

Selim, Sheikh

**Working Paper**

## Revisiting the Capital Tax Ambiguity Result

Cardiff Economics Working Papers, No. E2006/20

**Provided in Cooperation with:**

Cardiff Business School, Cardiff University

*Suggested Citation:* Selim, Sheikh (2006) : Revisiting the Capital Tax Ambiguity Result, Cardiff Economics Working Papers, No. E2006/20, Cardiff University, Cardiff Business School, Cardiff

This Version is available at:

<https://hdl.handle.net/10419/83940>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Cardiff Economics Working Papers



Working Paper No. E2006/20

Revisiting the Capital Tax Ambiguity Result

*Sheikh Selim*

April 2006, updated February 2010

Cardiff Business School  
Aberconway Building  
Colum Drive  
Cardiff CF10 3EU  
United Kingdom  
t: +44 (0)29 2087 4000  
f: +44 (0)29 2087 4419  
[business.cardiff.ac.uk](http://business.cardiff.ac.uk)

This paper can be downloaded from [econpapers.repec.org/RePEc:cdf:wpaper:2006/20](http://econpapers.repec.org/RePEc:cdf:wpaper:2006/20)

This working paper is produced for discussion purpose only. These working papers are expected to be published in due course, in revised form, and should not be quoted or cited without the author's written permission.

Cardiff Economics Working Papers are available online from: [econpapers.repec.org/paper/cdfwpaper/](http://econpapers.repec.org/paper/cdfwpaper/) and [business.cardiff.ac.uk/research/academic-sections/economics/working-papers](http://business.cardiff.ac.uk/research/academic-sections/economics/working-papers)

Enquiries: [EconWP@cardiff.ac.uk](mailto:EconWP@cardiff.ac.uk)

**Cardiff Economics  
Working Papers**

Sheikh Selim

*Revisiting the Capital Tax Ambiguity Result*

E2006/20

Cardiff Business School  
Cardiff University  
Colum Drive  
Cardiff CF10 3EU  
United Kingdom  
t: +44 (0)29 2087 4000  
f: +44 (0)29 2087 4419  
[www.cardiff.ac.uk/carbs](http://www.cardiff.ac.uk/carbs)

ISSN 1749-6101  
April 2006, updated February 2010

This working paper is produced for discussion purpose only. These working papers are expected to be published in due course, in revised form, and should not be quoted or cited without the author's written permission. Cardiff Economics Working Papers are available online from: <http://www.cardiff.ac.uk/carbs/econ/workingpapers>  
Enquiries: [EconWP@cardiff.ac.uk](mailto:EconWP@cardiff.ac.uk)

# Revisiting the Capital Tax Ambiguity Result

Sheikh Selim<sup>1</sup>  
*Cardiff University*

September, 2009.

## Abstract:

We provide a welfare based interpretation of the capital tax ambiguity result (due to Guo & Lansing, 1999). We show that the sign ambiguity of optimal capital tax rate in an imperfectly competitive economy is mainly due to the welfare cost of investment. The substitution and income effects of profit seeking investment reinforce each other which create a deadweight loss in welfare. Investors cannot perceive this effect and never invest at the right level. This loss is perceived only by the government which motivates capital taxation.

**Keywords:** Optimal taxation, Monopoly power, Ramsey policy.

**JEL Codes:** D42, E62, H21, H30.

---

<sup>1</sup> Correspondence:

Sheikh Selim, Economics Section, Cardiff Business School, Aberconway, Colum Drive,  
Cardiff University, CF10 3EU, United Kingdom; selimsT@cardiff.ac.uk.

## 1. Introduction.

In this paper we present a welfare based interpretation of the Guo and Lansing's (1999) capital tax ambiguity result. Guo and Lansing (1999) show that in an imperfectly competitive economy, in a steady state the sign of the optimal capital income tax is ambiguous. They argue that this ambiguity is mainly due to two effects that are opposite in sign: the profit effect, and the underinvestment effect. Their result extends Judd's (1997) main finding that in an imperfectly competitive economy, in a steady state the optimal capital income tax rate is negative. We show that the main difference between these two approaches is the way the profit tax is modelled. Judd (1997) primarily assumes that profits can be taxed separately, but Guo and Lansing (1999) assume that any change in capital tax affects profit taxation. This assumption stands as the key in deriving the optimal policy that has both the motivation to tax and to subsidize capital. The sign of this tax rate thus depends on their relative strengths<sup>2</sup>.

We show that the Guo and Lansing (1999) result can be reinterpreted from welfare point of view if one uses the primal approach to optimal taxation that identifies the welfare effect of profit seeking investment. In an imperfectly competitive economy, since factors earn less than the socially optimal returns, there is a general motivation to subsidize the returns to factors. We show that this motivation only depends on a single parameter that indexes the level of monopoly distortions. In a standard neoclassical growth model with imperfect competition, this effect is generally fixed. Subsidizing capital income on the basis of this effect is therefore unlikely to encourage investment. In essence this effect motivates a flat compensation for lost private returns. The capital tax ambiguity in an imperfectly competitive economy is mainly due to welfare cost of investment and the difference between government's perception and investors' perception about this welfare cost. This effect is analogous to what Guo and Lansing (1999) refers to as the profit effect, but their analysis leaves some room for the current interpretation to contribute. We show that since investment

---

<sup>2</sup> Judd (1997) also says that the tax on capital was ambiguous if one did not distinguish between taxing returns on new investment and taxing pure profits. His paper's main focus, however, was on the sub-optimality of a capital tax. In another paper, Judd (1999) argues that a tax on capital cannot be optimal since its distortions accumulate over time, a pattern that is inconsistent with the commodity tax principle. Later, Judd (2002) argues in favour of optimal capital subsidy with reference to the repealed Investment Tax Credit scheme in the US.

in an imperfectly competitive economy is primarily motivated by earning higher profits, the substitution and income effect of additional investment reinforces each other to worsen welfare. While this effect is perceived by the government, it is not perceived by investors. The adverse welfare effect of investment motivates the government to tax capital in order to discourage profit seeking investment. Mainly due to this motivation, the optimal capital income tax rate is ambiguous.

We argue that our interpretation is important since it distinguishes the fixed and variable effects that determine the steady state optimal capital income tax policy, which in turns assists in understanding how any change in tax code that affects investment decisions will affect the optimal policy in a steady state. Our analysis clearly shows that any change in tax code that affects investment decisions will change the motivation to tax capital, but will not affect the motivation to subsidize capital. In addition, we show that both these effects are strictly increasing in the level of monopoly distortions, implying that if one correctly identifies the level of monopoly distortions, the sign of optimal capital income tax rate remains ambiguous. We discuss some conditions under which this ambiguity may be resolved. We do this by establishing a correspondence of the optimal policy with the social cost of distorting taxes.

## **2. Capital Income Tax and Profit Tax.**

Tax reforms in most industrialized countries have shown clear tendency of moving towards simplistic capital tax policy involving lower (or no) amount of direct subsidy to capital and minimum amount of deductions. Various incentive schemes including investment tax credits and property related tax shelters have been moderated or abolished in numerous countries, such as Australia, Austria, Finland, Germany, Iceland, Ireland, Portugal, Spain and the USA<sup>3</sup>. In addition, several OECD countries have revised the allowances for depreciation of capital equipment that companies can use to cut down on taxable income.

---

<sup>3</sup> Important evidence includes the 1986 repeal of Investment Tax Credit Scheme in the USA, and more recently in the UK, replacing the 0% starting rate of corporation profit tax and the starting marginal relief of corporation profit tax by a single 19% small companies' profit tax for all companies with reported profit of £0-£300,000.

In addition, there is evidence of cutting down corporation tax rates with a purpose of increasing corporation tax revenue. The essential idea is that lower corporation tax rates provide lesser incentives for corporations to hide profits or to evade taxes. Examples of this trend include Ireland (38% to 12.5%), Australia (36% to 30%), Denmark (32% to 30%), France (37.8% to 35.4%), Germany (52% to 39%), Iceland (30% to 18%) and the Czech Republic (31% to 26%). Due to the cut in corporation tax rates, there has been a mixed response in the effective capital tax rates. For instance, this figure has increased from 18.6% to 18.7% for Ireland, from 19.2% to 23.1% in Czech Republic, and from 22.9% to 23.6% in France. By contrast, there has been a decline in the effective capital tax rate in Germany (21.1% to 19.9%), while in Australia it has remained unchanged at 28% (OECD data).

In this paper we do not intend to resolve the debate whether or not taxing/subsidizing capital is the right idea. The evidence we provide here says less about the exact relationship between profit tax rate and effective capital tax rate, but says clearly that providing direct subsidy to capital is something OECD countries are trying to avoid. We provide an interpretation of the capital tax ambiguity result based on the correspondence between the optimal policy in a steady state and the social cost of taxation. We argue that with profit and capital taxation in the scheme, a long run capital subsidy is optimal if capital can be taxed early and revenue can be frontloaded. If capital tax reforms affect profit taxation, or vice versa, subsidizing capital provides more than optimal returns to investors (in the form of profit subsidy). In an imperfectly competitive economy, a marginal increase in investment has a negative impact on welfare because it distorts welfare by two margins: one in terms of lost consumption (a substitution effect), and the other in terms of lost income from capital (an income effect). While the households' intertemporal consumption and saving decisions do not capture this effect, the planner's one does. The planner can perceive the negative impact of investment and thus will always have a motivation to discourage profit-seeking investment. We argue that the strength of this motivation will depend on the social cost of taxation, which in turns is determined by the government's policy of taxing capital along the transition.

### 3. The Model.

We nest the two results (Guo and Lansing, 1999, and Judd, 1997) in a single framework. We consider a perfectly competitive final goods sector and an imperfectly competitive intermediate goods sector. The two technologies are:

$$y_t = \left\{ \left( \int_0^1 z_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \right\}^\nu n_{yt}^{1-\nu}, \quad \nu \in (0,1); \sigma \in (0,1) \quad (1)$$

$$z_{jt} = k_{jt}^\alpha n_{zjt}^{1-\alpha}, \quad \alpha \in (0,1) \quad (2)$$

where  $y_t$  is the level of final good,  $z_{jt}$  is the level of intermediate good  $j \in [0,1]$ ,  $n_{yt}$  is working time in the final good sector, and  $n_{zjt}$  and  $k_{jt}$  are working time and capital used to produce intermediate good  $j \in [0,1]$ . The representative household solves:

$$\begin{aligned} \max_{\{c_t, n_{yt}, n_{zt}, k_{t+1}, b_{t+1}\}} \quad & \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_{yt} - n_{zt}) \\ \text{s.t.} \quad & c_t + k_{t+1} + R_t^{-1} b_{t+1} = (1 - \tau_{yt}) w_{yt} n_{yt} + (1 - \tau_{zt}) w_{zt} n_{zt} + [(1 - \tau_{kt}) r_t + (1 - \delta)] k_t + b_t + (1 - \kappa \tau_{kt}) \pi_t, \\ & k_0 > 0, b_0 \text{ given} \end{aligned} \quad (3)$$

where  $R_t$  is the rate of return on real government bonds  $b_t$ , and the tax rates are  $\tau_{kt}$ ,  $\kappa \tau_{kt}$ , and  $\tau_{st}$ , for  $s = y, z$ , for capital, profits and labour, respectively<sup>4</sup>. Given  $\{g_t\}_{t=0}^{\infty}$ ,  $b_0$  and  $k_0$ , the *symmetric equilibrium* consists of time path of allocations  $\{c_t, n_{yt}, n_{zt}, k_t, z_t, y_t\}_{t=0}^{\infty}$ , prices  $\{w_{yt}, w_{zt}, p_t, r_t, R_t\}_{t=0}^{\infty}$  and policy  $\{\tau_{yt}, \tau_{zt}, \tau_{kt}, b_t\}_{t=0}^{\infty}$  that are consistent with the standard transversality conditions and the following system (4):

$$0 < n_{yt} + n_{zt} \leq 1 \quad (\text{a})$$

<sup>4</sup>  $\kappa \in [0,1]$  implies the government's set of tax treatments [no tax, at par with capital tax] for distributed corporate profits. Here  $\tau_k$  is the average effective tax rate on capital income, and the parameter  $\kappa$  represents government's fiscal treatment of profits.



$$y_t = z_t^\nu n_{yt}^{1-\nu} \quad (b)$$

$$z_t = k_t^\alpha n_{zt}^{1-\alpha} \quad (c)$$

$$c_t + g_t + k_{t+1} = k_t^{\alpha\nu} n_{zt}^{\nu(1-\alpha)} n_{yt}^{1-\nu} + (1-\delta)k_t \quad (d)$$

$$p_t = \nu(y_t)^{1-\frac{1-\sigma}{\nu}} z_t^{-\sigma} (n_{yt})^{\frac{(1-\nu)(1-\sigma)}{\nu}} \quad (e)$$

$$w_{yt} = (1-\nu)(n_{yt})^{-1} y_t \quad (f)$$

$$w_{zt} = (1-\alpha)\nu(1-\sigma)(n_{zt})^{-1} y_t \quad (g)$$

$$r_t = \alpha(1-\sigma)\nu(k_t)^{-1} y_t \quad (h)$$

$$\pi_t = (\nu\sigma)k_t^{\alpha\nu} n_{zt}^{\nu(1-\alpha)} n_{yt}^{1-\nu} \quad (i)$$

$$-u_{ns}(t) = u_c(t)(1-\tau_{st})w_{st}, \quad s = y, z \quad (j)$$

$$\beta[(1-\tau_{kt+1})r_{t+1} + 1 - \delta] = R_t = \frac{u_c(t)}{u_c(t+1)} \quad (k)$$

$$\tau_{yt}w_{yt}n_{yt} + \tau_{zt}w_{zt}n_{zt} + \tau_{kt}(r_t k_t + \kappa\pi_t) + R_t^{-1}b_{t+1} - b_t - g_t = 0 \quad (l)$$

**Proposition 1:** The first best policy involves  $\tau_{zt} = \tau_{kt} = -\sigma(1-\sigma)^{-1} < 0$ ,  $\tau_{yt} = 0$ , and a lump sum tax equal to  $g_t + \nu\sigma(1-\sigma)^{-1} y_t [1 - \sigma(1-\kappa)]$ .

**Proof:** Say the social planner can implement a lump sum tax equal to  $\ell_t$ . The social planner's problem is to choose allocations  $\{c_t, n_{yt}, n_{zt}, k_{t+1}\}_{t=0}^{\infty}$  that maximizes discounted lifetime utility subject to resource constraint (4d). The first order conditions associated with this problem are consistent with the first best allocations. Together with (4), the social planner's optimum imply that one can replicate the first best allocations in this economy by implementing a policy that involves  $\tau_{zt} = \tau_{kt} = -\sigma(1-\sigma)^{-1} < 0$ , and  $\tau_{yt} = 0$ , which together with (4l) imply that  $\ell_t = g_t + \nu\sigma(1-\sigma)^{-1} y_t [1 - \sigma(1-\kappa)]$ . •

If profit tax was not linked to capital tax, and if profits could be taxed away each period, it could perform the role of a lump sum tax and capital could be subsidized at the first best subsidy rate. This is the main result of Judd (1997). Since we assume that there are no lump sum taxes or its equivalent, one needs to solve the Ramsey problem.

We use primal approach, due to Ljungqvist & Sargent (2000, ch.12). The government chooses the allocation that maximizes social welfare subject to resource constraint (4d) and the implementability constraint:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t + u_{n_y}(t)n_{y_t} + u_{n_z}(t)n_{z_t} - u_c(t)(1 - \kappa\tau_{kt})\pi_t] - \Omega(c_0, n_{y_0}, n_{z_0}, \tau_{k_0}) = 0 \quad (5.1)$$

where

$$(1 - \kappa\tau_{kt})\pi_t = \begin{cases} \nu\sigma(1 - \kappa)k_t^{\alpha\nu} n_{z_t}^{(1-\alpha)\nu} n_{y_t}^{1-\nu} + k_t \frac{\kappa\sigma}{\alpha(1 - \sigma)\beta u_c(t)} [u_c(t-1) - \beta u_c(t)(1 - \delta)], & \text{for } t \geq 1 \\ (1 - \kappa\tau_{k_0})(\nu\sigma)k_0^{\alpha\nu} n_{z_0}^{\nu(1-\alpha)} n_{y_0}^{1-\nu}, & \text{for } t = 0 \end{cases} \quad (5.2)$$

and  $\Omega(c_0, n_{y_0}, n_{z_0}, \tau_{k_0}) \equiv u_c(0)\{[(1 - \tau_{k_0})\alpha(1 - \sigma)\nu(k_0)^{-1}y_0 + (1 - \delta)]k_0 + b_0\}$ . We denote the Lagrange multipliers associated with (5) and (4d) by  $\Phi \geq 0$  and  $\{\chi_t\}_{t=0}^{\infty}$ , respectively. We seek an allocation  $\{c_t, n_{y_t}, n_{z_t}, k_{t+1}\}_{t=0}^{\infty}$ , and a multiplier  $\Phi$  that maximize discounted lifetime utility (i.e. the social welfare) subject to (5) and (4d). We define the second best welfare function as<sup>5</sup>:

$$V(c_t, n_{y_t}, n_{z_t}, k_t, \Phi) \equiv u(c_t, n_{y_t}, n_{z_t}) + \Phi[u_c(t)c_t + u_{n_y}(t)n_{y_t} + u_{n_z}(t)n_{z_t} - u_c(t)(1 - \kappa\tau_{kt})\pi_t] \quad (6)$$

where  $(1 - \kappa\tau_{kt})\pi_t$  is defined by (5.2). Here the nonnegative Lagrange multiplier  $\Phi$  measures the utility cost of raising government revenues through distorting taxes. Without distorting taxes, the household's present value budget constraint (5) would not exert any additional constraining effect on welfare maximization beyond what is present in the economy's technology, and  $\Phi$  would be equal to zero. In contrast,

---

<sup>5</sup> The second best level of welfare is equal to the first best level of welfare less the loss in welfare due to distorting taxes and after tax profits. The loss in welfare is measured in terms of the loss in allocations due to symmetric equilibrium reaction of taxpayers, which is multiplied by the shadow price of taxes,  $\Phi$ . This multiplier's value is representative of the amount (in terms of consumption) taxpayers are willing pay in order to replace a unit of distorting tax with a unit of lump sum tax.

when the government has to use some distorting taxes, the multiplier  $\Phi$  is strictly positive, and reflects the welfare cost of the distorted margins.

The Ramsey equilibrium condition for  $k_{t+1}$  is:

$$V_c(t) - \beta \left\{ V_k(t+1) + V_c(t+1) \left[ \frac{r_{t+1}}{(1-\sigma)} + (1-\delta) \right] \right\} = 0, \quad t \geq 1 \quad (7)$$

The derivatives  $V_k(t+1)$  and  $V_c(t+1)$  in (7) represent the marginal effects of capital accumulation and consumption on the second best level of welfare. Their ratio, therefore, is a measure of the relative effect of investment in physical capital on the second best level of welfare. Unlike a setting with economy-wide competitive markets, profits appear in the implementability constraint, implying that investment induces a direct effect on the second best level of welfare. This effect is not perceived by households. This can be verified by examining the household's Euler equation. The return to investment as perceived by the households is characterized by the Euler equation:

$$u_c(t) - \beta u_c(t+1) [(1-\tau_{kt+1})r_{t+1} + 1 - \delta] = 0 \quad (8)$$

The return to investment as perceived by the government is characterized by (7). The government's perception includes the term  $\frac{V_k(t+1)}{V_c(t+1)}$ , which is the ratio of the marginal effects of capital accumulation and consumption on the second best level of welfare, i.e. it is a measure of the relative effect of investment on the second best level of welfare. Notice that:

$$V_k(t+1) = \Phi \left[ \frac{\kappa\sigma}{\alpha(1-\sigma)} \left\{ u_c(t+1)(1-\delta) - \beta^{-1}u_c(t) \right\} - \nu\sigma(1-\kappa)u_c(t+1) \frac{r_{t+1}}{(1-\sigma)} \right] \quad (9)$$

The marginal effect of investment on the second best level of welfare includes the substitution effect,  $\Phi \frac{\kappa\sigma}{\alpha} \left[ -u_c(t+1)(1-\tau_{k,t+1}) \frac{r_{t+1}}{1-\sigma} \right]$ , which represents the loss in welfare due to the margin of loss of a period ahead consumption. The second effect,  $\left[ -\Phi \nu\sigma(1-\kappa)u_c(t+1) \frac{r_{t+1}}{(1-\sigma)} \right]$ , represents the income effect, i.e. the government's perceived loss in welfare due to a loss in capital income. As long as  $\tau_{k,t} \leq 1$ , both these effects are strictly negative, and therefore reinforce each other to reduce welfare.

#### 4. The Optimal Policy in a Steady State.

Assume that  $u : \mathbf{R}_+^3 \rightarrow \mathbf{R}$  is separable in consumption and labour, and linear in labour. Consider a steady state. The optimal tax policy consistent with the steady state versions of (7) and (8) is represented by:

$$1 - \tau_k = \frac{1}{(1-\sigma)} + \frac{V_k}{rV_c} \quad (10)$$

where

$$V_c = u_c + u_c \Phi \left[ 1 + \frac{u_{cc}c}{u_c} \left\{ 1 - \sigma \nu \frac{y}{c} \left[ (1-\kappa) - (1-\delta - \beta^{-1})r^{-1}\kappa \right] \right\} \right]$$

$$V_k = u_c \Phi \left[ \frac{\kappa\sigma}{\alpha(1-\sigma)} (1-\delta - \beta^{-1}) - \nu\sigma(1-\kappa) \frac{r}{(1-\sigma)} \right]$$

One can easily verify that  $V_c > 0$  and  $V_k < 0$ , which is why in a steady state the sign of the optimal capital tax rate is ambiguous. Our approach thus shows the underlying force of this ambiguity result: the welfare effect of investment. First, notice that the motivation to subsidize capital is simply one minus the mark up ratio, and this effect is completely independent of the level of investment. This motivation only depends on the level of monopoly distortions, and with mark up pricing this effect is always there. The motivation to tax capital is due to the link between profit and capital taxes,

and if they are not linked, this motivation is no longer there<sup>6</sup>. With no imperfect competition,  $V_k$  would just be equal to zero, i.e. investors would invest at the right level. In the current setting,  $V_k$  is strictly negative, i.e. the welfare effect of investment in terms of the consumption good is strictly negative. This effect is not perceived by the investors (i.e. it is not in their Euler equation). In a steady state, the Ramsey policy for capital taxation is therefore determined by the relative strengths of these two effects. If the welfare effect (distortion effect) dominates the distortion effect (welfare effect), the Ramsey policy is to tax (subsidize) capital income.

We now show that this ambiguity cannot be resolved even if one identifies the correct level of  $\sigma$ .

**Proposition 2:** Both the monopoly distortion effect and steady state welfare effect of investment are strictly increasing in  $\sigma$ . The capital tax ambiguity cannot be resolved if the magnitude of  $\sigma$  is identified.

**Proof:** Showing that the distortion effect is strictly increasing in  $\sigma$  is straightforward.

Consider  $\frac{\partial(-\frac{V_k}{rV_c})}{\partial\sigma}$ . The derivative is equal to  $\frac{SY - ZX[v^2\alpha\frac{y}{k}\Phi u_{cc}(1-\kappa)\frac{y}{c}]}{(Y)^2}$ , where,

$$S = u_c\Phi[v^2\alpha(1-\kappa)\frac{y}{k} - \alpha^{-1}\kappa(1-\delta-\beta^{-1})(1-\sigma)^{-2}] > 0;$$

$$Z = -V_k > 0;$$

$$Y = rV_c > 0; \text{ and}$$

$$X = \sigma^{-1}(\alpha v\frac{y}{k}[u_c(1+\Phi) + u_{cc}\Phi] - Y)$$

Since  $Y > 0$ , and  $Y = \alpha v\frac{y}{k}[u_c(1+\Phi) + u_{cc}\Phi] - X\sigma$ , it follows that  $X < 0$ . The proof is complete if one can show that  $(SY - ZX[v^2\alpha\frac{y}{k}\Phi u_{cc}(1-\kappa)\frac{y}{c}]) > 0$ . Say not, and say:

---

<sup>6</sup> If profits are taxed at a rate  $\tau_\Pi$ , say,  $V_k(t+1) = -\bar{\Phi}u_c(t+1)v\sigma(1-\tau_\Pi)r_{t+1}(1-\sigma)^{-1}$ , and  $V_k = 0$  vis a vis  $\tau_k = -\sigma(1-\sigma)^{-1}$ , if and only if  $\tau_\Pi = 1$ . With (10), this implies that the optimal policy is to subsidize capital if and only if profits can be taxed away, which is one of Judd's (1997) results.

$$\sigma \frac{S}{Z} < \sigma \frac{X [v^2 \alpha \frac{y}{k} \Phi u_{cc} (1-\kappa) \frac{y}{c}]}{Y} \quad (11.1)$$

Note that  $(S\sigma - Z) = \alpha^{-1} \kappa (\beta^{-1} - 1 + \delta) \sigma^2 (1-\sigma)^{-2} > 0$ , implying that  $\sigma \frac{S}{Z} > 1$ .

Condition (11.1) therefore implies that

$$\sigma \frac{X [v^2 \alpha \frac{y}{k} \Phi u_{cc} (1-\kappa) \frac{y}{c}]}{Y} > 1 \quad (11.2)$$

which cannot be true. This is because for  $[u_c(1+\Phi) + u_{cc}\Phi] > 0$ ,  $|v^2 \alpha \frac{y}{k} \Phi u_{cc} (1-\kappa) \frac{y}{c}| > Y$  which is a contradiction. Also, for  $[u_c(1+\Phi) + u_{cc}\Phi] < 0$ ,  $|v^2 \alpha \frac{y}{k} \Phi u_{cc} (1-\kappa) \frac{y}{c}| = Y$  which is a contradiction. •

Furthermore to proposition 1, the derivative of the distortion effect with respect to  $\sigma$  and the derivative of the welfare effect of investment with respect to  $\sigma$  both are infinitely large in the neighbourhood of  $\sigma = 1$ , but converges to a constant in the neighbourhood of  $\sigma = 0$ . The magnitude of monopoly distortion that necessitates optimal tax (or subsidy) on capital income is therefore ambiguous.

Since  $V_{ny} = u_{ny}(1+\Phi) - \Phi u_c v \sigma (1-\kappa) w_y$ ,  $V_{nz} = u_{nz}(1+\Phi) - \Phi u_c v \sigma (1-\kappa) \frac{w_z}{(1-\sigma)}$ ,

and in the Ramsey equilibrium,  $V_{ny}(t) = -V_c(t) w_{yt}$  and  $V_{nz}(t) = -V_c(t) \frac{w_{zt}}{(1-\sigma)}$ , the

optimal labour income tax rates that can be implemented in a decentralized equilibrium are consistent with:

$$(1-\tau_y) = \frac{\left[ 1 + \Phi \left( 1 + \frac{u_{cc} c}{u_c} \left\{ 1 - \sigma v \frac{y}{c} [(1-\kappa) - (1-\delta - \beta^{-1}) r^{-1} \kappa] \right\} \right) \right] - \Phi v \sigma (1-\kappa)}{(1+\Phi)} \quad (12.1)$$

$$(1 - \tau_z) = \frac{\left[ 1 + \Phi \left( 1 + \frac{u_{cc}c}{u_c} \left\{ 1 - \sigma \nu \frac{y}{c} \left[ (1 - \kappa) - (1 - \delta - \beta^{-1})r^{-1}\kappa \right] \right\} \right) \right] - \Phi \nu \sigma (1 - \kappa)}{(1 + \Phi)(1 - \sigma)} \quad (12.2)$$

and together they imply,  $\frac{(1 - \tau_z)}{(1 - \tau_y)} = \frac{1}{(1 - \sigma)}$ . In a steady state, the Ramsey policy prescribes that labour income in the monopoly sector should be taxed at a lower rate than labour income in the competitive sector. This is a classic result of differential taxation in the presence of market power, proposed primarily by Stiglitz and Dasgupta (1971).

We now return to the (steady state) optimal capital income tax policy. In particular, we discuss the properties of an implementable capital income subsidy.

**Proposition 3:** An implementable capital income subsidy with  $\kappa \neq 0$  overcompensates capital income at the cost of higher debt or higher labour income taxes.

**Proof:** The second term in (5.2) for  $t \geq 1$  is equal to  $\frac{\kappa\sigma}{\alpha(1 - \sigma)}(1 - \tau_{kt})r_t k_t$ .

Together with the household's budget constraint it implies that the (after tax) effective real return to capital is equal to  $\left[ (1 - \tau_{kt})r_t \left\{ 1 + \frac{\kappa\sigma}{\alpha(1 - \sigma)} \right\} + (1 - \delta) \right]$ . Thus a capital subsidy with  $\kappa \neq 0$  not only pushes buyer price up to social marginal return, but also pays capital an extra compensation, i.e. it overcompensates capital income at the cost of higher debt or higher labour income taxes. •

Notice from (7-9) that if profits and capital are taxed at the same rate (i.e.  $\kappa = 1$ ), along the transition it is possible to offset the welfare effect of investment by setting  $\tau_{kt} = 1$ ,  $t \geq 1$ . Given a set of initial tax rates and allocations, capital for  $t = 1$  is supplied inelastically. If the government taxes it away, there will not be any *welfare-worsening* investment at  $t = 1$ . What remains to be examined is how long does it take for the economy to reach the steady state and for how many periods the government

can implement a confiscating capital income tax in order to frontload revenue. Essentially, these depend on the parameters of the model and the set of initial conditions that determine the social cost of taxation. The present value of this social cost of distorting taxes is represented by the multiplier  $\Phi$ . If the government can frontload the preset revenue by confiscating capital income in a few initial periods, the present value of the social cost of taxes is high. This policy will confiscate profits away and thus will weaken the steady state relative welfare effect of investment. This in turns implies that in the long run the motivation to tax capital is weaker if it is possible to tax capital early.

For the tax code with  $\kappa = 0$ , consider a characterization of the optimal policy. Say utility is logarithmic in consumption and linear in leisure. Normalize output to one and set  $\kappa = 0$  in the steady state versions of (7-9), in order to derive:

$$\tau_k = -\sigma \left[ \frac{c(1 - \Phi\nu) + \nu\sigma\Phi}{(c + \nu\Phi)(1 - \sigma)} \right] \quad (13)$$

Given the set of parameters, the restriction  $\Phi \leq \nu^{-1}$  implies that  $\tau_k < 0$ . This restriction is a representative case where the optimal policy involves a capital income subsidy. The parameter  $\nu$  is associated with the profit ratio (see the decentralized equilibrium condition (4i)), and higher  $\nu$  implies higher profits. If the tax code involves no tax/subsidy on profits, and if profits are high (low), a long run capital income subsidy can be implemented by setting a transitional capital income tax policy that is associated with low (high) social cost of taxation.

## 5. Conclusion.

In this paper we provide an alternative interpretation of the capital tax ambiguity result. We show analytically that the ambiguity is mainly due to the link between the profit tax and the capital income tax. The motivation to subsidize capital income in the long run is generally fixed but the motivation to tax capital income in the long run depends on the social cost of taxation, which in turns is determined by the



government's policy of capital taxation along the transition. Our interpretation extends the primary interpretation by Guo & Lansing (1999) in two ways. First, our interpretation distinguishes the fixed and the variable effect (or the welfare-independent and welfare-dependent effect) which determine the long run policy. This is important since it allows one to examine how any change in tax code will affect the motivation to tax or subsidize capital income in the long run. Essentially, any change in tax code that alters the incentives to invest or consume will impute changes in the motivation to tax income from capital. Second, our interpretation provides the welfare implications of this ambiguity by relating it to the social cost of taxation.

### **References:**

Guo, J-T. & K. J. Lansing, 1999. Optimal Taxation of Capital Income with Imperfectly Competitive Product Markets, *Journal of Economic Dynamics and Control*, 23, 967-995.

Judd, K. L. 1997. The Optimal Tax on Capital Income is Negative, NBER Working Paper No. 6004.

Judd, K. L. 1999. Optimal Taxation and Spending in General Competitive Growth Models, *Journal of Public Economics*, 71, 1-26.

Judd, K. L. 2002. Capital Income Taxation with Imperfect Competition, *The American Economic Review*, 92, 417-421.

Ljungqvist, L. & Sargent, T. J. (2000). *Recursive Macroeconomic Theory*. London: The MIT Press, 2000.

Stiglitz, J. E. & P. Dasgupta, 1971. Differential Taxation, Public Goods, and Economic Efficiency, *The Review of Economic Studies*, 38, 151-174.