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The Decentralization Tradeoff for Complementary Spillovers

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Abstract:
We examine a symmetric two-district setting with spillovers of local public spending where a spill-in from the foreign spending is not a substitute, but a complement to domestic spending. Specifically, we assume production of two district-specific public goods out of two complementary district-specific inputs. We compare equilibria in non-cooperative decentralization and cooperative centralization for different spillovers, complementarities and cost-division rules, and control for the effects of strategic delegation and the feasibility of voluntary contributions to the input in the foreign district. We find that centralization welfare-dominates decentralization in most institutional settings and for a wide range of parameters, yet we can also identify necessary and sufficient conditions for decentralization to welfare-dominate centralization. The setup features three novelties: In the absence of transfers, welfare in decentralization increases in spillovers, strategic delegation in decentralization improves welfare, and centralized provision may be non-monotonic in spillovers.

Keywords: Spillover, Spill-in, Strategic complementarity, Decentralization theorem

JEL: H41; H73; H77

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1 Introduction

The existence of multiple governments, mobility of factors and consumers, and a large institutional variety makes fiscal federalism one of the richest subfields in public economics. In spite of proliferation of studies on the vast array of tax and revenue instruments, a fundamental design issue in fiscal federalism still remains whether to centralize or decentralize public expenditures. By the seminal Oates' decentralization theorem (Oates 1972), this particular tradeoff is relatively straightforward when it comes to spillovers: the benefits of centralization relative to decentralization increase in the level of spillovers and decrease in the taste differences.

In this paper, we reexamine the role of spillovers in the presence of taste homogeneity. A standard approach is to define the spillover from foreign public spending to be a pure substitute of the domestic public spending. In contrast, we analyze a complete class of complementary aggregations. Although complementary (weak-link and weakest-link) composition functions have been extensively studied for a single pure public good (Hirshleifer 1983; Vicary 1990; Vicary and Sandler 2002; Ray et al 2007; Cornes and Hartley 2007; Gregor 2011), complementarities, to our best knowledge, have not yet been embedded in a setup with cross-border spillovers. The major difference is the introduction of multiple composite public goods aggregated out of domestic and foreign spending by district-specific complementary aggregations.

We build a setup with two districts and two geographically-specific inputs, one per district, where each local input generates a positive spillover in the other district.[1] A level of an output produced in a district and consumed entirely in the district is determined by complementary aggregation of the domestic input and spill-in from the foreign input. We compare two regimes, a non-cooperative decentralized regime, and a cooperative centralized regime that assumes that delegates from the districts maximize joint surplus. For decentralization, we introduce the possibility of voluntary transfers, so that the government in a district i is permitted to contribute to the provision of both domestic input i and foreign input $-i$.

Another key option is whether a district may strategically delegate an agent with a different valuation of the public good. Strategic delegation is a natural phenomenon in models where delegates (politicians) bargain and their principals (representative voters) behave non-cooperatively; low-value delegation is typically used to strategically decrease the breakdown allocation, and induce relatively larger compensations (Segendorff 1998). Strategic delegation is one of the many extensions that the second-generation of fiscal federalism (Oates 2005; Lockwood 2006) introduces to investigate robustness of the decentralization theorem in realistic political economy settings; other possible extensions involve electoral accountability,

[1] Given that composite public goods are non-linear in the expenditures, the two-district assumption is a necessary first step to avoid complexities of multidistrict (or higher-order) spillovers (Bloch and Zenginobuz 2007).

lobbying (Cheikbossian 2008), bias of the median from the mean (Lockwood 2008), or endogenous centralization (Ruta 2010). Here, strategic delegation is the key strategic instrument that affects both decentralization and centralization, each through a different channel.

With complementarity, transfers and strategic delegation, we observe that taste homogeneity is no longer a sufficient condition for welfare dominance of centralization. This finding contributes to the literature on the lack of robustness of centralization to strategic delegation (Besley and Coate 2003; Dur and Roelfsema 2005). In the absence of transfers, we even observe that the welfare in decentralization paradoxically increase in the level of spillovers. In addition, the provision in centralization is generally non-monotonic in the spillovers. As a result, the relative welfare gain of centralization may decrease in the level of spillovers. To our best knowledge, the only paper that achieved the identical result for the relative benefits is Kothenbuerger (2008).

Specifically, our welfare assessment compares distortions associated with complete specialization in decentralization and distortions associated with strategic delegation in centralization. Centralization generally tends to welfare-dominate decentralization. Under some parameters, it is even welfare-superior irrespective of the levels of spillovers, exactly as in Oates (1972). Only if voluntary transfers are feasible, decentralization turns out to be unambiguously more attractive for low spillovers, as in Besley and Coate (2003). Our explanation is nevertheless alternative to the one considered for substitutes. In decentralization of complementary spillovers with transfers, the equilibrium features cross-district specialization upon the foreign inputs. An increase in spillovers makes spill-ins from the foreign inputs more abundant, and the low elasticity between the foreign spill-in and domestic input (a property of complementary aggregations) causes that the foreign input becomes less demanded in the production mix. Cross-specialization upon the foreign inputs thus becomes more distortive and welfare deteriorates. In the classic case of substitutes, the districts specialize on the domestic inputs. An increase in spill-ins makes foreign inputs relatively more productive, hence specialization upon domestic inputs is more distortive, and the districts become worse off.

This paper proceeds as follows. Section 2 builds the setup of the district-specific complementary production functions and solves for the social optimum. Section 3 shows how infeasibility of transfers to the foreign input generates a welfare loss in decentralization, plus a paradoxical effect that larger spillovers improve provision. Section 4 studies how introducing voluntary transfers leads to specialization across districts and improves welfare both with and without strategic delegation. Section 5 studies centralization, mainly in the presence of strategic delegation. It compares centralization and decentralization under various cost-division rules, and explores the decentralization tradeoff in the levels of spillovers. The final Section 6 concludes the paper.

2 Setup

2.1 Assumptions

We have districts $1, 2$ and inputs x, y with common price $p > 0$. Input $x \in \mathbb{R}_+$ is geographically specific for District 1 and input $y \in \mathbb{R}_+$ for District 2. An input represents, e.g., the number of cultural facilities or the size of police squad in each district. Out of each unit of any input, an exogenous parameter $\kappa \in (0, \frac{1}{2})$ represents the spillover into the foreign district and $1 - \kappa$ the domestic use of the input.[2] The effective amounts of inputs in District i, (X_i, Y_i), are $(X_1, Y_1) = ((1-\kappa)x, \kappa y)$ and $(X_2, Y_2) = (\kappa x, (1-\kappa)y)$. The effective domestic input aggregates with the effective foreign input (i.e., foreign spill-in) by a complementary technology with a constant elasticity of substitution $\frac{1}{1+\rho}$. The complementarity reflects properties of tastes or technologies, namely a strong preference for variety, or protection against an adversary (Hirshleifer 1983; Vicary 1990). Outputs (levels of the public consumption) in the districts write

$$g_i = (X_i^{-\rho} + Y_i^{-\rho})^{-1/\rho}.$$

The parameter $\rho \geq -1$ determines the shape of the aggregation: For $\rho = -1$, we have a classic case of perfect substitution (also coined 'strategic substitution', cf. Dur and Roelfsema, 2005) and for $\rho \in (-1, 0)$ imperfect substitution. Our interest is in $\rho > 0$, which reflects complementarity, converging with $\rho \to +\infty$ to perfect complementarity, i.e., $g_1 = \min\{(1 - \kappa)x, \kappa y\}$. We keep the parameter ρ instead of elasticity of substitution, bearing in mind that the elasticity is a strictly decreasing transformation of ρ.

The payments for inputs by District i are (px_i, py_i), hence total amounts of inputs are $(x, y) = (x_1 + x_2, y_1 + y_2)$. If voluntary transfers are feasible, then $x_2 \in \mathbb{R}_+$ and $y_1 \in \mathbb{R}_+$; if not, then the restriction $x_2 = y_1 = 0$ applies. We assume that District i is represented by a single Citizen i with a quasi-linear utility, where input costs (or, benefits from private good consumption) are linear while benefits from the district-specific public output is logarithmic with taste parameter $\lambda > 0$,

$$u_i = \lambda \ln g_i - p(x_i + y_i).$$

Thereby, we keep both production and valuation symmetric, disregard any within-district differences, and abstract from the income effects which allows us to separate the public goods allocation decision from distributional decisions (Bergstrom and Cornes 1983).

With strategic delegation, the game separates into two stages. In Stage 1, Citizens 1 and 2 simultaneously and non-cooperatively select arbitrary identities of their delegates, $\lambda_1 \geq 0$ and

[2] This division of the unit suggests rivalry in the use of each input, but equally we can presume non-rivalry, i.e., domestic use of full unit and foreign use of $\kappa < 1$ units. The advantage of our approach is that social optimum is invariant to κ, which largely simplifies exposition of the main results. The extreme cases of $\kappa \in \{0, \frac{1}{2}\}$ are discussed always in limits since some variables take improper values if evaluated in the extremes.

$\lambda_2 \geq 0$. In Stage 2, Delegates 1 and 2 provide the inputs (cooperatively or non-cooperatively). Costs are born by the corresponding citizen.[3] By strategic delegation, each citizen thus non-cooperatively manipulates allocation in the subgame played between the delegates. If $\lambda_1 = \lambda_2 = \lambda$ holds in an equilibrium with strategic delegation, we call the regime (under given exogenous parameters) *delegation-proof*. Without delegation, the game reduces to a single stage where citizens (cooperatively in centralization or non-cooperatively in decentralization) decide on payments for the inputs themselves.

Let $\phi := (\frac{1-\kappa}{\kappa})^\rho$ be a measure of *asymmetry* of input productivity that combines both the effect of spillover and shape of the aggregation. To see how it captures asymmetry, rewrite the public output, without loss of generality in District 1, as

$$g_1 = (1-\kappa)(x^{-\rho} + \phi y^{-\rho})^{-1/\rho}.$$

In a symmetric profile, a foreign input is clearly ϕ-times more productive than a domestic input. The level $\rho = 0$ separates two structurally different cases, substitutability and complementarity. For substitutability ($\rho < 0$), we get $\phi \in (0,1)$; each citizen thus cares relatively more for the *domestic* input. For complementarity ($\rho > 0$) which is the topic of our interest, there is $\phi \in (1, +\infty)$, and each citizen cares relatively more for the *foreign* input. A detailed analysis of the special importance of the foreign inputs in the complementary production functions with spillovers is relegated into the Appendix.

2.2 Social optimum

To derive the social optimum, we investigate symmetric profiles, $x = y$. Maximization of welfare $W := u_1 + u_2$ is invariant to the division of costs across districts, hence we solve the following first-order condition:

$$\left.\frac{dW}{dx}\right|_{x=y} = -p\left(1 + \frac{dy}{dx}\right) + \frac{\lambda}{g_1}\left(\frac{\partial g_1}{\partial x} + \frac{\partial g_1}{\partial y}\frac{dy}{dx}\right) + \frac{\lambda}{g_2}\left(\frac{\partial g_2}{\partial x} + \frac{\partial g_2}{\partial y}\frac{dy}{dx}\right) = 0.$$

We derive marginal products in symmetric allocations,

$$\left(\frac{\partial g_1}{\partial x}; \frac{\partial g_1}{\partial y}\right) = \left(\frac{1}{1+\phi}\ \frac{g_1}{x}; \frac{\phi}{1+\phi}\ \frac{g_1}{y}\right). \tag{1}$$

Inserting $\frac{dy}{dx} = 1$ that stems from symmetry and plugging (1) into the first-order condition, socially optimal inputs yield

$$x^* = y^* = \frac{\lambda}{p}.$$

[3] Besley and Coate (2003) offer political-economy microfoundations for motivation of the delegates through citizen-candidate assumptions whereby (i) a median citizen's preference represents average preferences in a heterogeneous district, (ii) district citizens by simple-majority elect identity of their delegate, (iii) only district citizens are eligible for delegation, and (iv) all district citizens pay identical tax. Then, the elected delegate is the median citizen's preferred delegate. To save for notation, we directly assume that a single citizen nominates a delegate with a preferred taste for the public good.

3 Decentralization, no transfers

3.1 No delegation

We start with the assumption that transfers to the other district are not feasible, hence each citizen can finance only its domestic input. To derive a symmetric Nash equilibrium, we solve the first-order condition:

$$\frac{du_1}{dx} = -p + \frac{\lambda}{g_1}\frac{\partial g_1}{\partial x} = 0. \tag{2}$$

The first result in this baseline specification is achieved simply by inserting marginal products from (1) into the first-order condition in (2).

Result 1 (Decentralization without transfers, no delegation) *The equilibrium amounts of inputs in decentralization without transfers and without delegation (baseline case) are* $x^b = y^b$*, where*

$$\frac{x^b}{x^*} = \frac{1}{1+\phi} \in \left(0, \tfrac{1}{2}\right).$$

Underprovision is a standard result, but comparative statics of the baseline equilibrium is not obvious. With an increase in the input asymmetry ϕ, the provision drops from $\frac{1}{2}$ to 0, and underprovision is increasingly more serious. Recall however that the increase in asymmetry is associated with an increase in complementarity or a decrease in spillover. Thus, by increasing spillovers, inputs become more symmetric, hence provision improves. Contrary to established findings, we observe in a baseline setting that an increase in spillovers improves provision of all local public goods. To restore an opposite prediction of the decentralization theorem, we will see that the citizens and delegates must not be constrained to specialize on the less productive inputs which are for complementary spillovers the domestic inputs.

3.2 Strategic delegation

Introduction of strategic delegation separates the game into two stages. In Stage 1, Citizens 1 and 2 simultaneously and non-cooperatively select their Delegates, characterized by tastes $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. In Stage 2 under decentralization, Delegates 1 and 2 simultaneously and non-cooperatively provide the inputs. By strategic delegation, each citizen non-cooperatively manipulates allocation in the subgame played between the delegates. The equilibrium in the non-cooperative subgame of delegates is located on the other delegate's best response, hence choosing an optimal delegate is equivalent to the optimization along the other delegate's best response. This is because the other delegate's best response is unchanged with manipulations of the taste of own delegate. Best responses of Delegates 1 and 2, denoted as $x_1^C(y)$ and $y_2^C(x)$, are constructed from the standard first-order conditions upon each delegate's utility function, denoted as u_i^D:

$$x_1^C(y): \frac{du_1^D}{dx} = -p + \frac{\lambda_1}{g_1}\frac{\partial g_1}{\partial x} = 0$$

$$y_2^C(x): \frac{du_2^D}{dy} = -p + \frac{\lambda_2}{g_2}\frac{\partial g_2}{\partial y} = 0$$

By expressing the marginal products in the general form (not only for symmetric allocations), we characterize implicit-form best responses of the delegates as follows:

$$x_1^C(y)\left[1 + \phi\left(\frac{x_1^C(y)}{y}\right)^\rho\right] = \frac{\lambda_1}{p} \tag{3}$$

$$y_2^C(x)\left[1 + \phi\left(\frac{y_2^C(x)}{x}\right)^\rho\right] = \frac{\lambda_2}{p} \tag{4}$$

Now, to identify an equilibrium pair $(\lambda_1, \lambda_2) = (\lambda', \lambda')$ and corresponding equilibrium inputs (x', y'), we check possible deviations of Citizen 2 in Stage 1 from an allocation $(x', y') = (x_1^C(y'), y')$ that is induced by delegates in the subsequent Stage 2. For an equilibrium, a unilateral deviation must not increase Citizen 2's utility,

$$\frac{du_2}{dy} = -p + \frac{\lambda}{g_2}\frac{dg_2}{dy} = -p + \frac{\lambda}{g_2}\left(\frac{\partial g_2}{\partial y} + \frac{\partial g_2}{\partial x}\frac{dx_1^C(y)}{dy}\right) = 0. \tag{5}$$

The important difference to the case without delegation is the presence of *strategic complementarity*, $dx_1^C(y)/dy > 0$, which gives an incentive to exploit the option of high-value strategic delegation. Intuitively, if citizens nominated delegates sincerely ($\lambda_1 = \lambda_2 = \lambda$), then delegates would implement $x^b = y^b$ as characterized by (2), but entering (2) into (5) reveals an opportunity for unilateral improvement, given that

$$\left.\frac{du_2}{dy}\right|_{x=x^b} = \frac{\lambda}{g_2}\frac{\partial g_2}{\partial x}\frac{dx_1^C(y)}{dy} > 0.$$

Using an implicit-function theorem upon the implicit-form best response of Delegate 1 in (3), in symmetry, we derive the exact magnitude of the strategic complementarity,

$$\frac{dx_1^C(y)}{dy} = \frac{\rho\phi}{1 + \rho\phi + \phi}. \tag{6}$$

Now, we exploit that a pair of symmetric delegates (λ', λ') plays a symmetric allocation and that the equilibrium in the subgame of Stage 2 is already characterized by Result 1, only with different tastes, $x' = \frac{1}{1+\phi}\frac{\lambda'}{p}$. The symmetry allows us to insert complementarity in (6) and marginal products in (1) into (5), and obtain the equilibrium identity of the delegates,

$$\frac{\lambda'}{\lambda} = \frac{(1+\phi)(1+\rho\phi)}{1 + \rho\phi + \phi} > 1.$$

Next, we use $\frac{x'}{x^b} = \frac{\lambda'}{\lambda}$, or $\frac{x'}{x^*} = \frac{\lambda'}{\lambda}\frac{x^b}{x^*}$, to obtain the next result.

Result 2 (Decentralization without transfers, delegation) *The equilibrium amounts of inputs in decentralization without transfers but with strategic delegation are $x' = y'$, where*

$$\frac{x'}{x^*} = \frac{\lambda'}{\lambda} \cdot \frac{1}{1+\phi} = \frac{1+\rho\phi}{1+\rho\phi+\phi} \in \left(\tfrac{\rho}{1+\rho}, \tfrac{1+\rho}{2+\rho}\right).$$

Like in the case without delegation, by increasing spillovers, inputs become more symmetric, hence provision improves: $\frac{dx'}{d\kappa} > 0$. Again, this non-intuitive property is a consequence of an inferior matching of inputs to districts, and the impossibility to cross-subsidize the foreign input.

4 Decentralization with voluntary transfers

Since the foreign inputs are more productive in symmetric allocation, $\phi > 1$, the introduction of voluntary transfers to the foreign input should motivate citizens to boost provision by relocating contributions from domestic to foreign inputs. To start with, we derive for each player[4] an optimal amount of each input, conditional on having the amount of the other input fixed and assuming that the player covers the full cost of the input. We call it a *single-input optimum* and denote as $x_i^C(y)$ and $y_i^C(x)$. This is consistent with our previous notation where $x_1^C(y)$ and $y_2^C(x)$ in the case without transfers have been defined as the citizens' best responses; these best responses are by definition their single-input optima for the domestic inputs.

We will show that a unique decentralized equilibrium with transfers features complete specialization on the foreign input, where $(x_1, y_1) = (0, y_1^C(x))$ and $(x_2, y_2) = (x_2^C(y), 0)$. That *cross-specialization* emerges in an equilibrium is not surprising in itself since the foreign input is, in any symmetric allocation, ϕ-times more productive than the domestic input. Given the mutual use of transfers across districts, we call each cross-specialization profile a T-profile (transfer profile), and then characterize explicitly the equilibrium T-profile.

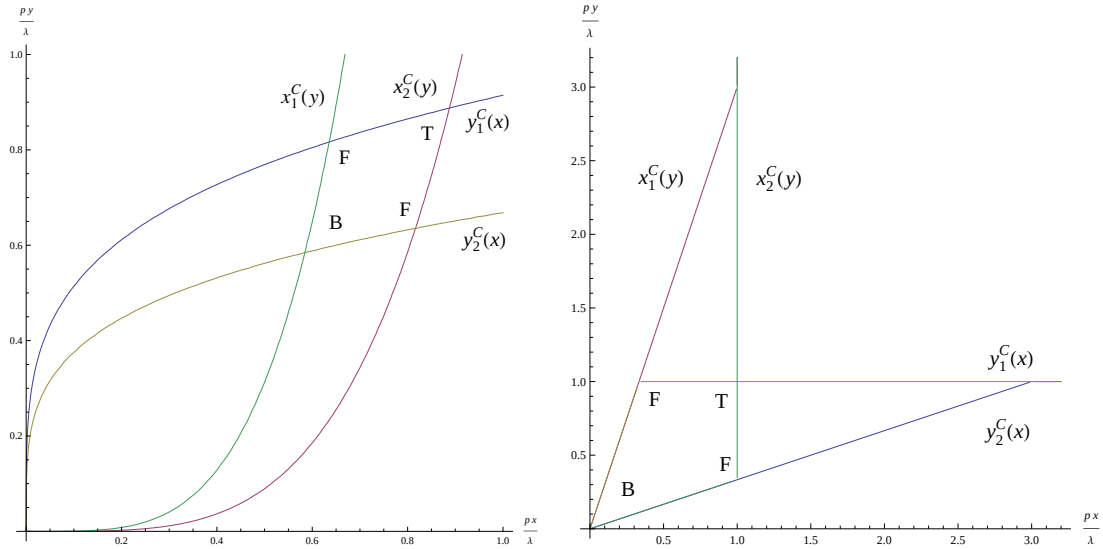
The single-input optima $x_i^C(y)$, and $y_i^C(x)$ of Delegate i are derived from the first-order conditions:

$$x_i^C(y) : \frac{du_i^D}{dx} = -p + \frac{\lambda_i}{g_i}\frac{\partial g_i}{\partial x} = 0$$

$$y_i^C(x) : \frac{du_i^D}{dy} = -p + \frac{\lambda_i}{g_i}\frac{\partial g_i}{\partial y} = 0$$

If the inputs are provided by citizens (i.e., in the absence of delegation), we use single-input optima with $\lambda_i = \lambda$. We plot these optima for imperfect complements in Fig. 1(a) and for perfect complements in Fig. 1(b). The implicit-form characterizations satisfy the following:

[4]The single-input optima are characterized for players who determine the provision, i.e. for citizens in regimes without delegation and delegates in regimes with strategic delegation.



(a) $0 < \rho < +\infty$ (b) $\rho = +\infty$

Figure 1: Citizens' single-input optima, baseline equilibrium (B) and transfer equilibrium (T) for imperfect and perfect complementarity

$$x_1^C(y)\left[1+\phi\left(\frac{x_1^C(y)}{y}\right)^\rho\right] = \frac{\lambda_1}{p} = y_1^C(x)\left[1+\frac{1}{\phi}\left(\frac{y_1^C(x)}{x}\right)^\rho\right] \tag{7}$$

$$x_2^C(y)\left[1+\frac{1}{\phi}\left(\frac{x_2^C(y)}{y}\right)^\rho\right] = \frac{\lambda_2}{p} = y_2^C(x)\left[1+\phi\left(\frac{y_2^C(x)}{x}\right)^\rho\right] \tag{8}$$

In the subgame of providers (citizens in the absence of delegation, delegates in the presence of strategic delegation), we apply the following necessary equilibrium conditions: For each input, (i) a provider of a positive amount of input must be in his or her single-input optimum, and (ii) a non-provider's single-input optimum must be lower or equal than the equilibrium amount of the input. Both are obvious: violation of (i) motivates the provider to unilaterally deviate by increasing or decreasing the amount, and violation of (ii) makes the non-provider to become a provider. The equilibrium conditions simplify to $x = \max\{x_1^C(y), x_2^C(y)\}$ and $y = \max\{y_1^C(x), y_2^C(x)\}$. In order to identify the maxima in symmetry, let us start with x-input and denote LHS and RHS from implicit characterizations of $x_1^C(y)$ and $x_2^C(y)$ in (7) and (8) for each player as $L_i(x,y)$, $R_i(x,y)$:

$$L_1(x,y) := x\left[1+\phi\left(\frac{x}{y}\right)^\rho\right] = \frac{\lambda_1}{p} =: R_1(x,y)$$

$$L_2(x,y) := x\left[1+\frac{1}{\phi}\left(\frac{x}{y}\right)^\rho\right] = \frac{\lambda_2}{p} =: R_2(x,y)$$

4.1 No delegation

Solving first the regime without delegation, $\lambda_1 = \lambda_2 = \lambda$, we have $L_1(x,y) > L_2(x,y)$, $R_1(x,y) = R_2(x,y)$, $\frac{\partial L_i}{\partial x} > 0$ and $\frac{\partial R_i}{\partial x} = 0$ for $i = 1,2$, thus we obtain $x_2^C(y) > x_1^C(y)$. Using superscript 't' for T-profile, hence $x^t = \max\{x_1^C(y^t), x_2^C(y^t)\} = x_2^C(y^t)$. By analogy, we obtain $y^t = \max\{y_1^C(x^t), y_2^C(x^t)\} = y_1^C(x^t)$. Both is easily seen on Fig. 1(a). By imposing symmetry into the implicitly derived single-input optima in (7) and (8), we derive the unique decentralized equilibrium with transfers and without delegation.

Result 3 (Decentralization with transfers, no delegation) *The equilibrium amounts of inputs in decentralization with transfers and without strategic delegation are $x^t = y^t$, where*

$$\frac{x^t}{x^*} = \frac{\phi}{1+\phi} \in (\tfrac{1}{2}, 1).$$

Clearly, abolishing the restriction to pay only for the domestic input increases provision, and also welfare, as $x^b < \frac{1}{2} < x^t < x^*$. With increasing asymmetry ϕ (due to increasing complementarity or decreasing spillover), the equilibrium converges to the social optimum, as the provision grows from $\frac{1}{2}$ to 1. Therefore, under transfers that exploit comparative advantages of cross-specialization, we happen to restore the standard observation that a larger spillover (i.e., lower asymmetry) worsens the decentralized provision. With increasing spillovers, Citizen 1 specializes on a decreasingly important (foreign) input, which magnifies distortion of decentralization.

We may shed even more light on the distortion associated with underprovision in this equilibrium. Both single-input optima are increasing in the other input. This implies strategic complementarity on both sides, and it is socially optimal to commit both districts to an allocation that exploits this strategic complementarity. Given that non-cooperative equilibrium does not account for strategic complementarity, there must be a wedge between the social optimum and the decentralized equilibrium. Only in the case of perfect complements where single-input optima are constant, as in Fig. 1(b), this opportunity for improvement is not present, and decentralized optimum is socially efficient.

To understand the extreme case of perfect complementarity more in detail, see that for Citizen 1 with output $g_1 = \min\{(1-\kappa)x, \kappa y\}$, marginal products are discontinuous. The marginal product of x-input is positive, $\partial g_1/\partial x = 1-\kappa$, if $(1-\kappa)x < \kappa y$ (the complementarity constraint does not bind) and zero otherwise (the complementarity constraint binds). Similarly, $\partial g_1/\partial y = \kappa$ if $(1-\kappa)x > \kappa y$ and zero otherwise. Single-input optima feature social optimum, $x_1^C(y) = y_1^C(x) = x^* = y^*$, only if the complementarity constraints do not bind:

$$x_1^C(y) = \begin{cases} \frac{\kappa}{1-\kappa} y & y < \frac{1-\kappa}{\kappa} x^* \\ x^* & y \geq \frac{1-\kappa}{\kappa} x^* \end{cases},$$

$$y_1^C(x) = \begin{cases} \frac{1-\kappa}{\kappa}x & x < \frac{\kappa}{1-\kappa}y^* \\ y^* & x \geq \frac{\kappa}{1-\kappa}y^* \end{cases}.$$

Now, the outcome for perfect complementarity dramatically differs depending on the feasibility of transfers. In the baseline case, for positive $x > 0$ and $y > 0$, we have $x_1^C(y) < y$ and $y_2^C(x) < x$. Thus, there is no equilibrium for positive (x, y), and we observe $x_1^C(y) = y_2^C(x)$ if and only if $x^b = y^b = 0$. In the transfer regime, we find that $x_2^C(y) \geq x_1^C(y)$ and $y_1^C(x) \geq y_2^C(x)$ (as in the case of imperfect complements), hence the symmetric equilibrium satisfies $x^t = \max\{x_1^C(y^t), x_2^C(y^t)\} = x_2^C(y^t)$ and $y^t = \max\{y_1^C(x^t), y_2^C(x^t)\} = y_1^C(x^t)$ and this is equilibrium if $x^t = y^t = x^*$.

4.2 Strategic delegation

By strategic delegation, each citizen can non-cooperatively manipulate allocation in the subgame played between the delegates. In the case without transfers, the only effect of strategic delegation was to exploit the strategic complementarity. The reason was that the structure of the equilibrium profile was always the same for any combination of tastes of the delegates; namely, in a subgame of delegate, each delegate specialized on the domestic input. With transfers and strategic delegation, cross-specialization is not generally warranted.

To start with, we easily observe that single-input optima of a delegate are increasing in the taste of the delegate. For x-input, by implicit function theorem upon $L_i(x, y) - R_i(x, y) = 0$,

$$\frac{dx_i^C(y)}{d\lambda_i} = \frac{-\frac{\partial L_i}{\partial \lambda_i} + \frac{\partial R_i}{\partial \lambda_i}}{\frac{\partial L_i}{\partial x} - \frac{\partial R_i}{\partial x}} = \frac{1}{\frac{\partial L_i}{\partial x}} > 0.$$

We analyze the problem from the perspective of Citizen 1. We know that $x_1^C(y) < x_2^C(y)$ if $\lambda_1 = \lambda_2$, and that $x_1^C(y)$ is increasing in taste λ_1. Thus, for each y, there is a critical level $\overline{\lambda_1} > \lambda_2$ where $x_1^C(y) \leq x_2^C(y)$ if $\lambda_1 \leq \overline{\lambda_1}$ and $x_1^C(y) \geq x_2^C(y)$ if $\lambda_1 \geq \overline{\lambda_1}$. Similarly, $y_1^C(x)$ is increasing in taste λ_1 and recall $y_1^C(x) > y_2^C(x)$ if $\lambda_1 = \lambda_2$. Thus, for each x, there is a critical level $\underline{\lambda_1} < \lambda_2$ where $y_1^C(x) \leq y_2^C(x)$ if $\lambda_1 \leq \underline{\lambda_1}$ and $y_1^C(x) \geq y_2^C(x)$ if $\lambda_1 \geq \underline{\lambda_1}$. In total, the equilibrium of the provision subgame varies in the three intervals: (i) $\lambda_1 \leq \underline{\lambda_1}$, where $(x, y) = (x_2^C(y), y_2^C(x))$ and Delegate 1 free rides on Delegate 2 (to be called F-profile); (ii) $\underline{\lambda_1} < \lambda_1 < \overline{\lambda_1}$, where $(x, y) = (x_2^C(y), y_1^C(x))$, hence delegates cross-specialize (T-profile); and (iii) $\lambda_1 \geq \overline{\lambda_1}$, where $(x, y) = (x_1^C(y), y_1^C(x))$ and Delegate 1 pays full cost alone (to be called all-pay, A-profile).[5] We proceed by analyzing only the subset of T-equilibria and then we check robustness of the resulting T-equilibrium to deviations to F-profiles and A-profiles.

To examine the set of T-profiles that occur in the subgame of delegates, we start with sincere delegation, $\lambda_1 = \lambda_2 = \lambda$. In this case, each delegate specializes on the foreign input in the equilibrium. Examine incentives of Citizen 1 to deviate from sincere delegation when

[5] In the border case $\lambda_1 = \underline{\lambda_1}$, there are equilibrium profiles where both district contribute positive amounts to y-input. Similarly, if $\lambda_1 = \overline{\lambda_1}$, there are multiple equilibria where both district contribute to x-input.

Citizen 2 delegates sincerely, $\lambda_2 = \lambda$. A marginal change in Delegate 1's taste does not violate that $x_2^C(y) > x_1^C(y)$ and $y_1^C(x) > y_2^C(x)$, hence the structure of the cross-specialization equilibrium is preserved, $(x, y) = (x_2^C(y), y_1^C(x))$. The manipulation of the identity of Delegate 1 amounts to moving along the best response of Delegate 2, which is by the structure of T-profile characterized by $(x, y) = (x_2^C(y), y)$, hence to manipulating y such that

$$\frac{du_1}{dy} = -p + \frac{\lambda}{g_1}\frac{dg_1}{dy} = -p + \frac{\lambda}{g_1}\left(\frac{\partial g_1}{\partial y} + \frac{\partial g_1}{\partial x}\frac{dx_2^C(y)}{dy}\right) = 0.$$

By strategic delegation, Citizen 1 now internalizes complementarity between the inputs, $dx_2^C(y)/dy > 0$, hence obtains an extra positive marginal benefit. This extra benefit motivates high-value strategic delegation. We apply the implicit function theorem upon the single-input demand of Delegate 2 (which is the best response along which Citizen 1 is optimizing), $L_2(x, y) - R_2(x, y) = 0$, and characterize the magnitude of the strategic complementarity as follows:

$$\Gamma(x, y) := \frac{dx_2^C(y)}{dy} = \frac{-\frac{\partial L_2}{\partial y} + \frac{\partial R_2}{\partial y}}{\frac{\partial L_2}{\partial x} - \frac{\partial R_2}{\partial y}} = -\frac{\frac{\partial L_2}{\partial y}}{\frac{\partial L_2}{\partial x}} = \frac{\rho}{1 + \rho + \phi} \in (0, 1) \tag{9}$$

The exact size of the complementarity will be later useful for the comparison with the strategic delegation in centralization. Notice that the positive effect of strategic complementarity is diminishing to zero for increasing asymmetry between inputs (increasing ϕ),[6] as we approach orthogonal best responses. The orthogonal single-input optima under perfect complementarity are visible in Fig. 1(b).

Conditional on subgames with specialization T-profiles, we can characterize the symmetric equilibrium in the strategic delegation game, denoted as $(\tilde{\lambda}, \tilde{\lambda})$. We use marginal products in (1) to see that in symmetry,

$$\frac{du_1}{dy} = -p + \frac{\lambda}{g_1}\frac{\partial g_1}{\partial x}\left(\phi + \frac{\rho}{1 + \rho + \phi}\right) = -p + \frac{\lambda}{\tilde{\lambda}}\frac{p}{\phi}\left(\phi + \frac{\rho}{1 + \rho + \phi}\right) = 0.$$

This yields the equilibrium strategic delegation at the level

$$\frac{\tilde{\lambda}}{\lambda} = 1 + \frac{\rho}{\phi(1 + \rho + \phi)} > 1.$$

Does the equilibrium T-profile still underprovide relative to the social optimum? We use Result 3 to see that the provision subgame features $\tilde{x} = \tilde{y} = \frac{\phi}{1+\phi}\frac{\tilde{\lambda}}{p}$. The inputs provided by $\tilde{\lambda}$-delegates indeed show underprovision, albeit vanishing with perfect complementarity or zero spillovers.

[6] Combining L'Hôpital's rule, $\frac{\partial \phi}{\partial \rho} > 0$, and $\lim_{\kappa \to 0} \phi = +\infty$, we observe $\lim_{\rho \to +\infty} \frac{\rho}{1+\rho+\phi} = \lim_{\rho \to +\infty} \frac{1}{1+\phi \log \frac{1-\kappa}{\kappa}} = 0$ and $\lim_{\kappa \to 0} \frac{\rho}{1+\rho+\phi} = \lim_{\kappa \to 0} \frac{1}{1+\frac{1}{\rho}+\frac{\phi}{\rho}} = 0$.

Result 4 (Decentralization with transfers and delegation) *Restricting to T-profiles in the provision subgames, the equilibrium amounts of inputs in decentralization with transfers and strategic delegation are $\tilde{x} = \tilde{y}$, where*

$$\frac{\tilde{x}}{x^*} = \frac{\phi}{1+\phi}\frac{\tilde{\lambda}}{\lambda} = \frac{\rho+\phi}{1+\rho+\phi} \in \left(\tfrac{1+\rho}{2+\rho}, 1\right).$$

Like in the regimes without delegation, abolishing the restriction to pay only for the domestic input increases provision and welfare, $x' < \tilde{x} < 1$. And again, we return from a paradoxical comparative statics of the baseline case to the standard case, namely that an increase in the spillovers worsens decentralized provision, $\frac{\partial \tilde{x}}{\partial \kappa} < 0$. The explanation rests again in the relative productivity of the domestic and foreign input. With an increase in spillovers, the domestic and foreign inputs converge in the productivity; in cross-specialization, the relative productivity of the paid input drops, hence the equilibrium is more distorted from the social optimum.

T-profile induced by delegation $(\tilde{\lambda}, \tilde{\lambda})$ in Stage 1 is stable to deviations to all other T-profiles, but it must be stable also to deviations to F-profiles and A-profiles. Lemma 1 proves that deviation from the T-profile to any A-profile cannot improve utility. As a result, when considering incentives for strategic delegation, we will only have to compare the case of strategically low delegation inducing free riding (F-profile) with the case of strategic delegation inducing the best profile among the cross-specialization profiles (T-profile). We discuss in the Appendix the parametric cases when a F-profile is sufficiently attractive to motivate deviation from the T-profile. To conclude, under some parameters, strategic delegation not only brings an opportunity to exploit strategic complementarities of cross-specialization, but also a potential instability vis-a-vis strategic free riding.

Lemma 1 *In T-profile with $(\lambda_1, \lambda_2) = (\tilde{\lambda}, \tilde{\lambda})$, a unilateral deviation to any (all-pay) A-profile is not in any citizen's best response.*

5 Cooperative centralization

We introduce cooperative centralization as in Besley and Coate (2003). They presume that the delegates can maximize joint surplus $W = u_1 + u_2$, and accordingly divide the costs by a pre-determined rule. For utility linear in private consumption, maximization of joint surplus is independent on the division of cost. Therefore, if delegates are symmetric, $\lambda_1 = \lambda_2$, the surplus-maximizing amounts of inputs for the delegates are derived analogically to the derivation of social optimum for the citizens, namely $x = y = \frac{\lambda_1}{p} = \frac{\lambda_2}{p}$. In the absence of strategic delegation, the delegates $\lambda_1 = \lambda_2 = \lambda$ of course implement the social optimum.

Result 5 (Centralization without delegation) *The equilibrium amounts of inputs in centralization without delegation are the first-best amounts (x^*, y^*).*

Cost division is irrelevant in the subgame of cooperative delegates, yet it crucially matters for the incentives of the non-cooperative citizens who use delegation as a strategic tool. We will explore the relation between the cost division and strategic delegation in a class of *symmetric linear rules*. Unlike Besley and Coate (2003) who examine a special case of the cost uniformity in the presence of provision non-uniformity, we find it natural to extend non-uniformity also to costs. Specifically, a linear symmetric rule defines $\sigma \in [0,1]$ to be a fixed share of the domestic inputs that is paid by each delegate. To recover all costs, each delegate and correspondingly also his/her citizen pays $1-\sigma$ share of the foreign inputs,[7] hence the costs write $p(\sigma x + (1-\sigma)y)$ in District 1 and $p((1-\sigma)x + \sigma y)$ in District 2.

5.1 Centralization and strategic delegation

To consider incentive of Citizen 1 to delegate strategically, Lemma 2 first derives how the structure of inputs provided in cooperative Stage 2 changes with a modification of taste of Delegate 1 in centralization. The change is measured by the effect that the more productive (foreign) input has upon the less productive (domestic) input. Lemma 2 also compares the change with a change in the structure of inputs in decentralization with transfers.

Lemma 2 (Strategic complementarity for centralization) *Modification of taste of Delegate 1 in centralization affects the complementary input by level $\psi(x,y) \in (0,1)$, where*

$$\psi(x,y) := \frac{dx}{dy} = \frac{1+\phi+2\rho\phi}{\phi\ (1+\phi+2\rho)} > \Gamma(x,y). \tag{10}$$

The value $\psi(x,y) \in (0,1)$ characterizes the strategic complementarity for any symmetric profile, not only the equilibrium one. With $\rho \to +\infty$ or $\kappa \to 0$, there is also $\phi \to +\infty$, and ψ falls from one to zero. An increase in asymmetry thus weakens strategic complementarity both in the non-cooperative and cooperative regime. Nevertheless, there is a difference in the magnitude of the complementarity. In non-cooperative decentralization with cross-specialization, a unilateral increase in one input leads to an opponent's increase of the other input that reflects only the *opponent's* extra benefits from having more inputs. In cooperative centralization, increases in the amounts of inputs reflect extra benefits of *both* delegates. Therefore, it is straightforward that strategic complementarity under centralization exceeds strategic complementarity under decentralization with transfers.

In cooperative centralization, the equilibrium level of strategic delegation critically depends not only upon the magnitude of strategic complementarity, but also upon the division of the costs. We denote the equilibrium tastes of delegates in the cooperative regime as $(\hat{\lambda}, \hat{\lambda})$. Examine incentive of Citizen 1 to deviate by strategic delegation from a symmetric profile $\hat{x} = \hat{\lambda}/p$, attained by cooperative delegates $(\hat{\lambda}, \hat{\lambda})$, using again marginal products in (1):

[7] Notice that the parameter is constant, hence this class of division rules does not account for the case when possible asymmetry in valuations, $\lambda_1 \neq \lambda_2$, modifies the cost share. Nevertheless, the symmetry in the rule does not necessarily imply symmetry (equality) in costs; if $x \neq y$ and $\sigma \neq \frac{1}{2}$, then $\sigma x + (1-\sigma)y \neq (1-\sigma)x + \sigma y$.

$$\frac{du_1}{dy} = \frac{\lambda}{g_1}\left(\frac{\partial g_1}{\partial x}\frac{dx}{dy} + \frac{\partial g_1}{\partial y}\right) - p\left(\sigma\frac{dx}{dy} + 1 - \sigma\right) = p\left(\frac{\lambda}{\hat{\lambda}}\frac{\psi+\phi}{1+\phi} - (\sigma\psi + 1 - \sigma)\right) \tag{11}$$

The equilibrium identity of delegates as a function of the rule σ is derived such that $\frac{du_1}{dy} = 0$:

$$\frac{\hat{\lambda}}{\lambda} = \frac{\psi+\phi}{1+\phi} \cdot \frac{1}{\sigma\psi + 1 - \sigma} \tag{12}$$

Lemma 3 exploits (11) to reveal that cooperative centralization with strategic delegation achieves the first-best allocation only if the delegates in the districts (and citizens as their principals) are required to pay relatively more for the foreign inputs, but are not forced to completely cross-specialize. In our setup, an ideal cost-division rule in centralization σ^* also deviates from the cost-uniformity which is proposed for centralization in the recent literature (c.f., Harstad 2007).

Lemma 3 (Delegation-proof centralization) *If $\rho > 0$ and $\kappa \in (0, \frac{1}{2})$, centralization with strategic delegation delivers $\hat{\lambda} = \lambda$ and the first-best allocation (x^*, y^*), if and only if $\sigma = \sigma^*$, where*

$$\sigma^* := \frac{1}{1+\phi}.$$

Next, it remains to derive the equilibrium provision as a function of the division rule. Since Stage 2 is cooperative, the delegates $(\hat{\lambda}, \hat{\lambda})$ implement $\hat{x} = \frac{\hat{\lambda}}{p} = \frac{\hat{\lambda}}{\lambda}x^*$. To present conveniently the next result, we use (12) as well as $\sigma^* = \frac{1}{1+\phi}$.

Result 6 (Centralization with delegation) *The equilibrium amounts of inputs in centralization with strategic delegation are $\hat{x} = \hat{y}$, where*

$$\frac{\hat{x}}{x^*} = \frac{\hat{\lambda}}{\lambda} = \frac{\sigma^*\psi + 1 - \sigma^*}{\sigma\psi + 1 - \sigma} \lessgtr 1. \tag{13}$$

This expression combining convex combinations of $\psi > 1$ and 1 allows us to clearly infer how the provision responds to a change in the division rule σ. For $\sigma = \sigma^*$, the level is socially optimal. For $\sigma > \sigma^*$, we have an incentive for high-value delegation and overprovision; in contrary, if $\sigma < \sigma^*$, we get strategic low-value delegation and underprovision. It is easy to see that the optimal cost division approaches zero with increasing complementarity or decreasing spillover; at the limit of perfect complementarity or zero spillover, and the only distortion associated with centralization can be overprovision, as $\sigma \geq \sigma^* = 0$. That the equilibrium amount increases in the division rule is also intuitive. By nominating a high-valuation delegate, the cooperation between delegates increases the foreign input more than the domestic input. For a citizen who is forced to pay more for the domestic input and less for the foreign input (increasing σ), the effective marginal cost of the incremental local output decreases and the citizen aims to attain more of the output by means of the strategic delegation.

5.2 Non-monotonocity in spillovers

In this subsection, we analyze the comparative statics properties in spillovers $\kappa \in (0, \frac{1}{2})$. Since $\psi(x, y)$ is monotonically increasing in spillovers, we may for convenience study how the equilibrium provision $\hat{x}$ changes in ψ instead of κ:

$$\frac{d\hat{x}/x^*}{d\psi} = \frac{\sigma^* - \sigma + (\psi - 1)\frac{d\sigma^*}{d\psi}}{(\sigma\psi + 1 - \sigma)^2}$$

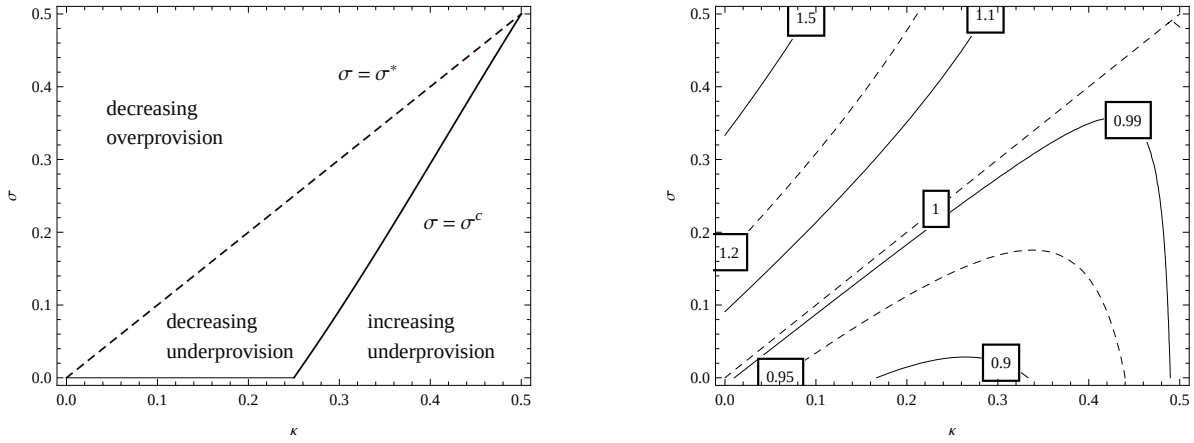
Clearly, the sign of the marginal effect is determined by the nominator. Let σ^c be the *cutoff division rule* for which the equilibrium level is unchanged, $\sigma^c := \sigma^* + (\psi - 1)\frac{d\sigma^*}{d\psi}$. Since ψ and σ^* are independent on the division rule, the cutoff level is also invariant in the division rule, and changes only with the parameters of the production function, κ and ρ. The cutoff rule separates the cases when (i) the equilibrium amount is *increasing* in spillovers ($\sigma < \sigma^c$), and when (ii) the amount is *decreasing* in spillovers ($\sigma > \sigma^c$). It is relatively straightforward that $\sigma^c < \sigma^*$.[8] We can also express the gap $\sigma^* - \sigma^c > 0$ for the extreme values of spillovers:

$$\lim_{\kappa \to \frac{1}{2}} \sigma^* - \sigma^c = 0, \quad \lim_{\kappa \to 0} \sigma^* - \sigma^c = \frac{1}{1 + 2\rho}$$

The equilibrium amount follows either of the following patterns: (i) If $\sigma \geq \frac{1}{2}$, then we always have $\sigma \geq \frac{1}{2} \geq \sigma^* > \sigma^c$, hence the equilibrium amount is *always* decreasing in the spillovers. (ii) If $\sigma < \frac{1}{2}$, then for $\kappa \to 0$ where $\sigma^* \to 0$, we must have $\sigma > \sigma^* \geq \sigma^c$, hence we have overprovision and the amount is decreasing in the spillovers. For $\kappa \to \frac{1}{2}$ where $\sigma^* \to \frac{1}{2}$, we must have $\sigma < \sigma^c < \sigma^*$, hence we have underprovision and the amount is increasing in spillovers. So, if $\sigma < \frac{1}{2}$, the equilibrium amount must be non-monotonic in spillovers, first decreasing from overprovision to underprovision, and then converging back to the social optimum.

Figure 2 illustrates the marginal effect of an increase in spillovers in space $\kappa \times \sigma$ for $\rho = 1$. For large division rules, $\sigma \geq \frac{1}{2}$, the provision is always excessive and decreasing, hence it converges to the efficient amount, and welfare always goes up. For $\sigma < \frac{1}{2}$, we have three intervals indeed: low spillovers where $\sigma > \sigma^*$, intermediate spillovers where $\sigma \in [\sigma^c, \sigma^*]$, and high spillovers where $\sigma < \sigma^c$. Intermediate spillovers are the most interesting; we have underprovision, but the amount is falling and the wedge between the equilibrium and the social optimum further grows. In contrast, for large spillovers, the insufficient amount goes up and converges to the social optimum. Interestingly, monotonicity under cost uniformity which is observed in Besley and Coate (2003) is for complementarities a knife-edge property that is invalidated by any, even infinitesimal decrease in the division rule from cost uniformity, $\sigma = \frac{1}{2}$.

[8] Formally, using $\psi < 1$, $\frac{d\sigma^*}{d\phi} = -(\sigma^*)^2 < 0$, and $\frac{d\psi}{d\phi} = -\frac{(1+\phi)^2 + 2\rho(1+\phi^2)}{\phi^2(1+\phi+2\rho)^2} < 0$, we obtain $\frac{d\sigma^*}{d\psi} = \frac{d\sigma^*}{d\phi}\frac{d\phi}{d\psi} > 0$, hence $\sigma^c < \sigma^*$.



(a) Changes in the equilibrium levels (b) The equilibrium levels relative to the social optimum

Figure 2: The equilibrium provision in centralization $\frac{\hat{x}}{x^*}$ as a function of spillovers and the division rule ($\rho = 1$)

It is also useful to evaluate the equilibrium provision in the extremes of spillovers. For maximal spillovers, the inputs are symmetric, there is no incentive for manipulation through strategic delegation, and the amount is efficient irrespective of the division rule. For zero spillovers, the optimal division is cross-specialization, $\sigma^* \to 0$, and any other division rule implies overprovision, $\lim_{\kappa \to 0} \frac{\hat{x}}{x^*} = \frac{1}{1-\sigma} \geq 1$.

5.3 Decentralization tradeoff

We now exploit Results 1-6 to generate the welfare comparison of decentralization vs. centralization. By Oates' theorem, for taste symmetry, centralization should welfare-dominate decentralization irrespective of the realizations of spillovers (to be called *absolute dominance*). In our setup, feasibility of transfers and strategic delegation are crucial for validity of the absolute dominance. These two options give us four different regimes. Performance of centralization additionally depends on the division of costs in centralization, hence the full comparison within each regime is a function of the triple (ρ, κ, σ). Our main interest is how, ceteris paribus, the tradeoff changes in the level of spillovers.

To start with, Lemma 4 shows that if centralization is tilted towards excessive cross-specialization and accordingly underprovides, then it underprovides less than decentralization. The explanation rests in the different levels of strategic complementarity in decentralization and centralization, as observed in Lemma 2. In other words, the true shortcoming of centralization relative to decentralization can not be underprovision, but only overprovision associated with excessive focus upon the domestic inputs and insufficient focus upon the foreign inputs.

Lemma 4 *For $\sigma \leq \sigma^*$, centralization dominates all decentralization regimes.*

For parameters and regimes where centralization does not dominate absolutely (for all levels of spilllovers), we examine whether it at least dominates for high spillovers, whereas decentralization dominates for low spillovers (to be called *relative dominance*). Besley and Coate (2003) prove relative dominance for pure substitution under cost uniformity ($\sigma = \frac{1}{2}$). Proposition 1 observes relative dominance for all admissible cost-division rules only in the regime 'Transfers, delegation'. In the regime 'No transfers, delegation', the division rule is a key variable for the presence of either absolute or relative dominance. For regimes without delegation, welfare dominance of centralization is absolute and unconditional on the division rule.

Proposition 1 (Decentralization tradeoff for imperfectly complementary spillovers)

1. *In the absence of strategic delegation, centralization of complementary spillovers welfare-dominates decentralization for any $\kappa \in (0, \frac{1}{2})$ (absolute dominance).*

2. *With delegation and without voluntary transfers,*

   (i) there is a level of the division rule $\underline{\sigma} > 0$, where for all $\sigma < \underline{\sigma}$, centralization welfare-dominates decentralization for any $\kappa \in (0, \frac{1}{2})$ (absolute dominance conditional on the low division rule);

   (ii) there is a level of the division rule $\overline{\sigma} < 1$, where for all $\sigma > \overline{\sigma}$, decentralization welfare-dominates centralization for low spillovers ($\kappa \to 0$) and centralization welfare-dominates decentralization for high spillovers ($\kappa \to \frac{1}{2}$) (relative dominance conditional on the high division rule).

3. *With delegation and voluntary transfers, there is a cutoff level $0 \leq K < \frac{1}{2}$ such that decentralization dominates centralization for $\kappa < K$, centralization dominates decentralization for $\kappa > K$, and provisions are identical for $\kappa = K$ (relative dominance).*

For convenient exposition, Table 1 evaluates the equilibrium levels of inputs for the extreme values of spillovers. The tradeoffs at these extremes are easily obtained, recalling $\rho > 0$ and $\sigma \in [0, 1]$. In the mid part of Table 1, the provisions are expressed in the closed form for the intermediate values of spillovers. The welfare comparisons corresponding to the intermediate levels refer to Proposition 1.

Table 1: The equilibrium amounts of inputs relative to social optimum and welfare comparison of centralization vs. decentralization

$\kappa \to 0$	Decentralization	Centralization	Tradeoff

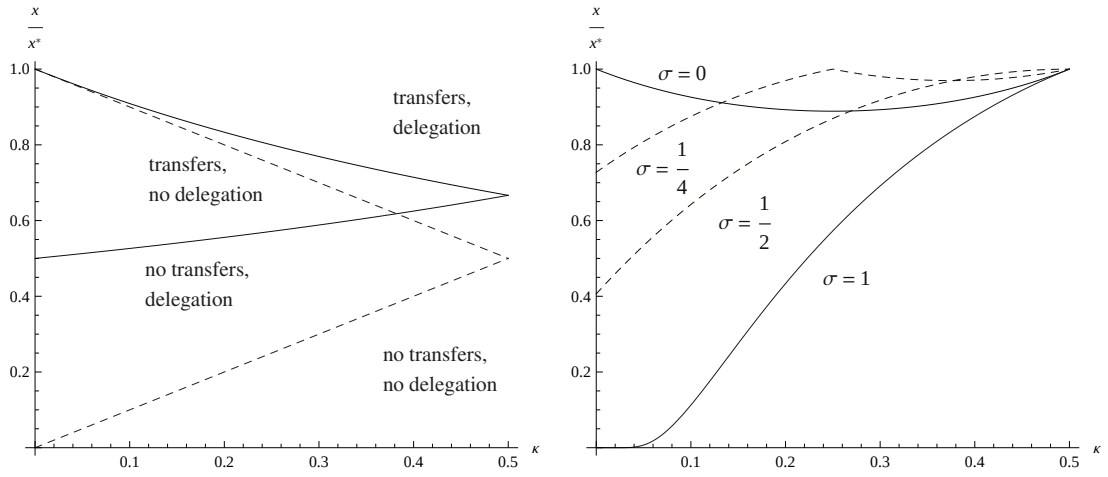
Table 1: The equilibrium amounts of inputs relative to social optimum and welfare comparison of centralization vs. decentralization

No transfers	No delegation	0	1	C
No transfers	Delegation	$\frac{\rho}{1+\rho} \in (0,1)$	$\frac{1}{1-\sigma} \in (1,+\infty)$	C or D
Transfers	No delegation	1	1	identical
Transfers	Delegation	1	$\frac{1}{1-\sigma} \in (1,+\infty)$	D
$\kappa \in (0, \frac{1}{2})$				
No transfers	No delegation	$\frac{1}{1+\phi} \in \left(0, \frac{1}{2}\right)$	1	absolute
No transfers	Delegation	$\frac{1+\rho\phi}{1+\rho\phi+\phi} \in \left(\frac{\rho}{1+\rho}, \frac{1+\rho}{2+\rho}\right)$	$\frac{\sigma^*\psi+1-\sigma^*}{\sigma\psi+1-\sigma} \lessgtr 1$	absolute or relative
Transfers	No delegation	$\frac{\phi}{1+\phi} \in \left(\frac{1}{2}, 1\right)$	1	absolute
Transfers	Delegation	$\frac{\rho+\phi}{1+\rho+\phi} \in \left(\frac{1+\rho}{2+\rho}, 1\right)$	$\frac{\sigma^*\psi+1-\sigma^*}{\sigma\psi+1-\sigma} \lessgtr 1$	relative
$\kappa \to \frac{1}{2}$				
No transfers	No delegation	$\frac{1}{2}$	1	C
No transfers	Delegation	$\frac{1+\rho}{2+\rho} \in (\frac{1}{2}, 1)$	1	C
Transfers	No delegation	$\frac{1}{2}$	1	C
Transfers	Delegation	$\frac{1+\rho}{2+\rho} \in (\frac{1}{2}, 1)$	1	C

To sum up, Oates' prediction of absolute dominance for taste symmetry is conditional on other features in the economy and politics. With strategic delegation, centralization under taste-symmetry is no longer wasteless. Like in Besley and Coate (2003), cooperative centralization distorts cooperative centralization as citizens tend to manipulate identities of their delegates to achieve marginal subsidies. We add that the distortion may be both into overprovision and underprovision, but only overprovision may invalidate Oates' prediction. Secondly, strategic delegation is not only detrimental to welfare; it boosts the level in decentralization as the citizens get an extra instrument to exploit strategic complementarity between the inputs and thereby internalize spillovers. Notice also that the strategic complementarity is unrelated to income effects of opponent's contributions and stems purely from the technical complementarity in the output functions.

Consequently, it is not surprising that a necessary condition for welfare-dominance of decentralization is strategic delegation that is largely distortive in centralization (see Corollary 1) and at the same time welfare-improving in decentralization. Specifically, the distortion in centralization abounds for large division rules and low spillovers, and the distortion in decentralization in the presence of transfers is small for low spillovers.

Corollary 1 *A necessary condition for welfare-dominance of decentralization is strategic delegation and overprovision in centralization associated with an excessive division rule ($\sigma > \sigma^*$).*



(a) Decentralization (b) Centralization with delegation

Figure 3: The equilibrium amounts of the inputs (for overprovision, a welfare-equivalent $\frac{h(x)}{x^*}$ replaces $\frac{x}{x^*}$) for $\rho = 1$

To shed even more light on the tradeoff in spillovers, Figure 3 illustrates the equilibrium levels of inputs[9] evaluated for a particular value of complementarity, $\rho = 1$. Panel (a) plots the levels under the four decentralized regimes, while Panel (b) plots the centralized levels in regimes in the presence of delegation and for the selected levels of the division rule.

6 Conclusions

This paper asks whether complementary aggregation of domestic spending and foreign spill-in in a symmetric two-district setting affects properties of the decentralization tradeoff as described by the seminal decentralization theorem (Oates 1972, 2005). We cover a complete class of complementary aggregations with a constant elasticity of substitution. We observe that high complementarity of a lower level of spillovers increase productivity of spending in the foreign district relative to domestic spending, which motivates specialization *across* the

[9]We use that for symmetric allocations, welfare strictly increases in the amounts of inputs if $x < x^*$ and strictly decreases in the amounts of inputs if $x > x^*$. When examining welfare in regimes that feature underprovision, we may therefore interchangeably use the levels of inputs. To compare overprovision with underprovision, we need to introduce welfare-equivalent input levels that correspond to underprovision. Namely, if $x > x^*$, we map the original variable x into a welfare-equivalent $h(x)$, where $\lambda \ln h(x) - ph(x) = \lambda \ln x - px$ and $h(x) < x^*$. (Or, we replace x/x^* by $h(x)/x^*$ because in the forthcoming analysis, every provision is normalized to the social optimum.) The welfare-equivalent $h(x)$ is also obtained as an implicit solution of

$$\frac{x}{x^*} - \frac{h(x)}{x^*} = \ln\left(\frac{h(x)}{x^*}\frac{x^*}{x}\right).$$

districts. This contrasts the standard case of substitutes, wherein a decrease in the level of spillovers makes a foreign spill-in relatively less important. We also demonstrate that an increase in spillovers is strategically equivalent to an increase in complementarity.

Like Besley and Coate (2003), we explicitly compare the decentralized equilibrium with cooperative centralization and control for the strategic delegation. Unlike Besley and Coate (2003), we investigate a complete class of linear cost-division rules instead of a special case of the uniform cost division. We examine four regimes that vary in the presence and absence of strategic delegation and voluntary transfers. A general message is that centralization dominates decentralization in most regimes under vast ranges of parameters. This echoes Oates' statement that in symmetric setting with positive spillovers, centralization is preferable. Importantly, we obtain that if centralization underprovides, it is still the second-best regime.

Nevertheless, we can also identify necessary and sufficient conditions for decentralization to welfare-dominate centralization. A key necessary condition is the presence of strategic delegation. Strategic delegation, combined with a division rule that is excessively oriented to payments for domestic inputs, distorts centralization to overprovision. The distortion is large with small spillovers. In contrast, underprovision in decentralization is partly remedied with strategic delegation and transfers. Therefore, with strategic delegation and transfers, decentralization always welfare-dominates centralization for low spillovers if centralization largely overprovides, namely if the division rule in centralization is tilted toward excessive payment for the less productive domestic input.

The decentralization tradeoff additionally reflects two structurally new phenomena. First, contrary to intuition, welfare in decentralization may grow if spillovers increase. This is associated with the absence of transfers. Second, welfare in centralization may be non-monotonic in spillovers. This is associated with strategic delegation and the fact that the optimal division rule is endogenous to spillovers. These two novel effects are combined in the regime 'No transfers, delegation', and lead to ambiguity of the general welfare comparison in this regime. In the other regimes, centralization either dominates absolutely (i.e., for any level of spillovers) or relatively (i.e., only for large spillovers).

This setup also incorporates the level of complementarity into the optimal design of centralization. We find that the first-best allocation can be implemented in centralization through a delegation-proof cost-sharing rule, yet the rule varies in the economy fundamentals. The less elastic output and the larger spillover, the less of the domestic input should be paid by each district. Thereby, we contribute to the literature on the pros and cons of the uniformity requirement in the interjurisdictional bargaining (c.f. Harstad 2007).

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A The production functions

First, see that in any in symmetric profile, $x = y$, the marginal rates of technical transformation of inputs satisfy, for each district-specific public good,

$$\frac{\partial g_1}{\partial y} \Big/ \frac{\partial g_1}{\partial x} = \phi = \frac{\partial g_2}{\partial x} \Big/ \frac{\partial g_2}{\partial y}.$$

Second, let $x_i^d(g_i)$ and $y_i^d(g_i)$ be conditional demands for inputs x and y, i.e. the cost-minimizing input-mix along an isoquant $g_i = g$. Irrespective of the isoquant, the ratio of conditional demands is constant,

$$\left(\frac{x_1^d(g_1)}{y_1^d(g_1)}, \frac{x_2^d(g_2)}{y_2^d(g_2)} \right) = \left(\phi^{-\frac{1}{1+\rho}}, \phi^{\frac{1}{1+\rho}} \right).$$

Under substitutability, we have $\phi \in (0,1)$ and elasticity of substitution satisfies $\frac{1}{1+\rho} > 1$, hence $x_1^d/y_1^d > 1 > x_2^d/y_2^d$. Each citizen thus cares relatively more for the *domestic* input. In contrast, for complementarity ($\rho > 0$), we have $\phi > 1$, while elasticity of substitution is still positive $\frac{1}{1+\rho} \in (0,1)$, hence the conditional demands feature $x_1^d/y_1^d < 1 < x_2^d/y_2^d$, i.e. $x_1^d < y_1^d$ and $x_2^d > y_2^d$. Each district cares relatively more for the *foreign* input.

The asymmetric interest in inputs increases in complementarity, $\frac{\partial \phi}{\partial \rho} = \phi \log \frac{1-\kappa}{\kappa} > 0$. It also varies in the level of spillovers,

$$\frac{\partial \phi}{\partial \kappa} = -\frac{\rho}{\kappa^2} \, \phi^{\frac{\rho-1}{\rho}} \lesseqgtr 0.$$

Under substitutability, $\frac{\partial \phi}{\partial \kappa} > 0$, while under complementarity, $\frac{\partial \phi}{\partial \kappa} < 0$. We know that inputs are more symmetric if $\phi \to 1$. For substitutability, this happens if ϕ increases, which is by having either a larger spillover ($d\kappa > 0$) or weaker substitutability ($d\rho > 0$). For complementarity, inputs become more symmetrically productive if ϕ decreases, which is by having either a larger spillover ($d\kappa > 0$) or weaker complementarity ($d\rho < 0$). To sum up, the effect of increasing spillovers upon input asymmetry is identical for substitutes and complements, but the effect of the increasing elasticity of substitution upon input asymmetry varies between substitutes and complements.

To shed more light on the effects of increasing spillovers, examine further the optimal production mix. In complementarity, we must take into account not only a standard substitution mechanism, but also the low elasticity of substitution between the inputs. We decompose the effects in the following way: Let $X := (1-\kappa)x$ and $Y := \kappa y$ be the *effective* amounts of inputs in District 1 that are purchased for prices $\left(\frac{p}{1-\kappa}, \frac{p}{\kappa}\right)$; specifically, for conditional demands, let $X^d := (1-\kappa)x^d$ and $Y^d := (1-\kappa)y^d$. An increase in spillover $d\kappa > 0$ is equivalent to a change in the relative prices of the effective inputs such that the domestic effective input becomes more expensive and the foreign effective input cheaper. With an increase in the relative price of the X-input, a standard technical substitution implies that for effective amounts in the optimal production mix, $d\frac{X^d}{Y^d} < 0$. In addition to this effect, the overall effect upon *nominal* amounts consist also from the change in the transformation rate from the effective amount to nominal amount. A larger spillover makes transformation of domestic effective input into domestic nominal input stronger as $x_1^d = \frac{X^d}{1-\kappa}$ and transformation of foreign effective input into nominal input weaker as $y_1^d = \frac{Y^d}{\kappa}$. The effect on transformation rate goes in the opposite direction to the technical substitution effect present for effective amounts. In total, we have an ambiguous sign for the nominal levels of inputs,

$$d\frac{x_1^d(g)}{y_1^d(g)} \Big/ d\kappa = -\frac{1}{1+\rho}\ \phi^{-1-\frac{1}{1+\rho}} \frac{\partial \phi}{\partial \kappa} \gtreqless 0.$$

Since elasticity of substitution is positive, $\frac{1}{1+\rho} > 0$, the sign is negative for substitutability, where $\frac{\partial \phi}{\partial \kappa} > 0$, and positive for complementarity, where $\frac{\partial \phi}{\partial \kappa} > 0$. This also corresponds to the fact that the elasticity of substitution is large for substitutes, $\frac{1}{1+\rho} \geq 1$, but small for complements, $\frac{1}{1+\rho} \in (0, 1)$. Under substitutability, to make an effective input more expensive implies a sharp decrease in the amount of the effective input, hence a strong technical substitution effect. Under complementarity, the amount of the effective input drops, but given complementarities between the effective inputs, not significantly. The weak technical substitution effect is overridden by the effect upon transformation rates. This results in an increase in the nominal amount. Somewhat paradoxically, with an increase in spill-in and decrease of the effective amount of the domestic input, the *nominal* foreign input is effectively *less important* for the production, consequently demanded at lower amounts. And vice versa, the nominal domestic input is effectively more important for the production. The intuition is that for complementarity, making one complementary input relatively less accessible means that it must be demanded at higher levels to compensate for its insufficiency.

B Stability of T-profile to F-profile

To identify free-riding F-profiles (labeled F in Fig. 1), let Delegate 1 be the free rider and Delegate 2 the provider. The exact characterization of the free-riding F-allocation is at an intersection of the provider's (Delegate 2's) single-input optima, $x^F = x_2^C(y^F), y^F = y_2^C(x^F)$,

namely

$$\left(\frac{x^F}{x^*}, \frac{y^F}{y^*}\right) = \left(\frac{\phi^{\frac{1}{1+\rho}}}{1+\phi^{\frac{1}{1+\rho}}}, \frac{1}{1+\phi^{\frac{1}{1+\rho}}}\right).$$

By entering into the local production function, the output for the free rider (Delegate 1) is

$$g_1^F(x^F, y^F) = \frac{\kappa}{1+\phi^{\frac{1}{1+\rho}}} \left(\frac{\phi^{\frac{\rho}{1+\rho}}}{\phi^{-1}+\phi^{\frac{\rho}{1+\rho}}}\right)^{\frac{1}{\rho}} \frac{\lambda}{p}.$$

An incentive to deviate from T-profile associated with $(\tilde{\lambda}, \tilde{\lambda})$ to F-profile associated with $(\underline{\lambda_1}, \tilde{\lambda})$ is weak enough if

$$\lambda \ln \tilde{g}_1 - p\tilde{y} \geq \lambda \ln g_1^F. \tag{14}$$

This property largely depends on the logarithmic shape of the utility function. Generally, notice that T-profile's disadvantage stems from the fact that *inframarginal* contributions have to be paid. A simple way to relax this disadvantage while keeping all marginal decisions unaffected is to modify the benefit function, for example into $\ln(g_i + b)$. If we substitute $G_i = g_i + b$, then $b \geq 0$ is to be interpreted as the baseline level of output that is provided exogenously. Marginal decisions over the additional amount g_i are equivalent to the marginal decisions over the total output G_i, only substitution applies. Thereby, free-riding associated with F-profile becomes less attractive since cost-saving relative to T-profile is lower. A similar way is to assume that some inputs are provided exogenously. Upon request, we can demonstrate that a deviation to F-profile in the case of baseline levels set to (x^b, y^b) occurs only in an interval around $\kappa = 0.4$. Notice that these baseline levels are the lowest interior equilibrium provisions in all decentralized regimes, hence introduction of maximally these levels implies extra constraints $(x \geq x^b, y \geq y^b)$ that however are not binding in an equilibrium.

C Proofs

Proof of Lemma 1. Without loss of generality, consider Citizen 1. We can eliminate deviations to A-profiles in three steps. First, let H-profile $(x^H, y^H) = (x_1^C(y^H), y_1^C(x^H))$ be the profile constructed from the single-input optima of Citizen 1. In other words, (x^H, y^H) are the amounts of total inputs that Citizen 1 wants to spend if he/she expects that nothing is provided by the other district; H-profile is the best out of all profiles where Citizen 1 bears full costs. Since in A-profiles, Citizen 1 pays all costs, H-profile (at least weakly) dominates all A-profiles.

Second, any A-profile is defined by the single-input optima of Delegate 1 with taste $\lambda_1 \geq \overline{\lambda_1} > \tilde{\lambda} > \lambda$. The Delegate 1 who implements A-profile is therefore a high-valuation delegate,

who implements (x^A, y^A), where $x^A > x^H$ and $y^A > y^H$. Therefore, for Citizen 1, all A-profiles overprovide, and the best out of A-profiles is the minimal one. It is associated with $\lambda_1 = \overline{\lambda_1}$. We can proceed with this profile only.

Third, if Citizen 1 delegates $\lambda_1 = \overline{\lambda_1} - \epsilon$, where $\epsilon > 0$ is very small, he/she must be better off because the level of the output g_1 changes very little (continuous loss), whereas Delegate 2 starts to participate by covering fully input x, and this implies a discrete gain for Citizen 1. Hence, even the best out of A-profiles is worse for Citizen 1 than a T-profile associated with $(\overline{\lambda_1} - \epsilon, \tilde{\lambda})$, and consequently also worse than a T-profile associated with $(\tilde{\lambda}, \tilde{\lambda})$. □

Proof of Lemma 2. To start with, we implicitly characterize the amounts of inputs that maximize joint surplus by means of the following first-order conditions:

$$W^x(\lambda_1, \lambda_2, x, y) := \frac{\partial W}{\partial x} = -p + \frac{\lambda_1}{g_1}\frac{\partial g_1}{\partial x} + \frac{\lambda_2}{g_2}\frac{\partial g_2}{\partial x}$$
$$W^y(\lambda_1, \lambda_2, x, y) := \frac{\partial W}{\partial y} = -p + \frac{\lambda_1}{g_1}\frac{\partial g_1}{\partial y} + \frac{\lambda_2}{g_2}\frac{\partial g_2}{\partial y}$$

For any modification of taste of Delegate 1 from the baseline level $\lambda_1 = \lambda$, i.e. for $\lambda_1 = \lambda + d\lambda_1$, the cooperative centralization must deliver $W^x(\lambda + d\lambda_1, \lambda, x + dx, y + dy) = 0$ and $W^y(\lambda + d\lambda_1, \lambda, x + dx, y + dy) = 0$. Using subscripts for partial derivatives of the implicit functions $W^x(\cdot), W^y(\cdot)$, total differentials yield

$$dW^x = W^x_\lambda d\lambda_1 + W^x_x dx + W^x_y dy = 0,$$
$$dW^y = W^y_\lambda d\lambda_1 + W^y_x dx + W^y_y dy = 0.$$

We re-write the differentials as follows:

$$\frac{dx}{d\lambda_1} = -\frac{W^x_\lambda}{W^x_x} - \frac{W^x_y}{W^x_x}\frac{dy}{d\lambda_1}$$
$$\frac{dy}{d\lambda_1} = -\frac{W^y_\lambda}{W^y_y} - \frac{W^y_x}{W^y_y}\frac{dx}{d\lambda_1}$$

By rearranging, we obtain how the amounts of inputs respond to strategic delegation of Citizen 1,

$$\frac{dx}{d\lambda_1} = \frac{W^y_x W^y_\lambda - W^y_y W^x_\lambda}{W^x_x W^y_y - W^y_x W^x_y},$$
$$\frac{dy}{d\lambda_1} = \frac{W^x_y W^x_\lambda - W^x_x W^y_\lambda}{W^x_x W^y_y - W^y_x W^x_y}.$$

The next step is to apply (1) (marginal products evaluated in symmetry) and obtain

$$(W^x_\lambda, W^y_\lambda) = \left(\frac{1}{x + \phi y}, \frac{\phi}{x + \phi y}\right).$$

Imposing into the equations describing the change in inputs, we derive how the *structure* of inputs varies with a change in taste of Delegate 1,

$$\frac{dx}{dy} = \frac{W_x^y W_\lambda^y - W_y^y W_\lambda^x}{W_y^x W_\lambda^x - W_x^x W_\lambda^y} = \frac{\phi W_y^x - W_y^y}{W_x^y - \phi W_x^x}. \tag{15}$$

In order to express the partial derivatives $W_x^x, W_y^x, W_x^y, W_y^y$, we express a general form and evaluate in symmetry $x = y$, where $g := g_1 = g_2$

$$\left(\frac{\partial g_1}{\partial x}, \frac{\partial g_1}{\partial y}\right) = \frac{g}{x}\left(\frac{1}{1+\phi\ (\frac{x}{y})^\rho}, \frac{\phi\ (\frac{x}{y})^\rho}{1+\phi\ (\frac{x}{y})^\rho}\right)$$

$$\frac{\partial^2 g_1}{\partial x \partial y} = \frac{\partial^2 g_1}{\partial y \partial x} = -\frac{\partial^2 g_1}{\partial x^2} = -\frac{\partial^2 g_1}{\partial y^2} = \frac{g}{x^2} \cdot \frac{\phi\ (1+\rho)}{(1+\phi)^2}$$

It is only a matter of algebra to obtain that for $\lambda_1 = \lambda_2$ and $x = y$

$$W_x^x = W_y^y = \frac{\lambda}{g^2}\left[-\left(\frac{\partial g_1}{\partial x}\right)^2 - \left(\frac{\partial g_2}{\partial x}\right)^2 + g\left(\frac{\partial^2 g_1}{\partial x^2} + \frac{\partial^2 g_2}{\partial x^2}\right)\right] = -\frac{(1+2\phi+\phi^2+2\rho\phi)\lambda}{x^2(1+\phi)^2}$$

$$W_x^y = W_y^x = \frac{\lambda}{g^2}\left[-2\left(\frac{\partial g_1}{\partial x}\frac{\partial g_1}{\partial y}\right) + g\left(\frac{\partial^2 g_1}{\partial x \partial y} + \frac{\partial^2 g_2}{\partial x \partial y}\right)\right] = \frac{2\phi\rho\lambda}{x^2(1+\phi)^2}$$

We plug all partial derivatives into (15) to obtain the level of strategic complementarity in (10). As a final step, we compare (9) and (10) for $\rho > 0$ and by rearranging obtain

$$\Gamma(x,y) = \frac{\rho}{1+\phi+\rho} < \frac{\frac{1}{\phi}+1+2\rho}{\phi+1+2\rho} = \frac{1+\phi+2\phi\rho}{\phi^2+\phi+2\phi\rho} = \psi(x,y). \ \square$$

Proof of Lemma 3. We evaluate (11) at sincere delegation, i.e., if $\hat{\lambda} = \lambda$:

$$\frac{du_1}{dy} = p\left(\sigma - \frac{1}{1+\phi}\right)(1-\psi) \tag{16}$$

For imperfect complementarity and $\kappa \in (0, \frac{1}{2})$, recall that by (10), $\psi(x,y) \in (0,1)$. Therefore, (16) equals zero (i.e., sincere delegation $\hat{\lambda} = \lambda$ is stable to unilateral strategic delegation) if and only if the term in the large brackets is zero. For other cost-division rules, $\sigma \neq \frac{1}{1+\phi}$, the symmetric equilibrium features $\hat{\lambda} \neq \lambda$, hence $\hat{x} = \frac{\hat{\lambda}}{p} \neq \frac{\lambda}{p} = x^*$. $\square$

Proof of Lemma 4. Without delegation, centralization is first-best. With delegation, the best decentralized outcome out of the four regimes is for transfers and delegation, $\tilde{x} = \max\{x^b, \bar{x}, x^t, \tilde{x}\}$. The outcome of the centralized regime is $\hat{x}$. The equilibrium in the decentralized regime (T-profile) is defined by

$$\left.\frac{du_1}{dy}\right|_{x=\tilde{x}} = -p + \frac{\lambda}{g_1}\left(\frac{\partial g_1}{\partial y} + \frac{\partial g_1}{\partial x}\Gamma(x,y)\right) = 0.$$

For centralization, start with $\sigma = 0$. Using $\psi(x, y) > \Gamma(x, y)$, we express the marginal utility of manipulating the input structure at the level corresponding to decentralization, $x = \tilde{x}$, as follows:

$$\left.\frac{du_1}{dy}\right|_{x=\tilde{x}} = -p + \frac{\lambda}{g_1}\left(\frac{\partial g_1}{\partial y} + \frac{\partial g_1}{\partial x}\psi(x, y)\right) = \frac{\lambda}{g_1}\frac{\partial g_1}{\partial x}\left(\psi(x, y) - \Gamma(x, y)\right) > 0$$

The first-order condition in centralization is satisfied only if there are more x-inputs, hence we have $\hat{x} > \tilde{x}$. Now, since since $\hat{x}$ is increasing in σ, and an increase in x up to σ^* must represent additional increase in the inputs and a welfare improvement. Therefore, for any $\sigma \in [0, \sigma^*]$, centralization welfare-dominates decentralization. □

Proof of Proposition 1. We prove each part separately.

1. By Result 5, centralization without delegation is social efficient, whereas by Results 1 and 3, decentralization with or without transfers is inefficient for $\kappa \in (0, \frac{1}{2})$.

2. We always use that the provisions and also welfare differences are continuous in the division rule and spillovers.

   (i) Low division rules: For $\sigma = 0$, we have by Lemma 4 that $\hat{x} > x'$, hence the welfare-difference between centralization and decentralization is positive irrespective of the level of spillovers. Thus, there is a non-empty neighborhood of $\sigma = 0$ where the difference is positive for $\kappa \in (0, 1/2)$.

   (ii) High division rules: We examine the welfare difference in the extreme; once we observe a positive (negative) welfare difference between decentralization and centralization, we use that the difference must be positive (negative) also for admissible values in the neighborhood of the extreme. In specific, in the extreme $(\sigma, \kappa) = (1, 1/2)$, we have in decentralization $\lim_{\kappa \to 1/2} x' = \frac{1+\rho}{2+\rho} < 1$ and in centralization $\lim_{\kappa \to 1/2} \hat{x} = 1$. Thus, for admissible elements in the neighborhood of $(\sigma, \kappa) = (1, 1/2)$, centralization welfare-dominates decentralization. Similarly, in the extreme $(\sigma, \kappa) = (1, 0)$, we have in decentralization $\lim_{\kappa \to 0} x' = \frac{\rho}{1+\rho} > 0$ and in centralization $\lim_{\kappa \to 0} \hat{x} = \infty$, hence its welfare-equivalent satisfies $\lim_{\kappa \to 0} h(\hat{x}) \to 0$. Thus, for admissible elements in the neighborhood of $(\kappa, \sigma) = (1/2, 1)$, decentralization welfare-dominates centralization.

3. First, notice that the provision in decentralization relative to the social optimum (and also welfare in decentralization) decreases from 1 to $\frac{1+\rho}{2+\rho} < 1$. In centralization, we know that provision is first (i) excessive but decreasing, then (ii) insufficient and decreasing, and finally (iii) insufficient and increasing. This means that welfare in centralization (i) grows, (ii) falls and again (iii) grows. By Lemma 4, on Intervals (ii) and (iii), we have $\sigma < \sigma^*$ and therefore centralization dominates decentralization. On Interval

(i), welfare in decentralization is decreasing, and welfare in centralization is increasing. Now, for the lowest spillover that satisfies Interval (i), i.e. $\kappa = 0$, decentralized amount is efficient and weakly-welfare-dominates the centralized amount, and for the highest spillover that satisfies Interval (ii), i.e. $\sigma^* = \sigma$, centralization weakly-welfare-dominates decentralization, there must be a cutoff level K on Interval (i) where the welfare is identical. □

Proof of Corollary 1. Let A is proposition that centralization welfare-dominates decentralization, B is proposition that $\sigma \leq \sigma^*$, and C is proposition that strategic delegation is not allowed. By Lemma 4, $B \Rightarrow A$. By Part (i) of Proposition 1, $C \Rightarrow A$. Therefore, $\neg A \Rightarrow (\neg B \wedge \neg C)$. □

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