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# Cardiff Economics Working Papers



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On the equality of Real Interest Rates across borders  
in Integrated Capital Markets

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# On the Equality of Real Interest Rates across borders in Integrated Capital Markets

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## Abstract

The purpose in this letter is first to review briefly the empirical results on the relationship between real interest rates and real exchange rates; this empirical literature provides little support for the hypothesis of Roll that expected real interest rates are equal in general. Our second aim is to discuss the theoretical conditions that have to be met for his hypothesis to hold.

*Keywords:* Real interest rates, Real Exchange rates, Roll.

*JEL classification:* F31, C22, C51

## 1 Introduction

A large number of papers have reported empirical results on the relationship between real interest rates and real exchange rates. The empirical results reported and their interpretation are often contradictory and there appears to be some confusion as to what relationships are implied by economic theory. Of particular relevance in this respect is the influential paper of Roll (1979). Roll argues that in capital markets which are efficient expected real interest rates should be equated across borders. As a consequence real exchange rates, from the standard uncovered interest parity condition, should follow a random walk.

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The purpose in this letter is first to review briefly the empirical results on the relationship between real interest rates and real exchange rates and second is to discuss the theoretical conditions that have to be met for the Roll hypothesis to hold.

## 2 The Empirical Literature

From the uncovered parity conditions in ex-post form, ignoring any time varying risk-premia for simplicity form

$$\rho_t^n = \rho_t^{n*} + q_{t+n} - q_t + \epsilon_{t+n} \quad (1)$$

where  $\rho^n$  and  $\rho^{n*}$  are the real ex post interest rates — domestic and foreign over  $n$  periods,  $q$  is the (log of the) real exchange rate. The error term  $\epsilon_t$  can exhibit up to  $MA(n-1)$  structure under the assumption of rational expectations.

Though ex-post real interest rates or ex-post real interest rate differentials between countries have often been reported to be non-stationary, see e.g. Rose (1988), Edison and Melick (1999), unboundedness of the real interest rate or real interest differential is not an appealing economic property. Also if the interest differential is  $I(1)$  then the real exchange rate has to be an  $I(2)$  process from (1) which is again problematical.

Despite the lack of theoretical grounds for expecting the real exchange rate to be cointegrated with the real interest rate differential, (see e.g. Baxter (1994)), a number of studies have examined the relationship in a cointegration framework. Some are unable to reject the null of no cointegration, e.g. Edison and Pauls (1993), Chortareas and Driver (2001) whilst others reject the null, see e.g. Edison and Melick (1995), MacDonald and Nagayasu (2000). These contradictory results might be explained by recent work by e.g. Lai (1997), and Smallwood and Norrbin (2003) who report that ex post real interest rates can be parsimoniously modeled by fractional processes.<sup>1</sup> In this regard the empirical finding that PPP deviations can be parsimoniously described by stationary fractional processes is significant see, e.g. Diebold et.al., (1991), Cheung and Lai (2000). If relative real interest rates (or real exchange rates) exhibit the fractional property then real exchange rates (or real interest rate differentials) will, in general, also exhibit them as an implication of (1). It is known that structural breaks in series can produce autocorrelations that mimic fractional processes. (See e.g. Diebold and Inoue (2001).) Perhaps an even more plausible explanation of the fractional property can be based on recent work on the time series properties of real exchange rates where real exchange rates have been modelled as a stationary nonlinear process of the smooth transition autoregression model form (ESTAR). (See e.g. Michaels et. al. (1997)). Data that is generated from an

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<sup>1</sup>Fractional processes allow the degree of integration of a series to be non-integer and standard unit root tests may exhibit low power against the fractional alternative. The fractional processes are given by members of the class of ARFIMA( $p, d, q$ ) processes,  $x_t(1-L)^d = u_t$ , where  $u_t$  is a stationary ARMA( $p, q$ ) process, and  $d$  is non-integer.

ESTAR process can appear to exhibit the fractional property (see e.g. Acosta and Granger (1995)).

Overall the empirical work provides little empirical support for the conjecture of Roll that expected real interest rates are equal. Does this imply that these asset markets are inefficient? We show now why this is not the case.

### 3 The Roll Model

Roll (1979) makes the surprising statement — Equation 6.7 on p.139 — that under efficiency where  $Q_t$  is available information,

$$E[r(i, j, t) | Q_{t-1}] = 0 \quad (2)$$

$r(i, j, t)$  is the real return — on a commodity (such as cotton or gold; hereafter ‘cotton’) for which continuous PPP holds across borders — when a resident of country  $j$  invests in cotton in country  $i$  and converts the proceeds back into  $j$  currency. Moving to a more familiar notation than Roll’s, like ours above, we will write this leading it one period as

$$E_t r_{ijt} = E_t (p_{it+1} + S_{t+1} - \pi_{jt+1}) - p_{jt} + \pi_{jt} \quad (3)$$

$E_t$  is the expectations operator conditioned on  $t$ -period information; all variables are in natural logs except  $r$  which is fractional returns per  $t$ -period;  $p$  is the price of cotton,  $\pi$  the general price index and  $S$  is units of  $j$ -currency per 1 unit of  $i$ -currency.

Because  $p_{it} + S_t = p_{jt}$  for all  $t$  by arbitrage or PPP it follows that

$$E_t r_{ijt} = E_t r_{jt} = E_t (p_{jt+1} - \pi_{jt+1}) - p_{jt} + \pi_{jt} \quad (4)$$

where  $r_{jt}$  is the real return on cotton held domestically for  $j$ -residents. Hence Roll is asserting that

$$E_t r_{ijt} = E_t r_{jt} = 0 \quad (5)$$

He offers as justification that one should not observe ‘consistently positive (or negative) rents from shipping... commodities...’ However note that because  $E_t r_{ijt} = E_t r_{jt}$  there is no rent for  $j$ -residents from shipping cotton as the return is the same if it is held at home or if it is shipped abroad and sold there after being held.

Of course if the expected real rate of interest in  $j$  is  $E_t \rho_{jt}$ , then by normal speculative equilibrium  $E_t r_{jt} = E_t \rho_{jt}$  which in general is non-zero.

Roll goes on in the next paragraph to say ‘the strictest interpretation of PPP would also imply equality of real interest rates’, before going on to consider matters of testing, various parallels with Fama’s work on interest rates and other issues. Though Roll’s remarks appear to have had a considerable influence, we will argue that the assertion  $E_t r_{ijt} = 0$  is incorrect; but that nevertheless it is true that strict (general) PPP implies equality of real rates.

Roll's maintained assumption which is normal, and which we retain, is that speculators in country  $i$  wish to consume their proceeds in  $i$ , those resident in  $j$  to consume theirs in  $j$ . It is simple to show that these assumptions imply, in the absence of general PPP, the inequality of real interest rates in general.

Thus begin from

$$E_t r_{ijt} = E_t r_{jt} = E_t \rho_{jt}, \quad E_t r_{jit} = E_t r_{it} = E_t \rho_{it} \quad (6)$$

As we saw

$$\begin{aligned} E_t r_{ijt} &= E_t (p_{it+1} + S_{t+1} - \pi_{jt+1}) - p_{jt} + \pi_{jt} \\ &= E_t (p_{it+1} + S_{t+1} - \pi_{jt+1}) - p_{it} - S_t + \pi_{jt} \\ &= E_t r_{it} + E_t \pi_{it+1} + E_t S_{t+1} - E_t \pi_{jt+1} - \pi_{it} - S_t + \pi_{jt} \\ &= E_t r_{jit} + E_t q_{t+1} - q_t \end{aligned} \quad (7)$$

where  $q_t = \pi_{it} + S_t - \pi_{jt}$  is the real exchange rate as above.

This last condition is that of Real Uncovered Interest Parity, usually written as

$$E_t \rho_{jt} = E_t \rho_{it} + E_t q_{t+1} - q_t \quad (8)$$

Clearly by intra-country equilibrium this condition can be generalised to the expected real return on any asset other than stocks of cotton (abstracting from risk-premia).

### 3.1 General PPP

It is clear from (8) that the sufficient and necessary condition for  $E_t \rho_{jt} = E_t \rho_{it}$  is  $E_t q_{t+1} - q_t = 0$ , in other words that the real exchange rate be either constant or if stochastic be a random walk.

How this might occur? In general consumer prices (and the exchange rate if floating) are endogenous variables in the economy. A mechanism is required to control their *relative* movement in such a way that either the expected or actual real exchange rate is fixed. We may identify at least 3 sorts of goods: exported, imported and non-traded with prices respectively  $p_x$ ,  $p_m$ ,  $p_n$ . Let us assume that goods of the traded industries share the same prices. Then in general the consumer price indices will be (we assume Cobb-Douglas utility for simplicity):

$$\pi_t = \alpha p_{xt} + \beta p_{mt} + (1 - \alpha - \beta) p_{nt} \quad (9)$$

$$\pi_t^* = \gamma p_{xt}^* + \delta p_{mt}^* + (1 - \gamma - \delta) p_{nt}^* \quad (10)$$

(home variables no subscript, foreign an asterisk).

Hence

$$q_t = S_t + \alpha p_{xt} + \beta p_{mt} + (1 - \alpha - \beta) p_{nt} - \gamma p_{xt}^* - \delta p_{mt}^* - (1 - \gamma - \delta) p_{nt}^* \quad (11)$$

From (11) a constant  $q_t$  implies

$$p_x = p_x^* + S = p_m = p_m^* + S = p_n = p_n^* + S. \quad (12)$$

This can be described as:

1. home traded goods must have their prices set in world markets: the classic ‘small open economy’ assumption.
2. home non-traded goods must be close substitutes (presumably in supply due to mobility of resources; or, less likely, in demand); call this high internal industrial integration.
3. foreign prices of all traded and non-traded goods should move in line with each other; that is, the foreign non-traded industry must also be in close substitution with the foreign traded industry and the foreign terms of trade should not move.

These are clearly very stringent conditions which we could describe as general PPP. In terms of analysing an economy in respect of domestic shocks or policy changes we could ignore 3 because foreign prices are exogenous to such events. This would leave 1 and 2. Notice that in addition to the small open economy assumption one also requires that non-traded industry prices move in tandem with traded. However this is *not* assumed in the bulk of analysis of small open economies where the relation of non-traded to traded prices permits the real exchange rate to move in reaction to shocks and policy changes.

In the case where these relationships are stochastic, we have to assume that they are subject to permanent shocks to these equilibrium relativities. Thus for example transport costs or quality differences or index weights might change. Subject to such changes the relationships then resulting would be expected to remain fixed by the same stringent conditions as above.

### 3.2 Complete asset markets

Another condition under which the real exchange rate would follow a random walk is that of complete state-contingent asset markets. These would enable perfect risk-sharing by households across borders (so that if a shock raising marginal utility in country A were to occur combined with a shock lowering marginal utility in country B, A-households would borrow from B-households contingent on that shock combination sufficiently to equate their marginal utilities). In this case the ratio A- and B-households’ consumption would be equal to the ratio of their permanent incomes; this ratio would be a random walk. This again would justify Roll’s assertion of real interest rate equality. However Roll did not suggest any such market completeness. Also in the asset market efficiency literature asset market completeness is not usually assumed. This is for the good reason that like the previous argument the most stringent conditions are required for it to hold.

### 3.3 Moveable residence

There is another mechanism which may be what Roll had in mind. Suppose that speculators could move residence as they sold home assets and bought foreign ones. For example if the asset were housing, one might sell a house in

New York and buy one in London — and live there. Then of course speculators are comparing real returns in each country — that is the nominal return in country A adjusted for inflation in A, with the nominal return in B adjusted for B-inflation. They will then continue to move assets between countries until these real returns are equalised.

One is tempted to believe that a large multinational company might approximate such flexibility of residence. However any such company's shareholders must live somewhere and consider the dividends they receive in terms of their purchasing power there. Thus any marginal speculation must be converted back prospectively into the currency of their residence; this indeed is found in the practice of multinational companies, which translate their returns into the company's principal home currency. Again therefore fixed residence dictates that the wedge of expected real exchange rate change must be driven between real returns. To avoid it shareholders' residence must be mobile.

Such mobility would be an unusual extension of speculative/arbitrage theory which assumes fixed residence — immobility of speculators. Presumably there are non-negligible transactions costs in moving residence in order to exploit real return differences; it seems reasonable to assume these would rule out the sort of instantaneous speculative equalisation of real returns postulated by Roll.

## 4 Conclusions

The empirical results reported in the literature on the relationship between real interest rates and real exchange rates are often contradictory, but provide little support for the argument of Roll (1979) that expected real interest rates should be equal. We show that Roll's assumption of equality between real interest rates across borders of integrated capital markets is in general inapplicable; for it to hold one would require either general continuous PPP, or complete asset markets, or flexible cross-border residence among investors. These conditions can be regarded, a priori, as highly unusual for short-run analysis, in line with the empirical results.

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