

Karádi, Péter; Reiff, Ádám

**Working Paper**

## Inflation asymmetry, menu costs and aggregation bias - A further case for state dependent pricing

MNB Working Papers, No. 2010/3

**Provided in Cooperation with:**

Magyar Nemzeti Bank, The Central Bank of Hungary, Budapest

*Suggested Citation:* Karádi, Péter; Reiff, Ádám (2010) : Inflation asymmetry, menu costs and aggregation bias - A further case for state dependent pricing, MNB Working Papers, No. 2010/3, Magyar Nemzeti Bank, Budapest

This Version is available at:

<https://hdl.handle.net/10419/83605>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



MAGYAR NEMZETI BANK

**MNB WORKING  
PAPERS**

**2010/3**

**PÉTER KARÁDI-ÁDÁM REIFF**

**Inflation asymmetry, menu costs and  
aggregation bias – A further  
case for state dependent pricing**



**Inflation asymmetry, menu costs and  
aggregation bias – A further  
case for state dependent pricing**

**April 2010**



The MNB Working Paper series includes studies that are aimed to be of interest to the academic community, as well as researchers in central banks and elsewhere. Starting from 9/2005, articles undergo a refereeing process, and their publication is supervised by an editorial board.

The purpose of publishing the Working Paper series is to stimulate comments and suggestions to the work prepared within the Magyar Nemzeti Bank. Citations should refer to a Magyar Nemzeti Bank Working Paper. The views expressed are those of the authors and do not necessarily reflect the official view of the Bank.

MNB Working Papers 2010/3

**Inflation asymmetry, menu costs and aggregation bias – A further case for state dependent pricing\***

(Inflációs aszimmetria, menüköltségek és aggregációs torzítás – egy újabb bizonyíték az állapotfüggő árazásra)

Written by: Péter Karádi\*\*–Ádám Reiff\*\*\*

Published by the Magyar Nemzeti Bank

Szabadság tér 8–9, H-1850 Budapest

<http://www.mnb.hu>

ISSN 1585 5600 (online)

\* The authors would like to thank Péter Benczúr, Mark Gertler, László Halpern, John Leahy, Virgiliu Midrigan and Attila Rátfai for insightful comments. All remaining errors are ours. The views expressed are those of the authors and do not necessarily reflect the views of the central bank of Hungary.

\*\* Researcher, Magyar Nemzeti Bank, e-mail: karadip@mnb.hu.

\*\*\* Researcher, Magyar Nemzeti Bank, e-mail: reiffa@mnb.hu.

# Contents

<b>Abstract</b>	5
<b>1 Introduction</b>	6
<b>2 Stylized facts of the Hungarian experiment</b>	9
<b>3 The model</b>	14
<b>4 The data</b>	19
<b>5 Results</b>	21
<b>6 Discussion</b>	25
<b>7 Conclusion</b>	30
<b>References</b>	31
<b>Appendix</b>	33



# Abstract

Asymmetric inflation response to aggregate shocks is an identifying macro-prediction of state dependent pricing models with trend inflation (Ball and Mankiw, 1994). The paper uses the natural experiment of symmetric value-added tax (VAT) changes in Hungary with highly asymmetric inflation responses to provide further evidence for state-dependent pricing and for the Ball-Mankiw conjecture.

The paper shows, furthermore, that while a standard menu cost model like that of Golosov and Lucas (2007) underestimates the observed asymmetry, a model of multi-product firms that takes sectoral heterogeneity explicitly into consideration can quantitatively account for the inflation asymmetry observed in the data. This aggregation bias of the standard model is the result of the strong interaction term between trend inflation and menu costs in determining asymmetry in the model, and the positive correlation between sectoral inflation rates and menu costs in the data. The paper implies that the real effects of negative monetary shocks can be substantial even in the standard Golosov and Lucas (2007) model if these additional factors are taken into consideration.

**JEL Classification:** E30.

**Keywords:** Aggregation Bias, Inflation Asymmetry, Menu Cost, Sectoral Heterogeneity, Value-Added Tax.

## Összefoglalás

Trendinfláció esetén az állapotfüggő árazási modellekben az aggregált sokkoknak aszimmetrikus inflációs hatása van (Ball-Mankiw, 1994). Ebben a tanulmányban az ilyen típusú modellek és a Ball-Mankiw-féle mechanizmus mellett érvelünk, a természetes kísérletként is felfogható magyarországi ÁFA-változások aszimmetrikus inflációs hatásának a felhasználásával.

A tanulmányban bemutatjuk, hogy míg a Golosov-Lucas (2007)-féle egyszerű étlapköltségmodell alulbecsli a megfigyelt aszimmetriát, a többtermékes vállalatokat feltételező és szektorális heterogenitást közvetlenül figyelembe vevő modell képes az adatokban megfigyelthez hasonló mértékű aszimmetria előállítására. Az alapmodellbeli aggregációs torzítás oka kettős: egyrészt a modellbeli aszimmetriát a trendinfláció és az étlapköltség *szorzata* is befolyásolja, másrészt pedig a szektorszintű adatokban a trendinfláció és az étlapköltség közötti korreláció pozitív. A tanulmány alapján a negatív monetáris sokkok reálhatása még a Golosov-Lucas (2007)-féle alapmodellben is jelentős lehet, amennyiben ezeket a megfontolásokat is figyelembe vesszük.

# 1 Introduction

This paper documents an asymmetric inflation response to transparent symmetric aggregate shocks and discusses its implications to the pricing debate. On the one hand, this observed asymmetry supports the state-dependent pricing assumption: as Ball and Mankiw (1994) showed, menu cost models with trend inflation predict asymmetry, while standard flexible or time-dependent pricing models (Calvo, 1983) would predict symmetry. On the other hand, it raises quantitative issues about the standard menu cost model of Golosov and Lucas (2007), that not only underestimates the frequency effects of the shocks, but also predicts the inflation-asymmetry to be small. A model that reproduces the frequency effects and the level of asymmetry observed in the data modifies essential predictions of these models: in the more realistic model, the real effects of a negative monetary policy shock can be substantial.

The natural experiment the paper capitalizing on happened in Hungary (we shall refer to it as the “Hungarian experiment”), and involved symmetric changes in the value added tax (VAT) rates with highly asymmetric inflation effects. The shocks happened in a similar macro-environment with a couple of months difference. In 2006, the Hungarian government sequentially closed the gap between two different VAT rates: it decreased 25% rate to 20% in January and increased the 15% rate to 20% in September. It did it in steps in hope of political gains in the April elections for the preelection tax cut, which ensures the exogeneity of the shocks. Therefore these VAT shocks provide textbook cases of easily measurable, exogenous aggregate cost push shocks to gross prices, that Hungarian firms are required by law to quote.

The experiment is highly suitable for analyzing sectoral and aggregate inflation asymmetry.<sup>1</sup> Gabriel and Reiff (2010) have documented that the tax changes – influencing similar proportion of products<sup>2</sup> – had highly asymmetric inflation effects: the immediate pass-through of the affected products for the increase was over 85%, while for the decrease it was less than 26%.

Asymmetric inflation response to transparent symmetric shocks (as the VAT changes, or monetary policy shocks) are unexplained by standard flexible or time-dependent pricing models, while they are natural macro-predictions of state-dependent pricing models with trend inflation (Ball and Mankiw, 1994). It is straightforward to see why the flexible price assumption would not interact with trend inflation: firms would adjust their nominal prices every instance to follow the aggregate inflation rate and their *additional* response to aggregate shocks would imply a full and symmetric pass-through. In the time-dependent model of Calvo (1983), the argument against aggregate asymmetry is more subtle because trend inflation introduces a *firm-level asymmetry*. This asymmetry rests on forward looking firms who set their prices for several periods in advance: their dynamically optimal price is larger than their static optimal price in the current period (“front loading”).<sup>3</sup> This trend inflation-induced bias would mean higher responses for positive shocks and lower responses to negative ones for every single firm. But this type of asymmetry is present even in the absence of aggregate shocks: the constant fraction of price-changing firms always front load, so there is no additional asymmetry from front loading when an aggregate shock hits. So the *additional* (i.e. shock-induced, as opposed to trend inflation-induced) response to symmetric aggregate shocks will not be asymmetric.

In menu cost models this kind of front loading is also present, but – as Ball and Mankiw (1994) show – there is an additional source of asymmetry after aggregate shocks. This additional source is *endogenous selection*: since the firms’ expected average relative price until the next adjustment is always below its desired level (except for the moments of adjustment), relatively small positive shocks will trigger adjustment (while similar negative shocks will not). In other words, facing a positive shock in an inflationary environment, the relative

---

<sup>1</sup> Although none of the products were affected by both tax changes, there are close substitutes affected by the different tax changes: e.g. cookies were hit by the tax increase, while chocolate chip cookies were hit by the tax decrease.

<sup>2</sup> 51% and 46.9% in the sample.

<sup>3</sup> This is because firms set their nominal price so that their average relative price until their next adjustment is optimal, and the relative price is declining with constant nominal price and positive trend inflation.

proportion of price-increasing firms to the price-decreasing ones is higher than the relative proportion of price-decreasing firms to price-increasing ones in case of a negative shock, which leads to asymmetry.<sup>4</sup>

Stylized facts from micro-level Hungarian Consumer Price Index (CPI) data indicate that the main reason of the observed asymmetry is indeed the interaction of trend inflation and menu costs. On the one hand, state dependent models with *menu cost* are a valid framework for us: flexible price models would have serious difficulty in explaining the fact that at the months of tax increase only a bit less than 60% of affected firms changed their prices,<sup>5</sup> but the same number would also embarrass a Calvo-type model with exogenous probability of price change (12% in our data). On the other hand, there are three stylized facts supporting that *trend inflation* has to be an unavoidable ingredient in any explanations of the Hungarian experiment. First, the two main sectors behave in line with the Ball and Mankiw (1994) model: the observed asymmetry is much higher in the one with both high inflation rate and high level of price stickiness (services) than in the one that has lower inflation rate and lower level of price stickiness (processed food). Second, the observed asymmetry decreases over time. For the tax increase, the pass-through is immediate as firms front load their responses, while for the tax decrease the cumulative pass-through is increasing as firms adjust by not changing their prices and letting their relative prices to adjust gradually with the trend inflation. Third, going down to a more disaggregated product level, the larger the product-specific trend inflation is, the larger is the observed asymmetry, a clear indication that trend inflation plays a crucial role in the observed asymmetries.

To provide quantitative evidence for the Ball-Mankiw conjecture, the paper develops and calibrates a structural model. It finds that even though the standard structural menu cost model of Golosov and Lucas (2007) incorporates the Ball-Mankiw channel, it seriously underestimates the aggregate asymmetry. There are two interacting causes of this.

The first is aggregation bias. As the paper shows, the cross effect of trend inflation and menu costs has a large and significant effect on the inflation-asymmetry. So, if sectoral trend inflation - that is a closer proxy for the competitors' prices than the aggregate inflation rate - is higher in sectors with high level of price stickiness, then models disregarding sectoral heterogeneity can substantially underestimate the aggregate asymmetry. Though no theoretical argument we know of explains the correlation between trend inflation and price stickiness, services sectors - with substantial weight in consumption - are generally the most sticky price sectors with also higher than average trend inflation rate,<sup>6</sup> as is the case in Hungary. Sectoral calibration of the menu cost model allows the paper to take sectoral heterogeneity into consideration, and also allows it to use the sectoral variation in trend-inflation, price stickiness and inflation asymmetry to better test the model's quantitative predictions.<sup>7</sup>

The second reason why the Golosov-Lucas (2007) model underestimates aggregate inflation asymmetry is that it underestimates the fraction of price changing firms in the months of the VAT changes (see figure 1 later), thereby it reduces the asymmetry-amplification effect coming from the more frequent price changes. To correct for this caveat, the paper assumes that changing prices at the months of VAT changes is cheaper than in normal times. Through this assumption, the model, by construction, can reproduce the observed frequency increase observed during the VAT shocks, but also gets closer to the *lower* average size of price changes observed during the VAT shocks (see figure 2 later). The lower average menu cost can be justified by the existence of multi-product firms with increasing returns-to-scale in their repricing technology as in Midrigan (2009).

The paper finds that the sectoral menu cost model calibrated to hit the sectoral trend inflation and standard pricing moments (frequency and size of price changes) at normal times and during VAT shocks quantita-

<sup>4</sup> Ball and Mankiw (1994) show that in fact there is yet another source of additional asymmetry: as firms know that their nominal price is more likely to remain in effect if they will be hit by a negative shock, rational firms set a bit lower initial price, which further increases the asymmetry.

<sup>5</sup> Moreover, only 68.2% of the affected firms changed their prices in the following 4 months of the VAT shock.

<sup>6</sup> A potential explanation for the relatively large trend inflation in the services sector is the Balassa-Samuelson effect.

<sup>7</sup> This choice, furthermore, allows the paper to control for the different sectoral composition of the observed tax changes. This is necessary as VAT-changes affected various sectors differently (the VAT-increase hitting sectors with more flexible prices disproportionately).

tively hits the inflation asymmetry observed in the data, so it provides quantitative evidence for the Ball and Mankiw (1994) conjecture.

There is a long line of research documenting asymmetric price developments to monetary and cost shocks using aggregate (see e.g. Cover, 1992, Ravn and Sola, 2004) and sectoral data (Peltzman, 2000) in reduced form estimations. Our paper is the first we know of, however, which uses micro-level structural calibration and VAT shocks to analyze the effects of the asymmetry, which are arguably more easily measurable and identifiable shocks than those used by the previous papers. As standard frictionless, flexible price models, and time dependent models would have difficulty in explaining these asymmetries, our paper is also a contribution to the growing literature using natural experiments – like the effect of euro introduction to restaurant prices in Hobijn, Ravenna and Tambalotti (2006) – and special environments – like the high inflation episodes in Mexico in Gagnon (2009) – to provide evidence to the validity of the sticky price assumptions in general and menu cost models in particular.

We believe that some alternative explanations of asymmetry cannot play a crucial role in the Hungarian experiment. First, consider an alternative state-dependent model assuming convex adjustment cost of price changes, like that of Rotemberg (1982). This model would also imply asymmetric inflation response, but with opposite sign: in an inflationary environment, the additional response of firms to positive shocks would be *smaller* than to a negative one, because the marginal cost of price increase (additional to the trend-inflation induced optimal positive change) would be higher. Moreover, a series of recent papers use search frictions to explain asymmetric price responses (Cabral and Fishman, 2008, and Yang and Ye, 2008). Their common key assumption, however, is that the marginal consumers are uninformed about the cost shocks faced by price setting firms, which is clearly not applicable to our case of VAT-shocks: these had easily measurable, widely publicized and uniform effects on all affected firms. Finally, the asymmetric inflation effect due to the asymmetric shape of the profit function (Devereux and Siu, 2007, Ellingsen et al, 2006) is in fact incorporated in our model, but numerical results showed that this type of asymmetry is negligible relative to the Ball-Mankiw-type asymmetry.

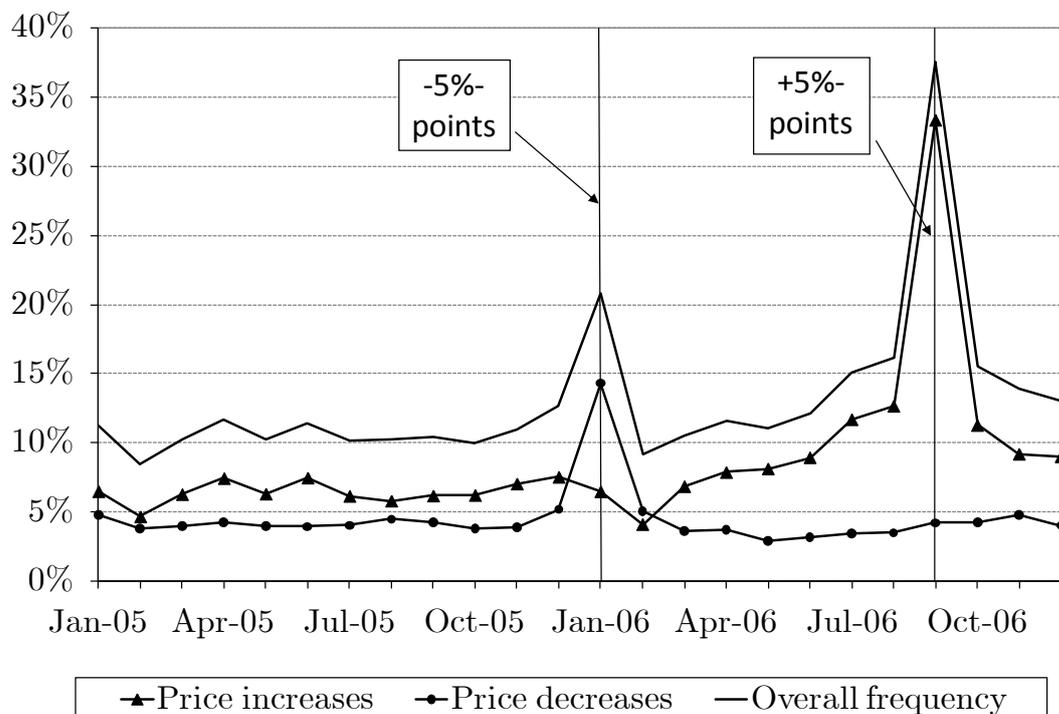
The paper is organized as follows. After presenting the main stylized facts of the Hungarian experiment in Section 2, Section 3 presents the model and explains the numerical algorithm used for its solution. Section 4 describes the data and the moments the paper is about to match. Section 5 presents the results for the aggregate and the sectoral calibrations and Section 6 concludes.

## 2 Stylized facts of the Hungarian experiment

In this section, we describe the Hungarian experiment and present some basic observations on the pricing responses to the VAT shocks. These observations suggest that a state dependent menu cost model is a valid framework to study the effects of the shocks and trend inflation is a necessary ingredient to explain the asymmetry observed in the data.

In the months when no VAT changes occurred, the Hungarian CPI data shows the standard pricing behavior documented for other countries (see e.g. Klenow and Krystov, 2008). In our sample of products affected by the VAT shocks, the fraction of price changing firms is fairly stable around 12%<sup>8</sup> (see figure 1). The average absolute size of the price changes – also in line with other studies – is high: it fluctuates around 11.5% supporting the application of a model with volatile idiosyncratic shocks like that of Golosov and Lucas (2007) (see figure 2).

**Figure 1**  
Frequency of price changes



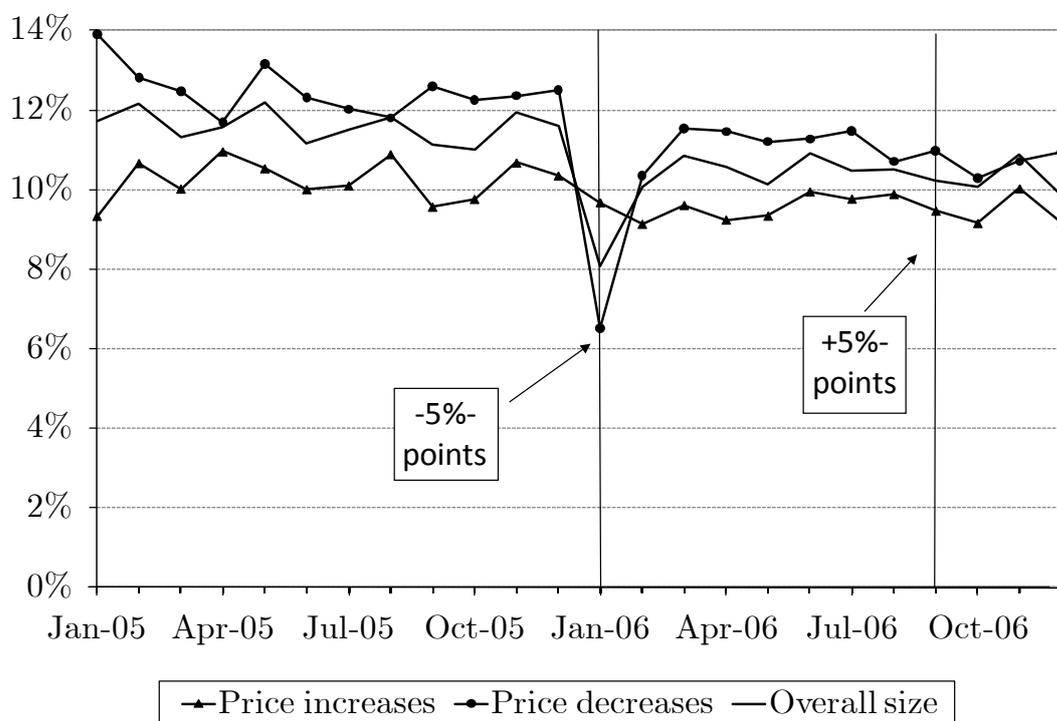
### THE VAT CHANGES

In 2006, The Hungarian authorities sequentially closed the gap between two VAT tax rates levied on two sets of products with each covering approximately half of our sample.<sup>9</sup> As table 1 shows, the top rate was decreased in January 2006 from 25% to 20% and the lower rate was increased in September 2006 from 15%

<sup>8</sup> Its relatively low level can be explained by the fact that some sectors with very flexible prices were excluded from the sample (e.g. fuel with special tax rate developing independently from VAT; with these sectors the average frequency were 18.5%).

<sup>9</sup> 51% and 46.9% CPI weights. There is also a third, very low tax rate levied on a few subsidized products (drugs, school books). As these products only constitute 2.1% in CPI-weights of our sample, we will ignore these.

**Figure 2**  
Average absolute size of price changes



to 20%. This tax increase in September could not be anticipated at the time of the tax decrease in January, as it was done by a new government elected in April 2006.

**Table 1**  
VAT rates in Hungary

VAT-rates in Hungary	Lower	Higher
- Dec 31, 2005	15%	25%
Jan 1, 2006 - Aug 31, 2006	15%	20%
Sep 1, 2006 -	20%	20%

The VAT changes are measurable and transparent cost push shocks directly influencing the gross prices of affected firms, which are the quoted prices in Hungary – differently from the U.S. practice regarding sales tax, but similarly to most other European countries. This practice is also reinforced by a consumer protection law requiring also that “consumers can not be forced to calculate prices in their head”.<sup>10</sup> This cost push shock dominated the inflation effects during these months as the inflation targeting central bank has expressed in advance that it was not planning to respond to the direct effects of the tax shocks, as these only affect the price level and have only temporary effects on the measured inflation rate.

The tax changes had substantial direct effects on the consumer prices. The aggregate fraction of price-changing firms increased substantially (to 42.7% on average from 12%) in the months of the tax changes (see figure 1), in line with international evidence (see Gagnon, 2009 and the references there.) This supports the use of state-dependent menu cost models that predict endogenous increase in the number of price-changing

<sup>10</sup> 1997.CLV. Law on Consumer Protection and 7/2001. (III. 29.) Ministry of Economy decree and its explanation.

firms. The average absolute size of price changes dropped during the tax-changing months (from 11.6% to 8.0%), as it can be seen in figure 2.

### ASYMMETRIC INFLATION EFFECT

The positive and negative VAT shocks of similar magnitude influencing similar weighted proportion of products had highly asymmetric inflation effects. Figure 3 depicts the monthly inflation rates around the VAT changes in 2006 in the processed food and services sectors and among all products. It is apparent from the figure that the 5 percentage point tax increase had much bigger impact on the monthly inflation rate than the similar tax decrease.

**Figure 3**  
Monthly inflation rates and VAT changes

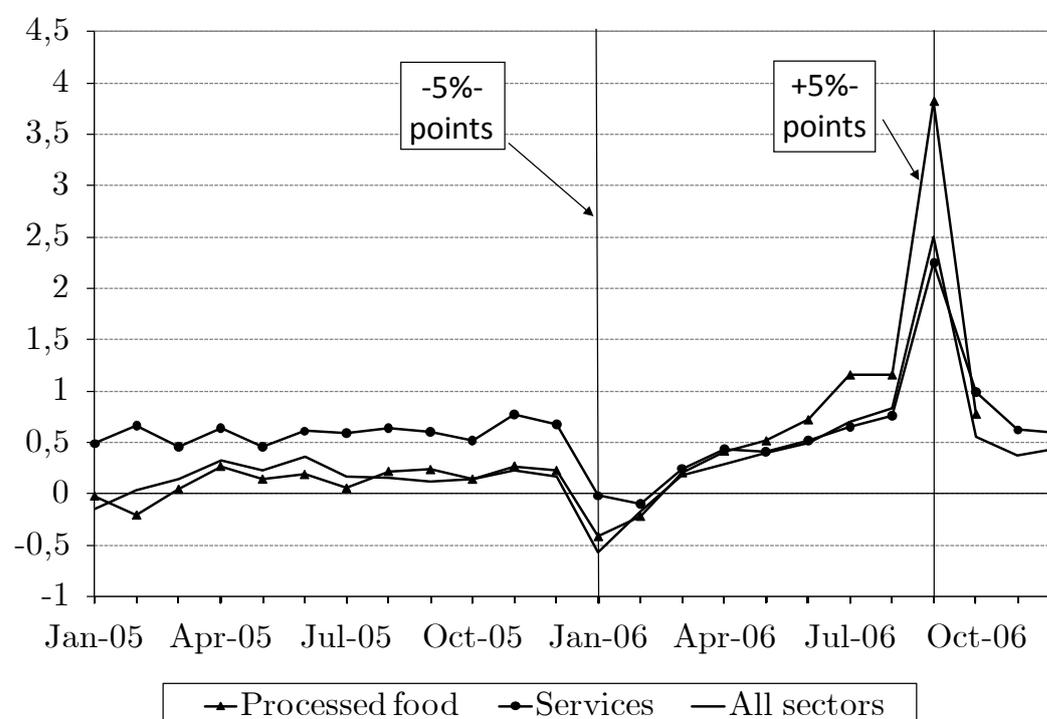


Table 2 shows the overall pass-through of unit tax changes that are calculated as the ratio of the inflation effect (of affected and non-affected products) and the change in the average sectoral tax rate.<sup>11</sup> For the whole economy the pass-through for the positive shock was 1.11, while that of the unit tax decrease was 0.41. The higher than 1 pass-through comes from the fact that there was also price increase among products not affected by the VAT change.

Stylized facts about sectoral asymmetries provide suggestive evidence that these asymmetries were caused by the interaction of trend inflation and price stickiness. Looking at two of the largest major sectors in the economy, we see that the sectoral asymmetry is higher in the services sector with higher trend inflation and higher price stickiness (lower fraction of price changing firms) than in the processed food sector with lower inflation and lower price stickiness, in line with the prediction of the Ball and Mankiw 1994 model.

<sup>11</sup> This is the moment we would like our model - with affected and non-affected products - to hit. In the simulations of Section 6, however, we consider only products facing the same tax rates and shocks uniformly affecting all the products.

**Table 2****Trend-inflation, price stickiness and inflation asymmetry**

Sector	Yearly		Inflation effect of unit tax	
	Inflation	Frequency	increase	decrease
Aggregate	3.4%	12.0%	1.11	0.41
Processed food	4.2%	13.5%	1.04	0.85
Services	8.3%	6.6%	1.15	0.33

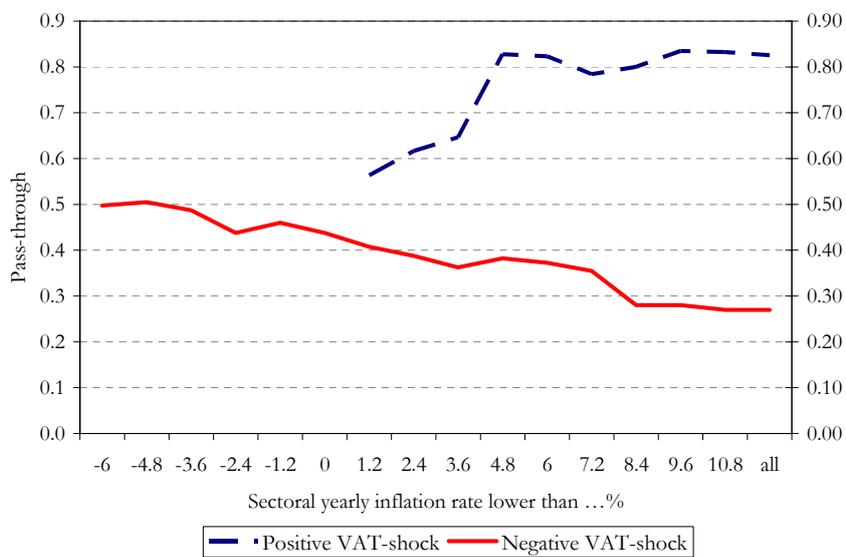
Also in line with the Ball-Mankiw argument, the observed asymmetry decreases over time. As table 3 shows, for the tax increase, the pass-through on the affected products is immediate, and this effect is almost entirely because of the increased frequency of price increases. For the tax decrease, on the other hand, the cumulative pass-through is increasing as firms choose not to respond to trend inflation. In this case, the first month's inflation effect was a result of the increased frequency of price decreases, but later it was the decreased frequency of price increases that led to additional inflation effects. This is exactly what happens in a model with menu costs and positive trend inflation.

**Table 3****Pass-through on affected products of VAT-changes over 1-3 months**

Horizon	VAT-increase	VAT-decrease
1 month	82.5%	-25.9%
2 months	82.1%	-38.9%
3 months	82.1%	-53.3%

A further stylized fact giving a clear indication that inflation plays a crucial role in explaining the observed asymmetry comes from going down to the product level. Here we can observe that the larger is the product-specific trend inflation, the larger is the observed asymmetry. Figure 4 shows the asymmetry estimates for products with different trend inflation rates. For example, if we estimate the asymmetry of the pass-through of the positive and negative tax shocks only for products with a moderate trend inflation (say, smaller than 2.4% per year), the result is 0.62 and 0.39. If we do the same for products with relatively larger trend inflation (say, for products with trend inflation smaller than 7.2% per year), the resulting asymmetry is much larger: 0.78 vs 0.36. Ideally, we should see asymmetry with the opposite sign for products with negative trend inflation. Unfortunately, there were no such products affected by the VAT increase, only by the VAT decrease.

**Figure 4**  
Trend inflation and asymmetric inflation effects



### 3 The model

The model is a sectoral version of a standard heterogeneous firm quantitative menu cost model like Golosov and Lucas (2007) with value added taxes (VAT). The formulation is similar to Klenow and Willis (2006).

#### THE CONSUMER

The representative consumer consumes a Dixit-Stiglitz aggregate ( $C$ ) of a basket of goods, holds real balances  $M/P$  and supplies sector specific labor  $L^s$  to maximize the expected present value of her utility<sup>12</sup>

$$\max_{\{C_t^s(i), L_t^s, M_t\}} E \sum_{t=0}^{\infty} \beta^t \left( \log \left[ C_t \cdot \left( \frac{M_t}{P_t} \right)^\nu \right] - \sum_{s=1}^S \frac{\mu^s}{1 + \psi^s} (L_t^s)^{1 + \psi^s} \right). \quad (1)$$

The assumption of sector specific labor supply leads to sectoral labor markets and allows us to assume away sectoral interactions between large sectors of the economy that is not necessary for our argument. For the same reason, we assume that the aggregate consumption basket  $C_t$  is obtained as a Cobb-Douglas aggregate of sectoral consumption baskets  $C_t^s$ . This assumption implies that fixed proportion of nominal expenditure is going to be spent on a certain sector, so sectoral inflation rates are not going to interact. Sectoral consumptions  $C_t^s$ , in turn, are constant elasticity of substitution aggregates of individual good consumptions  $C_t(i)$ :

$$C_t = \prod_{s=1}^S \left( \frac{C_t^s}{\alpha^s} \right)^{\alpha^s}, \quad C_t^s = \left( \int_{\mathcal{N}^s} (N^s)^{-\frac{1}{\theta}} C_t^s(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where  $\alpha^s$  are sectoral expenditure shares and  $\theta$  is the elasticity of substitution,  $\mathcal{N}^s$  is the set of the products in a certain sector, and  $N^s$  is their measure. Note that with  $S = 1$  and  $\alpha^1 = 1$  the model reduces to a standard non-sectoral model.

The consumer's budget constraints for all time period  $t$  and history  $h^t$  are given<sup>13</sup> by

$$\sum_{s=1}^S \int_{\mathcal{N}^s} P_t^s(i) C_t^s(i) di + \int_{h_{t+1}} B_{t+1}(h_{t+1}) d h_{t+1} + M_{t+1} = R_t B_t + M_t + \sum_{s=1}^S \tilde{\omega}_t^s L^s + \tilde{\Pi}_t + T_t, \quad (2)$$

where  $P_t^s(i)$  is the gross price,  $B_t(h_t)$  is a nominal Arrow-security with state dependent gross return  $R_t$ ,  $M_t$  is the nominal money balance and  $T_t$  is a lump-sum transfer.

In order to express the consumer's optimality conditions in a convenient form, it is useful to define the aggregate ( $P_t$ ) and the sectoral price levels ( $P_t^s$ ) by

$$P_t = \prod_{s=1}^S (P_t^s)^{\alpha^s}, \quad P_t^s = \left( \int_{\mathcal{N}^s} \frac{P_t^s(i)^{1-\theta}}{N_s} di \right)^{\frac{1}{1-\theta}},$$

which are standard definitions, implying that the aggregate and sectoral expenditures are given by  $P_t C_t$  and  $P_t^s C_t^s$  respectively.

The Cobb-Douglas formulation across sectors implies that the consumer will spend a constant  $\alpha^s$  fraction of his nominal expenditures on the sectoral composite good. His real demand can be expressed as

$$C_t^s = \alpha^s \left( \frac{P_t^s}{P_t} \right)^{-1} C_t, \quad (3)$$

<sup>12</sup> The subscript indices denote time ( $t$ ), superscripts the sector ( $s$ ) and individual firms and products are in brackets ( $i$ ).

<sup>13</sup> Dependence on history is suppressed for notational convenience.

and his demand for individual good  $i$  from sector  $s$ , in turn, is given by

$$C_t^s(i) = \frac{1}{N^s} \left( \frac{P_t^s(i)}{P_t^s} \right)^{-\theta} C_t^s. \quad (4)$$

The Euler equation of the consumer implies that the stochastic discount factor  $\frac{1}{R_{t+1}}$  is given by

$$\frac{1}{R_{t+1}} = \beta \frac{P_t C_t}{P_{t+1} C_{t+1}}, \quad (5)$$

the labor supply equation in each sector  $s = 1, \dots, S$  is given by

$$\mu^s (L_t^s)^{\psi_s} C_t = \frac{\tilde{w}_t}{P_t}, \quad (6)$$

and, finally, the money demand equation is going to be

$$\frac{M_t}{P_t} = \nu C_t \frac{i_t + 1}{i_t}, \quad (7)$$

where  $i_t$  is the nominal interest rate.

## THE GOVERNMENT AND THE CENTRAL BANK

In line with Hungarian practice, we assume that a subset of firms ( $i \in \mathcal{N}^{s,1}$ ) in each sectors face tax rate  $\tau_t^1$ , while the remaining firms face tax rate  $\tau_t^2$ . We assume that these rates ( $\tau_t(i)$ ) are given exogenously and firms do not expect future changes in their tax rates,<sup>14</sup> but can respond to the tax rate changes in the same month. Tax revenues are assumed to be redistributed in a lump sum fashion.<sup>15</sup>

$$\sum_{s=1}^S \sum_{i=1}^{n^s} P_t^s(i) \frac{\tau_t(i)}{1 + \tau_t(i)} C_t^s(i) = T_t^g.$$

As general in this literature, the central bank is assumed to follow a nominal income targeting rule by maintaining a predetermined growth rate ( $g_{PY}$ ) of the nominal aggregate output ( $P_t Y_t$ ), which we assumed to be equal to the money supply ( $M_t$ ). The exogenous nominal growth assumption substantially simplifies the analysis, allowing the paper to focus on firm level and sectoral incentives for responding to tax changes.<sup>16</sup> The resulting extra money supply  $M_t$  in the economy is redistributed in a lump sum way

$$M_t - M_{t-1} = T_t^m,$$

where  $T_t^m + T_t^g = T_t$  is the total transfer to the consumers.

<sup>14</sup> Changing this assumption of unexpected permanent tax shock to a model-consistent uncertainty of temporary tax shocks (with persistence parameter 0.95) had no numerically significant effects on our asymmetry results at the month of tax change.

<sup>15</sup> If the net prices  $P^n$  are taxed by  $\tau$  VAT, then the gross price is  $P = (1 + \tau)P^n$ , so the tax revenue equals  $\tau P^n C = \frac{\tau}{1 + \tau} P C$ .

<sup>16</sup> The constant nominal GDP growth assumption would be restrictive if we wanted to consider endogenous monetary policy responses to the tax shocks. As this was not the case in Hungary, this assumption is acceptable.

## THE FIRMS

In what follows, we characterize the firms' problem in sector  $s$ .<sup>17</sup> As there are no sectoral interactions, the firms' problem does not depend on any aggregate endogenous variables and, as a result of the fixed sectoral expenditure shares, the growth of nominal expenditure ( $g_{PY}$ ) is going to be constant across sectors.

We introduce value added taxes (VAT) to the framework in a straightforward way: as we do not model explicitly the production process, VAT in our framework is equivalent to a standard sales tax. We assume – in line with the Hungarian legal rules – that firms set gross prices and they need to pay a fixed menu cost in case they decide to change these.

We distinguish two types of firms: there are  $N^1$  measure of firms with tax rate  $\tau^1$  and  $N^2 = N - N^1$  measure of firms with tax rate  $\tau^2$ . By this, we are modeling the fact that firms producing competing goods might face different VAT-rates and the tax changes influence only a subset of firms in each sector.

Each firm  $i$  is assumed to produce product  $i$  monopolistically, post gross prices  $P_t(i)$  and satisfy all demand given this price. They face a small menu cost  $\tilde{\phi}_t$  if they choose to change their prices; these are assumed to be proportional to their revenues ensuring that the economy can not grow out of the price stickiness. We also allow the menu cost to be smaller for in the months of tax shocks as an implicit way of modeling multi-product firms with increasing returns to scale in their repricing technology. This is consistent with lower average menu costs during periods with frequent price changes (like the tax change periods).

The firms' problem is to maximize the expected discounted present value of their profits

$$\max E \sum_{t=0}^{\infty} \frac{1}{\prod_{q=0}^t R(q)} \tilde{\Pi}_t(i), \quad (8)$$

where the periodic profit level is given by

$$\tilde{\Pi}_t(i) = \frac{1}{1 + \tau_t(i)} P_t(i) Y_t(i) - \tilde{w}_t L_t(i). \quad (9)$$

We assume that the firms use a constant returns to scale technology with only labor as a factor to produce their differentiated good  $i$  and face idiosyncratic  $A_t(i)$  and sectoral technology shocks  $Z_t$ . The production functions of the firms are given by

$$Y_t(i) = Z_t A_t(i) L_t(i). \quad (10)$$

We assume that the sectoral technology shock  $Z_t$  grows at a constant rate  $g_Z$ . The difference between the growth rate of nominal expenditure and sectoral technology will determine the trend inflation. By assuming deterministic growth, we rule out inflation variability. The assumption makes the numerical algorithm substantially easier and has no significant effects on the results.<sup>18</sup>

Idiosyncratic productivity shocks  $A_i$  are the now standard assumption of Golosov and Lucas (2007) to reproduce the observed large size of average price changes. The log of the idiosyncratic productivity is assumed to follow an AR(1) process:

$$\ln A_t(i) = \rho_A \ln A_{t-1}(i) + \varepsilon_t(i),$$

where the innovations are i.i.d. normally distributed random variables with mean zero and standard deviation  $\sigma_A$ .

<sup>17</sup> For notational convenience we suppress the reference to this.

<sup>18</sup> Results with inflation variability using the Krusell-Smith (1998) algorithm are available from the authors upon request.

The production function (10) implies an individual labor demand

$$L_t(i) = \frac{Y_t(i)}{Z_t A_t(i)}, \quad (11)$$

which aggregates to a sectoral labor demand given by

$$L_t = \int_{\mathcal{N}} L_t(i) di. \quad (12)$$

Substituting the individual demand (equation (4)) and the labor demand (equation (11)) into the periodic profit function (9) and using the equilibrium condition  $Y_t(i) = C_t(i)$ , we obtain that

$$\tilde{\Pi}_t(i) = \frac{1}{1 + \tau_t} \frac{1}{N} P_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t - \tilde{w}_t \frac{\frac{1}{N} \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t}{Z_t A_t(i)}.$$

As we have constant nominal growth in the model, we can normalize the profit level by the average sectoral revenues

$$\Pi_t(i) = \frac{\tilde{\Pi}_t(i) N}{P_t Y_t}.$$

Let  $p_t(i) = \frac{P_t(i)}{P_t}$  be the (sectoral) relative price,  $w_t = \frac{\tilde{w}_t}{P_t Y_t}$  the normalized wage rate, and  $\phi_t = \frac{\tilde{\phi}_t N}{P_t Y_t}$  the normalized menu cost. Let  $\zeta_t = w_t \frac{Y_t}{Z_t}$  be a sectoral cost factor. Substituting these variables into the normalized periodic profit function we obtain that

$$\Pi_t(p_t(i), A_t(i), \zeta_t, \tau_t(i)) = \frac{1}{1 + \tau_t(i)} (p_t(i))^{1-\theta} - (p_t(i))^{-\theta} \zeta_t A_t(i)^{-1} - \phi_t, \quad (13)$$

where the firm needs to pay  $\phi_t$  only if it chooses to change its price.

The exogenous state variables of the normalized problem are given by  $(A_t(i), \tau_t^1, \tau_t^2, \phi_t)$ . The endogenous state variables of the problem are given by  $(p_{t-1}(i), \pi_t, \zeta_t, \Gamma_t)$ , where  $\pi_t$  is the sectoral inflation rate and  $\Gamma_t$  is the distribution of relative prices. To present the firms' Bellman equation, we express the set of state variables as  $(p_{-1}(i), \Omega(i))$ , where  $\Omega(i) = (A(i), \pi, \zeta, \tau^1, \tau^2, \phi, \Gamma)$ .

Given these state variables the value of firm  $i$  is determined by the maximum it can get by changing (C) its nominal price or keeping it constant (NC):

$$V(p_{-1}(i), \Omega(i)) = \max_{\{C, NC\}} \left[ V^{NC}(p_{-1}(i), \Omega(i)), V^C(p_{-1}(i), \Omega(i)) \right], \quad (14)$$

where the value function in case of no price change (NC) is given by

$$V^{NC}(p_{-1}(i), \Omega(i)) = \Pi(i) \left( \frac{p_{-1}(i)}{1 + \pi}, A(i), \zeta, \tau(i) \right) + \beta E V \left( \frac{p_{-1}(i)}{1 + \pi}, \Omega'(i) \right), \quad (15)$$

where we used the fact that if the firm decides to keep its nominal price constant, its relative price  $p(i)$  is going to depreciate by the inflation rate. The value function of firm  $i$  in case of price change (C) is given by

$$V^C(p_{-1}(i), \Omega(i)) = \max_{p(i)} \Pi(p(i), A(i), \zeta, \tau(i)) + \beta E V(p(i), \Omega'(i)). \quad (16)$$

The endogenous distribution of the relative prices  $\Gamma$  is, in general, a very complicated function of the last period price distribution  $\Gamma_{t-1}$ , the current distribution of the sectoral idiosyncratic technology distribution  $\Lambda_t$  and the development of the exogenous state variables  $\tau_t^1, \tau_t^2, \phi_t$ :

$$\Gamma_t = \Theta(\Gamma_{t-1}, \Lambda_t, \tau_t^1, \tau_t^2, \phi_t) \quad (17)$$

## THE EQUILIBRIUM

We consider a stochastic dynamic general equilibrium of the model. As there is no aggregate uncertainty in the model, firms know the steady-state values of the aggregate endogenous state variables  $\pi, \zeta, \Gamma$ . The equilibrium conditions are the following:

1. The representative consumer chooses  $C_t(i), L_t^s, M_t$  to maximize her utility function (1) given her budget constraint (2), taking goods prices  $\{P_t(i)\}$ , the interest rates  $R_t$  and the sectoral wages  $\{w^s\}$  as given.
2. The firms are assumed to set prices  $P_t(i)$  to maximize their value function (14), (15), (16), given the exogenous state variables  $(A, \tau^1, \tau^2, \phi)$  and the values of the endogenous aggregate state variables  $(\pi_t, \zeta_t, \Gamma_t)$ , and they also have correct beliefs about the random process of the idiosyncratic productivity shock  $A$ .
3. The central bank sets  $i_t, M_t$  to keep the nominal output growth  $g_{PY}$  constant, and the fiscal transfers are set in a way to imply balanced budget.
4. Market clearing in all goods markets  $C_t(i) = Y_t(i)$ .
5. Assets in zero net supply in nominal Arrow securities:  $B_t = 0$ .
6. Equilibrium in the sectoral labor markets implying sectoral wages  $w_t^s$  equating sectoral labor demand (11) and labor supply (6).

## NUMERICAL SOLUTION

Under flexible prices (zero menu costs) the model has a closed form solution. Inflation responds symmetrically with a full pass-through to the changes in the VAT rates (please see the appendix for the derivation).

Under positive menu costs, the model has no closed form solution, so we use numerical methods to obtain solutions for the equilibrium. The challenge of the framework is that the distribution of relative prices is a state variable which is an infinite dimensional object. To solve the value and the policy functions of individual firms, however, we only need to know about the development of two main moments of distribution: the inflation rate ( $\pi_t$ ) and the sectoral cost factor ( $\zeta_t$ ). To find the equilibrium, the algorithms need to find fixed points of these sequences such that the optimal response of the firms results in relative price distribution development that is consistent with these sequences.

To find these fixed points, we use standard heterogeneous agent iterative algorithms. The experiment is to find the equilibrium response to unforeseen permanent tax changes. The algorithms first obtain solutions for the endogenous variables in the steady states before and after the tax changes, then they obtain an equilibrium transitional inflation path between these steady states (please see the appendix for details).

## 4 The data

The data set we use contain store-level price quotes that are originally used to the monthly calculation of the Consumer Price Index (CPI) in Hungary. It spans from December 2001 to December 2006. In terms of product categories, it contains price information on 550 different representative items; the total CPI-weight of these items is 45.3% in 2006.<sup>19</sup>

The overall sample coverage by the sample and by some main CPI-categories is illustrated in Table 4. The Statistical Office categorizes products into 6 major sectors (processed and unprocessed food, clothes, durable goods, energy, services and other goods). From these, the two largest sectors (processed food and services) were the homogeneous sectors with products facing each VAT rates with substantial weight (the other goods sector was not homogeneous). These sectors are used to calibrate the model and test its predictions.

**Table 4**

Coverage of the data set by some CPI-categories

CPI category	CPI basket		Sample	
	Weight	Items	Weight	Items
Processed food	17.4%	139	16.1%	128
Services	25.1%	161	8.8%	78
<b>TOTAL</b>	<b>100%</b>	<b>896</b>	<b>45.4%</b>	<b>550</b>

The 550 representative items in the data set, that will form the units for our moments, can be regarded as 550 mini panels, containing time series of price quotes from different outlets. As an example, consider item 10001 “Bony pork rib with tenderloin”: the data set contains 7,922 observations from 162 different outlets, i.e. 48.9 price quotes per outlet. For 96 of the 162 stores we have data for each month ( $T = 60$ ). It is true for most of the representative items in the data set that the list of observed outlets is typically unchanged. On average, there are approximately 6,566 observations per representative item in the data set, which means that the total number of observations exceeds 3.6 million (3,611,335).

Our analysis focuses on regular prices, rather than sales prices. The price collectors of the Central Statistical Office use a sales flag to identify sales prices (i.e. prices that are temporarily low, and have a "sales" label), and we use these flags to filter out sales prices in the first round. After this we also filter out any remaining price changes that are (1) at least 10 %, (2) and are completely reversed within 2 months.

### DATA MOMENTS

We calibrate model parameters by matching simulated “theoretical” model moments to their analogues in the data. The moments are chosen such that they identify essential structural parameters of the model. We chose some standard moments in non-tax-changing and tax-changing months:

- sectoral yearly inflation rate ( $\pi$ );
- frequency of price changes in non tax-changing months ( $\bar{I}_{NT}$ );
- frequency of price changes in tax-changing months ( $\bar{I}_T$ );

<sup>19</sup> We do not use the whole CPI-basket for several reasons. First, regulated prices are excluded. Moreover, in some cases (e.g. used cars, computers) the Hungarian Central Statistical Office simply does not collect price data from different stores. We also exclude representative items for which the maximum length of price spells were limited for some reason (seasonal data collection – like gloves, cherries etc – or data collection began late – LCD TV-s, memory cards etc.). Finally, fuels, alcoholic beverages and tobacco were also dropped from the data set as frequent indirect tax changes make it very difficult to estimate the effect of VAT changes for these items. For more details on data collection and item exclusion, see Gabriel-Reiff (2010).

- average absolute size of non-zero price changes in non tax-changing months ( $\overline{\Delta P}_{NT}$ ).

We use the sectoral monthly inflation rate to calibrate the growth rate of aggregate technology ( $g_Z$ ). The other three moments identify the size of menu costs ( $\phi_T, \phi_{NT}$ ) and the variance of the idiosyncratic technology shocks ( $\sigma_A$ ).

After the model parameters are calibrated, the model's performance can be evaluated by observing how close it gets to some non-matched moments, from which the most important one is the theoretical inflation asymmetry. These *non-matched moments* are

- the inflation effect of a unit tax increase and decrease ( $\gamma^+$  and  $\gamma^-$ ), and their asymmetry.
- the average absolute size of non-zero price changes in tax-changing months ( $\overline{\Delta P}_T$ ).

The moments are calculated as weighted averages of moments calculated for each representative items, as we are interested in the behavior of an average “representative item” for the aggregate and each sectors. The exact calculation of the data moments is described in the appendix, here we only show, as an example, how the immediate inflation effects of VAT-changes are calculated. To be consistent with the model simulations, we calculate these effects from time-series data.<sup>20</sup> We estimate the following time-series regression for each representative item ( $j$ ):

$$\pi_t^j = \beta_0 + \sum_{k=1}^{11} \beta_k^j (MONTH = k)_t + \beta_{12}^j VAT04J_t + \beta_{13}^j VAT06J_t + \beta_{14}^j VAT06S_t + \varepsilon_t, \quad (18)$$

where the explanatory variables are month dummies, and other dummies corresponding to the three<sup>21</sup> 2004:01, 2006:01 and 2006:09 value-added tax changes. So the inflation effect of the 2006 January VAT-decrease and the 2006 September VAT-increase for item  $j$  are estimated by  $(\hat{\beta}_{13}^j, \hat{\beta}_{14}^j)$  respectively, and the overall inflation effects are given by

$$\gamma^+ = \sum_j w_j \hat{\beta}_{14}^j, \quad \gamma^- = \sum_j w_j \hat{\beta}_{13}^j, \quad (19)$$

with  $w_j$  being the relative consumer expenditure weights of the items.

Table 5 collects the calculated moments for the aggregate economy and the two main sectors we are about to match.

**Table 5**

**Estimated data moments**

Sector	Matched moments				Non-matched moments		
	$\pi$	$\bar{I}_{NT}$	$\bar{I}_T$	$\overline{P}_{NT}$	$\overline{P}_T$	$\gamma^+$	$\gamma^-$
<b>Aggregate</b>	<b>3.4%</b>	<b>12%</b>	<b>42.7%</b>	<b>11.6%</b>	<b>8.0%</b>	<b>1.11</b>	<b>0.41</b>
Food	4.2%	13.5%	54.5%	9.9%	0.089	1.04	0.85
Services	8.3%	6.6%	28.6%	14.0%	0.090	1.15	0.33

The two sectors we focus on are quite different: the services sector faces higher inflation rate and higher price rigidity (lower frequency of price changes) than the processed food sector.

<sup>20</sup> The VAT-effects calculated by Gabriel and Reiff (2010) using panel estimations and the time-series method used here provided very similar results.

<sup>21</sup> There was also a VAT change in January 2004, that was a 3% increase changing the tax rate to 15%. We filter out the effects of this shock, but do not analyze it in the current paper.

## 5 Results

We calibrate model parameters to hit some standard moments of the data, and our main interest is whether the calibrated model is able to explain the (asymmetric) response of the inflation rate to the tax changes. In this section, we present model calibrations first with the standard non-sectoral homogenous menu cost model, and then we introduce heterogeneous menu costs and sectoral calibrations in steps. The two major sectors for which we do the calibrations are the processed food and the services sectors. These two sectors are the largest in our sample – with 16.1% and 8.8% overall CPI-weights –, and both contain a mixture of tax increase- and tax decrease-affected products. The services sector has higher trend inflation and stickier prices, therefore our model predicts higher inflation asymmetry. We show that the sectoral calibration of the multi-product version of our model is able to quantitatively reproduce the observed asymmetry.

### PARAMETRIZATION

We fix some parameters exogenously. We set  $\beta = 0.96^{1/12}$  (implying 4% yearly real rate), and the mean aggregate nominal growth rate to  $g_{PY} = 0.0934 \cdot (1/12)$ , which is the average monthly nominal consumption growth in Hungary over the period 2002:01-2006:12. We set the value of  $\theta$  (which determines the level of competition within a sector) to 5, which is a usual number used in the industrial organization literature.<sup>22</sup> Further, we set the persistence of the idiosyncratic technology shock  $\rho_A$  equal to 0.7, a value calibrated by Klenow and Willis (2006). We found that the choice of  $\rho_A$  does not influence our results on asymmetry. Finally, the inverse of the Frisch-elasticity ( $\psi$ ) of the labor supply is set to zero, implying a perfectly elastic labor supply.

The other four parameters of the model ( $g_Z, \phi_{NT}, \phi_T, \sigma_A$ ) are calibrated to match the data moments presented in the previous section. In particular, the sectoral technology growth  $g_Z$  is calibrated to make the simulated inflation rate equal to the mean sectoral inflation  $\pi$ , using the steady-state condition  $\pi = g_{PY} - g_Z$ . The other three parameters do not have a clear one-to-one relationship with the targeted moments, but we have a good idea of how they influence the moments we would like to hit. The menu cost parameters  $\phi_{NT}$  and  $\phi_T$  decrease the frequency and increase the average absolute size of the price changes. The standard deviation of the idiosyncratic productivity shocks  $\sigma_A$  increases both the frequency and the average absolute size of price changes.

To sum up, we have four calibrated parameters ( $g_Z, \phi_{NT}, \phi_T, \sigma_A$ ) and we use them to hit four major moments of the data ( $\pi, \bar{I}_{NT}, \bar{I}_T, \overline{\Delta P}_{NT}$ ). In addition we have unmatched moments, for which we have no independent model parameters: these are the inflation effects of a unit positive and negative tax shock ( $\gamma^+$  and  $\gamma^-$ ) and the average absolute size of price changes in tax-changing months ( $\overline{\Delta P}_T$ ). These unmatched moments are used to evaluate the model's performance.

### CALIBRATION RESULTS

Table 6 contains the calibrated parameters for the different calibrations (for the aggregate sample: homogeneous and heterogeneous menu costs; for the sectoral samples: processed food and services). Table 7 presents the values of the moments which were directly used for the calibration ('matched moments') and the resulting values of the 'unmatched' moments including the asymmetry estimates.

---

<sup>22</sup> There is no agreement about the value of  $\theta$  in the menu cost literature, the values range from 3 (Midrigan, 2009) to 11 (Gertler-Leahy, 2008). The choice of  $\theta$  though influences our estimates of menu costs and the standard deviation of idiosyncratic shocks, but within this range it practically does not influence our estimates on the estimated inflation asymmetry.

Table 6

## Model calibrations

Fixed parameters		Non-sectoral		Sectoral	
		Hom.	Het.	Food	Services
$\beta$	Discount factor	0.96 <sup>1/12</sup>			
$\theta$	Elasticity of substitution	5			
$\rho_A$	Persistence of idiosyncratic shocks	0.7			
$g_{PY}$	Nominal growth	9.34%/12			
$\psi$	Inverse elasticity of labor supply	0			
Calibrated parameters		Non-sectoral		Sectoral	
		Hom.	Het.	Food	Services
$g_Z$	Sectoral technology growth	0.45%	0.45%	0.35%	0.08%
$\sigma_A$	Std. dev. of idiosyncratic shocks	5.9%	5.9%	5.1%	6.4%
$\phi_{NT}$	Menu costs during 'normal' times	3.2%	3.2%	2.2%	7.65%
$\phi_T$	Menu costs during tax changes	–	1.15%	0.56%	3.2%

Table 7

## Moments

Matched Moments			Non-sectoral		Sectoral	
			Hom.	Het.	Food	Services
$\pi$	Monthly inflation rate	<i>Data</i>	0.28%	0.28%	0.35%	0.69%
		Estimate	0.27%	0.27%	0.35%	0.65%
$\bar{I}_{NT}$	Frequency during 'normal' times	<i>Data</i>	12.0%	12.0%	13.5%	6.6%
		Estimate	12.0%	12.0%	13.4%	6.6%
$\bar{I}_T$	Frequency during tax changes	<i>Data</i>	42.7%	42.7%	54.5%	28.6%
		Estimate	16.4%	42.2%	54.5%	28.6%
$\Delta P_{NT}$	Size during 'normal' times	<i>Data</i>	11.6%	11.6%	9.9%	14.0%
		Estimate	11.6%	11.6%	10.0%	14.1%
Unmatched Moments			Non-sectoral		Sectoral	
			Hom.	Het.	Food	Services
$\gamma^+$	Inflation effect of a unit tax increase	<i>Data</i>	1.11	1.11	1.04	1.15
		Estimate	0.62	0.97	1.00	1.54
$\gamma^-$	Inflation effect of a unit tax decrease	<i>Data</i>	0.41	0.41	0.85	0.33
		Estimate	0.61	0.93	0.87	0.25
$\Delta P_T$	Size during tax changes	<i>Data</i>	8.0%	8.0%	8.9%	9.0%
		Estimate	12.2%	9.0%	8.5%	11.0%

## Standard model

The calibration results of the standard non-sectoral model with homogeneous menu costs are in the first columns of Tables 6 and 7. Results show that the model is able to hit the moments in 'normal' times with

reasonable parameter values, but the model predicts essentially no asymmetry, and is unsuccessful in hitting the frequency and the size of price changes at the months of the tax changes.

The parameter estimates are standard, similarly to previous menu cost models with idiosyncratic shocks, the model needs volatile idiosyncratic shocks ( $\sigma_A = 5.9\%$ ) to be able to hit the large average absolute size and frequency of the price changes. The menu cost is estimated to be 3.2% when paid, but note that it is only paid in case of price change which – under no tax change – happens with 12% probability. It means that the yearly menu cost proportional to the firms' revenue is estimated to be 0.38%, which is within the range of previous studies' empirical estimates (Levy et al, 1997 estimates menu costs to be 0.70% of yearly revenues, Klenow-Willis, 2006 estimates a yearly cost of 1.4%, while Nakamura-Steinsson, 2008a finds this measure to be 0.2%).

The standard model, however, is unable to explain the inflation asymmetry observed in the data. While the inflation effect of a unit VAT-increase and decrease are estimated to be 1.11 and 0.41 respectively, the model only predicts 0.62 and 0.61. Furthermore, the frequency of price changes at the months of VAT-changes (42.7%) is substantially underestimated (16.4%), and the average absolute size of price changes (8.0%) is overestimated (12.2%).

### Introducing heterogeneous menu costs

As a shortcut to assuming multiproduct firms with decreasing marginal menu costs as in Midrigan (2009), we assume that the average menu costs are smaller at the months of the VAT-changes. This assumption allows us to match the increased frequency of the price changes observed at the months of tax changes and get closer to the observed decrease in their average absolute size.

The results of the heterogeneous menu cost estimation is shown in the second column of tables 6 and 7. The results show that the best calibration is indeed one with lower menu costs during the VAT-shock: the 3.2% menu cost during normal times is estimated to drop approximately to its third: 1.15%. With this calibration, the model is still able to hit the moments during 'normal' times, and it is also able to reproduce the higher frequency and the lower size of the price change at the month of the tax changes (though it slightly overestimates the average absolute size in tax-changing month). The results also show, however, that though the model predicts somewhat higher asymmetry (0.97 and 0.93 positive and negative inflation effects, respectively) than in the homogeneous menu cost case, it still significantly underestimates the asymmetry observed in the data.

### Sectoral calibrations

In this subsection we calibrate the model to sectoral moments, and show that this way we are able to reproduce the asymmetry observed in the data.

As column 3-4 of Tables 6 and 7 show, sectoral estimations are successful in hitting the major moments. The major difference between the two sectors is that the services sector faces both higher monthly trend inflation (0.69% compared to 0.35%) and higher estimated menu costs (7.65% compared to 2.2%<sup>23</sup>) than the processed food sector. These differences can fully explain both the substantially lower fraction of price changing firms in the services sector (6.6% compared to 13.5%) and the substantially higher average absolute size of price changes in the services sector (14.0% compared to 9.9%) than in the processed food sector. The calibrated idiosyncratic technology processes are somewhat different: the standard deviation of the process is estimated to be relatively high (6.4%) in the services sector, while in the processed food sector this standard deviation is lower (5.1%).

The menu costs at the month of the VAT-shocks  $\phi_T$  are calibrated to reduce around third of its normal value  $\phi_{NT}$ : it is estimated to reduce to 0.56% (from 2.2%) for the processed food sector and to 3.2% (from 7.65%)

<sup>23</sup> Implying reasonable expected menu costs, 0.50% and 0.30% of revenues.

for the services sector. The average size the month of tax changes is somewhat underestimated in case of the processed food sector and overestimated in case of the services sector.

The sectoral calibrations, in line with the stylized facts, predict much lower asymmetry in the processed food sector than in the services sector. The model is also successful in quantitatively predicting the asymmetric inflation effects of the tax changes. In the processed food sector the inflation effects of unit tax increases are estimated to be 1.04 and 0.85 respectively, which are hit quite well by the model with similar numbers of 1.00 and 0.87. In the services sector, the coefficients of the unit tax changes are 1.15 and 0.33, respectively, and the same moments in the calibrated model are 1.54 and 0.25.

## 6 Discussion

The fact that the standard non-sectoral model seriously underestimated the observed asymmetry, while the sectoral estimates were able to get quantitatively close to the sectoral asymmetries suggests an aggregation bias of the aggregate calibration. We argue that this is a joint result of a strong theoretical interaction effect of inflation and price stickiness on the asymmetry and an empirical correlation between higher inflation and price stickiness in the sectoral level. Figure 5 presents the smoothed<sup>24</sup> asymmetry (as the difference between the unit effects of tax increase and decrease over their average) predicted by the model for different values of trend inflation and menu costs. It shows the asymmetry for three different levels of increasing returns to scale in the firms' repricing technology modeled as the proportion between the menu costs in the non-tax changing ( $\phi_{NT}$ ) and the tax changing months ( $\phi_T$ ). The flattest plane is the results with non-increasing returns to scale ( $\phi_{NT} = \phi_T$ ). Note, that the asymmetry is increasing in the cross-effect of inflation and menu costs, though the effect is quantitatively small. The asymmetry is increasing more with cross effects if the returns to scale is higher:  $\phi_T = 0.5\phi_{NT}$ , and  $\phi_T = 0.33\phi_{NT}$  respectively, and the relationship is relatively stable for the returns to scale values we find in the calibration exercise.

It is the cross-effect of inflation and menu costs that explains most of the variation in the observed asymmetry. Running a simple regression on a sample of 99 points simulated<sup>25</sup> for figure 5 with a constant and a cross-term of the form

$$u = \frac{\gamma^+ - \gamma^-}{0.5(\gamma^+ + \gamma^-)} = f(\pi, \phi) \approx \delta_0 + \delta_1 \pi \phi, \quad (20)$$

where  $u$  stands for the asymmetry, we find for the baseline case with  $\phi_{NT}/\phi_T = 0.33$  that the estimate of the cross term is positive, large ( $\hat{\delta}_1 = 0.32$ ) and highly significant (with a standard error of 0.012), and the equation explains 94% of the variance in the simulated asymmetry. The results are almost identical for the  $\phi_{NT}/\phi_T = 0.5$  case, but substantially smaller with the case with homogeneous menu costs: the cross effect is still highly significant, but its value is lower ( $\delta_1 = 0.078$ ) with standard error of 0.004 and  $R^2 = 0.81$ . This simulation, thus, provides numerical evidence on the amplification effect of lowering the menu cost at the month of the tax change and, thereby, increasing the frequency of price change in line with the evidence observed in the data. Unless sectoral inflation rates and the menu costs are uncorrelated, this strong cross-effect implies a potentially substantial bias in an aggregate model with average inflation rate and menu costs.

Assuming a relationship with cross term as in equation 20 and sectoral heterogeneity of inflation  $\pi$  and menu costs  $\phi$ , the expected aggregate asymmetry is given by:

$$E[u] = \delta_0 + \delta_1 E[\pi \phi] = \underbrace{\delta_0 + \delta_1 E[\pi] E[\phi]}_{\text{aggregate}} + \underbrace{\delta_1 \text{cov}[\pi, \phi]}_{\text{bias}}, \quad (21)$$

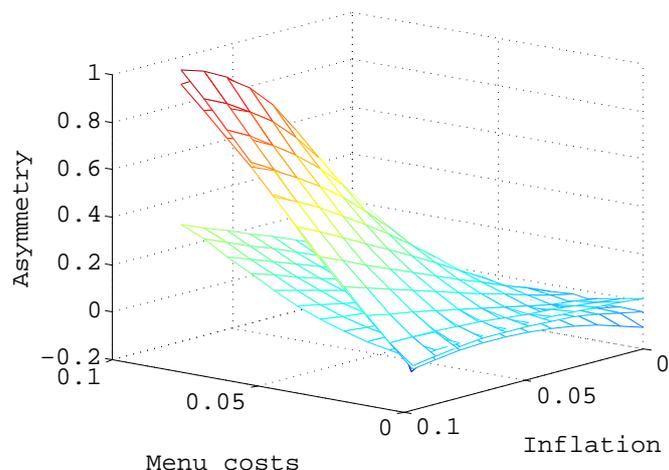
where the second equation comes from the definition of the sectoral covariance of inflation and menu costs. The first sum on the right hand side is the simulated asymmetry from the aggregate calibration using the average inflation and average price stickiness. The second product is the sectoral aggregation bias. The bias is higher the higher the parameter of the cross-term is, and the higher the covariance is between sectoral inflation rate and price stickiness.

The positive relationship between the inflation rate and the price stickiness is the result of the relative importance of sectors with both high inflation and high price stickiness. Services give a major part of the story: they have 34.2% weight in the Hungarian consumption basket. The frequency of services price changes is only 6.6% that is much lower than the average of 12% of our sample between 2002 and 2006. The services trend-inflation during the same period, on the other hand, is very high: it is 8.3% that is much higher than the average inflation rate of 3.4%.

<sup>24</sup> We used a flexible Chebyshev projection methods to obtain an estimate of the function disregarding simulation errors.

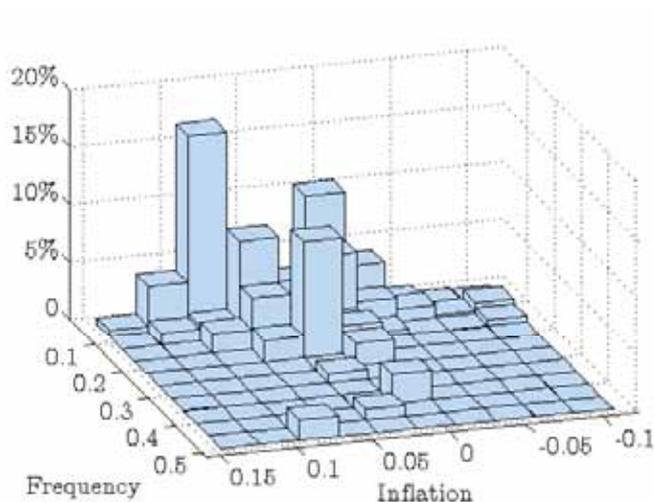
<sup>25</sup> With the simulation errors

**Figure 5**  
**Simulated asymmetry as a function of inflation and menu costs**



Looking at a more disaggregated level, Figure 11 plots a histogram of the distribution of 123 sectors over their trend inflation between 2002 and 2006 and their average frequency of price change during ‘normal’ times, each sector weighted by their relative consumption expenditure. Considering the frequency of price change as a proxy for price stickiness, we see that the distribution is not symmetric: there are more mass at the high inflation-low frequency quadrant. As frequency is increasing with the inflation rate, the figure actually underestimates the asymmetry of the inflation-menu cost distribution.

**Figure 6**  
**Weighted histogram of sectoral trend-inflation and frequency**

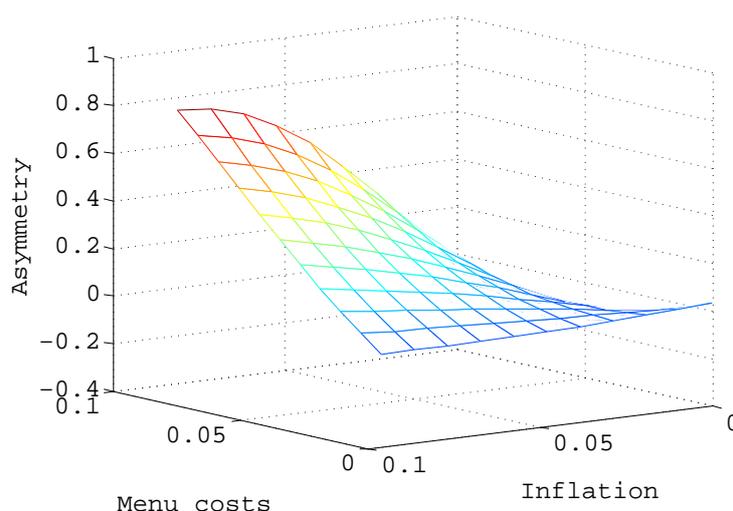


In the Appendix, we show by looking at US data that these relationships in the data are not particular to Hungary. Services, in the US, also have stickier than average prices and higher than average inflation rates. Furthermore, sectoral frequencies and inflations on a sample similar to ours show a positive relationship suggesting a stronger positive relationships between inflation and the (unobserved) menu costs that directly influences the aggregation bias.

## MONETARY SHOCK

A standard monetary shock in the model is a one-time permanent change in the nominal GDP ( $PY$ ), or, equivalently, a one time shock to the growth rate of nominal GDP ( $g_{PY}$ ). A demand shock like this has similar asymmetric effects on the inflation rate than the supply-side VAT shock. Figure 7 shows the simulated inflation asymmetry to a symmetric large (5%) positive and negative monetary shocks.<sup>26</sup>

**Figure 7**  
Simulated asymmetry of a monetary shock as a function of inflation and menu costs



The strong influence of the inflation-menu cost cross effect on the inflation asymmetry is very similar to the VAT-shock of a similar size. The regression equation 20 in this case has a cross-effect coefficient of  $\delta_1 = 0.226$ , that is somewhat lower than in the VAT case, but still high and highly significant.

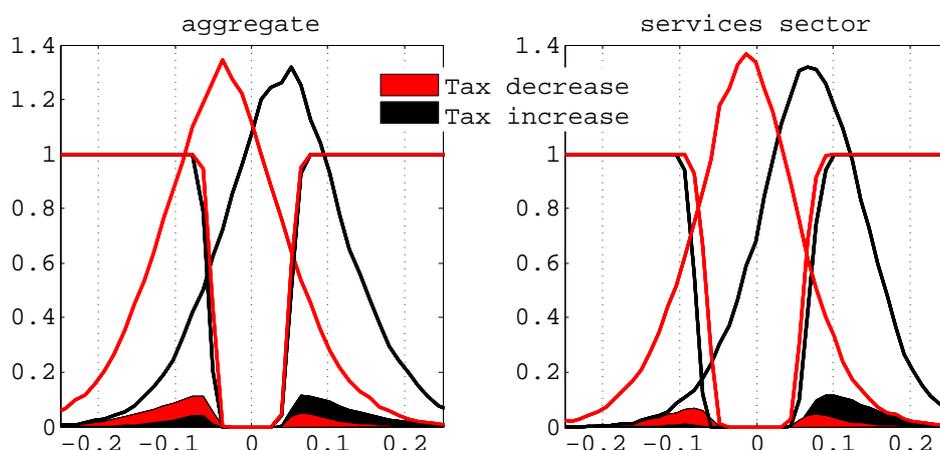
## ASYMMETRIC SELECTION

A major factor driving the effects of inflation and menu costs on the simulated inflation asymmetry is the asymmetry in the endogenous selection of firms that change their prices. It is instructive to divide the price change distribution  $f(x)$  to two parts: the distribution of desired price changes  $g(x)$  and the adjustment hazard of the firms  $h(x)$ , where  $x$  is the desired price change of a firm. The price change distribution comes from the multiplication of these two functions:  $f(x) = g(x)h(x)$ . Figure 8 show these functions for tax increases and tax decreases for the aggregate calibration (with moderate inflation rate and menu costs) and for the services sector calibration (with higher inflation rate and menu costs). Let's first look at the left panel with the aggregate calibration. The hump shaped functions show the desired price change distribution, that have, intuitively, higher median values for the tax increases than for the tax decreases. The tubby shaped functions show the adjustment hazards of the firms, and these are very close to each other. The solid areas show the distribution of price changing firms: it shows that - intuitively - the proportion of price increasing firms are higher for the tax increase and lower for the tax decrease. The symmetry of the distribution, i.e. that the position and distribution of price increases for the tax-increase are similar to the position and distribution of the price decreases for the tax-decrease (and similarly for the price decreases for the tax increase and price increases for the tax decrease) is the reason we observed no significant simulated inflation asymmetry for the aggregate case.

<sup>26</sup> The size of the monetary shocks are of similar size as the VAT shocks, because that is when we can expect similar frequency responses that are resulted by the lower menu costs at the months of the shock.

**Figure 8**

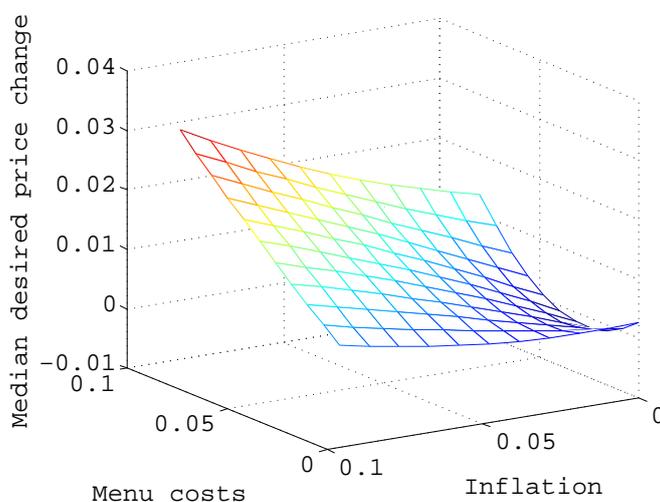
**Desired price change distributions and adjustment hazards for tax increases and decreases for calibrations with different menu costs and inflation rates**



The right panel shows the same functions for the services sector calibration with higher trend inflation and menu costs. The significant asymmetry we found is the result of the asymmetric distribution of price increases and decreases for the tax increase and tax decrease we can see on the figure. There are two important effects causing this. First, the higher inflation and menu costs pushes the distribution of desired price changes to the right for both the tax increase and the tax decrease case. The reason of this is fairly straightforward: firms are going to set their optimal nominal price above their static optimum, because in this way they can keep their relative price the closest to its optimum for the duration of their price spell. Higher menu costs increases the expected duration of a price spell, while higher inflation increases the pace of depreciation of the firm's relative price. Figure 9 shows the development of the median of the desired price change as a function of inflation and menu costs for a steady state distribution: the strong interaction term we saw for the simulated inflation asymmetry is very apparent here. The second effect is that higher menu costs and, to a lesser extent, higher inflation also increases the width of the inaction regions of the firms, but has no numerically significant effect on the central position of them.

**Figure 9**

**Median desired price change as a function of inflation and menu cost**



The asymmetry of the inflation effect comes from the interaction of the desired price distribution and the adjustment hazard: a desired price change distribution with higher median changes both the relative frequency

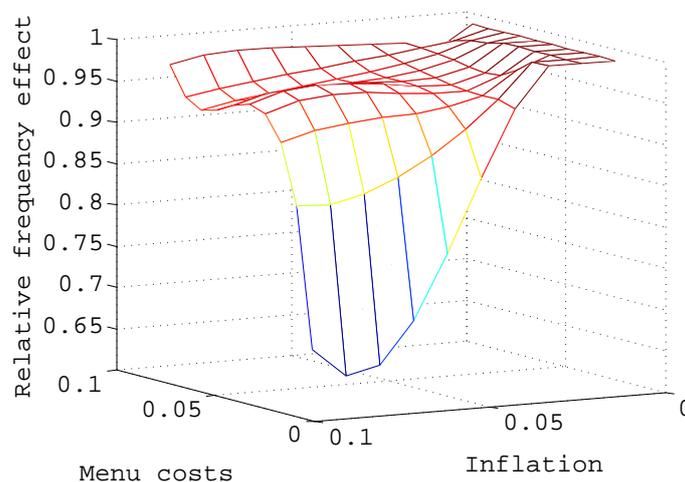
of price increases and decreases and their average size. The asymmetry can be decomposed into frequency and size effects in a straightforward way:

$$\gamma^+ - \gamma^- = \frac{d\pi^+ - d\pi^-}{d\tau} = \underbrace{\frac{\overline{\Delta P} (dI^+ - dI^-)}{d\tau}}_{\text{freq.effect}} + \underbrace{\frac{I (d\overline{\Delta P}^+ - d\overline{\Delta P}^-)}{d\tau}}_{\text{sizeeffect}}, \quad (22)$$

where, as before,  $I$  is the frequency,  $\overline{\Delta P}$  is the average size of price changes. The  $+$  and  $-$  superscripts refer to the positive and negative shock, respectively. Figure 10 shows the percentage proportion of the frequency effects for inflation and menu cost values with significant asymmetry (0 menu cost and 0 inflation cases are dropped). It shows that for most values of the parameter space the frequency effect explains over 90% of the asymmetry. It is lower in areas where, because of low menu costs and high inflation, the steady state frequency of price changes are already high and the asymmetry relatively low.

**Figure 10**

Relative contribution of the frequency effect on the asymmetry as a function of inflation and menu cost



## TREND-INFLATION IS NECESSARY

For our parametrization, positive trend-inflation is a necessary condition of significant inflation asymmetry. In a similar framework as ours, Devereux and Siu (2007) provided a different argument for asymmetry by showing that individual firms' strategic incentives are asymmetric: while prices are strategic complements in case of positive aggregate shocks to the (nominal) marginal cost, they are strategic substitutes in case of negative shocks. The Devereux and Siu (2007) channel would imply asymmetry even without positive trend-inflation. However, our simulations with 0 trend-inflation rate show no significant asymmetry for our parametrization.

Devereux and Siu (2007) found that this strategic asymmetry is larger with more intense competition (higher elasticity of substitution parameter  $\theta$ ). But our negative asymmetry results were true even if we increased the level of competition substantially ( $\theta = 11$ ), implying that the 5%-points tax changes were not large enough in our model to make the strategic asymmetry numerically significant.

## 7 Conclusion

The Hungarian experiment showed that asymmetric response to (large) symmetric aggregate shocks can be an empirically relevant issue. It provides a policy relevant macroeconomic reason supporting state dependent pricing models, that - contrary to standard flexible price and time-dependent pricing models - can explain the aggregate asymmetry. The paper found that the standard quantitative menu cost models similar to Golosov and Lucas (2007), however, seriously underestimate the observed asymmetry. It showed that introducing multi-product firms and calibrating the model on a sectoral level, the (sectoral) trend inflation can successfully account for the observed sectoral asymmetry.

The reason of the aggregation bias is the strong influence of the sectoral inflation-price stickiness cross-effect on the asymmetry in the model and the positive correlation between inflation and price stickiness in the data. The paper argues that real effects of even large negative monetary shocks can be substantial even in the standard Golosov and Lucas (2007) framework suggesting more serious real costs of disinflations relative to real gains of inflationary surprises. As long as these effects are present in other countries - which seems to be the case in the US - asymmetric aggregate response to symmetric shocks should be a serious consideration, and should be taken into account for optimal policy design.

# References

- BALL, L. AND G. N. MANKIW (1994): “Asymmetric Price Adjustment and Economic Fluctuations”, *Economic Journal*, March 1994, vol. 104, issue 423, pp. 247-261.
- CABRAL, L. AND A. FISHMAN (2008): *Business as Usual: A Consumer Search Theory of Sticky Prices and Asymmetric Price Adjustment*, manuscript.
- CALVO, G. A. (1983): “Staggered Prices in a Utility-Maximizing Framework”, *Journal of Monetary Economics*, September 1983, vol. 12, issue 3, pp. 383-398.
- CARVALHO, C. (2006): “Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks”, *Frontiers of Macroeconomics*, 2006, vol. 2, issue 1, article 1.
- COVER, J. (1992): “Asymmetric Effects of Positive and Negative Money Supply Shocks”, *Quarterly Journal of Economics*, November 1992, vol. 107, no. 4, pp. 1261-1282.
- DEVEREUX, M. B. AND H. E. SIU (2007): “State Dependent Pricing and Business Cycle Asymmetries”, *International Economic Review*, February 2007, vol. 48, issue 1, pp. 281-310.
- ELLINGSEN, TORE, RICHARD FRIBERG AND JOHN HASSLER (2006): “Menu Costs and Asymmetric Price Adjustment”, *CEPR Discussion Paper 5749*, July 2006.
- GÁBRIEL, P. AND Á. REIFF (2010): “Price Setting in Hungary – a Store-Level Analysis”, *Managerial and Decision Economics*, March-April 2010, vol. 31, issue 2-3, pp. 161-176.
- GAGNON, ETIENNE (2009): “Price Setting During Low and High Inflation: Evidence from Mexico”, *The Quarterly Journal of Economics*, August 2009, vol. 124, no. 3, pp. 1221-63.
- GERTLER, M. AND J. LEAHY (2008): “A Phillips Curve with an Ss Foundation”, *Journal of Political Economy*, June 2008, vol. 116, no. 3, pp. 533-572.
- GOLOSOV, M. AND R. E. LUCAS JR. (2007): “Menu Costs and Phillips Curves”, *Journal of Political Economy*, 2007, vol. 115, no. 2, pp. 171-199.
- HOBIIJN, B., F. RAVENNA AND A. TAMBALOTTI (2006): “Menu Costs at Work: Restaurant Prices and the Introduction of the Euro”, *The Quarterly Journal of Economics*, August 2006, vol. 121, issue 3, pp. 1103-1131.
- KEHOE, P. J. AND V. MIDRIGAN (2008): “Temporary Price Changes and the Real Effects of Monetary Policy”, *Federal Reserve Bank of Minneapolis Research Department Staff Report*, No. 413.
- KIMBALL, M. (1995): “The Quantitative Analytics of a Basic Neomonetarist Model”, *Journal of Money, Credit and Banking*, November 1995, vol. 27, issue 4, pp. 1241-1277.
- KLENOW, P. J. AND J. L. WILLIS (2006): “Real Rigidities and Nominal Price Changes”, *The Federal Reserve Bank of Kansas City Working Paper*, no. 06-03.
- KLENOW, P. J. AND O. KRYVSTOV (2008): “State-Dependent or Time-Dependent Pricing – Does it Matter for Recent US Inflation”, *Quarterly Journal of Economics*, August 2008, vol. 123, issue 3, pp. 863-904.
- KRUSELL, P. AND A. A. SMITH JR. (1998): “Income and Wealth Heterogeneity in Macroeconomics”, *Journal of Political Economy*, October 1998, vol. 105, issue 5, pp. 867-896.

LEVY, DANIEL, MARK BERGEN, SHANTANU DUTTA AND ROBERT VENABLE (1997): “The Magnitude of Menu Costs: Evidence from Large U.S. Supermarket Chains”, *The Quarterly Journal of Economics*, August 1997, vol. 112, no. 3, pp. 791-825.

MIDRIGAN, V. (2009): *Menu Costs, Multi-Product Firms and Aggregate Fluctuations*, manuscript.

NAKAMURA, E. AND J. STEINSSON (2008a): “Monetary Non-Neutrality in a Multi-Sector Menu Cost Model”, *NBER Working Paper* No. 14001 May 2008.

NAKAMURA, E. AND J. STEINSSON (2008b): “Five Facts About Prices: A Reevaluation of Menu Cost Models”, *Quarterly Journal of Economics*, November 2008, vol. 123, no. 4, pp. 1415-64.

PELTZMAN, S. (2000): “Prices Rise Faster than They Fall”, *Journal of Political Economy*, June 2000, vol. 108, no. 3, pp. 466-502.

RAVN, M. O. AND M. SOLA (2004): “Asymmetric Effects of Monetary Policy in the United States”, *Federal Reserve Bank of St. Louis Review*, pp. 41-60.

TAUCHEN, G. (1986): “Finite State Markov Chain Approximations to Univariate and Vector Autoregressions”, *Economic Letters*, 1986, vol. 20, issue 2, pp. 177-181.

YANG, H. AND YE, L. (2008): “Search with Learning: Understanding Asymmetric Price Adjustments”, *RAND Journal of Economics*, 2008, vol. 39, no. 2, pp. 547-564.

ZBARACKI, M., M. RITSON, D. LEVY, S. DUTTA AND M. BERGEN (2004): “Managerial and Customer Dimensions of the Cost of Price Adjustment: Direct Evidence from Industrial Markets”, *Review of Economics and Statistics*, May 2004, vol. 86, no. 2, pp. 514-533.

# Appendix

## CALCULATION OF THE FLEXIBLE PRICE EQUILIBRIUM

When describing our numerical solution technique, we noted that we start the iterative procedures from the fully flexible steady-states. In this subsection we show the analytical solution of the model under perfectly flexible prices.<sup>27</sup> We show how we can derive the inflation pass-through of tax shocks in this case.

Solving the firms' profit maximization problem, it is easy to derive that the optimal relative price is

$$p_i^*(t) = \frac{\theta \zeta_t}{\theta - 1} \frac{1 + \tau_t(i)}{A_t(i)}, \quad (23)$$

with  $\zeta_t = w_t \frac{C_t}{Z_t}$ , and  $w_t = \frac{\tilde{w}_t}{P_t C_t}$  being the normalized nominal wage.

Then the optimal relative consumptions are  $\frac{C_t^*}{C_t/N} = p_t^*(i)^{-\theta}$ , which implies

$$C_t^*(i) = \left( \frac{C_t}{N} \right)^{1-\theta} Z_t^\theta \left( \frac{\theta N w_t}{(\theta - 1)} \right)^{-\theta} \left( \frac{1 + \tau_t(i)}{A_t(i)} \right)^{-\theta}. \quad (24)$$

Aggregating these with the CES-aggregator  $C_t = \left[ \int_{\mathcal{N}} N^{-\frac{1}{\theta}} C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ , we can derive that

$$\frac{(\theta - 1)\eta}{\theta \zeta_t} = \left[ \int_{\mathcal{N}} N^{-1} \left( \frac{1 + \tau_t(i)}{A_t(i)} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \equiv 1 + \bar{\tau}_t, \quad (25)$$

where the summation is a CES-aggregate of individual "effective" tax rates  $\frac{1 + \tau_t(i)}{A_t(i)}$ , denoted as an average tax rate  $1 + \bar{\tau}(t)$ .

With this average tax rate we can write the optimal individual relative prices as

$$p_t^*(i) = \frac{(1 + \tau_t(i)) / A_t(i)}{1 + \bar{\tau}_t}, \quad (26)$$

and relative outputs as

$$\frac{C_t^*(i)}{C_t^*/N} = \left[ \frac{(1 + \tau_t(i)) / A_t(i)}{1 + \bar{\tau}_t} \right]^{-\theta} \quad (27)$$

which says that the optimal relative prices and relative outputs are determined by the relative effective tax rates (i.e. the ratio of the individual effective tax rates  $\frac{1 + \tau_t(i)}{A_t(i)}$  and the average tax rate  $1 + \bar{\tau}_t$ ).

The wage rate will be determined on the labor market by making labor demand and supply equal. Labor supply can be derived from the consumers' maximization problem. Rewriting equation (6) leads to

$$L_t = \left( \frac{\tilde{w}_t}{\mu P_t C_t} \right)^{\frac{1}{\psi}} = \left( \frac{w_t}{\mu} \right)^{\frac{1}{\psi}}, \quad (28)$$

<sup>27</sup> Sectoral subscripts  $s$  are suppressed for notational convenience.

while the labor demand equation (11) can be written as:

$$L_t(i) = \frac{C_t^*(i)}{Z_t A_t(i)} = \frac{(\theta - 1)}{\theta n \omega_t [1 + \bar{\tau}_t] A_t(i)} \left[ \frac{(1 + \tau_t(i)) / A_t(i)}{1 + \bar{\tau}_t} \right]^{-\theta}. \quad (29)$$

Aggregate labor demand is the sum of individual demands. A little algebra shows that the equilibrium wage rate is

$$\omega_t = \mu^{\frac{1}{1+\psi}} \left( \frac{(\theta - 1)}{\theta} \right)^{\frac{\psi}{1+\psi}} \left[ \int_{\mathcal{N}} \frac{1}{1 + \tau_t(i)} \frac{[(1 + \tau_t(i)) / A_t(i)]^{1-\theta}}{N [1 + \bar{\tau}_t]^{1-\theta}} \right]^{\frac{\psi}{1+\psi}}, \quad (30)$$

where the last term is a weighted average of  $\frac{1}{1+\tau_t(i)}$ -s (the weights sum to 1 by the definition of  $1 + \bar{\tau}_t$  in equation (25)), and can therefore be written as another average of individual tax rates:  $\frac{1}{1+\tilde{\tau}_t}$ . Therefore the equilibrium wage rate is simply

$$\omega_t = \mu^{\frac{1}{1+\psi}} \left( \frac{(\theta - 1)}{\theta (1 + \tilde{\tau}_t)} \right)^{\frac{\psi}{1+\psi}}, \quad (31)$$

a function of deep parameters and individual tax rates.

With this equilibrium wage we can derive the level of individual outputs and prices. Rearranging the aggregation equation (25), we can write  $\omega(t) \frac{C_t}{Z_t} = \zeta_t = \frac{(\theta - 1)}{\theta [1 + \bar{\tau}_t]}$ , which implies that the real GDP path is

$$C_t^* = N Z_t \frac{(\theta - 1)}{\theta N \omega_t [1 + \bar{\tau}_t]} \approx Z_t \left( \frac{(\theta - 1)}{\theta \mu [1 + \bar{\tau}_t]} \right)^{\frac{1}{1+\psi}}, \quad (32)$$

(where we used the approximation  $1 + \bar{\tau}_t \approx 1 + \tilde{\tau}_t$ ).

The aggregate price level is the ratio of the nominal GDP (which is exogenous) and the real GDP:

$$P_t^* = GDP_t / C_t^* \approx \frac{1}{Z_t} \left( \frac{\theta \mu [1 + \bar{\tau}_t]}{(\theta - 1)} \right)^{\frac{1}{1+\psi}}. \quad (33)$$

The expected growth rates can be calculated easily. From the wage equation (31), we have

$$E(g_{\omega,t}) = -\frac{\psi}{1+\psi} E(g_{1+\tilde{\tau},t}) \approx -\frac{\psi}{1+\psi} E(g_{1+\bar{\tau},t}). \quad (34)$$

From the real GDP-equation (32), it follows that

$$E(g_{Y_t}) = E(g_{C_t}) \approx E(g_{Z_t}) - \frac{1}{1+\psi} E(g_{1+\bar{\tau},t}). \quad (35)$$

Finally, from the price level equation (33), inflation is the difference between the nominal GDP-growth ( $g_{PY}$ , given exogenously) and real GDP-growth:

$$E(\pi_t) = E(g_{P_t}) = g_{PY} - E(g_{Y_t}) = g_{PY} - E(g_{Z_t}) + \frac{1}{1+\psi} E(g_{1+\bar{\tau},t}). \quad (36)$$

## NUMERICAL ALGORITHMS

### Calculating the steady-state

When calculating the steady-state, we solve for the aggregate cost factor  $\zeta$  and distribution of relative prices  $\Gamma$  simultaneously with the following iterative procedure:

1. We assume that the steady-state value of  $\zeta$  is equal to a specific value  $\zeta_0$ . (In the start of the iterative process we set  $\zeta_0$  to be equal with the flexible-price value of  $\zeta$ .)
2. Given this  $\zeta_0$ , and  $\tau^1, \tau^2, \phi, \pi$ , we solve for the value function of the firms with value function iteration. We do this over a discrete two-dimensional grid in  $(p_{-1}, A)$ .<sup>28</sup>
3. With the resulting policy functions, we simulate an artificial data set. The initial relative price distribution is the one under flexible prices, and we do the simulation for subsequent time periods until we reach the steady-state relative price distribution  $\Gamma$ .
4. Finally we calculate the resulting aggregate cost factor  $\zeta_1$ , and if this is different from the initially assumed value of  $\zeta_0$ , we start over this procedure (with  $\zeta_0 = \zeta_1$ ) until convergence in  $\zeta$ .

We solve for steady-state under the initial tax regime and also under the resulting tax regime after the tax change.

### Transition between steady-states

Our main focus is to study the inflation path between the steady-states corresponding to different tax regimes (before and after the tax change). We do this by iterating in the resulting inflation path until we find a fixed point in the path. The iteration entails the following steps:

1. We assume that the new steady-state will be achieved  $T$  periods after the tax change. Our notational convention is that the tax change occurs at  $t = 1$ , and the new steady state will be reached at  $t = T + 1$
2. We guess the transitional inflation path  $\pi_1, \pi_2, \dots, \pi_T$ . Our initial guess is the inflation path under fully flexible prices.
3. Given the assumed inflation path and the constantly growing nominal GDP and aggregate technology  $Z$ , we obtain the resulting path of the real GDP and aggregate cost factor  $\zeta = w \frac{Y}{Z}$ .
4. We calculate the series of value and policy functions with backward induction: we know that at time  $t = T + 1$  the value function of the firms ( $V_{T+1}$ ) is the value function under the after tax-change steady-state. Given  $\pi_T, \zeta_T$  and  $V_{T+1}$ , we can use equations (14), (15), (16) to solve for the value function  $V_T$ . Then given  $V_T$  and  $\pi_{T-1}, \zeta_{T-1}$ , we can use the same equations to obtain  $V_{T-1}$ , etc. With this method we can calculate the series of value and policy functions for the time periods  $t = 1, \dots, T$ .
5. We use the resulting series of value and policy functions to simulate an artificial data set about the transition period between the steady-states. From this artificial data set we calculate the resulting inflation path.
6. If the resulting inflation path is different from the one we assumed initially, we start over this procedure until convergence in the inflation path.

<sup>28</sup> We have 100 equidistant grids in the log relative price and 13 equidistant grids in the log idiosyncratic productivity shocks. We verified that the grid is fine enough: increasing the number of grids in either dimensions does not change the results.

## CALCULATING DATA MOMENTS

To describe the calculation of the *mean sectoral monthly inflation rate*, let us introduce some notation. We index time by  $t$ , representative items by  $s$ , and stores by  $i$ . Then the mean sectoral (i.e. representative item-level) inflation rate is

$$\pi_{st} = \sum_i \frac{\log P_{sit} - \log P_{si,t-1}}{N_i}, \quad (37)$$

where  $N_i$  is the number of stores observed both at time  $t$  and time  $t - 1$  in sector (representative item)  $s$ . From these we calculate average monthly inflation rates for the representative items by time aggregation:

$$\pi_s = \sum_t \frac{\pi_{st}}{T}, \quad (38)$$

and finally the mean monthly inflation rate for the whole economy (or broader CPI-categories) is obtained by aggregating over representative items:

$$\bar{\pi} = \sum_s w_s \pi_s, \quad (39)$$

where  $w_s$  are CPI-weights (we use the CPI-weights in 2006). Note that seasonal variation in monthly inflation rates does not affect our estimates as we are using price changes between January 2002 - December 2006 to calculate mean representative item-level inflation rates.

The second and third moment that we use for matching is the *frequency of price changes* (both in non tax-changing and tax-changing months). These are again calculated at the representative item-level, and then aggregated across representative items:

$$\bar{I}_s = \sum_t \sum_i \frac{I(\Delta P_{sit} \neq 0)}{N_s}, \quad (40)$$

where  $I(\Delta P_{sit} \neq 0)$  is a dummy for price changes, and  $N_s$  is the total number of observations for representative item  $s$ . The overall average frequency is then

$$\bar{I} = \sum_s w_s \bar{I}_s. \quad (41)$$

Again, seasonal variation in frequencies does not bias our frequency estimates as we use price change data between January 2002 - December 2006 for the frequency calculations.

The *average size of price changes* is calculated first at the representative item level:

$$\Delta P_s = \sum_{I(\Delta P_{sit} \neq 0)} \frac{|\Delta P_{sit}|}{N_{I_s}}, \quad (42)$$

where  $N_{I_s}$  is the total number of price changes for representative item  $s$ :  $\sum_t \sum_i I(\Delta P_{sit} \neq 0)$ . Then the average size across representative items is

$$\overline{\Delta P} = \sum_s w_s \Delta P_s. \quad (43)$$

The calculation of the inflation effect of tax changes is described in Section 3 of the paper.

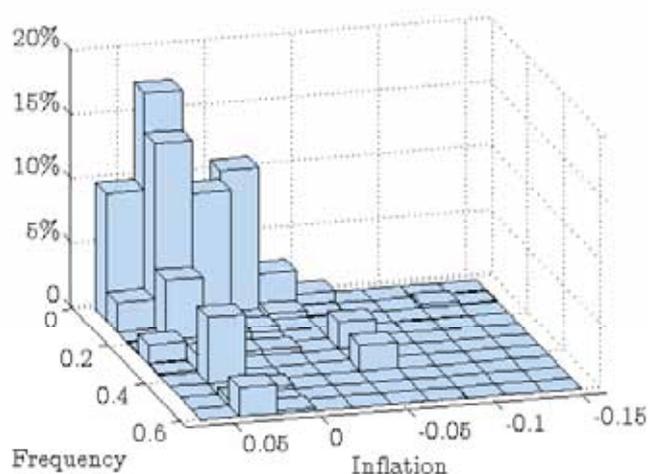
## SECTORAL INFLATION AND FREQUENCY IN THE US

An essential reason of the aggregation bias the paper identified is the positive sectoral correlation between inflation and price stickiness (menu costs). A substantial reason of this correlation comes from the prominent role of the services sector with both higher inflation and price stickiness. This is also true in the US (with services over 30% weight in the US consumption basket), though not as prominently as in the emerging country Hungary. To obtain a measure of price stickiness, we use sectoral frequencies of price changes reported by Nakamura and Steinsson (2008b) for 180 4-digit Entry Level Items (ELI) of US CPI. The weighted average of these frequencies for the services sector is 13.6% that is substantially lower than the 21.1% average price changing frequency. The trend inflation is 2.7% in the services sector between 1998-2005 while the average was 2.25% during this period.

The main text has also shown that in a weighted histogram of Hungarian sectors the frequency of price changes is indeed tend to be lower (meaning higher price stickiness) in sectors with high inflation rate. As inflation itself causes higher frequency, this observed relationship implies that the (weighted) correlation between inflation and the unobserved menu costs is indeed positive. Here we show a similar figure for the US to show that this relationship can be expected to be present in other countries as well.

We use the frequencies of 180 ELI sectors reported by Nakamura and Steinsson (2008b). The Bureau of Labor Statistics publish the sectoral inflation rates for only 111 of these sectors. We use the 2-digit ELI inflation rates for the remaining sectors - where available - to obtain the sample of 170 sectors. For valid comparison to our sample, we also dropped 10 additional sectors of tobacco, new and used cars and gasoline and energy items from the sample, as these were also missing from the Hungarian sample.

**Figure 11**  
**Weighted histogram of sectoral trend-inflation and frequency in the US**



The weighted histogram for the US data shows a somewhat smaller but still significant negative relationship between sectoral inflation and the frequency of price changes than the Hungarian data. It implies a stronger positive relationship between sectoral inflation rates and menu costs suggesting that the aggregation bias can be expected to be present in the US as well.

MNB Working Papers 2010/3  
Inflation asymmetry, menu costs and aggregation bias – A further case for state dependent pricing

Print: D-Plus  
H-1037 Budapest, Csillaghegyi út 19-21.

