

Hasnas, Irina; Lambertini, Luca; Palestini, Arsen

Working Paper

Open Innovation in a dynamic Cournot duopoly

DICE Discussion Paper, No. 111

Provided in Cooperation with:

Düsseldorf Institute for Competition Economics (DICE), Heinrich Heine University Düsseldorf

Suggested Citation: Hasnas, Irina; Lambertini, Luca; Palestini, Arsen (2013) : Open Innovation in a dynamic Cournot duopoly, DICE Discussion Paper, No. 111, ISBN 978-3-86304-110-6, Heinrich Heine University Düsseldorf, Düsseldorf Institute for Competition Economics (DICE), Düsseldorf

This Version is available at:

<https://hdl.handle.net/10419/83954>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

DISCUSSION PAPER

No 111

Open Innovation in a Dynamic Cournot Duopoly

Irina Hasnas,
Luca Lambertini,
Arsen Palestini

October 2013

IMPRINT

DICE DISCUSSION PAPER

Published by

düsseldorf university press (dup) on behalf of
Heinrich-Heine-Universität Düsseldorf, Faculty of Economics,
Düsseldorf Institute for Competition Economics (DICE), Universitätsstraße 1,
40225 Düsseldorf, Germany
www.dice.hhu.de

Editor:

Prof. Dr. Hans-Theo Normann
Düsseldorf Institute for Competition Economics (DICE)
Phone: +49(0) 211-81-15125, e-mail: normann@dice.hhu.de

DICE DISCUSSION PAPER

All rights reserved. Düsseldorf, Germany, 2013

ISSN 2190-9938 (online) – ISBN 978-3-86304-110-6

The working papers published in the Series constitute work in progress circulated to stimulate discussion and critical comments. Views expressed represent exclusively the authors' own opinions and do not necessarily reflect those of the editor.

Open Innovation in a Dynamic Cournot Duopoly

Irina Hasnas

Düsseldorf Institute for Competition Economics (DICE)
Gebäude 24.31, Universitätsstr. 1, 40225 Düsseldorf, Germany
irina.hasnas@dice.hhu.de

Luca Lambertini

Department of Economics, University of Bologna
Strada Maggiore 45, 40125 Bologna, Italy
ENCORE, University of Amsterdam
Roetersstraat 11, WB1018 Amsterdam, The Netherlands
luca.lambertini@unibo.it

Arsen Palestini

MEMOTEF, Sapienza University of Rome
Via del Castro Laurenziano 9, 00161 Rome, Italy
arsen.palestini@uniroma1.it

October 2013

Abstract

In recent years Open Innovation (OI) processes have been receiving growing attention from the empirical and theoretical economic literature, where a debate is taking place on the aspects of complementarity or substitutability between internal R&D and OI spillover. By means of a differential game approach, we analyze the case of substitutability in an OI setup in a Cournot duopoly where knowledge spillovers are

endogenously determined via the R&D process. The game produces multiple steady states, allowing for an asymmetric solution where a firm may trade off the R&D investment against information absorption from the rival. The technical analysis and the numerical simulations point out that the firm which commits to a higher level of OI absorption produces a smaller output and enjoys higher profits than its rival.

JEL codes: C73, L13, O31

Keywords: R&D, spillovers, dynamic games

1 Introduction

According to Chesbrough (2003), the economic system is entering a new era of Open Innovation (OI), where OI is defined as "the use of purposive inflows and outflows of knowledge to accelerate internal innovation, and expand the markets for external use of innovation, respectively". This fact is a logical consequence of a fast growing and highly competitive market for new technologies. At this rate of growth, the companies' internal resources are not sufficient to meet the new challenges, and they have to access external sources. What drives firms towards OI is the fact that many companies are obliged to innovate and develop new products under extremely tough time and resource constraints in order to stay in the market and keep being competitive.

OI is vital for companies whose products have short life cycles (for example software and consumer electronics) and extremely demanding criteria regarding quality, price and customers' expectations. Specifically, the typical example of OI is the Open Source software: Linux, Unix, Mozilla, OpenOffice, Android are products whose source code has been partially or completely released (for an exhaustive overview of the economics of Open Source and on its technological aspects, see Lerner and Tirole, 2002, and West and Gallagher, 2006). Open Source developed by Information Technology firms may be a convenient strategy, when companies expect to boost their profits in a complementary segment, or when they are so small that they cannot compete commercially in the primary segment (Lerner and Tirole, 2002).

The complexity and diversity of the knowledge structure, like nanotechnologies and biotechnologies, are another important factor. As long as "not all smart people work for you" (Chesbrough 2003), there is an increasingly dispersed distribution of useful knowledge in companies of all sizes. The

amount of available information is heterogeneously spread and companies cannot access and monitor all necessary networks. Those networks are built on several factors, including governmental and private organizations, academic research, synthetic knowledge, know-how and highly specialized knowledge based on experience and interaction among the agents. Hence, OI provides an access to these networks. Thus, in a fast developing and expanding environment, relying on the Closed Innovation approach is both insufficient and risky.

The OI policy allows the company to use external technologies (Spithoven *et al.*, 2011) and share its own knowledge with outside partners at a strictly managed and controlled level. The company boundaries become "permeable" (Mortara *et al.*, 2009). The process of learning, accumulation of outside knowledge and competence enables the company to be not only more innovative, but also innovate at a higher speed. It creates linkages among companies that stimulate the sharing of ideas, technology and experience. These strategies, as presented by Monica Beltrametti, Vice President of the Xerox Research Center Europe (XRCE) at Grenoble Innovation Fair (GIF) on October 2009, help share the risk, and reduce costs by using more suppliers. This also opens the way to new markets and explorations (ideas from new external participants, such as governmental organizations, universities, other large companies and individual representatives).

Furthermore, Open Innovation is an innovation in itself (Mortara *et al.*, 2009). It stimulates innovation by distributing the cost and risk, offering access to new contact networks through "intermediaries" (Mortara, 2010) and information pools, and also internalizes the unintended byproducts of innovation process, spillovers (Bogers, 2011). A recent study on the underlying factors affecting the degree of openness of a firm from a strategic point of view is provided by Drechsler and Natter (2012). An accurate description of the various kinds of OI and on the way it has been treated in literature can be found in Dahlander and Gann (2010). Assuming the distinction between inbound and outbound OI, they identify sourcing and acquiring as the typical inbound processes and revealing and selling as the typical outbound processes and investigate the advantages and the disadvantages of each form of openness.

The Closed Innovation model based on the traditional patent system discards unintended or irrelevant research results; however, it has long been considered as a necessary cost for enhancing technical progress. Conversely, the OI policy allows the company to sell those products that do not meet

the firm's capacity and development possibilities (in form of IP licenses) to other companies (Chesbrough 2003, Chesbrough *et al.* 2006). For example, in bio-pharmaceutical industry firms out-license for profit in different stages of R&D process, because they endogenously decide how much of innovation they want to disclose or sell (Bianchi *et al.*, 2011). If observed from this new angle, the firm turns out to have two functions: one is learning from the environment and the other is constructing it by sharing its own knowledge via a systematic spillover flow. Hence, no proof exists that OI generates less spillovers. The company must spend in order to innovate, it must create and achieve itself the new technologies or find them on the market in the opposite case. Thus, there should be a balance between the level of internal R&D and external knowledge resources accrued by the firms. Otherwise, due to spillover's negative effect, that reduces the gains, the firms have a lower investment level than socially desired (as in Arrow, 1962, *inter alia*). In other cases firms may not take into account the positive level their spillovers have on other firms' R&D and *vice versa*, leading again to suboptimal level of R&D (Romer, 1990). On the other hand, firms may be trapped into patent racing and overinvest, thus significantly affecting their profits levels. Moreover, they can disclose too much internal information by adopting a purely Open Innovation approach (Enkel *et al.*, 2009).

An important issue about OI is the ongoing debate on the complementarity or substitutability between the internal and external knowledge used for innovation in a given company. In order to maximize the profits and achieve better results, the company should be able to efficiently scan the environment and correctly assess the degree of complementarity between its R&D program and externally available technology in order to take advantage from the OI policy (Vanhaverbeke *et al.*, 2008). This aspect of a firm's overall strategy can be thought of as a dynamic capability which is built over time (Helfat *et al.*, 2007). Despite these arguments, there are scholars proving that, strategically, some firms rely on substitutability rather than complementarity, due to higher costs of the latter, industry specific qualities or budget constraints.

Moreover, it is worth stressing that OI can be achieved through different routes. One possibility is that the firm implements the policy as a "conscious" movement due to its internal necessities (Mortara *et al.*, 2009). Another possibility is that firms are pushed towards OI by some external factors, like globalization, knowledge-intensive environment, markets, or customer preferences. What happens if the firms are able to optimally determine the spillover delivered to others in the industry, and - in turn - rationally grab the

spillovers created by other firms? The effect is an increase in their gains, and a decrease in unnecessary competitiveness. According to Jaffe (1986), who analyzed the relevance of external R&D to individual companies, spillovers have a positive impact on productivity of own R&D, and a negative effect on competitiveness. This effect is confirmed in a recent empirical paper by Czarnitzki *et al.* (2012).

Since OI has been studied theoretically and empirically in the latest two decades, some models have already appeared in the economic literature, especially concerning Open Source software. The most common frameworks have been the static oligopoly games by far. Some aspects of technology transfers and differentiated Cournot duopolies leading to asymmetric equilibrium structures have been investigated by Zanchettin (2006), whereas Li and Ji (2010) have focused on the characteristics of cost-reducing innovation. Modica (2012) has proposed a model of Open Source development as a two-stage oligopoly game. Another two-stage game is the one constructed by Llanes and de Elejalde (2013) between Open Source and proprietary firms. To the best of our knowledge, one of the very few contributions on OI building on a dynamic model has been conceived by Caulkins *et al.* (2013), where the authors characterize the optimal strategy of a firm when it has to choose between proprietary and Open Source software. From a technical viewpoint, their model is an optimal control problem for a firm in the market subject to the evolutionary dynamics of the quality of its product. Their analysis focuses on the crucial role of R&D costs, and intends to explain when it is convenient to open the source code for the firm.

But why does a large company choose an OI policy? A very relevant example of large IT company which committed to OI development strategies is IBM. In 1993 IBM agreed to sell its industry-leading 2½-inch drives to Apple, which was one of its direct competitors, to use inside its PowerBook laptop computers (see Chesbrough, 2003). In 1999, IBM announced support for Linux, starting to invest financial and technical resources to foster growth, development and use of Linux Open Source technology (see the IBM website for official information). In particular, IBM simply explains its commitment to Open Source technology by listing some clear motivations: 'IBM established the Linux Technology Center (LTC) as the primary vehicle to participate in the Linux community. IBM and the LTC have established four goals for participation in the Linux community: make Linux better, expand Linux's reach for new workloads, enable IBM products to operate with Linux, increase collaboration with customers to innovate in ways IBM

cannot do by itself’.

In this paper, we will take into account a duopoly allowing for both symmetric and asymmetric optimal strategies where the two firms engage in a Cournot competition to maximize their discounted payoff flows over an infinite horizon. In our setup, the firms’ strategic variables are the quantities and the levels of R&D efforts, whereas the state variables are the marginal costs of production and the levels of positive technological spillover spreading from a firm to its rival. Namely, we will look upon OI for one player as depending on her opponent’s development of R&D, as if OI was the fraction of the research she takes or buys from outside, then resulting in a sort of substitute (not complement) with respect to her own R&D. Consequently, we will focus on the aspects of substitutability of OI rather than complementarity of OI. In this respect, Spithoven *et al.* (p. 240, 2012) state that ‘However, empirical evidence does not always support the idea of complementarity between internal and external R&D. Based on a cross-section of Dutch manufacturing firms, Audretsch *et al.* (1996) reported a substitution effect of external R&D activity in low and medium technology industries (the reverse was true in high-tech industries)’. Also Fu (2012) finds that the complementarity between internal R&D and OI is not always feasible, being too costly. It is relevant to remark that OI is not totally free, i.e. we will take into account an appropriation cost of external innovation for the firms.¹

The model we adopt builds upon Cellini and Lambertini (2005, 2009), with respect to which there are two fundamental differences. First, the knowledge spillovers are endogenously determined during the R&D process, because they are state variables which evolve over time as they are affected by the R&D efforts. This is a clear-cut complication of Cellini and Lambertini (2005, 2009), where the technology spillover is described by a constant parameter, identical across firms. Secondly, the model is extended in that we are not going to impose any symmetry assumption, thus allowing for the presence of both symmetric and asymmetric optimal trajectories and steady states. This is actually helpful to separately assess the results between the firm which spends more or less in internal R&D or in OI absorption.

¹This aspect is confirmed by some empirical papers such as Lokshin *et al.*(2008) and Fu (2012), which have analyzed the impact of internal and external R&D on productivity and incentives for further innovation. In particular, Lokshin *et al.* (2008) have examined the data on a 6-year panel of the Dutch manufacturing firms, finding that the costs of appropriation of external R&D were definitely smaller than the costs of development of internal R&D.

Besides being an improvement of a previous differential game, this model intends to shed some light on the effects of firms' strategic behavior when they can acquire both internal and external R&D resources. Given the substitutability of R&D and OI, it is interesting to assess the output levels and the profit levels at equilibrium so as to identify the most convenient investment strategy in a small market. The main findings of the paper can be outlined as follows:

- We prove the existence of multiple steady states, in particular a symmetric and an asymmetric one. In the asymmetric steady state, a firm may trade off the R&D investment against information absorption from the rival.
- We deduce from the technical results and from the numerical simulation that, interestingly, the firm investing less in R&D enjoys higher profits than the rival, thanks to the combining effects of savings upon investment costs and exploiting information transmission.
- The firm with a higher private R&D investment level can commit to a definitely smaller level of OI appropriation effort, and *vice versa*.

The structure of the paper includes Section 2, where the setup of our model is outlined, Section 3, containing the analytical study of the optimal strategies, and separately featuring the symmetric and the asymmetric case. Section 4 includes our conclusions and supplies hints for future developments.

2 The Setup

We consider an infinite horizon differential game modeling a duopoly with single-product firms in continuous time $t \in [0, \infty)$. For simplicity, assume firms supply homogeneous goods. Define the inverse market demand function as

$$p(t) = A - q_1(t) - q_2(t), \quad (1)$$

where $A > 0$ is the constant reservation price and $q_i(t)$ is the quantity produced by the i -th firm at time t .

Production takes place at constant returns to scale, with marginal cost $c_i(t)$ evolving over time according to the following dynamic equation:

$$\frac{dc_i}{dt} \equiv \dot{c}_i(t) = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t), \quad i, j = 1, 2; i \neq j \quad (2)$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t , and $\beta_i(t) \in (0, 1)$ is the level of positive technological spillover or the level of OI enjoyed by firm i (and transmitted by firm j).² Parameter $\delta \geq 0$ is a constant depreciation rate that results in decreasing returns due to aging of the technology.

Our approach to spillovers is precisely the feature of the model where we depart from Cellini and Lambertini (2005, 2009). Here, we allow for the spillover to be endogenously determined by the firms' instantaneous R&D efforts, whereby Open Innovation made available to firm i changes over time according to the following dynamic equation:

$$\frac{d\beta_i}{dt} \equiv \dot{\beta}_i(t) = \alpha k_j(t) - \eta \beta_i(t) \quad (3)$$

where α and η are positive parameters. The above equation refers to a situation where firm i has access to an amount of OI (a state variable) that deteriorates if firm j ceases to carry out any R&D.

(3) deserves some further explanations: this representation implies that the inflow of knowledge from the i -th firms stems from the R&D carried out by the j -th firm and *vice versa*. By acquiring, or buying, such knowledge from its opponent, a firm may substitute its own R&D with OI and the inflow of knowledge for one agent corresponds to the outflow for the other agent. Hence, this kinematic equation describes our idea of substitutability between R&D and OI, also corresponding to the endogenization of the spillover effect, contrary to Cellini and Lambertini (2005, 2009), where it is a constant parameter. α , which will be taken as the relevant parameter for the analysis of the equilibrium profits in Subsections 3.1.1 and 3.2.1, represents the rate at which the spillover effect on OI grows as it is driven by external R&D.

The cost of creating R&D by firm i is described by the convex function: $\Gamma_i(k_i(t)) = \frac{b[k_i(t)]^2}{2}$, where b is a positive parameter. We are also assuming that, in order for a firm to be able to absorb positive externalities from the environment, it has to bear some appropriation cost, which can be represented as $C_i(\beta_i(t)) = \frac{\epsilon[\beta_i(t)]^2}{2}$, where ϵ is a positive parameter. For example, it can be generated by the process of searching and assimilating new knowledge, and subsequently adapting it to the firms necessities and standards.

²In this model, productive technologies are perfect substitutes. For an alternative approach where complementarity is considered, see Scotchmer (2010).

Accordingly, the instantaneous profit function of the i -th firm will be written as follows:

$$\begin{aligned}\pi_i(t) &= (p(t) - c_i(t))q_i(t) - \Gamma_i(k_i(t)) - C_i(\beta_i(t)) = \\ &= [A - q_i(t) - q_j(t) - c_i(t)]q_i(t) - \frac{b[k_i(t)]^2}{2} - \frac{\epsilon[\beta_i(t)]^2}{2}.\end{aligned}$$

We shall analyze a fully noncooperative game in which each firm sets independently its own level of R&D (determining thus their productive efficiency and the respective degrees of information sharing), as well as the level of output it wants to sell. Hence, the problem of firm i is to

$$\max_{q_i, k_i \geq 0} \Pi_i \equiv \int_0^\infty \pi_i(t) e^{-\rho t} dt \quad (4)$$

subject to

$$\dot{c}_i(t) = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t) \quad (5)$$

$$\dot{c}_j(t) = -k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t) \quad (6)$$

$$\dot{\beta}_i(t) = \alpha k_j(t) - \eta \beta_i(t) \quad (7)$$

$$\dot{\beta}_j(t) = \alpha k_i(t) - \eta \beta_j(t) \quad (8)$$

whose initial conditions are $c_i(0) = c_{i0} \geq 0$, $\beta_i(0) = \beta_{i0} \geq 0$, for $i = 1, 2$. Along the game firms will discount the future profits, hence $\rho > 0$ is an intertemporal discount rate common to both firms.

3 The game

The Hamiltonian of the i -th player is a function depending on the arguments: t , q_1 , q_2 , k_1 , k_2 , c_1 , c_2 , β_1 , β_2 and on all the related costate variables. In turn, all arguments are dependent on time, but from now on we will omit the time arguments whenever possible to simplify notation. The current value Hamiltonian function \mathcal{H}_i takes the following form:

$$\begin{aligned}\mathcal{H}_i(\cdot) &= e^{-\rho t} \left\{ [A - q_i(t) - q_j(t) - c_i(t)]q_i(t) - \frac{b[k_i(t)]^2}{2} - \frac{\epsilon[\beta_i(t)]^2}{2} + \right. \\ &+ \lambda_{ii}(t)[-k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t)] + \lambda_{ij}(t)[-k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t)] + \\ &\left. + \mu_{ii}[\alpha k_j(t) - \eta \beta_i(t)] + \mu_{ij}[\alpha k_i(t) - \eta \beta_j(t)] \right\} \quad (9)\end{aligned}$$

where $\lambda_{ii}(t)$ and $\lambda_{ij}(t)$ are the current value co-state variables respectively associated to the state variables $c_i(t)$ and $c_j(t)$, and $\mu_{ii}(t)$ and $\mu_{ij}(t)$ are the current value co-state variables respectively associated to the state variables $\beta_i(t)$ and $\beta_j(t)$.

We are going to determine the open-loop information structure of this game by applying the Pontryagin's Maximum Principle. The procedure we are going to implement is standard in differential game theory applied to industrial economic models (analogous techniques can be found in Cellini and Lambertini, 1998, 2002, 2009).

The first-order conditions (FOCs) are

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = 0 \implies A - 2q_i - q_j - c_i = 0 \quad (10)$$

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = 0 \implies -bk_i - \lambda_{ii} - \beta_j \lambda_{ij} + \alpha \mu_{ij} = 0 \quad (11)$$

Note that the linear-quadratic structure of the Hamiltonian function ensures the concavity w.r.t. the control variables, and then the existence of a maximum point; indeed, the second order conditions read as:

$$\frac{\partial^2 \mathcal{H}_i}{\partial q_i^2} = -2 < 0 \quad (12)$$

$$\frac{\partial^2 \mathcal{H}_i}{\partial k_i^2} = -b < 0 \quad (13)$$

Before carrying out the standard procedure, we must point out that this game has degenerate features, in that (10) do not contain any costate variable. Hence, (10) represent a couple of policy functions which will be employed to subsequently determine the equilibrium structure. Differentiating (10) w.r.t. time we obtain:

$$\dot{c}_i(t) = -2\dot{q}_i(t) - \dot{q}_j$$

$$\dot{c}_j(t) = -2\dot{q}_j(t) - \dot{q}_i$$

Plugging the expressions obtained from the latter relations and from (10) into (5) and (6), we achieve the dynamic equations:

$$-2\dot{q}_i(t) - \dot{q}_j = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t) \quad (14)$$

$$-2\dot{q}_j(t) - \dot{q}_i = -k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t) \quad (15)$$

From (14) and (15) we deduce the following output dynamics:

$$\dot{q}_i(t) = \frac{1}{3}[(2 - \beta_j(t))k_i(t) + (-1 + 2\beta_i(t))k_j(t) + \delta(c_j(t) - 2c_i(t))] \quad (16)$$

$$\dot{q}_j(t) = \frac{1}{3}[(-1 + 2\beta_j(t))k_i(t) + (2 - \beta_i(t))k_j(t) + \delta(-2c_j(t) + c_i(t))] \quad (17)$$

By construction, (16) and (17) depend linearly on (5) and (6), so they are not going to provide any further information about equilibrium. The adjoint equations and transversality conditions for the costates λ_{ij} amount to:

$$\begin{cases} \dot{\lambda}_{ii}(t) = (\rho - \delta)\lambda_{ii}(t) + q_i(t) \\ \lim_{t \rightarrow +\infty} e^{-\rho t} \lambda_{ii}(t) c_i(t) = 0 \end{cases}, \quad (18)$$

$$\begin{cases} \dot{\lambda}_{ij}(t) = (\rho - \delta)\lambda_{ij}(t) \\ \lim_{t \rightarrow +\infty} e^{-\rho t} \lambda_{ij}(t) c_j(t) = 0 \end{cases}. \quad (19)$$

for $i \neq j$, (19) admits the solutions $\lambda_{ij} \equiv 0$. The adjoint equations and transversality conditions for the costates μ_{ij} are:

$$\begin{cases} \dot{\mu}_{ii}(t) = \epsilon\beta_i(t) + \lambda_{ii}(t)k_j(t) + (\rho + \eta)\mu_{ii}(t) \\ \lim_{t \rightarrow +\infty} e^{-\rho t} \mu_{ii}(t)\beta_i(t) = 0 \end{cases}, \quad (20)$$

$$\begin{cases} \dot{\mu}_{ij}(t) = \lambda_{ij}(t)k_j(t) + (\rho + \eta)\mu_{ij} \\ \lim_{t \rightarrow +\infty} e^{-\rho t} \mu_{ij}(t)\beta_j(t) = 0 \end{cases}. \quad (21)$$

Plugging the solutions to (19) into (21) implies that $\mu_{ij}(t) \equiv 0$ are solutions to (21), for $i \neq j$.

Before proceeding any further, some technical aspects are to be explained in detail. The peculiarity of this game relies upon the fact that, since the costate variables μ_{ii} , for $i = 1, 2$, do not enter the FOCs, they are not actually relevant for equilibrium. The standard procedure, i.e. the differentiation w.r.t. time of (10) and of (11) can only yield a system of control equations originated from (18). The corresponding policy implication suggests that the diffusion of the spillover does not affect the optimal policy of the i -th

firm, so that the spillover effects are completely exogenous with respect to the producer's strategy. In other words, although the beneficial consequences of OI on profit flows are obviously affecting both firms, the dynamics of its evolution is irrelevant on the strategic decisions.

Therefore, we will only take into account the transversality conditions for λ_{ii} . Since the involved optimal states and costates are:

$$c_i^*(t) = \left(c_i^*(0) - \int_0^t (k_i^*(s) + \beta_i^*(s)k_j^*(s))e^{-\delta s} ds \right) e^{\delta t},$$

$$\lambda_{ii}^*(t) = \left(\lambda_{ii}^*(0) + \int_0^t q_i^*(s)e^{(\delta-\rho)s} ds \right) e^{(\rho-\delta)t},$$

then the transversality conditions hold if and only if:

$$\lim_{t \rightarrow +\infty} \left(\lambda_{ii}^*(0) + \int_0^t q_i^*(s)e^{(\delta-\rho)s} ds \right) \left(c_i^*(0) + \int_0^t (k_i^*(s) - \beta_i^*(s)k_j^*(s))e^{-\delta s} ds \right) = 0,$$

i.e.

$$\lambda_{ii}^*(0) = - \int_0^\infty q_i^*(s)e^{(\delta-\rho)s} ds,$$

then the optimal costates must be:

$$\lambda_{ii}^*(t) = - \left(\int_t^\infty q_i^*(s)e^{(\delta-\rho)s} ds \right) e^{(\rho-\delta)t}.$$

The output control equations will follow from the differentiation of (10), entailing that they are linearly dependent on the dynamic constraints. Subsequently, we achieve the simple identity:

$$\dot{\lambda}_{ii}(t) = -b\dot{k}_i(t), \quad (22)$$

then, exploiting (11) and (22) in (18) we have:

$$\dot{k}_i(t) = (\rho - \delta)k_i(t) - \frac{q_i(t)}{b} \quad (23)$$

$$\dot{k}_j(t) = (\rho - \delta)k_j(t) - \frac{q_j(t)}{b} \quad (24)$$

leading to the following state-control dynamical system, consisting in eight ordinary differential equations:

$$\left\{ \begin{array}{l} \dot{c}_i(t) = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t) \\ \dot{c}_j(t) = -k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t) \\ \dot{\beta}_i(t) = \alpha k_j(t) - \eta \beta_i(t) \\ \dot{\beta}_j(t) = \alpha k_i(t) - \eta \beta_j(t) \\ \dot{k}_i(t) = (\rho - \delta)k_i(t) - \frac{q_i(t)}{b} \\ \dot{k}_j(t) = (\rho - \delta)k_j(t) - \frac{q_j(t)}{b} \\ \dot{q}_i(t) = \frac{1}{3}[(2 - \beta_j(t))k_i(t) + (-1 + 2\beta_i(t))k_j(t) + \delta(c_j(t) - 2c_i(t))] \\ \dot{q}_j(t) = \frac{1}{3}[(-1 + 2\beta_j(t))k_i(t) + (2 - \beta_i(t))k_j(t) + \delta(-2c_j(t) + c_i(t))]. \end{array} \right. \quad (25)$$

We are now going to search for the possible steady states of (25) by letting all its equations vanish, but the aforementioned linear dependence of $\dot{q}_i(t)$ and of $\dot{q}_j(t)$ leaves us with only six equations and eight unknowns. Consequently, we will make use of (10), thus expressing all state variables depending on the R&D efforts as follows:

$$c_i = A - b(\rho - \delta)(k_j + 2k_i), \quad (26)$$

$$c_j = A - b(\rho - \delta)(k_i + 2k_j), \quad (27)$$

$$\beta_i = \frac{\alpha}{\eta}k_j, \quad (28)$$

$$\beta_j = \frac{\alpha}{\eta}k_i. \quad (29)$$

By replacing the resulting expressions for c_i , c_j , β_i and β_j in the last two equations of (25) and imposing stationarity, we obtain the following two

nonlinear equations in k_i and k_j :

$$-k_i - \frac{\alpha}{\eta} k_j^2 + \delta [A - b(\rho - \delta)(k_j + 2k_i)] = 0 \quad (30)$$

$$-k_j - \frac{\alpha}{\eta} k_i^2 + \delta [A - b(\rho - \delta)(k_i + 2k_j)] = 0 \quad (31)$$

Then, subtracting (31) from (30) and subsequently factoring the equation:

$$\begin{aligned} k_j - k_i - \frac{\alpha}{\eta} (k_j^2 - k_i^2) + \delta b(\rho - \delta) [k_j - k_i] &= 0 \iff \\ \iff (k_j - k_i) \left[1 - \frac{\alpha}{\eta} (k_j + k_i) + \delta b(\rho - \delta) \right] &= 0, \end{aligned}$$

we obtain two kinds of different zeros:

$$k_i = k_j \quad \text{and} \quad k_i = \frac{(1 + b\delta(\rho - \delta))\eta}{\alpha} - k_j. \quad (32)$$

In the following Subsections, we shall investigate both the symmetric and the asymmetric equilibrium cases.

3.1 The symmetric equilibrium structure

The following Proposition illustrates the properties of the symmetric equilibrium point:

Proposition 1. *If*

1. $\rho > \delta$;
2. $\alpha < \frac{\eta}{\delta A} [2 + 3b\delta(\rho - \delta)]$,

the game admits a symmetric steady state $P = (c_1^, c_2^*, \beta_1^*, \beta_2^*, k_1^*, k_2^*, q_1^*, q_2^*)$,*³
where:

$$k_1^* = k_2^* = k^* = \frac{-\eta[1 + 3b\delta(\rho - \delta)] + \sqrt{\eta^2(1 + 3b\delta(\rho - \delta))^2 + 4\alpha\eta\delta A}}{2\alpha},$$

³Note that under symmetry, the 2 dynamics (3) collapse into a unique one, and the respective R&D efforts cannot be distinguished, then it may also be interpreted as the case in which the i -th firm's spillover is produced by its own R&D and not from its competitor's R&D.

$$\begin{aligned}
c_1^* &= c_2^* = c^* = A - 3b(\rho - \delta)k^*, \\
\beta_1^* &= \beta_2^* = \beta^* = \frac{\alpha}{\eta}k^*, \\
q_1^* &= q_2^* = q^* = b(\rho - \delta)k^*.
\end{aligned}$$

Proof. Consider the symmetric case, i.e. $k_i = k_j = k$ and substitute in (30):

$$-k - \frac{\alpha}{\eta}k^2 + \delta(A - b(\rho - \delta)(k + 2k)) = 0,$$

whose zeros are:

$$k_{1,2}^* = \frac{-\eta[1 + 3b\delta(\rho - \delta)] \pm \sqrt{\eta^2(1 + 3b\delta(\rho - \delta))^2 + 4\alpha\eta\delta A}}{2\alpha}.$$

If $\rho > \delta$, it can be easily observed that the smallest solution is negative, whereas the remaining one is positive, and consequently feasible.

Since the level of spillover β^* is supposed to belong to the interval $(0, 1)$, we need to find suitable assumptions so that this property be satisfied:

$$\begin{aligned}
\beta^* = \frac{\alpha}{\eta}k^* &= \frac{-\eta[1 + 3b\delta(\rho - \delta)] + \sqrt{\eta^2(1 + 3b\delta(\rho - \delta))^2 + 4\alpha\eta\delta A}}{2\eta} < 1 \iff \\
&\iff (\eta(3 + 3b\delta(\rho - \delta)))^2 - \eta(4A\alpha\delta + \eta(1 + 3b\delta(\rho - \delta))^2) > 0 \iff \\
&\iff \dots \iff \alpha < \frac{\eta}{\delta A}[2 + 3b\delta(\rho - \delta)],
\end{aligned}$$

hence if $\alpha < \frac{\eta}{\delta A}[2 + 3b\delta(\rho - \delta)]$, then $\beta^* \in (0, 1)$.

Moreover, since the remaining coordinates of P , achieved by the relations originated by the vanishing of (25), are positive for $\rho > \delta$, then P is a feasible steady state for the system (25). \square

As far as the dynamic features of the trajectories are concerned, we can get some information from the eigenvalues of the 4×4 Jacobian matrix of the dynamic system achieved from the symmetry assumptions, that is:

$$\mathcal{J}(P) = \begin{pmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial \beta} & \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial q} \\ \frac{\partial \dot{\beta}}{\partial c} & \frac{\partial \dot{\beta}}{\partial \beta} & \frac{\partial \dot{\beta}}{\partial k} & \frac{\partial \dot{\beta}}{\partial q} \\ \frac{\partial \dot{k}}{\partial c} & \frac{\partial \dot{k}}{\partial \beta} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial q} \\ \frac{\partial \dot{q}}{\partial c} & \frac{\partial \dot{q}}{\partial \beta} & \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial q} \end{pmatrix} = \begin{pmatrix} \delta & -k^* & -1 - \beta^* & 0 \\ 0 & -\eta & \alpha & 0 \\ 0 & 0 & \rho - \delta & -\frac{1}{b} \\ -\frac{\delta}{3} & \frac{k^*}{3} & \frac{1+\beta^*}{3} & 0 \end{pmatrix}. \quad (33)$$

Proposition 2. *The symmetric steady state P is a saddle point equilibrium.*

Proof. In order to check the stability of P , we have to evaluate the eigenvalues (or zeros) of the characteristic polynomial of (33). Calling λ the unknown of the equation, we have:

$$\begin{aligned} p(\mathcal{J}(P)) &= (\delta - \lambda) \left[(\eta\lambda + \lambda^2)(\rho - \delta - \lambda) - \frac{\alpha k^*}{3b} - \frac{(1 + \beta^*)(\eta + \lambda)}{3b} \right] + \\ &\quad + \frac{\delta}{3b} [\alpha k^* + (1 + \beta^*)(\eta + \lambda)] = \dots = \\ &= \lambda(\delta - \lambda)(\eta + \lambda)(\rho - \delta - \lambda) + \lambda \frac{k^* + (1 + \beta^*)(\eta + \lambda)}{3b}, \end{aligned}$$

having the zero $\lambda_1 = 0$. The remaining 3rd degree polynomial admits at least one real negative zero because the known term $\rho\eta(\rho - \delta) + \frac{k^* + (1 + \beta^*)\eta}{3b}$ is positive when P exists and is feasible by Proposition 1. Hence, P is a saddle point equilibrium whose stable manifold has at least dimension 1. \square

This analysis ensures that there exist trajectories heading towards that equilibrium point and also trajectories running away from it.

The expression of profit evaluated at P , expressed by means of k^* , is⁴

$$\begin{aligned} \Pi^* &= (A - 2q^* - c^*)q^* - \frac{b(k^*)^2}{2} - \frac{\epsilon(\beta^*)^2}{2} = \\ &= \dots = (k^*)^2 \left[b^2(\rho - \delta)^2 - \frac{b}{2} - \frac{\epsilon\alpha^2}{2\eta^2} \right]. \end{aligned} \quad (34)$$

The next Proposition intends to characterize the assumptions for the positivity of (34).

Proposition 3. *If*

1. $\rho > \delta$;
2. $b > \frac{1}{2(\rho - \delta)^2}$;
3. $\alpha < \min \left\{ \frac{\eta}{\delta A} [2 + 3b\delta(\rho - \delta)]; \eta \sqrt{\frac{2}{\epsilon} \left[b^2(\rho - \delta)^2 - \frac{b}{2} \right]} \right\}$,

⁴The complete calculations are available upon request to the authors.

then the profit function evaluated at P^* is positive.

Proof. A sufficient condition for the positivity of (34) is given by:

$$b^2(\rho - \delta)^2 - \frac{b}{2} - \frac{\epsilon\alpha^2}{2\eta^2} > 0,$$

which can be arranged by isolating α as in Proposition 1:

$$\alpha^2 < \frac{2\eta^2}{\epsilon} \left[b^2(\rho - \delta)^2 - \frac{b}{2} \right].$$

Thus, combining this condition with the one stated in Proposition 1 ensuring the existence and feasibility of P^* , we can conclude that if $b > \frac{1}{2(\rho - \delta)^2}$ and

$$\alpha < \min \left\{ \frac{\eta}{\delta A} [2 + 3b\delta(\rho - \delta)]; \eta \sqrt{\frac{2}{\epsilon} \left[b^2(\rho - \delta)^2 - \frac{b}{2} \right]} \right\},$$

then $\Pi^* > 0$. □

3.1.1 A numerical simulation

We chose the following values for parameters: $\delta = 0.01$, $\rho = 0.77$, $\epsilon = 0.05$, $A = 1$, $\eta = 0.001$, $b = 2.5$, verifying the conditions of Proposition 1. In particular, for $\alpha = 0.006$, P is feasible:

$$P = (0.9359, 0.9359, 0.0674, 0.0674, 0.0112, 0.0112, 0.0213, 0.0213).$$

Figure 1, sketched by employing Mathematica 5.0, is the outcome of a numerical simulation performed to illustrate the shape of $\Pi^*(\alpha)$. It shows that $\Pi^*(\alpha)$ keeps positive as $\alpha \in (0.002, 0.01)$.

It is worth noting that $\Pi^*(\alpha)$ is concave w.r.t. α , which can be explained on the basis of the balance between two opposite effects, i.e., the desirable gain generated by the transmission of technological knowledge through an increase in the spillover level, on the one hand, and the undesirable increase in the intensity of competition that the same fact brings about via a decrease in marginal costs and the resulting output expansion, on the other.

As we can infer from the simulation, $\Pi^*(\alpha)$ attains a maximum positive level and then decreases, becoming negative as α exceeds a certain threshold.

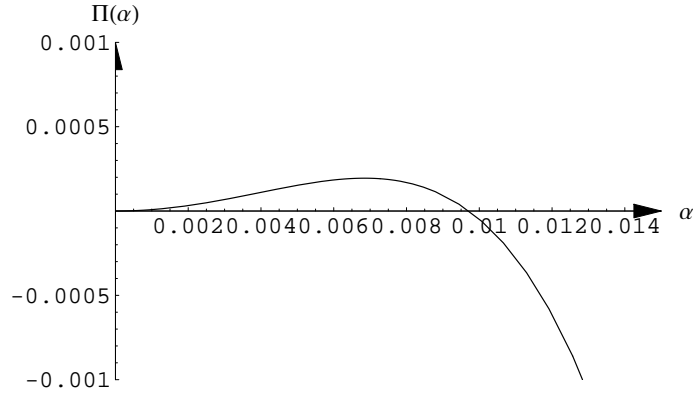


Figure 1: The graph of $\Pi^*(\alpha)$.

Substituting the values of parameters employed in Figure 1, the Jacobian matrix evaluated at the symmetric steady state reads as:

$$\mathcal{J}(P) = \begin{pmatrix} 0.01 & -0.0112 & -1.0674 & 0 \\ 0 & -0.001 & 0.006 & 0 \\ 0 & 0 & 0.76 & -0.4 \\ -0.0033 & 0.0037 & 0.3558 & 0 \end{pmatrix},$$

having the following eigenvalues:

$$\lambda_1 = 0, \quad \lambda_2 = -0.01, \quad \lambda_{3,4} = 0.385 \pm 0.0414i,$$

where $i = \sqrt{-1}$. Since λ_3 and λ_4 are complex conjugate with positive real parts, whereas λ_2 is negative, P turns out to be an unstable focus, with a 2-dimensional unstable manifold and a 1-dimensional stable manifold.

3.2 The asymmetric equilibrium structure

Here we focus on the possible arising of asymmetric outcomes. We are going to prove the following result:

Proposition 4. *If the following parametric hypotheses hold:*

1. $\rho > \delta$,

$$2. b < \frac{1}{\delta(\rho - \delta)},$$

$$3. \alpha \in \left(\frac{\eta[3 + 7b\delta(\rho - \delta)][1 + b\delta(\rho - \delta)]}{4A\delta}, \frac{\eta(1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2)}{A\delta} \right),$$

the game also admits a non-symmetric steady state

$$Q = (c_1^{NS}, c_2^{NS}, \beta_1^{NS}, \beta_2^{NS}, k_1^{NS}, k_2^{NS}, q_1^{NS}, q_2^{NS}),$$

where:

$$c_1^{NS} = A - b(\rho - \delta) \left[\frac{3\eta[1 + b\delta(\rho - \delta)] - \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha} \right],$$

$$c_2^{NS} = A - b(\rho - \delta) \left[\frac{3\eta[1 + b\delta(\rho - \delta)] + \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha} \right],$$

$$\beta_1^{NS} = \frac{\eta[1 + b\delta(\rho - \delta)] + \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\eta},$$

$$\beta_2^{NS} = \frac{\eta[1 + b\delta(\rho - \delta)] - \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\eta},$$

$$k_1^{NS} = \frac{\eta[1 + b\delta(\rho - \delta)] - \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha},$$

$$k_2^{NS} = \frac{\eta[1 + b\delta(\rho - \delta)] + \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha},$$

$$q_1^{NS} = b(\rho - \delta) \left[\frac{\eta[1 + b\delta(\rho - \delta)] - \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha} \right].$$

$$q_2^{NS} = b(\rho - \delta) \left[\frac{\eta[1 + b\delta(\rho - \delta)] + \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha} \right],$$

and where

$$\Omega(b, \eta, \delta, \rho, \alpha, A) := \eta^2(1 + b\delta(\rho - \delta))^2 - 4[(1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2)\eta^2 - \alpha\eta\delta A].$$

Proof. Consider the non-symmetric case, consisting in the following relation between the $R&D$ optimal strategies:

$$k_i = \frac{(1 + b\delta(\rho - \delta))\eta}{\alpha} - k_j.$$

We substitute k_i into (30) to obtain an equation having k_j as an unknown:

$$\begin{aligned} & -\frac{(1 + b\delta(\rho - \delta))\eta}{\alpha} + k_j - \frac{\alpha}{\eta}k_j^2 \\ & + \delta \left[A - b(\rho - \delta)(k_j + 2\frac{(1 + b\delta(\rho - \delta))\eta}{\alpha} - 2k_j) \right] = 0, \end{aligned}$$

which can be arranged as follows:

$$\alpha^2 k_j^2 - \alpha\eta(1 + b\delta(\rho - \delta))k_j + [1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2]\eta^2 - \alpha\eta\delta A = 0.$$

The two roots of the latter equation are:

$$k_{1,2}^{NS} = \frac{\eta[1 + b\delta(\rho - \delta)] \pm \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha}$$

where

$$\Omega(b, \eta, \delta, \rho, \alpha, A) := \eta^2(1 + b\delta(\rho - \delta))^2 - 4[(1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2)\eta^2 - \alpha\eta\delta A].$$

To check whether the $k_{1,2}^{NS}$ are feasible we need to assess the positivity of the function $\Omega(b, \eta, \delta, \rho, \alpha, A)$:

$$\begin{aligned} & \Omega(b, \eta, \delta, \rho, \alpha, A) > 0 \iff \\ \iff & \eta^2(1 + b\delta(\rho - \delta))^2 > 4[(1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2)\eta^2 - \alpha\eta\delta A] \iff \\ \iff & \dots \iff -7(b\delta(\rho - \delta))^2 - 10b\delta(\rho - \delta) - 3 + \frac{4\alpha\delta A}{\eta} > 0, \end{aligned}$$

implying that α should exceed the level denoted by $\hat{\alpha}$:

$$\alpha > \hat{\alpha} \equiv \frac{\eta}{4A\delta} [(3 + 7b\delta(\rho - \delta))(1 + b\delta(\rho - \delta))]$$

As in the asymmetric case, the feasibility of the steady state requires that $0 < \beta_{1,2}^{NS} < 1$. Since obviously $\beta_1^{NS} > 0$ and $\beta_1^{NS} > \beta_2^{NS}$, we have to determine sufficient conditions such that the following system of inequalities holds:

$$\begin{cases} \beta_1^{NS} < 1 \\ \beta_2^{NS} > 0 \end{cases} \iff \begin{cases} \sqrt{\Omega(\cdot)} < \eta[1 - b\delta(\rho - \delta)] \\ \sqrt{\Omega(\cdot)} < \eta[1 + b\delta(\rho - \delta)] \end{cases},$$

hence the sufficient conditions can be expressed by the following system:

$$\begin{cases} b < \frac{1}{\delta(\rho - \delta)} \\ \sqrt{\Omega(\cdot)} < \eta[1 + b\delta(\rho - \delta)] \end{cases}.$$

Rearranging the latter inequality, we have that:

$$\begin{aligned} 2(b\delta(\rho - \delta))^2 + 3b\delta(\rho - \delta) + 1 - \frac{\delta\alpha A}{\eta} > 0 &\iff \\ \iff \dots \iff \alpha < \bar{\alpha} \equiv \frac{\eta(1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2)}{A\delta}. \end{aligned}$$

If we compare the two levels, a direct computation yields that $\hat{\alpha} < \bar{\alpha}$ irrespective of all the remaining parameters' values, therefore a sufficient condition for α is $\alpha \in (\hat{\alpha}, \bar{\alpha})$. Combining this last constraint with the one for the feasibility of q_1^{NS} and q_2^{NS} , i.e. $\rho > \delta$, we obtain the three assumptions for all the coordinates of Q except c_1^{NS} and c_2^{NS} , whose expressions follow from the relations (26) and (27). Since $c_1^{NS} > c_2^{NS}$, it suffices to prove that $c_2^{NS} > 0$ under the same three assumptions. To begin with, we can rewrite it as follows:

$$c_2^{NS} > 0 \iff 2\alpha A - 3b\eta(\rho - \delta)[1 + b\delta(\rho - \delta)] - b(\rho - \delta)\sqrt{\Omega(\cdot)} > 0.$$

Then, employing the above inequality

$$\sqrt{\Omega(\cdot)} < \eta[1 + b\delta(\rho - \delta)] \iff -b(\rho - \delta)\sqrt{\Omega(\cdot)} > -\eta b(\rho - \delta)[1 + b\delta(\rho - \delta)],$$

the previous expression can be estimated:

$$\begin{aligned} &2\alpha A - 3b\eta(\rho - \delta)[1 + b\delta(\rho - \delta)] - b(\rho - \delta)\sqrt{\Omega(\cdot)} > \\ &> 2\alpha A - 3b\eta(\rho - \delta)[1 + b\delta(\rho - \delta)] - \eta b(\rho - \delta)[1 + b\delta(\rho - \delta)] = \\ &= 2\alpha A - 4b\eta(\rho - \delta) - 2b^2\delta\eta(\rho - \delta)^2 > 0 \end{aligned}$$

if and only if the following condition on α holds:

$$\alpha > \tilde{\alpha} := \frac{b\eta(\rho - \delta)[2 + b\delta(\rho - \delta)]}{A}.$$

Consequently, now it is sufficient to prove that $\tilde{\alpha} < \hat{\alpha}$ in order that $\alpha \in (\hat{\alpha}, \bar{\alpha})$ yields $c_2^{NS} > 0$. By using some algebra, we obtain that

$$\begin{aligned} \tilde{\alpha} < \hat{\alpha} &\iff \frac{b\eta(\rho - \delta)[2 + b\delta(\rho - \delta)]}{A} < \frac{\eta[3 + 7b\delta(\rho - \delta)][1 + b\delta(\rho - \delta)]}{4A\delta} \iff \\ &\iff \dots \iff 3 + 3b^2\delta^2(\rho - \delta)^2 + 2b\delta(\rho - \delta) > 0, \end{aligned}$$

and this completes the proof of the feasibility of c_1^{NS} and c_2^{NS} and finally of the asymmetric steady state Q . \square

As we stated in Section 3, the linear dependence of $\dot{q}_1(t)$ and of $\dot{q}_2(t)$ on the remaining kinematic equations does not provide us with additional information on dynamics. Therefore, we are going to neglect them and construct the Jacobian matrix in the 6-equation case, evaluated at Q :

$$\begin{aligned} \mathcal{J}(Q) &= \begin{pmatrix} \frac{\partial \dot{c}_1}{\partial c_1} & \frac{\partial \dot{c}_1}{\partial c_2} & \frac{\partial \dot{c}_1}{\partial \beta_1} & \frac{\partial \dot{c}_1}{\partial \beta_2} & \frac{\partial \dot{c}_1}{\partial k_1} & \frac{\partial \dot{c}_1}{\partial k_2} \\ \frac{\partial \dot{c}_2}{\partial c_1} & \frac{\partial \dot{c}_2}{\partial c_2} & \frac{\partial \dot{c}_2}{\partial \beta_1} & \frac{\partial \dot{c}_2}{\partial \beta_2} & \frac{\partial \dot{c}_2}{\partial k_1} & \frac{\partial \dot{c}_2}{\partial k_2} \\ \frac{\partial \dot{\beta}_1}{\partial c_1} & \frac{\partial \dot{\beta}_1}{\partial c_2} & \frac{\partial \dot{\beta}_1}{\partial \beta_1} & \frac{\partial \dot{\beta}_1}{\partial \beta_2} & \frac{\partial \dot{\beta}_1}{\partial k_1} & \frac{\partial \dot{\beta}_1}{\partial k_2} \\ \frac{\partial \dot{\beta}_2}{\partial c_1} & \frac{\partial \dot{\beta}_2}{\partial c_2} & \frac{\partial \dot{\beta}_2}{\partial \beta_1} & \frac{\partial \dot{\beta}_2}{\partial \beta_2} & \frac{\partial \dot{\beta}_2}{\partial k_1} & \frac{\partial \dot{\beta}_2}{\partial k_2} \\ \frac{\partial \dot{k}_1}{\partial c_1} & \frac{\partial \dot{k}_1}{\partial c_2} & \frac{\partial \dot{k}_1}{\partial \beta_1} & \frac{\partial \dot{k}_1}{\partial \beta_2} & \frac{\partial \dot{k}_1}{\partial k_1} & \frac{\partial \dot{k}_1}{\partial k_2} \\ \frac{\partial \dot{k}_2}{\partial c_1} & \frac{\partial \dot{k}_2}{\partial c_2} & \frac{\partial \dot{k}_2}{\partial \beta_1} & \frac{\partial \dot{k}_2}{\partial \beta_2} & \frac{\partial \dot{k}_2}{\partial k_1} & \frac{\partial \dot{k}_2}{\partial k_2} \end{pmatrix} = \\ &= \begin{pmatrix} \delta & 0 & -k_2^{NS} & 0 & -1 & -\beta_1^{NS} \\ 0 & \delta & 0 & -k_1^{NS} & -\beta_2^{NS} & -1 \\ 0 & 0 & -\eta & 0 & 0 & \alpha \\ 0 & 0 & 0 & -\eta & \alpha & 0 \\ 0 & 0 & 0 & 0 & \rho - \delta & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho - \delta \end{pmatrix} \end{aligned} \tag{35}$$

Because (35) is an upper triangular matrix, so its entries on the main diagonal are its eigenvalues. It is easy to prove the following:

Proposition 5. *The asymmetric steady state Q is a saddle point equilibrium.*

Proof. By the same procedure carried out in Proposition 2, the eigenvalues of (35) are δ , $-\eta$ and $\rho - \delta$, having algebraic multiplicity 2. Hence, Q is a saddle point whose stable manifold has dimension 2 and whose unstable manifold has dimension 4. \square

The profits of firm i at asymmetric equilibrium are:

$$\Pi_i^{NS} = [A - q_i^{NS} - q_j^{NS} - c_i^{NS}] q_i^{NS} - \frac{b(k_i^{NS})^2}{2} - \frac{\epsilon(\beta_i^{NS})^2}{2}. \quad (36)$$

In order to check that $\Pi_i^{NS} > 0$ for both firms, when the asymmetric equilibrium is feasible, we shall prove the following Proposition:

Proposition 6. *If*

1. *Proposition 4 holds with the further hypothesis $\rho > \frac{3\delta}{2}$;*
2. $b > \frac{1}{2(\rho - \delta)^2}$;
3. $\epsilon < \frac{b(2b(\rho - \delta)^2 - 1)\eta^2}{\alpha^2} \left(\frac{\eta(1 + b\delta(\rho - \delta)) - \sqrt{\Omega(\cdot)}}{\eta(1 + b\delta(\rho - \delta)) + \sqrt{\Omega(\cdot)}} \right)^2$,

then both players' profits are positive at the steady state Q .

Proof. Plugging the coordinates of Q into the expressions (36) and imposing positivity, we obtain two different inequalities which can be expressed by isolating parameter ϵ , i.e.:

$$\Pi_1^{NS} > 0 \iff \epsilon < \frac{b(2b(\rho - \delta)^2 - 1)\eta^2}{\alpha^2} \left(\frac{\eta(1 + b\delta(\rho - \delta)) - \sqrt{\Omega(\cdot)}}{\eta(1 + b\delta(\rho - \delta)) + \sqrt{\Omega(\cdot)}} \right)^2, \quad (37)$$

$$\Pi_2^{NS} > 0 \iff \epsilon < \frac{b(2b(\rho - \delta)^2 - 1)\eta^2}{\alpha^2} \left(\frac{\eta(1 + b\delta(\rho - \delta)) + \sqrt{\Omega(\cdot)}}{\eta(1 + b\delta(\rho - \delta)) - \sqrt{\Omega(\cdot)}} \right)^2, \quad (38)$$

where $\Omega(b, \eta, \delta, \rho, \alpha, A)$ is defined as in Proposition 4.

Since $\epsilon > 0$, a necessary condition for (37) and (38) to hold is given by: $b > \frac{1}{2(\rho - \delta)^2}$, which must be compliant with the assumption on b of Proposition 4. That can occur if we replace the assumption $\rho > \delta$ with $\rho > \frac{3\delta}{2}$.

Since the right hand side of (38) is larger than the right hand side of (37) irrespective of all parameters' values, the most restrictive inequality is (37). Hence, combining all the previous parametric assumptions, the positivity of both profits is verified. \square

3.2.1 A numerical simulation

Also in this case, we may carry out some numerical simulations for illustrative purposes. Choosing the parameter values $\delta = 0.05$, $\rho = 0.5$, $\epsilon = 1.2$, $A = 1$, $b = 2.5$, $\eta = 0.0005$, $\alpha = 0.011$, we can list the equilibrium levels of states, controls and profits in the next table and proceed to a comparison between the players' performances:

	c^{NS}	β^{NS}	k^{NS}	q^{NS}	Π^{NS}
1st firm	0.942068	0.979609	0.00348368	0.00391914	0.575781
2nd firm	0.895894	0.0766409	0.0445277	0.0500936	0.00355528

Figure 2 illustrates the behavior of the two firms' profits as functions of α . A general appraisal of the comparative performance of firms is that firm 1 attains higher profits by virtue of the following mechanism: a lower R&D effort yields a higher production cost, which in turn brings about an output restriction; hence, firm 1 essentially aims at reducing its own investment costs while free-riding over the rival's R&D activity. Overall, the cost-saving effect of shrinking the R&D investment more than offset the negative consequences of operating at a higher marginal cost and selling a lower quantity (which always amounts to bad news under Cournot competition).

$\mathcal{J}(Q)$ has the following eigenvalues: $\delta = 0.05$, $-\eta = -0.0005$, $\rho - \delta = 0.45$, all of them being double roots of the associated characteristic polynomial. By Proposition 5, Q is a saddle point too, hence as in the symmetric case there exist optimal trajectories heading towards Q .

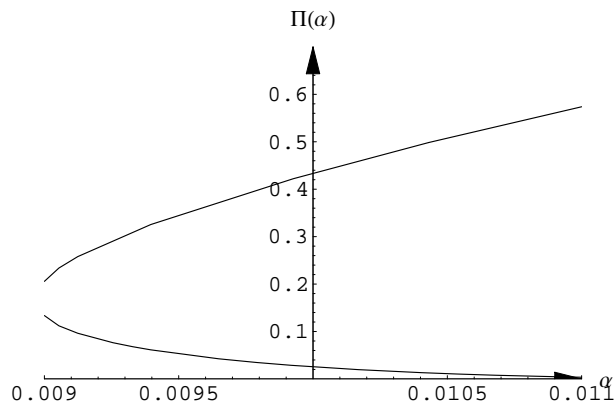


Figure 2: The upper graph represents $\Pi_1^*(\alpha)$ and the lower one represents $\Pi_2^*(\alpha)$ as $\alpha \in (0.009, 0.011)$. In this range of parameters, the difference between profits (as well as the prevalence of Π_1^*) is particularly evident.

4 Concluding remarks

This paper presents a model of dynamic Cournot duopoly where the effects of a spillover due to Open Innovation is taken into account. We described it in a differential game with cost-reducing R&D. The main goal was to investigate the nature of Open Innovation in a small oligopoly and make an analogy to extant economical and managerial issues. To do so, we have nested the endogeneity of knowledge spillovers into the setup dating back to Cellini and Lambertini (2005, 2009), transforming the spillover parameter into a state variable evolving endogenously under the effect of external R&D. This approach seems to be more realistic, in that the OI flow is dynamically driven by R&D evolution, as all the real world examples suggest.

By doing so, we have identified the open-loop equilibrium structure, where we have achieved multiple equilibria, with both symmetric and asymmetric steady states. In both cases, we have found the parametric conditions under which the steady states are feasible and the firms' profits are positive.

In the symmetric steady state solution, a numerical simulation shows that the equilibrium profit attains a unique maximum value corresponding to a growth rate of knowledge spillover under R&D effect.

On the other hand, the asymmetric solution is quite interesting as it is generated by a setup which, a priori, is fully symmetric. Numerical simulations show that the firm with a higher private R&D investment level has a considerably smaller level of OI absorption effort, and *vice versa*. Moreover, profits increase as OI absorption increases, even in presence of a lower level of production.

In view of the growing relevance of OI, research on this issue will plausibly intensify in the near future. Possible developments include investigating (i)

the possibility of selling R&D spillovers in the market, taking into account the issue of property rights; and (ii) determining the feedback equilibrium structure.

Acknowledgements

The authors would like to thank the anonymous referees and the editor for their suggestions for the improvement of the paper, Bruno Viscolani, Georges Zaccour and the audience at the 12th Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics in Wien, 2012, for valuable comments and suggestions. The usual disclaimer applies.

References

- [1] Arrow, K.J., *Economic Welfare and the Allocation of Resources for Invention*, *The Rate and Direction of Industrial Activity*, Edited by R. Nelson, Princeton University Press, Princeton, New Jersey, 1962.
- [2] Audretsch, D. B., Menkveld, A. J., and Thurik, A.R. *The Decision Between Internal and External R&D*, *Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift für die gesamte Staatswissenschaft*, pp. 519-530, 1996.
- [3] Bianchi, M., Cavaliere, A., Chiaroni, D., Frattini, F., and Chiesa, V. *Organisational modes for Open Innovation in the bio-pharmaceutical industry: An exploratory analysis*, *Technovation*, 31, pp. 22-33, 2011.
- [4] Bogers, M., *The Open Innovation Paradox: Knowledge Sharing and Protection in R&D Collaborations*, *European Journal of Innovation Management*, Vol. 14, No. 1, pp. 93-117, 2011.
- [5] Caulkins, J.P., Feichtinger, G., Grass, D., Hartl, R.F., Kort, P.M., and Seidl, A. *When to make proprietary software open source*, *Journal of Economic Dynamics & Control*, 37, pp. 1182-1194, 2013.
- [6] Cellini, R., and Lambertini, L., *A Dynamic Model of Differentiated Oligopoly with Capital Accumulation*, *Journal of Economic Theory* 83, pp. 145-155, 1998.

- [7] Cellini, R., and Lambertini, L., *A Differential Game Approach to Investment in Product Differentiation*, Journal of Economics Dynamics and Control, Vol. 27, pp. 51-62, 2002.
- [8] Cellini, R., and L. Lambertini, *R&D Incentives and Market Structure: A Dynamic Analysis*, Journal of Optimization Theory and Applications, 126, pp. 85-96: 2005.
- [9] Cellini, R., and Lambertini, L., *Dynamic R&D with Spillovers: Competition vs Cooperation*, Journal of Economics Dynamics and Control, 33, pp. 568-582, 2009.
- [10] Chesbrough H. *Open Innovation: The new imperative for creating and profiting from technology*, Harvard Business Press, 2003.
- [11] Chesbrough H., Vanhaverbeke, W., and West, J. *Open Innovation: Researching a New Paradigm*, Oxford University Press, 2006.
- [12] Czarnitzki, D., and Kraft, K. *Spillovers of innovation activities and their profitability*, Oxford Economic Papers, 64, pp. 302-322, 2012.
- [13] Dahlander, L., and Gann, P.M., *How open is innovation?*, Research Policy, 39, pp. 699-709, 2010.
- [14] Drechsler, W., and Natter, M. *Understanding a firm's openness decisions in innovation*, Journal of Business Research, 65, pp. 438-445, 2012.
- [15] Enkel, E., Gassmann, O. and Chesbrough, H.W., *Open R&D and open innovation: Exploring the phenomenon*, R&D Management, Vol. 39, No. 4, pp. 311-16, 2009.
- [16] Fu, X., *How does openness affect the importance of incentives for innovation?*, Research Policy, 41, pp. 512-523, 2012.
- [17] Helfat, C.E., Finkelstein, S., Mitchell, W., Peteraf, M.A., Singh, H., Teece, D.J. and Winter, S.G., *Dynamic Capabilities: Understanding Strategic Change in Organizations*, Blackwell Publishing, Malden, MA, 2007.
- [18] *Linux and IBM*, [http : //www – 03.ibm.com/linux/](http://www-03.ibm.com/linux/).

- [19] Jaffe, A.B. *Technological Opportunity and Spillovers of R&D: Evidence from Firms Patents, Profits and Market Value*, American Economic Review, 76, pp. 984-1001, 1986.
- [20] Lerner, J., and Tirole, J., *Some simple economics of open source*, The Journal of Industrial Economics, Vol. L, 2, pp. 197-234, 2002.
- [21] Li, C., and Ji, X. *Innovation, licensing and price vs. quantity competition*, Economic Modelling, 27, pp. 746-754, 2010.
- [22] Llanes, G., and de Elejalde, R. *Industry equilibrium with open-source and proprietary firms*, International Journal of Industrial Organization, 31, pp. 36-49, 2013.
- [23] Lokshin, B., Belderbos, R. and Carree, M. *The Productivity Effects of Internal and External R&D: Evidence from a Dynamic Panel Data Model*, Oxford Bulletin of Economics and Statistics, 70, pp. 399-413, 2008.
- [24] Modica, S. *Open Source without Free-Riding*, Economia Politica, XXX, 2, pp. 247-260, 2012.
- [25] Mortara L., Napp J.J., Slacik I., and Minshall T., *How to Implement Open Innovation: Lessons from studying large multinational companies*, University of Cambridge Institute for Manufacturing, 2009.
- [26] Mortara, L., *Getting help with open innovation*, Institute for Manufacturing, University of Cambridge, 2010.
- [27] Romer, P.M. *Endogenous Technological Change*, Journal of Political Economy, 98, S71-S102, 1990.
- [28] Scotchmer, S. *Openness, Open Source, and the Veil of Ignorance*, American Economic Review, 100 (P&P), pp. 165-171, 2010.
- [29] Spithoven, A., Clarysse, B., and Knockaert, M. *Building absorptive capacity to organise inbound open innovation in traditional industries*, Technovation, 31, pp. 10-21, 2011.
- [30] Spithoven, A., Teirlinck, P., and Frantzen, D. J. *Managing open innovation: Connecting the firm to external knowledge*, Edward Elgar Publishing, 2012.

- [31] Vanhaverbeke, W., Van de Vrande, V. and Cloudt, M. *Connecting Absorptive Capacity and Open Innovation*, available at SSRN: <http://ssrn.com/abstract=1091265>, 2008.
- [32] West, J., and Gallagher, S. *Challenges of open innovation: the paradox of firm investment in open-source software*, R&D Management, 36, pp. 319-331, 2006.
- [33] Zanchettin, P., *Differentiated duopoly with asymmetric costs*, Journal of Economics and Management Strategy, Vol. 15, No. 4, pp. 999-1015, 2006.

PREVIOUS DISCUSSION PAPERS

- 111 Hasnas, Irina, Lambertini, Luca and Palestini, Arsen, Open Innovation in a Dynamic Cournot Duopoly, October 2013.
- 110 Baumann, Florian and Friehe, Tim, Competitive Pressure and Corporate Crime, September 2013.
- 109 Böckers, Veit, Haucap, Justus and Heimeshoff, Ulrich, Benefits of an Integrated European Electricity Market, September 2013.
- 108 Normann, Hans-Theo and Tan, Elaine S., Effects of Different Cartel Policies: Evidence from the German Power-Cable Industry, September 2013.
Forthcoming in: Industrial and Corporate Change.
- 107 Haucap, Justus, Heimeshoff, Ulrich, Klein, Gordon J., Rickert, Dennis and Wey, Christian, Bargaining Power in Manufacturer-Retailer Relationships, September 2013.
- 106 Baumann, Florian and Friehe, Tim, Design Standards and Technology Adoption: Welfare Effects of Increasing Environmental Fines when the Number of Firms is Endogenous, September 2013.
- 105 Jeitschko, Thomas D., NYSE Changing Hands: Antitrust and Attempted Acquisitions of an Erstwhile Monopoly, August 2013.
- 104 Böckers, Veit, Giessing, Leonie and Rösch, Jürgen, The Green Game Changer: An Empirical Assessment of the Effects of Wind and Solar Power on the Merit Order, August 2013.
- 103 Haucap, Justus and Muck, Johannes, What Drives the Relevance and Reputation of Economics Journals? An Update from a Survey among Economists, August 2013.
- 102 Jovanovic, Dragan and Wey, Christian, Passive Partial Ownership, Sneaky Takeovers, and Merger Control, August 2013.
- 101 Haucap, Justus, Heimeshoff, Ulrich, Klein, Gordon J., Rickert, Dennis and Wey, Christian, Inter-Format Competition Among Retailers – The Role of Private Label Products in Market Delineation, August 2013.
- 100 Normann, Hans-Theo, Requate, Till and Waichman, Israel, Do Short-Term Laboratory Experiments Provide Valid Descriptions of Long-Term Economic Interactions? A Study of Cournot Markets, July 2013.
Forthcoming in: Experimental Economics.
- 99 Dertwinkel-Kalt, Markus, Haucap, Justus and Wey, Christian, Input Price Discrimination (Bans), Entry and Welfare, June 2013.
- 98 Aguzzoni, Luca, Argentesi, Elena, Ciari, Lorenzo, Duso, Tomaso and Tognoni, Massimo, Ex-post Merger Evaluation in the UK Retail Market for Books, June 2013.
- 97 Caprice, Stéphane and von Schlippenbach, Vanessa, One-Stop Shopping as a Cause of Slotting Fees: A Rent-Shifting Mechanism, May 2012.
Published in: Journal of Economics and Management Strategy, 22 (2013), pp. 468-487.
- 96 Wenzel, Tobias, Independent Service Operators in ATM Markets, June 2013.
Forthcoming in: Scottish Journal of Political Economy.

- 95 Coublucq, Daniel, *Econometric Analysis of Productivity with Measurement Error: Empirical Application to the US Railroad Industry*, June 2013.
- 94 Coublucq, Daniel, *Demand Estimation with Selection Bias: A Dynamic Game Approach with an Application to the US Railroad Industry*, June 2013.
- 93 Baumann, Florian and Friehe, Tim, *Status Concerns as a Motive for Crime?*, April 2013.
- 92 Jeitschko, Thomas D. and Zhang, Nanyun, *Adverse Effects of Patent Pooling on Product Development and Commercialization*, April 2013.
- 91 Baumann, Florian and Friehe, Tim, *Private Protection Against Crime when Property Value is Private Information*, April 2013.
Published in: *International Review of Law and Economics*, 35 (2013), pp. 73-79.
- 90 Baumann, Florian and Friehe, Tim, *Cheap Talk About the Detection Probability*, April 2013.
Forthcoming in: *International Game Theory Review*.
- 89 Pagel, Beatrice and Wey, Christian, *How to Counter Union Power? Equilibrium Mergers in International Oligopoly*, April 2013.
- 88 Jovanovic, Dragan, *Mergers, Managerial Incentives, and Efficiencies*, April 2013.
- 87 Heimeshoff, Ulrich and Klein Gordon J., *Bargaining Power and Local Heroes*, March 2013.
- 86 Bertschek, Irene, Cerquera, Daniel and Klein, Gordon J., *More Bits – More Bucks? Measuring the Impact of Broadband Internet on Firm Performance*, February 2013.
Forthcoming in: *Information Economics and Policy*.
- 85 Rasch, Alexander and Wenzel, Tobias, *Piracy in a Two-Sided Software Market*, February 2013.
Published in: *Journal of Economic Behavior & Organization*, 88 (2013), pp. 78-89.
- 84 Bataille, Marc and Steinmetz, Alexander, *Intermodal Competition on Some Routes in Transportation Networks: The Case of Inter Urban Buses and Railways*, January 2013.
- 83 Haucap, Justus and Heimeshoff, Ulrich, *Google, Facebook, Amazon, eBay: Is the Internet Driving Competition or Market Monopolization?*, January 2013.
Forthcoming in: *International Economics and Economic Policy*.
- 82 Regner, Tobias and Riener, Gerhard, *Voluntary Payments, Privacy and Social Pressure on the Internet: A Natural Field Experiment*, December 2012.
- 81 Dertwinkel-Kalt, Markus and Wey, Christian, *The Effects of Remedies on Merger Activity in Oligopoly*, December 2012.
- 80 Baumann, Florian and Friehe, Tim, *Optimal Damages Multipliers in Oligopolistic Markets*, December 2012.
- 79 Duso, Tomaso, Röller, Lars-Hendrik and Seldeslachts, Jo, *Collusion through Joint R&D: An Empirical Assessment*, December 2012.
Forthcoming in: *The Review of Economics and Statistics*.
- 78 Baumann, Florian and Heine, Klaus, *Innovation, Tort Law, and Competition*, December 2012.
Forthcoming in: *Journal of Institutional and Theoretical Economics*.

- 77 Coenen, Michael and Jovanovic, Dragan, Investment Behavior in a Constrained Dictator Game, November 2012.
- 76 Gu, Yiquan and Wenzel, Tobias, Strategic Obfuscation and Consumer Protection Policy in Financial Markets: Theory and Experimental Evidence, November 2012. Forthcoming in: Journal of Industrial Economics under the title "Strategic Obfuscation and Consumer Protection Policy".
- 75 Haucap, Justus, Heimeshoff, Ulrich and Jovanovic, Dragan, Competition in Germany's Minute Reserve Power Market: An Econometric Analysis, November 2012. Forthcoming in: The Energy Journal.
- 74 Normann, Hans-Theo, Rösch, Jürgen and Schultz, Luis Manuel, Do Buyer Groups Facilitate Collusion?, November 2012.
- 73 Riener, Gerhard and Wiederhold, Simon, Heterogeneous Treatment Effects in Groups, November 2012.
- 72 Berlemann, Michael and Haucap, Justus, Which Factors Drive the Decision to Boycott and Opt Out of Research Rankings? A Note, November 2012.
- 71 Muck, Johannes and Heimeshoff, Ulrich, First Mover Advantages in Mobile Telecommunications: Evidence from OECD Countries, October 2012.
- 70 Karaçuka, Mehmet, Çatik, A. Nazif and Haucap, Justus, Consumer Choice and Local Network Effects in Mobile Telecommunications in Turkey, October 2012. Published in: Telecommunications Policy, 37 (2013), pp. 334-344.
- 69 Clemens, Georg and Rau, Holger A., Rebels without a Clue? Experimental Evidence on Partial Cartels, April 2013 (First Version October 2012).
- 68 Regner, Tobias and Riener, Gerhard, Motivational Cherry Picking, September 2012.
- 67 Fonseca, Miguel A. and Normann, Hans-Theo, Excess Capacity and Pricing in Bertrand-Edgeworth Markets: Experimental Evidence, September 2012. Published in: Journal of Institutional and Theoretical Economics, 169 (2013), pp. 199-228.
- 66 Riener, Gerhard and Wiederhold, Simon, Team Building and Hidden Costs of Control, September 2012.
- 65 Fonseca, Miguel A. and Normann, Hans-Theo, Explicit vs. Tacit Collusion – The Impact of Communication in Oligopoly Experiments, August 2012. Published in: European Economic Review, 56 (2012), pp. 1759-1772.
- 64 Jovanovic, Dragan and Wey, Christian, An Equilibrium Analysis of Efficiency Gains from Mergers, July 2012.
- 63 Dewenter, Ralf, Jaschinski, Thomas and Kuchinke, Björn A., Hospital Market Concentration and Discrimination of Patients, July 2012.
- 62 Von Schlippenbach, Vanessa and Teichmann, Isabel, The Strategic Use of Private Quality Standards in Food Supply Chains, May 2012. Published in: American Journal of Agricultural Economics, 94 (2012), pp. 1189-1201.
- 61 Sapi, Geza, Bargaining, Vertical Mergers and Entry, July 2012.
- 60 Jentzsch, Nicola, Sapi, Geza and Suleymanova, Irina, Targeted Pricing and Customer Data Sharing Among Rivals, July 2012. Published in: International Journal of Industrial Organization, 31 (2013), pp. 131-144.

- 59 Lambarraa, Fatima and Riener, Gerhard, On the Norms of Charitable Giving in Islam: A Field Experiment, June 2012.
- 58 Duso, Tomaso, Gugler, Klaus and Szücs, Florian, An Empirical Assessment of the 2004 EU Merger Policy Reform, June 2012.
Forthcoming in: Economic Journal.
- 57 Dewenter, Ralf and Heimeshoff, Ulrich, More Ads, More Revs? Is there a Media Bias in the Likelihood to be Reviewed?, June 2012.
- 56 Böckers, Veit, Heimeshoff, Ulrich and Müller Andrea, Pull-Forward Effects in the German Car Scrappage Scheme: A Time Series Approach, June 2012.
- 55 Kellner, Christian and Riener, Gerhard, The Effect of Ambiguity Aversion on Reward Scheme Choice, June 2012.
- 54 De Silva, Dakshina G., Kosmopoulou, Georgia, Pagel, Beatrice and Peeters, Ronald, The Impact of Timing on Bidding Behavior in Procurement Auctions of Contracts with Private Costs, June 2012.
Published in: Review of Industrial Organization, 41 (2013), pp.321-343.
- 53 Benndorf, Volker and Rau, Holger A., Competition in the Workplace: An Experimental Investigation, May 2012.
- 52 Haucap, Justus and Klein, Gordon J., How Regulation Affects Network and Service Quality in Related Markets, May 2012.
Published in: Economics Letters, 117 (2012), pp. 521-524.
- 51 Dewenter, Ralf and Heimeshoff, Ulrich, Less Pain at the Pump? The Effects of Regulatory Interventions in Retail Gasoline Markets, May 2012.
- 50 Böckers, Veit and Heimeshoff, Ulrich, The Extent of European Power Markets, April 2012.
- 49 Barth, Anne-Kathrin and Heimeshoff, Ulrich, How Large is the Magnitude of Fixed-Mobile Call Substitution? - Empirical Evidence from 16 European Countries, April 2012.
- 48 Herr, Annika and Suppliet, Moritz, Pharmaceutical Prices under Regulation: Tiered Co-payments and Reference Pricing in Germany, April 2012.
- 47 Haucap, Justus and Müller, Hans Christian, The Effects of Gasoline Price Regulations: Experimental Evidence, April 2012.
- 46 Stühmeier, Torben, Roaming and Investments in the Mobile Internet Market, March 2012.
Published in: Telecommunications Policy, 36 (2012), pp. 595-607.
- 45 Graf, Julia, The Effects of Rebate Contracts on the Health Care System, March 2012,
Forthcoming in: The European Journal of Health Economics.
- 44 Pagel, Beatrice and Wey, Christian, Unionization Structures in International Oligopoly, February 2012.
Published in: Labour: Review of Labour Economics and Industrial Relations, 27 (2013), pp. 1-17.
- 43 Gu, Yiquan and Wenzel, Tobias, Price-Dependent Demand in Spatial Models, January 2012.
Published in: B. E. Journal of Economic Analysis & Policy, 12 (2012), Article 6.

- 42 Barth, Anne-Kathrin and Heimeshoff, Ulrich, Does the Growth of Mobile Markets Cause the Demise of Fixed Networks? – Evidence from the European Union, January 2012.
- 41 Stühmeier, Torben and Wenzel, Tobias, Regulating Advertising in the Presence of Public Service Broadcasting, January 2012.
Published in: Review of Network Economics, 11/2 (2012), Article 1.
- 40 Müller, Hans Christian, Forecast Errors in Undisclosed Management Sales Forecasts: The Disappearance of the Overoptimism Bias, December 2011.
- 39 Gu, Yiquan and Wenzel, Tobias, Transparency, Entry, and Productivity, November 2011.
Published in: Economics Letters, 115 (2012), pp. 7-10.
- 38 Christin, Clémence, Entry Deterrence Through Cooperative R&D Over-Investment, November 2011.
Forthcoming in: Louvain Economic Review.
- 37 Haucap, Justus, Herr, Annika and Frank, Björn, In Vino Veritas: Theory and Evidence on Social Drinking, November 2011.
In a modified version forthcoming in: European Journal of Law and Economics.
- 36 Barth, Anne-Kathrin and Graf, Julia, Irrationality Rings! – Experimental Evidence on Mobile Tariff Choices, November 2011.
- 35 Jeitschko, Thomas D. and Normann, Hans-Theo, Signaling in Deterministic and Stochastic Settings, November 2011.
Published in: Journal of Economic Behavior and Organization, 82 (2012), pp.39-55.
- 34 Christin, Cérence, Nicolai, Jean-Philippe and Pouyet, Jerome, The Role of Abatement Technologies for Allocating Free Allowances, October 2011.
- 33 Keser, Claudia, Suleymanova, Irina and Wey, Christian, Technology Adoption in Markets with Network Effects: Theory and Experimental Evidence, October 2011.
Published in: Information Economics and Policy, 24 (2012), pp. 262-276.
- 32 Çatik, A. Nazif and Karaçuka, Mehmet, The Bank Lending Channel in Turkey: Has it Changed after the Low Inflation Regime?, September 2011.
Published in: Applied Economics Letters, 19 (2012), pp. 1237-1242.
- 31 Hauck, Achim, Neyer, Ulrike and Vieten, Thomas, Reestablishing Stability and Avoiding a Credit Crunch: Comparing Different Bad Bank Schemes, August 2011.
- 30 Suleymanova, Irina and Wey, Christian, Bertrand Competition in Markets with Network Effects and Switching Costs, August 2011.
Published in: B. E. Journal of Economic Analysis & Policy, 11 (2011), Article 56.
- 29 Stühmeier, Torben, Access Regulation with Asymmetric Termination Costs, July 2011.
Published in: Journal of Regulatory Economics, 43 (2013), pp. 60-89.
- 28 Dewenter, Ralf, Haucap, Justus and Wenzel, Tobias, On File Sharing with Indirect Network Effects Between Concert Ticket Sales and Music Recordings, July 2011.
Published in: Journal of Media Economics, 25 (2012), pp. 168-178.
- 27 Von Schlippenbach, Vanessa and Wey, Christian, One-Stop Shopping Behavior, Buyer Power, and Upstream Merger Incentives, June 2011.

- 26 Balsmeier, Benjamin, Buchwald, Achim and Peters, Heiko, Outside Board Memberships of CEOs: Expertise or Entrenchment?, June 2011.
- 25 Clougherty, Joseph A. and Duso, Tomaso, Using Rival Effects to Identify Synergies and Improve Merger Typologies, June 2011.
Published in: Strategic Organization, 9 (2011), pp. 310-335.
- 24 Heinz, Matthias, Juranek, Steffen and Rau, Holger A., Do Women Behave More Reciprocally than Men? Gender Differences in Real Effort Dictator Games, June 2011.
Published in: Journal of Economic Behavior and Organization, 83 (2012), pp. 105-110.
- 23 Sapi, Geza and Suleymanova, Irina, Technology Licensing by Advertising Supported Media Platforms: An Application to Internet Search Engines, June 2011.
Published in: B. E. Journal of Economic Analysis & Policy, 11 (2011), Article 37.
- 22 Buccirosi, Paolo, Ciari, Lorenzo, Duso, Tomaso, Spagnolo Giancarlo and Vitale, Cristiana, Competition Policy and Productivity Growth: An Empirical Assessment, May 2011.
Published in: The Review of Economics and Statistics, 95 (2013), pp. 1324-1336.
- 21 Karaçuka, Mehmet and Çatik, A. Nazif, A Spatial Approach to Measure Productivity Spillovers of Foreign Affiliated Firms in Turkish Manufacturing Industries, May 2011.
Published in: The Journal of Developing Areas, 46 (2012), pp. 65-83.
- 20 Çatik, A. Nazif and Karaçuka, Mehmet, A Comparative Analysis of Alternative Univariate Time Series Models in Forecasting Turkish Inflation, May 2011.
Published in: Journal of Business Economics and Management, 13 (2012), pp. 275-293.
- 19 Normann, Hans-Theo and Wallace, Brian, The Impact of the Termination Rule on Cooperation in a Prisoner's Dilemma Experiment, May 2011.
Published in: International Journal of Game Theory, 41 (2012), pp. 707-718.
- 18 Baake, Pio and von Schlippenbach, Vanessa, Distortions in Vertical Relations, April 2011.
Published in: Journal of Economics, 103 (2011), pp. 149-169.
- 17 Haucap, Justus and Schwalbe, Ulrich, Economic Principles of State Aid Control, April 2011.
Forthcoming in: F. Montag & F. J. Säcker (eds.), European State Aid Law: Article by Article Commentary, Beck: München 2012.
- 16 Haucap, Justus and Heimeshoff, Ulrich, Consumer Behavior towards On-net/Off-net Price Differentiation, January 2011.
Published in: Telecommunication Policy, 35 (2011), pp. 325-332.
- 15 Duso, Tomaso, Gugler, Klaus and Yurtoglu, Burcin B., How Effective is European Merger Control? January 2011.
Published in: European Economic Review, 55 (2011), pp. 980-1006.
- 14 Haigner, Stefan D., Jenewein, Stefan, Müller, Hans Christian and Wakolbinger, Florian, The First shall be Last: Serial Position Effects in the Case Contestants evaluate Each Other, December 2010.
Published in: Economics Bulletin, 30 (2010), pp. 3170-3176.
- 13 Suleymanova, Irina and Wey, Christian, On the Role of Consumer Expectations in Markets with Network Effects, November 2010.
Published in: Journal of Economics, 105 (2012), pp. 101-127.

- 12 Haucap, Justus, Heimeshoff, Ulrich and Karaçuka, Mehmet, Competition in the Turkish Mobile Telecommunications Market: Price Elasticities and Network Substitution, November 2010.
Published in: Telecommunications Policy, 35 (2011), pp. 202-210.
- 11 Dewenter, Ralf, Haucap, Justus and Wenzel, Tobias, Semi-Collusion in Media Markets, November 2010.
Published in: International Review of Law and Economics, 31 (2011), pp. 92-98.
- 10 Dewenter, Ralf and Kruse, Jörn, Calling Party Pays or Receiving Party Pays? The Diffusion of Mobile Telephony with Endogenous Regulation, October 2010.
Published in: Information Economics and Policy, 23 (2011), pp. 107-117.
- 09 Hauck, Achim and Neyer, Ulrike, The Euro Area Interbank Market and the Liquidity Management of the Eurosystem in the Financial Crisis, September 2010.
- 08 Haucap, Justus, Heimeshoff, Ulrich and Schultz, Luis Manuel, Legal and Illegal Cartels in Germany between 1958 and 2004, September 2010.
Published in: H. J. Ramser & M. Stadler (eds.), Marktmacht. Wirtschaftswissenschaftliches Seminar Ottobeuren, Volume 39, Mohr Siebeck: Tübingen 2010, pp. 71-94.
- 07 Herr, Annika, Quality and Welfare in a Mixed Duopoly with Regulated Prices: The Case of a Public and a Private Hospital, September 2010.
Published in: German Economic Review, 12 (2011), pp. 422-437.
- 06 Blanco, Mariana, Engelmann, Dirk and Normann, Hans-Theo, A Within-Subject Analysis of Other-Regarding Preferences, September 2010.
Published in: Games and Economic Behavior, 72 (2011), pp. 321-338.
- 05 Normann, Hans-Theo, Vertical Mergers, Foreclosure and Raising Rivals' Costs – Experimental Evidence, September 2010.
Published in: The Journal of Industrial Economics, 59 (2011), pp. 506-527.
- 04 Gu, Yiquan and Wenzel, Tobias, Transparency, Price-Dependent Demand and Product Variety, September 2010.
Published in: Economics Letters, 110 (2011), pp. 216-219.
- 03 Wenzel, Tobias, Deregulation of Shopping Hours: The Impact on Independent Retailers and Chain Stores, September 2010.
Published in: Scandinavian Journal of Economics, 113 (2011), pp. 145-166.
- 02 Stühmeier, Torben and Wenzel, Tobias, Getting Beer During Commercials: Adverse Effects of Ad-Avoidance, September 2010.
Published in: Information Economics and Policy, 23 (2011), pp. 98-106.
- 01 Inderst, Roman and Wey, Christian, Countervailing Power and Dynamic Efficiency, September 2010.
Published in: Journal of the European Economic Association, 9 (2011), pp. 702-720.

Heinrich-Heine-University of Düsseldorf

**Düsseldorf Institute for
Competition Economics (DICE)**

Universitätsstraße 1_ 40225 Düsseldorf
www.dice.hhu.de

ISSN 2190-9938 (online)
ISBN 978-3-86304-110-6