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## Rare disasters and the equity premium in a two-country world

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# Cardiff Economics Working Papers



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## Rare Disasters and the Equity Premium in a Two-Country World

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March 2007

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# Rare Disasters and the Equity Premium in a Two-Country World

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## **Abstract**

Rare Disasters and the Equity Premium in a Two-Country World

Laurence Copeland and Yanhui Zhu

We extend the Barro (2006) closed-economy model of the equity risk premium in the presence of extreme events ("disasters") to a two-country world. In this more general setting, both the output risk of rare disasters and the associated risk of a default on Government debt, can be diversified. The extent to which agents in one country can diversify away the risk of extreme events depends on the relative size of the two countries, and critically on the probability of a disaster in one country conditional on a disaster in the other. We show that, using Barro's own calibration in combination with a broad range of plausible values for the additional parameters, the model implies levels of the equity risk premium far lower than those typically observed in the data. We conclude that the model is unlikely to explain the equity risk premium.

JEL Classification: F3, G1

Keywords: equity risk premium, default risk, international diversification

# 1 Introduction

In the years since Mehra and Prescott (1985) drew the attention of the economics profession to the fact that the return on equities has historically been far higher relative to the riskless rate than can be justified by reasonable levels of risk aversion, numerous attempts have been made to resolve the apparent puzzle.<sup>1</sup> Most recently, in an important contribution Barro (2006) demonstrated that, by making explicit allowance for rare disasters, it is possible to explain the scale of equity premium found in the USA over the same period. His analysis, however, is based on an extension of the Lucas (1978) closed-economy tree model, with disaster probabilities calibrated to match the frequency observed during the twentieth century. We show here that generalising the model to a two-country framework substantially reduces the predicted equity risk premium and therefore weakens the analysis as an explanation of the puzzle. The reason for this result is that, insofar as extreme events are less than perfectly correlated internationally, introducing a second country with its own tree-economy makes disasters potentially diversifiable, and thereby mitigates their effect on the required return on equity.

This strand of the literature starts with the work of Rietz (1988), who offers an analysis based on the possibility of rare disasters as a contribution to resolving the equity premium puzzle. He models the endowment as a three-state Markov process, where in the first and

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<sup>1</sup> See, for example, Brown, Goetzmann and Ross (1995), Jorion and Goetzmann (1999) who attribute the anomaly to survivorship bias among world stockmarkets, Campbell and Cochrane (1999) who point to habit formation in consumption patterns, Dumas (1989) on heterogeneous preferences, and more recently Lungu and Minford (2007), who present simulations based on an overlapping generations model of equity markets.

second states, endowment growth is slightly higher or lower than its steady-state rate, as in Mehra and Prescott (1985), while in the third state, output falls drastically. Calibration results show that the introduction of a potential disaster justifies a relatively low risk-free interest rate and a high equity premium. Intuitively, a rare disaster implies an increase in aggregate risk, which leads to stronger demand for risk-free assets, and hence a lower riskless rate. At the same time, high aggregate risk also requires an increase in the reward for holding risky assets. For both reasons, therefore, rare disasters generate a larger equity premium.

Into this framework, Barro (2006) introduces two modifications. First, he allows for partial default on government debt in the depression state. Second, he examines the empirical distribution of the size of 20th century output contractions across a wide range of countries. The endowment growth rate is modelled as a random walk with drift and two i.i.d. shocks. One shock captures the slight fluctuations in the endowment in normal, non-disaster states. Its function parallels the binominal Markov process in Mehra and Prescott (1985). The other shock models the drastic output contraction in disaster states, comparable to the third state in Rietz (1988). With probability close to zero, it takes a large negative value corresponding to the size of the output contraction in disaster. Otherwise, it is zero. The model also allows for the possibility of a partial default by the Government on its debts in disaster states. Results of computations based on calibrated parameter values show that the model can produce theoretical values in line with the observed returns data for both equities and Government debt.

We generalise this model to a two-country scenario. The countries are similar in the sense that both are subject to stochastic shocks drawn from identical distributions, so that there

are three relevant extreme events: a disaster in the home country, a disaster in the foreign country and simultaneous disasters in both, each state being associated with an appropriate stochastic discount factor (SDF). The results derive from the interactions between these three SDF's in the implied risk-adjusted returns.

To the extent that neither disasters nor consequent defaults are perfectly correlated across the two countries, the introduction of international diversification opportunities reduces aggregate risk. As a result, the equity risk premium is unambiguously lower than in the single country case, and furthermore our calibrations suggest that, for reasonable values of the key parameters, it falls well short of the levels found in Barro (2006) (and in the data), leading us to the conclusion that the puzzle remains unresolved, at least by this branch of the literature.

We set out our basic framework and assumptions in the next section. The model solution is given in the succeeding section, with the equations for the returns on equities and Government bills. Section 4 presents the results of calibrations, and the final section our conclusions.

## 2 The Two-Country Model

We start by postulating a setup similar to Barro (2006) in each of two countries. Each country is populated by a representative agent with an initial endowed income in the form of fruit of the local Lucas-tree. Equity claims on the time  $t+1$  stochastic endowment (dividends) are traded internationally at time  $t$ , and each agent holds a mix of claims on the endowed domestic output and on the foreign agent's endowment. In addition, the Government of

each country issues a bill in the form of a claim paying a fixed return. However, in the event of an economic disaster, the Government guarantee may fail, and there is therefore a nonzero probability of default on bills. As with equities, each country's bills are traded internationally.

A representative agent in the home country maximises a power utility function of the standard form:

$$E_t(U) = U(C_t) + \sum_{s=1}^{\infty} e^{-\rho s} E_t[U(C_{t+s})] \quad (1)$$

where:

$$U(C_{t+s}) = \frac{C_{t+s}^{1-\theta}}{1-\theta} \quad (2)$$

and similarly for the agent in the foreign country. Since we assume the output of the two trees is traded freely between the agents, it follows that the common worldwide stochastic discount factor will be:

$$M_{t+1} = e^{-\rho} \frac{U'(C_{t+1})}{U'(C_t)} = e^{-\rho} \frac{U'(C_{t+1}^*)}{U'(C_t^*)} = e^{-\rho} \left( \frac{A_{t+1}^W}{A_t^W} \right)^{-\theta} \quad (3)$$

where starred variables refer to the foreign country, and in place of Barro's single-country output/endowment,  $A$ , we have world output,  $A^W$ , so the constraint on world consumption is given by the condition:

$$C_t + C_t^* = A_t + A_t^* = A_t^W \quad (4)$$

For each country, the (log of the) endowment process is a random walk with drift  $\gamma, \gamma^*$  respectively, and subject to two types of disturbance at any time,  $t + 1$ :

$$g_{t+1} = \ln A_{t+1} - \ln A_t = \gamma + u_{t+1} + v_{t+1} \quad (5)$$

$$g_{t+1}^* = \ln A_{t+1}^* - \ln A_t^* = \gamma^* + u_{t+1}^* + v_{t+1}^* \quad (6)$$



In these processes,  $u_t, u_t^*$  are bivariate normal innovations with zero means, standard deviations  $\sigma, \sigma^*$  respectively, and correlation coefficient,  $\kappa$ . In addition, output is prone to occasional large negative shocks ("disasters"),  $v_t, v_t^*$  associated with recessions of size,  $b, b^*$  respectively, which are two independent, identically distributed random variables.

As in Barro (2006), we assume the probability of either country being hit by a disaster is  $1 - e^{-\rho}$  in any (short) period.<sup>2</sup> Critically, however, we allow for the fact that disasters are not independent events across countries, by assuming that the probability of a disaster in the home (foreign) country coinciding with one in the foreign (home) country is nonzero<sup>3</sup>,  $0 < \eta < 1$ . It follows that the extreme-event shocks,  $v_t, v_t^*$  both take the value zero (i.e no disaster occurs) with probability:

$$(e^{-\rho} - 1)(1 - \eta) + e^{-\rho} \quad (7)$$

Both countries go into a slump of size  $b, b^*$  so that  $v_t = \ln(1 - b), v_t^* = \ln(1 - b^*)$  with probability:

$$(1 - e^{-\rho})\eta \quad (8)$$

and a disaster occurs in just one country while the other is untouched ( $v_t = \ln(1 - b), v_t^* =$

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<sup>2</sup> Note that we assume disasters are one-off events, so that there is no prospect of two disasters in any holding period. Also, although we allow for the scale of a disaster to vary across countries, we assume the same probability of a disaster in either country. Allowing the probabilities to differ would complicate the algebra without providing additional insight.

<sup>3</sup> It might be interesting to examine the consequences of allowing for asymmetry here i.e. a situation where the probability of a disaster in the home country conditional on a disaster in the foreign country is not the same as the reverse.

0 or vice versa) with probability:

$$(1 - e^{-\rho})(1 - \eta) \quad (9)$$

Given these assumptions, the stochastic discount factor (3) becomes:

$$M_{t+1} \approx \exp [-\rho - \theta w g_{t+1} - \theta(1 - w)g_{t+1}^*] \quad (10)$$

where  $w = A_t/A_t^W$  is the home country's share of world output. This pricing kernel implies a notional riskless rate which can be written as follows:

$$\ln R_{t+1}^f \approx \Phi_1 - \Phi_2 \quad (11)$$

The first term in this decomposition represents the standard components of the riskless rate, attributable to time preference, output growth and normal shocks,  $u_t$ :

$$\Phi_1 = \rho + \theta w \gamma + \theta(1 - w)\gamma^* - \frac{1}{2}\theta^2 [w^2\sigma^2 + (1 - w)^2\sigma^{*2}] - \theta^2 w(1 - w)\sigma\sigma^*\kappa \quad (12)$$

which is the riskless rate when the probability of disaster is zero. Note that  $\Phi_1$  is decreasing in  $\kappa$ , the correlation between normal shocks, because when volatility is more synchronised between the two countries, everyday risks are harder to diversify, hence the demand for a riskless asset (if one exists) would be greater at any given interest rate.

The second term in (11) represents the component attributable to the prospect (albeit remote) of a disaster:

$$\begin{aligned} \Phi_2 &= p(1 - \eta) \{ [E(1 - b)^{-\theta w} - 1] + [E(1 - b)^{-\theta(1-w)} - 1] \} \\ &\quad + p\eta [E(1 - b)^{-\theta w} E(1 - b)^{-\theta(1-w)} - 1] \end{aligned} \quad (13)$$

$$= p(1 - \eta) [(E\beta - 1) + (E\beta^* - 1)] + p\eta [E\beta E\beta^* - 1] > 0 \quad (14)$$

where:

$$\beta = (1 - b)^{-\theta w} \quad \text{and} \quad \beta^* = (1 - b^*)^{-\theta(1-w)} \quad (15)$$

are the values taken by the stochastic discount factor in those states of the world when the domestic and foreign countries respectively suffer disasters.  $\Phi_2$  is the risk adjustment for disasters i.e the sum of the SDF's (less one) in each of the three possible disaster states: in the home country alone and in the foreign country alone, weighted by the probability of these two outcomes,  $p(1 - \eta)$ , plus the joint-disaster state, which has probability  $p\eta$ . Hence,  $\Phi_2$  must be positive, reflecting the fact that disaster risk increases the attraction of riskless investment. Moreover, the higher is  $p$ , the greater is the probability of disaster in either or both countries, so it is increasing in  $p$ , and likewise in  $\eta$ , since greater disaster correlation also makes the world riskier and therefore reduces the riskless rate.

In the single-country case i.e. when  $w = 1$  or  $w = 0$ , (13) reduces either to:

$$\Phi_2 = p[E\beta - 1] \quad \text{or} \quad \Phi_2 = p[E\beta^* - 1] \quad (16)$$

in which, of course,  $\eta$  plays no part. On the other hand, when  $w = \frac{1}{2}$ ,  $\Phi_2$  is at its minimum level:

$$\Phi_2 = p \left\{ 2(1 - \eta) \left[ E(1 - b)^{-\frac{\theta}{2}} - 1 \right] + \eta \left[ E(1 - b)^{-\frac{\theta}{2}} - 1 \right]^2 \right\} > 0 \quad (17)$$

so that in this case the riskless rate is maximised, because with equally-sized countries, diversification opportunities are greater than with unequal sizes, minimising aggregate global risk and thereby the demand for riskless assets other things being equal. Market clearing therefore requires the highest riskless rate.

This allows us to state our first conclusion: under the assumptions made here, the effect on the riskless rate of extreme events is always smaller in a two-country world than in a

single-country setting. In general, allowing for disasters increases the demand for a riskless asset and thereby reduces the riskless rate. However, it is important to note that, because both ordinary risk and disaster risk are potentially diversifiable in the two-country world, aggregate risk is less than in a single country model, so the riskless rate is higher in the former than in the latter. Specifically, diversification opportunities are greater on all fronts. Insofar as normal volatility is less than perfectly correlated across the two countries, the risk associated with  $u_t$  can be reduced. As far as disasters are concerned, even in the limiting case, when  $\eta = 1$  so that with absolute certainty disasters coincide in the two countries, there are still gains to be made from diversification because the scale of the realized output contractions in the disaster state,  $b$  and  $b^*$ , are not necessarily the same in the two countries.

### 3 Security Returns

As in Barro (2006), there is no completely riskless asset in the model, but each government issues a bill at  $t$  promising the payment of a unit of consumption in period  $t + 1$ . In non-disaster states, the actual return on the bill equals the face return. In the disaster state, a partial default on the government bill occurs with probability  $q$ . (Note that  $q$  is the same for both countries, and moreover defaults are assumed to be independent events, in the sense that a default in one country has no bearing on the probability of default in the other.) In the event of default, the shortfall is a percentage of the face value equal to the scale of the drop in output,  $b$  or  $b^*$ .<sup>4</sup> Default does not directly affect output. The proceeds from default

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<sup>4</sup> This assumption is primarily intended to avoid further complicating the model, but Barro (2006) provides evidence of its plausibility.

are returned to the representative agent through lump-sum transfer.

These assumptions can be summarised for the home country by saying that, with probability  $e^{-p}$ , no disaster occurs, and hence there is no danger of default. With probability  $(1 - e^{-p})(1 - q)$ , a disaster occurs, but with no default. Default only occurs in the disaster state with probability  $(1 - e^{-p})q$ , in which case the debt payoff is  $1 - b$ . For the foreign country, the probabilities of the three states are the same, but the actual payoffs in the disaster state, while drawn from the same distribution, are not necessarily the same.

The solution for the return on the home country's Government bill is<sup>5</sup>:

$$\ln E_t R_{t+1}^b = \ln R_{t+1}^f + pq [(1 - \eta) Eb (\beta - 1) + \eta Eb (\beta\beta^* - 1)] \quad (18)$$

The expression in the bracket is the expected value of the risk-adjusted loss in the default scenario in the two states where that can occur. With probability  $pq(1 - \eta)$ , the default occurs only in the home country, and the relevant SDF is  $\beta$ . With probability  $pq\eta$ , the Government defaults in both countries, in which case the relevant SDF is  $\beta\beta^*$ .

Since we assume that  $b$  and  $b^*$  have the same expected values, it follows that the SDF's differ only as a result of the relative size of the two countries. When the two countries are of equal size, we have  $E\beta = E\beta^* = E(1 - b)^{-\frac{\theta}{2}}$ .

In the limiting cases, if  $\eta = 0$  - in other words, if there is no chance of a slump in the foreign country when the home country defaults - the return is simply:

$$\ln E_t R_{t+1}^b = \ln R_{t+1}^f + pq Eb (\beta - 1) \quad (19)$$

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<sup>5</sup> Note that we need only consider one-period claims because, under the assumptions made here, the term structure of interest rates is flat and moreover, the return on a claim on a one-period dividend is identical to the return on the infinite stream represented by the tree asset (see Barro (2006))

so the bill rate is the riskless rate plus the expected value of the risk-premium in a home country disaster.<sup>6</sup> On the other hand, when  $\eta = 1$ , so that disasters are certain to coincide,

$$\ln E_t R_{t+1}^b = \ln R_{t+1}^f + pqEb(\beta\beta^* - 1) \quad (20)$$

which is of course higher than in (19), because agents need compensation for their Government's default in a situation where the payout in both countries is at slump level.<sup>7</sup>

As far as equities are concerned, agents are assumed to trade the two classes of share at time  $t$ , which entitle them to a dividend at time  $t + 1$  equal to the output of either the domestic or the foreign-country tree. For the moment, we assume there is no leverage, so that equity buyers receive the total output of each country.

Subject to these assumptions, the log of the return on equity can be written:

$$\ln E_t R_{t+1}^E = \ln R_{t+1}^f + \theta w \sigma^2 + \theta(1 - w) \sigma \sigma^* \kappa + p[(1 - \eta)Eb(\beta - 1) + \eta Eb(\beta\beta^* - 1)] \quad (21)$$

This tells us that, in addition to the standard components attributable to the variance and covariance of normal shocks to growth rates, the excess return over the riskless rate is the risk-adjusted expected value of the output loss in a disaster, given in the final term as the probability-weighted average of the payoffs in the single-country and two-country disaster states. It is worth noting that the disaster premium for equity and for Government bills differs only by the default probability,  $q$ , a point which helps to explain the determination of the risk premium in this model.

The equity premium is clearly driven by two sorts of factor: the normal growth shocks, which impinge solely on equity returns, and extreme events, which likewise affect shares

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<sup>6</sup> Recall that we are following Barro (2006) in assuming  $b$  is not only the proportionate fall in output in the disaster state, but also the shortfall in the payout on Government debt if default occurs.

<sup>7</sup> Note that the expression in square brackets is positive, given that  $0 < b, w < 1$ , and  $\theta > 0$

alone and leave bills untouched, unless there is a default, in which case bills and shares are affected equally. Comparing (18) and (21), we can see that the equity premium will be given by:

$$E_t x_{t+1} = \ln E_t R_{t+1}^E - \ln E_t R_{t+1}^b \quad (22)$$

$$= \theta w \sigma^2 + \theta(1-w)\sigma\sigma^* \kappa + p(1-q) [(1-\eta) Eb(\beta-1) + \eta Eb(\beta\beta^*-1)] \quad (23)$$

i.e. the sum of the reward for carrying normal risks plus the expected value of the loss in a disaster when there is no default. (Clearly, the loss in states where the Government defaults is rewarded equally in the returns on bills and equities).

The analysis so far has been based on the assumption that the Government is the only borrower. In fact, corporate leverage can easily be incorporated in this model in the same stylised form as in Barro (2006). i.e. agents in both countries are assumed to finance their initial equity purchase in part by the issue of  $N_t$  units of private debt. The privately-issued "corporate" debt has exactly the same characteristics and payoffs in the different states as Government debt i.e. it offers the same face return in normal states and pays  $1-b$  or  $1-b^*$  in disaster states.

Under these conditions, the equity premium for leveraged shares is, as in Barro (2006):

$$E_t^L x_{t+1} = (1 + \lambda_t) E_t x_{t+1} \quad (24)$$

where  $\lambda_t$  is the leverage ratio, measured as the time  $t$  value of the debt relative to the equity.

## 4 Calibration

In the absence of data to test hypotheses relating to rare events, the only obvious approach is to calibrate the model based on what little evidence can be found. We rely as far as possible on the parameter values used in Barro (2006) based on a dataset relating to the twentieth century and covering the two world wars. However, we have introduced two critical additional parameters,  $w$ , the weight of the domestic country in the global economy, and  $\eta$ , the probability of disaster in one country, given its occurrence in another. In each case, we examine a range of values. But first, it is worth considering the theoretical importance of these parameters.

As far as  $w$  is concerned, its impact on aggregate risk is greatest when it is exactly 0.5. Other things being equal, the smaller (larger) is  $w$ , the greater the benefits of diversification for domestic (foreign) agents. Hence, it is not surprising that for values of  $w$  in the middle range, our model tends to generate far higher values for the expected bill return than in the single-country case, for the simple reason that, with lower aggregate risk, the attraction of a safer (but not totally safe) security is greatly reduced. Only for very small values of  $w$  do we find that the home country's contribution to diversification is so small that its securities need to pay a lower return than in the Barro (2006) model.

The effect of  $\eta$ , the conditional probability of disaster, is more subtle. On the one hand, higher levels of this parameter clearly imply less scope for diversifying disaster risk, and therefore greater risk at world level (and hence a lower riskless rate). On the other hand, higher  $\eta$  also means greater country-specific disaster risk associated with lower levels of correlation between each country and the world price of risk. The former effect will tend to



increase the demand for bills and equities, as agents save more - the usual stochastic discount factor mechanism at work. It should be noted that, since this effect impinges equally on the bill and equity returns, it has no net effect on the equity premium. At the same time, the increase in the riskiness of country-specific investment will raise returns on all securities, but most of all on the assets whose payoff is most closely tied to the output of the individual country i.e. equities. The impact on the equity return depends on the size of  $w$  relative to  $\theta$ . When  $w < \theta^{-1}$ , higher levels of  $\eta$  result in higher equity returns, but the implication for bills is indeterminate.<sup>8</sup> However, comparing the two, it is clear that the increase in equity returns predominates, so that higher values of  $\eta$  are unambiguously associated with a greater equity premium.

The actual calibration results are given in some detail in the figures and summarised in Tables 1 and 2. Since the size of non-disaster shocks is very small, the correlation coefficient  $\kappa$  has very little effect on the results, so it is fixed at 0.5. Note that in the figures the results of the calibrations in Barro (2006) are given by the intercepts of the curves with the right hand vertical, at the point where  $w = 1$ .

Figures 1 to 3 use Barro (2006)'s values for all the single-country parameters i.e we set:  $p = 0.017, q = 0.4, \theta = 4, \rho = 0.03, \lambda = 0.5$ . Each of the three graphs plots the values implied by our model for the expected return on equity and on bills and the implied risk premium for different values of  $m$ . Figure 1 shows that, a low value of  $\eta = 0.1$  results in a return on bills which is far too high relative to the long run observed value of around 1.5%, at least when the countries are of roughly equal size. Consequently, our risk premium is far too

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<sup>8</sup> Note that, if the coefficient of risk aversion is in the range 3 to 5, as is commonly believed, this condition is only satisfied for small values of  $w$ , certainly far lower than 0.5.

low. Higher levels of  $\eta$  do raise the equity premium to around the 2%-3% range, as can be seen in Figures 2 and 3, but they also raise the return on bills to unrealistically high levels. Figure 4 shows that raising the coefficient of risk aversion to  $\theta = 5$  rather than 4 produces higher values of the risk premium (nearly 4% when  $w = 0.5$ ), but also gives values of the bill return which are far too high. Finally, in Figure 5, we plot the same relationships under the assumption that the default probability is only 5% (compared to the 40% frequency used by Barro (2006)). This again produces plausible values for the equity return, but bill returns which are far too high.<sup>9</sup>

## 5 Conclusions

Starting from the analysis in Barro (2006), which claims to explain the apparently excessive equity premium, we have built a model of extreme events in a two-country world and shown that, for a wide range of parameter values, including the most plausible values, it is impossible to reconcile this more general model with the well-established facts regarding the equity risk premium. We conclude that the puzzle remains unresolved, at least by this branch of the literature. Moreover, we conjecture that a further generalisation to an  $n$ -country world would yield even more extreme results, because the diversification opportunities would be greatly increased.<sup>10</sup>

A number of avenues remain unexplored. For example, it would be interesting to know

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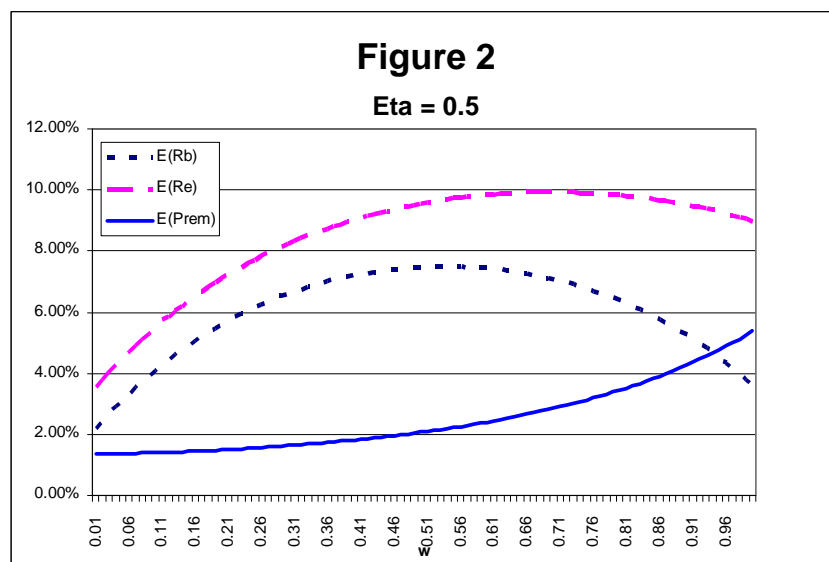
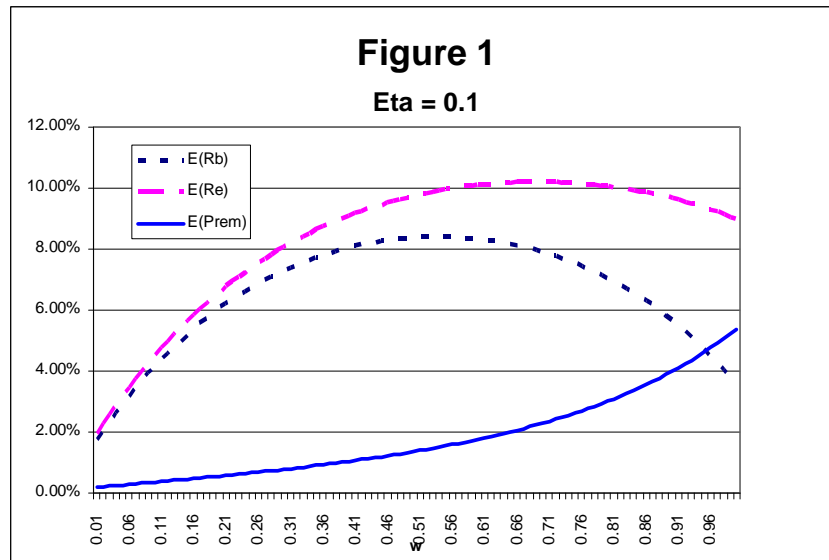
<sup>9</sup> Other results (not presented here, but available from the authors) were generated by taking different values of  $p$  and  $q$ , but none came close to replicating the values seen in the data.

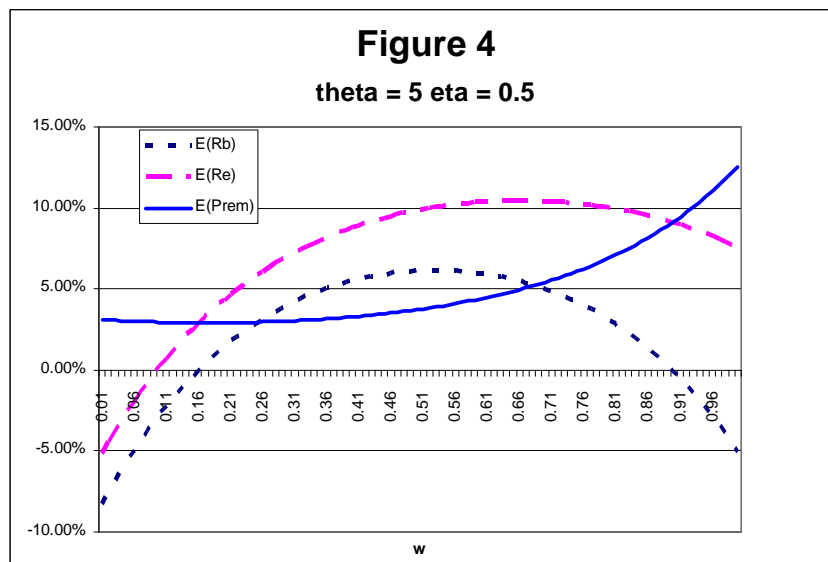
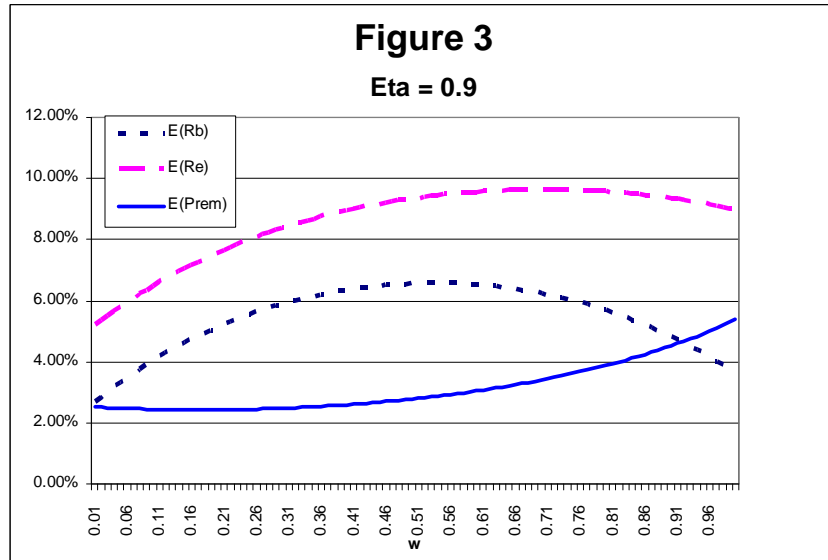
<sup>10</sup> A hint of the type of results which might result from this approach can be gained by looking at the calibrations for low values of  $w$  in the graphs given here.

whether the results are robust to a change in the assumptions regarding default. Following Barro (2006), we have assumed here that the size of a Government default is equal to the percentage of output lost in the disaster state. This assumption is somewhat arbitrary, and it would be interesting to see the effect of replacing it. The results given here are also dependent on the assumed symmetry in the probability of disaster i.e. the probability of an extreme event in the home country conditional on one in the foreign country is the same as the conditional probability of disaster in the foreign country, which may be unrealistic if the countries are of unequal size.

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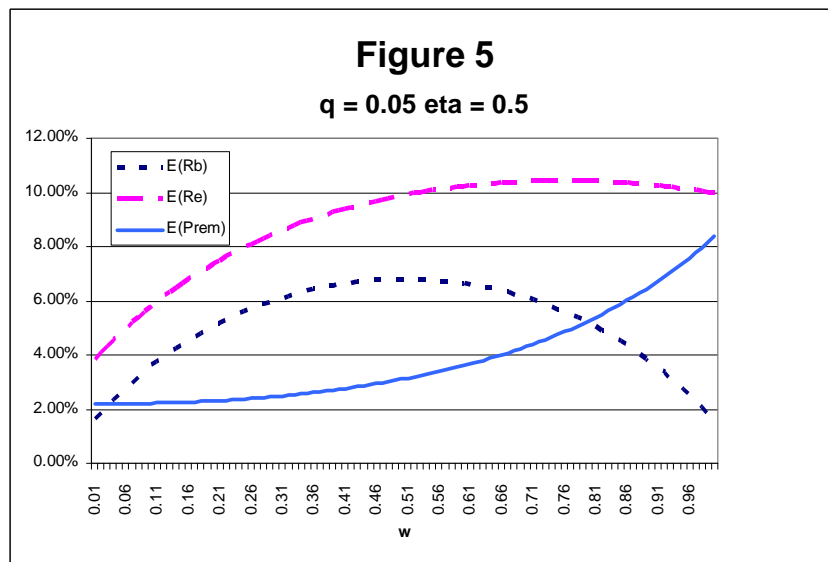


TABLE 1

Expected return of equity (ERe); Expected return of government bill (ERb); Equity premium (EPrem)

Assumptions:  $\theta = 4$ ;  $\sigma = 0.02$ ;  $\rho = 0.03$ ;  $\gamma = 0.025$ ;  $p = 0.017$ ;  $q = 0.4$ 

$w$	$w = 0.1$			$w = 0.5$			$w = 0.9$			1.0
$\eta$	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9	(Barro)
E(Re)	4.52%	5.49%	6.46%	9.72%	9.52%	9.32%	9.64%	9.49%	9.33%	8.96%
E(Rb)	4.16%	4.09%	4.02%	8.36%	7.45%	6.54%	5.65%	5.23%	4.81%	3.59%
E(Prem)	0.36%	1.40%	2.44%	1.36%	2.07%	2.78%	4.00%	4.26%	4.52%	5.38%

TABLE 2

Expected return of equity (ERe); Expected return of government bill (ERb); Equity premium (EPrem)

Assumptions:  $w = 0.5$ ;  $\eta = 0.5$ ;  $\sigma = 0.02$ ;  $\rho = 0.03$ ;  $\gamma = 0.025$ 

$\theta$	4	5	4	4
$p$	0.017	0.017	0.03	0.017
$q$	0.4	0.4	0.4	0.05
E(Re)	9.52%	9.81%	6.82%	9.87%
E(Rb)	7.45%	6.08%	3.39%	6.76%
E(Prem)	2.07%	3.73%	3.42%	3.10%