the last few years, many examples of sub-diffusive and super-diffusive behavior have been observed experimentally and understood theoretically by using microscopic models to derive the values of parameters which give phenomenologically the macroscopic behavior (for an example see Schlesinger et al 1987).

Recently a lot of progress has been made in the description of systems that have more complicated interactions. Some issues that have become influential in other sciences and even our everyday language should be mentioned here:

a) Complex systems, described by non-linear differential equations (the double-pendulum is probably the simplest example of this kind) and, in general, systems with many degrees of freedom display fascinating phenomena such as chaos (Feigenbaum

1978, Lorentz 1963, for an introduction see Gleick 1987) and turbulence (Takayasu 1984, Schlesinger et al 1987).

b) Under certain conditions (called critical domains), some macroscopic systems such as fluids or chemical reactants display self-organization (pattern formation). Ice dendrites on a window in winter time and the Belusov-Shabotinsky chemical reaction, where oscillations take place at the rate of concentrations of the catalyst of citric acid by bromate, are some examples (Glansdorff and Prigogine 1971, Nicolis and Prigogine 1977, for an introduction see Prigogine and Stengers 1980). Today there are mathematical models which give precise descriptions of these phenomena.

c) "Fat tails" are observed in many apparently unrelated phenomena such as the distribution of the sizes of oilfields (Laherre and Sornette 1998), the magnitude of earthquakes (Johnston et al 1985), the size of fish colonies, the extinction rate of species (Raup 1984) – the list goes on and on (for an easily understandable introduction see Bak 1996).

The mathematical description of a socio-economic system is obviously much more complicated than that of physical systems. There are many more kinds of "actors" with many more kinds of interactions than those encountered in physical systems.

In the framework of the theories of endogenous growth (for a review see Barro et al 1995), the growth process is described by systems of differential equations which relate various economic observables (like capital, labor, human capital). However, this description is only possible by assuming equilibrium and steady state (the meaning of equilibrium and steady state is different from the one used in physics). This picture corresponds to the macroscopic, the deterministic as opposed to the stochastic, approach in natural sciences. In the deterministic approach, microscopic aspects which lead to fluctuations are not taken into account. Labor, capital and human capital are rigid quantities without any structure. In fact the whole structure is substituted by "representatives" (for detailed discussion see Harcourt 1976 and references therein). The microscopic approach to endogenous growth would consider the interactions between the different components of labor, capital and human capital. That means it becomes important to compute quantities like auto-correlation and various other correlation functions (in stochastic processes, for example, events which at different times are dependent on each other produce non-vanishing auto-correlations functions).

In the last twenty years the concepts and methods mentioned above have been exported from physics to some fields of economics:

a) Using less simplifying assumptions, the macroscopic (i.e. macro-economic) equations become nonlinear. The equations have solutions showing chaotic behavior of the variables. A nice example of this kind is the Benhabib/Day 1981 model of rational

choice. For an overview on chaotic dynamics in economics see Barnett et al 1989, Goodwin 1990 and Lorenz 1989.

b) The concept of self-organization has found its way into economic thinking.

c) Methods used in non-equilibrium statistical physics were used in the analysis of the growth of enterprises (Nunes-Amaral et al 1997) as mentioned above and in the analysis of financial markets. It may be observed that financial prices exhibit some universal characteristics that resemble the scaling laws characterizing physical systems in which large numbers of units interact (see Lux et al 1998, Mantegna et al 1995, Galluccio et al 1997).

3 From the Random Walker to the Levy Flier

In this section the time evolution of systems which lead to Gauss distribution (random walk) and distributions with fat tails (Levy flight) will be discussed briefly. In this way the importance of the microscopic level for an understanding of the macroscopic evolution will be illustrated.

As mentioned above, the Gauss distribution

$$P_G = \frac{1}{\sqrt{2\pi^1\sigma}} \exp\left(-\frac{\left(x-\bar{x}\right)^2}{2\sigma^2} \right) \tag{1}$$

not only gives the error distribution but is also the solution of the time dependent diffusion equation. The diffusion equation appears in a large class of phenomena (see West 1985) and its application to random processes dates back to the beginning of this century (Perrin 1910).

In a random walk a particle on a one-dimensional lattice, in which a lattice site is labeled by an integer n, can hop to the left or to the right with the probability P. The particle stays in the same place with a probability $1-2P$. The probability of finding the particle on the lattice site n at time $t+\Delta t$ is given by the recurrence relations:

$$P_n(t+\Delta t) = (1-2P)P_n(t) + P\left(P_{n-1}(t) + P_{n+1}(t)\right) \tag{2}$$

In the limit $\Delta t \to 0$ the recurrence relation becomes the master equation, a set of ordinary linear differential equations.

$$\frac{dP_n(t)}{dt} = \frac{a}{2}\left[P_{n+1}(t) + P_{n-1}(t) - 2P_n(t)\right] \tag{3}$$

where $a := P\Delta t$.

Moreover, if the lattice is long enough one can take the limit $\Delta n \to 0$ and obtain the diffusion equation:

$$\frac{\partial P(x,t)}{\partial t} = \frac{a}{2}\frac{\partial^2 P(x,t)}{\partial x^2} \tag{4}$$

This is a partial linear differential equation which has the Gauss distribution given by Eq. (1) as solution. If the particle started at $t=0$ on the lattice site $n=0$ the mean value $\bar{x}$ equals

zero, independent of time. If the hopping rates are asymmetric (one prefers for example to hop more to the left than to the right), one obtains a non-vanishing mean value with a linear time dependence. Moreover, the standard deviation has, both in the symmetric as well as in the asymmetric case, the time dependence

$$\sigma \approx \sqrt{t} \tag{5}$$

As opposed to the random walker a Levy flier can not only walk to the next site but can also "fly" to a distant site m with the probability $1/x^{(a-1)}$ (Schlesinger et al 1993). These long flights give a probability distribution very different from the Gaussian one. One obtains the standard deviation with the time dependence

$$\sigma = t^{\mu} \tag{6}$$

where

$$\mu = 2 - \frac{2}{a} \tag{7}$$

For large values of x the probability distribution does not decrease exponentially but as a power law

$$P(x) \approx \frac{1}{x^a} \tag{8}$$

The slow decrease with x given by Eq.(8) is an example of a fat tail. Levy flights have been observed in nature (Solomon et al 1993) and have been used to analyze the stock market (Mantegna 1991, Bouchaud and Potter 1997). The Levy distributions are not the only ones used in financial analysis. The so-called truncated Levy distributions (Koponen 1995) take into account, in an empirical way, the difference between socio-economical sciences and natural sciences. Unlike in the latter, variables in the former can never take infinitely large values (the variation of the value of a share cannot reach the value of the enterprise). Another probability distribution, the stretched exponential one

$$P(x) = c\frac{x^{c-1}}{x_0} e^{-\left(\frac{x}{x_0}\right)^c} \tag{9}$$

(here c and x_0 are parameters) was found to be relevant for a variety of phenomena (Laherre and Sornette 1998). The dynamics behind this distribution, which plays the role of the random walker for the Gaussian one, is based on multiplicative processes (Sornette 1997). These processes are more complicated and this paper is not the place to introduce them.

An indicator for non-Gaussian behavior of a probability distribution is the kurtosis defined by the equation

$$\kappa = \int_{-\infty}^{\infty} \frac{\left(x - \bar{x}\right)^4}{\sigma^4} P(x)dx - 3 \qquad (10)$$

The kurtosis is given by the first four moments of the probability function (the variance is given by the first two). For practical purposes a value larger than one for the kurtosis indicates that fat tails exist. This is the case for all the non-Gaussian distributions described above (for the Gauss distribution the kurtosis is equal to zero).

We believe that the modern mathematical approaches used for complex systems and described above can also be relevant for development research since they allow, for example, the effects of fluctuations and the interdependence of different countries to be taken into account. These fluctuations should change the formulation of growth models and will affect the estimates of growth rates. This approach could also be useful for decisions about investments in developing countries and hopefully give indicators about the danger of imminent crises. That this approach is not far-fetched can be seen in models in the field of finance. In these models one takes into account the actors at the micro level and looks at the effect of their interactions on the observables on the macro level (price changes). For example in the Lux et al 1998 model, the pool of traders is divided into 2 groups: the “fundamentalists” who follow the efficient market hypothesis and the “noise traders” who, instead of focusing on demands, identify price trends and charts.

4 Applications in Development Research: The Example of p.c. GNP Probability Distribution Function

Looking for possible ways in which the methods of non-equilibrium statistical physics could be applied to development research, we have looked at the global distribution of the p.c. GNP. The p.c. GNP of a country is considered as an indicator for its ability to interact with other countries through trade and financial markets. This preliminary study can be a first step in the direction of formulating a model describing the involvement of a country in the world markets. In the context of the global economical development of the last 20 years, the analysis of the time dependence of the distribution of p.c. GNP might provide us with information on the extent of participation in the world economy of various countries. Moreover if the p.c. GNP is indeed an indicator of the wealth of a country, the analysis of the time dependence of the distribution can show the extent to which countries profit from global wealth.

Dealing with small samples and the p.c. GNP is such a case; the function $P(x)$ approximated through binning and polynomial fitting (this implies determining the coefficients of a polynom such that the function thus obtained goes through the experimentally measured points) gives a poor description of the data for large and small values of x (where the fat tails occur). The advantage of using the $P_c(x)$ defined below is that one generally obtains a tidier representation of the data. As will be shown, this is also valid in our case.

The statistical ensemble is made up of countries and what is observable is the p.c. GNP, denoted by x in units of constant 1987 US$. The probability distribution is $P(x)$ and the cumulative probability $P_c(x)$ defined as:

$$P_c(x) = \int_x^\infty P(y)dy \qquad (11)$$

where $P_c(0) = 1, \quad P_c(\infty) = 0$

We have used the World Bank's world development indicators for the years 1975, 1980, 1985, 1990 and 1995 (see Appendix). One problem that must be faced is that the number of countries for which data are available changes over the years. For 1975 the p.c. GNP of 127 countries was reported, for 1995 the data of 160 countries. This is due to the changes in Eastern Europe and in the Balkans as well as to the increase in data from emerging economies. In order to deal with this problem, we have looked at the data in two ways. First we have taken the data at

face value and next we have taken an ensemble where we have considered only the 116 countries which appear in all the years.

We have first considered all the countries for which the data are known (one country can appear in one year but not in another; the countries of the former Soviet Union are an example). Surprisingly, the average, the standard deviation (σ) and the kurtosis (K) show no significant time dependence:

$$\bar{x} = 4220 \pm 350, \sigma = 6400 \pm 500, \kappa = 3.2 \pm 0.4 \qquad (12)$$

The time independence of lower moments of the probability distribution given in (12) suggest, but do not prove (higher moments were not considered), a stationary state. The latter would imply that the whole probability function is time independent.

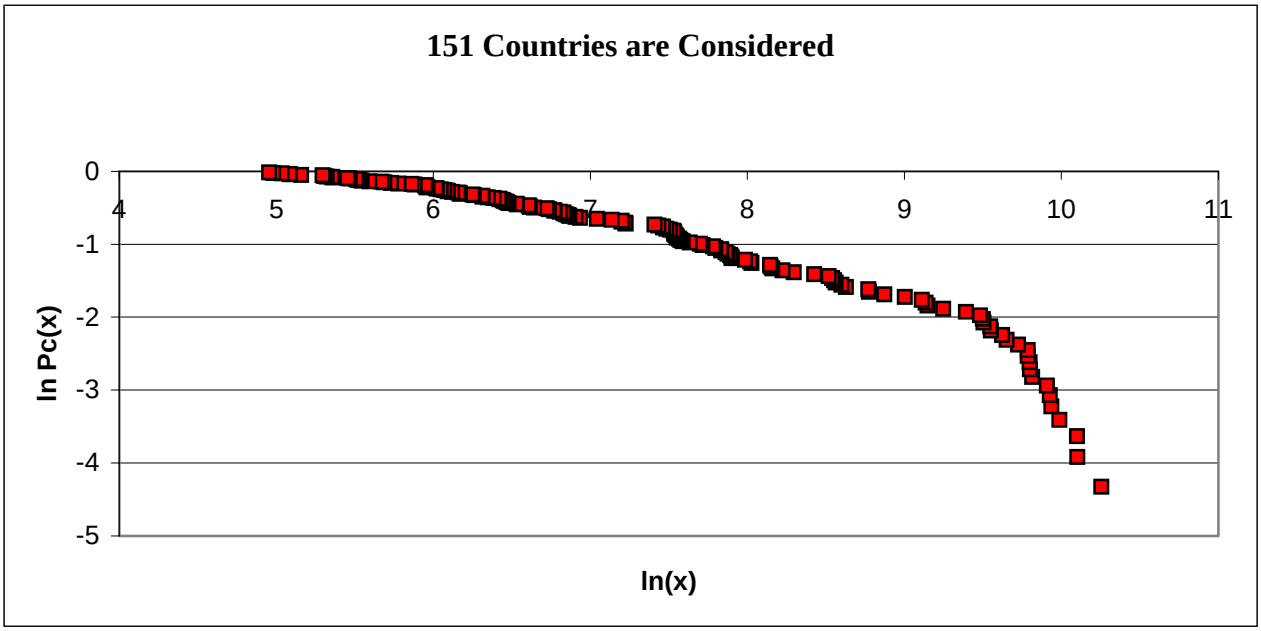
As discussed before, a value of the kurtosis larger than one indicates that the distribution is far from a Gaussian one and we have to look at the whole probability distribution $P(x)$ or the cumulative one. There is another aspect related to the consideration of only the low moments given by (12). Large changes in the p.c. GNP in the small x domain, which can reflect significant social changes, affect the values given in (12) very little.

The procedure for constructing $P_c(x)$ is standard: if in a given year one has N countries, one ranks them according to their p.c. GNP. The one with the largest p.c. GNP is ranked one, the second ranked two etc. The cumulative is then given by

$$P_c(Y_k) = \frac{k}{N} \qquad (k = 1,2,....N) \qquad (13)$$

where Y_k is the p.c. GNP of the country of rank k. In this way one builds from the "measured" values for each country the whole probability distribution.

Figure 1: The cumulative distribution for 1995



In Fig. 1 we show in a double-log plot the function $P_c(x)$ for the year 1995. This plot shows a remarkable phenomenon. For values of x smaller than 13000 \$, the function is smooth then there is a sudden change in the behavior. For values larger than 13000 \$, as suggested by the data, (this is the reason why it has been elected to show a double log-plot), the function can be parameterized by the simple expression.

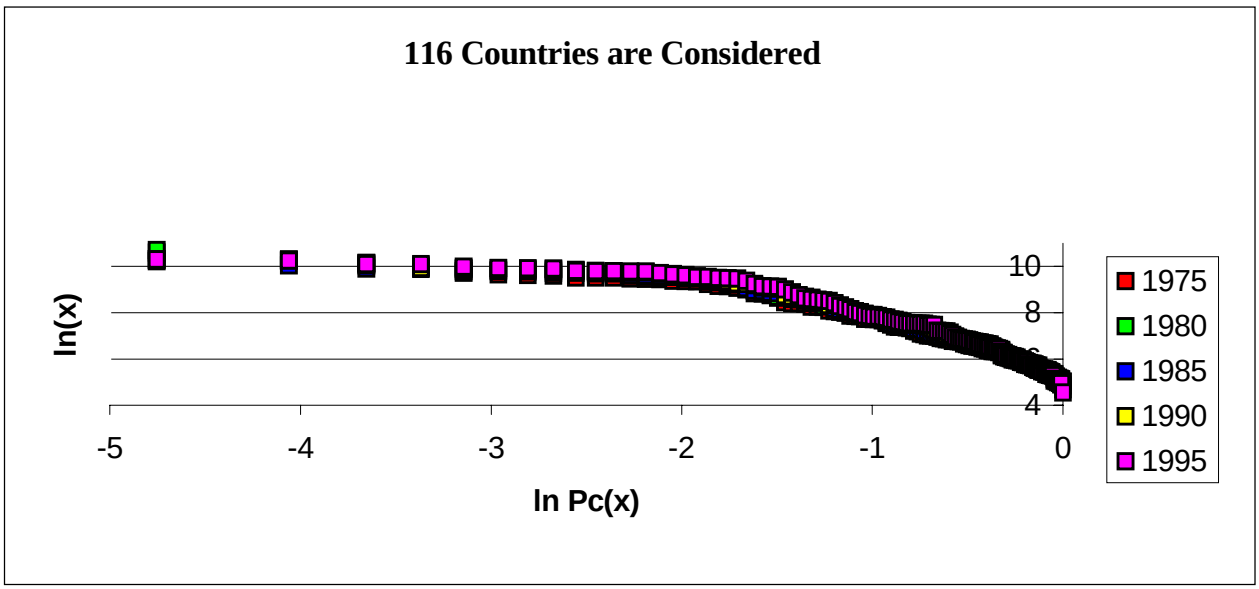
$$P_c(x) = \frac{c_1}{x^b} \tag{14}$$

where c_1 is a constant and $b = 3.9 \pm 0.3$. The values of b for the five years considered are shown in Table 1. This implies (using Eq. 11) that, where p.c. GNP is high, the probability distribution can be expressed thus:

$$P(x) = \frac{c_1 (b+1)}{x^{b+1}} \tag{15}$$

It is interesting to note that power law behavior of the probability distribution is common to a large variety of socio-economic phenomena. A recent study of interest rates (Nyuts et al. 1998) shows for example behavior similar to the one given by Eq. (15). In order to see if the fat tail given by Eq.s (14) and (15) is robust to change of the sample, in Fig. 2 we show the five cumulative distributions spanning twenty years, taking as samples only the 116 countries appearing in all those years.

Figure 2: The cumulative distribution function for the years 1975-1995.



The restricted sample shows no significant time dependence. The change in behavior of the $P_c(x)$ function remains the same as for the sample where all the countries were taken (compare with Fig. 1).

Table 1

Year	b	λ **in const. 1987 US $**
1995	3.9 ± 0.3	560 ± 30
1990	3.7 ± 0.4	615 ± 30
1985	5.0 ± 1.0	530 ± 40
1980	3.6 ± 1.2	560 ± 60
1975	3.0 ± 0.5	470 ± 10

The double-log plots presented above are perfect for studying the behavior in the large x domain. They are however inadequate if one is interested in the small x domain. One way to reveal the properties of probability distribution in this domain (which is relevant if one is dealing with the problem of poverty) is to introduce a new variable $u = 1/x$ (the inverse of p.c. GNP) and to study the probability distribution of this variable $P^1(u)$ and the corresponding cumulative $P_c^1(u)$.

Obviously the two probability distributions are related:

$$P^1(u) = \left|\frac{dx}{du}\right| P\left(\frac{1}{u}\right) = \frac{1}{u^2} P\left(\frac{1}{u}\right) \tag{16}$$

Figure 3: The cumulative distribution for the year 1995

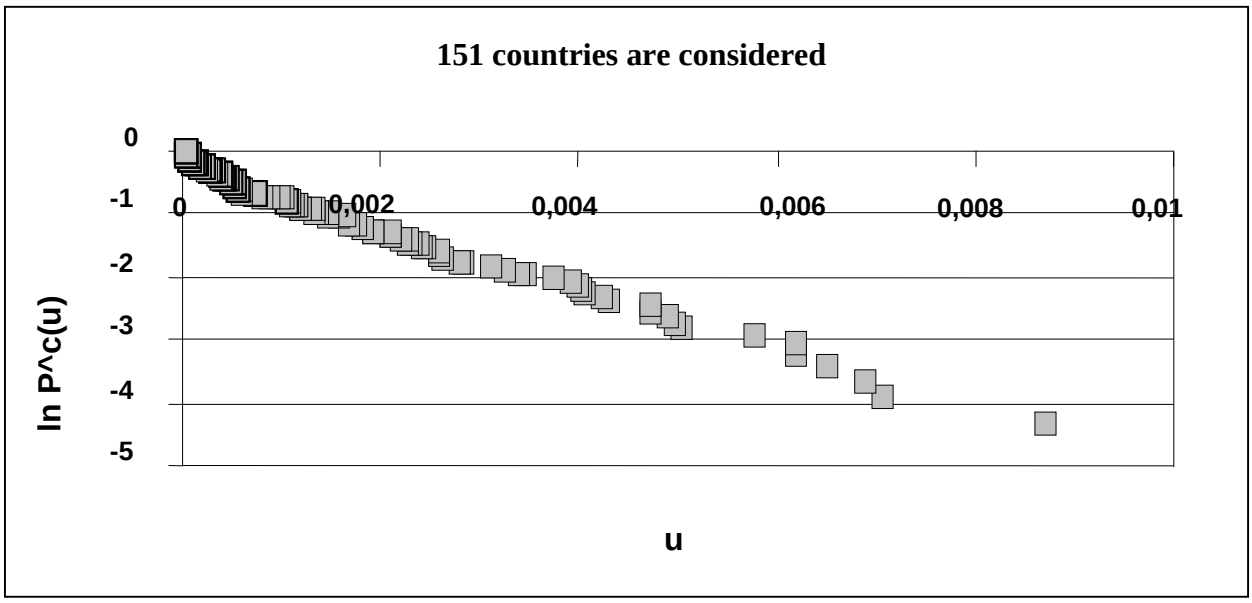


Fig. 3 shows the simple log-plot of the function $P_c^1(u)$ for the year 1995 where, as for Fig. 1, we have taken all 160 countries. (In order to obtain this plot, we have to apply the same procedure as the one used to obtain Fig.1, but the ranks have to be taken in the reverse order.)
It may be seen that the data are practically on a straight line, which implies that, for large values of u, one has

$$P_c^1(u) = c_2 e^{-\lambda u} \tag{17}$$

where c_2 is a constant and λ a characteristic length which indicates how the countries with low p.c. GNP participate in the global economy.

We should make an important observation at this point. Although the parameterization (17) is very different from the one given by Eq. (14), the result is again a fat tail. From Eq. (17) we obtain for the probability distribution itself the following behavior:

$$P^1(u) = \lambda c_2 e^{-\lambda u} \tag{18}$$

A Gauss distribution would imply an exponential decrease $\exp(-u^2)$ which is much faster than the one given by (18).

We have again checked if the parameterization (18) is robust, repeating the procedure which took us from Fig.1 to Fig.2. The parameterization holds over the twenty years. The value of the parameter λ being $(550 \pm 30)\$$ for the ensemble of 116 countries indicates, again, no significant time dependence, as can be seen in Table 1.

Obviously a more careful analysis of all the 20 distributions for the whole period can and should be done (for a first approach we took only one for each five years). This will settle the question of whether one has indeed a time independent, i.e. stationary, state. Such analysis would produce a reasonable parameterization of the whole probability function (up to now we have considered only its tails). The present data suggest a parameterization given by the function:

$$P(x) = C\frac{e^{-\lambda/x}}{x^b} \qquad (19)$$

where C is a constant and the parameters λ and b are around 550\$ and 4.8 respectively. Notice that the function (19) automatically gives the fat tails (15) and (18) in the algebraic respective exponential domains.

One can naturally ask the question: What are the practical consequences of the observation of the probability distribution with the properties described above? A few are listed below:

1. The power law behavior in the domain of rich countries is remarkable since a power law behavior implies no scale (this is at variance with the situation for poor countries where we have a scale given by λ). In physics, phenomena without scales are often associated with the so-called self-organized criticality (see for introduction Bak 1996). The name self-organized criticality (SOC) comes from the observation that, as opposed to the usual critical phenomena where a physical parameter like the temperature has to be tuned to a special value (like the temperature of the boiling of water), in SOC phenomena there is no need for tuning. This process of self-organization takes place over a relatively long period. Without going into details, this suggests that the rich countries participating in the fat tail interact essentially among each other. In fact, in the physical phenomena where SOC is observed (large earthquakes or large avalanches in sandpiles) the power law behavior occurs as a result of the complex interaction between the components of the system (faults or grains). On the other end of the distribution, the poor countries, where one has a scale, should have a negligible interaction with each other but a sizeable interaction with the rich ones. The intensity of this interaction is roughly given by λ. This picture can be useful if one intends to build a mathematical model which would deliver the observed probability distribution and gives inspiration, for example, for an non-equilibrium trade model.

2. It is a known fact that over the last 20 years trade among countries has increased; the same is true for the financial market. How can we explain the existence of a probability distribution which is essentially time independent? A possible way out is that the structural change within the countries has offset the effects of changes in this intercourse between countries. This can be verified by calculating the regional contributions to p.c. GNP and the income distributions.

3. If a more detailed study reveals a time dependence, λ can be used as an indicator for social change. For a given "level of poverty" or a given level of p.c. GNP with a certain infrastructure $\tilde{u}$, $P_c^1(\tilde{u})$, we can obtain the number of countries below this level. If λ varies with time, the number of poor countries changes accordingly.

5 Conclusions and Outlook

The motivation behind the research presented in this discussion paper was to explore the extent to which methods and concepts used in non-equilibrium physics can be suitably applied to development research.

Since the paper is written for researchers with a heterogeneous scientific background, it starts with a short review about physical methods and concepts in order to facilitate the exchange of ideas between different disciplines.

Motivated by the discussion about “one world” and globalization, the p.c. GNP of all countries was brought into one “picture”, the probability distribution function. This can facilitate the study of interactions between different countries.

The example of p.c. GNP shows that, despite the smallness of the statistical sample, that we can obtain smooth probability distribution functions. Moreover these functions have meaningful properties; they show fat tails.

For a physicist interested in socio-economic phenomena, this result is fascinating. Fat tails are observed in quite different problems that occur in nature as well as in the financial market. This suggests that completely different phenomena may be described mathematically in a similar way and that mathematical models can be transferred from one discipline to another. It also implies that a physicist can consider this to be a relevant subject.

We hope that an economist or a social scientist who is introduced to the ideas and concepts of non-equilibrium physics and to the first results concerning p.c. GNP distribution will be motivated to engage in multi-disciplinary discussion of models and the interpretation of the data. The question of p.c. GNP is only one example of the applicability of non-equilibrium physics. The same approach can of course be used for other observables (GNP, income, labor etc.) and other statistical ensembles (enterprises, regions, individuals) depending on the problem of interest.

The present paper is intended as an exploratory exercise. Since we believe that this exploration has been successful, the following further investigations should be considered:

1. Using all the p.c. GNP data available for the considered period of time, the statistics can be considerably improved. The first results concerning the time independence can be better checked and the whole p.c. GNP distribution can be studied in detail (here only the tails were considered)

2. The position of an individual country in the distribution can be “traced” and the change in its position followed. Its trajectory may be related to changes in its internal structure.

3. The same procedure can be applied to the GNP distribution to obtain more detailed information.

4. Distributions of various observables can be considered in order to obtain indicators for social change.

5. A more difficult question is the appearance of crises. The type of crisis concerned is not one such as that experienced in South East Asia in 1998 but the simpler case of a country which interacts with a stable environment. Its trajectory (see point 2) should show drastic changes, but how drastic they have to be in order to have a crisis is an open question.

6. The consequences of conceptional changes (predictable macroscopic systems, deterministic chaos, self-organization, self-organized criticality) on the reflection about development and on the mathematical description of development problems could be elaborated in a more detailed way.

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