

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Dixon, Huw; Kara, Engin

## **Working Paper**

# Persistence and nominal inertia in a generalized Taylor economy: How longer contracts dominate shorter contracts

Cardiff Economics Working Papers, No. E2007/1

**Provided in Cooperation with:** Cardiff Business School, Cardiff University

*Suggested Citation:* Dixon, Huw; Kara, Engin (2007) : Persistence and nominal inertia in a generalized Taylor economy: How longer contracts dominate shorter contracts, Cardiff Economics Working Papers, No. E2007/1, Cardiff University, Cardiff Business School, Cardiff

This Version is available at: https://hdl.handle.net/10419/83917

#### Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

#### Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



# WWW.ECONSTOR.EU

# **Cardiff Economics Working Papers**





# Working Paper No. E2007/1

## Persistence and Nominal Inertia in a Generalized Taylor Economy: How Longer Contracts Dominate Shorter Contracts

Huw Dixon and Engin Kara

January 2007

Cardiff Business School Aberconway Building Colum Drive Cardiff CF10 3EU United Kingdom t: +44 (0)29 2087 4000 f: +44 (0)29 2087 4419 business.cardiff.ac.uk

This paper can be downloaded from econpapers.repec.org/RePEc:cdf:wpaper:2007/1

This working paper is produced for discussion purpose only. These working papers are expected to be published in due course, in revised form, and should not be quoted or cited without the author's written permission.

Cardiff Economics Working Papers are available online from: econpapers.repec.org/paper/cdfwpaper/ and business.cardiff.ac.uk/research/academic-sections/economics/working-papers

Enquiries: EconWP@cardiff.ac.uk

# Persistence and Nominal Inertia in a Generalized Taylor Economy: How Longer Contracts Dominate Shorter Contracts<sup>\*</sup>

Huw Dixon<sup>†</sup>and Engin Kara<sup>‡</sup>

January 20, 2007

#### Abstract

We develop the Generalized Taylor Economy (GTE) in which there are many sectors with overlapping contracts of different lengths. In economies with the same average contract length, monetary shocks will be more persistent when longer contracts are present. Using the Bils-Klenow distribution of contract lengths, we find that the corresponding *GTE* tracks the US data well. When we choose a *GTE* with the same distribution of completed contract lengths as the Calvo, the economies behave in a similar manner.

JEL: E50, E24, E32, E52

Keywords: Persistence, Taylor contract, Calvo.

<sup>\*</sup>This paper is a revised version of the 2005 European Central Bank working paper no:489. This work was started when Huw Dixon was a visiting Research Fellow at the European Central Bank in 2002. We would like to thank Guido Ascari, Vitor Gaspar, Michel Juillard, Simon Price, Neil Rankin, Luigi Siciliani, Frank Smets, Peter N. Smith, Zheng Liu and participants at the 2004 St. Andrews Macroconference, the 2005 World Congress of the Econometric Society, the 2005 SCE conference, the 2006 Royal Economic Society, and seminars at the Bank of England, Cardiff Business School, City University, Essex, Leicester, Lisbon (ISEG), Monetary Policy Strategy division at the European Central Bank, National University of Singapore, Nottingham, Warwick and York for their comments. Faults remain our own.

<sup>&</sup>lt;sup>†</sup>Corresponding author: Cardiff Business School, Aberconway Building, Column Drive, Cardiff, CF10 3EU. Email: DixonH@cardiff.ac.uk.

<sup>&</sup>lt;sup>‡</sup>Economics Department, University of Birmingham, Birmingham, B15 2TT. Email: e.kara@bham.ac.uk

# 1 Introduction

"There is a great deal of heterogeneity in wage and price setting. In fact, the data suggest that there is as much a difference between the average lengths of different types of price setting arrangements, or between the average lengths of different types of wage setting arrangements, as there is between wage setting and price setting. Grocery prices change much more frequently than magazine prices - frozen orange juice prices change every two weeks, while magazine prices change every three years! Wages in some industries change once per year on average, while others change per quarter and others once every two years. One might hope that a model with homogenous representative price or wage setting would be a good approximation to this more complex world, but most likely some degree of heterogeneity will be required to describe reality accurately."

Taylor (1999).

There are two main approaches to modelling nominal wage and price rigidity in the dynamic general equilibrium (DGE) macromodels: the staggered contract setting of Taylor (Taylor (1980)) and the Calvo model of random contract lengths generated by a constant hazard (reset) probability (Calvo (1983)). This paper proposes a generalization of the standard Taylor model to allow for an economy with many different contract lengths: we call this a Generalized Taylor Economy - GTE for short. The standard approach in the literature has been to adopt a simple Taylor economy, in which there is a single contract length in the economy: for example 2 or 4 quarters<sup>1</sup>. As the above quote from John Taylor indicates, in practice there is a wide range of wage and price setting behavior resulting in a variety of contract lengths. We can use the GTE framework to evaluate whether the hope expressed by John Taylor that a representative sector approach "is a good approximation to this more complex world".

<sup>&</sup>lt;sup>1</sup>This is not to ignore some recent papers: Carvalho (2006), Coenen, Christoffel and Levin (2006) and Carlstrom, Fuerst, Ghironi and Hernandez (2006) consider economies with multiple contract lengths. See also Taylor (1993) for what we believe to be the first instance. Other papers that allow for two sectors with different contract durations are Aoki (2001), Benigno (2004), Erceg and Levin (2005), Carlstrom, Fuerst and Ghironi (2006) and Mankiw and Reis (2003). However, these studies are mainly concerned with computing optimal monetary policy in a dynamic general equilibrium setting.

An additional advantage of the GTE framework is that it includes the Calvo model as a special case, in the sense that we can set up the GTE to have the same distribution of contract lengths as the Calvo model. This is an important contribution in itself since the two approaches have until now appeared to be distinct and incompatible at the theoretical level even if they are sometimes claimed to be empirically similar (see for example Kiley (2002) for a discussion). As we shall show, a simple Taylor economy can indeed be a good approximation to a Calvo model, but only if the two are calibrated in a consistent manner.

We develop our approach in a DGE setting following the approach of Ascari (2000). The issue we focus on is the way a monetary shock can generate changes in output through time, and in particular the degree of persistence of deviations of output from steady-state. Much recent attention has been devoted to the ability of the staggered contract approach of Taylor to generate enough persistence in the sense of being quantitatively able to generate Two influential papers in this are the persistence observed in the data. Chari, Kehoe and McGrattan (2000) (CKM hereafter) and Ascari (2000). Both papers are pessimistic for staggered contracts. CKM develop a microfounded model of staggered price-setting and find that they do not generate enough persistence and conclude that the "mechanism to solve persistence problem must be found elsewhere". Ascari focusses on staggered wage setting, and finds that whilst nominal wage rigidities lead to more persistent output deviations than with price setting, they are still not enough to explain the data. Based on these conclusions, it is commonly inferred that in a dynamic equilibrium framework, staggered contracts cannot generate enough persistence.

We show that by allowing for an economy with a range of contract lengths, the presence of longer contracts can significantly increase the degree of persistence in output following a monetary shock. We calibrate the model in a way which either the CKM or Ascari setting would not generate much persistence. We show that even a small proportion of longer contracts can significantly increase the degree of persistence. For example, we consider the case of a economy where 90% of the economy consist of simple 2-period Taylor contracts, and 10% have 8-period Taylor contracts (the average is 2.6 quarters) and show that the economy has a marked increase in output persistence. The intuition behind this finding is that there is a spillover effect or strategic complementarity in terms of wage or price-setting through the price level. The presence of longer contracts means that the general price level is held back in response to monetary shocks. This in turn means that the wage setting of shorter contracts is influenced and hence they adjust by less than they otherwise would. We also compare the impulse response function estimated by CKM with the one from a simulated GTE based on the actual distribution of contract lengths for the US based on the Bils-Klenow data set (Bils and Klenow (2004)). We find that the impulse response for this distribution is very similar to the impulse response that CKM estimated from US data.

It has long been observed that in the Calvo setting there can be a significant backlog of old contracts: for example, with a reset probability of  $\omega = 0.25$  (a common value used with quarterly data), there is a probability of over 10% that a contract will survive for 8 periods (see for example Erceg (1997), Wolman (1999)). We construct a GTE which has exactly the same distribution of completed contract lengths as the Calvo distribution (as derived in Dixon and Kara (2006a)). We find that this Calvo-*GTE* has similar persistence to the Calvo economy. The remaining difference between the Calvo economy and the Calvo-GTE is in the wage-setting decision. We find that Calvo reset firms are more forward looking on average than in the Calvo-GTE. This is because short contracts are more predominant amongst wage-resetters in the Calvo-GTE than in the economy as a whole, simply because wage-setters with long contracts reset wages less frequently. However, for the calibrated values this does not make a big difference and indicates that the two approaches of Taylor and Calvo can be brought together in the framework of the GTE.

The outline of the paper is as follows. In section 2 we outline the basic structure of the Economy. The main innovation here is to allow for the GTE contract structure. In section 3 we present the log-linearized general equilibrium and discuss the calibration of the model in relation to recent literature. In section 4 we explore the influence of longer term contracts on persistence as compared to the simple Taylor economy and apply this to US data. In section 5 we apply our methodology to evaluating persistence in the Calvo model.

# 2 The Model Economy

The approach of this paper is to model an economy in which there can be many sectors with different wage setting processes, which we denote a Generalized Taylor Economy (GTE). As we will show later, an advantage of the GTE approach is that it includes as special cases not only the standard Taylor case of an economy where all wage contracts are of the same length, but also the Calvo process.

The model in this section is an extension of Ascari (2000) and includes a number of features essential to understanding the impact of monetary shocks on output in a dynamic equilibrium setting. The exposition aims to outline the basic building blocks of the model. However, the novel aspects of this paper only begin with the wage setting process. Firstly, we describe the behavior of firms which is standard. Then we describe the structure of the contracts in a GTE, the wage-setting decision and monetary policy.

### 2.1 Firms

There is a continuum of firms  $f \in [0, 1]$ , each producing a single differentiated good Y(f), which are combined to produce a final consumption good Y. The production function here is CES with constant returns and corresponding unit cost function P

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$$
(1)

$$P_t = \left[ \int_0^1 P_{ft}^{1-\theta} df \right]^{\frac{1}{1-\theta}}$$
(2)

The demand for the output of firm f is

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\theta} Y_t \tag{3}$$

Each firm f sets the price  $P_{ft}$  and takes the firm-specific wage rate  $W_{ft}$  as given. Labor  $L_{ft}$  is the only input so that the inverse production function is

$$L_{ft} = \left(\frac{Y_{ft}}{\alpha}\right)^{\frac{1}{\sigma}} \tag{4}$$

Where  $\sigma \leq 1$  represents the degree of diminishing returns, with  $\sigma = 1$  being constant returns. The firm chooses  $\{P_{ft}, Y_{ft}, L_{ft}\}$  to maximize profits subject

to (3) and (4) yields the following solutions for price, output and employment at the firm level given  $\{Y_t, W_{ft}, P_t\}$ 

$$P_{ft} = \left(\frac{\theta}{\theta - 1}\right) \frac{\alpha^{-1/\sigma}}{\sigma} W_{ft} Y_{ft}^{\frac{1 - \sigma}{\sigma}}$$
(5)

$$Y_{ft} = \kappa_1 \left(\frac{W_{ft}}{P_t}\right)^{-\sigma\varepsilon} Y_t^{\frac{\varepsilon\sigma}{\theta}}$$
(6)

$$L_{ft} = \kappa_2 \left(\frac{W_{ft}}{P_t}\right)^{-\varepsilon} Y_t^{\frac{\varepsilon}{\theta}}$$
(7)

where  $\varepsilon = \frac{\theta}{\theta(1-\sigma)+\sigma} > 1$   $\kappa_1 = \left(\frac{\theta}{\theta-1}\right)^{-\sigma\varepsilon} \sigma^{-\sigma\varepsilon} \alpha^{-\varepsilon}$   $\kappa_2 = \left(\frac{\theta}{\theta-1}\right)^{-\varepsilon} \sigma^{\varepsilon} \alpha^{\varepsilon\left(\frac{\theta-1}{\theta}\right)}$ .

Price is a markup over marginal cost, which depends on the wage rate and the output level (when  $\sigma < 1$ ): output and employment depend on the real wage and total output in the economy.

## 2.2 The Structure of Contracts in a GTE

In this section we outline an economy in which there are potentially many sectors with different lengths of contracts. Within each sector there is a standard Taylor process (i.e. overlapping contracts of a specified length). The economy is called a Generalized Taylor Economy (GTE). Corresponding to the continuum of firms f there is a unit interval of household-unions (one per firm)<sup>2</sup>. The economy consists N sectors i = 1...N. The budget shares of the N sectors with uniform prices (when prices  $p_f$  are equal for all  $f \in [0, 1]$ ) are given by  $\alpha_i \geq 0$  with  $\sum_{i=1}^{N} \alpha_i = 1$ , the N vector  $(\alpha_i)_{i=1}^{N}$  being denoted  $\boldsymbol{\alpha}$ , where  $\boldsymbol{\alpha} \in \Delta^{N-1}$ . Without loss of generality, we suppose that in sector i there are i-period contracts, so that the longest contracts are N periods. If there are no j period contracts, then  $\alpha_j = 0$ . Whilst this notation is valid for all GTEs, in some cases where only a few contract lengths are present it is easier to list the lengths and shares: in this case: the vector of contract lengths  $T_i$  is  $\mathbf{T} = (T_i)_{i=1}^N$ , the resultant GTE being denoted  $GTE(\mathbf{T}, \boldsymbol{\alpha})$ . Thus an economy that has 30% 2-period contracts and 70% 4-period contracts is GTE((2, 4), (0.3, 0.7)) as well as GTE((0, 0.3, 0, 0.7).

<sup>&</sup>lt;sup>2</sup>Following Taylor, we will present the model as one of wage-setting. However, the framework also holds for price-setting. The distinction between wage and price-setting rests primarily when we come to calibration, as we discuss in some detail below.

We can partition the unit interval into sub-intervals representing each sector. Let us define the cumulative budget share of sectors k = 1...i.

$$\hat{\alpha}_i = \sum_{k=1}^i \alpha_k$$

with  $\hat{\alpha}_0 = 0$  and  $\hat{\alpha}_N = 1$ . The interval for sector *i* is then  $[\hat{\alpha}_{i-1}, \hat{\alpha}_i]$ .

Within each sector i, each firm is matched with a firm-specific union: there are i equally sized cohorts j = 1...i of unions and firms. Each cohort sets sets the wage which lasts for i periods: one cohort moves each period. We can partition the interval  $[\hat{\alpha}_{i-1}, \hat{\alpha}_i]$  into cohort intervals: cohort j in sector i is then represented by the interval

$$\left[\hat{\alpha}_{i-1} + \frac{(j-1)\left(\hat{\alpha}_{i} - \hat{\alpha}_{i-1}\right)}{i}, \hat{\alpha}_{i} + \frac{j\left(\hat{\alpha}_{i} - \hat{\alpha}_{i-1}\right)}{i}\right]$$

The general price index P can be defined in terms of sectors, or subintervals  $[\hat{\alpha}_{i-1}, \hat{\alpha}_i]$  for each sector i.

$$P = \left[\sum_{i=1}^{N} \int_{\hat{\alpha}_{i-1}}^{\hat{\alpha}_i} P_f^{1-\theta} df\right]^{\frac{1}{1-\theta}}$$

This can be further broken down into intervals for each cohort, where we note that all firms in the same cohort face the same wage and hence set the same price  $p_f = p_{ij}$  for  $f \in \left[\hat{\alpha}_{i-1} + \hat{\lambda}_{ij-1}\alpha_i, \hat{\alpha}_{i-1} + \hat{\lambda}_{ij}\alpha_i\right]$ 

$$P = \left[\sum_{i=1}^{N} \sum_{j=1}^{N_i} \int_{\hat{\alpha}_{i-1} + \frac{j}{i}(\hat{\alpha}_i - \hat{\alpha}_{i-1})}^{\hat{\alpha}_{i-1} + \frac{j}{i}(\hat{\alpha}_i - \hat{\alpha}_{i-1})} P_{ij}^{1-\theta} df\right]^{\frac{1}{1-\theta}}$$
(8)

We can log linearize the price equations around the steady state , given the wages. All firms with the same wage will set the same price: define  $P_{ij}$  as the price set by firms in sector *i* cohort *j*. This yields the following log-linearization in terms of deviations from the steady state (where we assume  $P^* = 1$ ):

$$p = \sum_{i=1}^{N} \sum_{j=1}^{N_i} \frac{\alpha_i}{i} p_{ij} \tag{9}$$

Note that there is an important property of CES technology. The demand for an individual firm depends only on its own price and the general price index (see equation(3)). There is no sense of location: whilst we divide the unit interval into segments corresponding to sectors and cohorts within sectors, this need not reflect any objective factor in terms of sector or cohort specific aspects of technology or preferences. The sole communality within a sector is the length of the wage contract: the sole communality within a cohort is the timing of the contract. This is an important property which will become useful when we show that a Calvo economy can be represented by a *GTE*.

### 2.3 Household-Unions and Wage Setting

Households  $h \in [0, 1]$  have preferences defined over consumption, labour, and real money balances. The expected life-time utility function takes the form

$$U_h = E_t \left[ \sum_{t=0}^{\infty} \beta^t u(C_{ht}, \frac{M_{ht}}{P_t}, \underbrace{1 - H_{ht}}_{L_{ht}}) \right]$$
(10)

where  $C_{ht}$ ,  $\left(\frac{M_{ht}}{P_t}\right)$ ,  $H_{ht}$ ,  $L_{ht}$  are household h's consumption, end-of period real money balances, hours worked, and leisure respectively, t is an index for time,  $0 < \beta < 1$  is the discount factor, and each household has the same flow utility function u, which is assumed to take the form

$$U(C_{ht}) + \delta \ln(\frac{M_{ht}}{P_t}) + V\left(1 - H_{ht}\right)$$
(11)

Each household-union belongs to a particular sector and wage-setting cohort within that sector (recall, that each household is twinned with firm f = h). Since the household acts as a monopoly union, hours worked are demand determined, being given by the (7).

The household's budget constraint is given by

$$P_t C_{ht} + M_{ht} + \sum_{s_{t+1}} Q(s^{t+1} \mid s^t) B_h(s^{t+1}) \le M_{ht-1} + B_{ht} + W_{ht} H_{ht} + \pi_{ht} + T_{ht}$$
(12)

where  $B_h(s^{t+1})$  is a one-period nominal bond that costs  $Q(s^{t+1} | s^t)$  at state  $s^t$  and pays off one dollar in the next period if  $s^{t+1}$  is realized.  $B_{ht}$  represents the value of the household's existing claims given the realized state of nature.  $M_{ht}$  denotes money holdings at the end of period t.  $W_{ht}$  is the nominal wage,  $\pi_{ht}$  is the profits distributed by firms and  $W_{ht}H_{ht}$  is the labour income. Finally,  $T_t$  is a nominal lump-sum transfer from the government.

The households optimization breaks down into two parts. First, there is the choice of consumption, money balances and one-period nominal bonds to be transferred to the next period to maximize expected lifetime utility (10) given the budget constraint (12). The first order conditions derived from the consumer's problem are as follows:

$$u_{ct} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} u_{ct+1} \right) \tag{13}$$

$$\sum_{s_{t+1}} Q(s^{t+1} \mid s^t) = \beta E_t \frac{u_{ct+1} P_t}{u_{ct} P_{t+1}} = \frac{1}{R_t}$$
(14)

$$\delta \frac{P_t}{M_t} = u_{ct} - \beta E_t \frac{P_t}{P_{t+1}} u_{ct+1} \tag{15}$$

Equation (13) is the Euler equation, (14) gives the gross nominal interest rate and (15) gives the optimal allocation between consumption and real balances. Note that the index h is dropped in equations (13) and (15), which reflects our assumption of complete contingent claims markets for consumption and implies that consumption is identical across all households in each period  $(C_{ht} = C_t)^3$ .

The reset wage is for household h in sector i is chosen to maximize lifetime utility given labour demand (7) and the additional constraint that nominal wage will be fixed for  $T_i$  periods in which the aggregate output and price level are given $\{Y_t, P_t\}$ . From the unions point of view, we can collect together all of the terms in (7) which the union treats as exogenous by defining the constant  $K_t$  where:

$$K_t = \kappa_2 P_t^{\varepsilon} Y_t^{\frac{\varepsilon}{\overline{\theta}}}$$

Since the reset wage at time t will only hold for  $T_i$  periods, we have the following first-order condition:

 $<sup>^{3}</sup>$ See Ascari (2000).

$$X_{it} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left[\frac{\sum_{s=0}^{T_i - 1} \beta^s \left[V_L \left(1 - H_{it+s}\right) \left(K_{t+s}\right)\right]}{\sum_{s=0}^{T_i - 1} \beta^s \left[\frac{u_c(C_{t+s})}{P_{t+s}} K_{t+s}\right]}\right]$$
(16)

Equation (16) shows that the optimal wage is a constant mark-up (given  $by_{\varepsilon-1}^{\varepsilon}$ ) over the ratio of marginal utilities of leisure and marginal utility from consumption within the contract duration, from t to  $t + T_i - 1$  When  $T_i = 2$ , this equation reduces to the first order condition in Ascari (2000).

### 2.4 Government

There is a government that conducts monetary policy via lump-sum transfer, that is,

$$T_t = M_t - M_{t-1}$$

The demand for money is given by a simple Quantity Theory relation:

$$\frac{M_t}{P_t} = Y_t \tag{17}$$

which is a reference case in much of this literature (see for example Dotsey and King (2006), Mankiw and Reis (2002)).

We model the growth rate of money supply as an AR(1) process<sup>4</sup>:

$$\ln(\mu_t) = v.\ln(\mu_{t-1}) + \xi_t \tag{18}$$

where  $0 < v \leq 1$  and  $\xi_t$  is a white noise process with zero mean and a finite variance. In this paper we consider two values of  $\nu$ . The first case is  $\nu = 0$  so that the money supply is a random walk: As Huang and Liu (2001) argue, this enables us to focus on the role of the *GTE* in generating the output persistence in itself rather than with serially correlated money growth. However, the assumption of  $\nu = 0$  does not fit the data well, so where appropriate we may consider a second calibrated case with some serial correlation. Authors who have worked with this specification have considered slightly different calibrations for  $\nu$ . CKM estimate  $\nu$  to be 0.57, Mankiw and Reis (2002) use the value of  $\nu = 0.5$ , Christiano, Eichenbaum

<sup>&</sup>lt;sup>4</sup>This specification for monetary shocks is in line with that found in empirical studies (see for example Christiano, Eichenbaum and Evans (1999)).

and Evans (2005) estimate  $\nu = 0.5$ , Huang, Liu and Phaneuf (2004) assumes a value of  $\nu = 0.68$ . We will take the value of  $\nu = 0.5$  to reflect the serial correlation of monetary growth.

# 3 General Equilibrium

In this section, we characterize equilibrium of the economy. We first describe the equilibrium conditions for sector i and then the equilibrium conditions for the aggregate economy. To compute an equilibrium, we reduced the equilibrium conditions to four equations, including the household's first order condition for setting its contract wage, the pricing equation, the household's money demand equation, and an exogenous law of motion for the growth rate of money supply. We then log-linearize this equilibrium conditions around a steady state. The steady state which we choose is the zero-inflation steady state, which is a standard assumption in this literature. The linearized version of the equations are listed and discussed below. We follow the notational convention that lower-case symbols represents log-deviations of variables from the steady state.

The linearized wage decision equation (16) for sector *i* is given by

$$x_{it} = \frac{1}{\sum_{s=0}^{T_i - 1} \beta^s} \left[ \sum_{s=0}^{T_i - 1} \beta^s \left[ p_{t+s} + \gamma y_{t+s} \right] \right]$$
(19)

The coefficients on output in the wage setting equation in all sectors is given by

$$\gamma = \frac{\eta_{LL} + \eta_{cc}(\sigma + \theta(1 - \sigma))}{\sigma + \theta(1 - \sigma) + \theta\eta_{LL}}$$
(20)

Where  $\eta_{cc} = \frac{-U_{cc}C}{U_c}$  is the parameter governing risk aversion,  $\eta_{LL} = \frac{-V_{LL}H}{V_L}$  is the inverse of the labour elasticity,  $\theta$  is the elasticity of substitution of consumption goods.

Using equation (5) and aggregating for sector i, we get

$$p_{it} = w_{it} + \left(\frac{1-\sigma}{\sigma}\right) y_{it} \tag{21}$$

where

$$w_{it} = \sum_{j=1}^{N_i} \lambda_{ijt} w_{ijt}$$

Using equation (3) and aggregating for sector i yields

$$y_{it} = \theta(p_t - p_{it}) + y_t \tag{22}$$

Log-linerazing (17) yields the following

$$y_t = m_t - p_t \tag{23}$$

Finally, the linearized price index in the economy is simply a weighted average of the ongoing prices in all sectors and is given by

$$p_t = \sum_{i=1}^N \alpha_i p_{it} \tag{24}$$

# 3.1 The Calibration of Simple Taylor Economies with Wage and Price setting

In this section, we examine whether our model can account for a contract multiplier. Since the novel aspect of our paper is the incorporation of generalized wage setting, it is useful to compare our results with identical models that makes the standard assumption of a simple Taylor economy. However, before presenting our main results by using the chosen parameter values, it useful to discuss possible alternatives found in the literature and illustrate their implications in simple Taylor economies. The parameters of the model include the discount factor,  $\beta$ , the elasticity of substitution of labour,  $\eta_{LL}$ , the elasticity of substitution of consumption,  $\eta_{CC}$ , the elasticity of substitution of consumption goods,  $\theta$ , the monetary policy parameter,  $\xi_t$ .

The utility is additively separable and for simplicity, we assume  $\beta = 1$ . Empirical studies reveal that intertemporal labour supply elasticity,  $1/\eta_{LL}$ , is low and is at most 1. In particular, the survey by Pancavel (1986) suggests that  $\eta_{LL}$  is between 2.2 and infinity. Following the literature, we set  $\eta_{LL} =$ 4.5, which implies that intertemporal labour supply elasticity,  $1/\eta_{LL}$ , is 0.2. Following Ascari (2000), we set  $\theta = 6$ . Finally, we set  $\eta_{CC} = 1$  and  $\sigma = 1$ , which are all standard values used in the literature (see for example Huang and Liu (2002)). Finally, we assume that at time t there is 1% shock to the disturbance term corresponding to the money growth rate,  $\xi_t$ , so that  $\xi_t = 1$ and  $\xi_s = 0$  for all s > t. As we discussed earlier, we will take two values of serial correlation in monetary growth:  $\nu = 0$  (random walk) and the more reasonable empirical value  $\nu = 0.5$ .

### **3.2** The Choice of $\gamma$

The key parameter determining aggregate dynamics is  $\gamma$ . The magnitude of  $\gamma$ is important since it governs how responsive household-unions are to current and future changes in output (see equation 19). When there is an increase in aggregate demand, households face higher demand for their labour and therefore the marginal disutility of labour increases. With higher income they consume more and marginal utility of consumption falls. The combination of an increase in the marginal disutility of labour and the fall in the marginal utility of consumption leads household-unions to increase their wage. The coefficient  $\gamma$  determines how wages change in response to changes in current and future output. If  $\gamma$  is large, then wages respond a lot to changes in output which implies faster adjustments and a short-lived response of output. On the other hand, if  $\gamma$  is small, then unions are not sensitive to changes in current and future output. In response to an increase in aggregate demand, the wage would not change very much and hence wages are more rigid. In the limit, if  $\gamma = 0$ , there will be no relationship between output and wages, so that shocks are permanent. Hence the smaller  $\gamma$ , the more wages are rigid and hence the more persistent are output responses.

Estimating  $\gamma$  as an unconstrained parameter, Taylor found that for the US  $\gamma$  is between 0.05 and 0.1. However, in a general equilibrium framework  $\gamma$  is derived so as to conform to micro-foundations. CKM find that with reasonable parameter values,  $\gamma$  will be bigger than one in a staggered price setting, whilst with staggered wage setting Ascari finds the value of  $\gamma$  to be 0.2. Both CKM and Ascari argue that the microfounded value of  $\gamma$  is too high generate the observed persistence following a monetary shock, hence raising doubts over the Taylor model in this respect. In a general equilibrium setting,  $\gamma$  is determined by the fundamental parameters of the model according to (20). In particular, its magnitude depends on the parameter governing risk aversion,  $\eta_{cc}$ , the labour supply elasticity,  $\eta_{LL}^{-1}$  and the elasticity of substitution of consumption goods  $\theta$  (which determines the elasticity of firm demand and the markup from (3) and hence the markup (5)).

With staggered price setting, CKM find that with reasonable parameter

values, the value of  $\gamma$  is bigger than one: in particular with  $\sigma = 1$ 

$$\gamma^{CKM} = \eta_{LL} + \eta_{cc} = 1.2 > 1$$

However, for CKM the value of  $\gamma^{CKM}$  could reasonably be much higher<sup>5</sup>: for example with  $\eta_{_{LL}} = 4.5$  and  $\eta_{_{cc}} = 1$ ,  $\gamma^{CKM} = 5.5$ . Huang and Liu (2002) choose to set  $\eta_{_{LL}} = 2$ , so that  $\gamma^{CKM} = 2$ .

The value of  $\gamma$  with wage-setting is much smaller. In our model, as in Ascari, with  $\sigma = 1$ ,

$$\gamma^A = \frac{\eta_{\scriptscriptstyle LL} + \eta_{\scriptscriptstyle cc}}{1 + \theta \eta_{\scriptscriptstyle LL}} = \frac{\gamma^{\scriptscriptstyle CKM}}{1 + \theta \eta_{\scriptscriptstyle LL}}$$

Under our preferred calibration,  $\gamma^{CKM} = 5.5$  and  $1 + \theta \eta_{LL} = 27$ , so that  $\gamma^A = 0.2$ . The value of  $\gamma$  under wage setting could arguably be much smaller: some authors set  $\theta = 10$  and combined with a smaller  $\eta_{LL} = 2$ ,  $\gamma = 1/7 = 0.14$ . The lower value of  $\gamma$  is significant and means that in Ascari's wage setting model the aggregate price level changes more slowly than in CKM's price setting model. In fact, this finding is the main reason behind the conclusion of Huang and Liu (2002), who argue that staggered price setting by itself is incapable of generating sufficient persistence, whilst staggered wage setting has a greater potential.

However, Edge (2002) argues that price-setting is also consistent with lower values of  $\gamma$ . Huang and Liu rely heavily on the assumption that all firms use identical inputs. Edge shows that if one assumes a firm specific labour market, the staggered price setting model is as capable as the staggered wage model of generating persistence. In fact, she demonstrates that if CKM were to assume firm specific labour market, as in Ascari, then they would have obtained a similar value for  $\gamma^6$ . This is most easily seen by considering the flexible wage sector in our model. The log-linearized version of equation (5) is given by

$$p_{it} = mc_{it} = w_{it} \tag{25}$$

Log-linearizing equation (16) and noting that  $T_i = 1$ , one obtains

$$w_{it} - p_t = \eta_{LL} h_{it} + \eta_{cc} c_t \tag{26}$$

<sup>&</sup>lt;sup>5</sup>Since CKM were aiming to show that the staggered price model did not generate enough peristence, they chose a value of  $\gamma^{CKM}$  which was low to make the model as persistent as it could reasonably be.

 $<sup>^{6}</sup>$ See also Ascari (2003) and Woodford (2003).

Combining the two equations, along with the log-linearized versions of (7), the relation  $Y_t = C_t$  and noting that  $\sigma = 1$ , price level in sector *i* can be expressed as

$$p_{it} = p_t + \gamma^A y_t$$

If we instead assume common labour market, then all firms in the economy face the same marginal cost and  $\gamma$  is rather different. Once again using the log-linearized versions of (7) and the relation  $Y_t = C_t$  and nothing that  $P_t = W_t$  and  $W_{it}/W_t = 1$ , the optimal price setting rule is

$$p_{it} = p_t + \gamma^{CKM} y_t$$

Based on this finding, it can be concluded that wage and price setting have very similar implications, if one assumes firm-specific labour markets.

However, whilst Ascari (2000) shows that the output is more persistent in a model with a firm specific labour market, he also shows it is still not persistent enough to generate the observed persistence in output.

### Figure 1 here.

We can illustrate how the magnitude of  $\gamma$  can affect the result by comparing the impulse responses using the values of  $\gamma$  from CKM, Ascari (2000) and Taylor (1980). We assume a simple Taylor economy with T = 2 (wages last 6 months). All other decisions are made quarterly. We display the impulseresponse functions for output after a one percent monetary shock. As we can see from Figure 1, in response to the one percent monetary shock, output displays similar patterns in the case of  $\gamma^{CKM} = 1.22$  and  $\gamma^A = 0.20$ . For both cases, output increases when the shock hits and quickly returns to its steady state level. For the case of  $\gamma = 1.22$ , output returns to steady state level when both unions have had the chance to reset wages, i.e. two quarters. Output is certainly more persistent with  $\gamma = 0.20$ , but not significantly. Finally, the impulse response of output in the case with  $\gamma = 0.05$  originally used by Taylor (1980), which yields a level of persistence more in line with the evidence, but not the microfoundations.

## 4 Persistence in a GTE

The existing literature has tended to focus on the value  $\gamma$  in generating persistence. We want to explore another dimension: for a given  $\gamma$ , we allow for different contract lengths in the *GTE* framework we have developed. Having more than one type of contract length thus is necessary if the model is to generate output persistence beyond the initial contract period. In what follows, we show that including longer term contracts can significantly increase persistence. Of course, this is in a sense obvious: longer contracts lead to more persistence, and we can achieve any level of persistence if contracts are long enough (so long as  $\gamma > 0$ ). However, we want to show that even a small proportion of long-term contracts can lead to a significant increase. Throughout this section, we will take the value of  $\gamma = 0.2$  and explore how persistence changes when we allow for a range of contract lengths. We do this in three stages: first we simply illustrate our case with a simple two sector example. Second, we use the Bils-Klenow dataset on price-data to calibrated model of the US economy allowing for contract lengths from 1-20 quarters. In the next section we consider the Calvo contract process with the corresponding distribution of contract lengths from 1 to infinity.

### 4.1 Two-sector GTEs.

First, let us consider the simple case of a two sector uniform GTE,  $\{\mathbf{T}, \boldsymbol{\alpha}\} = \{(2, 8), (0.9, 0.1)\}$ : in sector 1 there are two period contracts, in sector 2 there are 8 period contracts: the short contract sectors produce 90% of the economies output, the long-contracts 10%. The average contract length in the whole economy (weighted by  $\alpha_i$ ) is 2.6 quarters.

### Figure 2 here

In Figure 2 we show both the simple Taylor economy with only 2-period contracts alongside the GTE with 10% share of 8-period contracts. We report the impulse response of aggregate output after a one-percent shock in money supply as in Figure 1<sup>7</sup>. As can be seen from the Figure 2, the GTE and simple Taylor economy have dramatically different implications for persistence. In the simple Taylor economy with 2-quarter contracts, changes in money supply have a potentially large but short-lived effect on output. In the GTE, the presence of long-term contracts means that not only does aggregate output rise following a increase in the money supply, but it is considerably more persistent.

<sup>&</sup>lt;sup>7</sup>We use Dynare to compute the impulse response functions. See Juillard (1996).

What is the intuition behind this finding? We believe that the presence of the longer term contracts influences the wage-setting behaviour of the shortterm contracts. This can be seen as a sort of "strategic complementarity". A monetary expansion means that the new steady state price is higher. When setting wages, unions trade off the current price level and the future. The fact that the long-contracts will adjust sluggishly means that the shorter contracts will also react more sluggishly, since their wage setting is influenced by the general price level which includes the prices of the more sluggish sectors. There is a spillover effect from the sluggish long-contract sectors to the shortcontract sectors via the price level, a mechanism identified previously in Dixon (1994).

### Figure 3a and b here.

We can perhaps best illustrate the contrast in terms of mean-equivalent GTEs. In Figure 3a we have the output response compared in two GTEs with a mean contract length of 2: one is a simple Taylor economy, the other consists of mainly flexible wages and 1/7 are 8 period contracts. The presence of the perfectly flexible one period contracts leads to a dampened impact relative to the 2-period Taylor. However, it is clear that although the economy consists mainly of flexible wages, the output dies away slowly and after the second quarter output is larger in the mixed economy. This is because the 8 period contracts are holding back the general price level and hence influencing the wage-setting of the flexible sector. In Figure 3b we have a simple 3 Taylor economy with a mixed one of 2 and 8 period contracts. Again the impact is less in the mixed economy but soon becomes more persistent.

## 4.2 An Application to U.S. Data.

In the previous section we have considered some hypothetical two sector GTEs and compared them to the simple Taylor model. In this section we consider an empirical distribution of contract lengths derived from the Bils and Klenow (2004) data set based on U.S. Consumer Price Index microdata. Although this is for *price* data, we use it as an illustrative data set. We will then examine the impulse response function (under a plausible money supply process) and compare it to the actual behaviour of US output taken from CKM.

The data is derived from the US Consumer Price Index data collected by the *Bureau of Labor statistics*. The period covered is 1995-7, and the 350 categories account for 69% of the CPI. The data set gives the average proportion of prices changing per month for each category. We assume that this is generated by a simple Calvo process within each sector. We then generate the distribution of durations within each sector, and aggregate across sectors to obtain the distribution in the economy<sup>8</sup>. Figure 4 plots the distribution in terms of quarters.

#### Figure 4 here.

The mean contract length is 4.4 quarters. Perhaps the most striking aspect of this distribution is the high share of short-term contracts. The share of 1 and 2 period contracts are about 50% (see Dixon and Kara (2006b) for a more detailed discussion).

CKM estimated the dynamic response of output to a policy shock by fitting an AR(2) process to quadratically detrended log of real GDP:

$$y_t = 1.30y_{t-1} - 0.38y_{t-2} + \xi_t$$

### Figure 5 here.

The impulse response of output to a unit shock in  $\xi_t$  is plotted in Figure 5<sup>9</sup>. As the figure shows, the estimated output response is persistent: the half life of output is 10 quarters. Another important feature of this response is its hump shape: the response peaks three quarters after the shock The pattern is consistent with other empirical studies such as Christiano et al. (2005).

#### Figure 6 here.

Figure 6 reports the impulse response functions for output in BK-GTEand the simple Taylor with T = 4, with the CKM's estimated output response from Figure 5 superimposed. As discussed earlier, we assume that v, serial correlation in monetary growth, to be 0.5. For comparison purposes, the

<sup>&</sup>lt;sup>8</sup>Note, the mean is much longer than stated by Bils and Klenow themselves. This is for two reasons. First, our mean is the distribution of contract lengths across firms, whereas BK are inferring the average length of contracts; see Dixon (2006) for a full discussion. Second, they are using continuous time: the average allows for firms to reset prices more than once per discrete period.

<sup>&</sup>lt;sup>9</sup>Note that CKM find little evidence for serial correlation of the residuals.

responses are normalized in the sense that the impact is set at 1. As the figure indicates incorporating empirically relevant contract structure into a dynamic general equilibrium model has a significant effect on dynamic response of output. We can see that the BK-GTE predictions and the CKM's estimated output response have almost identical characteristics. More specifically, the BK - GTE generates a hump-shaped persistent output response and the half life is about 10 quarters. In this sense, the GTE framework with the BK distribution is able to explain the observed patterns of output.

The figure also shows the difference between the GTE framework and the simple Taylor economy. Although both settings have similar mean contract lengths, the simple Taylor economy generates much less persistence. This can be most easily seen by comparing areas under the impulse response functions. The area in the BK - GTE is twice the area in the simple Taylor.

For robustness, we also examine the implications of the BK - GTE in terms of two measures of persistence proposed in the literature. One is the "contract multiplier" proposed by CKM, which is defined as the ratio of the half life of output to one-half length of exogenous stickiness. The other one is the "mean lag" measure suggested by Dotsey and King (2006). Mean lag is

defined as the ratio of  $\left(\sum_{j=0}^{\infty} j * \kappa_j\right) / \sum_{j=0}^{\infty} \kappa_j$ , where  $\kappa_j$  is the impulse response coefficient for output at lag  $j^{10}$ .

As the table shows, both the contract multiplier and "the mean lag" measures increase significantly in the case of BK - GTE compared with the simple Taylor economy. In fact, both measures indicate that the BK - GTE generates twice as much persistence than the simple Taylor economy. The table further indicates that the mean lag of the BK - GTE is very close to the mean lag of the CKM's estimated response.

|                     | BK-GTE | Taylor; $T = 4$ | CKM IR |
|---------------------|--------|-----------------|--------|
| Contract Multiplier | 4.4    | 2.6             |        |
| Mean Lag            | 5.7    | 2.7             | 6.6    |

 Table 1: Persistence Measures

<sup>&</sup>lt;sup>10</sup>We trancate the sum in these expensions at 35 quarters. Adding more terms does not significantly affect the results.

As noted by CKM, the contract multiplier does not vary a lot with the contract length in the simple Taylor. We calculate the contract multiplier in the simple Taylor setting in our model for contract lengths T = 2, 6, 8: the resulting multipliers are 2.50, 2.55, 2.48, respectively. The only way to match the observed pattern of output with a simple Taylor contract is to assume implausibly long contract lengths or implausible parameter values. In order to get the same degree of persistence in simple Taylor, contract length of 8 quarters is required. Alternatively, if we keep  $T = 4, \sigma = \eta_{CC} = 1$  and  $\eta_{LL} = 4.5$ , then a value of  $\theta = 27$  is required to obtain the same degree of persistence in the Taylor model, which is implausibly high. As discussed earlier, a reasonable range of  $\theta$  is from 6 to  $10^{11}$ . It is interesting to note that the value of  $\theta = 27$  implies that  $\gamma = 0.05$ , which takes us back to the value of  $\gamma$  put forwarded by Taylor (1980).

# 5 Comparison with a Calvo Economy

It has long been noted that Calvo contracts appear to be far more persistent than Taylor contracts. In this section, we will show that if we focus on the structure of contracts (as opposed to the wage-setting rule), the Calvo economy is a special case of the *GTE*. Two main features of the Calvo setup stand out as different form the standard Taylor setup. First its "stochastic" nature: at the firm or household level, the length of the wage contract is random. Second, that the model is described in terms of the "age" of contracts (which includes uncompleted durations) and the hazard rate (the reset probability  $\omega$ ). On the first issue, the stochastic nature of the Calvo model at the firm level does affect the wage setting decision. However, apart form the wage setting decision we can describe the Calvo process in deterministic terms at the aggregate level because the firm level randomness washes out. At the aggregate level, the precise identity of individual firms does not matter: what matters is population demographics in terms of proportions of firms setting contracts of particular lengths at particular times. Because there is a continuum of firms, the behavior of contracts at the aggregate level can be seen as a purely deterministic process.

The second difference is one of perspective. As shown in Dixon (2006), any steady state distribution of contracts can be looked at equivalently in

<sup>&</sup>lt;sup>11</sup>We calculate T = 8 and  $\theta = 27$  by matching the autocorrelation fuctions of output in the simple Taylor to that of the esimated output response in CKM.

terms of the age distribution/hazard rate, or as the distribution of completed contract lengths across firms. In Dixon and Kara (2006a) we apply this idea to the comparison of Calvo and simple Taylor contracts.

With a reset probability the cross-sectional distribution is represented by the vector of proportions  $\alpha_i^s$  of firms surviving at least *i* periods:

$$\alpha_i^s = \omega \left(1 - \omega\right)^{i-1} : i = 1..\infty \tag{27}$$

with mean  $\bar{s} = \omega^{-1}$ . In demographic terms, *i* is the *age* of the contract:  $\alpha_i^s$  is the proportion of the population of age *s*;  $\bar{s}$  is the average age of the population. The corresponding distribution of completed contract lengths is given by<sup>12</sup>:

$$\alpha_{i} = \omega^{2} i \left( 1 - \omega \right)^{i-1} : i = 1..\infty$$
(28)

with mean  $\overline{T} = \frac{2-\omega}{\omega}$ . In demographic terms,  $\alpha_i$  gives the distribution of ages at death (for example as reported by the registrar of deaths) for the same cross-section:  $\alpha_i$  being the proportion of the steady state population who will live to die at age *i*.

Assuming that we are in steady state (which is implicit in the use of the Calvo model), we can assume that there are in fact ex ante *fixed* contract lengths. We can classify household-unions by the duration of their "contract". The fact that the contract length is fixed is perfectly compatible with the notion of a reset probability if we assume that the wage-setter does not know the contract length. We can think of the wage-setter having a probability distribution over contract lengths given by  $\alpha_i^s$  in (27): Nature chooses the contract length, but the wage-setters do not know this when they have to set the wage (when the contract begins)<sup>13</sup>.

Having redefined the Calvo economy in terms of completed contract lengths, we can now define the GTE with exactly the same distribution of completed contract lengths:  $GTE(\alpha)$  where  $\alpha_i$  are given by (28).

Let us just check that this will yield a contract structure equivalent to the Calvo process. Since the wage setting process is uniform, we can consider the representative period. In the sector i, a proportion  $\alpha_i/i$  contracts come to an end. Hence, using (28) and summing across all sectors the total measure

<sup>&</sup>lt;sup>12</sup>At any time t, of those aged i a proportion  $\omega$  in that cohort will terminate. From (27) those who complete their contract at time t aged i are  $\omega^2 (1-\omega)^{i-1}$ . There are a total of i cohorts aged  $j \leq i$  at time t, which gives (28).

<sup>&</sup>lt;sup>13</sup>In game-theory terms wage-setting is done under incomplete information.

of all contracts in the economy coming to an end in any period is  $\omega$ , since:

$$\sum_{i=1}^{\infty} \frac{\alpha_i}{i} = \sum_{i=1}^{\infty} \omega^2 \left(1 - \omega\right)^{i-1} = \omega$$
(29)

Figure 7 here.

In Figure 7 we have the distribution of completed contract lengths in the Calvo model with  $\omega = 0.25$ , the distribution of contract ages (i.e. steady-state durations, complete and incomplete). As can be seen, the modal completed contract lengths are 3 and 4 quarters which have exactly the same proportions (just over 10%), and the distribution beyond that tails off, with the mean being 7 quarters.

### 5.1 Wage-setting in the Calvo-GTE

We have defined the Calvo-GTE in terms of the structure of completed contract lengths. The only difference between the Calvo economy and the Calvo-GTE is in the wage-setting decision (exactly the same arguments and observations apply to price-setting). In the Calvo economy, the wage-setter is uncertain of the contract length: the wage-setting decision must be made "ex ante", that is before the firm knows which length nature has chosen. This yields the standard Calvo wage-setting decision. Once the wage is set, the firm finds out its contract length in due course<sup>14</sup>. By contrast, in the Calvo-GTE, the wage-setters know which sector they belong to when they set the wage. Hence, wages in each sector of the Calvo-GTE will be different. Taking the simple case of  $\beta = 1$ , from (19) the reset wage in sector *i* is then the average "optimal" price over the following *i* periods:

$$x_{it} = \frac{1}{i} \sum_{s=0}^{i-1} \left( p_{t+s} + \gamma y_{t+s} \right)$$
(30)

Thus, in sector i, the wage-setter does not need to look forward more than i periods.

If we take the mean reset-wage in the Calvo-GTE we need to measure the mean *conditional on the wage being reset*, across the subset of firms who

<sup>&</sup>lt;sup>14</sup>It does not matter when: either straight after the pricing decision or at the last moment when it gets the Calvo phone call that it is time to reset the wage.

are resetting price:

$$\bar{x}_t = \frac{1}{\omega} \sum_{i=1}^{\infty} \frac{\alpha_i}{i} x_{it} = \sum_{i=1}^{\infty} \omega \left(1 - \omega\right)^{i-1} x_{it}$$
(31)

This is different from the unconditional mean, using the sectoral weights:

$$\hat{x}_t = \sum_{i=1}^{\infty} \alpha_i x_{it} \tag{32}$$

Within the sector with *i* period contracts only  $i^{-1}$  reset their prices each period. Hence if we weight each sector using the proportions resetting using (29), then the less frequent price setters are under-represented relative to their share in the total population. A union that resets every period (i = 1) is counted every period, whilst a union that resets every 10 periods is only counted once every 10 periods.

There are thus two main differences between the Calvo and the Calvo-GTE wage-setting rules. First, in the Calvo-GTE there is a distribution of sector specific reset wages  $x_{it}$  in each period. Hence, in addition to the distribution of prices across cohorts (defined by when they last reset prices) as in the Calvo model, the GTE has a distribution across sectors within the cohort.

Second, the Calvo-GTE puts more weight on the immediate future than the Calvo rule. If we expand equations (31) and (32)using (30), we can write the conditional and unconditional mean reset wages in terms of current and future outputs and prices  $(p_{t+s} + \gamma y_{t+s})$ .

#### **Proposition 1** Let $\beta = 1$ .

(a) The unconditional mean reset wage at time t in the Calvo-GTE is

- -

$$\hat{x}_{t} = \sum_{s=0}^{\infty} C_{s} \left( p_{t+s} + \gamma y_{t+s} \right)$$
$$C_{s} = \omega \left( 1 - \omega \right)^{s}$$

(b) The conditional mean reset price is

$$\bar{x}_t = \sum_{s=0}^{\infty} b_s \left( p_{t+s} + \gamma y_{t+s} \right)$$
$$b_s = \omega \sum_{T=s+1}^{\infty} \frac{(1-\omega)^{T-1}}{T}$$

Clearly, the unconditional mean reset wage in the Calvo-GTE is equal to the standard Calvo reset wage, with familiar Calvo weights  $C_s$ . However, whilst this is a useful reference point, it is not the correct comparison, because it is weighting by sector size, including those who do not reset wages. The conditional mean gives the *average reset wage across those who are resetting* the wage. The weights of the conditional mean reset reset wage are  $b_s$  and can be expressed in terms of the corresponding Calvo weights  $C_s$ :

$$b_s = C_s - \frac{s}{s+1}C_s + \sum_{i=s+1}^{\infty} \frac{C_i}{i}$$

If we look at the Calvo-GTE weights  $b_s$ , they are a simple transformation of the Calvo weights. Calvo weights on future prices are "passed back" along the line. The general term  $b_s$  has three elements: the Calvo weight  $C_s$ ; the weight it passes back equally to all the previous s weights  $\frac{s}{s+1}C_s$ ; and the weight it receives from the subsequent Calvo weights  $\sum_{i=s+1}^{\infty} \frac{C_i}{i}$ . Thus we can see that the Calvo-GTE puts a much bigger weight on the more immediate future than the Calvo rule. This is intuitive: in every sector  $i \geq 1$  there is a proportion  $i^{-1}$  weight on the current period t: in every sector  $i \geq 2$  there is a weight of  $i^{-1}$  period t + 1 and so on.

We can define the degree of *forward-lookingness* as the weighted mean of future dates in the log-linearised reset wage equation. In Calvo this is simply<sup>15</sup>  $\omega^{-1}$ :

$$FL^{C} = \sum_{s=0}^{\infty} C_{s} \left(s+1\right) = \frac{1}{\omega}$$

In the Calvo-GTE this is derived from (31). Note that the wage set by the sector *i* cohort  $x_{it}$  is the mean over periods 1...*i*,so that the mean forward lookingness in sector *i* is (i + 1)/2. Hence, from (31) the mean forward-lookingness in the Calvo-GTE is

$$FL^{CGTE} = \sum_{i=1}^{\infty} C_{i-1} \left(\frac{i+1}{2}\right) = \frac{1+\omega}{2\omega}$$

Note that since  $\omega < 1$ ,  $FL^C > FL^{CGTE}$ . Hence in the Calvo-GTE, the forward-lookingness of wage-resetters as a whole is less than in the equivalent Calvo, with the ratio  $FL^C/FL^{CGTE} = \frac{2}{1+\omega}$ . With  $\omega = 0.25$ , the Calvo reset

<sup>&</sup>lt;sup>15</sup>We follow the convention of saying that the present (s = 0) is period 1 and so on.

price looks forward on average 4 periods, whilst the Calvo-GTE, the average reset wage looks forward 2.5 quarters. If we choose a simple Taylor process with contract lengths  $T = 2\omega^{-1} - 1$ , the mean forward-lookingness of the cohort that resets its wage is

$$FL^T = \frac{T+1}{2} = \frac{1}{\omega}$$

Hence  $FL^T = FL^C$ . This reinforces the insight that the reason that wagesetting in the Calvo-GTE is more myopic than both the simple Taylor and Calvo economies with the same mean contract length, is that in the Calvo-GTE the longer contracts are "under-represented" in the wage-resetters because they reset wages less frequently.

### 5.2 Persistence in the Calvo and Calvo-GTE compared

We now compare the Calvo-GTE and the standard Calvo economy in terms of the impulse-response functions. In theory, the Calvo-GTE and the Calvo economy are exactly the same in terms of contract structure. However, for computational purposes whilst the Calvo economy effectively has an infinite lag structure (via the Koyck transform), the Calvo-GTE has to be truncated. Hence we also introduce a Calvo-Calvo-GTE: that is the GTE with the same contract structure and wage-setting rule as the Calvo model, but truncated as in the Calvo-GTE. For the simulations, we truncated the distribution of contract lengths to 20 quarters T = 1, ...20. with the 20 period contracts absorbing all of the weight from the longer contracts. When we apply the standard Calvo pricing rule to this truncated distribution, it yields a perceptible but negligible difference; hence all of the visually apparent differences between the Calvo-GTE and the standard Calvo model are due almost entirely to the difference in wage-setting behaviour.

#### Fig 8 here

In Figure 8 we compare the impulse response for the Calvo-GTE which has the same distribution of completed contract lengths as the Calvo distribution, with the standard Calvo economy for  $\omega = 0.25$ . We find that Calvo-GTE has very similar persistence to the Calvo economy. The effect is as little larger for 6 quarters and a little less subsequently, reflecting the less forward looking pricing behaviour. We also show the standard Taylor economy with the same mean contract length  $\overline{T} = 7$ . Although the effect is a greater for the first 5 quarters, the effect dies down and is significantly less thereafter. This reflects the fact that although the mean contract lengths are the same, the longer contracts in the Calvo and Calvo-*GTE* generate the extra persistence.

#### Figure 9 here.

To understand the difference between the Calvo and Calvo-GTE we can focus on wage-setting behavior as depicted in Figure 9. In Fig 9a, we depict the price level in the two cases. We see that the price level rises a bit more in the Calvo case early on (for the first 6 quarters) and a bit less later on (hence mirroring the comparison in terms of output we saw in Figure 8). In Fig 9b we depict the trajectory of the reset wage in both cases: again the Calvo reset wage is a little higher early on (for the first 4 quarters) and a The effect of the permanent increase in the money little lower later on. supply is to lead to an upward trajectory in prices. In the Calvo economy the wage-resetters are more forward looking and so raise wages more in the initial period in anticipation of the future price rises. This leads to a slightly smaller increase in output in the first few periods. As the new steady state is approached, the Calvo resetters slow down the increase in wages, whilst the more myopic Calvo-GTE wage resetters keep up the momentum of wage increases, so that the output becomes a little larger in the Calvo case

### 5.3 Multiple Calvo Economy and GTE

In the previous section, we have interpreted the BK data set in a particular way. We assume (as do Bils and Klenow) that in each sector there is a sectorspecific reset probability. We then generate the distribution of completed contract lengths that corresponds to the sector specific reset propability and then aggregate over sectors using the sectoral weights. However, in order to generate the impulse response in Figure 6, we worked on "Taylor" basis, that firms or unions know the length of their contract when they set the nominal wage rate. However, an alternative is to assume that the "Calvo" story goes through: in each sector, when firms set wages, they do not know the length of the contract. This is the "Multiple Calvo" model as defined in Dixon (2006). Carvalho (2006) has developed the multiple Calvo approach using the Bils-Klenow data set. So, we can compare exactly the same distribution of contract lengths, but under two different assumptions about the pricing behaviour: Taylor and (multiple) Calvo. The impulse responses are shown on Figure 10.

Figure 10

As we can see from the figure, the IR are very similar for the two cases: output is a little higher earlier on in the BK - GTE case, but both peak together, and after 18 months the output is a little larger in the multiple Calvo case. This is what we would expect form the fact that the GTE is more myopic than the MC with the same distribution of contract lengths: on impact wages and prices will increase on average a little more in the MCcase, but the BK - GTE will catch up and overtake later on.

# 6 Conclusions

In this paper we have developed a general framework, the *GTE* which unifies the previously disparate approaches of modelling dynamic price and wage setting: Calvo and Taylor. The approach is a generalization of the simple Taylor model to take into account the presence of a range of different contract lengths. We use this approach to focus on the effect of the presence longer term contracts on the persistence of impulse-response functions generated by a monetary shock.

- A small proportion of long-term contracts can generate a significant increase in persistence.
- We apply the idea to US data using the Bils-Klenow dataset to generate the distribution of contract lengths. We find that the impulse response for this distribution is very similar to the impulse response that CKM estimated from US data.
- In general, if we want to model an economy with many different contract lengths using a simple Taylor economy, we should choose a contract length which is greater than the average. This is because the presence of contracts with longer duration leads to more persistence despite having a similar mean. In the case of the Bils-Klenow distribution (which has a mean of just over 4 quarters), we would need a simple Taylor model to have 8 quarters to generate the equivalent persistence.

- As shown in Dixon and Kara (2006a), the average length of contracts in the Calvo model has been seriously underestimated, because the age and life-time of contracts have been confused. If modelers want an average contract length of 4-quarters, they should choose a reset probability of  $\omega = 0.4$ . The often used value of 0.25 generates an average contract length of 7 quarters.
- When we compare the standard Calvo model with the corresponding Calvo-*GTE* (which has exactly the same distribution of contract lengths), we find that although the wage-setting behavior differs, the persistence of the two is very similar.
- The main difference in wage or price setting behaviour between Calvo and Calvo-*GTE* is in the forward-lookingness of the wage or price setting decision. In the *GTE* setting, longer contracts reset wages less frequently and so are under-represented amongst wage-resetters relative to their share in the economy. This means reset wages are on average less forward looking than in either the Calvo or simple Taylor economy with same mean contract life.

# References

- Aoki, K.: 2001, Optimal monetary policy response to relative price changes, Journal of Monetary Economics 48(1), 55–80.
- Ascari, G.: 2000, Optimising agents, staggered wages and the persistence in the real effects of money shocks, *The Economic Journal* **110**, 664–686.
- Ascari, G.: 2003, Price/wage staggering and persistence: A unifying framework, Journal of Economic Surveys 17(4), 511–540.
- Benigno, P.: 2004, Optimal monetary policy in a currency area, Journal of International Economics 63(2), 293–320.
- Bils, M. and Klenow, P.: 2004, Some evidence on the importance of sticky prices, *Journal of Political Economy* 112(5), 947–985.
- Calvo, G. A.: 1983, Staggered prices in a utility-maximizing framework, Journal of Monetary Economics 12(3), 383–398.
- Carlstrom, C., Fuerst, T. and Ghironi, F.: 2006, Does it matter (for equilibrium determinacy) what price index the central bank targets?, *Journal of Economic Theory* **128**, 214–231.
- Carlstrom, C., Fuerst, T., Ghironi, F. and Hernandez, K.: 2006, Relative price dynamics and the aggregate economy, *Boston College, mimeo*.
- Carvalho, C.: 2006, Heterogeneity in price stickiness and the real effects of monetary shocks, *Frontiers of Macroeconomics* (forthcoming).
- Chari, V. V., Kehoe, P. J. and McGrattan, E. R.: 2000, Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem?, *Econometrica* 68(5), 1151–79.
- Christiano, L. J., Eichenbaum, M. and Evans, C. L.: 1999, Monetary Policy Shocks: what have we learned and to what end? in:J.B. Taylor and M. Woodford, eds, Handbook of Macroeconomics, Vol. 1, North-Holland, Amsterdam.
- Christiano, L. J., Eichenbaum, M. and Evans, C. L.: 2005, Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of Political Economy* 113(1), 1–45.

- Coenen, G., Christoffel, K. and Levin, A. T.: 2006, Identifying the influences of nominal and real rigidities in aggregate price-setting behavior, *Journal* of Monetary Economics (forthcoming).
- Dixon, H.: 1994, Macroeconomic price and quantity responses with heterogeneous product markets, Oxford Economic Papers 46, 385–402.
- Dixon, H.: 2006, The distribution of contract durations across firms: a unified framework for understanding and comparing dynamic wage and price setting models, *Working paper series 676, European Central Bank*.
- Dixon, H. and Kara, E.: 2006a, How to compare Taylor and Calvo contracts: a comment on Michael Kiley, *Journal of Money, Credit and Banking* **38**(2).
- Dixon, H. and Kara, E.: 2006b, Understanding inflation persistence: a comparison of different models, *Working paper series 672, European Central Bank*.
- Dotsey, M. and King, R. G.: 2006, Pricing, production, and persistence, Journal of the European Economic Association 4(5), 893–928.
- Edge, R.: 2002, The equivalence of wage and price staggering in monetary business cycle models, *Review of Economic Dynamics* 5, 559–585.
- Erceg, C. J.: 1997, Nominal wage rigidities and the propagation of monetary disturbances, *International Discussion Papers No. 590, Federal Reserve Board of Governors*.
- Erceg, C. J. and Levin, A. T.: 2005, Optimal monetary policy with durable consumption goods, *Journal of Monetary Economics* (forthcoming).
- Huang, K. X. D., Liu, Z. and Phaneuf, L.: 2004, Why does the cyclical behavior of real wages change over time?, *American Economic Review* 94(4), 836–856.
- Huang, K. X. and Liu, Z.: 2001, Production chains and general equilibrium aggregate dynamics, *Journal of Monetary Economics* **48**(2), 437–462.
- Huang, K. X. and Liu, Z.: 2002, Staggered price-setting, staggered wagesetting, and business cycle persistence, *Journal of Monetary Economics* 49(2), 405–433.

- Juillard, M.: 1996, Dynare: A program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm, CEPREMAP, Working Paper No. 9602. (http://www.cepremap.cnrs.fr/dynare).
- Kiley, M. T.: 2002, Partial adjustment and staggered price setting, *Journal of Money, Credit, and Banking* 34(2), 283–98.
- Mankiw, N. G. and Reis, R.: 2002, Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve, *The Quarterly Journal of Economics* 117(4), 1295–1328.
- Mankiw, N. G. and Reis, R.: 2003, What measure of inflation should a central bank target?, Journal of the European Economic Association 1(5), 1058–1086.
- Pancavel, J.: 1986, Labor supply of men: a survey in: Ashenfelter, O. and Layard P.R.G, eds., Handbook of Labor Economics, Vol. 1, North-Holland, Amsterdam.
- Taylor, J. B.: 1980, Staggered wage and price setting in macroeconomics, Journal of Political Economy 88(1), 1–23.
- Taylor, J. B.: 1993, Macroeconomic Policy in a World Economy: From Econometric Design to Practical Operation, W W Norton.
- Taylor, J. B.: 1999, Staggered Wage and Price Setting in Macroeconomics in: J.B. Taylor and M. Woodford, eds, Handbook of Macroeconomics, Vol. 1, North-Holland, Amsterdam.
- Wolman, A. L.: 1999, Sticky prices, marginal cost, and the behavior of inflation, Economic Quarterly, Federal Reserve Bank of Richmond 85(4), 29– 48.
- Woodford, M.: 2003, Interest and prices: Foundations of a theory of monetary policy, *Princeton University Press, Princeton, NJ*.

# 7 Appendix

Proof of Proposition 1(a)

$$\begin{aligned} \hat{x}_{t} &= \sum_{T=1}^{\infty} \omega^{2} T \left( 1 - \omega \right)^{T-1} \left[ \sum_{s=0}^{T} \frac{(p_{t+s} + \gamma y_{t+s})}{T} \right] \\ &= \sum_{T=1}^{\infty} \omega^{2} \left( 1 - \omega \right)^{T-1} \left[ \sum_{s=0}^{T-1} (p_{t+s} + \gamma y_{t+s}) \right] \\ &= \omega^{2} \left( p_{t} + \gamma y_{t} \right) \sum_{T=1}^{\infty} (1 - \omega)^{T-1} + \omega^{2} \left( p_{t+1} + \gamma y_{t+1} \right) \sum_{T=2}^{\infty} (1 - \omega)^{T-1} \\ &= \omega \left( p_{t} + \gamma y_{t} \right) + \omega^{2} (1 - \omega) \left( p_{t+1} + \gamma y_{t+1} \right) \sum_{T=1}^{\infty} (1 - \omega)^{T-1} \\ &= \sum_{s=1}^{\infty} \omega \left( 1 - \omega \right)^{s-1} \left( p_{t+s} + \gamma y_{t+s} \right) \end{aligned}$$

Proof of Proposition 1(b)

$$\bar{x}_{t} = \sum_{T=1}^{\infty} \omega (1-\omega)^{T-1} \left[ \sum_{s=0}^{T-1} \frac{(p_{t+s} + \gamma y_{t+s})}{T} \right]$$

$$= \omega (p_{t} + \gamma y_{t}) + \omega (1-\omega) \left[ \sum_{s=0}^{1} \frac{(p_{t+s} + \gamma y_{t+s})}{2} \right]$$

$$+ \omega (1-\omega)^{2} \left[ \sum_{s=0}^{2} \frac{(p_{t+s} + \gamma y_{t+s})}{3} \right] + \dots$$

$$= (p_{t} + \gamma y_{t}) \omega \sum_{T=1}^{\infty} \frac{(1-\omega)^{T-1}}{T} + (p_{t+1} + \gamma y_{t+1}) \omega \sum_{T=2}^{\infty} \frac{(1-\omega)^{T-1}}{T} + (p_{t+1} + \gamma y_{t+1}) \omega \sum_{T=2}^{\infty} \frac{(1-\omega)^{T-1}}{T} + (p_{t+1} + \gamma y_{t+1}) \omega \sum_{T=1}^{\infty} \frac{(1-\omega)^{T-1}}{T} + (p_{t+1} + \gamma y$$



Figure 1: Output response for alternative  $\gamma's$ 



Figure 2: More persistence with a few longer contracts



Figure 3: A mean preserving spread increases persistence



Figure 4: The  $U\!S$  distribution of price contract lengths derived from the Bils-Klenow data set



Figure 5: CKM's estimated output response



Figure 6: Output response in the BK-GTE and CKM's estimated output response



Figure 7: The distribution of contract lengths( $\alpha$ )



Figure 8: Output responses of the Calvo Economy, the corresponding GTE and the simple Taylor economy with same mean contract length



Figure 9: Responses of price level and average reset wage for the Calvo Economy and the Calvo-GTE with  $\omega = 0.25$ 



Figure 10: Responses of output in the BK - MC and in the BK - GTE