

Bokor, László

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## Optimality criteria of hybrid inflation-price level targeting

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MAGYAR NEMZETI BANK

# **MNB WORKING PAPERS**

**2010/8**

**LÁSZLÓ BOKOR**

**Optimality criteria of hybrid inflation-price  
level targeting**



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MNB Working Papers 2010/8

**Optimality criteria of hybrid inflation-price level targeting\***

(A hibrid inflációs árszínvonal-célkitűzés optimalitási feltételei)

Written by: László Bokor\*\*

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\*\* E-mail: bokor@eik.bme.hu.

# Contents

<b>Abstract</b>	5
<b>1 Introduction</b>	6
<b>2 The model</b>	9
2.1 The economy	9
2.2 The regimes	9
2.3 The three-period analytical framework	10
<b>3 Comparing the regimes</b>	14
3.1 No exogenous persistence ( $\rho=0$ )	14
3.2 The role of the exogenous persistence ( $\rho>0$ )	22
<b>4 Inflation and output gap variability</b>	28
<b>5 Notes on empirical issues</b>	30
<b>6 Conclusions</b>	32
<b>Appendix</b>	33
A Model solutions	33
B Social preference weight and the 'slope' of the Phillips curve	34
<b>References</b>	36



# Abstract

This paper provides a sensitivity analysis of the relative performance of inflation targeting, price level targeting, and hybrid targeting, the combination of these two. A simple, three-period, steady state to steady state economy is presented, where monetary policy is facing various sets of forward and backward looking expectations, social preferences on inflation and output gap stabilization, and degrees of cost push shock persistence. We derive optimal policy mix under the whole spectrum of these economic conditions, reporting also the criteria of the replicability of the theoretically optimal solution. The main intention of the examination is to reveal the nature of each interrelation between economic and policy parameters. The results show that (i) the relative strength of regimes depends heavily on the preconditions, and that (ii) the relationships of parameters related to the performance are non-linear and occasionally non-monotonic as well. Our model specification is somewhat restrictive, however, contrary to the related literature, the examination, even in the intermediate cases, can be conducted analytically.

**JEL:** E50, E52, E58.

**Keywords:** hybrid inflation-price level targeting, hybrid New Keynesian Phillips curve, cost push shock persistence.

## Összefoglalás

Ebben az írásban az inflációs célkövetés, az árszínvonal-célkitűzés, valamint ezek kombinációjának, a hibrid célkitűzésnek az érzékenységvizsgálatát követhetjük nyomon. Egy egyszerű, háromperiódusos, egyensúlyi helyzettől egyensúlyi helyzetig mozgó gazdaságot mutatunk be, melyben a monetáris politika az előre és hátratekintő várakozásoknak, az infláció és az output gap stabilizációjára vonatkozó társadalmi preferenciának, valamint a cost push sokk perzisztenciának különböző összetételeivel szembesül. Levezetjük az optimális politikai mixet ezen gazdasági paraméterek valamennyi lehetséges kombinációja mellett, jelezve az elvi szinten optimális politika reprodukálhatóságának kritériumait. A vizsgálat legfőképp arra irányul, hogy feltárjon minden egyes összefüggést a gazdasági és policy paraméterek között. Az eredmények azt mutatják, hogy (i) a rezsimek relatív ereje nagyban függ az előfeltételektől, valamint hogy (ii) az egyes paraméterek viszonya a teljesítmény vonatkozásában nem lineáris, és bizonyos esetekben nem is monoton. Modellspecifikációnk némileg korlátozó, mindazonáltal így a vizsgálatokat – a vonatkozó irodalomtól eltérően – köztes esetekben is analitikusan tudjuk végezni.



# 1 Introduction

The difference between the policy of inflation targeting and price level targeting can be captured in their attitude to shocks affecting price level. In most general terms, inflation targeting attempts to maintain a targeted inflation path, and does not care for unanticipated misses in the past which implies the rising uncertainty around the future price level. Price level targeting attempts to maintain a targeted price path which implies that uncertainty around the price level does not increase with the progress of time.<sup>1</sup> In other words, thinking in terms of price level, inflation targeting is a regime without memory, while price level targeting is a history-dependent policy.

A fixed price level has reasonable benefits. Planning and contracting becomes easier as nominal values become real values. The information learnt from prices is without any distortion, since their realignments would purely reflect scarcity, which enhances the resource allocation mechanism. The continuous transfer of welfare from the cash holders to the government using inflation device (not just surprise inflation!) is also wiped out. However, the idea of price level targeting has been criticized from both practical and theoretical standpoint.<sup>2</sup> The “conventional wisdom”, as Svensson (1999) named it, states that the consequence of price level stabilization is the higher volatility of inflation and output gap.<sup>3</sup> “The intuition is straightforward: In order to stabilize the price level under price-level targeting, higher-than-average inflation must be succeeded by lower-than-average inflation. This should result in higher inflation variability than inflation targeting, since in the latter case, base level drift is accepted and higher-than-average inflation need only be succeeded by average inflation. Via nominal rigidities, the higher inflation variability should then result in higher output variability.” Svensson (1999, p. 278) Svensson (1999) pointed out that the root of the conventional results arises from the usage of postulated reaction functions instead of endogenous decision rules. He showed, using a New Classical Phillips curve, that under discretion price level targeting provides lower inflation variability, than inflation targeting does, without affecting the output gap variability at the same time, if there is sufficiently high persistence in the output gap. He called this “free lunch”. All other things being equal, Dittmar and Gavin (2000) used ‘New Keynesian expectations’. Their analysis showed that price level targeting always outperforms inflation targeting both in inflation and output gap variability. They also demonstrated that the rise of the output gap persistence further amplifies this advantage. Vestin (2006) made his comparison in a ‘New Keynesian economy’ with cost push shock persistence. He also demonstrated that free lunch result holds, even if there is no endogenous output gap persistence, and that if there is no persistence in cost push shocks, price level targeting can implement commitment solution. With exogenous inflation persistence, price level targeting can be also better than inflation targeting, though the key issue is the assignment of proper preference weight in the loss function of the central bank.<sup>4</sup>

As Woodford (2000) emphasized, the optimal policy under commitment is history-dependent in the case of forward looking expectations. However, since it is generally time-inconsistent, it does not provide a too realistic solution.<sup>5</sup> The point is to implement such a discretionary policy, that can incorporate the past in the decision making process. A predetermined price

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<sup>1</sup> With a constant inflation target, inflation is stationary, while the (log) price level has a unit root. If the inflation target is zero, the (log) price level follows random walk, if it is a positive value, then the (log) price level follows a stochastic trend. With a constant price level target, the price level is stationary around the targeted value, and inflation is stationary, too, around zero. If the targeted (log) price path has a constant positive slope, then the (log) price level is trend stationary as it follows a deterministic trend, and inflation becomes stationary around the slope of the (log) targeted price path.

<sup>2</sup> Fischer (1994) argues that since indexed financial assets and nominal contracting are given, targeted price level has not too much sense. McCallum (1999) also claims that the abovementioned benefits would not be significant in the United States.

<sup>3</sup> Svensson (1999) referred to the writings of Lebow, Roberts and Stockton (1992), Fischer (1994), and Haldane and Salmon (1995).

<sup>4</sup> Svensson (1999) and Dittmar and Gavin (2000) derived their results delegating the same preference weight into the loss functions of the central banks. It is important to emphasize that considering nothing but the value of the preference weights, a regime targeting inflation and a regime targeting price level cannot be compared to figure out how much they are “conservative”. In inflation targeting, the weight indicates the relative significance of the variability of output gap compared to the variability of inflation, while in price level targeting, it signs the same to the variability of price level.

<sup>5</sup> Woodford’s (1999) concept of “timeless perspective” ignores the initial conditions of the regime, eventuate in commitment policy that is time-consistent.

level target operates as a solid nominal anchor, and incorporates history dependent policy.<sup>6</sup> Under New Classical Phillips curve, current inflation-output gap trade off is not affected by inflation expectations, as they are predetermined. However, as Barnett and Engineer (2001) explained, with the existence of sufficiently high persistence in the output gap, rational agents indirectly form their expectations in a forward looking manner, as they know the future persistency effects of the current output gap affecting the trade off. In a New Keynesian economy the relation is more straightforward, as here inflation expectations affect inflation-output gap trade-off right in the present, and so the gains of a credible price level target arise immediately.<sup>7</sup>

Since efficiency of price level targeting and inflation targeting is sensitive to the key assumptions on, for instance, expectations, several examinations were concluded with creating more generalized economic environments and implementing new hybrid policies. Nessén and Vestin (2005) demonstrated, using a New Keynesian Phillips curve, that a policy targeting the average inflation of several forthcoming periods provides better performance than inflation targeting, but worse than price level targeting.<sup>8</sup> They also showed, using a hybrid Phillips curve with exogenous inflation persistence, that if backward pricing more and more characterizes the economy, then the benefits of price level targeting deteriorates, and from one point average inflation targeting will offer the best solution, while approaching rather backward looking economy, inflation targeting becomes the best out of the discretionary regimes. Under similar conditions, Walsh (2003) also came to these results examining an alternative policy aimed at stabilizing the change in the output gap (so called “speed limit” policy). Batini and Yates (2003) analyzed such a hybrid regime that combines the characteristics of inflation targeting and price level targeting, incorporating the weighted average of inflation and price level target into the central bank’s loss function. Analyzing different policy rules (optimal, contemporaneous, and forward looking rules) under two sets of inflation expectation structure (a partially and a fully forward looking set), they came to totally different results in point of inflation, output gap, and price level variability. At a certain expectation mix that contains forward and backward looking elements, they concluded that hybrid targeting is better than unmixed policies when policy rules are set in a forward looking manner, instead of using simple rules with smaller information set such as Taylor’s (1993). On the contrary, in the fully forward looking case, pure policies always dominate hybrid policies. Analyzing open economy aspects, they found antagonistic forces. The familiar effect is that price level targeting helps to lower inflation variability through the expectational channel, on the other hand, the higher variability of interest rate under price level targeting increases the variability of exchange rate, thus increases variability of inflation more than under inflation targeting, eventuating in an unknown resultant. In point of output gap persistence, the results of Batini and Yates (2003) contradict those of Svensson’s (1999), since in their model the rise of persistency leads to the deterioration of inflation stabilizing performance of price level targeting.<sup>9</sup> Cecchetti and Kim (2005) showed that in an economy represented by a New Classical Phillips curve with sufficiently high output gap persistence, an appropriately chosen hybrid target according to the degree of persistence results in optimal policy.<sup>10</sup> Contrary to the results of Batini and Yates (2003), they did not find significantly different outcomes when moving from a closed to an open economy model.

<sup>6</sup> *Commitment* means that the central bank sticks to the plan, whatever events may come. On the contrary, in the case of *discretionary* decision-making, an optimization process takes place in every period based on the new circumstances – a new plan is settled. Kydland and Prescott (1977), and Barro and Gordon (1983) have revealed that in the latter case, the central bank quite often has the temptation to raise output over the potential level by using surprise inflation. However, actors of the economy cannot be cheated on a long run, which results in the fall of the output to the original level with higher inflation. The difference between the experienced average inflation rate and the inflation target is the so-called *inflation bias*. Even if the central bank had no such intention, it would suffer from credibility deficit due to the lack of a long-term commitment, which results in higher loss due to the more drastic interventions needed. This is the so-called *stabilization bias*. All these phenomena originate in the problem of time-inconsistency, i.e. a decision which looked good before fails to be optimal after the expectations formed. Commitment solution excludes cheating possibility, thus results in the achievable social optimum (In other words, expected loss is the lowest under commitment; inflation bias and stabilization bias under discretion can be interpreted relatively to this benchmark.) However, the problem is that this solution is not realistic, because it is time-inconsistent. The advantage of the discretionary decision-making is that it gives space to react to non-foreseeable economic changes. The goal is to set such “guaranties” that do not hinder the operation flexibility of the central bank, but where it can also increase its credibility, hence it is able to reduce the aforementioned biases. Inflation targeting and price level targeting under discretion bear precommitment features in a different extent, which is a key factor in their relative performance.

<sup>7</sup> In Svensson’s New Classical model, inflation is the linear function of the output gap under inflation targeting, while under price level targeting, it is (with equal functional form) the linear function of the change in the output gap. Since output movement is defined as an AR(1) process, the variance of the gap is lower, than the variance of the first difference of the gap, if the autoregressive coefficient is larger than 0.5. In the New Keynesian case, the result comes from the microfoundation that the price setting opportunities of the firms are stochastically and exogenously given. This kind of staggered price setting ensures that rational agents ‘damp’ their price setting sensitivity on marginal cost changes in the case of a credible price level target.

<sup>8</sup> Note that price level targeting is a policy that targets average inflation of infinite periods, while inflation targeting aims the ‘average’ of one period ahead only. Properties of the intermediate case (i.e. average inflation targeting) are extensively examined using New Classical Phillips curve by Nessén (2002).

<sup>9</sup> Batini and Yates (2003) argued it is the reason that in their model output gap depends on other variables, while in the model of Svensson (1999) it follows an AR(1) process, thus in their model a more persistent output gap is harder to control.

<sup>10</sup> In the model of Cecchetti and Kim (2005) the output gap follows an AR(1) process, just like in Svensson’s (1999). Under same conditions, Kobayashi (2005) presented that hybrid targeting always gives better solution than price level targeting, but it outperforms inflation targeting only when the autoregressive coefficient is larger than 1/3. Recall that Svensson (1999) showed that price level targeting dominates inflation targeting when the coefficient is larger than 0.5.

In this paper, the examination extended to all intermediate cases of expectation spectrum, fully highlighting the interrelationships between the hybrid policy weight-expectation weight-social preference weight-Phillips curve slope-cost push shock persistence dimensions and their related social loss implications. Furthermore, utilizing a three-period analytical framework, we are able to do this analytically. We provide the optimal hybrid policy weight and also the existence criteria of the replicability of the model's theoretical benchmark, and examine the underlying factors of inflation and output gap variability as well.

Batini and Yates (2003) applied only a fixed hybrid expectation structure, thus they did not analyse all of the intermediate cases and did not derive optimal hybrid policy weight analytically. Cecchetti and Kim (2005) derived optimal hybrid policy analytically, but under New Classical Phillips curve with output gap persistence, thus did not examine the impact of various expectation structures.<sup>11</sup> By construction, none of them analyzed the implications of the interdependence of endogenous and exogenous persistence related to the relative performance issues of the regimes, which we will.

Our results show that nonlinearities like the one experienced in performance when moving on the spectrum of regimes described by Batini and Yates (2003) and Cecchetti and Kim (2005) are also perceivable when examining the spectrum of expectations, as well as other variables. Thus, it is important to unfold as much of these connectivities as possible.

Briefly, the paper proceeds as follows. The model is introduced in Section 2. Section 3 and 4 provides comparative under different economic circumstances, beginning with the least realistic case up to the most general case. Section 5 gives a brief empirical outlook related to the results, while Section 6 concludes.

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<sup>11</sup> Recalling the analogous examinations, in the case of average inflation targeting, Nessén and Vestin (2005) also did not offer analytical solutions when the considered horizon of calculating the average inflation rate is longer than two periods, because, as they noted, those cases are analytically intractable. Walsh (2003) also used numerical methods in his comparative analysis.

# 2 THE MODEL

## 2.1 THE ECONOMY

Suppose that current inflation in the economy is given by factors presented in the following system of hybrid Phillips curve and aggregate demand,<sup>12</sup>

$$\begin{aligned}\pi_t &= \phi\pi_{t+1|t} + (1-\phi)\pi_{t-1} + \delta x_t + u_t, \\ x_t &= f(\text{instruments}_{t-1}),\end{aligned}\tag{1}$$

where (in a logarithm)  $\pi_t$  is the inflation rate,  $\pi_{t+1|t}$  is the forward looking rational inflation expectation,  $\pi_{t-1}$  is the backward looking adaptive inflation expectation,  $x_t$  is the output gap (the difference between the actual and the natural level of output),  $u_t$  is an AR(1) disturbance term, where is an i.i.d. with zero mean and variance of  $\sigma^2$ , and  $\phi, \delta, \rho$  are constants ( $0 \leq \phi \leq 1, \delta > 0$ );  $\phi$  gives the composition of the expectations of the agents in economy,  $\delta$  denotes the ‘slope’ coefficient of the Phillips curve (accurately, the relation of the output gap to the difference of inflation realized and the one expected), and  $\rho$  indicates the persistence of cost push type supply shock.<sup>13</sup>

What does this Phillips curve consider and what does it not cover? Inflation is influenced by three factors on a general basis: expectations, shocks, and cyclic factors. The model captures various expectation structures, persistent cost push shock, and it also considers a one-period monetary control lag. As shocks affecting potential output and aggregate demand are not modelled<sup>14</sup>, the value  $x_t$  of is unambiguously determined by the monetary instruments (usually specified as policy interest rate) set before the period, namely on the basis of the information in period  $t-1$ . Although  $x_t$  is under the perfect control of the central bank, actually, it can respond to a concurrent shock in the next periods only. It follows that contrary to the more prevalent assumption of the topic literature, the central bank has not perfect control over inflation.<sup>15</sup>

Both endogenous and exogenous inflation persistence stand in accordance with the general perception that the inflation process has inertia, however, they presume widely different policy implications.<sup>16</sup> Furthermore, the lowered reaction capability of the central bank reflects that monetary actions exert their full impact in a longer time.

## 2.2 THE REGIMES

We consider four regimes, the theoretical benchmark and three discretionary solutions, namely inflation targeting with commitment, inflation targeting, price level targeting, and the hybrid regime of the latter two. Standard quadratic loss functions used in the literature generally incorporate the inflation, output gap, and seldom the nominal interest rate variability.

<sup>12</sup> Notation  $q_{ij}$  is used instead of  $E q_i$ . Both denote the expected value of the variable  $q$  at time  $i$  conditional upon information available in time  $j$ .

<sup>13</sup> As Galí and Gertler (1999) discussed its microfoundations, this form of the Phillips curve is evolving from the price setting behaviour of the firms. In their staggered price setting model, the competition is monopolistic, thus the ‘benchmark’ output level (the natural level of output) implies equilibrium under imperfectly competitive markets without nominal rigidities. Coefficient  $\delta$  comprises the sensitivity of marginal cost to changes in output gap and the sensitivity of firms’ price adjustment in relation to marginal cost. Term  $u_t$  captures movements in marginal cost due to factors other than output gap; it disappears eventually if no other market frictions exist than nominal price rigidity. More complicated quantitative models introduce wage rigidity as well (usually entering staggered wage contracts à la Erceg et al. (2000) in order to endogenize the run of the cost push shock. In our model, we treat  $u_t$  as exogenously given.

<sup>14</sup> Nevertheless, it is not a shortcoming at all, if we consider the study of Coletti et al. (2008). Utilizing a form of the IMF’s full-scale, multi-country, dynamic stochastic general equilibrium model (Global Economic Model, GEM), they showed that price level targeting performs always better than inflation targeting considering demand shocks, independently from the expectation structure. They also showed, however, that IT begins to dominate PT, when price and wage mark-up shocks hit the economy, if the proportion of rule-of-thumb price and wage setters shifts from 40 per cent to 50 per cent. Relying on this result, we may focus solely on the impacts of the latter kind of shocks that, in the short run, create antagonistic movements between inflation and output gap, offering a trade-off between inflation stabilization and output gap stabilization.

<sup>15</sup> Svensson (1999), Vestin (2006), Dittmar and Gavin (2000), Nessén and Vestin (2005), Cecchetti and Kim (2005) all assumed that the central bank has perfect control over inflation with the concurrent change of the output gap, and supposed that the output gap is the instrument. Our approach is analogous with the one of Batini and Yates (2003), who used backward looking IS function; however, with the elimination of control uncertainty, we can utilize the simplicity of the previous method.

<sup>16</sup> Properties of endogenous inflation persistence, as well as autoregressive exogenous shocks are extensively examined in Clarida et al. (1999).

Under inflation targeting with commitment (ITC), the central bank makes a precommitment to its future actions, and optimizes in the initial period only. This theoretical benchmark solution minimizes the intertemporal social loss function itself, which is

$$E \sum_{t=1}^{\infty} \beta^{t-1} L_t = E \sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} [(\pi_t - \pi_t^*)^2 + \lambda(x_t - x_t^*)^2], \quad (2)$$

where  $\lambda$  reflects the relative importance of output gap variability compared to the importance of inflation variability.<sup>17</sup>

Since central banks to be examined supposedly cannot credibly precommit, their delegated objective functions differ from society's preference representation for better outcomes in terms of social loss. In the case of inflation targeting (IT) the central bank tries to pursue the targeted value of inflation and output gap, namely to minimize the expected loss

$$E \sum_{t=t_0}^{\infty} \beta^{t-1} \hat{L}_t = E \sum_{t=t_0}^{\infty} \beta^{t-1} \frac{1}{2} [(\pi_t - \pi_t^*)^2 + \hat{\lambda}(x_t - x_t^*)^2] \quad (3)$$

in every period.<sup>18</sup> Svensson (1999) replaced the inflation target to price level target in this 'standard' loss function. In price level targeting (PT) regime, the central bank tries to neutralize the divergence from the targeted price level and output gap, namely aims to minimize the loss function

$$E \sum_{t=t_0}^{\infty} \beta^{t-1} \tilde{L}_t = E \sum_{t=t_0}^{\infty} \beta^{t-1} \frac{1}{2} [(p_t - p_t^*)^2 + \tilde{\lambda}(x_t - x_t^*)^2] \quad (4)$$

in every period. If inflation, output gap, and price level are incorporated in such a way that the weighted mixture of the inflation and price level targets are used in the loss function, we obtain hybrid targeting (HT). In this case, the loss function to be minimized in every period is

$$E \sum_{t=t_0}^{\infty} \beta^{t-1} \check{L}_t = E \sum_{t=t_0}^{\infty} \beta^{t-1} \frac{1}{2} [((1-\theta)\pi_t + \theta p_t) - ((1-\theta)\pi_t^* + \theta p_t^*)]^2 + \check{\lambda}(x_t - x_t^*)^2] \quad (5)$$

where  $0 < \theta < 1$ .<sup>19</sup> It is perceivable that if  $\theta=0$ , we get to IT, and if  $\theta=1$ , then we get to PT.<sup>20</sup>

## 2.3 THE THREE-PERIOD ANALYTICAL FRAMEWORK

The framework to be presented is examining a steady state to steady state economy. The economy is hit by a single shock ( $\varepsilon_t$ ) at the beginning of the first period. For simplicity, suppose that the significance of the events in each period is equivalent ( $\beta=1$ ).<sup>21</sup> In period 0, the economy is in steady state, i.e. the state variables and expected values of them ( $\pi_{t+1|t}$ ,  $\pi_t$ ,  $\pi_{t-1}$ , and  $x_t$ ) are equal to zero, and for simplicity, price level ( $p_t$ ) is normalized also to zero. Again for convenience, inflation target ( $\pi_t^*$ ) in the case of ITC and IT, and price level target ( $p_t^*$ ) in the case of PT are also set to zero.<sup>22</sup> Supposing that economy works on its long time potential, neither discretionary regimes endeavour to aim an output level differing from the potential one in any case creating surprise inflation ( $x_t^*=0$ ), namely there is no inflation bias. Thus, the scope of the examination is on stabilization bias, originating also from dynamic inconsistency.

<sup>17</sup> This specification can be deduced from microeconomic basis, usually utilizing general equilibrium models with infinite lived agents and monopolistically competitive firms with staggered price setting. In detail, for instance, Rotemberg and Woodford (1998) defined such a general equilibrium model, where consumers has CES utility function to optimize under infinite time horizon, while the production function was the one presented by Calvo (1983), where the possibilities of firms to modify prices are given stochastically and exogenously. Output gap term catches the disutility of consumption fluctuation, while inflation term represents the drawback of price dispersion affecting production. Note, however, that this model implies purely forward looking agents, thus the exact Taylor approximation of utility will more or less differ from loss function (2), when the coexistence of firms with rule of thumb price setting behaviour is also presumed. Nevertheless, since the literature ubiquitously applies it in the case of similar examinations of policy issues, we will stick to this form.

<sup>18</sup> Though it is not marked, the central banks' target values of the variables can differ as well.

<sup>19</sup> As Batini and Yates (2003) noted referring to Larry Ball and Frank Smets, this weighting method incorporates the covariance term between inflation and price level into the loss function.

<sup>20</sup> The analysis of Söderström (2001) and Steinsson (2003) showed that optimal policy under discretion puts significantly lower weight on output gap stabilisation than society, moreover, under certain specifications, this weight is very close to zero. Recall, however, that revealing the real preference of a society is impossible; its theoretical estimations provide significantly different values, firmly depending on model specifications.

<sup>21</sup> Though starting-up from another basis, in their examination, Cecchetti and Kim (2005) concluded that results of comparative welfare analysis are not sensitive to the choice of  $\beta$ . We will demonstrate that there are analogies that allow us to rely upon this result.

<sup>22</sup> Note that setting the initial logarithmic price level and target to zero theoretically implies the existence of negative logarithmic prices. Anyhow, prices remain always positive.

In practice, none of the monetary regimes can temporize the pursuing of previously communicated targets without deteriorating credibility, but the scale is largely depend on what the economic agents surmise on the reliability of the central bank.<sup>23</sup> Eventually, excess recalibration of the path of the targeted variable erodes the trust in the declared policy for sure. In this sense, the three-period model covers the *reasonable* time horizon where declared targets should be achieved in order to maintain high credibility in the absence of commitment technology<sup>24</sup>, that is, where the immanent characteristics (i.e. fundamental differences) of the regimes are clearly revealed; however, only in ‘low resolution’ ( $\hat{\lambda} = \tilde{\lambda} = \bar{\lambda} = 0$ ).<sup>25</sup> Thus, suppose that discretionary regimes have the credibility in point of reaching their final goals considering rational agents, that is the forward looking economic agents fully understand the nature of the regime, and trust in the pursuing of the declared target.<sup>26</sup> The expectations can be formalized to

$$\pi_{t+1|t} = p_{t+1|t} - p_t, \quad (6)$$

where  $p_{t+1|t}$  means the expected price level of the next period. These expectations manifest in different manner, depending on the characteristics of the discretionary regime. In IT, the agents expect that a zero inflation target is pursued, namely that the price level of next period will be the same as in the concurrent period ( $p_{t+1|t} = p_t$ ). In PT, it is believed that monetary actions are in order to assure the targeted price level, namely zero ( $p_{t+1|t} = p^* = 0$ ). What do the agents of the economy expect in HT? It depends what emphasis the price level target bears, namely from the grade of ‘history dependency’. In every period, they expect that  $\theta$  proportion of the inflation occurred in the first period will be undone, or in other words, their expected price level target will be

$$p_{t+1|t} = (1 - \theta)^t p_1. \quad (7)$$

With this model specification, expectations are driven by policy framework and formed by past actions at the same time. At latest in the fourth period, economy is again in steady state. It is in accordance with the model setup, that is, there is no long run trade-off between inflation and output gap in the absence of cost push disturbance, i.e. the output gap converges to zero. Since the examined discretionary regimes supposedly exclude inflationary policies, steady state means zero inflation and fully closed output gap. Model calibration has been summarized in Table 1, while the solutions of the model are presented by Table 2 (details in Appendix A).

Two restraints of the model should be emphasized. First, in period 3, because of the existence of lagged price term, inflation should be zero in order to ensure steady state in period 4 (for further details see Appendix A). Second, since the disturbance term is an AR(1) process, and so it calms down within the progress of time, its effects from period 4 are disregarded.

<sup>23</sup> In the 1990s, early inflation targeters were criticized because inflation target, which meant disinflation at that time, was in the foreground causing higher unemployment rate; however that was the way of gaining credibility. After successful disinflation, secondary goals (e.g. output, interest rate, exchange rate) started to move into the foreground. With built-up credibility, the counter-actions to mitigate a potential shock as soon as possible were and are used less frequently, and gradual approach is emphasized.

<sup>24</sup> Cecchetti and Kim (2005) gave an approximation over the optimal evaluation horizon for the central banks, namely the horizon, where desired price path should be achieved, which they define in  $(1 - \theta)^{-1}$  (plus transmission lag). As we will see, it is in accordance with our ‘reasonable’ time horizon approach, since in our model, first IT, later HT, and at latest, PT provide the proper adjustment to the desired price path.

<sup>25</sup> Note again that putting weight solely on inflation/price level targets in the loss functions of the central banks is the reason of the low resolution model structure, rather than the consideration of “strict” targeting. Instead, in our analysis central banks can be considered as “conservatives” in the sense that their preference weight on output gap is equal or lower than that of society.

<sup>26</sup> As Cecchetti and Kim (2005) noted, hybrid policy has the disadvantage that it is very hard to be communicated, which would be a key issue of conducting credible monetary policy. More transparency may result more credibility, however we do not draw any distinction between the examined discretionary regimes in this regard.

**Table 1****Set of variables**

Variable \ Period (t)	0	1	2	3	4
$\pi_t$	0	$\pi_1$	$\pi_2$	0	0
$x_t$	0	0	$x_2$	$x_3$	0
$p_t$	0	$\pi_1$	$\pi_1 + \pi_2$	$\pi_1 + \pi_2$	$\pi_1 + \pi_2$
$\pi_t^*, x_t^*, p_t^*$	0	0	0	0	0
$\varepsilon_t$	0	$\varepsilon_1$	0	0	0
$u_t$	0	$\varepsilon_1$	$\rho\varepsilon_1$	$\rho^2\varepsilon_1$	0

The presented model is a thinking frame and definitely not a full-scale model to be estimated. The most powerful attribute of its unfamiliar build is, however, it can serve analytical deductions in such cases where models with infinite periods would fail, while, in spite of its simplicity in methodology, it does serve consistent results with those prevalent models. We utilize this feature in the exploration of new results.

**Table 2****Results**

	IT	PT	HT
$\pi_1$	$\varepsilon_1$	$\frac{\varepsilon_1}{1+\phi}$	$\frac{\varepsilon_1}{1+\theta\phi}$
$\pi_2$	0	$-\frac{\varepsilon_1}{1+\phi}$	$-\varepsilon_1 \left( \frac{\theta}{1+\theta\phi} \right)$
$x_2$	$-\frac{\varepsilon_1}{\delta}(1-\phi+\rho)$	$-\frac{\varepsilon_1}{\delta} \left( \frac{2-\phi}{1+\phi} + \rho \right)$	$-\frac{\varepsilon_1}{\delta} \left( \frac{\theta+1-\phi}{1+\theta\phi} + \rho \right)$
$x_3$	$-\frac{\varepsilon_1}{\delta}\rho^2$	$\frac{\varepsilon_1}{\delta} \left( \frac{1-\phi}{1+\phi} - \rho^2 \right)$	$\frac{\varepsilon_1}{\delta} \left( \frac{\theta(1-\phi)}{1+\theta\phi} - \rho^2 \right)$
ITC			
$\pi_1$	$\varepsilon_1 \left( \phi \left( \frac{1-\phi}{1-\phi(1-\phi)} - \delta \frac{\frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)}(1-\phi)+\phi}{\delta(1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2)+\frac{\lambda}{\delta}(1-\phi(1-\phi))^2} + \rho \left( \frac{1}{1-\phi(1-\phi)} - \delta \frac{\frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} + \rho \frac{\lambda}{\delta^2}(1-\phi)}{\delta(1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2)+\frac{\lambda}{\delta}(1-\phi(1-\phi))^2} \right) \right) + 1 \right)$		
$\pi_2$	$\varepsilon_1 \left( \frac{1-\phi}{1-\phi(1-\phi)} - \delta \frac{\frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)}(1-\phi)+\phi}{\delta(1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2)+\frac{\lambda}{\delta}(1-\phi(1-\phi))^2} + \rho \left( \frac{1}{1-\phi(1-\phi)} - \delta \frac{\frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} + \rho \frac{\lambda}{\delta^2}(1-\phi)}{\delta(1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2)+\frac{\lambda}{\delta}(1-\phi(1-\phi))^2} \right) \right)$		
$x_2$	$-\varepsilon_1 \left( \frac{\frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)}(1-\phi)+\phi + \rho \left( \frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} + \rho \frac{\lambda}{\delta^2}(1-\phi) \right)}{\delta \frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} + \frac{\lambda}{\delta}(1-\phi(1-\phi))} \right)$		
$x_3$	$\frac{\varepsilon_1}{\delta} \left( (\phi-1) \left( \frac{1-\phi}{1-\phi(1-\phi)} - \delta \frac{\frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)}(1-\phi)+\phi}{\delta(1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2)+\frac{\lambda}{\delta}(1-\phi(1-\phi))^2} + \rho \left( \frac{1}{1-\phi(1-\phi)} - \delta \frac{\frac{1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} + \rho \frac{\lambda}{\delta^2}(1-\phi)}{\delta(1+\phi^2+\frac{\lambda}{\delta^2}(1-\phi)^2)+\frac{\lambda}{\delta}(1-\phi(1-\phi))^2} \right) \right) - \rho^2 \right)$		



## 3 Comparing the regimes

When judging various regimes, we let society decide, therefore, when the social loss function indicates a lower value, it is considered to be the better policy. For good perspicuity, solely in the domain of our framework, the best discretionary policy is denoted to be the optimal if it can replicate the commitment solution, and (sub)optimal if it can not. First, we examine results under specific conditions, and then step by step loosening constraints, eventually we obtain general results. Keeping in mind that  $\lambda$  and  $\delta$  have inverse relationship to social loss, suppose for simplicity that a change in the output gap puts an equal impact on the inflation ( $\delta=1$ ). Thus, examination implicitly follows the implications of the altering ‘slope’ of the Phillips curve (see Appendix B).<sup>27</sup>

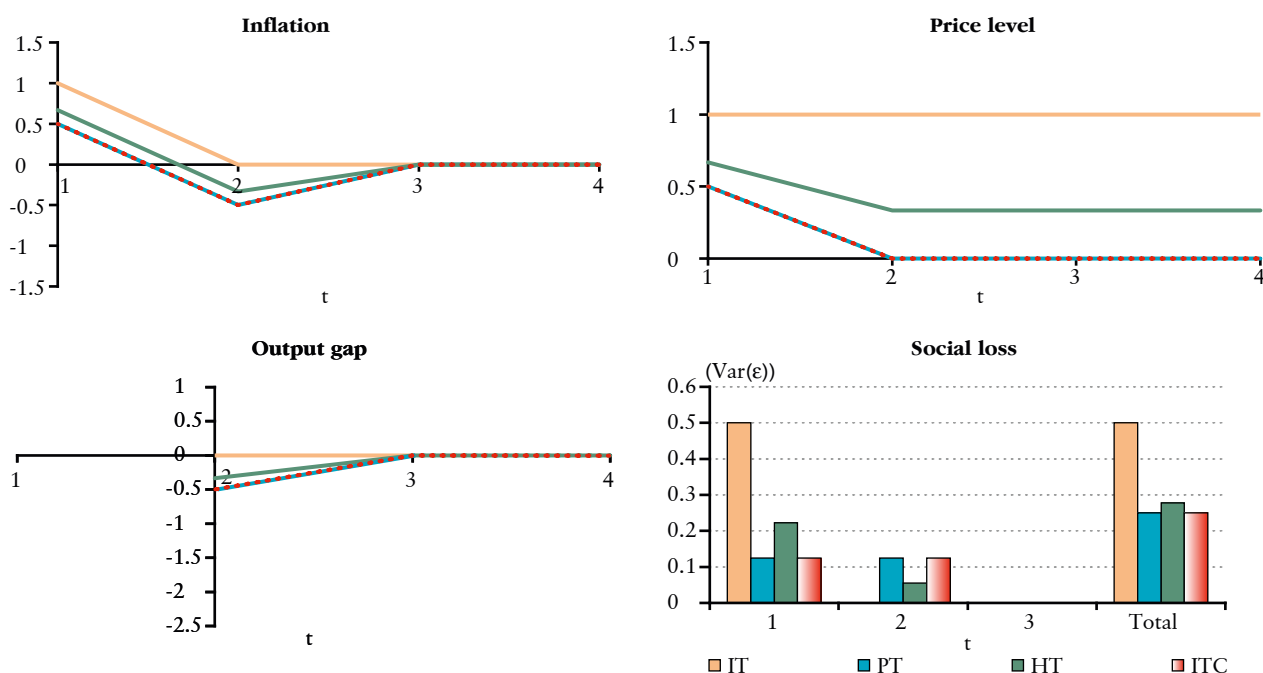
### 3.1 NO EXOGENOUS PERSISTENCE ( $\rho=0$ )

#### 3.1.1 Only inflation matters ( $\lambda=0$ )

If society focuses solely on inflation variability, the dynamic response of inflation, output, price level, and the level of loss, when expectation structure is forward looking, is showed in Chart 1 ( $\theta=0.5$  is used in HT).<sup>28</sup>

**Chart 1**

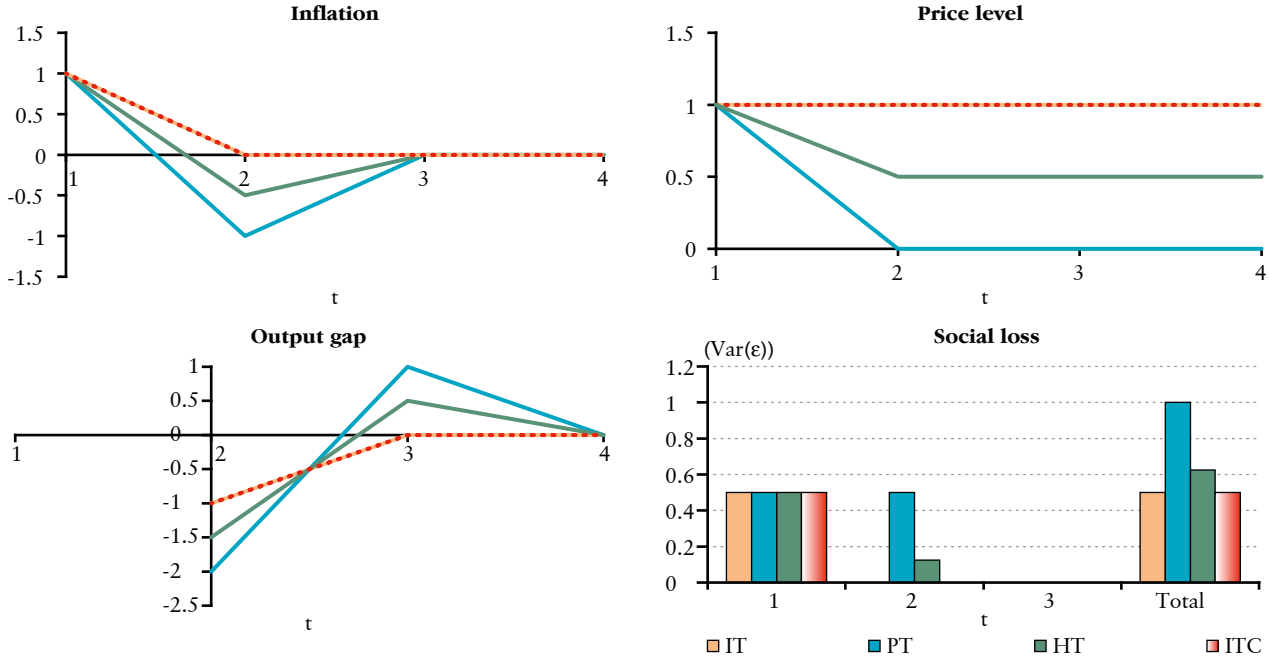
**Dynamic response of variables and social loss ( $\phi=1, \lambda=0, \rho=0$ )**



If expectations are forward looking, ITC, the theoretical benchmark implements price stability. Along such expectations, PT provides the best performance out of discretionary solutions; it achieves a total expected loss equal to the benchmark, which is half the size than in the case of IT. Due to the forward looking expectations, price level target proved itself to be useful, and just like in the case of ITC, only the half of the shock appeared in the inflation of the first period ( $0.5\epsilon_1$ ). In the case of HT, the inflation of the first period was higher ( $0.667\epsilon_1$ ), the correction in the second period was the half of it ( $-0.334\epsilon_1$ , since  $\theta=0.5$ ), and resulted in a moderate price level drift. Conspicuous, that the expected loss was largely diminished by the partial presence of price level target.

<sup>27</sup> It follows that during examinations we will run the value of  $\lambda$  up to empirically implausible ranges. Since the estimated values of  $\delta$  are close to zero (see Chapter 5), large values of  $\lambda$  has its rationale.

<sup>28</sup> In the response diagrams, the scales of the ordinates are fixed in order to ease comparability of the different cases.

**Chart 2****Dynamic response of variables and social loss ( $\varphi=0, \lambda=0, \rho=0$ )**

If the expectation structure is purely backward looking, we have totally different result (Chart 2). The order turns around with PT performing at its low by creating an expected loss double as much as IT does. In this case, ITC means a price level drift equal to the size of the shock, which is the same as what happened in the IT regime. What is the root of this? The PT regime has to maintain the price level target. Due to the purely backward looking expectations, price level target plays no orienting role at all; therefore, the shock gets integrated in the inflation of the first period at maximum extent. In the second period, a price rise of the same magnitude would occur due to lagged pricing. The task of the monetary authority is eventually to neutralize this inflationary pressure and to undo the price rise of the previous period, which means higher inflation volatility compared to IT.<sup>29</sup>

Inflation expectations perceivable in reality are not characterized by these extreme structures.<sup>30</sup> In order to conduct comparative analysis, let us take a look at Chart 3, which shows the losses of regimes as a function of expectations, more precisely as the degree of forward lookingness;  $\hat{S}(\phi)$ ,  $\tilde{S}(\phi)$ ,  $\check{S}(\phi)$ , and  $S(\phi)$  denote social loss indicated under IT, PT, HT, and ITC, respectively.<sup>31</sup>

<sup>29</sup> At the same time, it means higher output gap volatility, too, but now it has no relevance for society.

<sup>30</sup> Chapter 5 provides a broader empirical outlook.

<sup>31</sup> Variability of inflation and output gap are discussed extensively in Chapter 4. Note that period social loss reflects the linear combination of the variance of inflation and output gap.

Proof: The two variables are

$$\pi_t = c_t \varepsilon_t \text{ and}$$

$$x_t = d_t \varepsilon_t,$$

The expected period loss is

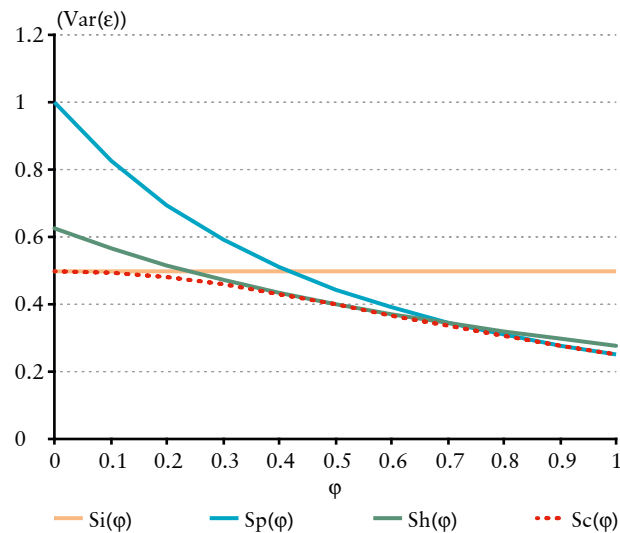
$$E(L_t) = \frac{1}{2} [E(\pi_t^2) + \lambda E(x_t^2)] = \frac{1}{2} [E(c_t^2 \varepsilon_t^2) + \lambda E(d_t^2 \varepsilon_t^2)]$$

Since  $\text{Var}(\varepsilon_t) = E(\varepsilon_t^2) - E^2(\varepsilon_t) = E(\varepsilon_t^2)$ , the loss will be

$$E(L_t) = \frac{1}{2} [c_t^2 \sigma^2 + \lambda d_t^2 \sigma^2]$$

where both terms are the appropriate variances, namely

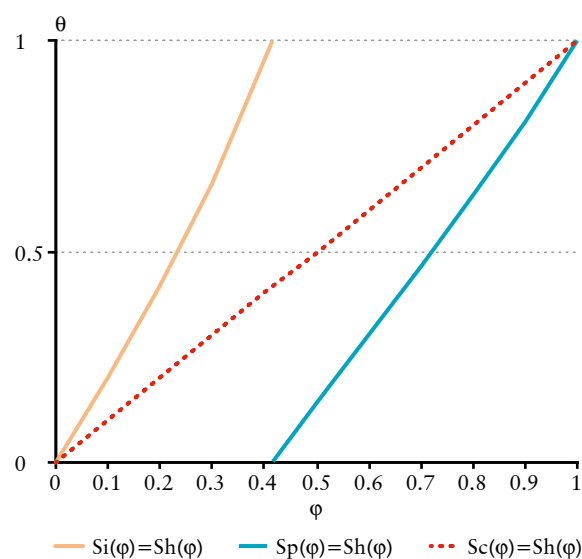
$$E(L_t) = \frac{1}{2} [\text{Var}(\pi_t) + \lambda \text{Var}(x_t)]. \text{ Q.E.D.}$$

**Chart 3****Social loss ( $\lambda=0, \rho=0$ )**

This chart shows that, depending on expectations, what discretionary policy is more adequate and where they are optimal. With the circumstances given, expected loss of IT is not influenced by the expectations, and in the case  $\phi=0$ , its performance coincides with the one perceived by ITC. The more forward looking economy agents are, the better PT performs, and in the case of  $\phi=1$ , it will be equivalent with the benchmark. The performance of HT depends on the value of  $\theta$ :  $\theta \rightarrow 1$  if, then  $\tilde{S}(\phi)$  embeds in the  $\tilde{S}(\phi)$  curve; if  $\phi=0$ , then it embeds in the  $\hat{S}(\phi)$  curve.

Table 3 sums up the order of regimes under various expectations and policy mixes, i.e. when it is better to apply a certain policy, and gives the optimality criteria.

In the case of IT or PT with the circumstances given, the reproduction of the commitment solution will occur only at the two extreme expectation structures. In the case of hybrid expectation, HT stands for solution. With the proper balance between price level and inflation target, it is possible to create a result that is equivalent to ITC under every value of  $\phi$ . With the current

**Chart 4****Order of strength and optimal policies ( $\lambda=0, \rho=0$ )**

**Table 3****Order of strength and optimal policies ( $\lambda=0, \rho=0$ )**

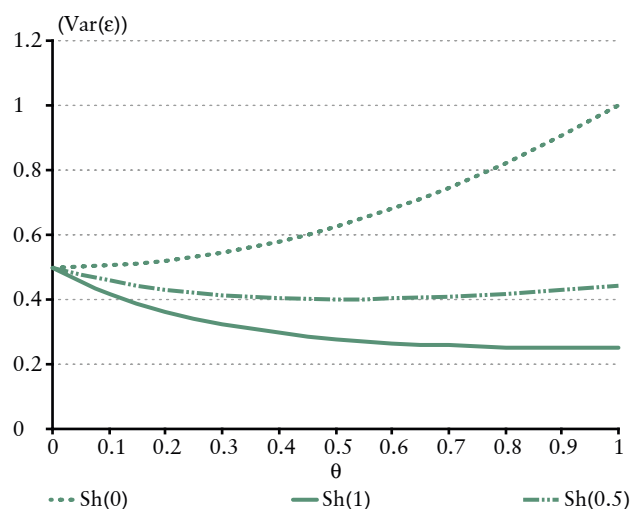
Expectation	Policies	Loss
$0 < \phi < \sqrt{2} - 1$	$\frac{2\phi}{1-\phi^2} < \theta < 1$	$\hat{S}(\phi) < \tilde{S}(\phi) < \tilde{S}(\phi)$
	$\theta = \frac{2\phi}{1-\phi^2}$	$\hat{S}(\phi) = \tilde{S}(\phi) < \tilde{S}(\phi)$
	$0 < \theta < \frac{2\phi}{1-\phi^2}$	$\tilde{S}(\phi) < \hat{S}(\phi) < \tilde{S}(\phi)$
$\phi = \sqrt{2} - 1$	$\exists \theta$	$\tilde{S}(\phi) < \hat{S}(\phi) = \tilde{S}(\phi)$
$\sqrt{2} - 1 < \phi < 1$	$\frac{\phi^2 - 2\phi - 1}{1 - \phi^2 + 2\phi} < \theta < 1$	$\tilde{S}(\phi) < \tilde{S}(\phi) < \hat{S}(\phi)$
	$\theta = \frac{\phi^2 - 2\phi - 1}{1 - \phi^2 + 2\phi}$	$\tilde{S}(\phi) = \tilde{S}(\phi) < \hat{S}(\phi)$
	$0 < \theta < \frac{\phi^2 - 2\phi - 1}{1 - \phi^2 + 2\phi}$	$\tilde{S}(\phi) < \tilde{S}(\phi) < \hat{S}(\phi)$
$\phi = 0$	$\theta = 0$	$\hat{S}(\phi) = S(\phi)$
$0 < \phi < 1$	$\theta = \phi$	$\tilde{S}(\phi) = S(\phi)$
$\phi = 1$	$\theta = 1$	$\tilde{S}(\phi) = S(\phi)$

criteria given, emphasis shall be taken on price level target exactly at the same extent as the degree of forward lookingness (e.g. see Chart 3). What stated above can be seen graphically in Chart 4.

The grey curve shows what mix leads IT and HT to equivalent results. The black curve means the same relation between PT and HT. The area above the grey curve means the unambiguous dominance of IT. The area below the grey curve and above the black curve shows the superiority of hybrid policies over IT and PT regimes, while the area below the black curve means the unambiguous superiority of PT. The relative effectiveness of PT and IT depends on expectations, with the previous one being a better option if the ratio of forward looking expectations is a bit over 40 per cent ( $\phi > \sqrt{2} - 1$ ). ITC can be achieved in any expectation structure, and optimal solutions fall on the diagonal.

One important question left to be cleared is the relation of HT policies under different expectation structures, which is shown in Chart 5.

In the forward looking case, adding some price level target to an IT loss function results in a notable decline in social loss. In the extremely backward looking case, incorporating some inflation target into a PT loss function decreases the loss as well; however, this latter decline is more significant, as we put more and more weight on the newly incorporated target. These relations are not linear, since the gains are decreasing. Under hybrid expectations, this monotonicity disappears: to some extent, adding new targets reduces, but from a certain point, it increases social loss again. Moreover, concerning expectations, linearity does not stand either: if expectations of the economic agents shift from fully backward looking

**Chart 5****Social loss under different policy mixes ( $\lambda=0$ ,  $\rho=0$ )**

behaviour, it ameliorate the performance of any HT more, than a same size shift would do it closer to the fully forward looking case.<sup>32</sup>

It is interesting to see the connections between our results and the ones founded on different basis. In the relations of inflation volatility under different hybrid regimes, similar result were reported by Gavin et al. (1999) under New Classical Phillips Curve with high output gap persistence. Running simulations with four different values of hybrid policy weight, they demonstrated that delegating even a low weighted price level target aside inflation target into the loss function of the central bank results in large drop in the uncertainty around future price path, which outcome, recall, is in line with the decreasing inflation variability in that kind of model.<sup>33</sup> This result corresponds to ours in the cases of dominantly forward looking expectations. To reveal its background, one needs to look at how the output gap persistence affects different regimes. Under New Classical Phillips curve, the empirical estimations of Cecchetti and Kim (2005) demonstrated that with the rise of output gap persistence, the ab ovo small social welfare advantage of optimal hybrid policy, in contrast with price level targeting, is smoothly decreasing, while this advantage, in contrast with inflation targeting, is exponentially growing above a given degree of persistence. The more and more persistent the output gap is, i.e. the more and more the expectations implicitly forward looking are, then the benefits of regime switching to an optimal hybrid policy from price level targeting becomes less conspicuous than a similar regime switch from inflation targeting. Thus, it explicitly sheds light on the issue that the examination of the behaviour of HT (as well as of IT and PT) under New Classical Phillips curve with output gap persistence comes to similar results, as under a hybrid, at least moderately forward looking New Keynesian environment.

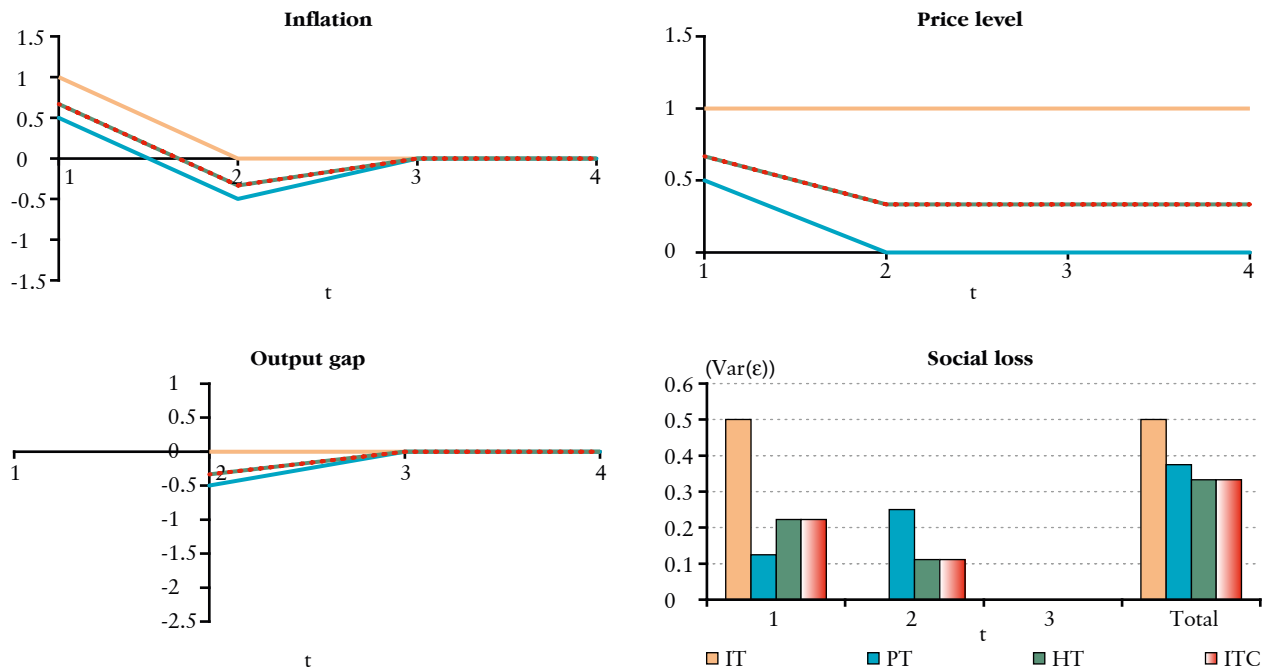
### 3.1.2 Multigoal society ( $\lambda > 0$ )

First, we are about to examine a society that considers the inflation and the output gap divergence from their preferred values equally harmful ( $\lambda=1$ ).

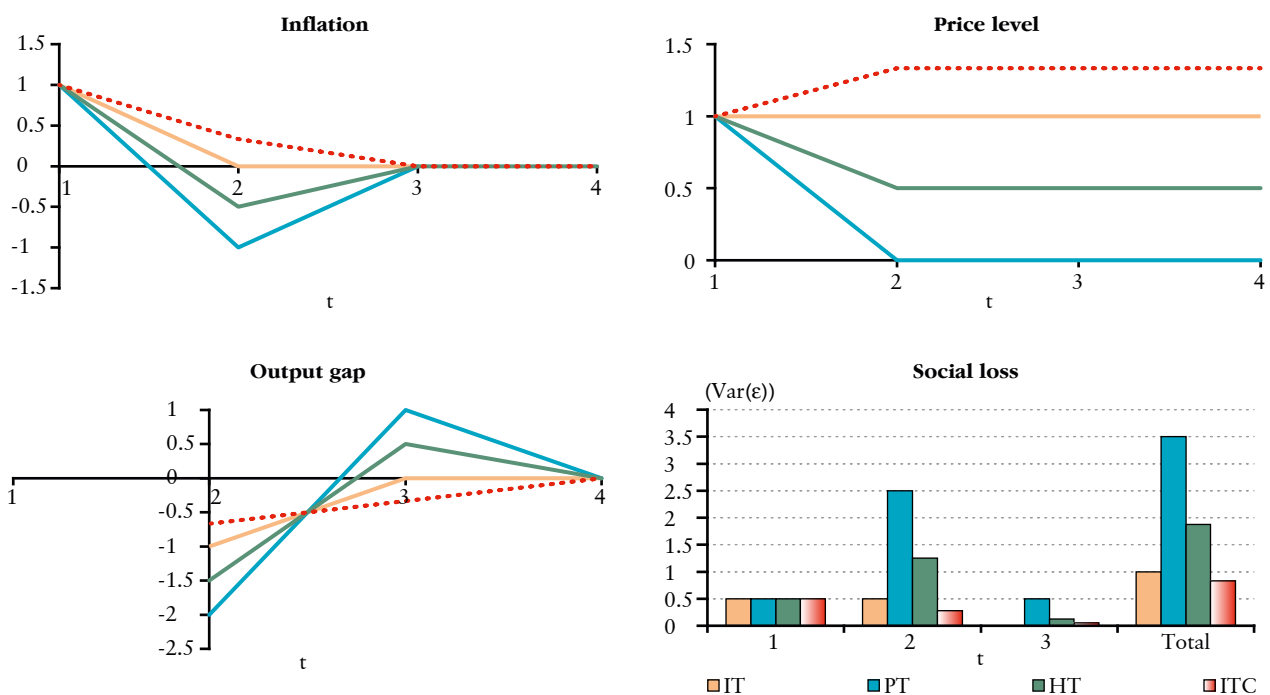
We knew right at the beginning that the dynamic response of the variables of the three discretionary regimes will not change compared to the preceding but their social loss levels. However, in the case of ITC the optimal values of variables and the loss are also affected, since it considers ‘real’ social preference.

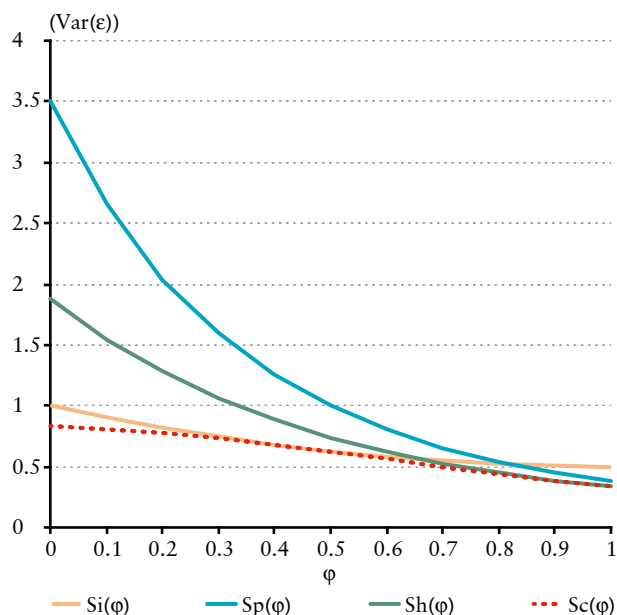
<sup>32</sup> As presented previously, Chart 5 also reveals that in the forward looking case it is PT, in the backward looking case it is IT, and in the mean it is HT with equal weight on price level and inflation target, that is the best discretionary policy, and that different set of expectations affect performance of PT the most, and do not affect IT at all.

<sup>33</sup> Similar result were reported by Batini and Yates (2003) in most cases of their analysis, namely that the most reduction in price level variability can be achieved by moving slightly away from pure inflation targeting, but at the same time in their model, it did not always couple with lower inflation variability.

**Chart 6****Dynamic response of variables and social loss ( $\phi=1, \lambda=1, \rho=0$ )**

In the case of purely forward looking expectations, inflation in the first period became higher than in the case of  $\lambda=0$  ( $0.667\epsilon_1$  instead of  $0.5\epsilon_1$ ), and inflation shock was adjusted only partially ( $-0.334\epsilon_1$  instead of  $-0.5\epsilon_1$ ), since the opening of the output gap was dampened. The result is a price level drift that is less than the shock itself ( $0.334\epsilon_1$ ).

**Chart 7****Dynamic response of variables and social loss ( $\phi=0, \lambda=1, \rho=0$ )**

**Chart 8****Social loss ( $\lambda=1, \rho=0$ )**

In the case of fully backward pricing, the total shock builds into price level in the first period, and in the second period, ITC neutralized inflation persistence thereof only partially ( $-0.334\varepsilon_1$ ), thus resulted in a further price level drift.

Chart 8 shows the social loss in the environment of the hybrid Phillips curve.

The order of strength among regimes has been reset. The disadvantages of the PT regime are plainer to see, especially when expectations are more and more backward looking. The reason behind is that the readjustment of the price level needed heavy intervention in the case of significant endogenous persistence: larger output gap had to be made, which is now penalized by the social loss function. Adding the importance of output gap volatility, IT has gained a relatively better position over PT. What even more important is that PT cannot replicate the commitment solution under any expectation structure.

However, the reproduction of the benchmark result by IT exists on a theoretical level. With any given  $\lambda$ , there is only one expectation structure where IT can perform this, i.e. only by certain dot pairs  $(\lambda', \phi')$ . The probability of the existence of a proper pair is zero, while there are very limited instruments of the economic policy to influence these variables.

HT could mean a solution to this dilemma. This regime has the advantage to unbind the constraints of the expectation structure by using weighted inflation and price level target mix, thus only the social preference weight remains the independent variable. Under certain circumstances by picking the right  $\theta$ , it enables to achieve the ITC solution at a positive social preference weight. The freedom of the hybrid policy is limited by the position of IT, which means that with a given  $\lambda$ , it is capable to do so where

$$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi \leq 1. \quad (8)$$

If the significance of the variance of the output gap becomes higher, the lower bound of inequality (8) will be satisfied by higher values of  $\phi$ , which means that the latitude of HT keeps on diminishing,

$$\hat{S}^{-1}(\phi') = S^{-1}(\phi') \rightarrow 1, \text{ when } \lambda \rightarrow \infty,$$

and so it is necessary that  $\theta \rightarrow 0$ . The rising  $\lambda$  of enables HT to achieve the ITC solution on a shrinking (more and more forward looking) spectrum. To sum it up: in the range of values of  $\phi$  where inequality (8) is not satisfied, IT is the (sub)optimal

**Table 4****Optimal policies ( $\lambda > 0$ ,  $\rho = 0$ )**

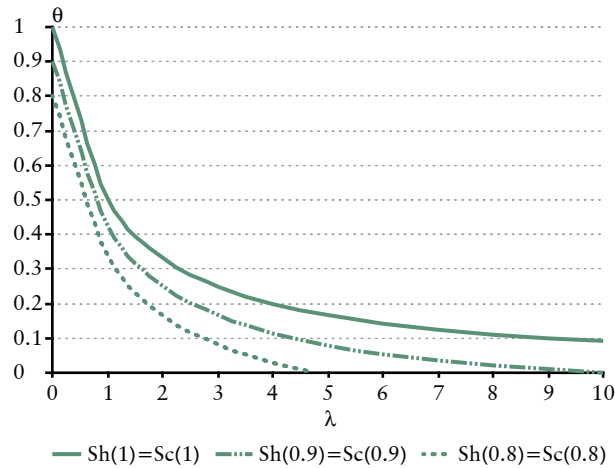
Expectations	Best policy	Remarks
$0 < \phi < \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, (sub)optimal	$\hat{S}(0) \neq S(0)$ , if $\lambda > 0$
$\phi = \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, optimal	
$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi \leq 1$	HT, optimal	
$\hat{S}^{-1}(\phi') = S^{-1}(\phi') = 1$	IT, optimal	$\hat{S}(1) \rightarrow S(1)$ , if $\lambda \rightarrow \infty$

policy, and in the range  $\phi$  of satisfying the inequality, HT is the optimal policy. In the asymptotic case,  $\hat{S}(1) = S(1)$ , i.e. IT becomes the optimal policy in the fully forward looking case.<sup>34</sup> Table 4 summarizes optimal policies.

Considering what has been said before, let us take a look again at Chart 8. It shows that in the case of purely forward looking expectations ( $\phi = 1$ ) and that of inflation and output gap volatility having the same importance to society ( $\lambda = 1$ ), the optimal combination to achieve the theoretical minimal loss is  $\theta = 0.5$ . Generally, in the fully forward looking case, the optimal values of  $\theta$  belonging to the various values of  $\lambda$  are

$$\theta = \frac{\lambda^2 + 3\lambda + 2}{\lambda^3 + 4\lambda^2 + 5\lambda + 2}, \quad (9)$$

which is shown in Chart 9, adding optimal solutions under other expectation structures.<sup>35</sup>

**Chart 9****Optimal policies at various expectations and social preference weights ( $\rho = 0$ )**

This chart highlights that the rise of the preference weight, particularly by its lower values, (and/or the decline in the ‘slope’ of the Phillips curve) drastically worsen the usefulness of incorporating significant price level target aside inflation target into the loss function of the central bank, even in the forward looking case. The reason is that society does not like larger output gap variability needed for eliminating price level drift, and this is even more obvious with the increase of lagged pricing at the expense of forward looking behaviour, as the interventions required are even heavier. Two of the previous results can

<sup>34</sup> With elementary calculus,  $(\hat{S})I = S(1)$ , if  $\lambda \rightarrow \infty$  or  $\lambda = -2$ .

<sup>35</sup> Although these relations can be derived analytically at any given values of  $\phi$  satisfying inequality (8), they are a bit less tractable under hybrid expectations. Thus, the latter cases were solved numerically.



also be seen from a different perspective. In the fully forward looking case without exogenous persistence, PT can replicate ITC only if society does not concern the output gap variability, and under same expectations, IT can replicate the commitment solution only if preference weight tends to infinity (and/or the ‘slope’ of the Phillips curve tends to zero).

### 3.2 THE ROLE OF THE EXOGENOUS PERSISTENCE ( $\rho > 0$ )

In the previous analysis, the exogenous persistence effect of the shock was not considered. There is the question whether its presence changes our previous results, and if it does, then what way. One should not forget that the already perceived lagged inflation is endogenously determined, while cost push shock persistence is exogenously given, since it is not affected by policy. Let us see what our model indicates with moderate persistence ( $\rho = 0.5$ ).

#### 3.2.1 The output does not matter ( $\lambda = 0$ )

First, the more simple case is considered when society cares about the inflation variability only.

**Chart 10**

**Dynamic response of the output gap ( $\phi = 1$ , and  $\phi = 0$ ,  $\lambda = 0$ ,  $\rho = 0.5$ )**

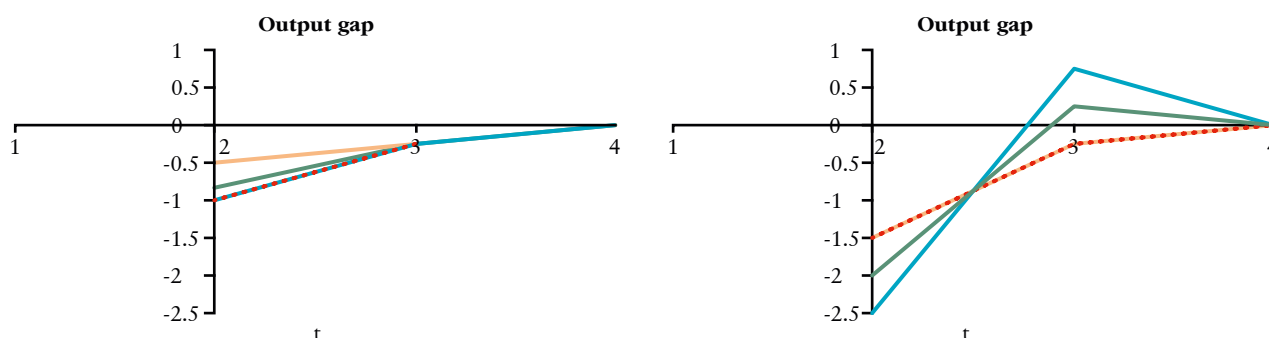
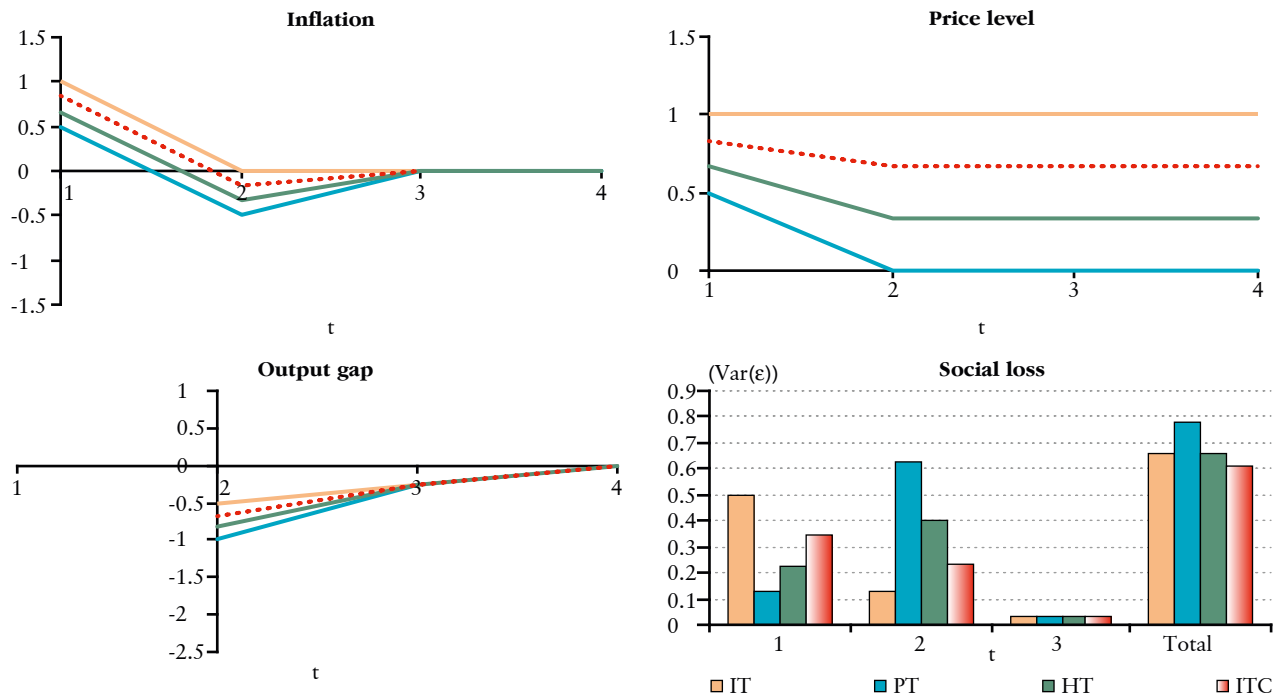
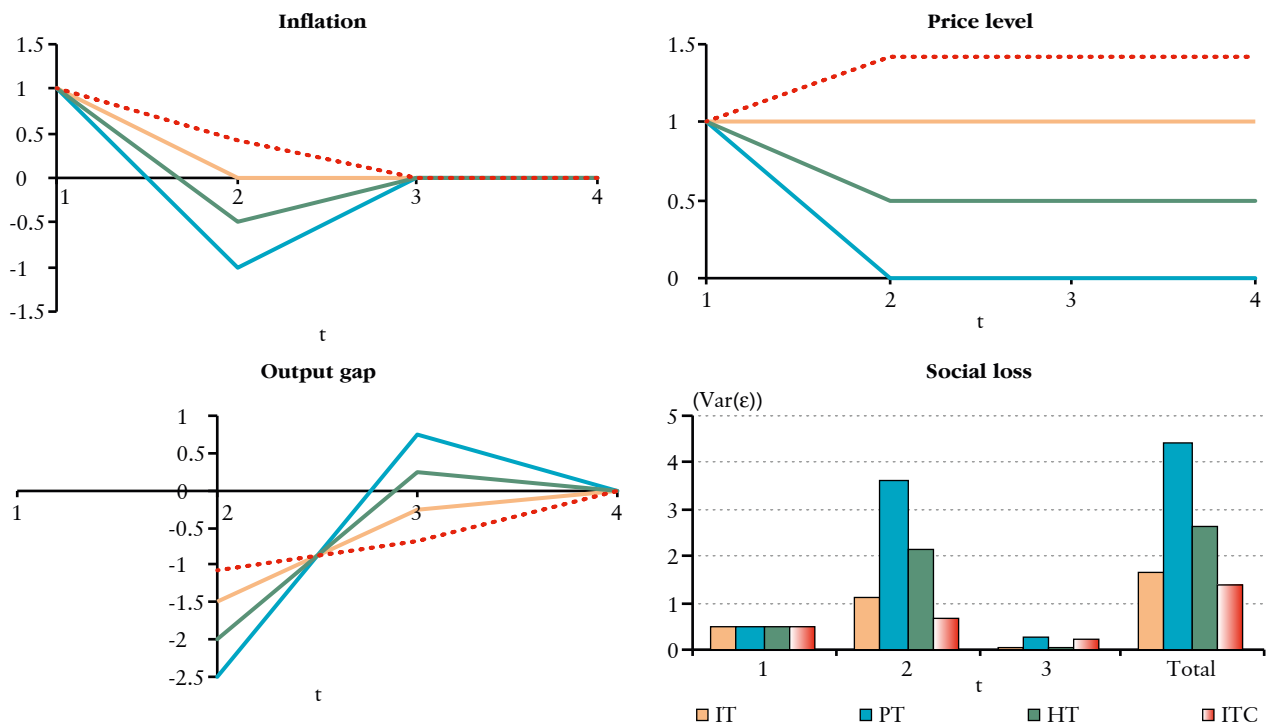


Chart 10 shows the dynamic response of the output gap only, since there was no quality shift at all when compared to the case without persistence *ceteris paribus*. If we take a look again at the results of the model (Table 2), exogenous persistence effects can apparently be identified in every solution. The effect of the exogenous persistence increasing inflation would have been  $0.5\epsilon_1$  in the second period, and  $0.25\epsilon_1$  would have been in the third period. Every regime had to intervene at a higher scale when compared to the case without persistence: its absolute value is as much as higher the persistence would have contributed to the increase of inflation, thus the result is the shifting of the output gap into negative direction. Since it does not affect losses, they are equivalent to the case without persistence.

#### 3.2.2 The output gap matters, too ( $\lambda > 0$ )

Again, suppose that the inflation and output gap variability have the same importance to society,  $\lambda = 1$ . It is clear that this change affects only the dynamics of the variables of ITC and, naturally, the loss of all regimes.

In the case of fully forward looking expectations, although inflation is set higher by ITC when compared to the case without persistence *ceteris paribus*, its increase is smaller than the pressure from persistence (the inflation in the first period is  $0.8667\epsilon_1$  instead of  $0.667\epsilon_1$ , and in the second it is  $-0.1667\epsilon_1$  instead of  $-0.334\epsilon_1$ ). The reason behind this is that commitment solution has countered the shock by widening the output gap ( $-0.667\epsilon_1$  instead of  $-0.334\epsilon_1$ ). Eventually, price level moved higher, although by the two third of the persistence effect of the first period only, when compared to the price level drift without persistence.

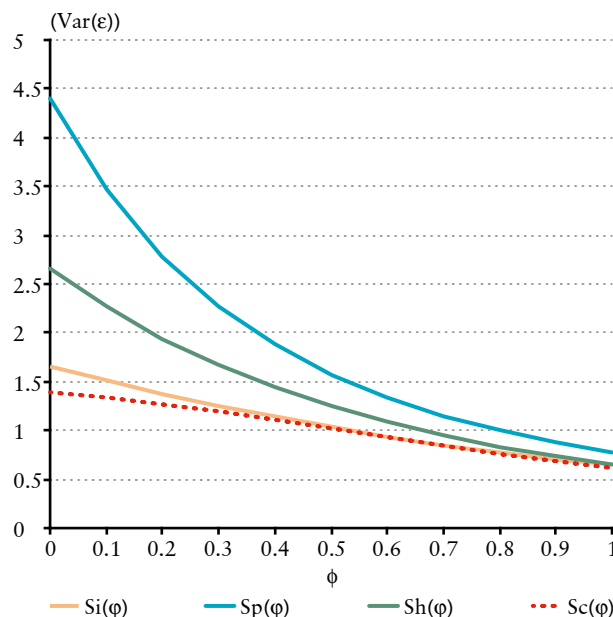
**Chart 11****Dynamic response of variables and social loss ( $\phi=1, \lambda=1, \rho=0.5$ )****Chart 12****Dynamic response of variables and social loss ( $\phi=0, \lambda=1, \rho=0.5$ )**

In the case of purely backward looking inflation expectations, the shock appears in the price level of the first period completely in the commitment solution. In the second period, ITC neutralizes around the three fourth of the inflation pressure originating from exogenous persistence and backward pricing (which is totally  $1.5\varepsilon_1$ ). This means that the drift in price level is larger by one sixth of the persistence effect when compared to the case without persistence.

In order to analyze the hybrid expectation structure, let us take a look at Chart 13.

### Chart 13

**Social loss ( $\lambda=1, \rho=0.5$ )**



Larger interventions due to the exogenous persistence have the similar observational loss effect as if the importance of the output gap variance ( $\lambda$ ) had increased (and/or the 'slope' of the Phillips curve had declined). Contrary to the case without persistence, ITC cannot be reproduced by a discretionary regime at all if the increase of  $\lambda$  goes beyond a certain point, while this point appears during the persistence increase at a lower  $\lambda$ , i.e. its higher value nullifies the latitude of monetary policy concerning hybrid strategies sooner. The opportunity for HT is limited by the position of the IT which is affected by the change in  $\lambda$  and  $\rho$  at different scale. Theoretical optimum can be achieved by HT, where

$$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi \leq 1 \quad (10)$$

can be satisfied. If  $\lambda \leq 1$ , solutions always exist under any degree of persistence; however, if  $\lambda > 1$ , inequality (10) can not be satisfied unconditionally. This criterion implicitly determines a proper subset of  $W$  the vector space  $V = \{v = (\lambda, \rho, \delta) : \lambda \in \mathbb{R}_0^+, 0 \leq \rho < 1, \delta \in \mathbb{R}^+\}$ , where IT and HT have the capability to implement the commitment solution. Combinations generating the boundary of this subset in the critical interval of  $\lambda > 1$  are satisfying equality (11),

$$\rho = g(\lambda) = \frac{2 + \lambda}{\lambda^2 + 2\lambda} . \quad (11)$$

In such situations when the economy is characterized by these combinations, only IT can reproduce the benchmark solution, namely in the fully forward looking case. Beyond this boundary, none of the discretionary regimes can achieve ITC, and in that case, it is IT that provides the best policy, even though (sub)optimal one only, in the whole range of expectations. Table 5 summarizes optimal policy criteria, while Chart 14 shows the abovementioned subset.

**Table 5****Optimal policies ( $\lambda > 0$ ,  $\rho > 0$ )**

Expectations	Best policy	Criteria	Remarks
$0 \leq \phi < \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, (sub)optimal	$\forall \rho$ , if $\lambda \leq 1$ ; $\rho < g(\lambda)$ , if $\lambda > 1$	If $\rho = g(\lambda)$ and $\phi = 1$ ,
$\phi = \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, optimal		IT is optimal ( $\hat{S}(1) = S(1)$ );
$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi \leq 1$	HT, optimal		If $g(\lambda) < \rho$ , IT is (sub)optimal, $0 \leq \forall \phi \leq 1$

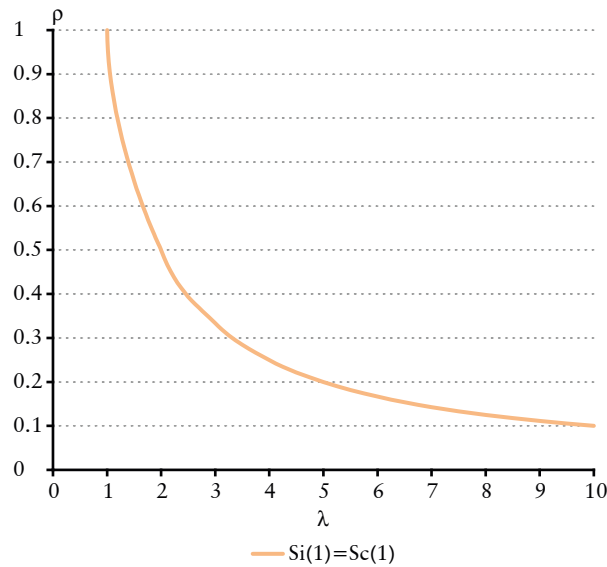
**Chart 14****Boundary of subset W ( $\delta=1$ )**

Chart 14 shows, if  $\lambda \leq 1$ , then persistence per se is not a constrain at all for the existence of optimal hybrid policies.<sup>36</sup> For instance, in the case of  $\rho=0.5$ , HT has a relevance, if  $\lambda < 2$ , which means that it is capable, through a narrowing expectation range with the increase of  $\lambda$  at the same time, to achieve the theoretical optimum. If  $\lambda=2$ , then  $\hat{S}(1)=S(1)$ , i.e. only IT can reproduce ITC, and only in the fully forward looking case; however, if  $\lambda > 2$ , then none of the discretionary regimes is capable of that. Hence, IT gives the solutions closest to ITC throughout the whole expectation spectrum.

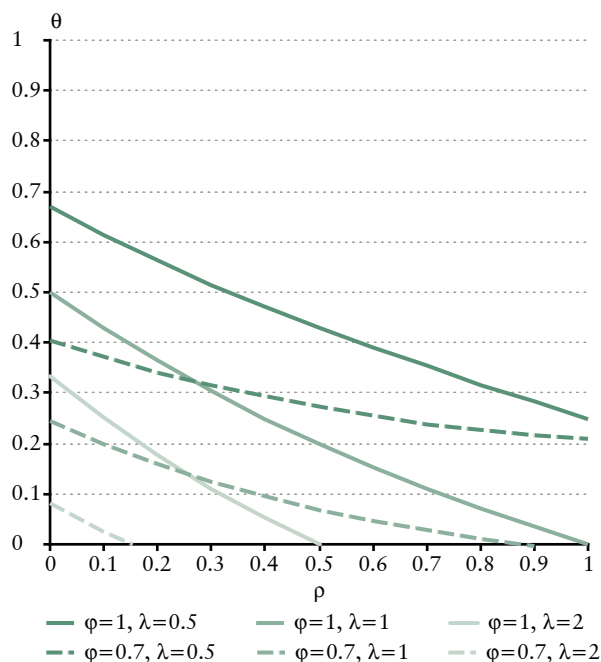
Demonstrating the relationship between optimal policy mix and persistence under fully forward looking expectations, we analytically derive the borderline case ( $\lambda=1$ ), where HT is not constrained to be optimal by any value of  $\rho$  (and/or by any value of  $\delta$ ). Optimal policy mixes are

$$\theta = \frac{2 - \rho^2 - \rho}{(\rho + 2)^2},$$

which is shown in Chart 15 next to other numerically computed solutions.<sup>37</sup>

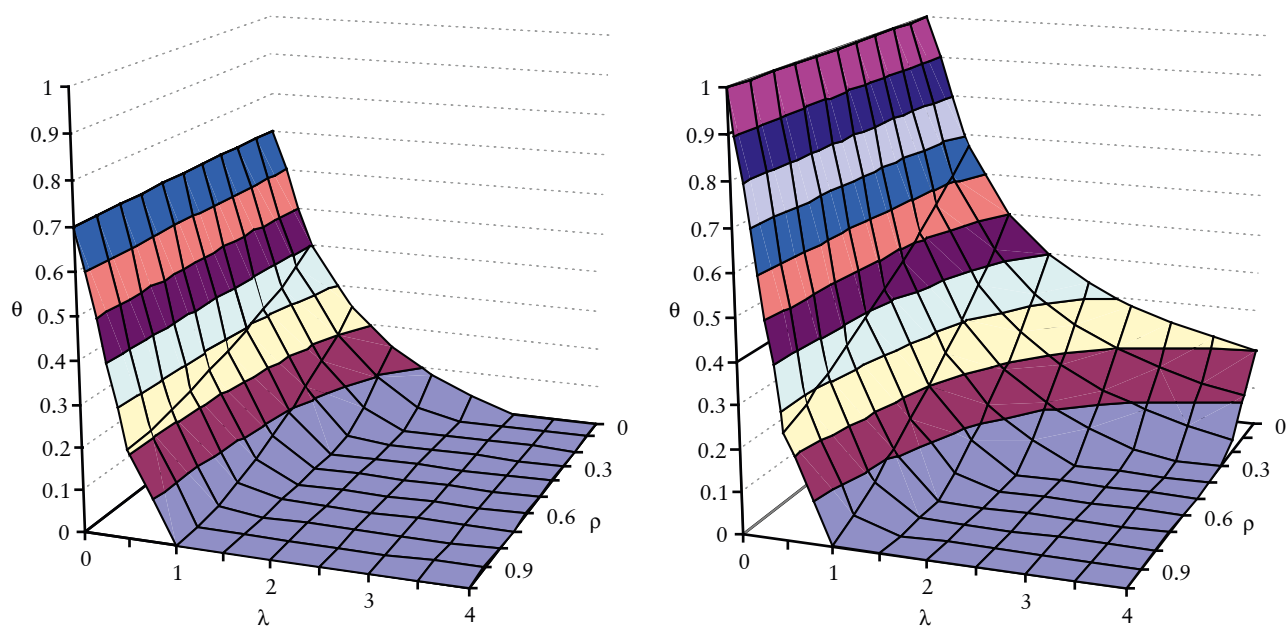
<sup>36</sup> At  $\lambda=1$ ,  $\hat{S}(1)=S(1)$  would require  $\rho=1$ , what can not be.

<sup>37</sup> They can be derived analytically; however, again, those cases are a bit clumsier.

**Chart 15****Optimal policies at various measures of cost push shock persistence<sup>38</sup>**

Larger cost push shock persistence calls for larger emphasis on inflation rather than price level target. The result is not surprising: larger intervention requirements, thus larger output fluctuations caused by more persistent cost push shocks arise more strikingly under PT than under IT. Just as we have already discussed, inflation variability, thus the inflation stabilizing effect of (partial) price level target gradually loses its significance with the rise of  $\lambda$ , and therefore, optimal policy response is to put less weight on the price level target as well. On the other hand, we can also see that these proportionalities are different if inflation is endogenously persistent, too, since it also affects social loss, thus optimal policy choices. In the next section, we take a closer look at the underlying forces driving the variability of inflation and output gap. The composite visualization of our previous discussion is shown in Chart 16.

<sup>38</sup> In the forward looking case, optimal policy at zero ordinate (that is IT) comprises combinations of  $(\lambda, \rho)$  that satisfy equality (11) (the defined boundary of subset  $W$ ). When  $\lambda \rightarrow \infty$ , then these curves collapse to a point into the origin.

**Chart 16****Optimal and (sub)optimal policies ( $\phi=1$  and  $0.7$ )<sup>39</sup>**

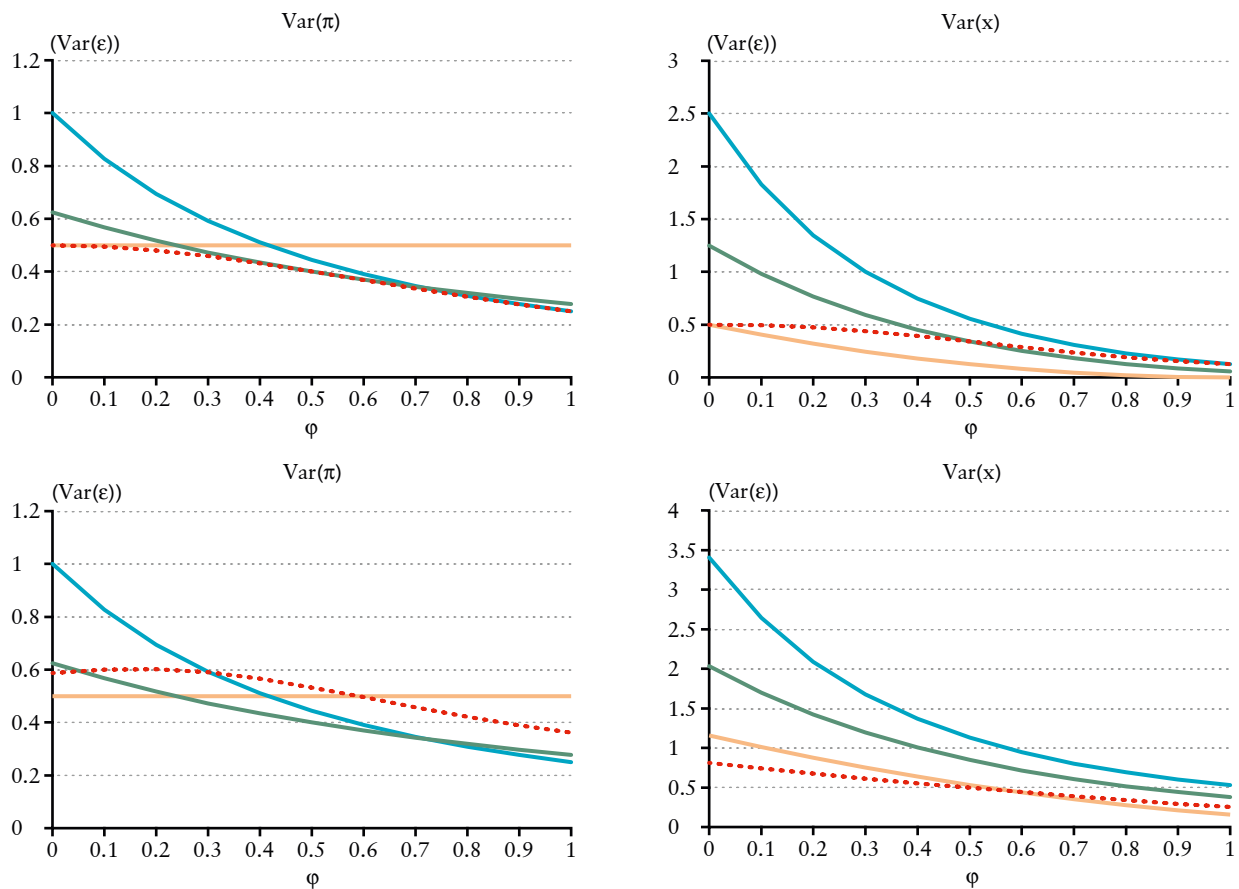
<sup>39</sup> (Sub)optimal policy comprises combinations of  $V \setminus W = \{x = (\lambda, \rho, \delta) \mid x \in V \wedge x \notin W\}$ .

## 4 Inflation and output gap variability

The presented model always indicates higher output gap variability in the case of PT related to IT, however, it shows lower inflation variability under certain circumstances.<sup>40</sup> The reason is the implementation of the control lag. In the fully forward looking case, IT creates no output gap in the period following the cost push shock, as it has no reason, while PT must shepherd the price level back to its targeted value. It is more straightforward in those cases, where the expectations are more and more backward looking, since the expectation driving effect of using a price level target deteriorates more and more, hence the initial jump in inflation is higher.<sup>41</sup>

**Chart 17**

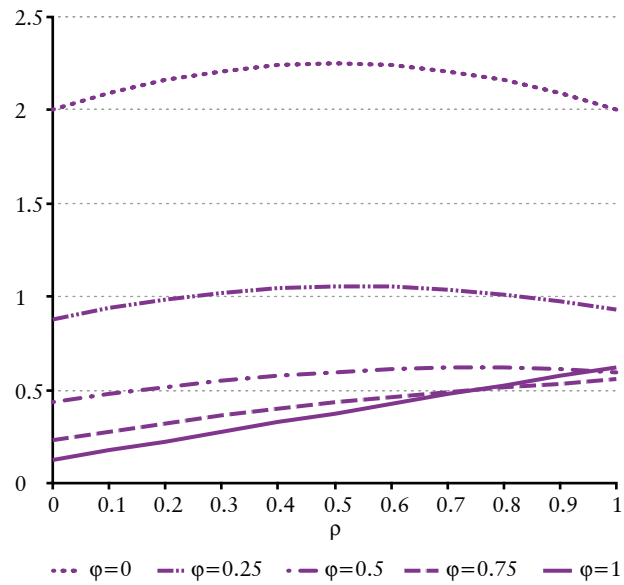
**Inflation and output gap variability,  $\lambda=0$ ,  $\rho=0$  and  $\lambda=1$ ,  $\rho=0.5$  ( $\delta=1$ )**



The output gap variability moves inversely to the ‘slope’ of the Phillips curve which has the same effect under IT and PT, and moves along with exogenous inflation persistence, but, on the contrary, this latter one does not have the same impact on IT and PT. According to PT, it causes additional intervention requirement in period 2, however, it *helps* to counter the backward pricing effect for one period after the deflationary phase. On the other hand, when expectations are rather forward looking, it may mean *burden* in every period. The situation in the case of IT is simpler, since exogenous inflation persistence always means additional intervention requirement on the whole range of expectations. Thus, with the rise of the exogenous persistence, the difference between output gap variances is the dependent of these interdependence impacts (see Chart 18).

<sup>40</sup> The simulation of Fillion and Tetlow (1994) also reported that PT creates lower inflation variability but higher output gap variability than IT, but as Svensson (1999) already noted, they did not give explanation beyond that these results indicate strong serial correlation of the price level. In most setups, Batini and Yates (2003) also found larger output gap variability under price level stabilization.

<sup>41</sup> All else being equal, if there was no control lag, the output gap variability would be lower in PT compared to IT in the fully forward looking case, and would be the same in the fully backward looking case.

**Chart 18****Differences in output gap variability of PT and IT, ( $\delta=1$ )**



## 5 Notes on empirical issues

The estimation of Fuhrer and Moore (1995) using a sticky price model showed near equivalent forward and backward looking behaviour, while Galí and Gertler (1999) demonstrated that the forward looking behaviour is more dominant.<sup>42</sup> Also, Galí and Gertler (1999) emphasized the sluggish behaviour of real marginal cost, which might be a good explanation of the slow inflation response to output gap, hence, the high and costly output gap needed for making inflation move. This flattening tendency of the Phillips curve is also demonstrated by Sbordone (2007). She found that global competition affecting US economy decreases the sensitivity of inflation to marginal cost. Continuous cost push shocks due to oil and food prices seem to be inevitable, too. Backward pricing, declining ‘slope’ of the Phillips curve, and persistent cost push shocks are not too favourable background for targeting a constant price level, though could be for hybrid policy according to the presented model.<sup>43</sup>

However, these conditions are not petrified. Even so, from a rather practical point of view, “conventional wisdom” always builds its arguments not to target constant price level upon perceived conditions that are considered as if they were exogenously given. At an early stage, the reason mentioned was transmission uncertainty<sup>44</sup>, but later the upward measurement bias in Consumer Price Index (CPI), downward nominal rigidities, the zero bound problem on nominal interest rates, and the threat of deflationary spiral became the main obstacles emphasized.<sup>45</sup> They happened to be the main arguments in favour of defining “price stability” as an inflation rate of consumer prices around 2 per cent. Upward bias in CPI is an exact category<sup>46</sup> and can be a reasonable counterargument of targeting its constant level, however, large magnitude of downward nominal rigidity cannot be proved with no doubt even in current monetary systems.<sup>47</sup> Though employees typically resist nominal wage reductions, in the case of prices, Amirault et al. (2004) found that firms in Canada actually modify prices *more* flexible downward than upward. They also pointed that the absolute flexibility of prices grew over time due to stronger competition and larger recline upon information technology. They draw the conclusion that flexible prices may mean larger inflation response to interest rate movements, thus targets may be achieved with shorter lags and smaller real side effects. The remainder arguments are also disputable in the sense of Lucas critique, since these phenomena may reflect the policy-affected economy of its era.<sup>48</sup> The Swedish episode of the 1930s showed that maintaining a constant price level target is feasible without falling into the pit of the zero bound problem. As Berg and Jonung (1999) emphasized the lessons learned, price level targeting helped to raise inflation expectations despite the persistent worldwide deflationary pressure.<sup>49</sup> This historical evidence seems to be a fine support of Lucas critique as the change in the policy modified the expectations of the economic agents, thus it was revealed that some of the ‘axiomatic causalities’ were only the manifest of a reigning paradigm. Fujiwara

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<sup>42</sup> Galí and Gertler (1999) argued that therefore the New Keynesian Phillips curve gives a good first approximation of the inflation dynamics. Rudd and Whelan (2005) and Lindè (2005) claimed that it is a result of specification bias, while Kurmann (2005) pointed out the uncertainty around the estimation procedure. In a recent paper, Galí et al. (2005) stand out for their results. Dufour et al. (2006) found that hybrid New Keynesian Phillips curve is not supported unconditionally, and emphasized the importance of using identification-robust inference methods.

<sup>43</sup> The latitude and applicability of hybrid policy is also restricted by all of these tendencies, and at the same time, as we have already noted referring to Cecchetti and Kim (2005), this regime has the disadvantage that a hybrid target is very hard to be communicated, which would be a key issue of conducting credible monetary policy. Moreover, we may add, that that an inflation target or a constant price level target is easier to be communicated than a positive-slope price level target or even more than an average inflation target or a hybrid target.

<sup>44</sup> In the 1960s-70s, uncertainty around monetary transmission was high, since the lag was long and variable. That was the reason why Friedman (1968) emphasized the impossibility of price stabilization, but added that it could be otherwise, if the “understanding of monetary phenomena advances”. Au contraire, it was not the case at the beginning of the 20th century. According to the observation of Fisher (1920) on the US economy, the monetary transmission reliably exerted its full effect on prices in one to three months, again contrary to the experienced 1.5-2 years of our time.

<sup>45</sup> See Fischer (1994) and Mishkin (2001). Mishkin (2006) reconsidered his earlier sceptical view contemplating the case of Japan, and concluded that PT can be favourable in an economy experiencing deflationary pressure.

<sup>46</sup> See estimations on the bias of US CPI by Boskin et al. (1996), Lebow and Rudd (2003), and Gordon (2006), and of HCIP by Wynne (2005).

<sup>47</sup> We can also find historical examples looking at the 3 per cent fall of consumer prices in Japan between 1998-2002, or an approximate 20 per cent fall of consumer prices in Great Powers and other developed countries between 1890-1896. Two rather different monetary regimes stand behind the two data cited: the first one from fiat and credit money system, and the second one is from the gold standard, respectively. The causes of price decline differ completely. In the case of Japan, it reflects inadequate demand originating in policy difficulties that will be discussed later, while the latter time series reflects the effect of gold scarcity under gold standard; since the price of a certain amount of gold was fixed, other prices, i.e. price level had to move down, restoring the appropriate parity between gold and other commodities.

<sup>48</sup> In the price level targeting literature, the necessity of this kind of approach was emphasized first by Black and Gavin (1990).

<sup>49</sup> Straumann and Woitek (2009) argued that as a matter of fact, Riksbank pursued exchange rate targeting policy and that achieving price level stability was due to coincidence of multiple economic factors. Berg and Jonung (1999) claimed that although the policy was never ‘pure’ price level targeting, exchange rate stabilization was only a secondary objective.

et al. (2005) summarized recent knowledge about escaping possibilities from the liquidity trap, referring to the models of Eggertsson and Woodford (2003), Wolman (2005), and others, and concluded the same, namely, that some form of price level targeting needed that can flatten yield curve, helping to evade hitting the zero bound.

On small open economy aspects, there are also appealing prospects in favour of price level targeting. Utilizing a comprehensive (two-country) dynamic stochastic general equilibrium model, Coletti et al. (2008) argued that a small, open economy like Canada could benefit from the implementation of price level targeting, since it would provide larger inflation stability at the cost of only slightly higher output gap variability. The results of their model demonstrated that if the proportion of rule-of-thumb price and wage setting behaviour is less than 40 per cent (which is highly realistic), then price level targeting always dominates inflation targeting, otherwise this relation is hanging on the source of the shocks. Only shocks that cause inverse movements in the output gap and inflation can make inflation targeting more favourable, but only if backward price setting is prevalent. If the resultant of different kind of shocks affecting terms of trade generates positive covariance between inflation and the output gap, like in Canada, then it makes price level targeting appealing for a highly open economy. Coletti et al. (2008) also demonstrated that a switch in the monetary policy framework of a large partner country (which is the United States in their model) does affect neither the relative merits of inflation targeting and price level targeting, nor the parameterisation of the Canadian monetary policy rule.<sup>50</sup>

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<sup>50</sup> Since only Bank of Canada deals with the possibility of the implementation of price level targeting in very serious manner, we rely upon their results with more emphasis.

## 6 Conclusions

In the presented three-period steady state to steady state framework, it is showed that inflation, price level and hybrid targeting can all become the best solution under certain circumstances. Considering control lag, the model indicated that price level targeting always creates higher output gap variability than inflation targeting, whereas the relation in inflation variability, and so the social loss implication, is an open issue, sensitively depending on the examined conditions.

Inflation targeting proved far more robust than price level targeting, while hybrid targeting had the best adaptability. It is showed that, without exogenous inflation persistence, inflation targeting and hybrid targeting can always reproduce commitment solution on a descending, more and more forward looking range of expectations with the rise of the social preference weight on the output gap (and/or with the decline of the 'slope' of the Phillips Curve), while in the most general and realistic case, the existence of exogenous persistence makes the possibility of reproduction to the function of the social preference weight (and/or the 'slope' of the Phillips curve).

The examination demonstrated the non-linear interrelations of economic and policy parameters. The impacts of changing parameter values on inflation variability, output gap variability and social loss manifest in different proportionalities, moreover, not always in monotonic fashion.

# Appendix

## A MODEL SOLUTIONS

### A.1 Inflation targeting with commitment

The expected loss to be minimized subject to the constraints given by Phillips curve is

$$E_0 \sum_{i=1}^{\infty} \frac{1}{2} [\pi_i^2 + \lambda x_{i+1}^2]$$

which is  $E_0 \frac{1}{2} [\pi_1^2 + \pi_2^2 + \lambda x_2^2 + \lambda x_3^2]$ . Since the central bank has full credibility, it endogenizes inflation expectations during its optimizing process, thus  $\pi_{t+1|t} = \pi_{t+1}$ . The Lagrangian is

$$\begin{aligned} \Omega(\pi_1, \pi_2, x_2, x_3) = & \frac{1}{2} [\pi_1^2 + \pi_2^2 + \lambda x_2^2 + \lambda x_3^2] - \eta_1 (\phi \pi_2 + \varepsilon_1 - \pi_1) - \\ & - \eta_2 ((1-\phi)\pi_1 + \delta x_2 + \rho \varepsilon_1 - \pi_2) - \eta_3 ((1-\phi)\pi_2 + \delta x_3 + \rho^2 \varepsilon_1) \end{aligned}$$

whose first order conditions are

$$\frac{\partial \Omega}{\partial \pi_1} = \pi_1 + \eta_1 - \eta_2 (1-\phi) = 0,$$

$$\frac{\partial \Omega}{\partial \pi_2} = \pi_2 - \eta_1 \phi + \eta_2 - \eta_3 (1-\phi) = 0,$$

$$\frac{\partial \Omega}{\partial x_2} = \lambda x_2 - \eta_2 \delta = 0,$$

$$\frac{\partial \Omega}{\partial x_3} = \lambda x_3 - \eta_3 \delta = 0,$$

$$\frac{\partial \Omega}{\partial \eta_1} = \phi \pi_2 + \varepsilon_1 - \pi_1 = 0,$$

$$\frac{\partial \Omega}{\partial \eta_2} = (1-\phi)\pi_1 + \delta x_2 + \rho \varepsilon_1 - \pi_2 = 0,$$

$$\frac{\partial \Omega}{\partial \eta_3} = (1-\phi)\pi_2 + \delta x_3 + \rho^2 \varepsilon_1 = 0.$$

Simple rearrangements and substitutions lead to the optimal solutions.

### A.2 Inflation targeting

The expected loss to be minimized is

$$E_t \sum_{i=1}^{\infty} \frac{1}{2} [\pi_{t+i}^2]$$

which is  $E_1 \frac{1}{2} [\pi_2^2 + \pi_3^2]$  in period 1, and  $E_2 \frac{1}{2} [\pi_3^2]$  in period 2. Inflation values minimizing the loss are

$$\pi_2=0 \text{ and } \pi_3=0.$$

According to equations (6) and (7),  $\pi_{t+1|t}=0$ . With simple substitutions into the conditions given by the Phillips curve, solutions are obtained.

### A.3 Price level targeting

The expected loss to be minimized is

$$E_t \frac{1}{2} \sum_{i=1}^{\infty} p_{t+i}^2 = E_t \frac{1}{2} \sum_{i=1}^{\infty} \left( \sum_{j=1}^{t+i} \pi_j \right)^2$$

which is  $E_1 \frac{1}{2} [p_2^2 + p_3^2] = E_1 \frac{1}{2} [(\pi_1 + \pi_2)^2 + (\pi_1 + \pi_2 + \pi_3)^2]$  in period 1,  $E_2 \frac{1}{2} [(\pi_1 + \pi_2 + \pi_3)^2]$  and in period 2. Inflation values minimizing the loss are

$$\pi_1 + \pi_2 = 0 \text{ and } \pi_3 = 0.$$

According to equations (6) and (7),  $\pi_{t+1|t} = p^* - p_t = -p_t$ . With simple substitutions into the conditions given by the Phillips curve, solutions are obtained.

### A.4 Hybrid targeting

Using the transformation of

$$\theta p_t + (1-\theta)\pi_t = \theta p_{t-1} + \theta\pi_t + \pi_t - \theta\pi_t = \pi_t + \theta p_{t-1},$$

the loss function to be minimized is

$$E_t \frac{1}{2} \sum_{i=1}^{\infty} (\pi_{t+i} + \theta p_t)^2 = E_t \frac{1}{2} \sum_{i=1}^{\infty} \left( \pi_{t+i} + \theta \sum_{j=1}^i \pi_j \right)^2$$

which is  $E_1 \frac{1}{2} [(\pi_2 + \theta p_1)^2 + (\pi_3 + \theta p_2)^2] = E_1 \frac{1}{2} [(\pi_2 + \theta\pi_1)^2 + (\pi_3 + \theta(\pi_1 + \pi_2))^2]$  in period 1, and  $E_2 \frac{1}{2} [(\pi_3 + \theta(\pi_1 + \pi_2))^2]$  in period 2. Inflation values minimizing the loss are

$$\pi_2 + \theta\pi_1 = 0 \text{ and } \pi_3 = 0.$$

According to equations (6) and (7),  $\pi_{t+1|t} = (1-\theta)\pi_1 - p_t$ . With simple substitutions into the conditions given by the Phillips curve, solutions are obtained.

On an infinite horizon, price level under HT converges to the same optimal path as under PT, since this path is solely defined by the price level target component. However, the adjustment of price level is more moderate under HT than under PT, since its deviation from that path is less penalized (hence, the larger the emphasis on price level target, the closer the adjustment speed is). Since our model captures the ‘reasonable’ time horizon where the initial drift of price level under PT is completely eliminated and then closed by construction, price level adjustment can happen only partially under HT. Still, high credibility assumption holds, since HT performs the expected time-proportional correction of the initial drift.

## B SOCIAL PREFERENCE WEIGHT AND THE ‘SLOPE’ OF THE PHILLIPS CURVE

Considering the commitment solution, one can recognize the relation scheme<sup>51</sup>

$$\frac{1}{\delta + \frac{\lambda}{\delta}} = k$$

<sup>51</sup> For perspicuity, the scheme of the new fully forward looking case is demonstrated. Similar would be the case of the discretionary regimes in a higher resolution (e.g. infinite-period) model, where weights on output gap are not set to zero.

in the case of the output gap, and

$$\frac{\delta}{\delta + \frac{\lambda}{\delta}} = \delta k$$

in the case of inflation.

What does that mean? If the output gap does not matter at all ( $\lambda=0$ ), and the central bank should focus on the inflation target alone, then only the ‘slope’ of the Phillips curve determines the size of the output gap necessary to achieve the goals. If it is not just the inflation, but the output gap matters ( $\lambda>0$ ), then it can be seen that the rules reduce the output level divergence from the potential one.

Considering the inflation rule, there are equivalent outcomes in any cases where  $\frac{\lambda}{\delta}=0$ , since  $\delta k=1$  at the same time. If  $\delta=\infty$ , then the central bank can make, with minimal intervention, the inflation move infinitely, and therefore, the preference weight on the output gap is not relevant. If  $\lambda=0$ , the inflation rule is independent from the ‘slope’ of the Phillips curve, as the scale of the intervention does not matter. These situations result in the same social loss; the only difference between the two cases can be captured in the size (and the variability) of the output gap. In discretionary regimes, because of the model structure, the social loss implications of these relations are more straightforward.

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