

Carlsson, Mikael; Eriksson, Stefan; Gottfries, Nils

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Product Market Imperfections and Employment Dynamics*

Mikael Carlsson^a, Stefan Eriksson^b, and Nils Gottfries^c

3 February 2011 (typo corrected)

How important is imperfect competition in the product market for employment dynamics?

To investigate this, we formulate a theoretical model of employment adjustment with imperfect competition in the product market, search frictions, and convex adjustment costs. From this model, we derive a structural equation for employment that we estimate on firm-level data. We find that product market demand shocks have significant and quantitatively large effects on employment. Thus, product market imperfections seem to be important for employment dynamics. However, we find no evidence that the tightness of the local labor market affects job creation in existing firms.

Keywords: Search-Matching, Job Creation, Unemployment, Labor Demand

JEL classification: E24, J23, J64.

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a) Research Department, Sveriges Riksbank, SE-103 37, Stockholm, Sweden, mikael.carlsson@riksbank.se.

b) Department of Economics, Uppsala University, PO Box 513, SE-751 20, Uppsala, Sweden, stefan.eriksson@nek.uu.se.

c) Department of Economics, Uppsala University, PO Box 513, SE-751 20, Uppsala, Sweden, CESifo and IZA, nils.gottfries@nek.uu.se.

1. Introduction

Current research on labor market dynamics typically relies on some variant of search-matching theory. This theory emphasizes frictions in the labor market but the product market is often assumed to be perfectly competitive. A firm can sell any amount it desires at the going price, so the notion of “product market demand” has no meaning on the microeconomic level. But many firms sell their products in markets, which seem to be characterized by imperfect competition, and this is also the approach taken in New Keynesian macroeconomic models, where the product market is modeled as monopolistically competitive.¹

The nature of competition in the product market is of fundamental importance for employment dynamics. When the product market is characterized by monopolistic competition, not only wages and prices, but also the position of the demand curve, matter for the employment decisions of individual firms. Since so much analysis of the labor market disregards product market imperfections, a natural question is whether imperfect competition matters for employment dynamics. The purpose of this paper is to investigate the importance of imperfect competition and search frictions for employment dynamics using firm-level data.

Starting from a theoretical model of employment with search frictions, convex adjustment costs, and imperfect competition in the product market, we derive a structural equation for employment growth, which we estimate on firm-level data. The equation includes firm-specific measures of real wage cost per worker, product demand, as well as unemployment and vacancies in the local labor market area. Special cases of the model include a search-matching model with perfect competition in the product market, and a labor demand model with adjustment costs. Testable predictions are derived from the model. If imperfect competition in the product market is important, product market demand shocks

¹ Recently, several researchers have incorporated variants of the search-matching model into new Keynesian models with sticky prices; see e.g. Walsh (2005), Krause and Lubik (2007), Christoffel and Kuester (2008), Trigari (2009), and Gertler and Trigari (2009). For a recent review, see Christoffel et al. (2008).

should be important for employment dynamics. Search-matching theory implies that the number hires depends on and the tightness of the local labor market.

The use of firm-level data allows us to separate the effects of different factors in a sharp way, relying on variation in demand and labor market conditions between firms and over time. To minimize the risk of spurious correlations, we estimate a difference-in-difference specification. Some firms export a large fraction of their production while others sell in the domestic market, some firms produce investment goods while others make consumption goods, and labor market developments differ between regions. These differences allow us to evaluate the importance of product demand and labor market tightness for firm-level employment dynamics. The data refers to Sweden in the 1990s.

We find that product demand shocks have statistically significant and quantitatively large effects on employment at the firm level. Based on our theoretical model, we conclude that imperfect competition in the product market is important for employment dynamics. In contrast, we find no evidence that the tightness of the local labor market affects job creation in existing firms. This result may reflect the slack labor market in this period. When job openings attract a large number of applicants, finding workers is less of a problem for firms, so matching frictions play a less important role. This idea has recently been developed theoretically by Michaillat (2010).

The main problem that we face in this investigation is potential simultaneity. Unobserved shocks, which affect employment in many firms, could affect the right-hand side variables in the equation and lead to biased estimates. To eliminate the effects of unobserved *aggregate shocks*, we include time dummies. Since we also have fixed effects, the estimation relies on *changes* of the right hand side variables differing *across* firms (difference-in-difference estimation). To avoid simultaneity arising from *industry-specific shocks*, we do not use industry production to construct our measure of industry demand. Instead, we construct a firm-specific demand variable by weighing together international demand with domestic

aggregate demand components (private consumption, investment, production) using fixed firm- and industry-specific weights. There may also be unobserved *local shocks*, which affect all firms, and hence vacancies and unemployment, in a local labor market area. Therefore, we instrument local unemployment and vacancies by demand and (lagged) price indexes, which reflect the industry structure of the local labor market area.

The paper by Burgess (1993) is similar in spirit to our paper. He estimates a labor demand model, but allows the speed of employment adjustment to depend on labor market tightness. He finds that both product demand factors and labor market tightness affect employment dynamics. Our paper adds evidence on the importance of demand and supply factors for employment dynamics, but differs in important ways. First, our theoretical specification is closer to the textbook search-matching model. Second, we use firm-level data while Burgess used aggregate time series data. Third, the use of firm-level data allows us to deal with simultaneity in a better way by relying on cross-industry and cross-firm variation.

Christiano, Trabandt and Walentin (2009) estimate a macroeconomic model where the hiring cost per hired worker may potentially depend on labor market tightness. They find a similar result as ours: there is no evidence that the hiring cost (per hired worker) depends on labor market tightness. This result, like ours, supports the quadratic hiring cost specification used by Gertler and Trigari (2009).

In Section 2, we derive a structural equation for employment growth. In Section 3, we present the data and discuss identification and estimation issues. The results are presented in Section 4, and Section 5 concludes.

2. Theory and Empirical Specification

In this section, we formulate a model of employment in an individual firm, which allows both supply and demand factors to have direct effects on employment. The model incorporates search frictions, quadratic adjustment costs, and imperfect competition in the product market. It contains a search-matching model with a perfectly competitive goods market and a labor demand model without search frictions as special cases. From this model, we derive a structural employment equation, which we estimate on firm-level data. The model is deliberately kept simple; it serves only to clarify the interpretation of the regression results in Section 4.

2.1 The Theoretical Model

The model is a large-firm version of the standard search-matching model (e.g., Pissarides, 2000). In the simplest search-matching model, firms are identical and hire at most one worker. In reality, firms sell in different markets, and most of them employ many workers. To derive an employment equation that can be implemented empirically on firm-level data, we consider a search-matching model with multi-employee firms facing different market prices. Linear and identical vacancy costs across firms would, under perfect competition, imply that all vacancies were opened by the most profitable firm. To avoid this extreme and counterfactual implication, we assume vacancy costs to be quadratic.

In the theoretical model, we assume that firms and workers are situated in local labor markets and cannot move between them. In a local labor market, indexed n , there is a large number of firms, indexed i . Firms belong to different production sectors and sell in different product markets, so they face different competitors' prices, denoted $P_{i,t}^C$. To keep the model simple, we assume that firms take wages as given and model employment dynamics

conditional on wages.² In the empirical implementation, we instrument for the wage to take account of possible simultaneity.

In each period, the following events take place:

- (i) At the start of a period, firms choose the number of vacancies to open. Firm i opens $V_{i,t}$ vacancies, and incurs total vacancy costs given by $P_{i,t}^C c V_{i,t}^2 / 2$.³
- (ii) Matching of unemployed workers ($U_{n,t}$) and total vacancies ($V_{n,t}$) takes place in the local labor market, indexed n . The matching process between vacancies and unemployment is described by a matching function: $M_{n,t} = \Phi U_{n,t}^\alpha V_{n,t}^{1-\alpha}$ where $M_{n,t}$ is the total number of matches in period t . The probability of filling a vacancy is $Q_{n,t} = M_{n,t} / V_{n,t} = \Phi (U_{n,t} / V_{n,t})^\alpha$.
- (iii) A fraction λ of the previously employed workers leave for exogenous reasons. This fraction is assumed to be sufficiently large that the firm will always open some vacancies.
- (iv) Production takes place with the CRS technology $Y_{i,t} = N_{i,t}$.
- (v) The firm sells its product in a monopolistically competitive market. Demand is determined by the Dixit-Stiglitz demand function $Y_{i,t} = (P_{i,t} / P_{i,t}^C)^{-\eta} D_{i,t}$, where $P_{i,t}$ is the firm's price, $D_{i,t}$ is a firm-specific demand-shifter, and $\eta > 1$. We abstract from nominal price rigidity so the firm adjusts its price until all its output is sold.

Firm i chooses vacancies so as to solve the following profit maximization problem:

² In Sweden, most wages are set in branch-level union contracts and there is evidence of high nominal wage rigidity in the Nordic countries, so wages in a particular period are largely predetermined (Forslund, Gottfries, and Westermark 2008).

³ In order to avoid introducing another price, we assume that the vacancy cost is proportional to the market price. The results are qualitatively unchanged if we assume that the vacancy cost is proportional to the wage.

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[(P_{i,\tau} - W_{i,\tau}) N_{i,\tau} - \frac{c P_{i,\tau}^C}{2} V_{i,\tau}^2 \right]$$

$$s.t. N_{i,\tau} = Q_{n,\tau} V_{i,\tau} + (1-\lambda) N_{i,\tau-1}, \quad (1)$$

$$N_{i,\tau} = \left(\frac{P_{i,\tau}}{P_{i,\tau}^C} \right)^{-\eta} D_{i,\tau}.$$

Deriving the Euler equation, taking a log-linear approximation, solving the resulting difference equation, and using the expression for $Q_{n,t}$ we get an equation for net job growth in firm i (see the Appendix):

$$\begin{aligned} \Delta \hat{n}_{i,t} = & \frac{\kappa_1 Q^2}{c(1-\lambda)N} E_t \left\{ \sum_{\tau=t}^{\infty} \left(\frac{1}{\kappa_2} \right)^{\tau-t} \left[\frac{\eta-1}{\eta^2} \frac{P}{P^C} \hat{d}_{i,\tau} - \frac{W}{P^C} \hat{w}_{i,\tau} \right] \right\} + \frac{2\kappa_1 \alpha \lambda}{1-\lambda} (\hat{u}_{n,t} - \hat{v}_{n,t}) \\ & + \frac{2\kappa_1 \alpha \lambda (1-\kappa_2 \beta (1-\lambda))}{(1-\lambda) \kappa_2} E_t \left\{ \sum_{\tau=t+1}^{\infty} \left(\frac{1}{\kappa_2} \right)^{\tau-t-1} (\hat{u}_{n,\tau} - \hat{v}_{n,\tau}) \right\} - (1-\kappa_1) \hat{n}_{i,t-1}, \end{aligned} \quad (2)$$

where $\kappa_2 \geq 1/(\beta(1-\lambda)) > 1/\beta$ and $\kappa_1 = 1/(\beta\kappa_2) < 1$. Capital letters without time subscripts

denote steady state values and $\hat{n}_{i,t}, \hat{d}_{i,t}, \hat{w}_{i,t}, \hat{u}_{n,t}, \hat{v}_{n,t}$ denote log deviations of

$N_{i,t}, D_{i,t}, W_{i,t} / P_{i,t}^C, U_{n,t}, V_{n,t}$ from their steady-state values. Employment growth in a firm

depends on current and expected future real wage costs $\hat{w}_{i,\tau}$ and demand $\hat{d}_{i,\tau}$. The level of

(local) unemployment has a positive effect on job creation for two reasons. First, high

unemployment means that vacancies are filled more quickly. Second, the firm opens more

vacancies when they are easy to fill. Vacancies have the opposite effect: if there are many

vacancies in the local labor market, job creation in firm i should decrease because it is harder

to find workers (congestion). Note also that expected future labor market tightness has a

positive effect on hiring: if it becomes harder to hire tomorrow, the firm will open more vacancies so as to speed up hiring today. Finally, employment in period $t-1$ is a state variable which has a negative effect on employment growth.

It is instructive to consider two special cases of the model. If there is *perfect competition in the goods market*, $\eta \rightarrow \infty$, $\kappa_2 = 1/(\beta(1-\lambda))$, $\kappa_1 = 1 - \lambda$, and equation (2) becomes:

$$\Delta \hat{n}_{i,t} = -\frac{Q^2}{cN} E_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} (1-\lambda)^{\tau-t} \frac{W}{P^C} \hat{w}_{\tau} \right\} + 2\lambda\alpha(\hat{u}_t - \hat{v}_t) - \lambda \hat{n}_{i,t-1}. \quad (3)$$

With perfect competition in the goods market, there is no role for product demand shocks beyond their effects on market prices, which affect real wage costs. Current labor market tightness is important, but future labor market tightness plays no role in this case. Under perfect competition, the firm can sell any amount it wants at a given price, so hiring in one period is independent of hiring in future periods. One way to see this is to note that, under perfect competition, the firm opens vacancies until the marginal cost of opening one more vacancy is equal to the marginal benefit of opening one more vacancy:

$$cV_{i,t} = Q_{n,t} \sum_{\tau=t}^{\infty} \left[\beta^{\tau-t} (1-\lambda)^{\tau-t} E_t \left(1 - \frac{W_{i,\tau}}{P_{i,\tau}^C} \right) \right]. \quad (4)$$

This equation shows that the optimal level of vacancies is independent of past and expected future employment levels under perfect competition.

Another special case occurs if there are *no search frictions*. In this case we replace $Q_{n,t} = (U_{n,t}/V_{n,t})^\alpha$ by $Q = 1$, and equation (2) is reduced to:

$$\Delta \hat{n}_{i,t} = \frac{\kappa_1}{c(1-\lambda)N} E_t \left\{ \sum_{\tau=t}^{\infty} \left(\frac{1}{\kappa_2} \right)^{\tau-t} \left(\frac{\eta-1}{\eta^2} \frac{P}{P^C} \hat{d}_{\tau} - \frac{W}{P^C} \hat{w}_{\tau} \right) \right\} - (1-\kappa_1) \hat{n}_{i,t-1}. \quad (5)$$

This is a standard labor demand model with adjustment costs, as discussed by Sargent (1979), Nickell (1986), and Hamermesh (1993). Labor market tightness does not affect hiring because the firm can always hire the workers they want. Job growth is determined by product demand conditions and real wage cost. Note, however, that this model is fully consistent with *some* frictions in the labor market. The quadratic adjustment cost may represent delays and costs of hiring, but the key difference compared to the standard search-matching model is that these costs are independent of labor market tightness.

Comparing the two special cases above, we see that real wage cost and lagged employment play a role in both cases. If there is imperfect competition in the goods market, product demand shifts are important. If there are search frictions in the labor market, unemployment and vacancies have direct effects on job creation.

2.2 Empirical Specification

Our baseline empirical specification is:

$$\Delta \ln N_{i,t} = \alpha_i + \beta_1 (\ln W_{i,t} - \ln P_{i,t}^C) + \beta_2 \ln D_{i,t} + \beta_3 \ln U_{n,t} + \beta_4 \ln V_{n,t} + \beta_5 \ln N_{i,t-1} + \varepsilon_{i,t}. \quad (6)$$

Based on our theoretical model, we interpret this equation as a *structural* equation describing employment choice by an individual firm. In the theoretical model above, employment depends on weighted averages of current and expected future real wage costs and demand. Since we have a rather short panel, we simply use current values and include extra lags if

necessary in the empirical specification.⁴ The intercept α_i is firm-specific. General productivity growth and other aggregate factors are captured by time dummies. We have added a shock $\varepsilon_{i,t}$ which represents unobserved variations in cost, demand, job destruction, productivity etc. and which is assumed to be i. i. d with mean zero. Our theoretical model implies $\beta_1 < 0$, $\beta_2 \geq 0$, $\beta_3 \geq 0$, $\beta_4 \leq 0$ and $-1 < \beta_5 < 0$. The coefficient β_2 reflects the importance of imperfect competition in the goods markets, while coefficients β_3 and β_4 reflect the importance of search frictions for employment dynamics.

In the theoretical model above, separations are taken as exogenous. A model with forward-looking quit and layoff decisions would be much more complicated. Empirical studies show that quits are procyclical and more important for separations than layoffs. Hence, total separations have been found to be either procyclical or uncorrelated with the cycle (Akerlof et al., 1988, Hall, 2006). If separations were to depend negatively on unemployment (λ depended negatively on U), this would reinforce the positive effect of unemployment on *net* job creation. A slack labor market would not only make it more profitable to open vacancies, and easier to fill them, but also reduce separations.

⁴ The period in the models is shorter than one year, which is the frequency of the data. Ideally, we would like to have data on higher frequency, but given the high persistence in the explanatory variables, it is unlikely that such data would lead to qualitatively different results.

3. Data and Estimation

In this section, we first describe the data and the construction of the variables in equation (6) from firm-specific, industry-specific, and aggregate variables. We then turn to identification and estimation.

3.1 Data

Firm-specific variables are taken from a firm-level dataset provided by Statistics Sweden and administered by the Swedish central bank, Sveriges Riksbank. This dataset contains annual information for the years 1990-2000 on all Swedish *industrial firms* with 20 or more employees and a sample of smaller firms.⁵ The database is constructed by merging information from several sources: Registry Based Labor Market Statistics (RAMS), Survey Based Statistics for Industrial Plants (*Industristatistiken*), and Survey Based Statistics for Firms (*Finansstatistiken* 1990-1996, *Företagsstatistiken* 1997-2000). Since we want to identify the labor market area where the firm is situated, we consider only firms with a single plant, which did not move during the sample period.

To construct our firm-specific demand index, we use the export share of the firm, which is only available for firms with 50 employees or more. An alternative would be to use the export share for the industry, but this is less appealing because export shares vary substantially between firms within an industry. In the baseline estimation, we use only data for firms for which we have all the relevant information and these constraints limit the sample, leaving us with a sample of 461 ongoing firms. We now explain the construction of the variables which are included in the equation.

Employment $N_{i,t}$ is the average number of workers employed in firm i in year t .

⁵ Unfortunately, a change of data collection methods makes it impossible to extend the sample in time.

Real wage cost is $\ln W_{i,t} - \ln P_{i,t}^C$ where $W_{i,t}$ is the firms' total labor cost per employee (including social insurance fees). A firm-specific competitor price for firm i in industry j is calculated as a weighted average of domestic and foreign prices

$\ln P_{i,t}^C = (1 - \delta_i) \ln P_{j,t}^D + \delta_i \ln P_{j,t}^{IC}$, where δ_i is the average export share over the sample period for firm i and $P_{j,t}^D$ is the industry j producer price index for the domestic market (domestic deliveries plus imports, SNI92 two-digit industry classification). The international competitors' price is calculated as $\ln P_{j,t}^{IC} = \sum_m \omega_{j,m} (\ln E_{m,t} + \ln P_{j,m,t}^F)$, where $\omega_{j,m}$ is the average share of industry j 's exports that went to country m during 1990-1994 ($j \in \{31, 32, \dots, 38\}$, SNI69 industry classification). The countries, indexed m , are Sweden's 13 main trading partners.⁶ The shares are computed using the available trade data for the classification of goods (varusni69) matching the SNI69 industry (production) classification. The competitor product price in foreign currency, $P_{j,m,t}^F$, is the implicit value-added deflator for industry j in country m taken from the OECD industrial database STAN. $E_{m,t}$ is the exchange rate (SEK per country m 's currency) taken from the OECD Annual National Accounts. Throughout, we use fixed weights (export shares, etc.) because time-varying weights may introduce simultaneity due to firm- or industry-specific shocks.

The *demand* variable for firm i in sector j is constructed as

$\ln D_{i,t} = (1 - \delta_i) [\phi_j^C \ln C_t + \phi_j^I \ln I_t + (1 - \phi_j^C - \phi_j^I) \ln Y_t] + \delta_i \ln D_{j,t}^I$, where again δ_i is the firm's average export share, ϕ_j^C is the industry-specific share of output going to final consumption in total domestic use, ϕ_j^I is the corresponding share going to investment, and $1 - \phi_j^C - \phi_j^I$ is the corresponding share used as intermediate goods (SNI92 two-digit industry classification).

These shares are computed as the average value from the 1995 and 2000 Input-Output tables

⁶ That is, Germany, France, Italy, Netherlands, Belgium, United Kingdom, Denmark, United States, Canada, Japan, Norway, Finland, and Austria. These countries absorb about 80 percent of Sweden's exports.

provided by Statistics Sweden. Y , C , and I are all aggregate variables. Y_t is a volume index of industrial production, C_t is real private consumption, and I_t is real private sector gross fixed investment. The international demand component is calculated as $\ln D_{j,t}^I = \sum_m \omega_{j,m} \ln Y_{j,m,t}$, where $Y_{j,m,t}$ is real value-added for industry j in country m taken from the OECD industrial database STAN and used as proxy for industry demand; the weights are defined above.

Unemployment $U_{n,t}$ is defined as the total number of unemployed workers in the local labor market area n at the end of the previous year (in November), as provided by the Swedish Labour Market Board (AMS).⁷ Local labor market areas consist of one or more municipalities and are constructed by Statistics Sweden using commuting patterns. We use the 1993 definition with 109 labor market areas. Johansson and Persson (2000) report that 80-90 percent of hired workers come from the local labor market area.

We have no data on the branch composition of unemployment and we assume that workers can move between sectors. Although some specialized workers may find it difficult to move between sectors, large groups of workers such as secretaries, business administrators, and low-skilled workers are not tied to any particular sector. Provided that a substantial fraction of workers can move between sectors, total unemployment in the local labor market should be a good indicator of the availability of applicants. Also, total unemployment is the standard variable used when estimating matching functions; see Petrongolo and Pissarides, (2001).

Vacancies $V_{n,t}$ in the local labor market area are constructed using monthly vacancy data from the Swedish Labour Market Board (AMS), which measures the number of unfilled

⁷ Since hiring goes on continuously, one may argue that unemployment throughout the year should affect net job creation. As an alternative, we measured unemployment as an average during the year rather than the level at the end of the previous year. This did not affect the results.

vacancies at the start of the month in each local labor market area. We take the average over the year as our measure of vacancies.

The general productivity trend is captured by time dummies. We also construct a firm-specific productivity trend as $\tau_i T_t$ where $\tau_i = (\ln(Y_{i,2000} / N_{i,2000}) - \ln(Y_{i,1990} / N_{i,1990}))$. $Y_{i,t}$ is the firm's real sales and T_t is a linear time trend.

Table 1 shows how firms in the sample are distributed across industries. We see that export shares and the composition of demand vary considerably across industries. *Figure 1* illustrates the severity of the Swedish recession, with investment and consumption falling substantially in 1992-1993, and then recovering quickly. This cycle affected primarily firms selling in the domestic market. *Figure 2* shows that the large depreciation of the currency in 1992-1993 had a substantial effect on real wage cost (competitiveness) in the machine industry, with a 64 percent export share, but did not affect the food industry, with a 11 percent export share, very much. *Figure 3* shows substantial co-movement of unemployment rates across local labor markets, but there is also some cross-section variation. Vacancies show more dispersion across local labor markets (*Figure 4*).

For our difference-in-difference estimation to work, it is important that there is substantial cross-firms variation in the right-hand side variables. To investigate this, we ran a panel regression for each right hand side variable with fixed effects and time dummies as explanatory variables. The standard deviation of the residuals was 0.034 for demand, 0.087 for the wage, 0.095 for unemployment, and 0.258 for vacancies. Since all variables are in logs, these numbers give the approximate percentage variation in each variable across firms and time. Hence there is economically significant variation in all right hand side variables.

Table 1. Industry Distribution of Firms in the Sample. Average Export Share and Share Used for Consumption, Investment, and Intermediate Goods for Each Industry

Industry (SNI92)	Number of firms	Average export share	Consumption	Investment	Intermediate goods
15	22	0.11	0.58	0	0.42
16	0	0.06	1	0	0
17	15	0.63	0.28	0	0.72
18	7	0.79	0.89	0	0.11
19	4	0.73	0.39	0	0.61
20	44	0.52	0.02	0	0.98
21	22	0.65	0.03	0	0.97
22	18	0.04	0.16	0	0.84
23	3	0.50	0.40	0	0.60
24	19	0.69	0.28	0	0.72
25	26	0.52	0.07	0	0.93
26	5	0.30	0.03	0	0.97
27	17	0.54	0.01	0	0.99
28	68	0.28	0.02	0.14	0.84
29	92	0.64	0.02	0.40	0.58
30	4	0.74	0.04	0.80	0.16
31	9	0.70	0.06	0.17	0.77
32	7	0.78	0.04	0.35	0.61
33	12	0.62	0.02	0.49	0.49
34	28	0.66	0.28	0.20	0.52
35	9	0.69	0.12	0	0.88
36	29	0.46	0.38	0.20	0.42
37	1	0	0	0	1

Figure 1: Demand Components

Domestic Consumption, Investment, and Production, and International Demand (the latter is for sni69=38), log changes.

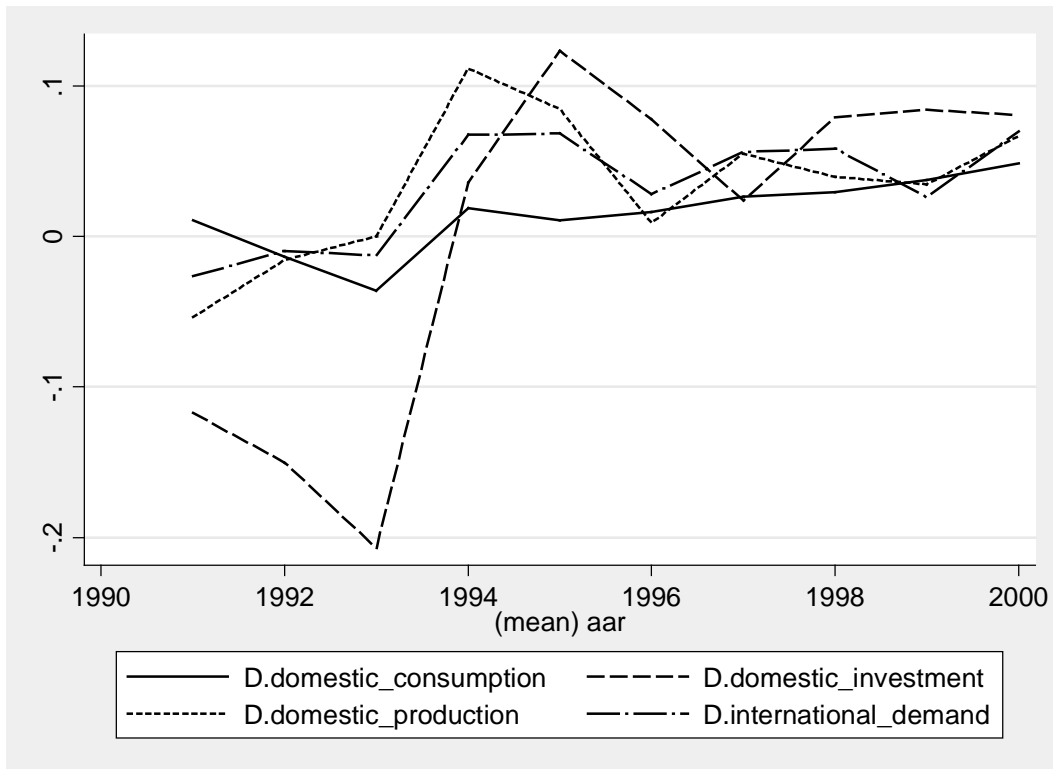


Figure 2: Real Wage Cost

Food (sni92=15) and Machine Industries (sni92=29), log changes.

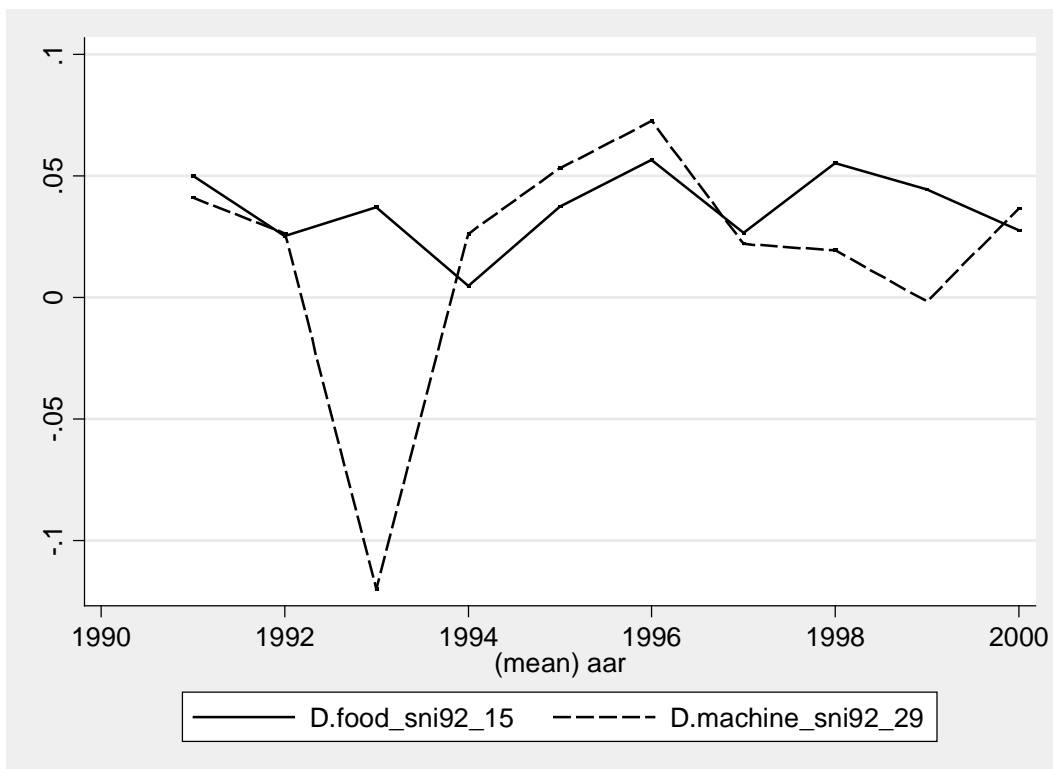
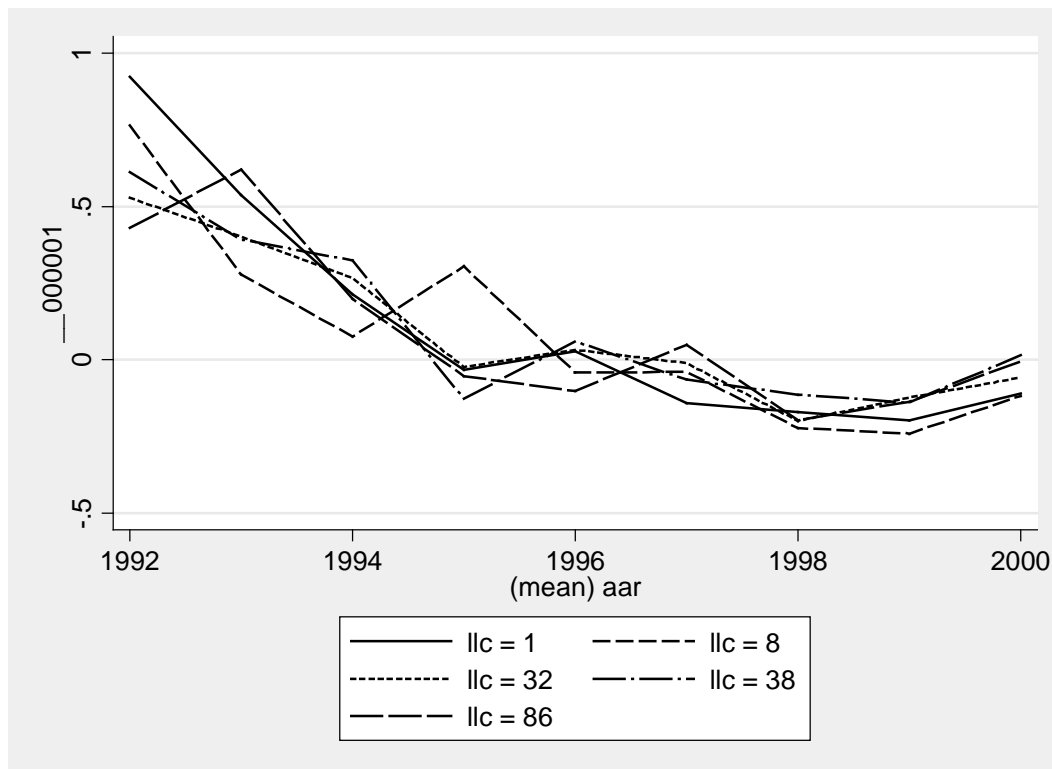
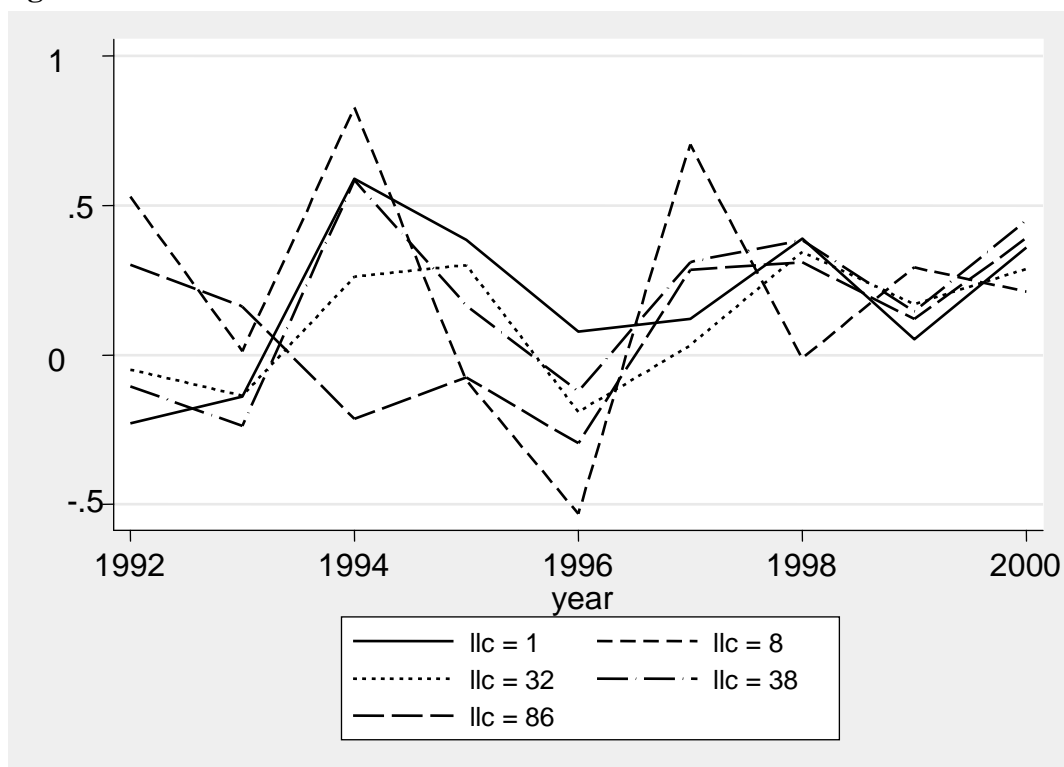


Figure 3: Unemployment for Some Local Labor Markets



Note: Log changes. The local labor market areas used for this illustration are Stockholm (llc=1), Gnosjö (llc=8), Malmö (llc=32), Göteborg (llc=38), and Örnsköldsvik (llc=86).

Figure 4: Vacancies for Some Local Labor Markets



Note: Log changes. The local labor market areas used for this illustration are Stockholm (llc=1), Gnosjö (llc=8), Malmö (llc=32), Göteborg (llc=38), and Örnsköldsvik (llc=86).

3.2 Identification and Estimation

The effects of product demand shocks are identified in our difference-in-difference estimation because firms differ in two ways. First, firms belong to different industries and the demand indexes for different industries have different weights for (aggregate) consumption, investment and production. Second, individual firms have different export shares and thus the weight of foreign (sector-specific) demand differs between firms.

To understand how we can identify the effects of labor market tightness (local unemployment and vacancies) on employment, consider a region A, which depends heavily on the steel industry, and a region B, which has no steel industry. Suppose that there is a downturn in international demand for steel, so that employment in the steel industry falls and unemployment increases particularly strongly in region A. How will this affect hiring by non-steel firms in region A? Clearly, wages in region A will tend to fall, and that this will stimulate firms to hire more workers. But according to standard search-matching theory, there is also a more *direct* effect. As unemployment increases, firms in region A can fill their vacancies more quickly, and this makes it profitable to open more vacancies. If the availability of workers matters for job creation, non steel-producing firms in region A should increase their employment relative to firms in region B, which was less affected by the downturn in steel demand.

Thus, time series variation across industries and regions allows us to estimate a structural employment equation with real wage cost, demand, unemployment, and vacancies as explanatory variables. But can we plausibly identify the parameters of our structural employment equation? The main problem is that unobserved shocks, which affect firm i , may also affect other firms, aggregate demand and wages, as well as unemployment and vacancies in the local labor market area. More specifically, the shock $\varepsilon_{i,t}$ in equation (6) may have *aggregate*, *industry-specific*, and *local* components, all of which could potentially cause correlation between $\varepsilon_{i,t}$ and the right-hand side variables. To eliminate the effects of

unobserved *aggregate shocks*, such as changes in the general business cycle outlook, we include time dummies. Since we also have fixed effects, our identification relies on differences in differences across firms.

An unobserved *industry-specific shock* to the steel industry will not only affect the labor demand of each individual steel producer, but also unemployment in regions with many steel producers. To avoid simultaneity arising from such shocks, we do not use industry production to construct the demand variable. Instead, the demand variable is constructed from aggregate and foreign data as described above. The industry price, which is used to calculate the real wage cost, is instrumented using suitably chosen lags.

There may also be unobserved *local shocks*, such as changes in local government taxes and policies, which affect all firms in the local labor market. To eliminate simultaneity arising from such shocks, we instrument local unemployment and vacancies by demand and (lags of) competitors' price indexes which reflect the industry structure in the local labor market area. A demand variable for the local labor market n is constructed as

$\ln D_{n,t} = \sum_j \kappa_{j,n} \ln D_{j,t}$ where the weights $\kappa_{j,n}$ reflect the local labor market's industry

composition. These weights are constructed using RAMS data on the number of employees in each sector (by local labor market SNI92 two-digit industry classification). Industry demand is calculated in the same way as for individual firms:

$\ln D_{j,t} = (1 - \delta_j)[\phi_j^C \ln C_t + \phi_j^I \ln I_t + (1 - \phi_j^C - \phi_j^I) \ln Y_t] + \delta_j \ln D_{j,t}^I$ where δ_j is now the

industry's average export share. An index of competitors' prices for local producers is

constructed analogously as $\ln P_{n,t}^C = \sum_j \kappa_{j,n} \ln P_{j,t}^C$ where $\ln P_{j,t}^C = (1 - \delta_j) \ln P_{j,t}^D + \delta_j \ln P_{j,t}^{IC}$.

By using instruments for vacancies, we also address the problem of measurement errors in vacancy data. Since many job openings are not officially registered, vacancies are a poor measure of the number of job openings. This would lead to a downward bias if we did not instrument. Wages and prices may be simultaneously determined and there are also

measurement errors in wages because of variation in hours. We use lagged values of the real wage cost variable, $\ln W_{i,t} - \ln P_{i,t}^C$, as instruments for the real wage cost.

Since our empirical specification includes lagged dependent variables as well as fixed effects, we use an Arellano-Bond (1991) estimator.⁸ Fixed effects are eliminated by taking first differences. This procedure introduces an MA(1) process in the residual ($\Delta \varepsilon_{i,t}$), so that the first difference of the lagged dependent variable and the residual are correlated. But provided that $\varepsilon_{i,t}$ is not serially correlated, we can use (suitably chosen) lags of the dependent variable as instruments.

As is generally the case for an asymptotically efficient GMM estimator, the instrument set grows with the number of time periods, but as the lag order increases, lags become less informative as instruments. To avoid including irrelevant instruments and overfitting in the first stage regressions, we do not include the full history of lags. Thus we do not use instruments further back than five years relative to the variable that is to be instrumented. The results are not sensitive to including the full history of lags, however.

Taking account of all the considerations above, we use the following instrument set: $\ln N_{i,t-s}$ where $s = 2, \dots, 5$, $\ln W_{i,t-s} - \ln P_{i,t-s}^C$ where $s=2, \dots, 5$, $\ln P_{n,t-s}^C$ where $s = 2, 3, \dots, 6$, $\ln D_{n,t-s}$ where $s = 0, \dots, 6$. Demand, $\ln D_{i,t}$, and the productivity trend are treated as exogenous.

⁸ We also tried using the System-GMM estimator suggested by Blundell and Bond (1998). However, the Hansen test indicates that the data does not square well with the restrictions imposed on the initial conditions process.

4. Results

When we estimate equation (6) as it stands, the AR(2) test indicates that we have second-order serial dependence in the residuals (see column I in *Table 2*). Our stylized theoretical model may not fully capture the dynamic adjustment, or there may be some omitted variables. By including two additional lags of employment in the regressions, we are able to remove any signs of serial dependence in the residual.⁹ The Hansen test does not reject the joint hypothesis that the model is correctly specified and that the instruments are valid. To find out if the instrument set is relevant, i.e. if the equation is identified, we use the Kleibergen-Paap rk Wald statistic (see Kleibergen and Paap, 2006). This test forcefully rejects the null of under-identification (p-value = 0.003).¹⁰

Column II in *Table 2* shows the estimates for the full model. Real wage costs and product demand shocks have the expected effects on employment, and both coefficients are statistically significant. They are also economically significant: in both cases, the long-run elasticity is close to unity. As explained in the theoretical analysis, product demand shocks would not affect employment if the product market was characterized by perfect competition. In such a case, the product price would be a sufficient description of product market conditions. Based on the theory presented above, we conclude that imperfect competition in the product market is very important for employment dynamics.

In contrast, unemployment and vacancies do not have significant effects on job creation. We find no evidence that unemployed workers contribute to new matches being formed, or that many vacancies in the local labor market make it harder to recruit workers. The results are unchanged if we include labor market tightness, i.e. we impose that the coefficients for unemployment and vacancies are equal but with opposite signs (column III).

⁹ We cannot eliminate the serial dependence mentioned above by adding lags of any other explanatory variable than employment.

¹⁰ Here we use a static instrument set.

In columns IV and V, we show estimates of the two special cases discussed in Section 2, the labor demand model without matching frictions (column IV), and the matching model with perfect competition in the product market (column V). These results confirm the results for the full model.

Obviously, the right-hand side variables are related. Unemployment and vacancies depend on demand and wages, and conversely. This would be a problem if the correlation was so high that coefficients were poorly identified because of multicollinearity. But highly significant coefficients for the wages and demand, and the fact that estimates and standard errors change very little when we leave out some variables, indicate that multicollinearity is unlikely to be a serious problem.

Table 2: Results (Dependent variable: $\Delta \ln N_{i,t}$)

	I	II	III	IV	V	VI	VII
$\ln W_{i,t} - \ln P_{i,t}^C$	-0.190 (0.100)	-0.263** (0.102)	-0.265** (0.103)	-0.270** (0.105)	-0.275** (0.105)	-0.345** (0.109)	-0.178** (0.044)
$\ln W_{i,t-1} - \ln P_{i,t-1}^C$	-	-	-	-	-	0.125 (0.110)	-
$\ln D_{i,t}$	0.215* (0.091)	0.319** (0.101)	0.317** (0.100)	0.302** (0.098)	-	0.459** (0.160)	0.285** (0.059)
$\ln D_{i,t-1}$	-	-	-	-	-	-0.398* (0.159)	-
$\ln U_{n,t}$	0.025 (0.057)	-0.012 (0.057)	0.017 (0.019)	-	-0.018 (0.059)	-0.006 (0.060)	-0.039 (0.027)
$\ln U_{n,t-1}$	-	-	-	-	-	-0.031 (0.053)	-
$\ln V_{n,t}$	-0.027 (0.020)	-0.020 (0.019)	-0.017 (0.019)	-	-0.015 (0.020)	-0.023 (0.018)	-0.007 (0.008)
$\ln V_{n,t-1}$	-	-	-	-	-	0.012 (0.016)	-
$\ln N_{i,t-1}$	-0.174** (0.036)	-0.269** (0.071)	-0.263** (0.070)	-0.264** (0.070)	-0.220** (0.067)	-0.218** (0.083)	-0.211** (0.032)
$\ln N_{i,t-2}$	-	-0.060 (0.036)	-0.061 (0.037)	-0.063 (0.037)	-0.062 (0.037)	-0.080* (0.039)	-0.071* (0.035)
$\ln N_{i,t-3}$	-	0.035 (0.030)	0.036 (0.030)	0.039 (0.030)	0.034 (0.031)	0.028 (0.028)	0.018 (0.024)
Productivity	0.008* (0.004)	0.006 (0.004)	0.006 (0.004)	0.006 (0.004)	0.007 (0.004)	0.005 (0.004)	0.006 (0.003)
Time Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Period	1992-2000	1994-2000	1994-2000	1994-2000	1994-2000	1994-2000	1993-2000
Number of obs.	4149	3227	3227	3227	3227	3227	3688
AR(2) (P-value)	0.010	0.170	0.173	0.197	0.163	0.144	-
Hansen(P-value)	0.144	0.263	0.294	0.301	0.168	0.374	-

Note: The sample is 461 firms. ** and * denote significance at the 1 and 5 percent levels, respectively. Estimation in columns I-VI is performed using the Arellano and Bond (1991) GMM estimator calculated with DPD 1.21 for Ox. Second-step coefficients with robust Windmeijer (2005) finite-sample corrected standard errors in parentheses. AR (2) denotes the p-value for the test of second-order autocorrelation in the first differenced residuals. Hansen denotes the p-value of the joint test of the model specification and instrument validity. The instruments used are: $\ln N_{i,t-s}$ where $s = 2, \dots, 5$, $\ln W_{i,t-s} - \ln P_{i,t-s}^C$ where $s = 2, \dots, 5$, $\ln P_{n,t-s}^C$ where $s = 2, 3, \dots, 6$, $\ln D_{n,t-s}$ where $s = 0, \dots, 6$. $\ln D_{i,t}$ is treated as exogenous and we treat the productivity trend as deterministic. The estimation in column VII is performed using the OLS within estimator, and robust standard errors are in parentheses.

To evaluate the robustness of the results, we have estimated several variants of the model. These are discussed briefly below.

Dynamics: Our theoretical model has relatively simple dynamics. To allow for more complicated dynamics, we estimated the model including one additional lag on real wage cost, demand, unemployment, and vacancies. As seen in Column VI, the only lag that is significant is lagged demand.¹¹

Estimation method: To test if our results are sensitive to the choice of estimation method, we also estimated the model by OLS using a within transformation to handle fixed effects. Column VII shows the results. The results are qualitatively similar but the coefficients for wages and demand are somewhat smaller compared to the GMM estimates. This may be due to measurement errors for wages, or the bias that arises with fixed effects in a short panel.

Sample size: The choice to use firm-specific export shares limited the analysis to firms with more than 50 employees, and the number of firms to 461. If we instead use export shares calculated at the two-digit industry level, we can include smaller firms (down to 10 employees) and the sample size rises to 938 firms. The results remain qualitatively unchanged.

Local interactions: One possible reason why labor market tightness does not have the expected effect on employment may be that firms cater to the local labor market. Suppose that some firms in the local labor market area are hit by negative shocks, so local unemployment increases. If firm i is a sub-contractor to those firms, it will be directly affected by the shock. This may bias the coefficient estimate for unemployment. Put differently, local unemployment may reflect not only the availability of workers, but also demand in the local labor market. Note, however, that we include only firms with more than

¹¹ The coefficient is almost as large as the contemporary effect, but with an opposite sign, implying that employment growth is related to growth of demand. This is evidence of an immediate effect of demand on employment. Customer market theory offers a natural explanation to why effects of demand shocks are fast while price effects take time; see Gottfries (2002) and Lundin, Gottfries, Bucht and Lindström (2009).

50 employees. Firms of this size are likely to sell their products national and international product markets. One way to check whether local interdependencies affect the results is to limit the analysis to firms with a substantial export share. Such firms should be less dependent on the local product market. Estimating equations for firms with export shares above 25 or 50 percent, we still do not find any significant effect of labor market tightness on employment growth.¹²

5. Conclusion

Our main result is that product market demand shocks have statistically significant and quantitatively large effects on employment at the firm level. Imperfect competition in the product market seems to be very important for understanding employment fluctuations.

Another result is that labor market tightness seems to have little effect on job growth at the firm level. Imperfect competition in the product market seems to be more important for employment dynamics than search frictions in the labor market. This result raises doubts about the importance of search frictions, as described by the matching function, for short run employment dynamics. This does not mean that there are no costs associated with finding and recruiting workers, but according to our estimates, those costs are unrelated to labor market tightness.

We should be cautious, however, when drawing conclusions about the labor market in general. First, we have looked at medium size firms. Job creation in small firms, and creation of new firms, may be more sensitive to labor market conditions. Second, the Swedish labor market was very weak in the 1990s, with open unemployment rising to 8 percent, and another 5 percent in labor market programs. In such a situation, firms receive many applicants

¹² In this case, we cannot identify the effect of demand because almost all firms with high export share belong to the same industry. Hence there is hardly any cross-firm variation in the demand variable.

for their job openings, so finding workers is less of a problem. Search frictions may be more important in a tight labor market.

The idea that search frictions are less important in a slack labor market has recently been formalized by Michailat (2010). He develops a model with search frictions, where wages are determined by an empirically calibrated wage schedule. “Rationing unemployment” is defined as the unemployment that would arise (for a given wage schedule) if there were no search frictions; the remaining unemployment is defined as “frictional”. He shows theoretically, and with numerical simulations, that the importance of matching frictions varies with labor market tightness. When unemployment is below 5 percent, all unemployment is frictional. When unemployment is 8 percent, only one quarter of the unemployment is frictional. In such a situation, jobs are filled quickly, the cost per required worker is small, and only a small fraction of unemployment can be ascribed to vacancy costs, and delays in filling vacancies. These results are well in line with ours since the Swedish labor market was very weak in the 1990’s. Using the approach developed here, firm-level data could be used to investigate how the relative importance of search frictions and product demand vary with the state of the labor market. Unfortunately, our dataset cannot be extended to study this issue, but it an interesting avenue for future research.

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Appendix. Derivation of the Job Growth Equation

To simplify notation, we omit the indices for the firm (i) and the local labor market (n). The firm knows the current state of the labor market, so by choosing vacancies, it effectively chooses employment. Inserting the constraints, we get the following maximization problem:

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\left\{ \left(\frac{N_{\tau}}{D_{\tau}} \right)^{-1/\eta} P_{\tau}^C - W_{\tau} \right\} N_{\tau} - \frac{cP_{\tau}^C}{2} \left\{ \frac{N_{\tau} - (1-\lambda)N_{\tau-1}}{Q_{\tau}} \right\}^2 \right].$$

Taking the first-order condition with respect to N_t and dividing by P_t^C , which is assumed to be known by the firm, we get:

$$E_t \left\{ \frac{\eta-1}{\eta} \left(\frac{D_t}{N_t} \right)^{1/\eta} - \frac{W_t}{P_t^C} - c \left(\frac{N_t - (1-\lambda)N_{t-1}}{Q_t} \right) + \beta c \left(\frac{N_{t+1} - (1-\lambda)N_t}{Q_{t+1}} \right) (1-\lambda) \right\} = 0.$$

Log-linearizing we get:

$$\begin{aligned} & \frac{\eta-1}{\eta^2} \left(\frac{D}{N} \right)^{1/\eta} \hat{d}_t - \frac{\eta-1}{\eta^2} \left(\frac{D}{N} \right)^{1/\eta} \hat{n}_t - \frac{W}{P^C} \hat{w}_t - \frac{cN}{Q^2} \hat{n}_t + \frac{c(1-\lambda)N}{Q^2} \hat{n}_{t-1} + \\ & \frac{2c\lambda N}{Q^2} \hat{q}_t + \frac{\beta c(1-\lambda)N}{Q^2} \hat{n}_{t+1} - \frac{\beta c(1-\lambda)^2 N}{Q^2} \hat{n}_t - \frac{2\beta c\lambda(1-\lambda)N}{Q^2} \hat{q}_{t+1} = 0. \end{aligned}$$

To simplify notation, we omit the expectation, taking for granted that future values are expectations. Capital letters denote steady state values and hats denote log deviations from steady state values (\hat{w}_t is the log deviation of W_t / P_t^C). Using the fact that

$(D/N)^{1/\eta} = P/P^C$, this can be rewritten as:

$$\beta \hat{n}_{t+1} + \phi \hat{n}_t + \hat{n}_{t-1} = \frac{Q^2}{c(1-\lambda)N} \left[-\frac{\eta-1}{\eta^2} \frac{P}{P^C} \hat{d}_t + \frac{W}{P^C} \hat{w}_t - \frac{2c\lambda N}{Q^2} \hat{q}_t + \frac{2\beta c\lambda(1-\lambda)N}{Q^2} \hat{q}_{t+1} \right]$$

where $\phi = -\left[\frac{\eta-1}{\eta^2} \frac{P}{P^C} \frac{Q^2}{c(1-\lambda)N} + \frac{1}{(1-\lambda)} + \beta(1-\lambda) \right]$ or, using lag operators:

$$\beta \left[1 + \frac{1}{\beta} \phi L + \frac{1}{\beta} L^2 \right] \hat{n}_{t+1} = -\frac{\eta-1}{\eta^2} \frac{P}{P^C} \frac{Q^2}{c(1-\lambda)N} \hat{d}_t + \frac{Q^2}{c(1-\lambda)N} \frac{W}{P^C} \hat{w}_t - \frac{2\lambda}{(1-\lambda)} [\hat{q}_t - \beta(1-\lambda)\hat{q}_{t+1}].$$

The same condition holds (in expectation) for future periods. Solving for $\hat{n}_{i,t}$ we get:

$$\hat{n}_t = \kappa_1 \hat{n}_{t-1} + \kappa_1 \sum_{\tau=t}^{\infty} \left(\frac{1}{\kappa_2} \right)^{\tau-t} \left(\frac{\eta-1}{\eta^2} \frac{P}{P^C} \frac{Q^2}{c(1-\lambda)N} \hat{d}_\tau - \frac{W}{P^C} \frac{Q^2}{c(1-\lambda)N} \hat{w}_\tau + \frac{2\lambda}{(1-\lambda)} [\hat{q}_\tau - \beta(1-\lambda)\hat{q}_{\tau+1}] \right),$$

where κ_1 and κ_2 are given by: $\kappa = \frac{\phi}{2\beta} \pm \sqrt{\frac{\phi^2}{4\beta^2} - \frac{1}{\beta}}$. This can be rewritten as:

$$\hat{n}_t = \kappa_1 \hat{n}_{t-1} + \frac{\kappa_1 Q^2}{c(1-\lambda)N} \sum_{\tau=t}^{\infty} \left\{ \left(\frac{1}{\kappa_2} \right)^{\tau-t} \left(\frac{\eta-1}{\eta^2} \frac{P}{P^C} \hat{d}_\tau - \frac{W}{P^C} \hat{w}_\tau \right) \right\} + \frac{2\kappa_1 \lambda}{1-\lambda} \hat{q}_t + \frac{2\kappa_1 \lambda (1-\kappa_2 \beta (1-\lambda))}{(1-\lambda) \kappa_2} \sum_{\tau=t+1}^{\infty} \left(\frac{1}{\kappa_2} \right)^{\tau-t-1} \hat{q}_\tau,$$

where $\kappa_2 \geq 1/(\beta(1-\lambda))$. Log-linearization of $Q_t = \Phi(U_t/V_t)^\alpha$ yields $\hat{q}_\tau = \alpha(\hat{u}_\tau - \hat{v}_\tau)$.

Inserting this, we get the equation in the text.