"A disaggregated approach to modelling UK labour force participation"

by Joanne Cutler and Kenny Turnbull
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A disaggregated approach to modelling UK labour force participation

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Abstract

Fluctuations in the labour force participation rate in the short to medium run are an important determinant of the cyclical behaviour of the unemployment rate, and hence wage pressures in the economy. This paper takes a closer look at the cyclical behaviour of labour force participation in the United Kingdom over the last 15 years with a view to producing short term projections for the next two years.

We model labour participation rates at a disaggregated level, for males and females separately. Because the trends in male and female participation rates have been quite different, a disaggregated approach is more likely to pick up reliable trends and therefore cyclical variations. We find evidence of procyclical movements in participation rates for both men and women, and the response to changes in the output gap is significant. Conditional on projections for the output gap made at the end of 2000, we estimate that the participation rate will continue to increase - by around +0.2pp over the next two years - but at a slower pace than in the recent past.

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**Introduction**

The decision to participate in the labour market is a key determinant of labour supply. A number of authors have investigated the labour force participation rate and found some evidence that it is influenced by both cyclical and structural factors, see for example Pencavel (1986), Killingworth and Heckman (1986), and Darby, Hart and Vecchi (1998). If the participation rate rises during cyclical upswings that will limit the decline in the unemployment rate thereby leading to lower wage growth than if there had been no change in participation. So fluctuations in the labour force participation rate in the short to medium run are an important determinant of the cyclical behaviour of the unemployment rate, and hence wage pressures in the economy.

This paper takes a closer look at the cyclical behaviour of labour force participation with a view to producing short term projections for the next two years. The structure of the paper is as follows. In Section 1, we look at recent movements in the aggregate participation rate. Section 2 explains the framework used to model participation. A key feature is that we model male and female participation rates separately. Because these have moved differently over the past, a disaggregated modelling approach is more likely to capture reliable trend and cyclical movements in labour participation rates. Section 3 describes the assumptions used to produce short term projections from these disaggregated models. Section 4 shows how the disaggregated equations can be combined to produce a single aggregate equation which can be used to endogenise the participation rate in a macroeconomic model. Section 5 reports the results of an even more disaggregated approach which models participation rates across different age groups as well as gender, and Section 6 concludes.

1. **Recent movements in the participation rate**

Chart 1 shows the United Kingdom participation rate over the last 15 years against a proxy of the output gap. ¹ The participation rate appears to have varied in a pro-cyclical way around an upward trend. It increased sharply in the late 1980’s in response to a large and positive output gap and then fell back in the early 1990’s as the economy went into recession. This cyclical pattern seemed to disappear in the early years of the subsequent recovery as the output gap closed between 1993 and 1995 but the participation rate

¹ We run the following regression:

\[ \log (GDP) = \alpha + \beta \times (time \ trend) + 0.7 \times \log (population \ of \ working \ age) + 0.3 \times \log (non \ residential \ capital \ stock) \]

and use the estimated \( \alpha = 0.005702 \) and \( \beta = 0.003196 \) to help generate trend GDP.
carried on declining. It began increasing again in the second half of the 1990s, and rose especially sharply towards the end of the decade.

Chart 1

The UK participation rate and output gap

In the two years to 2000 Q2 the participation rate increased by 0.6 percentage points to 63.4%\(^3\) - not quite as sharply as in the late-1980s when the (positive) output gap was larger, but still a strong rally. This has helped employment to rise to record levels without a concomitant increase in wage pressures. Employment rose by 696,000 between 1998 Q2 and 2000 Q2. In an accounting sense, 24% of this increase was met by a decline in inactivity, 30% was met by a decline in unemployment, and 46% was met by an increase in 16+ population (see Table 1).

<table>
<thead>
<tr>
<th>Table 1: Decomposition of change in employment: 1998 Q2 – 2000 Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Of which:</td>
</tr>
<tr>
<td>Change in unemployment</td>
</tr>
<tr>
<td>Change in 16+ population</td>
</tr>
<tr>
<td>Change in inactive</td>
</tr>
</tbody>
</table>

\(^2\) Data are linearly interpolated prior to 1992.

\(^3\) These data are seasonally adjusted and relate to calendar quarters.
2. Modelling participation rates

2.1 Background

In order to model the labour force within the context of a macro model, for given population, there is a choice between modelling employment and unemployment or, alternatively, employment and the participation rate, deriving unemployment as a residual.\(^4\) The approach taken in the Bank of England’s main macro model (denoted MM hereafter) is the latter. In fact, the MM models the inverse of the participation rate, the inactivity rate, but the two are linked by identity.

As suggested by Chart 1 the participation/inactivity rate appears to be determined by both trend and cyclical factors. Consequently, the MM equation models the inactivity rate as a constant, a time trend to pick up structural influences, lags of the employment rate to proxy cyclical effects, and also a lagged dependent variable. The estimated equation is shown below:\(^5\)

\[
INWA_t = 0.06 - 0.25 \left( \frac{EMP_{t-1}}{POWA_{t-1}} \right) + 0.20 \left( \frac{EMP_{t-2}}{POWA_{t-2}} \right) + 0.00006TIME + 0.87INWA_{t-1} \\
(2.3) \quad (-5.6) \quad (4.2) \quad (2.9) \quad (16.6)
\]  

Where:
INWA = inactivity rate
EMP = number of people employed
POWA = population of working age (exogenous)
TIME = time trend

The equation does not appear to fit the data very well. In particular, the cyclical response of inactivity is very small according to this equation: the sum of the coefficients on the Lags of EMP is 0.25 + 0.20 = 0.45.

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\(^4\) By identity, the population of working age is equal to the numbers employed and unemployed plus the number of inactive people i.e. POWA = EMP + UN + INACTIVE. The participation rate is defined as the number of people employed and unemployed expressed as a proportion of the population of working age i.e. \([\text{EMP + UN}]/\text{POWA}\) so by identity the inverse of the participation rate is the inactivity rate.
lagged employment rate terms is only –0.05. The coefficient on the time trend is small as well.

The lack of a significant trend and cyclical effect may be related. The MM equation models the aggregate participation rate, that is it combines participation rates across males and females, and different age groups. If the trends in the participation rates of these different groups move in different and offsetting ways, it may be difficult to identify a significant trend effect in an aggregate equation. This is indeed the case (see, for example, Nickell, 2001) - while male participation rates have trended downwards over the last 15 years or so, female participation rates have moved in the opposite direction (Chart 2). If aggregate equations cannot reliably capture movements in the trend rate of participation, this may affect their ability to pick up cyclical fluctuations as well.

Chart 2

Male and female activity rates

In the remainder of this paper we investigate whether it is possible to improve on the MM equation by modelling male and female participation rates separately, with a view to producing an alternative projection.

---

We model the participation rate directly (instead of the inactivity rate), and because of data availability estimate annual rather than quarterly models. But we maintain the same form of the MM equation described earlier. The participation rate depends on a constant, a trend term to capture structural influences, a cyclical variable, and a lagged dependent variable, as in equation (5) below.

\[ ACT = \alpha + \beta ACT(-1) + \gamma Trend + \phi Gap \]  

(5)

2.2 Data

Our dataset comprises annual LFS data on participation rates for the 16+ population disaggregated by age group and gender for the period 1971–1999. To estimate trends we fit a flexible non-linear function to the participation rate of each age/gender group in such a way so as to minimise the sum of squared deviations of observations (see Annex A for plots of these). The use of a non-linear function should allow us to fit past data better than the linear time trends used in other work. This gives us estimates of trend movements in the participation rates for each age group which we weight together using population shares to construct trends for males and females as a whole. It should be emphasised that we have not attempted to model the factors affecting these disaggregated ‘trends’, such as changes in the tax and benefit systems, in a structural way, but hope to pick up their net effect in a cruder way using a statistical filter.

To pick up cyclical influences on male/female participation, we include a proxy for the output gap – an estimate of the difference between actual output and trend output. This

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6 Quarterly LFS data on participation rates are not published before 1992.

7 The advantage of modelling the participation rate of individuals aged sixteen and over, instead of working age, is that we do not need to make an adjustment for individuals who are above working age but still employed and therefore participating in the labour market. Since the participation rates of these age groups is fairly flat, and our aim is to project changes in the aggregate participation rate over the short term, the distinction is unlikely to matter.

8 Data are published in “Labour Market Trends, June 1998” to 1997 and we update these to 1999 using Quantime. Disaggregated age data, for our entire sample period, are only available for GB. The GB/UK distinction is unlikely to be important in forecasting the change in the aggregate participation rate since the weight of Northern Ireland is small. An additional point to note is that the age data do not take account of the most recent population projections, but this has had a very small effect. The data also contain a number of discontinuities, which are detailed in the above Labour Market Trends article.

9 The function is of the form \( \alpha 0 + \alpha 1/(\alpha 2 + \alpha 3 * \exp(\alpha 4 + \alpha 5 t)) \). We are grateful to Nick Vaughan of HMT for this suggestion.
is a different cyclical variable to that used in the MM equation, which is the employment rate. The output gap is a broader measure of cyclical pressures and we found it to be more significant in our equations.

Because estimates of trend output are uncertain, we experiment with two different ways of estimating this:

(i) applying a Hodrick Prescott filter to the level of GDP over the past (with a smoothing parameter of 1600) and;
(ii) a production function approach whereby trend GDP is a weighted combination of the population of working age and the capital stock, with the weights equal to their average GDP shares, with an adjustment for estimated total factor productivity.\(^\text{10}\)

This gives us two different measures of the output gap, though these move in a similar way qualitatively (see Chart 3).

**Chart 3**

*Alternative measures of the output gap*

2.3 Results

The male and female participation rate equations are shown in Table 2 below (equations I and II) together with an aggregate equation estimated on annual data for comparison (equation III). All of the terms in the male and female equations are significant. The fit of the equations, as measured by the R-squared, is better than for the aggregate equation

\(^{10}\) See footnote 1.
providing support for disaggregation by gender. Also, the disaggregated equations appear to be stable according to the results from a Chow test for the presence of a structural break in the 1990s (1992 and 1995).

These results imply that the cyclical component of changes in the participation rate is reasonably large. Using an HP-based output gap measure, a 1% permanent step increase in the output gap raises the male participation rate by 0.23pp and the female participation rate by 0.13pp after 1 year; and by 0.33pp and 0.29pp after 2 years. Using a production function approach gives a slightly smaller response, of 0.22pp and 0.14pp respectively after 2 years.

Table 2: Dependent variable: participation rate
Sample period 1984-2000

<table>
<thead>
<tr>
<th></th>
<th>(I) Male equation</th>
<th>(II) Female equation</th>
<th>(III) Aggregate equation</th>
<th>(IV) MM equation(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.70 (-0.26)</td>
<td>4.33 (3.87)</td>
<td>1.05 (0.16)</td>
<td>0.06 (2.26)</td>
</tr>
<tr>
<td>Lagged participation rate</td>
<td>0.82 (17.02)</td>
<td>0.76 (13.51)</td>
<td>0.80 (20.16)</td>
<td>0.88 (18.33)</td>
</tr>
<tr>
<td>Non linear trend</td>
<td>0.19 (2.53)</td>
<td>0.16 (2.70)</td>
<td>0.18 (1.62)</td>
<td></td>
</tr>
<tr>
<td>Linear time trend (x 10^{-3})</td>
<td></td>
<td></td>
<td></td>
<td>0.06 (2.99)</td>
</tr>
<tr>
<td>Output Gap (production function)</td>
<td>0.12 (11.33)</td>
<td>0.08 (3.81)</td>
<td>0.11 (8.31)</td>
<td>-0.25 (-5.39)</td>
</tr>
<tr>
<td>Employment rate(-1)</td>
<td></td>
<td></td>
<td></td>
<td>0.20 (3.99)</td>
</tr>
<tr>
<td>Employment rate(-2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.990</td>
<td>0.982</td>
<td>0.934</td>
<td>0.974</td>
</tr>
<tr>
<td>LM(4)</td>
<td>0.11</td>
<td>0.03</td>
<td>0.78</td>
<td>0.26</td>
</tr>
</tbody>
</table>

3. Forecasting

To produce a forecast for the participation rate we need to project forward the output gap measures and the estimated trends in male and female participation rates. For the output gap projections, we use Consensus Forecasts of GDP growth for 2001 and 2002 made in

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11 In a quarterly version of the male/female equations, the trend terms were significant in the male and female equations but not significant in an aggregate counterpart equation, providing support for disaggregation. We estimate annual equations because quarterly LFS data are not available before 1992.
December 2000, combined with an assumption that trend output grows at a constant annual rate of 2.5% in the future.

To project male and female trends in participation rates, we use Department for Education and Employment (DfEE) projections for participation rates by age/gender out to 2011 applying the same technique as the one we used to derive trends over the past (described above).\footnote{12} Importantly, the DfEE projections are based on the stylised assumption that unemployment remains at its September 1997 level of 1.4 million, so our projected trends are driven by demographic rather than cyclical movements in participation. This leaves our output gap variable to pick up cyclical influences.

Table 3 shows projections for the male/female participation rate over the period 2000 Q2 – 2002 Q2 using the two measures of the output gap. While the female participation rate is projected to continue rising by between +0.4pp to +0.7pp over the next two years, according to which measure of the output gap is chosen, the male participation rate declines. Weighting these projections by population shares gives a projected change in the aggregate participation rate of -0.1 to +0.3pp, or a mid-point of +0.1pp.

<table>
<thead>
<tr>
<th></th>
<th>Females</th>
<th>Males</th>
<th>Population-weighted total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast 2000 Q2 – 2002 Q2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP output gap</td>
<td>+0.4</td>
<td>-0.6</td>
<td>-0.1</td>
</tr>
<tr>
<td>Production function output gap</td>
<td>+0.7</td>
<td>-0.0</td>
<td>+0.3</td>
</tr>
</tbody>
</table>

Chart 4 puts this rise in the context of past increases and shows that while we expect continued increases in the aggregate participation rate over the next two years this is likely to be less than the increases in 1998 and 1999 reflecting, among other things, an expected slowing in GDP growth.

\footnote{12 It makes little difference to the projections whether the trend lines take account of the DfEE projections out to 2011, or use a shorter cut off point of 2002 as in the results in Table 1.}
4. **Deriving an aggregate equation**

Our work on modeling male/female participation rates can be used to derive an aggregate equation which could be used to endogenise participation in a macro model (see Annex B). This would involve re-estimating the gender equations on a quarterly basis. The advantage is that projections for the participation rate will be consistent with other forecasting assumptions, in particular, those affecting the output gap. In particular, the forecast can take account of interactions between changes in the participation rate and the output gap. For instance, a structural increase in the participation rate, for given population and employment, will lead to higher unemployment than if there were no change in labour participation. This would lead to lower consumption reducing some of the initial rise in participation.

Chart 5 shows the (in-sample) one-step ahead forecast using the derived aggregate equation on an annual basis, which seems to track the data reasonably well over the past.
5. **A more disaggregated approach**

In addition to the approach outlined above we tried modelling labour supply at an even more disaggregated level, by age as well as gender group. Again the form of the equations is similar to equation (5) for comparability, and we use the same estimates of trend movements in the participation rate of each age/gender group. But this time we estimate separate equations for each age/gender group, which gives 21 equations; 11 for males and 10 for females.

The mean squared error from a one-step ahead forecast using the equations which disaggregate by age is higher than for the equations which disaggregate by gender alone, suggesting that the latter may be adequate for our purposes. Moreover, the projections produced from this more disaggregated approach are similar to those from the gender equations (compare table 3 and table 4, Annex A).

6. **Conclusions**

The aim of this research was to try to capture cyclical influences, proxied by measures of the output gap, on United Kingdom labour participation rates. By modelling male and
female participation separately, we find a significant and positive effect from changes in the output gap on labour participation rates. The cyclical response is significant: a 1% increase in the output gap leads to an increase of around +0.1pp to +0.2pp in the aggregate participation rate after two years in our equations. This contrasts with the past assumption of a very small cyclical effect in the Bank’s MM equation. A disaggregated approach overcomes a possible shortcoming of aggregate equations. In this case, because male and female participation rates have moved in different directions, it is difficult for an aggregate equation to pick up reliable trend and cyclical variations.

These equations were used to produce short term forecasts of the aggregate participation rate. Conditional on the projections for the output gap made at the end of 2000, which run of Consensus Forecasts of GDP growth, we project a small increase of around 0.2pp in the aggregate participation rate over 2001-2002. This is somewhat smaller than the increase in recent years. This work was used as an input to the November 2000 Inflation Report projection, and to produce a variant equation for the participation rate, which will be used in future forecast rounds to help the MPC form a judgement about prospective movements in participation.

Finally, it is worth noting the limitations of the approach we have used. It is unlikely that our simple equations can adequately capture all of the structural factors that determine the participation rate. Other work, for example, Schweitzer (2001) has shown that individual characteristics such as education are important in determining the probability of whether a person joins the labour force. At a macroeconomic level, government reforms of the tax and benefit system are likely to be important. Our statistical approach will only capture these influences in a crude way. While we find that the output gap does play an important role in influencing cyclical movements in the participation rate, the dominant variable in the equations is the lagged dependent variable. This may simply tell us that participation rate is a highly persistent variable or it may indicate that we are omitting important variables.

---

13 This is the mid-point of the range of estimates suggested by the disaggregated approaches in this paper of +0.1 to +0.2 pp (see Table 3 and Table 4, Annex A), which relies on different measures of the output gap than those used in the original work.
References


ANNEX A

Participation rates by age/gender

In addition to disaggregating by gender we also estimated disaggregated equations by age and gender. The disaggregated trends for each group are shown in the attached charts. This approach produces 21 equations, 10 for females and 11 for males. The equations are generally of the form outlined above, although in a number of the female equations we add variables to account for the number of young dependents.

Results

The 21 disaggregated equations are shown in Tables 5a and 5b. In general, the equations are well specified: the trend term is significant in most cases, and the output gap term where significant is correctly signed so that an increase in the gap raises participation rates. The output gap terms have been tested to allow for different lags in the cyclical responses of different age/sex groups. In general, older age groups have a slower and smaller response to changes in the output gap. As with the male/female equations, most of the disaggregated age equations pass a Chow test for a structural break in either 1992 or 1995.14

Forecast

Aggregating across age groups shows that while the female participation rate is projected to continue rising by 0.7pp to 0.8pp over the next two years, there is a partial off-set from a continued decline in male participation rates, of −0.2pp to −0.6pp. The aggregate participation rate, derived by population-weighting the forecasts from the disaggregated equations, is projected to increase by +0.1pp to +0.3pp over the two years to 2002 Q2 (see Table 4) – slightly higher than the range from the male/female equations in Table 3.

14 The equations which fail the Chow test for a break in 1992 are 55-59, 60-64 and 65+ (females) and 16-19 for non-students and 45-54 (males). In testing for a 1995 break, the equations which fail are 65+ (females) and 65-69 (males). These are among the smallest groups by population share.
<table>
<thead>
<tr>
<th></th>
<th>Females</th>
<th>Males</th>
<th>Population-weighted total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast 2000 Q2 – 2002 Q2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP output gap</td>
<td>+0.7</td>
<td>-0.6</td>
<td>+0.1</td>
</tr>
<tr>
<td>Production function output gap</td>
<td>+0.8</td>
<td>-0.2</td>
<td>+0.3</td>
</tr>
</tbody>
</table>
Table 5a: Male participation rates by age\textsuperscript{15} \textsuperscript{16}

<table>
<thead>
<tr>
<th></th>
<th>Students</th>
<th>Non Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>16-19</td>
<td>20-24</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.03</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(-1.13)</td>
<td>(1.13)</td>
</tr>
<tr>
<td>Lagged dep.</td>
<td>0.31</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(2.27)</td>
<td>(2.85)</td>
</tr>
<tr>
<td>Trend</td>
<td>0.59</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>(4.34)</td>
<td>(5.20)</td>
</tr>
<tr>
<td>Output Gap (production function) x 10\textsuperscript{-1}</td>
<td>0.28</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(4.58)</td>
<td>(5.21)</td>
</tr>
<tr>
<td>Output Gap (production function)(-2) x 10\textsuperscript{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy 1978</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.55)</td>
<td></td>
</tr>
<tr>
<td>Dummy 1979</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.64)</td>
<td></td>
</tr>
<tr>
<td>Dummy 1980</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy 1983</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy 1988</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy 1995</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy 1996</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-bar- squared</td>
<td>0.98</td>
<td>0.94</td>
</tr>
<tr>
<td>F-statistic</td>
<td>429.18</td>
<td>66.80</td>
</tr>
<tr>
<td>LM (4)</td>
<td>0.93</td>
<td>0.17</td>
</tr>
</tbody>
</table>

\textsuperscript{15} Dummy variables were used to where problems with the error terms were found.

\textsuperscript{16} Newey-West standard errors are used where appropriate.
### Table 5b: Female participation rates by age

<table>
<thead>
<tr>
<th></th>
<th>Students</th>
<th>Non Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>16-19</td>
<td>20-24</td>
</tr>
<tr>
<td>Constant</td>
<td>1.35</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(-3.99)</td>
<td>(-0.98)</td>
</tr>
<tr>
<td>Lagged dep.</td>
<td>0.42</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>(3.01)</td>
<td>(2.36)</td>
</tr>
<tr>
<td>Trend</td>
<td>0.54</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>(3.31)</td>
<td>(1.45)</td>
</tr>
<tr>
<td>Output Gap (production function) x 10$^{-1}$</td>
<td>0.17</td>
<td>0.09</td>
</tr>
<tr>
<td>Output Gap (production function)(-1) x 10$^{-1}$</td>
<td>0.27</td>
<td>0.12</td>
</tr>
<tr>
<td>Child (0-4)</td>
<td>-23.09</td>
<td>(-4.11)</td>
</tr>
<tr>
<td>Child (5-9)</td>
<td>-1.46</td>
<td>-2.87</td>
</tr>
<tr>
<td>Dummy 1976</td>
<td>-0.03</td>
<td>(-0.32)</td>
</tr>
<tr>
<td>Dummy 1977</td>
<td>0.17</td>
<td>(2.73)</td>
</tr>
<tr>
<td>Dummy 1984</td>
<td>-0.39</td>
<td>0.37</td>
</tr>
<tr>
<td>Dummy 1987</td>
<td>-0.07</td>
<td>(-2.22)</td>
</tr>
<tr>
<td>Dummy 1989</td>
<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
<td>F-statistic</td>
<td>58.89</td>
<td>48.93</td>
</tr>
<tr>
<td>LM(4)</td>
<td>0.06</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Trends in male participation rates

16-19 years old

20-24 years old

25-34 years old

35-44 and 45-54 years old

55-59 and 60-64 years old

65-69 and 70+ years old
Trends in female participation rates

16-19 years old

Activity rate (%)


20-24 years old

Activity rate (%)


25-34 years old

Activity rate (%)


35-44 and 45-54 years old

Activity rate (%)


55-59 and 60-64 years old

Activity rate (%)


65+ years old

Activity rate (%)

ANNEX B

Aggregating male and female participation rate equations

Male equation:
\[ a_t = \lambda_t a_{t-1} + \alpha_a + \alpha_1 OG_t + \alpha_2 TRM_t \]  
Equation (A)

Female equation:
\[ a_{2t} = \lambda_{2t} a_{2t-1} + \beta_0 + \beta_1 OG_t + \beta_2 TRF_t \]  
Equation (B)

where
\[ a_t = \text{activity rate (males)} \]
\[ a_{2t} = \text{activity rate (females)} \]

OG = Output Gap

TRM = estimated trend (males)

TRF = estimated trend (females)

\[ \alpha_{1.2}, \beta_{1.2} \] are estimated coefficients

\[ \alpha_0 \text{ and } \beta_0 \] = estimated constants

The aggregate participation rate is given by the population-weighted (\( \theta \)) disaggregated rates:
\[ a_t = \theta a_{1t} + (1-\theta)a_{2t} \]

To add (A) and (B) multiply by population weights and use lag operator:

\[ (1-\lambda_1 L) a_{1t} = \theta \alpha_0 + \theta \alpha_1 OG_t + \theta \alpha_2 TRM_t \]

\[ (1-\lambda_2 L)(1-\theta) a_{2t} = (1-\theta) \beta_0 + (1-\theta) \beta_1 OG_t + (1-\theta) \beta_2 TRF_t \]

Multiply by \( (1-\lambda_2 L) \) and \( (1-\lambda_1 L) \) respectively:

\[ (1-\lambda_2 L)(1-\lambda_1 L) \theta a_{1t} = (1-\lambda_2 L)(\theta \alpha_0 + \theta \alpha_1 OG_t + \theta \alpha_2 TRM_t) \]  
(A1)

\[ (1-\lambda_1 L)(1-\lambda_2 L)(1-\theta) a_{2t} = (1-\lambda_1 L)((1-\theta) \beta_0 + (1-\theta) \beta_1 OG_t + (1-\theta) \beta_2 TRF_t) \]  
(B1)

\[ (1-\lambda_2 L)(1-\lambda_1 L) = (1-\lambda_1 L)(1-\lambda_2 L) \]

Adding A1 + B1 and re-arranging gives the aggregate equation:

\[ a_t = \left(\lambda_1 + \lambda_2\right) a_{t-1} - \lambda_1 \lambda_2 a_{t-2} + \theta \left(1-\lambda_2\right) \alpha_0 + \left(1-\theta\right) \left(1-\lambda_1\right) \beta_0 + \left(\theta \alpha_1 + \left(1-\theta\right) \beta_1\right) OG_t + \left(\theta \alpha_2 + \left(1-\theta\right) \beta_2\right) TRM_t - \left(\theta \alpha_1 + \left(1-\theta\right) \beta_1\right) OG_{t-1} - \left(\theta \alpha_2 + \left(1-\theta\right) \beta_2\right) TRM_{t-1} - \left((1-\theta) \lambda_1 \beta_1\right) TRF_{t-1} \]