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Rates in Mexico**

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# An Affine Model of the Term Structure of Interest Rates in Mexico\*

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## Abstract

We develop and estimate an affine model that characterizes the dynamics of the term structure of interest rates in Mexico. Moreover, we provide empirical evidence on the relationship between the term structure factors and macroeconomic variables. First, we show that the model fits the data remarkably well. Second, we show that the first factor captures movements in the level of the yield curve, while the second factor captures movements in the slope of the curve. Third, the variance decomposition results show that the level factor accounts for a substantial part of the variance at the long end of the yield curve at all horizons. At short horizons, the slope factor accounts for much of the variance at the short end of the yield curve. Finally, we show that movements in the level of the yield curve are associated with movements in long-term inflation expectations, while movements in the slope of the curve are associated with movements in the short-term nominal interest rate.

**Keywords:** No-Arbitrage, Latent Factors, Term-Structure.

**JEL Classification:** C13, E43, G12

## Resumen

Se desarrolla y estima un modelo afín que caracteriza la dinámica de la estructura temporal de tasas de interés en México. Adicionalmente, se presenta evidencia empírica sobre la relación entre los factores del modelo afín y algunas variables macroeconómicas. Primero, se demuestra que el modelo se ajusta muy bien a los datos. Segundo, se demuestra que el primer factor captura movimientos en el nivel de la curva de rendimientos, mientras que el segundo captura movimientos en la pendiente. Tercero, los resultados de descomposición de la varianza muestran que el factor de nivel explica gran parte de la varianza en la parte larga de la curva en todos los horizontes. En horizontes de corto plazo, el factor de pendiente explica gran parte de la varianza en la parte corta de la curva. Finalmente, se muestra que los movimientos en el nivel de la curva de rendimientos están asociados a movimientos en las expectativas de inflación de largo plazo, mientras que los movimientos en la pendiente están asociados a movimientos en la tasa de interés de corto plazo.

**Palabras Clave:** No-Arbitraje, Factores Latentes, Estructura-Temporal.

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# 1 Introduction

We develop and estimate an affine model that characterizes the dynamics of the term structure of interest rates in Mexico. Moreover, we provide some empirical evidence on the relationship between the term structure factors and macroeconomic variables. Understanding the term structure of interest rates is important in finance and macroeconomics for different reasons. For monetary economists, the extent to which changes in the short-term policy rate affect long-term yields is important since it represents a key part of the transmission mechanism of monetary policy by affecting the spending, saving and investment behavior of individuals and firms in the economy. Moreover, the yield curve has been found to be a good predictor of future real activity and inflation (see Harvey 1988; Mishkin 1990; and Estrella and Hardouvelis 1991). The term structure also contains information about future short-term interest rates and term premiums. Monetary economists have focused on understanding the relationship between interest rates, monetary policy and macroeconomic variables. They have typically used the expectations hypothesis to describe bond yield dynamics. The expectations hypothesis assumes that term premiums are constant. However, there is strong empirical evidence that suggests that term premiums are time-varying. Financial economists, on the other hand, have mainly focused on forecasting and pricing interest rate related securities. They have developed models on the assumption of absence of arbitrage opportunities, but typically left unspecified the relationship of the term structure with other economic variables. This research has found that almost all movements in the yield curve can be captured in a no-arbitrage framework in which yields are affine functions of a few unobservable or latent factors (e.g., Duffie and Kan 1996, Litterman and Scheinkman 1991, and Dai and Singleton 2000). Litterman and Scheinkman (1991) show for the US that only three factors are needed to explain almost all of the variation in bond yields, and Dai and Singleton (2000), show that an affine arbitrage-free three factor model of the term structure is successful in accounting for features of the data that represent a puzzle for the expectations hypothesis.

We begin our analysis in section 2, where we introduce the affine no-arbitrage model of the term structure of interest rates. The model has two latent factors to reflect the fact that two factors account for much of the variation on the yield curve in Mexico. In section

3 we describe the estimation method, and in section 4 we present the results. First, we show that the model fits the data remarkably well. Second, we show that the first latent factor captures movements in the general level of interest rates, while the second latent factor captures movements in the slope of the yield curve. A positive shock to the first latent factor raises the yields of all maturities by a similar amount. This effect induces an essentially parallel shift in the yield curve, so this factor is called the level factor. A positive shock to the second latent factor increases short-term yields by much more than the long-term yields, thus the yield curve becomes less steep after a positive shock to this factor, so this factor is called the slope factor. Third, the variance decomposition results show that the level factor accounts for a substantial part of the variance at the long end of the yield curve at all horizons, and at the short and middle ranges of the yield curve at medium to long horizons. At short horizons, the slope factor accounts for much of the variance at the short end of the yield curve. Finally, we show that movements in the level of nominal interest rates are associated with movements in long-term inflation expectations, while movements in the slope of the yield curve are associated with movements in the short-term nominal interest rate. In Section 5, we present the conclusions.

## 2 A term structure model with latent factors

To develop a baseline model of the yield curve in Mexico, we estimate an affine no-arbitrage term structure model using zero-coupon bond yields. The term structure of interest rates can be characterized by affine term structure models.<sup>1</sup> These models impose a no-arbitrage condition that links yields at every maturity of the term structure, thereby increasing the efficiency of estimation, and allowing us to forecast the entire yield curve as a function of a few state variables. Affine term structure models start from the assumption of the absence of arbitrage and, thus, have an explicit economic content that puts restrictions on the cross-section and time series behavior of bond prices and interest rates. The models of Vasicek (1977) and Cox, Ingersoll and Ross (1985) are the pioneers of the class of affine term structure models. In the simplest versions of such models, the one-factor models, the

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<sup>1</sup>See Piazzesi (2003) for an excellent overview.

short-term interest rate is the single factor that drives the movements of the term structure.

However, one-factor models have some unrealistic properties. First, they are not able to generate all the shapes of the yield curve that are observed in practice. Second, one factor models do not allow for the twist of the yield curve, i.e. yield curve changes where short-maturity yields move in the opposite direction of long-maturity yields. This is because all yields are driven by a single factor, meaning that they have to be highly correlated. Multifactor models are more flexible and are able to generate additional yield curve shapes and yield curve dynamics. In multifactor models several observed or unobserved risk factors govern the dynamics of the term structure.

The standard affine no-arbitrage term structure model contains three basic equations. The first is the transition equation for the state vector relevant for pricing bonds. We assume that the state vector has two latent factors  $X_t = (X_{1t}, X_{2t})'$ . We choose two latent factors, because they appear to be sufficient to account for most of the variation in the yield curve in Mexico during the sample period considered. In particular, we conducted a principal-components analysis to identify the common factors that drive the dynamics of the Mexican term-structure of interest rates.<sup>2</sup> We found that the first principal component captures 79 percent of the variation in yields, and that the first and second principal components together capture 95 percent of such variation. That is, just two components can account for essentially all of the movements in the yield curve. We assume that the latent factors follow a VAR(1) process:

$$X_t = \Phi X_{t-1} + \Sigma \varepsilon_t \quad (1)$$

where  $\varepsilon_t$  are the shocks to the unobservable factors. We assume that the shocks are IID  $N(0, I_2)$ , that  $\Sigma$  is diagonal, and that  $\Phi$  is a  $2 \times 2$  lower triangular matrix. The second equation defines the one-period short-rate to be an affine function of the state variables:

$$i_t = \delta_0 + \delta_1' X_t \quad (2)$$

We work with montly data, so we use the one-month yield  $y_t^1$  as the short-term interest

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<sup>2</sup>Cortés, Ramos-Francia and Torres (2008) also find that two factors explain 95% of the variation in the yield curve.

rate  $i_t$ . We assume that there is no-arbitrage in the bond market, implying that a positive stochastic discount factor or pricing kernel determines the values of all fixed-income securities. The main result from modern asset pricing states that in an arbitrage-free environment there exists a positive stochastic discount factor  $M$  that gives the price at date  $t$  of any traded financial asset providing nominal cash-flows  $P$  as its discounted future pay-off. Specifically, the value of an asset at time  $t$  equals  $E_t [M_{t+1} D_{t+1}]$ , where  $M_{t+1}$  is the stochastic discount factor, and  $D_{t+1}$  is the asset's value in  $t + 1$  including any dividend or coupon paid by the asset. Because we will be considering zero-coupon bonds, the payout from the bonds is simply their value in the following period, so that the following recursive relationship holds:

$$P_t^n = E_t [M_{t+1} P_{t+1}^{n-1}] \quad (3)$$

where  $P_t^n$  represents the price of an  $n$ -period zero-coupon bond, and the terminal value of the bond  $P_{t+n}^0$  is normalized to 1.

$M$  is also known as the pricing kernel, given that it is the determining variable of  $P$ . Solving forward the pricing equation (3) by the law of iterated expectations and noting that the bond pays exactly one unit at maturity ( $P_{t+n}^0 = 1$ ) yields:

$$P_t^n = E_t [M_{t+1} \dots M_{t+n}] = E_t \left[ \prod_{i=1}^n M_{t+i} \right] \quad (4)$$

so that a model of bond prices could also be expressed as a model of the evolution of the pricing kernel. It follows that we can model  $P_t^n$  by modeling the stochastic process of  $M_{t+i}$ . The bond prices are a function of those state variables that are relevant for forecasting the process of the pricing kernel. Arbitrage free models are equilibrium models, i.e. only equilibrium prices of financial assets are determined. This means that a market that allows for arbitrage is not in equilibrium. Hence, we can exploit no-arbitrage conditions when solving for equilibrium prices.

We can also employ the pricing equation (3) to characterize the compensation for risk that an investor demands for holding a risky bond. If we denote the nominal gross return of

an asset  $(P_{t+1}^{n-1}/P_t^n)$  as  $(1 + i_{t+1}^r)$  we can rewrite (3) and get:

$$1 = E_t [M_{t+1} (1 + i_{t+1}^r)] = E_t [M_{t+1}] E_t [(1 + i_{t+1}^r)] + Cov_t [i_{t+1}^r, M_{t+1}] \quad (5)$$

It follows that:

$$E_t [(1 + i_{t+1}^r)] = \frac{1}{E_t [M_{t+1}]} (1 - Cov_t [i_{t+1}^r, M_{t+1}]) \quad (6)$$

Since the covariance term has to be zero for a risk-free asset, its rate of return has to satisfy:<sup>3</sup>

$$1 + i_t = \frac{1}{E_t [M_{t+1}]} \quad (7)$$

Thus, the excess return of any asset over a risk-free asset, measured as the difference between (6) and (7) is:

$$E_t [i_{t+1}^r] - i_t = - (1 + i_t) Cov_t [i_{t+1}^r, M_{t+1}] \quad (8)$$

Equation (8) illustrates a basic result in finance theory: the excess return of any asset over the risk-free asset depends on the covariance of its rate of return with the pricing kernel. Thus an asset whose pay-off has a negative correlation with the pricing kernel pays a risk premium. In consumption-based equilibrium models, the pricing kernel is equal to the marginal utility of consumption. When consumption growth is high the marginal utility of consumption is low. Therefore if returns are negatively correlated with the pricing kernel, low returns are associated with states of low consumption. A risk premium must be paid for investors to hold such assets because they fail to provide wealth when it is more valuable for the investor.

Following Ang and Piazzesi (2003), we assume that the pricing kernel is conditionally log-normal, as follows:

$$M_{t+1} = \exp \left( -i_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right) \quad (9)$$

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<sup>3</sup>The risk free rate is often referred to as the short rate.



where  $\lambda_t$  are the market prices of risk associated with the innovations of the state variables. In addition, we make the standard assumption of affine models of the term structure, that the prices of risk are affine functions of the state variables. With this assumption, the entire yield curve can be priced from the factor estimates.

$$\lambda_t = \lambda_0 + \lambda_1 X_t \quad (10)$$

Equations (9) and (10) relate shocks in the underlying state variables to the pricing kernel and therefore determine how factor shocks affect all yields. This model belongs to the affine class of term structure models (Brown and Schaefer, 1994; Duffie and Kan, 1996). The affine prices of risk specification in equation (10) has been used by, among others, Constantinides (1992), Fisher (1998), Duffie (2002) and Dai and Singleton (2002) in continuous time and by Ang and Piazzesi (2003), Ang, Piazzesi, and Wei (2005), and Dai and Philippon (2005) in discrete time. As Dai and Singleton (2002) show, the flexible affine price of risk specification is able to capture patterns of expected holding period returns on bonds that we observe in the data.

We take equation (9) to be a nominal pricing kernel which prices all nominal assets in the economy. This means that the total gross return process  $R_{t+1}$  of any nominal asset satisfies:

$$E_t [M_{t+1} R_{t+1}] = 1$$

The state dynamics of  $X_t$  (*equation 1*), the dynamics of the short rate  $i_t$  (*equation 2*) together with the pricing kernel (*equation 9*) and the market prices of risk (*equation 10*) form a discrete-time Gaussian 2-factor model. Since this model falls within the affine class of term structure models, we can show that bond prices are exponential affine functions of the state variables. More precisely, bond prices are given by:

$$P_t^n = \exp \left( \bar{A}_n + \bar{B}_n' X_t \right) \quad (11)$$

where the coefficients  $\overline{A}_n$  and  $\overline{B}_n$  follow the difference equations:

$$\overline{A}_{n+1} = \overline{A}_n - \overline{B}'_n \Sigma \lambda_0 + \frac{1}{2} \overline{B}'_n \Sigma \Sigma' \overline{B}_n + \overline{A}_1 \quad (12)$$

$$\overline{B}'_{n+1} = \overline{B}'_n (\Phi - \Sigma \lambda_1) + \overline{B}_1 \quad (13)$$

$n=1,2,\dots,N$ , with  $\overline{A}_1 = -\delta_0$  and  $\overline{B}_1 = -\delta_1$ . These difference equations can be derived by induction using equation (3). The continuously compounded yield  $y_t^n$  on an  $n$ -period zero coupon bond is given by:

$$y_t^n = -\frac{p_t^n}{n} = A_n + B'_n X_t \quad (14)$$

where  $p_t^n = \log P_t^n$ ,  $A_n = -\frac{\overline{A}_n}{n}$ , and  $B_n = -\frac{\overline{B}_n}{n}$ . The yields are affine functions of the state, so that equation (14) can be interpreted as being the observation equation of a state space system.

Let  $Y_t$  represents the vector containing the zero-coupon bond yields. Then,

$$Y_t = A_y + B_y X_t \quad (15)$$

The holding-period return on an  $n$ -period zero coupon bond for  $\tau$  periods, in excess of the return on a  $\tau$  period zero coupon bond, is given by:

$$rx_{t+\tau}^n = p_{t+\tau}^{n-\tau} - p_t^n - \tau y_t^T = \overline{A}_{n-\tau} + \overline{B}'_{n-\tau} X_{t+\tau} + \overline{A}_\tau + \overline{B}'_\tau X_t - \overline{A}_n - \overline{B}'_n X_t$$

so that the expected excess return is given by:

$$E_t (rx_{t+\tau}^n) = A_n^{\tau x} + B_n'^{\tau x} X_t \quad (16)$$

where  $A_n^{\tau x} = \overline{A}_{n-\tau} + \overline{A}_\tau - \overline{A}_n$ , and  $B_n'^{\tau x} = \overline{B}'_{n-\tau} \Phi^\tau + \overline{B}'_\tau - \overline{B}'_n$ . Using the recursive equations for  $\overline{B}'_n$ , the slope coefficients can be computed explicitly and are given by:

$$B_n'^{\tau x} = \overline{B}'_{n-\tau} [\Phi^\tau - (\Phi - \Sigma \lambda_1)^\tau] \quad (17)$$

Consequently, the one-period expected excess return can be computed using:

$$E_t (rx_{t+\tau}^n) = A_n^x + B_n^{lx} X_t \quad (18)$$

where  $A_n^x = \overline{B}_{n-1}' \Sigma \lambda_0 - \frac{1}{2} \overline{B}_{n-1}' \Sigma \Sigma' \overline{B}_{n-1}$ , and  $B_n^{lx} = \overline{B}_{n-1}' \Sigma \lambda_1$ .

From equation (18), we can see directly that the expected excess return comprises three terms: (i) a Jensen's inequality term  $-\frac{1}{2} \overline{B}_{n-1}' \Sigma \Sigma' \overline{B}_{n-1}$ , (ii) a constant risk premium  $\overline{B}_{n-1}' \Sigma \lambda_0$ , and (iii) a time-varying risk premium  $\overline{B}_{n-1}' \Sigma \lambda_1$ . The time variation is governed by the parameters in the matrix  $\lambda_1$ . This relation basically says that the expected excess log return is the sum of two risk premium terms and a Jensen's inequality term. The term premium is governed by the vector  $\lambda$ . A negative sign leads to a positive bond risk premium. This can be reasoned as follows. Consider a positive shock  $\varepsilon_{t+1}$  which increases a state variable. According to (11), (12) and (13) this lowers all bond prices and drives down bond returns. When  $\lambda$  is positive, the shock also drives down the log value of the pricing kernel (9), which means that bond returns are positively correlated with the pricing kernel. As explained above, this correlation has a hedge value, so that risk premia on bonds are negative. The same reason applies to the case when  $\lambda$  is negative, which leads to a positive risk premia.

Since both bond yields and the expected holding period returns of bonds are affine functions of  $X_t$ , we can easily compute variance decompositions following standard methods. The dynamics of the term structure depend on the risk premia parameters  $\lambda_0$  and  $\lambda_1$ . A non-zero vector  $\lambda_0$  affects the long-run mean of yields because this parameter affects the constant term in the yield equation (14). A non-zero matrix  $\lambda_1$  affects the time-variation of risk-premia, since it affects the slope coefficients in the yield equation (14). A model with a non-zero  $\lambda_0$  and zero matrix  $\lambda_1$ , allows the average yield curve to be upward sloping, but does not allow risk premia to be time-varying.

If investors are risk neutral,  $\lambda_0 = 0$  and  $\lambda_1 = 0$ . This case is usually called the Expectations Hypothesis. Macro models, such as Fuhrer and Moore (1995), usually impose the Expectation Hypothesis to infer long term yield dynamics from short rates.

In general, the yields on zero coupon bonds are determined by two components: (1) the

expected future path of one-period interest rates and (2) the excess returns that investors demand as compensation for the risk of holding longer-term instruments.

### 3 Estimation method

For a given set of observed yields, the likelihood function of this model can be calculated, and the model can be estimated by maximum likelihood. The yields themselves are analytical functions of the state variables  $X_t$ , which will allow us to infer the unobservable factors from the yields. To do this, we follow Chen and Scott (1993) and assume that as many yields as unobservable factors are measured without error, and the remaining yields are measured with error. We estimate this model using monthly data from January 2001 to June 2007 on five zero-coupon yields that have maturities of 1, 12, 36, 60 and 120 months. Since there are two latent factors but five observable yields, we assume that the 12, 36 and 60 month yields are measured with error, as in Ang and Piazzesi (2003).

#### 3.1 Innovations Representation

Constructing an innovations representation is a key step for evaluating the likelihood function. The state-space of the model is the following:

$$\tilde{X}_{t+1} = A\tilde{X}_t + B\varepsilon_{t+1} \quad (19)$$

$$Y_t = C\tilde{X}_t + w_t \quad (20)$$

$$w_t = Dw_{t-1} + \eta_t \quad (21)$$

where  $\tilde{X}_t = [X_{1t}, X_{2t}, 1]'$ ,  $Y_t = [y_t^1, y_t^{12}, y_t^{36}, y_t^{60}, y_t^{120}]'$ , and

$$A = \begin{bmatrix} \Phi & 0_{2 \times 1} \\ 0_{1 \times 2} & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} \Sigma \\ 0_{1 \times 2} \end{bmatrix}$$

$$C = \begin{bmatrix} B'_1 & A_1 \\ B'_{12} & A_{12} \\ B'_{36} & A_{36} \\ B'_{60} & A_{60} \\ B'_{120} & A_{120} \end{bmatrix}$$

The elements of  $D$  are the parameters governing serial correlation of the measurement error. We assume that  $E_t \eta_t \eta'_t = R$ , and  $E_t \epsilon_t \eta'_s = 0$  for all periods  $t$  and  $s$ . We define the quasi-differenced process as:

$$\bar{Y}_t = Y_{t+1} - DY_t \quad (22)$$

Then we can rewrite the system as:

$$\tilde{X}_{t+1} = A\tilde{X}_t + B\varepsilon_{t+1} \quad (23)$$

$$\bar{Y}_t = \bar{C}\tilde{X}_t + CB\varepsilon_{t+1} + \eta_{t+1} \quad (24)$$

where  $\bar{C} = CA - DC$ . The innovation vector  $u_t$  and its covariance  $\Omega_t$  are defined as follows:

$$\begin{aligned} u_t &= \bar{Y}_t - E[\bar{Y}_t | \bar{Y}_{t-1}, \bar{Y}_{t-2}, \dots, \bar{Y}_0, \hat{X}_0] = Y_{t+1} - E[Y_{t+1} | Y_t, Y_{t-1}, \dots, Y_0, \hat{X}_0] \\ &= Y_{t+1} - DY_t - \bar{C}\hat{X}_t \end{aligned}$$

which depends on the predicted state  $\hat{X}_t$ :

$$\hat{X}_t = E[\tilde{X}_t | Y_t, Y_{t-1}, \dots, Y_0, \hat{X}_0]$$

$$\Omega_t = Eu_t u'_t = \bar{C}\Sigma_t \bar{C}' + R + CBB'C'$$

The predicted state evolves according to:

$$\hat{X}_{t+1} = A\hat{X}_t + K_t u_t$$

where  $K_t$ , and  $\Sigma_t$  are the Kalman gain and state covariance associated with the Kalman filter

$$\begin{aligned} K_t &= (BB'C' + A\Sigma_t\bar{C}')\Omega_t^{-1} \\ \Sigma_{t+1} &= A\Sigma_tA' + BB' - (BB'C' + A\Sigma_t\bar{C}')\Omega_t^{-1}(\bar{C}\Sigma_tA' + CBB') \end{aligned}$$

Then an innovations representations for the system is:

$$\hat{X}_{t+1} = A\hat{X}_t + K_t u_t \quad (25)$$

$$\bar{Y}_t = \bar{C}\hat{X}_t + u_t \quad (26)$$

Initial conditions for the system are  $\hat{X}_0$  and  $\Sigma_0$ . We can use this innovations representation recursively to compute the innovation series, and then calculate the log-likelihood function.

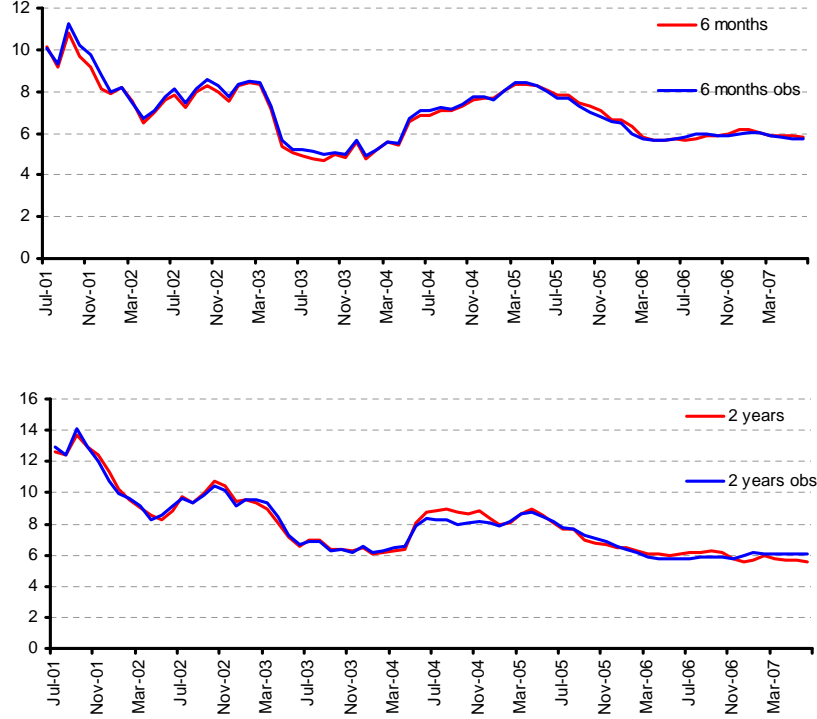
$$\ln L(\Theta) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=0}^{T-1} \ln |\Omega_t| - \frac{1}{2} \sum_{t=0}^{T-1} u_t' \Omega_t^{-1} u_t$$

where the parameters to be estimated are stacked in the vector  $\Theta$ , the innovation vector is  $u_t$ , and its covariance matrix is  $\Omega_t$ . The parameters that are estimated are the elements of  $\Phi, \Sigma, \delta_0, \delta_1, \lambda_0, \lambda_1$ , and  $R$ .

## 4 Results

As noted earlier, related models, such as those of Ang and Piazzesi (2003) and Rudebusch and Wu (2005), explain some important features of the term-structure of interest rates in the U.S. by using latent factors. Moreover, these models have found that a few latent factors drive most of the dynamics of the yield curve in the U.S. Before examining the dynamic properties of the model, it is useful to analyze how well the model fits the data. Figures 1 and 2 compare the fitted and actual time series for the, 6 month, 2-year, 3-year and 7-year yields. As we can see from these figures, the model predicts yields on zero coupon bonds reasonably well.

Figure 1

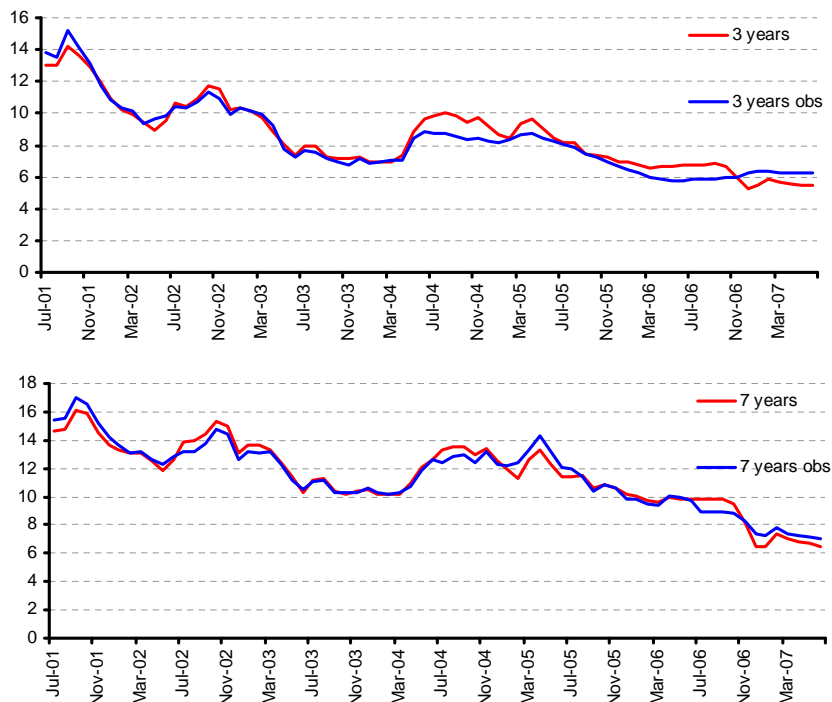


The parameter estimates of the model are reported in Table 1. As is typically found in empirical estimates, the latent factors differ somewhat in their time-series properties as shown by the estimated  $\Phi$ . The first latent factor is very persistent, while the second latent factor is mean-reverting. There is also small but significant cross-correlation between these factors. The prices of risk  $\lambda_0$  and  $\lambda_1$ , appear significantly as well. Negative parameters in  $\lambda_0$  induce long yields to be on average higher than short yields. Time-variation in risk premia is driven by  $\lambda_1$ . Thus, negative values of  $\lambda_1$  induce long yields to increase relative to short yields in response to positive shocks to the state variables.

Table 1  
Parameter estimates with Standard Errors

$\Phi_{11}$	$\Phi_{21}$	$\Phi_{22}$	$\delta_0$	$\Sigma_{11}$	$\Sigma_{22}$
0.992 (0.00014)	0.053 (0.0013)	0.828 (0.0039)	6.61 (0.02)	0.271 (0.0098)	0.443 (0.0077)
$\lambda_{0,1}$	$\lambda_{0,2}$	$\lambda_{1,11}$	$\lambda_{1,21}$	$\lambda_{1,12}$	$\lambda_{1,22}$
-6 (0.04)	-0.04 (0.0011)	-0.02 (0.0013)	-0.04 (0.0014)	-0.06 (0.0016)	-0.09 (0.0008)

Figure 2

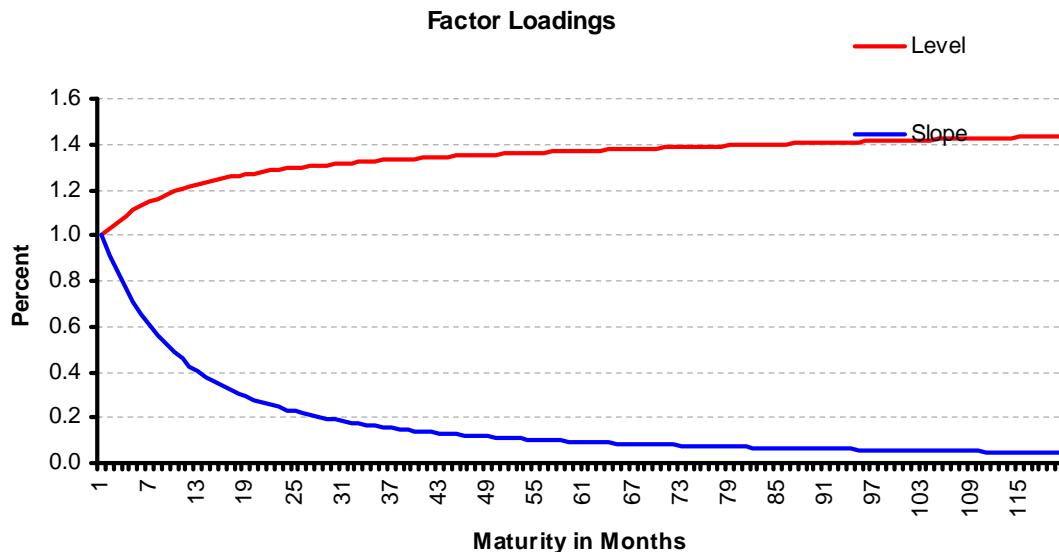


From equation (13), the effect of each factor on the yield curve is determined by the weights  $B_n$  that the term structure model assigns on each yield of maturity  $n$ , these weights  $B_n$  are also called factor loadings. These loadings show the initial response of yields of various maturities to a one standard deviation increase in each factor. Figure 3 plots these weights as a function of yield maturity. A positive shock to the first latent factor raises the yields of all maturities by a similar amount. This effect induces an essentially parallel shift in the yield curve, so this factor is called the level factor. A positive shock to the second latent factor increases short-term yields by much more than the long-term yields, thus the yield curve becomes less steep after a positive shock to this factor, so this factor is called the slope factor.

Litterman and Scheinkman (1991) label these latent factors level and slope respectively because of the effects of these factors on the yield curve. To show these effects, Figure 4 plots the first latent factor and a "level " transformation of the yield curve. We measure



Figure 3



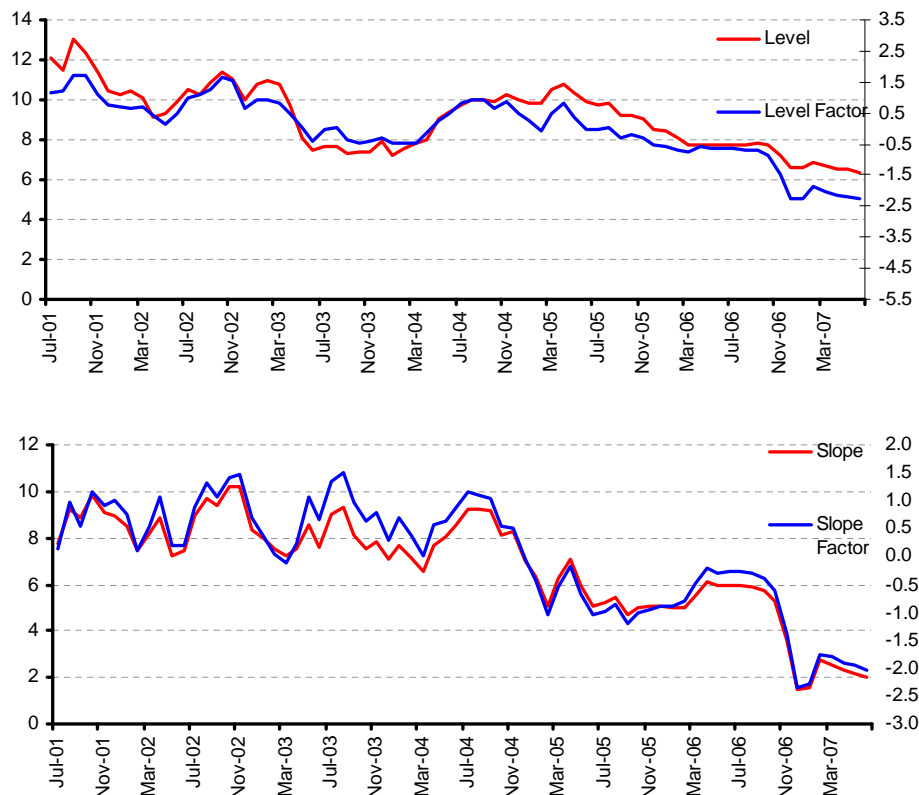
the level as the equally weighted average of the 1 month rate, 1 year and 10 year yields  $(y_t^1 + y_t^{12} + y_t^{120}) / 3$ . The correlation coefficient between the first factor and the level transformation is 92.5%. Figure 4 also plots the second latent factor and the slope of the yield curve, defined as the 10 year spread  $(y_t^{120} - y_t^1)$ , the correlation coefficient between the second factor and the slope of the yield curve is 98.5%.

To determine the relative contributions of the latent factors to forecast variances we construct variance decompositions. These show the proportion of the forecast variance attributable to each factor. Table 2 reports the variance decomposition for the 1-month, 12-month, 3-year, 5-year and 10-year yields at different forecast horizons. The level factor accounts for a substantial part of the variance at the long end of the yield curve at all horizons and at the short and middle ranges of the yield curve at medium to long horizons. At short horizons, the slope factor accounts for much of the variance at the short end of the yield curve. The level factor dominates the variance decompositions at long horizons across the yield curve.

Table 2  
Variance Decomposition

<b>Forecast-Horizon</b>	<b>Level</b>	<b>Slope</b>
<b>1-month yield</b>		
1 month	10.92	89.08
12 months	49.70	50.30
36 months	89.73	10.27
60 months	96.29	3.71
120 months	99.15	0.85
<b>12-month yield</b>		
1 month	13.47	86.53
12 months	56.64	45.36
36 months	91.30	8.70
60 months	96.88	3.12
120 months	99.28	0.72
<b>36-month yield</b>		
1 month	70.70	29.30
12 months	93.33	6.67
36 months	99.09	0.91
60 months	99.68	0.32
120 months	99.93	0.07
<b>60-month yield</b>		
1 month	80.56	19.44
12 months	95.94	4.06
36 months	99.45	0.55
60 months	99.81	0.19
120 months	99.93	0.04
<b>120-month yield</b>		
1 month	89.41	40.59
12 months	97.93	2.07
36 months	99.72	0.28
60 months	99.90	0.10
120 months	99.98	0.02

Figure 4

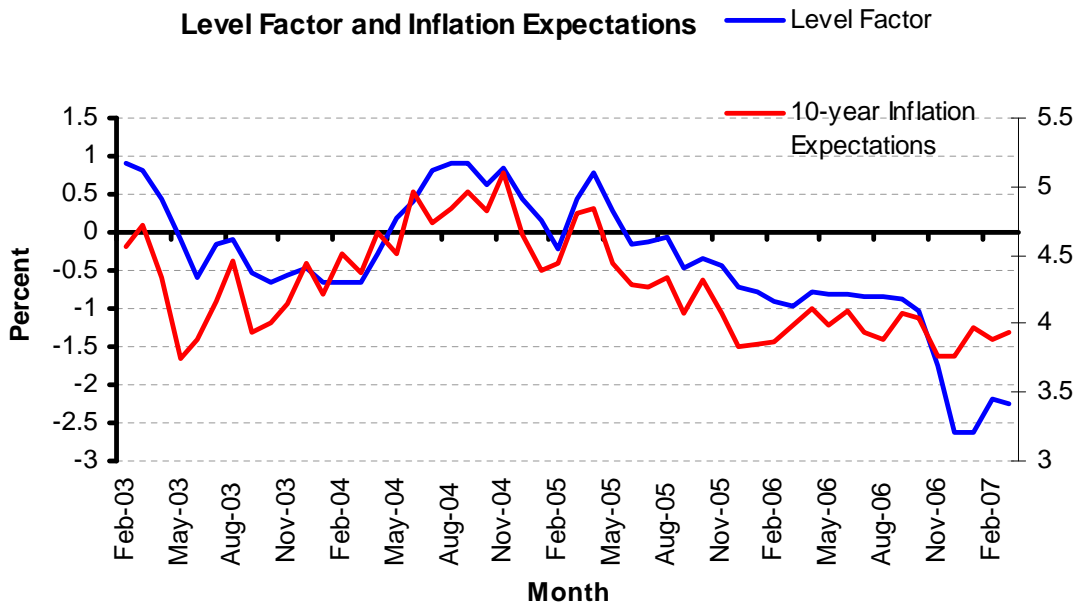


The correlation coefficient between the first factor and the level is 0.925  
The correlation coefficient between the second factor and the slope is 0.985

We have shown that the first latent factor captures movements in the general level of nominal interest rates, while the second latent factor captures movements in the slope of the nominal yield curve. Rudebusch and Wu (2004) identify movements in the level factor with changes in long-term inflation expectations. They also relate movements in the slope factor with the business cycle. In particular, they claim that the slope factor varies as the central bank moves the short end of the yield curve up and down during expansions and recessions respectively. To analyze if these relationships hold in the Mexican yield curve as well, figure 5 displays the level factor, and a measure of long-run inflation compensation or long-term inflation expectations, which is measured as the spread between 10-year nominal and indexed debt.<sup>4</sup> Figure 5 shows that the estimated level factor appears to be closely linked to long-

<sup>4</sup>This indicator also includes an inflation risk premium.

Figure 5



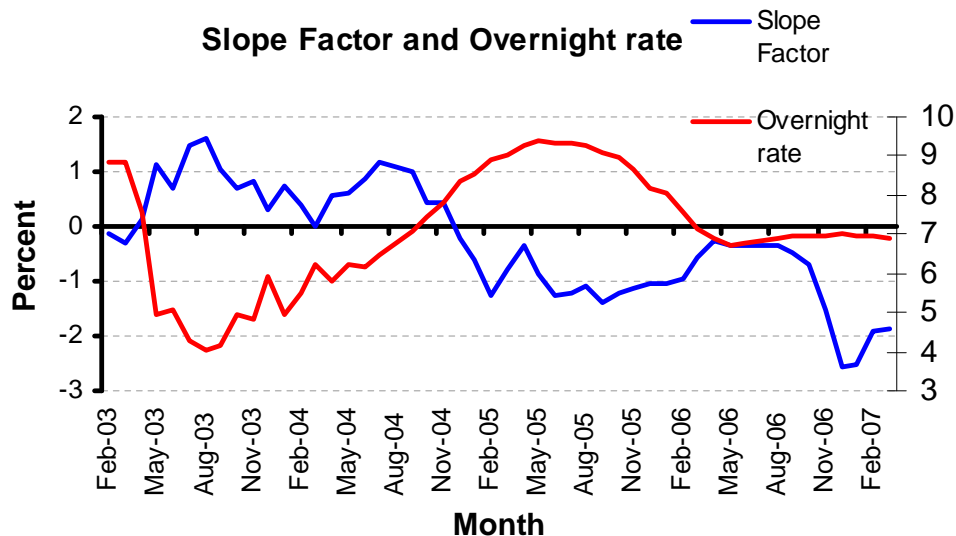
The correlation coefficient is 0.81

term inflation expectations, the correlation coefficient between these time series is 81%. Thus, this figure suggests that movements in the general level of nominal interest rates are associated with movements in long-term inflation expectations. This evidence is consistent with previous studies in the literature, for example, Barr and Campbell (1997) conclude that almost 80% of the movement in log-term nominal rates appears to be due to changes in expected long-term inflation. Figure 6 displays the slope factor and the overnight rate, the correlation coefficient between these series is -65%. This empirical evidence is consistent with Rudebusch and Wu (2004), who find a negative correlation between the policy rate and the slope factor in the US.

## 5 Conclusions

We have developed and estimated an affine model that characterizes the dynamics of the term structure of interest rates in Mexico. Moreover, we have provided some empirical evidence

Figure 6



The correlation coefficient is -0.65

on the relationship between the term structure factors and macroeconomic variables. We find that the affine model with two latent factors fits the data remarkably well. Moreover, our estimation results, based on Mexican zero-coupon bond yields, show that the first latent factor captures movements in the general level of interest rates, while the second latent factor captures movements in the slope of the yield curve. A positive shock to the first latent factor raises the yields of all maturities by a similar amount. This effect induces an essentially parallel shift in the yield curve, so this factor is called the level factor. A positive shock to the second latent factor increases short-term yields by much more than the long-term yields, thus the yield curve becomes less steep after a positive shock to this factor, so this factor is called the slope factor. The variance decomposition results show that the level factor accounts for a substantial part of the variance at the long end of the yield curve at all horizons, and at the short and middle ranges of the yield curve at medium to long horizons. At short horizons, the slope factor accounts for much of the variance at the short end of the yield curve. We also show that movements in the level of nominal interest rates

are associated with movements in long-term inflation expectations, while movements in the slope of the yield curve are associated with movements in the short-term nominal interest rate.

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