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Exchange Rates and Interest Rates in an Economy with Regime Shifts and Sticky Prices

Hans Dillén and Hans Lindberg⁰
Sveriges Riksbank, September, 1998

Abstract

This paper examines how expectations of a monetary regime shift to a high inflation regime may influence interest and exchange rates in an economy with sticky prices. A country in which monetary policy lacks credibility will experience an upward bias in inflation expectations and long term interest rates, but also a weak exchange rate. Furthermore, a shift to an inflationary monetary regime will be accompanied by a jump in the exchange rate. Expectations of such a jump will put an upward pressure on short-term interest rates in the economy. The effects on prices and financial variables are formalized as regime shift premia that are explicitly characterized.

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The views in this paper are those of the authors and do not necessarily reflect those of Sveriges Riksbank.

1. Introduction

It is generally recognized that short run movements of exchange rates mainly are driven by financial factors, e.g. interest rates and monetary policy actions. It is, however, non-trivial to relate actual short run movements in exchange rates to mechanisms found in theoretical models. The problem of finding empirical support for uncovered interest parity (UIP), which is a cornerstone in many theoretical models, illustrates this. In recent years it has been observed that countries with poor record of inflation and vulnerable fiscal positions have experienced a weak and unstable currency accompanied with high and volatile nominal long term interest rates.¹ In the public debate it is often argued that such developments reflect imperfect credibility for the economic policy. Consequently, a fragile confidence for the future developments of the economy will tend to induce a positive correlation between the exchange rate and the interest rate differential, which appears to contradict UIP.²

Some partial explanations to these patterns have been provided in the literature. For instance, Bertola and Svensson (1993) show that a fluctuating devaluation risk can explain a positive correlation between interest rate differentials and the exchange rate in a target zone exchange rate regime, but it is not clear how to carry over the analysis to a free floating exchange rate regime and the model cannot explain the volatility of long term interest rates. On the other hand, Dillén (1997) and Dillén and Hopkins (1998) show that regime shifts models, in which imperfect credibility is represented by investors' fears that the economy will switch to a high inflation regime, can explain high and volatile long-term interest rates. However, these regime shift models do not incorporate exchange rate effects.

The aim of this paper is to complement previous studies by examining the effects of monetary regime shifts in a free floating exchange regime within a sticky price framework and in particular to analyze the behavior of interest rates and exchange rates in such an environment. Moreover, the presence of exchange rate effects introduces additional term structure effects that cannot be found in previous interest rate models with regime shifts, e.g. Dillén (1997), Dillén and Hopkins (1998), and Evans and Lewis (1995).

Regime shifts have been analyzed earlier in the exchange rate literature, e.g. see Froot and Obstfeld (1991). However, we prefer to view our framework as a modification of

¹ See Inflation Report 1998:2, Sveriges Riksbank for a discussion of how credibility problems have affected the exchange rate and the long term interest rate in Sweden.

² Throughout the paper the exchange rate is defined as the price of one unit of foreign currency in terms of domestic currency units. With this definition UIP normally implies a negative correlation between the exchange rate and the interest rate differential, i.e. a shock that temporarily weakens the domestic currency (an increase of the exchange rate) leads to corrective appreciation expectation which tends to lower domestic interest rates relative foreign interest rates according to UIP.

exchange rate models with price inertia such as the models of Dornbush (1976) and Miller and Weller (1991). The modifications are: (i) The proposed model considers changes in the rate of money growth rather than a change in the money stock as in the Dornbush (1976) model. (ii) Agents have fluctuating expectations of (drastic) monetary shocks, which affects exchange and interest rates even if a shock does not occur. Both these modifications seem to be steps towards increased realism.

The outline of the paper is as follows. Section 2 presents a structural monetary model with trends in money and prices. Section 3 allows for switching money supply regimes and incredible inflation targets with a focus on the implications for the exchange rate. Section 4 elaborates on the term structure effects of the model. The interaction between interest rate differentials and the exchange rate is analyzed in section 5. Section 6 concludes.

2. A structural monetary model

The model is an extension of the structural model of Miller and Weller (1991). The economy is described by the following set of equations:

$$y = -\gamma(i - \pi) + \eta(s + p^* - p), \quad \gamma, \eta > 0 \quad (1)$$

$$m - p = \kappa y - \lambda i, \quad \kappa, \lambda > 0 \quad (2)$$

$$ds = (i - i^*)dt + du_s \quad (3)$$

$$dp = \bar{\pi} dt + \phi(y - \bar{y})dt + du_p, \quad du_p = \sigma dW_p, \quad \phi, \sigma > 0, \quad (4)$$

where y, \bar{y}, p, p^*, s , denotes logarithms of output, potential output, the home price level, the foreign price level and the nominal exchange rate (price of one unit of foreign currency in terms of domestic currency units), respectively, i and i^* are the home and foreign nominal short term interest rates, $\bar{\pi}$ denotes the core inflation of the economy, du_s and du_p are exchange rate and supply shocks. The shock to the price level follows a Wiener process dW_p with an instantaneous standard deviation equal to σ . The exchange rate shock, du_s , will typically be a function of the supply shock, du_p , but it may also include components that make the exchange rate jump. This means that the exchange rate might jump when new important information arrives, whereas the adjustment of prices is sluggish (i.e. the price level does not jump) due to price inertia.

Equation (1) is an IS-curve and shows that output increases when the real interest rate declines or when the real exchange rate depreciates. Equation (2) is a LM-curve and equation (3) is the uncovered interest parity condition. Equation (4) describes a Phillips-curve, which shows that inflation above the core inflation occurs when actual output exceeds potential output. The core inflation of the economy is determined by the money supply policy rule, which is

$$m = \bar{\pi} t, \quad (5)$$

where $\bar{\pi}$ is the money growth rate. Hence, the long-term inflation target of the central bank is $\bar{\pi}$. Equations (1) to (4) imply that

$$\begin{pmatrix} dp \\ ds \end{pmatrix} = A \begin{pmatrix} p \\ s \end{pmatrix} dt + \frac{1}{\Delta} \begin{pmatrix} \phi\gamma \\ \phi\gamma - 1 \end{pmatrix} m dt + \frac{1}{\Delta} \begin{pmatrix} \kappa\gamma + \lambda \\ \kappa\gamma \end{pmatrix} \bar{\pi} dt + \frac{1}{\Delta} \begin{pmatrix} g_p \\ g_s \end{pmatrix} dt + \begin{pmatrix} du_p \\ du_s \end{pmatrix} \quad (6)$$

where

$$A = \frac{1}{\Delta} \begin{pmatrix} -\phi(\gamma + \lambda\eta) & \phi\lambda\eta \\ 1 - \kappa\eta - \phi\gamma & \kappa\eta \end{pmatrix}, \quad \begin{pmatrix} g_p \\ g_s \end{pmatrix} = \begin{pmatrix} -\phi\bar{y}[\kappa\gamma + \lambda] + \lambda\phi\eta p^* \\ -\phi\bar{y}\kappa\gamma + \kappa\eta p^* - \Delta i^* \end{pmatrix}$$

and $\Delta = \kappa\gamma + \lambda - \phi\gamma\lambda$.

The variables p and s are state variables, g_p and g_s are functions of exogenous variables whereas the money supply variable m is to be considered as a policy variable. Thus, the money growth rate may change over time and the pattern of the state variables, p and s , when the money supply rule changes is a main concern of this paper. In the special case of a constant money supply rule ($\bar{\pi} = 0$) we want the system to have the saddle point property as in Miller and Weller (1991), which is guaranteed by the assumption that $\Delta > 0$. We will without loss of generality henceforth set the exogenous variables, g_p and g_s , equal to zero.

A complicated feature of system (6) is that the dynamics of p and s interact. Therefore, we transform the system by replacing the price level, p , by a new state variable, f , of the form

$$f = p + \omega s \quad (7)$$

and choose ω such that f and s can be determined recursively. Given solutions for f and s the price level is easily obtained as $p = f - \omega s$. To be more specific, by choosing ω to be the solution to the quadratic equation

$$Q(\omega) \equiv \omega^2 - \frac{1}{\theta} [\kappa\eta + \phi\gamma + \phi\lambda\eta]\omega - \frac{1}{\theta} \phi\lambda\eta = 0, \quad \theta = 1 - \kappa\eta - \phi\gamma \quad (8)$$

the dynamics of f and s become

$$\begin{pmatrix} df \\ ds \end{pmatrix} = C \begin{pmatrix} f \\ s \end{pmatrix} dt + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} m dt + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \bar{\pi} dt + \begin{pmatrix} du_f \\ du_s \end{pmatrix} \quad (9)$$

where

$$C = \begin{pmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} \omega\theta - \phi(\gamma + \lambda\eta) & 0 \\ \theta & \kappa\eta - \omega\theta \end{pmatrix},$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} \phi\gamma + \omega(\phi\gamma - 1) \\ \phi\gamma - 1 \end{pmatrix}, \quad \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} (1 + \omega)\kappa\gamma + \lambda \\ \kappa\gamma \end{pmatrix}$$

and where $u_f = u_p + \omega u_s$. Furthermore, we assume that $\theta > 0$, which Miller and Weller (1991) refer to as the "overshooting" case. Given this assumption it is straightforward to show that the two roots satisfying (8) are real and of opposite signs, where the negative root is greater than -1. By choosing the negative root in (8) we see that

$$c_{11} < 0 \text{ and } c_{22} > 0 \quad (10)$$

The point of this transformation is that the dynamics of f often can be determined independently of the exchange rate, s , since $c_{12} = 0$. Given f we see from (9) that the exchange rate satisfies the equation

$$s = h + \mu E[ds]/dt \quad (11)$$

where

$$h = -[\theta f + (\phi\gamma - 1)m + \kappa\gamma\bar{\pi}] / [\kappa\eta - \omega\theta] \quad (12)$$

and $\mu = 1/c_{22} = \Delta / [\kappa\eta - \omega\theta] > 0$. It is well known that a bubble free solution to (11) is given by³

$$s(t) = \frac{1}{\mu} E_t \left[\int_t^\infty e^{(u-t)} h(u) du \right] \quad (13)$$

First we consider a constant money supply growth, $\bar{\pi}$, in which case (see appendix A) the price level and the exchange rate are given by

$$p = \hat{p} + \bar{\pi} t \quad (14)$$

and

³ It should be noticed that expression (13) is more complicated than similar expressions obtained in the target zone literature since the function h might be very complex. For instance, it is not generally possible to assume a specific stochastic process for h without considering the exchange rate, s , and (13) is in fact an *equation* for the exchange rate rather than a formula. By guessing the nature of the exchange rate process it is possible to solve this equation by first characterizing the process h and then solve (13) as demonstrated in the appendix.

$$s = \hat{s} + \bar{\pi} t \quad (15)$$

where \hat{p} and \hat{s} follow Ornstein-Uhlenbeck processes according to

$$d\hat{p} = \rho[\hat{p}_0 - \hat{p}]dt + \sigma dW_p, \quad \hat{p}_0 = \lambda \bar{\pi} \quad (16)$$

$$d\hat{s} = \rho[\hat{s}_0 - \hat{s}]dt + \sigma_s dW_s, \quad \hat{s}_0 = \lambda \bar{\pi} \quad (17)$$

where

$$\rho = -c_{11} > 0, \quad (18)$$

and

$$\sigma_s = \frac{\sigma}{\omega z} < 0, \quad z = \frac{\phi \lambda \eta}{\omega^2 \theta} > 0 \quad (19)$$

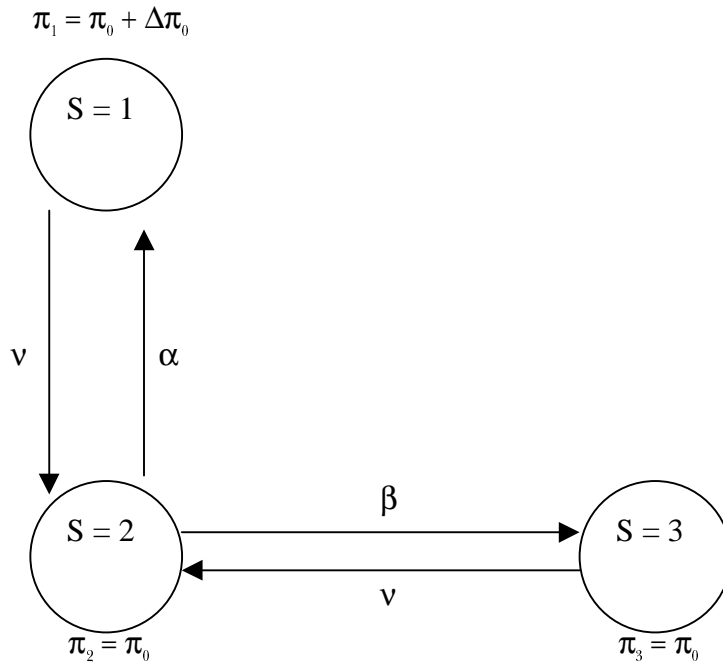
We see from (19) that, as in Miller and Weller (1991), the exchange rate and the price level are negatively correlated because of the overshooting of the exchange rate. For instance, a positive price shock increases interest rates, which makes home assets more attractive. This will result in an incipient capital inflow, and thus cause the exchange rate to appreciate. The extent of that appreciation has to be sufficient to give rise to depreciation expectations at just sufficient rate to offset the increased home interest rate. The increased interest rate and the stronger exchange rate will together lead to a decline in output, and hence to a negative output gap. This negative output gap will put a downward pressure on prices and the price level will eventually return to its equilibrium path, which is determined by the money growth rate in the economy. Both the exchange rate and interest rate will then be back in equilibrium.

3. Switching money supply regimes and imperfectly credible inflation targets

The inclusion of a constant trend in the money supply to the Miller and Weller (1991) model is a quite innocent extension that does not add any particular interesting economic mechanisms to the analysis. However, if the growth rate of the money supply may change, a change that represents a new monetary regime, then expectations of such a regime shift will affect expected future inflation and depreciation rates. It is particularly interesting to analyze an imperfectly credible inflation target, i.e., there are expectations of a shift to a more expansionary regime, in which the money growth rate as well as the core inflation rate is higher. The exchange rate will also tend to depreciate at a higher rate if such a regime occurs, but the role of expectations for the exchange rate is somewhat different. For instance, if a shift to an expansionary regime of the kind just mentioned occurs then this will cause a drastic revision of expected future money growth rates and hence an immediate depreciation of the domestic currency according to (13). Moreover, in light of

the presumption of sticky prices, an imperfectly credible inflation target will generate depreciation expectations of the exchange rate, which in turn lead to higher short term interest rates. It should be clear that this setup also is relevant as a description of the inflation process and inflation expectations in economies without explicit inflation targets. For instance, Evans and Wachtel (1993) found clear evidence of different inflation regimes in US. Moreover, the inflation scarce problems in US discussed by Goodfriend (1993) are very similar to the credibility problems that arise from expectations of a high inflation regime in this set-up.

Figure 1. A high/low inflation switching model



We will analyze the consequences on the exchange and interest rate of an imperfectly credible inflation target by adopting the regime shift model suggested by Dillén and Hopkins (1998). The economy is assumed to be in either of three different states. The state follows a continuous Markov chain ($S = 1, 2$ or 3) with switching probabilities (α, β and v), see figure 1. The money growth rate will henceforth be denoted as $\bar{\pi} = \pi_i$, where the subscript indicates the state.⁴ States 2 and 3 represent low inflation regimes of low and high credibility respectively. The money growth rate (and thus the average inflation rate) π_0 in these regimes can be interpreted as an inflation target. The regime 2 lacks credibility

⁴ If one wants to emphasize that the state is a random variable (for instance one considers a random state in the future) the subindex will be the random variable S .

in that the economy can switch to a high inflation regime, in which the money supply growth and hence the average inflation rate will increase with $\Delta\pi_0$. The motive of state 3 is mainly to have the possibility to consider credibility shocks, i.e. switches between regime 2 and 3. In what follows we will mainly focus the analysis on regime 2 in order to save space. Given this set-up it is shown in the appendix that the price level and the exchange rate can be written in the form

$$p = \hat{p}_i + J_i(t) \quad (20)$$

$$s = \hat{s}_i + J_i(t) + \Gamma_i, \quad i = 1, 2, 3 \quad (21)$$

where a hat indicates an Ornstein-Uhlenbeck process (with continuous sample path), $J_i(t)$ is a trend component of the form

$$J_i(t) = \pi_0 t + \Delta\pi_0 \int_0^t \chi_1(S(u)) du, \quad S(t) = i, \quad (22)$$

where we have introduced the indicator function

$$\chi_1(x) = \begin{cases} 1 & \text{if } x=1 \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

and where Γ_i is the jump component (with discontinuous sample path). It is shown in the appendix that the jump components are given by

$$\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} = \frac{\Phi \Delta\pi_0}{(1+\mu\nu)(1+\mu\phi)} \begin{pmatrix} 1+\mu\phi+\mu^2\alpha\nu \\ (1+\mu\nu)\mu\alpha \\ \mu^2\alpha\nu \end{pmatrix} \quad (24)$$

where

$$\phi = \alpha + \beta + \nu, \quad \Phi = \lambda \left(1 - \frac{1}{\omega z} \right) \quad (25)$$

are expressions frequently used in the following analysis. For future reference we notice that the expected rate of change of the jump component, $q_i \equiv E[d\Gamma_i]/dt$, can be written as

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \nu(\Gamma_2 - \Gamma_1) \\ \alpha(\Gamma_1 - \Gamma_2) + \beta(\Gamma_3 - \Gamma_2) \\ \nu(\Gamma_2 - \Gamma_3) \end{pmatrix} = \frac{\Phi \Delta\pi_0}{(1+\mu\nu)(1+\mu\phi)} \begin{pmatrix} \mu\alpha\nu - (1+\mu\phi)\nu \\ \alpha(1+\mu\nu) \\ \mu\alpha\nu \end{pmatrix} \quad (26)$$

Moreover, the Ornstein-Uhlenbeck processes, \hat{p}_i and \hat{s}_i , are of the form

$$d\hat{p}_i = \rho[\hat{p}_{0i} - \hat{p}_i]dt + \sigma dW_p, \quad \hat{p}_{0i} = \lambda\pi_i + \Lambda_i^p \quad (27)$$

and

$$d\hat{s}_i = \rho[\hat{s}_{0i} - \hat{s}_i]dt + \sigma_s dW_p, \quad \hat{s}_{0i} = \lambda\pi_i + \psi_i \quad (28)$$

where

$$\Lambda_i^p = \frac{\omega z(\Phi\Delta\pi_0\chi_1(i) - (q_i/\rho + \Gamma_i))}{1+z}, \quad \psi_i = -\frac{(z\Phi\Delta\pi_0\chi_1(i) + q_i/\rho + \Gamma_i)}{1+z} \quad (29)$$

Λ_i^p can be interpreted as the regime shift premium of the price level and inspection of (29) reveals that $\Lambda_2^p > \Lambda_3^p > 0 > \Lambda_1^p$. Hence, the regime shift premium of the price level is positive in a low inflation regime ($i = 2$ or 3) and the premium is higher in the regime that lacks credibility ($i = 2$) because there is probability of an immediate shift to the high inflation regime. However, in the high inflation regime the premium is negative due to expectations of shifts to the low inflation regimes sometime in the future.

It is worth emphasizing that there is a degree of arbitrariness how to decompose deviations from the trend into an Ornstein-Uhlenbeck process and a jump component. We have chosen to define the jump component in such way that it takes the value zero in the case of perfect credibility of the low inflation regimes ($\alpha = 0$). Thus, to obtain the total effect of the regime shift expectations on the exchange rate (the regime shift premium of the exchange rate) we have to add the jump component to ψ_i , i.e. $\Lambda_i^s = \psi_i + \Gamma_i$. Especially we have that the regime shift premium of the exchange rate in the low inflation regime of low credibility is of the form

$$\Lambda_2^s = \frac{\alpha\Phi\Delta\pi_0[\mu\rho z - 1]}{(1+z)(1+\mu\phi)\rho} > 0 \quad (30)$$

which can be shown to be positive.⁵

The change of the mean reverting level of the Ornstein-Uhlenbeck component when the economy shifts to the high inflation regime can be shown to be

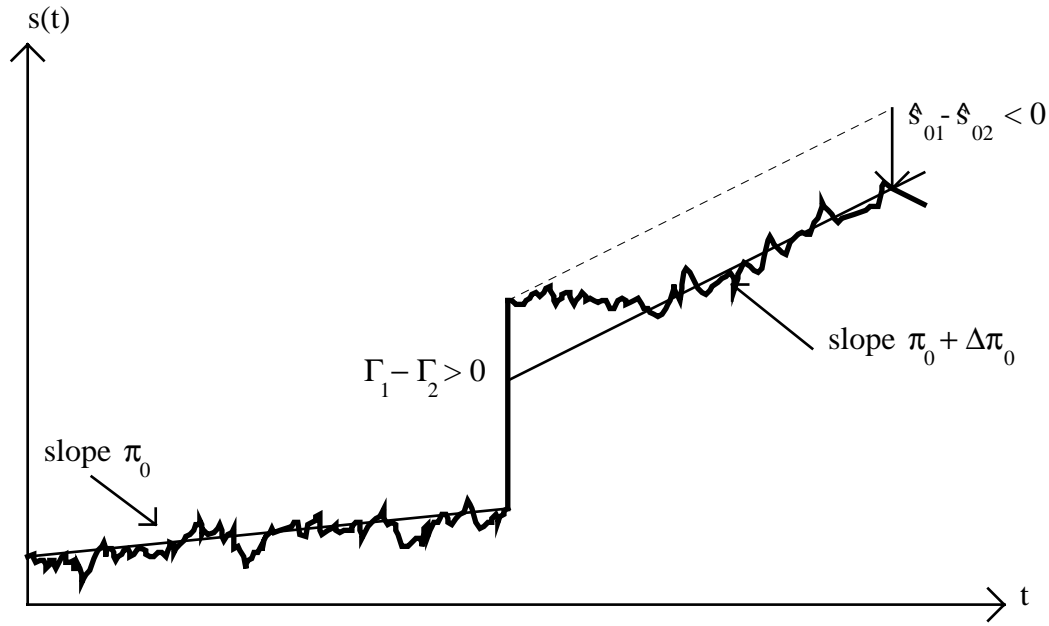
$$\hat{s}_{01} - \hat{s}_{02} = \frac{\Delta\pi_0}{(1+z)(1+\mu\phi)\rho} \left\{ \frac{\lambda(1+z)}{\omega z} \left(\rho + \frac{\phi\gamma}{\kappa\eta - \theta\omega} \right) - \frac{\Phi\beta(1+\mu\rho)}{1+\mu\nu} \right\} < 0 \quad (31)$$

and the negative sign implies that the exchange rate overshoots its long run trend in regime 1. Graphically the overshooting of the exchange rate is displayed in figure 2. Since the Ornstein-Uhlenbeck component does not jump, the downward shift of the mean reversion level implies that the appreciating tendency of the Ornstein-Uhlenbeck component increases (or that the depreciating tendency decreases). Expectations of such an

⁵ It can also be shown that the regime shift premium of the exchange rate is positive in regime 3 as well, but negative in regime 1.

appreciating scenario will give rise to a downward sloping effect on the yield curve in regime 2. This effect is counterbalanced by the effect that expectations of a higher inflation rate will give rise to. The latter effect increases with the term as in the model of Dillén (1997), and it constitutes the dominating effect on long-term yields, but for short maturity yields the effect is ambiguous as we will see in the next section.

Figure 2. Exchange rate effects of a switch to the high inflation regime

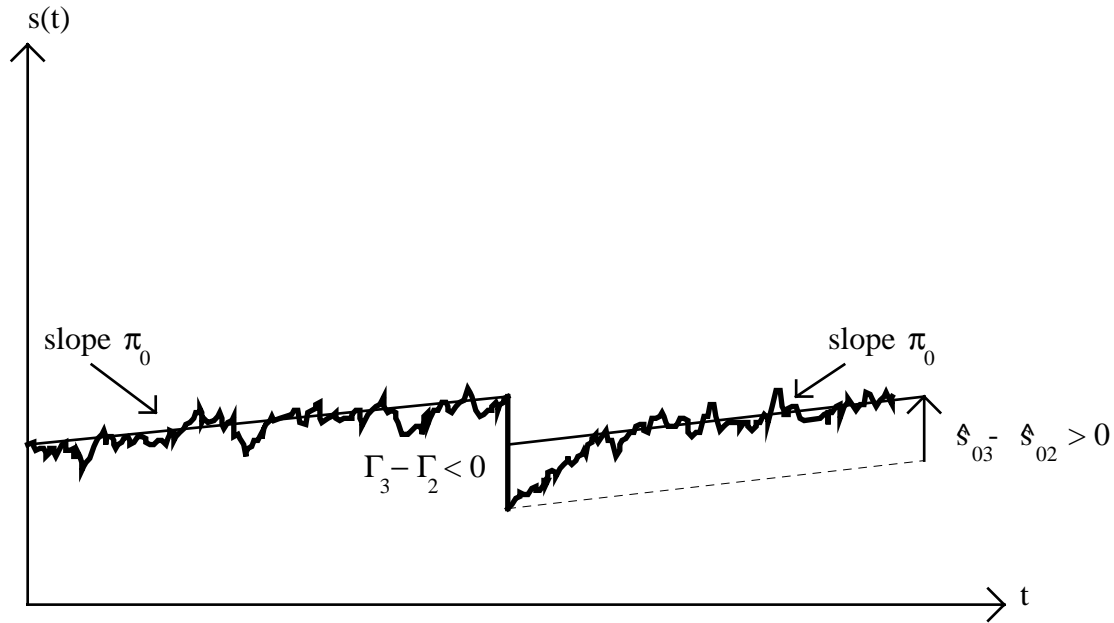


It is also interesting to analyze a shift to regime 3, which is illustrated in figure 3. This regime shift can be viewed as a positive credibility shock in the sense that the probability of leaving the low inflation policy (switch to the high inflation regime) is smaller in regime 3. Credibility shocks of this type will not only appreciate the domestic currency, but also squeeze depreciation expectations (thanks to reduced fears of switching to the high inflation regime), which in turn lowers interest rates. Thus, fluctuations in credibility in this manner provide an explanation why the exchange rate might be positively correlated with the interest spread relative the foreign country.⁶ Notice, however, that the domestic currency overshoots ($\hat{s}_{03} - \hat{s}_{02} > 0$) in an appreciating direction in this case, implying a depreciating tendency after a positive credibility shock. The reason for this overshooting can be explained as follows. When credibility improves there is a decline in

⁶ Fluctuations in the expected depreciation rate are the general mechanism why the exchange rate may be positively correlated with the interest rates differential). For instance, in the model of Bertola and Svensson (1993) this is modeled as fluctuating devaluation expectations in a target zone system. In our model fluctuating regime shift expectations explain why the expected depreciation rate fluctuates in a floating exchange rate system.

depreciation expectations originating from the risk of a more expansionary policy in the future, which indicates that interest rates will fall. Prices will also decline, since the regime risk premium of the price level is lower in the credible regime. However, prices adjust slowly and some depreciation expectations within the regime are therefore needed to keep the short term interest rate on a level that is sufficient to clear the money market during the adjustment. In the next section we will study term structure effects of the model more closely.

Figure 3. Exchange rate effects of a positive credibility shock



4. Expected depreciation of the exchange rate and term structure effects

Expectations of regime shifts heavily affect the expected depreciation of the domestic currency, which in turn has important implications for the term structure of interest rates. In the appendix the expressions for the expected depreciation in all regimes can be found, but in order to save space we will mainly focus on the effects on forward interest rates in the low inflation regime of low credibility (regime 2). The reason for focusing on forward interest rates, apart from that this simplifies the analysis, is that forward interest rates reflect expected future economic conditions and the expected future short term interest rate in particular, which makes the interpretation of the result more straightforward. Assuming π_0 equals zero the expected depreciation in regime 2 during a period of length τ can be shown to be

$$D_2(\tau) = -\hat{s}_2(t) (1-e^{-\rho\tau}) + \frac{\alpha\Delta\pi_0}{\phi} X(\tau) \quad (32)$$

where

$$X(\tau) = \tau - \frac{1-e^{-\varphi\tau}}{\varphi} + \frac{\lambda}{\omega z} B_\varphi(\tau) - \frac{\Phi(1+\mu\rho)\varphi}{(1+z)(1+\mu\varphi)\rho} L(\varphi, \tau) + \frac{\Phi(1-e^{-\varphi\tau})}{1+\mu\varphi} \quad (33)$$

and where $B_\varphi(\tau)$ and $L(\varphi, \tau)$ are positive functions of τ

$$B_\varphi(\tau) = \frac{\rho(1-e^{-\varphi\tau}) - \varphi(1-e^{-\rho\tau})}{\rho - \varphi} \quad (34)$$

$$L(\varphi, \tau) = \frac{\rho}{\rho - \varphi} [e^{-\varphi\tau} - e^{-\rho\tau}] \quad (35)$$

Moreover, the derivative of X with respect to τ is given by

$$X'(\tau) \equiv \frac{dX(\tau)}{d\tau} = 1 - e^{-\varphi\tau} + R_1 e^{-\varphi\tau} + R_2 \frac{e^{-\varphi\tau} - e^{-\rho\tau}}{\rho - \varphi} \quad (36)$$

$$R_1 = \frac{\Phi\varphi(z-\mu\rho)}{(1+z)(1+\mu\varphi)} > 0, \quad R_2 = \rho\varphi\lambda \left(\frac{(1-\xi)}{\omega z} + \xi \right) \quad (37)$$

where $\xi = (1+\mu\rho)/[(1+z)(1+\mu\varphi)]$. Inspection of R_1 reveals that it can be rewritten as $R_1 = (1+z)^{-1}(z-\mu\rho)q_2$, where q_2 is the expected rate of change of the jump component in regime 2, see equation (26). It is harder to interpret the last term, but it can be noticed that it is close to zero for very short time horizons (τ small) as well as for long time horizons (τ large). Uncovered interest parity implies that the forward interest rate differential, $\delta^f(\tau)$, is given by

$$\delta^f(\tau) = \frac{dD_2(\tau)}{d\tau} = \rho e^{-\rho\tau}(\lambda\pi_0 - \hat{s}_2(t)) + \frac{\alpha\Delta\pi_0}{\phi} X'(\tau) \equiv \hat{\delta}^f(\tau) + \delta_{rs}^f(\tau) \quad (38)$$

The interest rate differential is decomposed into a supply shock component, $\hat{\delta}^f(\tau)$, capturing the effects of supply shocks (i.e. expectations of future reversion towards the stationary level that supply shocks (dW_p cause)), and a regime shift premium reflecting expectations of regime shifts. It can easily be shown that if the semi-elasticity of money demand, λ , is zero then we have that $R_1 = R_2 = 0$ and the regime shift component reduces to the expression derived by Dillén and Hopkins (1998). Moreover, the short term and asymptotic regime shift premia can be written as

$$\delta_{rs}^f(0) = \frac{\alpha\Delta\pi_0}{\phi} R_1 > 0 \quad \text{and} \quad \delta_{rs}^f(\infty) = \frac{\alpha\Delta\pi_0}{\phi} > 0 \quad (39)$$

In contrast to the model of Dillén and Hopkins (1998), regime shift expectations give rise to a positive effect on the short interest rate ($\tau = 0$). This effect arises since there are expectations of a jump in the exchange rate that a shift to the high inflation regime will cause and investors will demand a compensation for this according to UIP. The shape of the regime shift premium (over maturities) can be analyzed by considering its derivative, which after some rearrangements takes the form

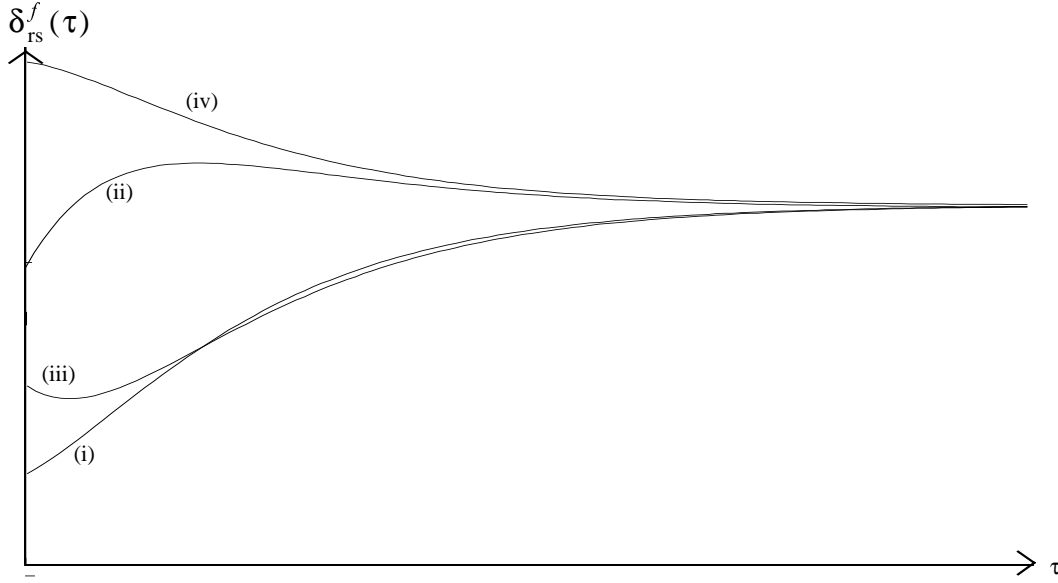
$$\frac{d\delta_{rs}^f(\tau)}{d\tau} = \frac{\alpha\Delta\pi_0}{\phi} X''(\tau) = \frac{\alpha\Delta\pi_0}{\phi} [\phi(1-X'(\tau)) + R_2 e^{-\rho\tau}] \quad (40)$$

There are four basic alternatives for the shape of the regime shift premium of the term structure, see Figure 4: (i) $R_1 < 1 + R_2/\phi$, $R_2 \leq 0$. In this case the regime shift premium is monotonically increasing in τ . This alternative includes as a special case ($R_1 = R_2 = 0$) the expression of the regime shift premium derived by Dillén and Hopkins (1998). (ii) $R_1 < 1 + R_2/\phi$, $R_2 > 0$. This alternative differs from alternative (i) in that the term premium curve is hump shaped. The maximum size of the regime shift premium is obtained by setting the derivative in (40) equal to zero, which renders

$$\delta_{rs}^f(\tau^*) = 1 + [R_2/\phi] e^{-\rho\tau^*} \quad (41)$$

where τ^* is the maturity that satisfies $X''(\tau) = 0$. Alternatives (i) or (ii) are likely to occur if the semi-elasticity of money demand, λ , is small.⁷ (iii) $R_1 > 1 + R_2/\phi$, $R_2 < 0$. The regime premium curve is U-shaped in this case and the expression for the minimum is once again given by (42). Notice that if the magnitude of R_2 is large (but negative) this alternative is feasible also for modest values of λ . This example shows that expectations of a monetary expansion might result in higher long term interest rates (due to increased inflation expectations) and at the same time expectations of an initial reduction in short-term rates. (iv) $R_1 > 1 + R_2/\phi$, $R_2 > 0$. The fourth alternative exhibits a regime shift premium that monotonically decreases with the term, but it appears to be less plausible since it requires a very high semi-elasticity of money demand.

⁷ Notice that R_1 is the product of Φ and a ratio that is bounded from above by one. Furthermore, equation (25) indicates that the size of Φ (and hence R_1) mainly depends on λ .

Figure 4 Different shapes of the regime shift premium of the term structure

Evidence, see Dillén and Hopkins (1998), suggests that alternative (i) is the most probable shape of the regime shift premium in Sweden, i.e. the regime shift premium appears to be monotonically increasing in τ , although some forms of alternative (ii) and (iii) cannot be excluded. Alternative (iv) appears, however, to be less probable. The observation that other countries with credibility problems normally exhibit a forward rate curve with a positive slope reinforces the judgement that alternative (iv) is unlikely.

It is possible to present explicit expressions for the regime shift premium in the high inflation regime (regime 1) and the low inflation regime of high credibility (regime 3), but these are very complicated and instead we will only report the main features in these cases. Considering the term structure effects of switching to the high inflation regime first, it can be shown that (under the maintained assumption of UIP) the change in the instantaneous regime shift premium is of the form

$$\delta_{rs}^f(0;1) - \delta_{rs}^f(0;2) = \Delta\pi_0 + \frac{\rho\lambda}{\omega z} \Delta\pi_0 + \frac{(z - \mu\rho)}{1+z} (q_1 - q_2) \lesssim 0 \quad (42)$$

where the second argument indicates the regime. The first term represents a positive Fisher effect, i.e. a switch to a regime characterized by a higher inflation rate will increase the nominal return requirements correspondingly. The remaining terms are, however, negative representing changes in regime shifts expectations and the effects of the appreciation that the overshooting of the exchange rate gives rise to. The total effect on short-term interest rates is ambiguous. However, the effect on the short real interest rate is negative which

together with a real depreciation of the domestic currency implies that a switch to an inflationary regime will have a stimulative effect on the economy in the short run.

It is probably more interesting to analyze the effects of a positive credibility shock, i.e. switch from the low inflation regime of low credibility (regime 2) to the low inflation regime of high credibility (regime 3), since credibility shocks appear to be a more frequent phenomenon than regime shifts⁸. Concerning effects on the short-term interest rate it can be shown that

$$\delta_{rs}^f(0;3) - \delta_{rs}^f(0;2) = \frac{(z - \mu\rho)}{1+z}(q_3 - q_2) < 0 \quad (43)$$

i.e. a positive credibility shock leads to lower short-term interest rates. There is a possibility that this effect declines as time to maturity increases and for long maturities the (forward) interest rate might even increase somewhat. However, it can also be the case that a positive credibility shock leads to a reduction of interest rates that initially increases with the term, i.e. the impact on interest rates from credibility shocks increases with the term. This appears to be the typical case in Sweden, see Dillén and Hopkins (1998). Of course, the magnitude of the regime shift premium will converge to the stationary level $\alpha\Delta\pi_0/\phi$ in both regime 2 and 3, but this mechanism might be relevant only for the very long term if the credibility shock is very persistent (v small). In the extreme case when regime 3 is an absorbing state ($v = 0$), which for countries like Sweden might be thought of as an EMU-regime, also the asymptotic level will change when the economy switches to regime 3.

5. The relationship between yield differentials and the exchange rate

Since the spot rate curve is related to the forward rate curve as the average cost curve is related to the marginal cost curve most of the features of the forward rate curve that the regime shift expectations give rise to carry over to the yield curve. We will focus on the relation between the yield differential and the exchange rate. A positive credibility shock, i.e., a switch to a regime 3, will not only lead to an appreciation of the domestic currency, but it is also probable that the interest rate differential will decrease as mentioned in section 4. Thus, the presence of credibility shocks can explain why we often observe a positive correlation between exchange rate and the yield differential. An easier way of

⁸ A regime shift in Sweden would probably be an observable event where the current inflation target is given up.

understanding why the exchange rate might be positively correlated to the yield differential is to notice that yield differential is given by

$$\delta(\tau) = F(\tau)(\lambda\pi_0 - \hat{s}_2(t)) + \delta_{rs}^\infty G(\tau) \equiv \hat{\delta}(\tau) + \delta_{rs}(\tau) \quad (44)$$

where

$$F(\tau) = (1 - e^{-\rho\tau})/\tau, \quad \delta_{rs}^\infty = \frac{\alpha\Delta\pi_0}{\phi}, \quad G(\tau) = X(\tau)/\tau$$

The term $\hat{\delta}(\tau)$ is the expected depreciation rate in absence of regime shift expectations and the function $F(\tau)$ shows how deviations from the stationary level, $\lambda\pi_0$, are expected to decline over time. The term δ_{rs}^∞ shows the effects of regime shift expectations and the function $G(\tau)$ shows how these effects vary with maturity. Thus, the yield differential can be seen as the sum of depreciation expectations within the regime and depreciation expectations that regime shift expectations give rise to. In a more general model, in which the switch intensity α is allowed to fluctuate, we see from (44) that such fluctuations might explain a positive correlation between the yield differential and the exchange rate.⁹ This can be seen as a variant of the result for target zones models with devaluation risk derived by Bertola and Svensson (1993) saying that the correlation between the interest rate differential and the exchange rate position within the band tends to be positive if the variability of the devaluation risk is large in relation to the variability of fundamentals.

The regime shift component of the yield differential in (44) can be viewed as a product of a credibility factor (δ_{rs}^∞) and a credibility sensitivity factor ($G(\tau)$), showing the credibility effect on the yield differential for different horizons, τ . This is a rather general feature of regime shift models of this kind. Moreover, notice that $G(\tau)$ tends to 1 while the term related to expectations within the regime ($\hat{\delta}(\tau)$) tends to zero as τ approaches infinity suggesting that in empirical work the long-term interest rate differential relative a credible country can be used as a proxy for the credibility factor (δ_{rs}^∞), see Dillén and Hopkins (1998)¹⁰. It is also possible to explain non-trivial phenomena such as a positive correlation between the exchange rate and interest rate differential by a time-varying exchange rate risk premium. In our view our attempt to explain irregularities on financial markets with fluctuating regime shift premium is a more promising device. The regime shift premium is directly related to credibility problems that often are discussed in the public debate whereas risk premia of various kinds are more vaguely related to uncertainty (often approximated by some measure of volatility in financial prices). Moreover, the empirical results reported by Dillén and Hopkins (1998) show that regime shift premia can have a far more substantial impact on financial prices than more traditional risk premia.

⁹ There are other (and maybe more straightforward) ways of modeling fluctuating credibility. One possibility is to introduce a stochastic the jump size in the money growth rate, $\Delta\pi_t$. Another possibility is to introduce more low inflation regime with different degree of credibility (i.e. the probability to be in a high inflation regime in the future differs between states) and switches between low inflation states can then be seen as credibility shocks.

¹⁰ Dillén and Hopkins (1998) use the forward rate differential (relative Germany) as a proxy for the credibility factor.

6. Conclusions

This paper analyses a model with sticky prices and expectations of an inflationary monetary regime shift. Such expectations will not only increase inflation expectations and long term interest rates, but also weaken the domestic currency. Moreover, if the economy shifts to the high inflation regime, the exchange rate will jump upward. Expectations of such an upward jump of the exchange rate will increase short-term interest rate as well. Expectations of a shift to an inflationary regime hence give rise to regime shift premia in interest rates and exchange rates. Explicit expressions for these effects are presented in the paper. Moreover, fluctuations in regime shift expectations of this kind provide an explanation why the exchange rate often appears to be positively correlated with the interest rate differential.

Thus, the model has non-standard implications that can explain nontrivial phenomena observed on financial markets in countries, where economic policy suffers from credibility problems. It is, however, worth emphasizing that the presented framework is not restricted to the analysis of credibility effects on financial markets, but can be used for addressing other interesting issues arising in monetary economics, e.g. analysis of monetary supply rules suitable for inflation targeting.

A. Characterization of a Miller-Weller economy with nominal trends

In this case the only source of uncertainty is supply shocks, σdW_p , and therefore we postulate that exchange rate shocks in this case can be written of the form $du_s = \sigma_s dW_p$. Assuming that the money supply rule is given by (13) the dynamics of f is given by (8) to be

$$df = [c_{11}f + b_1 \bar{\pi} t + d_1 \bar{\pi}]dt + \sigma_f dW_p \quad (A1)$$

where $\sigma_f = \sigma + \omega \sigma_s$. Now we show that f is trend stationary, i.e., f can be written as

$$f = \hat{f} + \phi_f t, \quad (A2)$$

where $\phi_f = -b_1 \bar{\pi} / c_{11} = (1+\omega) \bar{\pi}$ and where \hat{f} is an Ornstein-Uhlenbeck process¹¹

$$d\hat{f} = \rho[\hat{f}_0 - \hat{f}]dt + \sigma_f dW_p, \quad \rho = -c_{11}, \quad \hat{f}_0 = (1+\omega)\lambda \bar{\pi} \quad (A3)$$

(A3) can be verified by comparison with (A1) after differentiating (A2). Combining (12) and (A2) yields

$$h = \hat{h} + \bar{\pi} t \quad (A4)$$

$$d\hat{h} = \rho[\hat{h}_0 - \hat{h}]dt + \sigma_h dW_p, \quad \hat{h}_0 = \Omega_0 \hat{f}_0 + \Omega_1 \bar{\pi} \quad (A5)$$

where

$$\Omega_0 = -\theta/[\kappa\eta - \omega\theta], \quad \Omega_1 = -\kappa\gamma/[\kappa\eta - \omega\theta], \quad \text{and } \sigma_h = \Omega_0 \sigma_f \quad (A6)$$

To calculate integrals like (13) we notice for future reference that

$$\int_t^\infty e^{-k(u-t)} (u-t) du = k^{-1}(n+1) \quad (A7)$$

Assuming that one can move the expectation operator inside the integral in (12) and using that $E_t[\hat{h}(u)] = \hat{h}_0 + e^{-\rho(u-t)}[\hat{h}(t) - \hat{h}_0]$, straightforward application of (A7) renders

$$s = (1-x)\hat{h}_0 + x\hat{h} + \mu \bar{\pi} + \bar{\pi} t \equiv \hat{s} + \bar{\pi} t \quad (A8)$$

where $x = [\mu\rho + 1]^{-1}$. The stationary level of \hat{s} is given by

$$\hat{s}_0 = \hat{h}_0 + \mu \bar{\pi} = \lambda \bar{\pi} \quad (A9)$$

(A8) implies that $\sigma_s = x\sigma_h = -x\theta\sigma_f/[\kappa\eta - \omega\theta]$, which together with the definition $\sigma_f = \sigma + \omega\sigma_s$ yields

$$\sigma_s = -x\theta\sigma/[\kappa\eta - (1-x)\omega\theta] = \sigma/[\omega z] \quad (A10)$$

¹¹ When simplifying the expressions in the appendix we have besides (8) used that the following facts can be proven to be true: (i) $[1 - \phi\gamma + \theta b_1 / c_{11}] / [\kappa\eta - \omega\theta] = 1$, and (ii) $(1+\omega) + b_1 / c_{11} = 0$.

where the expression for z is given by (A30) in appendix B.

Finally, the price level can be expressed as $p = f - \omega s$, which according to the expressions derived above, takes the form

$$p = \hat{p} + \bar{\pi} t \quad (\text{A11})$$

$$d\hat{p} = \rho[\hat{p}_0 - \hat{p}]dt + \sigma dW_p, \quad \hat{p}_0 = \lambda \bar{\pi} \quad (\text{A12})$$

B. Characterization of the exchange rate in the regime shift model.

The characterization of the exchange rate in the regime shift model is presented in section 3 is a four step procedure.

Step 1. First we characterize the distribution properties of the Markov chain governing the regime switches. Let $Q_{ij}(\tau) = \text{Prob}[S(t+\tau) = j | S(t) = i]$. Furthermore, define $Q(\tau)$ as the matrix whose elements are $Q_{ij}(\tau)$. From the theory of continuous Markov Chains (see e.g. Karlin and Taylor (1975) p.150-152) we have that Q satisfies

$$\frac{dQ}{dt} = AQ, \quad A = \begin{pmatrix} -v & v & 0 \\ \alpha & -(\alpha+\beta) & \beta \\ 0 & v & -v \end{pmatrix}, \quad Q(0) = I. \quad (\text{A13})$$

As easily verified, the solution to system (A13) is

$$Q(\tau) = \frac{1}{\varphi} \begin{pmatrix} \alpha & v & \beta \\ \alpha & v & \beta \\ \alpha & v & \beta \end{pmatrix} + \frac{e^{-v\tau}}{\alpha+\beta} \begin{pmatrix} \beta & 0 & -\beta \\ 0 & 0 & 0 \\ -\alpha & 0 & \alpha \end{pmatrix} + \frac{e^{-\varphi\tau}}{(\alpha+\beta)\varphi} \begin{pmatrix} \alpha v & -(\alpha+\beta)v & \beta v \\ -(\alpha+\beta)\alpha & (\alpha+\beta)^2 & -(\alpha+\beta)\beta \\ \alpha v & (\alpha+\beta) & \beta v \end{pmatrix} \quad (\text{A14})$$

where $\varphi = \alpha + \beta + v$.

Step 2. Next we postulate that the exchange rate and the state variable f are of the form

$$s_i = \hat{s}_i + J_i(t) + \Gamma_i \quad i = 1, 2, 3 \quad (\text{A15})$$

$$f_i = \hat{f}_i + (1+\omega)J_i(t) + \omega\Gamma_i \quad i = 1, 2, 3 \quad (\text{A16})$$

where a hat indicates the Ornstein-Uhlenbeck component (with continuous sample path) and where the trend component is given by (22). Γ_i is a jump component that jumps upon regime shifts. It is worth emphasizing that there is a degree of arbitrariness how to decompose deviations from the trend into an Ornstein-Uhlenbeck process and a jump component. In what follows we define the jump component in such way that it takes the value zero in the case of perfect credibility of the low inflation regimes, i.e., $\alpha = 0$.

Since the expected jump size typically differs from zero we have to adjust for this so that the expected shock to the exchange rate, du_s , equals zero. Thus, du_s is of the form

$$du_s = \sigma_s dW_p + dM_i \quad (\text{A17})$$

where

$$dM_i = d\Gamma_i - q_i dt, \quad q_i = E[d\Gamma_i]/dt, \text{ is an increment of a Martingal implying } E[dM_i] = 0.$$

Moreover, q_i is the expected rate of change of the jump component in regime i , and it is given by

$$q_1 = v(\Gamma_2 - \Gamma_1), \quad q_2 = \alpha(\Gamma_1 - \Gamma_2) + \beta(\Gamma_3 - \Gamma_2), \text{ and } \quad q_3 = v(\Gamma_2 - \Gamma_3) \quad (\text{A18})$$

From (9), (A16) and (A17) the dynamics of f is

$$df_i = [c_{11}(\hat{f}_i + (1+\omega)J_i(t) + \omega\Gamma_i) + b_1 J_i(t) + d_1 \pi_i - \omega q_i]dt + \sigma_f dW_p + \omega d\Gamma_i \quad (\text{A19})$$

It is straightforward to show that the component \hat{f} is given by

$$d\hat{f}_i = \rho[\hat{f}_{0i} - \hat{f}_i]dt + \sigma_f dW_p, \quad \rho = -c_{11} \quad (\text{A20})$$

$$\hat{f}_{0i} = (1+\omega)\lambda\pi_i - \omega(\Gamma_i + q_i/\rho) \quad (\text{A21})$$

The logical situation at this stage is that we shown that (A16) with the Ornstein-Uhlenbeck process given by (A20) satisfies (A19). Moreover, the price level $p = f - \omega s$ does not have a jump component according to (A15) and (A16). Notice, however, that the state variable, f , depends on the jump component, Γ_i , which has to be determined.

Step 3. To determine the jump component, Γ_i , we let E^p and E^S denote expectations with respect to supply uncertainty (the Wiener process W_p) and regime uncertainty (the Markov chain S) respectively. It can be shown that

$$E_t^p [\hat{f}_i(t+\tau)] = \hat{f}_i(t) e^{-\rho\tau} + \rho \int_0^\tau \hat{f}_{0S(t+u)} e^{-\rho\tau} du \quad (\text{A22})$$

where the second term is stochastic due to regime uncertainty indicated by the subindex $S(u)$. By substituting (A16) and (A20) using (A22) into (12) via (13) we obtain the following expressions for the components of the exchange rate

$$\hat{s}_i(t) = \frac{\Omega_0}{1+\mu\rho} \hat{f}_i(t) + \left[\frac{\mu\rho\Omega_0\lambda(1+\omega)}{1+\mu\rho} + \Omega_1 + \mu \right] \pi_0 \quad (\text{A23})$$

$$J(t) = \pi_0 t + \Delta\pi_0 \int_0^t \chi_1(S(u))du, \quad S(t) = i \quad (\text{A24})$$

$$\Gamma_i = E_t^S \left[\frac{1}{\mu} \int_0^\infty e^{-\tau/\mu} (\Delta\pi_0 \Theta(\tau) + \Omega_0 \Xi(\tau)) d\tau | S(t) = i \right] \quad (\text{A25})$$

where

$$\Theta(\tau) = \Omega_1 \chi_1(t+\tau) + \int_0^\tau \chi_1(S(t+u)) du + (1+\omega)\lambda \int_0^\tau \chi_1(S(t+u)) e^{-\rho(\tau-u)} du \quad (\text{A26})$$

$$\Xi(\tau) = \omega \Gamma_{S(t+u)} - \omega \rho \int_0^\tau (\Gamma_{S(t+u)} + q_{S(t+u)} / \rho) e^{-\rho(\tau-u)} du \quad (\text{A27})$$

(A27) can be simplified by integrating the first term in the integral by parts, which after rearrangements and usage of (A17) renders

$$\Xi(\tau) = \omega e^{-\rho\tau} \Gamma_{S(t)} + \omega \int_0^\tau e^{-\rho(\tau-u)} dM_{S(u)} \quad (\text{A28})$$

Making use of (A7) and the assumption that one can move the expectation operator within the integrals in an evaluation (A25) leads to the following expression for the regime components:

$$\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} = \frac{U_0 \Delta \pi_0}{(1+\mu\nu)(1+\mu\phi)} \begin{pmatrix} 1+\mu\phi+\mu^2\alpha\nu \\ (1+\mu\nu)\alpha\mu \\ \mu^2\alpha\nu \end{pmatrix} + U_1 \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} \quad (\text{A29})$$

where $U_0 = \frac{\mu\rho(1+\omega)\lambda\Omega_0}{1+\mu\rho} + \mu + \Omega_1$, and $U_1 = \frac{\omega\Omega_0}{1+\mu\rho}$.

It is straightforward to show that the solution to (A29) is given by (21), where we have used the following expressions in order to simplify the notation.

$$\Phi \equiv \frac{U_0(1+z)}{z} = \lambda \left(1 - \frac{1}{\omega z} \right), \quad z = U_1^{-1} - 1 = \phi\lambda\eta/[\theta\omega^2] \quad (\text{A30})$$

Step 4. Expected depreciation and verification that the exchange rate expression satisfies (11).

Logically, the derived expressions for the exchange rate and the state variable, f , are candidates for being the solution to the dynamic equations in (9) when the growth rate of money supply is given by the Markov switching model presented in section 3. Moreover, from step 2 above it is clear that the state variable, f , satisfies the dynamics in the first row in (9). However, the calculation of the exchange rate expression relies on the unjustified assumption that one can move the expectation operator within the integrals, and it remains to be verified that the exchange rate expression satisfies (11). For this purpose we use the derived exchange rate expression and the probabilities given in (A14) one can calculate expected depreciation as $D_i(\tau) \equiv E_t[s(t+\tau) - s_i(t)]$. It turns out that

$$\begin{pmatrix} D_1(\tau) \\ D_2(\tau) \\ D_3(\tau) \end{pmatrix} = (1-e^{-\rho\tau}) \begin{pmatrix} \lambda\pi_0 - \hat{s}_1(t) \\ \lambda\pi_0 - \hat{s}_2(t) \\ \lambda\pi_0 - \hat{s}_3(t) \end{pmatrix}$$

$$\begin{aligned}
& + \frac{\Delta\pi_0}{(\alpha+\beta)\varphi} \left\{ \alpha(\alpha+\beta) \frac{\lambda}{\omega z} L(0, \tau) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \varphi \left[\frac{\lambda}{\omega z} + l_v \right] L(v, \tau) \begin{pmatrix} \beta \\ 0 \\ -\alpha \end{pmatrix} + \alpha \left[\frac{\lambda}{\omega z} + l_\varphi \right] L(\varphi, \tau) \begin{pmatrix} v \\ -(\alpha+\beta) \\ v \end{pmatrix} \right\} \\
& + \frac{V}{\varphi} \left\{ \begin{pmatrix} \alpha v \mu - (\beta + v)(1 + \mu \varphi) \\ \alpha(1 + \mu v) \\ \alpha(1 + \mu(\varphi + v)) \end{pmatrix} + \frac{(1 + \mu \varphi)\varphi}{\alpha + \beta} \begin{pmatrix} \beta \\ 0 \\ -\alpha \end{pmatrix} e^{-v\tau} + \frac{\alpha(1 + \mu v)}{\alpha + \beta} \begin{pmatrix} v \\ -(\alpha + \beta) \\ v \end{pmatrix} e^{-\varphi\tau} \right\} \\
& + \Delta\pi_0 \left\{ \frac{1}{\varphi} \tau \begin{pmatrix} \alpha \\ \alpha \\ \alpha \end{pmatrix} + \frac{1}{(\alpha + \beta)v} (1 - e^{-v\tau}) \begin{pmatrix} \beta \\ 0 \\ -\alpha \end{pmatrix} + \frac{\alpha}{(\alpha + \beta)\varphi^2} (1 - e^{-\varphi\tau}) \begin{pmatrix} v \\ -(\alpha + \beta) \\ v \end{pmatrix} \right\} + \pi_0 \tau \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (A31)
\end{aligned}$$

where

$$V = \frac{\Phi \Delta\pi_0}{(1 + \mu v)(1 + \mu \varphi)}, \quad L(x, \tau) = \frac{\rho(e^{-x\tau} - e^{-\rho\tau})}{\rho - x}, \quad l_x = \frac{\Phi(1 + \mu\rho)x}{(1 + z)(1 + \mu x)\rho} \quad x = v, \varphi.$$

The first two terms in (A31) represent the expected change in the Ornstein-Uhlenbeck component where the second term is the expected change of mean reverting level, \hat{s}_0 . The third term is the expected change of the jump component. The two last terms represent the expected change of the drift term. (A31) implies that the instantaneous rate of depreciation is given by

$$d \equiv \begin{pmatrix} E[ds_1]/dt \\ E[ds_2]/dt \\ E[ds_3]/dt \end{pmatrix} = \hat{d} + d^\Gamma \quad (A32)$$

where

$$\hat{d} = (\rho\lambda + 1)\pi_0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \rho \begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{pmatrix} \quad (A33)$$

and

$$d^\Gamma = (1 + \rho \frac{\lambda}{\omega z}) \Delta\pi_0 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{(z - \mu\rho)}{1 + z} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \quad (A34)$$

Next we notice that the expression for h appearing in (12) takes the form (cf. (A4))

$$h = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \hat{h} + h^\Gamma \quad (A35)$$

where the i th components of \hat{h} and h^Γ are given by $\Omega_0 \hat{f}_i + \Omega_1 \pi_0$, and $\omega \Omega_0 \Gamma_i + \Omega_1 \Delta\pi_0 \chi_1(i)$ respectively. Finally, it is straightforward to verify that $h + \mu d$ equals the exchange rate expression (21), which was to be shown.

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