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Forecasting Short-Run Inflation Volatility using Futures Prices: An Empirical Analysis from a Value at Risk Perspective*

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Abstract: In this research paper ARCH-type models are applied in order to estimate the Value-at-Risk (VaR) of an inflation-index futures portfolio for several time-horizons. The empirical analysis is carried out for Mexican inflation-indexed futures traded at the Mexican Derivatives Exchange (MEXDER). To analyze the VaR with time horizons of more than one trading day bootstrapping simulations were applied. The results show that these models are relatively accurate for time horizons of one trading day. However, the volatility persistence of ARCH-type models is reflected with relatively high VaR estimates for longer time horizons. These results have implications for short-term inflation forecasts. By estimating confidence intervals in the VaR, it is possible to have certain confidence about the future range of inflation (or extreme inflation values) for a specified time horizon.

Keywords: Bootstrapping, inflation, inflation-indexed futures, Mexico, Value at Risk, volatility persistence.

JEL Classification: C15, C22, C53, E31, E37.

Resumen: En el presente documento de investigación modelos tipo-ARCH se utilizan para estimar el Valor-en-Riesgo (VaR) de un portafolio de futuros indizados a la inflación para distintos horizontes en el tiempo. El análisis empírico se realiza para los futuros indizados a la inflación en México los cuales son negociados en la Bolsa Mexicana de Derivados (MEXDER). Para analizar el VaR con horizontes en el tiempo de más de un día de negociación simulaciones de remuestreo fueron utilizadas. Los resultados muestran que ese tipo de modelos son relativamente certeros para horizontes en el tiempo de un día de negociación. Sin embargo, la persistencia en la volatilidad de los modelos tipo-ARCH se ve reflejada con estimados relativamente altos de VaR para horizontes en el tiempo más largos. Estos resultados tienen implicaciones para pronósticos de volatilidad de la inflación de corto plazo. Al estimar intervalos de confianza en el VaR, es posible tener cierta confianza sobre el rango de posibles valores de la inflación (ó valores de inflación extremos) para un horizonte del tiempo específico.

Palabras Clave: Remuestreo, inflación, futuros indizados a la inflación, México, Valor en Riesgo, persistencia en la volatilidad.

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I. Introduction

Nowadays it is important to measure financial risks in order to make better-informed decisions relevant to risk management. It is well documented that volatility is a measure of financial risk. Measuring financial volatility of asset prices is a way of quantifying potential losses due to financial risks. An important tool for this measure is to forecast price return volatilities. As today, there are a significant number of research projects about forecasting futures price return volatility. These studies have covered a great variety of futures contracts, which are financial and non-financial. In terms of financial futures the studies have analysed exchange rates, interest rates, stocks, stocks indexes, among many others.

There is, however, significantly less work about analysing price return volatilities of inflation-indexed futures. This is probably because these types of futures contracts have relatively less trading if compared with its financial counterparts. In other words, inflation-indexed futures are less popular among traders and investors. In developed countries short-term inflation volatility is, usually, not a major concern. This is because inflation is relatively stable. In these types of countries it is normally not a priority to forecast short-term inflation volatility.¹ In most of them inflation-indexed futures do not even exist. But, the situation is different for developing countries. These usually have relatively higher inflation levels. Inflation volatility is commonly higher in the short-run. As a consequence, a great number of financial commitments (either short or long-term obligations) are

¹ In this paper short-term inflation refers to a period of less than three-months or its equivalent in trading days.

affected by inflation uncertainty. Nominal interest rates tend to be higher when that occurs. Therefore, investment decisions in both money and capital markets are obviously affected.

Under inflation targeting (IT) regimes, however, there is an idea that inflation has become relatively stable. With time series econometric techniques Chiquiar, Noriega and Ramos-Francia (2010) show that the inflation persistence in Mexico changed around the date Banco de México adopted an IT regime. It changed from a persistent process to a stationary one. However, Capistrán and Ramos-Francia (2009) show for a group of Latin American Countries that the dates of structural breaks in the inflation series (i.e. change in inflation persistence) not always match the dates IT was adopted. For some countries the series switch from unit root to a stationary process years before the IT regime was implemented, for instance, in the late eighties and early nineties. In the present research paper it is intended to contribute to the literature of inflation dynamics for emerging economies. It is expected that the results of an analysis of short-run inflation dynamics should be relevant under the Mexican IT regime.

In this research project, inflation volatility for Mexico is analysed using Mexican inflation-indexed futures prices from 13th October 2003 to 30th June 2010. An analysis of inflation can be performed from a different perspective by using futures prices. This is because these types of derivative contracts have daily trading and daily data is available. Up to date inflation-indexed futures have not been analysed with sufficient detail. They are rarely mentioned in the literature because these are not popular and few countries actually

trade them.² Research papers that use inflation-indexed futures hoping to accurately predict the Consumer Price Index (CPI) volatility are virtually non-existent.

It is a belief that futures prices hold relevant information about the future level of prices. As Working (1958) explained, expectations of the future level of prices are implicit in futures markets. Mexico is one of the few countries in the world, which has futures contracts for its Consumer Price Index (CPI). The analysis presented here is considered important given that this country has experienced periods of relatively high inflation volatility in the past. The relevance is related to financial risk management decisions as explained above.

The main objective of this paper is to analyse if Autoregressive Conditional Heteroscedasticity (ARCH-type) models, which include a proxy variable for volatility persistence, accurately predict risks caused by inflation volatility from a Value at Risk (VaR) perspective.³ This is done by considering a theoretical portfolio of inflation-indexed-units 'Unidades de Inversión' (UDI) futures.⁴ VaR is estimated using ARCH-type models and then their accuracy is formally tested with back-testing (Kupiec: 1995, Jorion: 2000, 2001). The procedure is to find out how accurate is the VaR with daily UDI futures observations. The time horizons considered are from one trading day up to three months equivalent in trading days. For one trading day a parametric approach is applied. For ten

² Besides Mexico, other countries that have inflation-indexed futures are the US (Consumer Price Index Futures Contracts) and Brazil.

³ Volatility persistence in this project refers to the financial volatility that takes a long time to die away.

⁴ 'Unidad de Inversión (UDI)' is a measure in units of account of the Mexican inflation, which is published by the Mexican Central Bank (Banco de México). These have a constant real value. Usually, these are used as inflation-indexed financial instruments in short-term and long-term investments and obligations. During periods of high inflation investors choose to invest in the UDI considering that it is a hedge against relatively high inflation.

trading days and more Bootstrapping simulations (Enfron: 1982) are carried out (non-parametric approach). If the number of daily violations or 'exceptions' is reasonable according to VaR models performance criteria, then the models are considered accurate. Otherwise, the ARCH-type models are rejected. The n -day forecast horizon is also interpreted as the probability that future inflation will be within certain statistical confidence interval i.e. the 95% confidence interval VaR. It is expected that these results could have forecast implications for the future range of inflation measured through the Mexican Consumer Price Index.⁵

The layout of this paper is as follows. The literature review is presented in Section II. The motivation and contribution of this work are presented in Section III. Section IV presents the definition of futures prices. The models are explained in Section V. Data is detailed in Section VI. Section VII presents the descriptive statistics. The results are analysed in Section VIII. Finally, Section IX concludes (figures and tables are included in the Appendix).

II. Literature Review

Historical volatility is described by Brooks (2002) as simply involving the calculation of the variance or standard deviation of returns in the usual statistical way over some long period (time frame). This unconditional variance or standard deviation may become a volatility forecast for all future periods (Markowitz: 1952). However, in this type of

⁵ The Mexican Consumer Price Index is called in Spanish 'Índice Nacional de Precios al Consumidor' (INPC) and it is published by Banco de México.

calculation there is a drawback. This is because unconditional volatility is assumed constant for a specified period of time. Nowadays, it is well known that financial prices have time-varying volatility i.e. volatility changes through time.⁶ This is also the case for commodity prices that are usually included in inflation indexes. It is well documented that non-linear ARCH models can provide accurate estimates of time-varying price volatility. See for example, Engle (1982), Taylor (1985), Bollerslev, Chou and Kroner (1992), Ng and Pirrong (1994), Susmel and Thompson (1997), Wei and Leuthold (1998), Engle (2000), Manfredo et. al. (2001), among others.⁷ However, the out-of-sample forecasting accuracy of these types of non-linear models could be, in some cases, questionable (see Park and Tomek: 1989, Schroeder et. al.: 1993, Manfredo et. al.: 2001, Benavides: 2003, 2006, Pong et. al.: 2003).⁸

Nonetheless, there is a growing literature of the implications of non-linear dynamics for financial risk management (Hsieh: 1993). In the light of this topic some researchers have extended the work for the application of time-varying volatility models, specifically ARCH-type models, in VaR estimations (Brooks, Clare and Persaud: 2000; Manfredo: 2001; Engle: 2003; Giot: 2005; Mohamed: 2005; among others). Most of these findings enhance the use of time-varying models in risk management applications using VaR. Even though, there are several research papers, which used these types of models for

⁶ The volatility that it is consider here is the conditional volatility of a financial asset.

⁷ For an excellent survey about applications of ARCH models in finance the reader can refer to Bollerslev, Chou and Kroner (1992).

⁸ All of them found that the explanatory power of these out-of-the-sample forecasts is relatively low. In particular, Pong et al. (2003) find that option implied volatility forecasts performed at least as well as forecasts from Autoregressive Fractional Integrated Moving Average Models (ARFIMA) for time horizons of one and three months. These were superior forecasts to those from ARCH-type models.

financial time series there is, however, no current literature that analyses a VaR for inflation-indexed futures in an emerging economy.

III. Motivation and Contribution

As it was explained in the literature review, there has been a significant amount of research done about forecasting financial time series volatility. A great number of works used futures markets data. However, there are no extensions of inflation forecasting within a risk-management framework, specifically VaR. Furthermore, there are no works that use inflation-index futures to try to predict future range-levels (confidence intervals) for inflation. Again, this is clearly a 'gap' in the literature.

Previous works have applied non-linear models within a VaR framework in order to estimate Minimum Capital Risk Requirements (MCRRs) (Hsieh: 1991; Brooks, Clare and Persand: 2000). MCRR is defined as the minimum amount of capital needed to successfully handle all but a pre-specified percentage of possible losses (Brooks, Clare and Persand: 2000). This concept is relevant to banks and bank regulators. For the latter it is important to require banks to maintain enough capital so banks could absorb unforeseen losses. These regulatory practices go back to the original Basle Accord of 1988. Even though there is a broad accord about the need of MCRRs there is, however, significantly less agreement about the method to calculate them.⁹ By estimating the VaR

⁹ According to Brooks, Clare and Persand (2000) the most well known methods are the *Standard/International Model Approach* of the Basle Accord (1988), the *Building-Block Approach* of the EC Capital Adequacy Directive (CAD), the *Comprehensive Approach* of the Securities Exchange Commission (SEC) of the US, the *Pre-commitment Approach* of the Federal Reserve Board (FED) and the *Portfolio Approach* of the Securities and Futures Authority of the UK.

of their financial portfolios banks are able to calculate the amount of MCRRs needed to meet bank supervision requirements.¹⁰

In this project the works of Hsieh (1991) and Brooks, Clare and Persaud (2000) are extended. The addition here is that MCRRs are estimated for futures contracts that are relatively rare, but at the same time, a different specification for ARCH-type models is used. The specification includes a proxy variable for volatility persistence, which is measured in a way different than before. This also has implications for inflation forecasts. By considering a similar methodology as the one used in Hsieh (1991) and Brooks, Clare and Persaud (2000) it is possible to have an idea of future inflation range-levels with certain statistical confidence. For example, if a 95% confidence level VaR with a time horizon of one month is applied, it is possible to quantify the range of possible inflation in one month with 95% statistical certainty. By the same token, it is possible to quantify what are the chances of observing extreme values (those outside the 95% interval in a parametric and non-parametric distribution), the latter by applying bootstrap methods. Furthermore, rigorous accuracy tests for ARCH-type models estimating VaR are carried out when these include a volatility persistence component. These models are assessed with back-testing in terms of the number of violations that occurred within the confidence intervals. The null hypothesis to test is the following, H_0 : Non-linear ARCH-type models are not accurate to estimate VaR even when a volatility persistence variable is

¹⁰ According to Basel Bank Supervision Requirements of 1988, banks have to hold capital (as a precautionary action) at least three times the equivalent to the VaR for a time horizon of 10 trading days at the 99% confidence level. There are no significant changes to this rule in the Basel II accord. The only change is that for repo-notes the time horizon must be 5 trading days. The interested reader can consult the previously mentioned information at the BIS webpage: <http://www.bis.org/publ/bcbs107.htm>

considered. In order to test the null the results will be analysed according to Kupiec (1995) and Jorion (2001) back-testing methodology.

Thus, these findings contribute with new knowledge to the existing academic literature given that the models are applied to inflation. The results could be for the interest of agents involved in risk management decisions related to inflation forecasts. These groups of persons could be private bankers, policy makers, investors, futures traders, central banks, academic researchers, among others. In particular, this topic could be of interest to policymakers in countries that have relatively high inflation volatility. Normally, these countries are developing countries.

An additional contribution is that an analysis of the inflation dynamics is carried out for the Mexican economy in derivative and spot markets. As it is known the inflation process in Mexico may have been impact by several monetary policy decisions. Even though the objective of this paper is not to study the effects of such policies in the inflation process, the present research could shed some light to relevant studies, which are related to Mexican inflation dynamics. This is because real-world densities (for VaR measurement) are estimated in the present research.

IV. Definition of Futures Prices

According to Hull (2003, pg. 706) a futures price is the 'delivery price currently applicable to a futures contract.' A futures contract obliges the participants to buy or sell an asset (depending on his or her position, i.e. long or short respectively) at a predetermined delivery price during a specified future time period. These contracts can be

use to hedge financial exposure by taking specific positions.¹¹ These are marked-to-market daily, which means that profits and losses are realized every trading day through a clearinghouse. The settlement price is usually a weighted average of the prices nearer to the end of the trading day. The settlement price calculation varies between the underlying asset and the futures (derivatives) exchanges.¹²

In the Mexican Derivatives Exchange (MEXDER) there are several ways to calculate the settlement price for the inflation-indexed (UDI) futures contract. For example, one way to calculate it is by obtaining a weighted average of the prices for the last five minutes of trading. However, these UDI futures are characterised by having relatively low trading volume. A common method when there is no trading (there is no volume for a specific trading day) is used by MEXDER that calculates a theoretical futures price according to the following formula:

$$F_0 = UDI_t \left(\frac{1 + i_{i,T}^N \left(\frac{T}{360} \right)}{1 + i_{i,T}^R \left(\frac{T}{360} \right)} \right) \quad (1)$$

where F_0 is the current futures price, UDI represents the value of the UDI at day t , which is published by Banco de México at the Mexican Official Gazette ('Diario Oficial de

¹¹ Even though futures contracts can be used to hedge financial risk it is common to observe that, in some cases, there is not an optimal demand for them. For example see Benavides and Snowden (2006) for details.

¹² For a good reference about the mechanics of futures markets the reader could refer to Fink and Feduniak (1988).

la Federación'),¹³ $i_{t,T}^N$ represents the nominal interest rate observed in day t calculated for Mexican Government Certificates of Deposits (CETES) with a maturity equivalent to the life of the specific futures contract in days (T), $i_{t,T}^R$ is the real interest rate observed for day t calculated from UDI bonds with a relevant maturity published by ValMer (a Mexican Market Valuation Company).¹⁴ Finally, T represents the number of days remaining for maturity of the futures contract.¹⁵ MEXDER publishes in its web page the time series futures prices for every UDI contract.¹⁶ Information about the UDI contract can be observed in more detail in **Table 1** in the appendix. Given that these are observed (market) prices, there is no need to obtain additional interest rate data since interest rates are implicit in the theoretical futures price calculation (Equation 1).

V. The Models

V.1. The GARCH Specification

The volatility of the time series under analysis is estimated with historical data. It is known that ARCH models (Engle: 1982) are accurate estimators of time-varying volatility. A well known model within the family of ARCH models is the univariate Generalized

¹³ The 'Diario Oficial de la Federación' is published every working weekday by the Mexican Federal Government and has relevant information to the general public. These are usually fiscal and economic policies.

¹⁴ ValMer is an acronym in Spanish for 'Valuación de Mercado S.A de C.V'.

¹⁵ In finance textbooks it is common to see that the theoretical futures (forward) price is expressed in continuous time, (Hull: 2003, pg. 46): $F_0 = S_0 e^{rT}$. Where F_0 is the current futures (or forward) price, S_0 is the current spot price, e equals the $e(\cdot)$ function, r is the risk-less rate of interest per annum expressed with continuous compounding and T is the time to maturity in years. For the previous formula it is assumed that the underlying asset pays no income. For the research purposes of this project F_0 equals the observed inflation-index futures price as reported by MEXDER (in discrete time) and S_0 equals the observed inflation-index spot price published by Banco de México.

¹⁶ The MEXDER web page is <http://www.mexder.com.mx/MEX/paginaprincipal.html>

Autoregressive Conditional Heteroscedasticity, GARCH(p, q) model. This model is estimated applying the standard procedure as explained in Bollerslev (1986) and Taylor (1986).¹⁷ The formulae for the GARCH(p, q) are presented below. For the model there are two main equations. These are the mean equation and the variance equation:

Mean equation,

$$\begin{aligned} \Delta y_t &= \mu + e_t & (2) \\ e_t | I_{t-1} &\sim N(0, \sigma_t^2), \end{aligned}$$

and the variance equation,

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 . \quad (3)$$

where Δy_t are the first differences of the natural logarithm (logs) of the series under analysis at time t (the inflation spot or futures-index), e_t is the error term at time t , I_{t-1} is the information set at time $t-1$, σ_t^2 is the variance at time t . $\mu, \omega, \alpha_i, \beta_j$ are parameters and it is considered the assumption that the log returns are normally distributed. In other words, assuming a constant mean μ (the mean of the series Δy_t) the distribution of e_t is assumed

¹⁷ The ARCH-type models presented in this paper were estimated using Eviews computer language.

to be Gaussian with zero mean and variance σ_t^2 . The parameters are estimated using maximum likelihood methodology applying the Marquardt algorithm.¹⁸

Considering that the assumption of normality of the residuals stated above does not hold, the Bollerslev and Wooldridge (1992) methodology is used in order to estimate consistent standard errors. With this method the results have robust standard errors and covariance. This method derives that the estimators are from Quasi-Maximum Likelihood Estimation. Thus, the coefficients are robust even though the normality assumption is not met by the data.¹⁹ The estimated coefficients are reliable once they are statistically significant and the sum of the $\alpha + \beta < 1$ (otherwise the series are considered explosive or equivalently non-mean reverting, Taylor: 1986).

V.2. The Threshold GARCH Model

Another model used in this paper is the Threshold GARCH model. This model is also known as the TARARCH model. It was postulated by Glosten, Jaganathan, and Runkle (1993) and Zakoian (1994). Compared with the GARCH(p, q) model the specification of the TARARCH model involves an additional term in the variance equation,

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{k=1}^r \delta_k \varepsilon_{t-k}^2 I'_{t-k} . \quad (4)$$

¹⁸ This algorithm modifies the Gauss-Newton algorithm by adding a correction matrix to the Hessian approximation. This allows to handle numerical problems when the outer products are near singular thus, increases the chance of improving the convergence of the parameters.

¹⁹ For more details about Quasi-Maximum Likelihood Estimation the interested reader can refer to Bollerslev and Wooldridge (1992).

where $I'_t = 1$ if $\varepsilon_t < 0$ and 0 otherwise. The intuition for this model is that bad news $\varepsilon_t < 0$ will have a different impact on the conditional variance compared to good news $\varepsilon_t > 0$.²⁰ In case of good news the impact is on α_i and for bad news the impact is on $\alpha_i + \delta_i$. If $\delta_i > 0$, there will be a higher increase in volatility driven by the bad news. If $\delta_i \neq 0$, then the news impact is asymmetric. This model is normally applied for estimating stock price volatility considering the leverage effect on stocks.²¹

For the case of the inflation-index futures, the asymmetric TARARCH model is applied in the opposite way. Here the bad news are considered increases in inflation thus, the news impact will be magnified if $\varepsilon_t > 0$. In other words, the model is adjusted to give a volatility asymmetry with positive inflation returns. This is more relevant for countries that have a history of high inflation. It is known that when there are high inflation periods it is common to observe high inflation volatility.

It is important to point out that in order to control for seasonality (spot price data) an indicator variable is included in the mean and variance equation. This dummy variable takes the value of one every time there is an inflation announcement and zero otherwise. Also, for this spot return series an autoregressive component of second order (order chosen by Information Criterion) is also included in the mean equation in order to have a better specification for the structure of the series.

²⁰ Good news refers to positive financial assets returns. Bad news is just the opposite.

²¹ The leverage effect on stocks refers to asymmetric volatility considering that a bear market sentiment has higher price volatility if compared with a bull market sentiment. In a bear market higher uncertainty about the cash flow stream could cause the stock price to decrease and the company increases its leverage ratio, which is undesirable (Brooks: 2002).

Other procedures were used in order to deal with seasonality. A moving average term was included in the mean-equation. The idea is to smooth out as possible the process. The previously mentioned dummy variable was included both in mean and variance equations. The objective is to observe if the estimated coefficient is statistically significant. In the Descriptive Statistics Section below the interpretation of some of the estimates is presented as well as the implications for controlling for seasonality.

V.3. The VaR model

The VaR is a useful measure of risk.²² It was developed in the early 1990s by the JP Morgan Corporation. According to Jorion (2001) 'VaR summarizes the expected maximum loss over a target horizon with a given confidence interval.' Even though it is a statistical figure, most of the times are presented in monetary terms. The intuition is to have an estimate of the potential change in the value of a financial asset resulting from systemic market changes over a specified time horizon (Mohamed: 2005). It is also normally used to obtain the probability of losses for a financial portfolio of futures contracts. Assuming normality, the VaR estimate is relatively easy to obtain from GARCH models. For example, for a one trading day 95% confidence interval VaR, the estimated GARCH standard deviation (for the next day) is multiplied by 1.645. If the standard deviation forecast is, let's say, 0.0065, the VaR is approximately 1.07%. To interpret this result it could be said that an investor can be 95% sure that he or she will not lose more

²² Value at Risk is normally abbreviated as VaR. The lower case "a" letter differentiates this abbreviation to that of Vector Autoregressive Models, which are usually abbreviated as VAR (with a capital A).

than 1.07% of asset or portfolio value in that specific day. However, a problem with the parametric approach is that if the observed asset returns depart significantly from a normal distribution the applied statistical model may be incorrect to use (Dowd: 1998).

So, as it was said, when using VaR models it is necessary to make an assumption about the distribution of the returns. Although normality is often assumed for price returns series, it is known in practice that this assumption is highly questionable (Mandelbrot: 1963, Fama: 1965, Engle: 1982, 2003). If the daily returns are divided by the (adjusted) TARCH standard deviations, the new series will have a constant volatility of one with a non-normal distribution (Engle: 2003). For these 'standardized residuals' or 'de-volitized returns' the kurtosis must be above normal, thus a non-normal distribution is therefore assumed in the VaR. The volatility asymmetries estimated within the TARCH model allow for this non-normality. This method will be considered for the estimation of VaR for time horizons of one trading day. However, there is also another approach which will also be applied in this project for time horizons of more than one trading day. The latter is explained next.

For time horizons of more than one trading day (ten, thirty and ninety trading days), the bootstrapping methodology of Enfron (1982) will be applied.²³ The fact that the returns of the series are non-normally distributed motivates the use of a non-parametric procedure as the bootstrapping. The procedure used in Hsieh (1993) and Brooks, Clare and Persaud (2000) is considered here. In the latter they empirically tested the

²³ The bootstrap is a resampling method for inferring the distribution of a statistic, which is derived by the data in the population sample. This is normally estimated by simulations. It is said to be a nonparametric method given that it does not draw repeated samples from well-known statistical distributions. On the other hand, a Monte Carlo simulation draws repeated samples from assumed distributions. In this research project the bootstrap methodology was implemented using Eviews computer language.

performance of that VaR model for futures contracts traded in the London International Financial Futures Exchange (LIFFE).²⁴ A similar paradigm is applied here for inflation-indexed (UDI) futures contracts. Thus, a hypothetical portfolio of UDI futures is considered and MCRRs will be estimated. These estimated MCRRs values for the UDI portfolio are compared to the observed (historical) inflation. This analysis allows to evaluate how accurate are the ARCH-type models in terms of estimating MCRRs for inflation-indexed futures. Yet, another objective is to analyse the performance of these in terms of how accurate are they for providing an upper threshold for inflation i.e. what are the statistical chances that inflation will be high enough to be outside the upper (positive) confidence interval.

In order to calculate an appropriate VaR estimate it is necessary to find out the maximum loss that a position might have during the life of the futures contract. In other words, by replicating with the bootstrap the daily values of a long futures position it is possible to obtain the possible loss during the sample period. This will be obtained with the lowest replicated value. The same reasoning applies for a short position. But in that case the highest possible loss will be obtained with the highest replicated value.²⁵ Following Brooks, Clare and Persaud (2000) and Brooks (2002) the formulae is as follows. The maximum loss (L) is given by

$$L = (P_0 - P_1) \times \text{Number of contracts} \quad (5)$$

²⁴ These futures contracts were the FTSE-100 stock index futures contract, the Short Sterling contract and the Gilt contract.

²⁵ As it is well known in futures market mechanics decreases in futures prices mean losses for long positions and increases in futures prices mean losses for short positions.

where P_0 represents the price at which the contract is initially bought or sold; and P_1 is the lowest (highest) simulated price for a long (short) position, respectively, over the holding period. Without loss of generality it is possible to assume that the number of contracts held is one. Algebraically, the following can be written,

$$\frac{L}{P_0} = \left(1 - \frac{P_1}{P_0}\right). \quad (6)$$

Given that P_0 is a constant, the distribution of L will depend on the distribution of P_1 . It is reasonable to assume that prices are lognormally distributed (Hsieh: 1993) i.e. the log of the ratios of the prices are normally distributed. However, this assumption is not considered here given that empirical distributions of the series under study are not normal. However, the log of the ratios of the prices is transformed into a standard normal distribution following JP Morgan Risk-Metrics (1996) methodology. This is done by matching the moments of the log of the ratios of the prices' distribution to a distribution from a set of possible ones known (Johnson: 1949). Following Johnson (1949) a standard normal variable can be constructed by subtracting the mean from the log returns and then divide it by the standard deviation of the series,

$$\frac{\ln\left(\frac{P_1}{P_0}\right) - \mu}{\sigma}. \quad (7)$$

The expression above is approximately normally distributed. It is known that the 5% lower (upper) tail critical value is -1.645 (1.645).

From Equation 6 the following can be expressed,

$$\frac{L}{P_0} = 1 - \exp[-1.645\sigma + \mu] \quad (8)$$

when the maximum loss for the long position is obtained. For the case of finding the maximum possible loss for the short position the following formula applies,

$$\frac{L}{P_0} = \exp[1.645\sigma + \mu] - 1. \quad (9)$$

The MCRRs of the short position can be interpreted as an upper threshold for inflation. This will be the threshold of interest given that in the Mexican economy it was common to observe increases in inflation²⁶. On the other hand, it was relatively rare to see deflation events (for deflation events the long position is the one of most interest). MCRRs for both positions are reported in this paper. However, the MCRRs for the long position estimates will not be analysed. Only the MCRRs for the short position are of interest and these are going to be interpreted and analysed. The latter will give a forecast value for extreme inflation for a n -day period with 95% confidence. The estimates are the ones from the positive side of the distribution (the right-hand tail) i.e., which are positive levels of inflation.

The simulations were performed in the following way. The GARCH and TARARCH models were estimated with the bootstrap using the standardized residuals from the whole sample (instead of residuals taken from a normal distribution as it was written in Equation 2). The UDI variable was simulated, with the bootstrap as well, for the relevant time horizon (10, 30 or 90 trading days) with 10,000 replications. The formula used was

²⁶ For the period under study only in few occasions deflation was observed. This occurred in 2004, 2009, 2010.

$UDI_t = UDI_{t-1}e^{returnT}$ (where UDI could be the futures or spot price, the rest of the notation is the same as specified above). From the UDI simulations the maximum and minimum values were taken in order to have the MCRRs for the short and long positions respectively.

VI. Data

VI.1. Data Sources

The data consists of daily spot and futures closing prices of the UDI obtained from Banco de México and MEXDER respectively. The sample period under analysis is more than seven years from 13/10/2003 to 30/06/2010. The sample size consists of 1,753 daily observations. The sample period was chosen according to availability of UDI futures data. These started formally trading on 13 October, 2003. This is the starting date for the sample period used in this project. The sample size is considered large enough for the estimation task at hand. Given that the time horizon for these simulations is relatively short (up to three months ahead) there is no need for a larger sample size.²⁷ The UDI contracts have delivery dates for up to five years. The periodicity of the maturities of the contracts is monthly up to one year and quarterly for the remaining four years. The MEXDER is relatively new compared to other derivatives exchanges around the world. It began operations in 1998.

²⁷ Nonetheless, an update for the sample was made up to October 2010. The estimations (available upon request) show no qualitatively changes.

VI.2. Data Transformation

When creating a time-series of futures prices a significant number of researchers use the prices of the futures contract closer to maturity or the one with higher trading volume. These procedures have the inconvenience of creating a pattern of ‘jumps’ in the price series when switching prices from one futures contract to another. This type of ‘jumps’ are unrealistic. Even though, ‘jumps’ are observable in futures prices there is, usually, no clear pattern as the one it is created using both methods. In order to avoid these unrealistic ‘jumps’ when creating a time-series of futures prices from different contracts (Pelletier, 1983; Wei and Leuthold: 1998), synthetic futures prices were created.²⁸ These were calculated by a ‘roll-over’ procedure that is basically an interpolation of futures prices from different maturity futures contracts (Herbst et. al. 1989, Kavussanos and Visvikis: 2005). This procedure creates a constant maturity weighted average futures price based upon the futures prices and the days to maturity of the two near-by-expiration contracts. The formula used to obtain the synthetic futures price is shown in Equation 12 below,²⁹

$$SYN_T = F_j \left[\frac{(T - T_i)}{(T_j - T_i)} \right] + F_i \left[\frac{(T_j - T)}{(T_j - T_i)} \right] \quad (10)$$

²⁸ The synthetic futures prices were calculated using Visual Basic for Applications computer language.

²⁹ The terms synthetic futures price and futures price are synonymous for the rest of this paper.

where SYN_T is the synthetic futures price for delivery at T , F_j is the contract j futures price expiring at T_j , F_i is the contract i futures price expiring at T_i , T equals 30, the chosen constant maturity in number of days, T_i is the contract i expiration in days remaining, T_j is the contract j expiration in days remaining, $j = i + 1$, with $T_i \leq T \leq T_j$.

The time to expiration of the synthetic futures prices calculated is T equals 30 days. This means that a constant 30-day maturity synthetic futures price was calculated. This is considered an appropriate time-to-expiration given that a shorter time-to-expiration could have higher expected volatility. This situation is observed in empirical research papers, which have found that volatility in futures prices increases, as a contract gets closer to expiration (Samuelson: 1965). This could be the case for futures contracts of less than 30 days remaining. A higher expected volatility due to time-to-expiration could bias the results of this analysis.

VII. Descriptive Statistics

This section presents the descriptive statistics for the daily (observed) volatilities of the UDI spot and futures returns and the forecast volatility from the models. Prior to fitting the GARCH and TARARCH models an ARCH-effects test was conducted for the series under analysis. This was done in order to see if these types of models are appropriate for the data (Brooks: 2002). The test conducted was the ARCH-LM following the procedure of Engle (1982).³⁰ According to the results both series under study have ARCH effects.

³⁰ These tests were conducted by using ordinary least squares regressing the logarithmic returns of the series under analysis against a constant. The ARCH-LM test is performed on the residuals of that regression. The test consists on regressing, in a second regression, the square residuals against constant

Under the null of homoscedasticity in the errors the F -statistics were 31.6153 for the spot and 7.8217 for the futures prices (the critical value is 3.84 for 378 degrees of freedom). Both statistics clearly reject the null in favour of heteroscedasticity on those errors.

The series were also tested for non-linear dependence using the BDS test (Brock et. al. 1996). This type of test is asymptotically distributed for a standard normal variate. The null is that the underlying series has independent and identical distribution (*i.i.d*). These tests can detect several types of non-*i.i.d* behaviour (Hsieh, 1991). If the null hypothesis is rejected then it is appropriate to use the GARCH models. **Table 2** presents the results for the BDS tests. It can be observed in Table 2 that the null hypothesis is rejected for both futures and spot returns. Following Hsieh (1991) and Brooks, Clare and Persaud (2000) the number of embedded dimensions (m) and the ϵ chosen are within the range of 2 to 5 and 0.50 to 1.50 respectively. In the test m refers to the number of consecutive points in the set and ϵ the distance between the data. If the observations of the series are truly *i.i.d*, then for any pair of points, the probability of the distance between these points being less than or equal to epsilon will be constant. There is no unique formula to use in order to choose the optimal values of m and ϵ . This is the reason why the proposed range is used following similar tests found in the literature as mentioned above. Nonetheless, robustness checks were applied by using values in the vicinity of the proposed range. There are no qualitative changes in the results, i.e., the analysed data are not independent.

and lagged values of the same square residuals. The null hypothesis is that the errors are homoscedastic. An F -statistic was used in order to test the null. The test was carried out with different lags 2 to 10. All have the same qualitative results. Only the cases for 5 lags are reported in the main text above.

Figure 1 presents the logs of the spot and futures prices of the UDI and their respective daily volatilities for the time frame under analysis.³¹ It can be observed that the futures price is usually above the spot price. This could be an indication of the expected inflation reflected in futures prices (Working: 1958). Also, it can be observed that the futures volatility is considerably higher than the spot volatility.

Table 3 shows the descriptive statistics for the daily volatility and the volatility from the forecasting models. The parsimonious specifications GARCH(1,1) and TARCH(1,1) were chosen according to results obtained from information criteria (Akaike Information Criterion and Schwarz Criterion tests). The model parameters were positive and statistically significant at the 1% level. The sum of $\alpha_1 + \beta_1$ was less than one. Diagnostic tests on the models were applied to ensure that there were no serious misspecification problems. The Autocorrelation Function as well as the BDS test was applied on the standardised residuals obtained from the forecast models. Both show that these residuals were *i.i.d.*³²

As it can be observed in Table 3 the means of the futures UDI series are the ones with higher values (the daily volatilities and the volatility forecasts). These findings are consistent with Figure 1 where the daily volatility of the futures was normally seen higher than the spot's volatility. The distributions in that table are highly skewed and leptokurtic indicating non-normality of the returns and the forecast estimates. This is consistent with the work of Wei and Leuthold (1998) that analysed volatility in futures markets and had

³¹ The daily volatility is simply defined as the absolute value of the log-return.

³² These results are available upon request.

similar findings with daily futures price volatility for agricultural commodities.³³ **Table 4** presents the autocorrelation coefficients of the UDI returns. It can be observed that there is autocorrelation (up to ten lags) in the UDI spot series and significant autocorrelation was also found for the futures UDI series. The absolute returns series show some evidence of serial correlation for several lags showing time-varying volatility. This further justifies the use of ARCH-type models for the modelling of these series.

Lastly, **Figures 2** and **3** present the observations of the daily volatility (top line) and the estimates of the volatility forecast models for the futures and spot series respectively (bottom lines). It can be observed in both graphs that the models captured the volatility clustering shown for the daily volatility. For the UDI spot return series it is also possible to observe seasonality. There are systematic periods when the volatility of the series is relatively stable. The correlogram in Table 4 shows that the portmanteau test with ten lags, $Q(10)$, is statistical significant, which corroborates the presence of seasonality for the spot returns series. The reason we observe this pattern is that inflation is published by the Central Bank of Mexico twice a month.³⁴ This occurs in day 10th and 25th of each month. If these days are non-working days then the publication is done the previous working day. In between these dates inflation is scaled by multiplying the last observed figure by \sqrt{h} , where h is the time horizon before the next inflation announcement day. Given the nature of this scaled-inflation forecast we can observe the seasonality pattern. That is, every time

³³ It is important to point out that most of the Mexican agricultural commodities prices are part of the Mexican CPI thus, their price movements are considered in the UDI.

³⁴ For more information about inflation publication procedures the interested reader can refer to the Mexican Official Gazette ('Diario Oficial de la Federación) of 25th June, 2002.

there is a publication about inflation (days 10th and 25th) the volatility observed for those days is higher compared to the periods there is no publication i.e. the scaled-adjustment was made. The latter shows relatively stable volatility, which can be observed in Figure 3 as segments of nearly horizontal lines. Thus, it can be said that seasonality in inflation spot UDI prices are explained by new information arrivals. However, seasonality apparently is not present for the futures series given the nature of this market-tradable financial instrument, which contrary to the spot it is not scaled (Figure 2). This can also be corroborated in Table 4 as well for the futures return series where we can observe smaller Q values for futures returns relative to spot returns whilst testing for statistical significance of the Portmanteau test.

Including a dummy variable to control for seasonality as explained in Section V.2 above shows an improvement in the volatility forecast. This can be observed in graphs by comparing the forecast with no dummy variables (not presented in this paper) and with the inclusion of seasonal dummies. The model without dummies shows a forecast with significant less 'stable volatility' i.e. those days when there is no inflation announcement. It is important to point out that the coefficient for the dummy variable was statistically significant in the variance equation but not in the mean equation (ARCH-type models). This can be interpreted as evidence of seasonality in inflation (market) risk. There is no agreement in the literature about how to deal with the problem of seasonality or which dummy variables should be included, however, in this research project some econometric tools are implemented in order to minimize the possible problem of the presence of seasonality in the series and its influence on the volatility forecast. As mentioned before

these econometric tools include autoregressive and moving average terms in the mean equation as well as dummy variables for both.

The implications of these forecasts are that they capture fairly well the dynamics of the daily volatility for both series under study. That is the GARCH(1,1) and adjusted TAR(1,1) models show forecasts that predict high volatility when in fact the actual daily volatility was high and predict low volatility when the actual daily volatility was low. The forecasts are relatively consistent in terms of capturing the dynamics for basically all the days in the sample. Again, this can be observed in Figures 2 and 3 in the Appendix.

VIII. Results

VIII.1 Parametric Method

Once the next-day volatility estimate is obtained the 95% confidence intervals are created by multiplying 1.645 by the forecasted conditional standard deviation (from the GARCH model). An analysis is made about the number of times the observed UDI spot return was above that 95% threshold (a violation or an exception). Again, the positive part (right tail of the distribution) is the one of most interest given that it is positive inflation what it causes more concern to relatively high inflation economies thus, the interest on predicting it.³⁵ **Figure 4** shows the spot inflation returns and the futures confidence

³⁵ Although for some economies it may be of interest to predict deflation. For that case it is important to see the negative side of the distribution. This is equivalent to taking a long position on the portfolio.

intervals. It can be observed that the UDI spot returns were mostly within the 95% confidence level for the daily forecasts. However, there were violations in 72 days, which represent 4.12% of the total number of observations. Considering that a 95% confidence level is applied the model should not exceed the VaR more than 5% (Jorion: 2001). The null hypothesis in this case is not to reject the model because it has fewer than 5% violations. The situation is different when spot prices are used to calculate the 95% confidence intervals. **Figure 5** shows the same UDI spot returns but with confidence intervals constructed with the spot UDI. For this case the number of violations is 141, which represents 8.05% of the total number of observations. The model is then rejected. Applying the Kupiec test as explained by Jorion (2000), the non-rejection region (interpolating) is $50 < x < 131$. So, the model is still not rejected for futures prices but rejected when using spot prices. The ambiguity of the results using different data does not permit to make conclusive answers about the acceptance of the GARCH model.

As explained before a normal distribution is highly questionable therefore the 'de-volatized returns' from the asymmetric TARARCH model are also applied. The procedure to apply this method was explained in Section V.2 above. For this case the Kurtosis of the series is 6.8535 (above normal) and there are 2.65 standard deviations away from the mean in order to reach the 95% confidence level. Therefore, the conditional standard deviation forecast is multiplied by 2.65 to construct the new intervals. Given these intervals the number of violations using futures prices is 57, which represents about 3.30% of the total number of observations. For spot prices the number of violations is 135, which represents 7.7% of the total number of observations. Thus, by considering a non-

normal distribution of the returns the VaR is higher and the number of violations decreases for both futures and spot prices. However, for the UDI spot series the decrease was not enough in order to be below the 5% threshold (or to be non-rejected in the Kupiec: 1995 test). The same conclusion as the one above for the GARCH(1,1) model applies here.³⁶

VIII.2 Bootstrapping Simulations

The methodology to carry out the simulations was explained in Section V.3 above. Following Brooks, Clare and Persaud (2000), a new variable is included in the conditional variance equation of the GARCH(1,1) in order to capture the volatility persistence, which is common in ARCH-type models (Gallo and Pacini: 1997). The proxy variable is calculated in the following way,

$$VP_t = \left| \ln \frac{close_t}{close_{t-1}} \right| \quad (11)$$

where VP represents volatility persistence and $close$ represents the UDI closing futures price at time t .³⁷ Including this proxy variable it is expected that the new specification will capture the volatility persistence common in financial price series. **Tables 5** and **6** present the VaR for the bootstrap simulations performed in the futures and spot series respectively. The numbers of n -days ahead considered in the simulations were 10, 30 and

³⁶ The figures for these results are available upon request.

³⁷ Now the conditional variance equation in the GARCH(p, q) model is,

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{k=1}^n \phi_k VP_{t-k}, \text{ where notation is the same as defined above.}$$

90 trading days. The simulations were done applying the GARCH(1,1), the adjusted TARCH(1,1) and the GARCH that includes the proxy for volatility persistence represented as GARCH(1,1, VP_{t-1}).

Considering the fact that the UDI spot returns have autocorrelation (see Table 4) it is necessary to do the bootstrap adjusting for this autocorrelated process.³⁸ The procedure postulated by Politis and Romano (1994) is applied here. This is basically a method in which the autocorrelated returns are grouped into non-overlapping blocks. For this case the size of these blocks is fixed during the estimation.³⁹ With the bootstrap the blocks are resampled. During the simulation of the UDI spot prices the returns are taken from the resample blocks. The intuition is that if the autocorrelations are negligible for a length greater than the fixed size of the block, then this Moving Block Bootstrap will estimate samples with approximately the same autocorrelation structure as the original series (Brownstone and Kazimi: 1998). Thus, with this procedure the autocorrelated process of the residuals is almost replicated and it is possible to obtain a more accurate simulated UDI spot series.

From Table 5 it can be observed that for ten trading days short positions (second, third and fourth rows) the MCRRs for all models are above the observed inflation during the simulated period from 17/06/2010 until 30/06/2010. The same qualitative results to those of ten trading days are observed for thirty trading days short positions (fifth, sixth

³⁸ I am thankful to Alejandro Díaz de León and Daniel Chiquiar for pointing this out. I also want to thank Arnulfo Rodríguez for his assistance in helping me to incorporate the Politis and Romano (1994) methodology in the Eviews computer code.

³⁹ It is also possible to have random size blocks. For a more detailed explanation please refer to Politis and Romano (1994).

and seventh rows) during the simulated period from 20/05/2010 until 30/06/2010. It can be observed that excluding the GARCH model, which includes the volatility persistence component (VP_{t-1}), the MCRRs are above the observed inflation for ninety trading days short positions (eight, ninth and tenth rows), which covers the period from 23/02/2010 until 30/06/2010. An explanation for these results is that the volatility persistence component in the variance equation is allowing the volatility persistence to die away rapidly instead of slowly. In this situation, the MCRRs are calculated with less volatility clustering showing less extreme values in the distribution. Thus, significantly lower MCRRs are calculated. In terms of the number of violations it can be observed that there are few in terms of VaR analysis, however, most of the estimated requirements look significantly conservative (they over-estimate the VaR). The results for the scenarios (time frames) in Table 6 with the spot simulated series are qualitatively similar to the futures results. However, the MCRRs tend to be lower than those from the futures series. This is explained because of the higher observed volatility of the futures prices in relation to the observed volatility of the spot prices (see Figure 1 and Table 3).

Obtaining relatively high values for the MCRRs for time horizons of 90 trading days could be explained by the volatility persistence that is part of the ARCH-type models. In this sense, these results are consistent with Brooks, Clare and Persaud (2000), where it was concluded that ARCH-type models tend to 'over-estimate' the VaR. In portfolio analysis the overestimation is considered costly. This is because unnecessary quantities of capital are set aside to meet MCRRs. However, it was observed in this project that for shorter-term VaR time horizons of one trading day, the models are relatively accurate.

Specifically, for the case of the UDI futures series in which the model was accepted, although it was not the case for the UDI spot series. These ambiguities in the results make it difficult to draw conclusive answers about ARCH-type models. However, with the inclusion of a volatility persistence variable these models become more accurate for periods of less than 10 trading days using futures prices (see Table 5). Therefore, it is concluded that ARCH-type models can be helpful in giving some insights about future inflation volatility in some cases, but not in all. Also, if a GARCH model includes a proxy variable for volatility persistence for a 10 day time horizon using futures data the results could be accurate. Thus, in line with other works in the literature it was observed that ARCH-type models tend to over-estimate the VaR of more than ten-trading days because of volatility persistence.

IX. Conclusions

Research about forecasting price return volatility in futures markets has been done extensively. However, research about forecasting short-term indexed-inflation futures returns is non-existent. Compared with other financial futures inflation-indexed futures are less common and have less trading overall. For developed countries inflation is usually stable and analysing its volatility is normally not a priority. However, for emerging economies inflation is more volatile and relatively higher. Having some insights about what could be the possible levels of inflation (or inflation volatility) is an important issue for emerging economies.

In this research project an analysis of Mexican short-term inflation volatility was presented. The research on this project differs from that found in the literature in that inflation-indexed futures are examined for a developing country. The volatility forecasts were estimated using ARCH-type models. For a hypothetical case of a portfolio of inflation-indexed futures, VaR estimates were presented as percentages of Minimum Capital Risk Requirements (MCRRs). The results show that the ARCH-type models can accurately estimate MCRRs for one-trading day ahead time horizons if futures prices are used. However, for time horizons of more than ten trading days the MCRRs were relatively high compared with the observed inflation. Highly conservative (over-estimated VaR) MCRRs are costly given that investors need to set aside more capital to meet the capital requirements. There is an opportunity cost of capital. It was argued that volatility persistence in ARCH-type models could explain the high MCRRs estimates for ninety trading days time horizons.

In terms of forecasting short-term inflation the VaR framework provides confidence intervals, which can give an insight about the expected range for future inflation. In other words, the expectations as a percentage about the future level of inflation considering a certain confidence interval. Thus, futures markets can give valuable information for one trading day time horizons. Using a GARCH model, which includes a volatility persistence variable in its conditional variance specification, is also helpful for ten or less trading days. The latter allows the volatility persistence element to be modelled within the conditional variance equation. Using this type of specification it is possible to have a relatively accurate certainty about the possible extreme values of inflation up to ten trading days.

However, it should be kept in mind that for longer periods the accuracy of the model declines. Therefore, it is concluded for the series under study not to reject the null hypothesis that the set of analysed models are not accurate to estimate VaR for long time horizons even when a volatility persistence variable is considered within the model. This conclusion is in line with previous studies in the literature specifically, Brooks et. al. (2000) and Hsieh (1993).

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Appendix

TABLE 1 UDI FUTURES CONTRACT SPECIFICATIONS

Underlying asset	UDI (inflation-indexed-units).
UDI quotes	The value of the UDI is multiplied by 100. For example, if the UDI is 3.258746 then the quote is 325.874.
Contract size	Each contract is for 50,000 UDIS.
Settlement	Cash settlement. To pay for one UDI futures contract the value of the UDI spot is multiplied by 50,000. For example, if the UDI is 3.258746 the payment will be \$162,937.30MXN ($3.258746 * 50,000 = \$162,937.30MXN$)
Symbol	UDI.
Maturity months	Every month for the following twelve months and every quarter afterwards.
Price limits	There are no price limits.
Negotiations mechanics	Electronically through the MEXDER's electronic trading system.
Trading hours	Weekdays from 7:30 until 15:00 hrs Mexico City time.
Last trading day	The 10 th day of the delivery month. If this is a holiday then the last trading day is the preceding one (assuming is non-holiday).
Settlement day	The following trading day (non-holiday) after the last trading day.
Marking-to-market	Applies according to the rules established by MEXDER. Daily profit/losses are daily by the clearinghouse.
Contract rollover date	First trading day (non-holiday) following the last trading day.
Quotes in decimals	UDIS are expressed up to six decimals, 0.000001.

This table presents detail information about the UDI futures contract. MXN = Mexican pesos (Mexican currency). The source of the information is MEXDER. The web page where this information was obtained is: http://www.mexder.com.mx/MEX/Contratos_Futuros.html (the information is also available in English).

TABLE 2 BDS TEST STATISTIC FOR THE UDI RETURNS

ε/σ	Futures or spot	Embedding dimensions			
		2	3	4	5
0.50	Futures	22.8462***	23.9924***	23.4041***	22.3983***
	Spot	33.1704***	32.9486***	29.8475***	25.4933***
1.00	Futures	17.0510***	16.6355***	14.7990***	13.3027***
	Spot	23.8851***	20.9489***	16.4849***	11.8408***
1.50	Futures	9.3828***	9.0084***	7.7855***	6.9862***
	Spot	14.7317***	12.0049***	9.0582***	6.7004***

This table presents the BDS test for the futures and the spot returns series. H_0 : Returns are *i.i.d.* /*** represents rejection of the null hypothesis at the 1% level. The embedding dimensions are 2, 3, 4 and 5. The epsilon has the range between 0.50 and 1.50. The sample size is 1,753 daily observations from 13th October 2003 to 30th June 2010.

FIGURE 1 LOG FUTURES AND SPOT UDIs AND THEIR DAILY VOLATILITIES (LEFT AXIS CORRESPONDS TO THE DAILY VOLATILITIES)

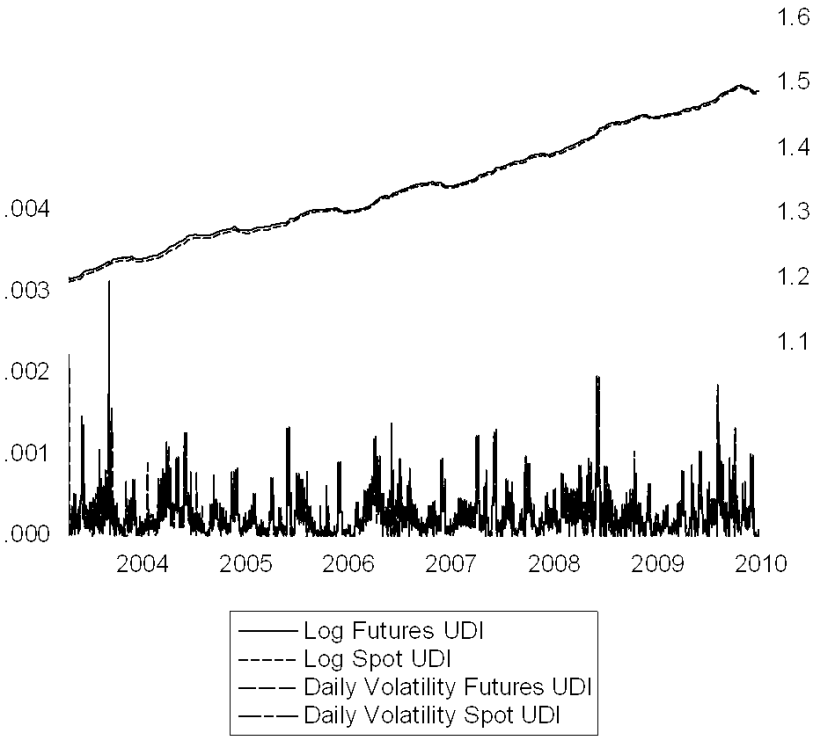


TABLE 3 DESCRIPTIVE STATISTICS FOR THE DAILY VOLATILITY OF THE SPOT AND FUTURES INFLATION-INDEX (UDI) AND THE FORECASTING MODELS

Model/Series	Mean	Variance	Skewness	Kurtosis	<i>N</i>
<u>Spot</u> daily volatility series	1.65×10^{-4}	6.92×10^{-8}	1.1270	10.0931	1,752
<u>Futures</u> daily volatility series	1.66×10^{-4}	9.18×10^{-8}	0.1062	18.8591	1,752
GARCH(1,1) model for the <u>spot</u> series	7.59×10^{-8}	6.12×10^{-15}	3.5611	21.6266	1,752
GARCH(1,1) model for the <u>futures</u> series	9.93×10^{-8}	2.76×10^{-14}	8.7740	111.9538	1,752
TARCH(1,1) model for the <u>spot</u> series	7.57×10^{-8}	6.02×10^{-14}	3.4902	20.6078	1,752
TARCH(1,1) model for the <u>futures</u> series	9.97×10^{-8}	2.69×10^{-14}	8.2401	97.9738	1,752

This table reports the descriptive statistics of the daily volatility and the volatility forecasting models for the daily UDI (Inflation-indexed) spot and futures returns. The sample size is 1,753 daily observations (adjusted sample 1,752 daily observations) from 13th October 2003 to 30th June 2010. *N* = Number of observations.

TABLE 4 AUTOCORRELATION OF THE UDI RETURNS

Lag length	Futures		Spot	
	Returns	Absolute returns	Returns	Absolute returns
1	0.326	0.252	0.476	0.285
2	0.335	0.249	0.469	0.289
3	0.349	0.250	0.429	0.254
4	0.328	0.182	0.405	0.239
5	0.460	0.455	0.625	0.578
6	0.195	0.138	0.304	0.131
7	0.174	0.110	0.257	0.078
8	0.189	0.066	0.215	0.036
9	0.149	0.049	0.189	0.030
10	0.207	0.213	0.313	0.255
Q(10)	1,457.8***	899.9***	2,678.5***	1,250.7***

Q(x) represents the Ljung-Box statistic, which has a χ^2 distribution. For this case there are 1,752 degrees of freedom (d.f.). The critical values at the 5% and 1% levels are 287.89 and 304.95 respectively. /** and *** represents statistical significance at the 5% and 1% level respectively. The sample size is 1,753 daily observations (adjusted sample 1,752 daily observations) from 13th October 2003 to 30th June 2010.

FIGURE 2 UDI FUTURES DAILY VOLATILITY AND THE VOLATILITY FORECASTS OF THE GARCH(1,1) AND ADJUSTED TARCH(1,1) MODELS

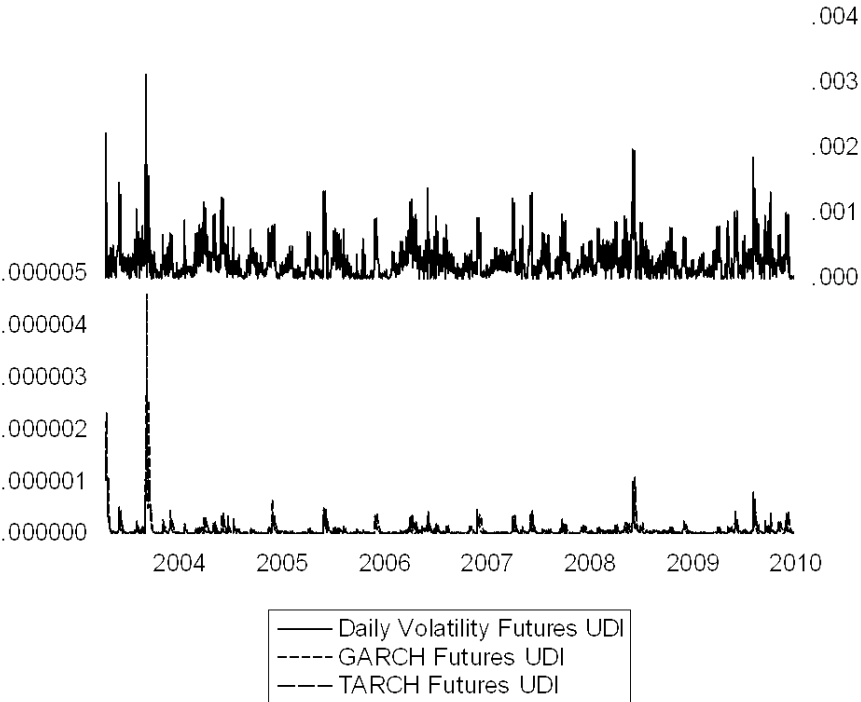


FIGURE 3 UDI SPOT DAILY VOLATILITY AND THE VOLATILITY FORECASTS OF THE GARCH(1,1) AND ADJUSTED TARCH(1,1) MODELS

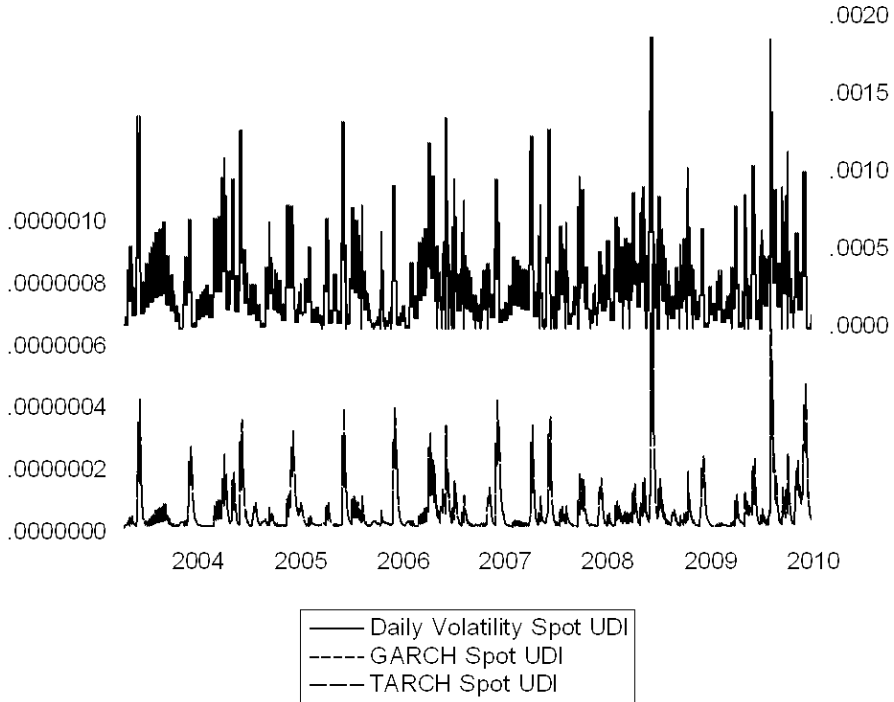


FIGURE 4 UDI SPOT RETURN AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH UDI FUTURES PRICES – GARCH(1,1) MODEL

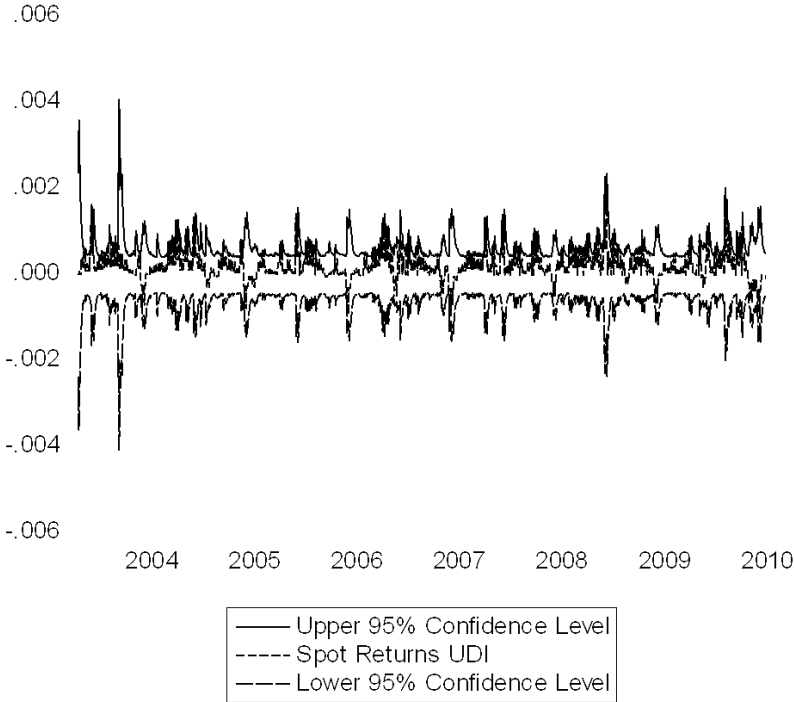


FIGURE 5 UDI SPOT RETURN AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH UDI SPOT PRICES – GARCH(1,1) MODEL

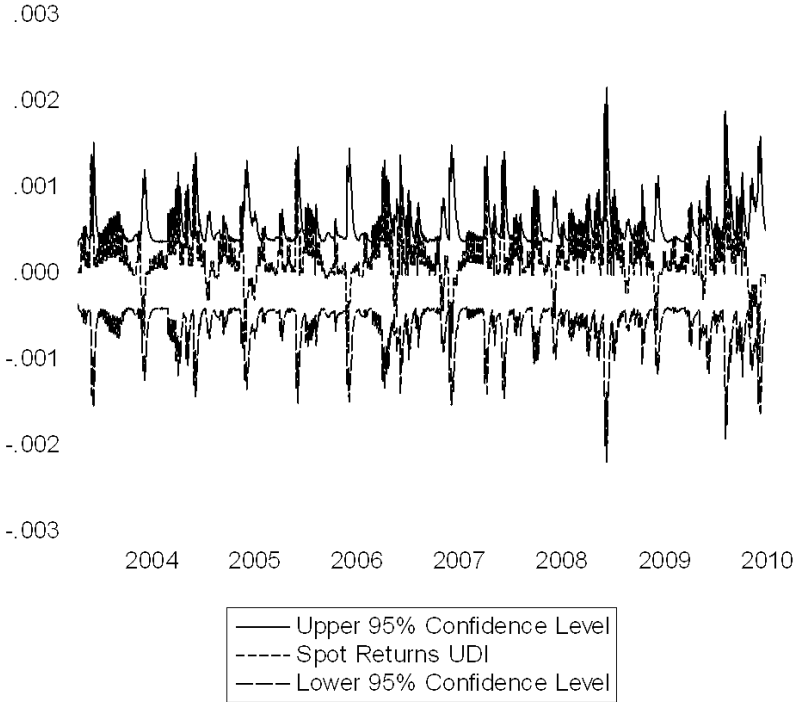


TABLE 5 VaR FOR THE UDI FUTURES PORTFOLIO OBTAINED WITH BOOTSTRAPPING SIMULATIONS

Model	VaR t -day horizon (trading days)	Minimum capital risk requirement <i>long</i> position	Minimum capital risk requirement <i>short</i> position	Observed inflation
GARCH(1,1)	10 trading days (from 17/06/2010 until 30/06/2010).	0.1096%	1.3169%	-0.0163%
TARCH(1,1)		0.1742%	1.2741%	-0.0163%
GARCH(1,1, VP_{t-1})		-0.2152%	0.5203%	-0.0163%
GARCH(1,1)	30 trading days (from 20/05/2010 until 30/06/2010).	- 0.0163%	1.9209%	-0.6163%
TARCH(1,1)		0.0588%	1.9182%	-0.6163%
GARCH(1,1, VP_{t-1})		-0.1832%	0.4756%	-0.6163%
GARCH(1,1)	90 trading days (from 23/02/2010 until 30/06/2010).	-0.0216%	3.6149%	0.1528%
TARCH(1,1)		0.0387%	3.6891%	0.1528%
GARCH(1,1, VP_{t-1})		- 0.1962%	0.4889%	0.1528%

This table presents the results of the bootstrap simulations. 10,000 replications were applied to simulate the UDI price. The time horizons are 10, 30 and 90 trading days. UDI futures prices are used for this table. The models applied are GARCH(1,1), TARCH(1,1) and the GARCH (1,1, VP_{t-1}). The last one includes a volatility persistence component. The sample size is 1,753 observations from 13th October 2003 to 30rd June 2010.

TABLE 6 VaR FOR THE UDI SPOT OBTAINED WITH BOOTSTRAPPING SIMULATIONS ADJUSTED FOR THE AUTOCORRELATION OF THE RETURNS

Model	VaR t -day horizon (trading days)	Minimum capital risk requirement <i>long</i> position	Minimum capital risk requirement <i>short</i> position	Observed inflation
GARCH(1,1)	10 trading days (from 17/06/2010 until 30/06/2010).	- 0.0003%	0.1838%	-0.0163%
TARCH(1,1)		0.0002%	0.1830%	-0.0163%
GARCH(1,1, VP_{t-1})		- 0.0002%	0.0182%	-0.0163%
GARCH(1,1)	30 trading days (from 20/05/2010 until 30/06/2010).	-0.0130%	0.6463%	-0.6163%
TARCH(1,1)		-0.0009%	0.6932%	-0.6163%
GARCH(1,1, VP_{t-1})		0.0009%	0.0546%	-0.6163%
GARCH(1,1)	90 trading days (from 23/02/2010 until 30/06/2010).	0.0288%	2.0245%	0.1528%
TARCH(1,1)		0.0292%	1.8679%	0.1528%
GARCH(1,1, VP_{t-1})		0.0431%	0.0175%	0.1528%

This table presents the results of the bootstrap simulations adjusted for autocorrelation of the UDI spot returns. 10,000 replications were applied to simulate the UDI price. The time horizons are 10, 30 and 90 trading days. The models applied are GARCH(1,1), TARCH(1,1) and the GARCH (1,1, VP_{t-1}). The last one includes a volatility persistence component. The sample size is 1,753 observations from 13th October 2003 to 30rd June 2010.