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# Optimal Non-Linear Income Taxation in Search Equilibrium\*

Per Engström<sup>†</sup>

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## Abstract

The paper extends the basic Stiglitz (1982) model of optimal income taxation into general search equilibrium. When we extend the basic taxation model to include a more realistic treatment of the labor market, a number of new interesting mechanisms arise. When wages are fixed we find that a "work hour effect" gives the government incentives to lower the marginal tax rate for both high and low skilled workers. The optimal marginal tax on high skilled is thus negative, and the sign for the low skilled marginal tax is ambiguous. With wages determined by bargaining between firm and worker the results are changed. Both marginal tax rates are of ambiguous sign. The tax systems' effects on the wage formation and the unemployment rates may result in new intricate redistribution channels. Simulations show that the marginal tax rate for high skilled is increasing in the level of redistribution when wages are fixed, but decreasing in the level of redistribution when wages are determined by bargaining.

JEL-classification: H21, J22, J41, J64

Keywords: Optimal non-linear income taxation, search, unemployment

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# 1 Introduction

In the last 20 years there has been numerous extensions of Stiglitz (1982) model of optimal non-linear income taxation. His basic model features two types of workers: high skilled and low skilled. The government seeks to find the Pareto optimal set of tax/transfer regimes. The tax-function can take on any form, and generally the tax functions solving the problem are non differentiable. The model also features the realistic assumption that the government cannot observe each worker's labor supply, only the gross labor income is observed. This opens up for the workers to engage in "mimicking", which restricts the government's possibilities to redistribute income among the different workers. If the government redistributes a lot of income from the high skilled to the low skilled, the high skilled may choose to mimic the low skilled by reducing their labor supply. If the high skilled workers earn the same income as the low skilled workers, there is no way the government can distinguish among the different types.

Alongside Mirrlees (1971) groundbreaking continuous skill model, the Stiglitz (1982) model has become be the natural choice in the study of non-linear income taxation. Extensions have been made in many directions, such as: heterogeneous preferences for leisure (Boadway et al, 2001), endogenous wages (Stiglitz, 1982) and the inclusion of commodity taxation (Edwards et al, 1994), just to mention a few. But in these studies the labor market has typically been Walrasian; there has been no risk of unemployment in the economy.<sup>1</sup> In this paper we will try to combine the Stiglitz (1982) model of optimal income taxation and the Pissarides/Mortensen search/matching framework (Pissarides, 2000). The paper will thus extend the optimal income tax model into a general equilibrium model with involuntary unemployment.

The Pissarides (2000) search model has been applied to a large number of issues that relates to taxes. When dealing with tax issues in labor economics

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<sup>1</sup>One exception is Aronsson and Sjögren (2001). They develop an optimal taxation model with imperfect competition in the labor market, resulting from union influence over the wage formation.

one usually specifies a functional form of the tax function. In this framework one usually asks questions of positive nature, e.g.: “how does the income tax affect the unemployment rate?” (Mortensen and Pissarides, 2001) and (Pissarides, 1998). There are however examples of normative studies as well, e.g.: “What is the optimal degree of income tax progressivity?” (Sørensen, 1999). The normative studies however, are not concerned with equality (the workers typically do not differ ex ante) but rather with efficiency. The present study does thus diverge from the existing search/matching literature in that the workers differ in ability, which makes the government concerned with both efficiency and equality aspects.

When we extend the basic model of optimal income taxation to include a more realistic treatment of the labor market, a number of new interesting mechanisms arise. In the case with exogenous wages we find that a positive external effect from longer work hours gives the government incentive to lower the marginal tax for both skill types. In the endogenous wage model the new mechanisms are more intricate, since the tax system now also affect the bargained wages and thereby also the unemployment rates. Perhaps the most surprising insight is that the government now will, to some extent, use the tax system to redistribute through the unemployment rates. Lower risk of unemployment gives – *ceteris paribus* – higher expected utility. To transfer utility in the form of low risk of unemployment is a very refined way to redistribute, since it does not cause any adverse behavioral effects; the potential mimicker does not enter the low skilleds’ labor market and she therefore cannot benefit from lower low skilled unemployment.

The paper is outlined as follows. In section 2 the model with exogenous wages is presented. We derive analytical expressions for the marginal tax rates for the set of Pareto optimal tax regimes. In a numerical exercise we let the government’s egalitarian preferences be determined by a strictly concave welfare function. We can then study the optimal redistribution as well as the optimal marginal tax rates. In section 3 the model is extended by endogenizing the wages. The wages are determined by bargaining between firms and workers. In numerical exercises we compare the results of the two models. Section 4 concludes.

## 2 Exogenous Wages

### 2.1 The Labor Market

There are two types of workers in the economy:  $j = H, L$ , where  $H$  and  $L$  denote high and low skill respectively. The two different types are matched in two separate labor markets of Pissarides/Mortensen type (Pissarides 2000). The workers live forever and the time is continuous. At any given time a specific worker is either employed or unemployed. The workers of type  $j$  are identical apart from being employed or unemployed. The matched jobs are dissolved at the exogenous rate  $\phi$ , equal for high and low skill.<sup>2</sup> For both types of workers there is a standard constant returns to scale (CRS) function that relates the number of new matches ( $M_j$ ), to the total number of efficient search hours ( $s_j u_j$ ) and the number of vacancies ( $v_j$ ); i.e.,

$$M_j = M(s_j u_j, v_j) = s_j u_j M(1, \frac{v_j}{s_j u_j}) = s_j u_j \alpha(\theta_j), \quad (1)$$

where  $\theta_j \equiv \frac{v_j}{s_j u_j}$  is the measure of overall labor market tightness in the  $j$ -type labor market,  $s_j$  is the search intensity and  $u_j$  denotes the number of unemployed. We have defined  $\alpha(\theta_j)$  according to:

$$\alpha(\theta_j) \equiv \frac{M_j}{s_j u_j}, \quad (2)$$

where  $s_j \alpha(\theta_j)$  is the rate at which unemployed individuals find jobs. We analogously define  $q(\theta_j)$  as the rate at which vacancies are filled; i.e.,

$$q(\theta_j) \equiv \frac{M_j}{v_j} = \frac{s_j u_j}{v_j} \alpha(\theta_j) = \frac{\alpha(\theta_j)}{\theta_j}. \quad (3)$$

Since  $M_j$  is increasing in both arguments, it follows that  $\alpha'_j > 0$  and  $q'_j < 0$ .

We normalize the total number of workers of each type to unity and can thus interpret  $u_j$  as the type specific unemployment rate. In equilibrium the total number of new matches is equal to the total number of separations, for both types separately. This generates a "flow equilibrium" for each type of

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<sup>2</sup>The matched jobs are dissolved according to a Poisson process with intensity  $\phi$ .

worker. When solving this for the type specific unemployment rate we arrive at:

$$u_j = \frac{\phi}{\phi + s_j \alpha(\theta_j)}. \quad (4)$$

## 2.2 The Workers

The employed workers derive utility from consumption ( $C_j$ ) and disutility from market work ( $L_j$ ). The unemployed workers derive disutility from search ( $s_j$ ). We ignore unemployment benefits, so the unemployed individuals have no income. We let the workers' utility function be quasi linear in consumption and for simplicity we normalize the marginal utility of consumption to unity. Hence:

$$v(C_j, L_j) = C_j - g(L_j), \quad (5)$$

if employed and

$$v(0, s_j) = -g(s_j), \quad (6)$$

if unemployed. For  $g(\cdot)$  holds that  $g'(\cdot) > 0$  and  $g''(\cdot) > 0$ .

The utility function is thus the same for all workers in the economy. The linearity in consumption eliminates the incentives for smoothing the consumption pattern over time. There is thus no incentive for saving in the economy; we therefore make the assumption that all workers consume their net income in every period.<sup>3</sup> The employed worker of type  $j$  receives the exogenously given wage  $w_j$ , with  $w_H > w_L$ .

### 2.2.1 The Value Functions

Let  $U_j$  be the value of being in the unemployed state for a worker of type  $j$  and let  $E_j$  be the corresponding state value for an employed worker. With

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<sup>3</sup>All the analytical results apply also for a general utility function (with the same normality assumptions as in Stiglitz, 1982), but with a general (concave) utility function there would be incentives to smooth the consumption over time.

the above assumptions, the value functions for the workers take the form:

$$rU_j = -g(s_j) + s_j\alpha(\theta_j)(E_j - U_j), \quad (7)$$

$$rE_j = C_j - g(L_j) + \phi(U_j - E_j). \quad (8)$$

Solving for  $E_j - U_j$  using (7) and (8) yields:

$$E_j - U_j = \frac{C_j - g(L_j) + g(s_j)}{r + \phi + s\alpha(\theta_j)}. \quad (9)$$

### 2.2.2 The Employed Workers' Maximization Problem

Let  $T(Y_j)$  be the total tax paid by a worker with gross income  $Y_j = w_j L_j$ . The workers' maximization problem then takes the following form:

$$\max_{L_j} [rE_j = Y_j - T(Y_j) - g(L_j) + \phi(U_j - E_j)].$$

The first order condition for this problem can be written as:

$$T'_j = 1 - \frac{g'(L_j)}{w_j}. \quad (10)$$

### 2.2.3 The Unemployed Workers' Maximization Problem

The unemployed workers maximize today's fraction of the total state value ( $rU_j$ ), w.r.t. the search intensity. To a single worker  $\theta_j$  and  $E_j$  are exogenous, hence the problem takes the following form:

$$\max_{s_j} rU_j = -g(s_j) + s_j\alpha(\theta_j)(E_j - U_j).$$

The first order condition for this maximization problem is given by:

$$g'(s_j) = \alpha(\theta_j) \frac{C_j - g(L_j) + g(s_j)}{r + \phi + s_j\alpha(\theta_j)}, \quad (11)$$

where we have used (9). From (11) we can solve for the optimal search intensity as a function of  $\theta_j$  and  $v_j \equiv C_j - g(L_j)$ ; hence  $s_j(v_j, \theta_j)$ . It is

straightforward to show that  $\frac{\partial s_j}{\partial v_j} > 0$ , which means that the search intensity increases with the instantaneous utility of being employed.

## 2.3 The Firms

The cost of holding a vacancy is  $\kappa y_j$ , where  $y_j$  is the exogenous productivity of a worker of type  $j$  and  $\kappa$  is an exogenous parameter. This specific form of vacancy cost can be rationalized using a model of a large firm that allocates its workforce between production and recruitment activities; see Holmlund (2001).

Let  $J_j$  be the total value of a filled job of type  $j$  and  $V_j^{vac}$  the total value of a vacancy of type  $j$ . The value functions for the firms can then be expressed as:

$$rJ_j = L_j(y_j - w_j) + \phi(V_j^{vac} - J_j), \quad (12)$$

and

$$rV_j^{vac} = -\kappa y_j + (J_j - V_j^{vac})q(\theta_j). \quad (13)$$

We assume that there is no additional cost associated with opening up a vacancy. The value of an additional vacancy will thus be zero in equilibrium, i.e.  $V_j^{vac} = 0$ . This "free entry" condition together with (12) and (13) gives:

$$(r + \phi) \frac{\kappa y_j}{q(\theta_j)} = L_j(y_j - w_j). \quad (14)$$

From (14) we can solve for the equilibrium tightness, i.e.,

$$\theta(Y_j; y_j, w_j, \kappa). \quad (15)$$

where  $\frac{\partial \theta(Y_j; y_j, w_j)}{\partial Y_j} > 0$ . One interpretation of (15) is that it expresses the "feasible" tightness conditional on the productivity, the wage and the parameter  $\kappa$ ; it gives the maximum number of vacancies that is affordable, given the workers' gross income. Since the wage is fixed, increased gross income implies



longer work hours. Longer work hours increase the return to the firm, which thus lowers the vacancy cost per unit of firm revenue. In this way there is a positive effect on the feasible tightness from longer work hours. We will refer to this effect as "the work hour effect" in the subsequent analysis.

## 2.4 The Government

The government's only objective is redistribution. Hence, all the tax that is collected is redistributed back as transfers. The government seeks to find the set of Pareto optimal solutions to the tax problem. As noted above, we do not consider unemployment benefits; only the employed workers are exposed to taxes and transfers.<sup>4</sup> As in the basic Stiglitz (1982) model, we assume that the government can only tax income; each worker's ability is not revealed to the government.

In order to simplify the government's role we make the assumption that the interest rate ( $r$ ) approaches zero. As we will see, this means that the government needs not to care about all four different types/states of workers. When  $r$  tends to zero, it turns out that today's fraction of the total state value will be equal across states; the workers only differ according to type, not according to state. Formally (9), (7) and (8) together with (4) imply that:

$$\lim_{r \rightarrow 0} rU_j = \lim_{r \rightarrow 0} rE_j = (1 - u_j) \underbrace{[C_j - g(L_j)]}_{v_j} + u_j \underbrace{[-g(s_j)]}_{v_{uj}}. \quad (16)$$

The intuition for (16) is rather clear cut. We can think of the right hand side as an expected value, where  $(1 - u_j)$  is the probability of being employed and  $u_j$  the probability of being unemployed. With no discounting

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<sup>4</sup>The rationale for ignoring benefits is twofold: i) It simplifies the model substantially ii) There are no unemployed – hence no benefits – in the Stiglitz (1982) model. In order to make the analogy as complete as possible we therefore ignore benefits. This equalizes the choice set of the tax instruments in our model with the choice set in the Stiglitz (1982) model.

and infinitely lived workers, today's fraction of the total state value is simply a weighted average of the instantaneous utilities in the potential states. Today's instantaneous utility does not enter the expression since it is of infinitesimal size compared to the total state value.<sup>5</sup>

#### 2.4.1 The Optimal Tax Problem – Pareto Efficient Taxation

Define today's fraction of the total value of being of type  $j$  according to:

$$V_{ej} \equiv \lim_{r \rightarrow 0} rU_j = \lim_{r \rightarrow 0} rE_j = (1 - u_j) \left[ C_j - g\left(\frac{Y_j}{w_j}\right) \right] + u_j [-g(s_j)]. \quad (17)$$

The Pareto efficient tax regimes are then given by maximizing  $V_{eH}$  w.r.t.  $C_H, Y_H, C_L$  and  $Y_L$ , subject to  $V_{eL} = \bar{V}_L$ , where  $\bar{V}_L$  is the "promised" utility (or promised fraction of total value) to the low skilled workers, plus the two additional constraints that we discuss below.<sup>6</sup>

The self selection constraint (SSC)

Since the government's information set is limited, the tax function can only be conditioned on income and not on the type of worker. The government does not observe each worker's wage or work hours separately, it can only observe each worker's gross earnings. This opens up for workers to engage in mimicking. Specifically a high ability worker can pretend to be a low ability worker, by working less in order to earn the same gross income as a low ability worker.<sup>7</sup> The government needs thus to ensure that the high ability worker prefers the income that was intended for her before any other income. One needs only consider one critical point on the gross income scale,

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<sup>5</sup>Note however that when  $r$  tends to zero, it still holds that  $E_j > U_j$  even though  $rE_j = rU_j$ .

<sup>6</sup>When we let  $r$  approach zero and take  $V_{ej}$  as the government's objective function, there is no need to "care" for the unemployed workers in the economy; in the government's notion they are as well off as the employed. This eliminates one intuitive reason for giving unemployment benefits.

<sup>7</sup>In principle it is possible also for the low ability worker to mimic the high ability worker. In this paper however, we choose to ignore this possibility and focus on the "normal" case, which also most of the existing literature has focused on.

namely the income that was intended for the low ability worker. To ensure that the high ability worker will have no incentive to engage in mimicking, the following must hold:

$$C_H - g\left(\frac{Y_H}{w_H}\right) \geq C_L - g\left(\frac{Y_L}{w_H}\right). \quad (18)$$

Since only the employed workers have the option to mimic, it is  $v_H$  that enters (18) instead of  $V_{eH}(\cdot)$ . However, the presence of mimickers will affect the value of a job ( $J_H$ ) and hence the tightness ( $\theta_H$ ). This means that it is not obvious that (18) rules out the possibility of a Nash equilibrium in which some workers are engaging in mimicking, that is Pareto dominant to the equilibrium with no mimicking. The critical case we need to look closer at is when (18) holds with equality. The workers are thus indifferent between mimicking or not, on the individual level; hence every outcome, in which a fraction  $\alpha \in [0, 1]$  of the workers engage in mimicking, is a Nash equilibrium. But the presence of mimicking has general equilibrium effects – working through the tightness – on the workers' utility.  $V_{eH}(\cdot)$  is thus not independent of  $\alpha$ . However, it is straightforward to show that  $\frac{dV_{eH}(\cdot)}{d\alpha} < 0$ , which means that the Nash equilibrium with no mimicking is Pareto dominating all other equilibria. We assume in the subsequent analysis that  $\bar{V}_L$  is large enough to make (18) hold with equality. The self selection constraint will then be:

$$C_H - g\left(\frac{Y_H}{w_H}\right) = C_L - g\left(\frac{Y_L}{w_H}\right). \quad (19)$$

The government's budget restriction (GBR)

The second constraint is simply the government's budget restriction. We assume that the government has no other goal than redistribution, so total consumption should equal total income in the economy.<sup>8</sup> That is:

$$(1 - u_H) Y_H + (1 - u_L) Y_L = (1 - u_H) C_H + (1 - u_L) C_L. \quad (20)$$

We can now express the government's problem as:

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<sup>8</sup>Since the interest rate is zero there are no profits made in the economy.

$$\max_{C_H, Y_H, C_L, Y_L} V_{eH} \quad (21)$$

s.t.

$$V_{eL} = \bar{V}_L \quad (22)$$

$$C_H - g\left(\frac{Y_H}{w_H}\right) = C_L - g\left(\frac{Y_L}{w_H}\right) \quad (23)$$

$$(1 - u_H) Y_H + (1 - u_L) Y_L = (1 - u_H) C_H + (1 - u_L) C_L \quad (24)$$

The Lagrange function for this problem is:

$$\begin{aligned} \Psi = & V_{eH} + \mu [V_{eL} - \bar{V}_L] + \lambda_H \left[ C_H - g\left(\frac{Y_H}{w_H}\right) - C_L + g\left(\frac{Y_L}{w_H}\right) \right] + \\ & \gamma [(1 - u_H) Y_H + (1 - u_L) Y_L - (1 - u_H) C_H - (1 - u_L) C_L], \end{aligned} \quad (25)$$

where all shadow prices,  $\mu$ ,  $\lambda_H$  and  $\gamma$ , are positive.

When we let the government control the workers' gross income and consumption in this way, we implicitly assume that there exists a tax function that renders the intended consumption decisions. For any given set of  $[C_j, Y_j]$ , the effective marginal tax rate is given by (10), and the total tax paid by a worker of type  $j$ , is  $Y_j - C_j$ . In general there is indeed an infinite number of tax functions corresponding to each set of  $[C_H, C_L, Y_H, Y_L]$ , provided that  $[C_H, C_L, Y_H, Y_L]$  satisfies (20) and (18).<sup>9</sup>

The first order conditions for the government's optimal tax problem can be written as:

$$\begin{aligned} \frac{\partial \Psi}{\partial C_H} &= (1 - u_H)(1 + \lambda_H) - \\ &\quad \gamma(1 - u_H) - \gamma(Y_H - C_H) \frac{\partial u_H}{\partial s_H} \frac{\partial s_H}{\partial V_H} \\ &= 0, \end{aligned} \quad (26)$$

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<sup>9</sup>This method of solving the government's maximization problem follows Stiglitz (1982) and it is described in greater detail in that paper.

$$\begin{aligned}
\frac{\partial \Psi}{\partial Y_H} &= -(1 - u_H)(1 + \lambda_H) \frac{g'(\frac{Y_H}{w_H})}{w_H} \\
&\quad - (1 + \lambda_H) \frac{\partial u_H}{\partial \theta_H} \frac{\partial \theta_H}{\partial Y_H} \left( C_H - g(\frac{Y_H}{w_H}) + g(s_H) \right) \\
&\quad + \gamma(1 - u_H) + \gamma(Y_H - C_H) \frac{\partial u_H}{\partial s_H} \frac{\partial s_H}{\partial v_H} \frac{g'(\frac{Y_H}{w_H})}{w_H} \\
&\quad - \gamma(Y_H - C_H) \frac{\partial u_H}{\partial \theta_H} \frac{\partial \theta_H}{\partial Y_H} \\
&= 0,
\end{aligned} \tag{27}$$

$$\begin{aligned}
\frac{\partial \Psi}{\partial C_L} &= \mu(1 - u_L) - \lambda_H(1 - u_H) - \\
&\quad \gamma(1 - u_L) - \gamma(Y_L - C_L) \frac{\partial u_L}{\partial s_L} \frac{\partial s_L}{\partial v_L} \\
&= 0,
\end{aligned} \tag{28}$$

$$\begin{aligned}
\frac{\partial \Psi}{\partial Y_L} &= -\mu(1 - u_L) \frac{g'(\frac{Y_L}{w_L})}{w_L} \\
&\quad - \mu \frac{\partial u_L}{\partial \theta_L} \frac{\partial \theta_L}{\partial Y_L} \left( C_L - g(\frac{Y_L}{w_L}) + g(s_L) \right) \\
&\quad + \lambda_H(1 - u_H) \frac{g'(\frac{Y_L}{w_H})}{w_H} \\
&\quad + \gamma(1 - u_L) + \gamma(Y_L - C_L) \frac{\partial u_L}{\partial s_L} \frac{\partial s_L}{\partial v_L} \frac{g'(\frac{Y_L}{w_L})}{w_L} \\
&\quad - \gamma(Y_L - C_L) \frac{\partial u_L}{\partial \theta_L} \frac{\partial \theta_L}{\partial Y_L} \\
&= 0.
\end{aligned} \tag{29}$$

**Proposition 1** The first order conditions of the optimal tax problem imply that  $T'_H < 0$  and  $T'_L \geq 0$ .

**Proof** Combining (26) with (27) and using (10) gives:

$$T'_H = \underbrace{\frac{\frac{\partial u_H}{\partial \theta_H} \frac{\partial \theta_H}{\partial Y_H}}{\gamma(1-u_H)}}_{-} \underbrace{\left[ \gamma(Y_H - C_H) + C_H - g\left(\frac{Y_H}{w_H}\right) + g(s_H) \right]}_{+} < 0. \quad (30)$$

Combining (28) with (29) and using (10) gives:

$$T'_L = \underbrace{\frac{-\frac{g^0(\frac{Y_L}{w_L})}{w_L} + 1}{1 + \frac{\gamma(1-u_L)}{\lambda_H}}}_{+/-} + \underbrace{\frac{\mu \frac{\partial u_L}{\partial \theta_L} \frac{\partial \theta_L}{\partial Y_L} \left( C_L - g\left(\frac{Y_L}{w_L}\right) + g(s_L) - \gamma(Y_L - C_L) \right)}{\lambda_H + \gamma(1-u_L)}}_{-} \geq 0. \quad (31)$$

■

We have now established that the marginal tax on the high ability workers should be negative and that we can not rule out negative marginal tax on the low income group. For a Walrasian model the corresponding result is  $T'_H = 0$  and  $T'_L > 0$ . The result for the high ability group stems from the positive external effect of working long hours that we described above. Long work hours increase the tightness, since it has a positive effect on the firms' revenues; long work hours reduce the vacancy cost per work hour. This "work hour effect" is however not internalized in the employed workers' utility maximizing consumption choices, so the government has incentives to use the tax instruments to encourage extended work hours. This can be achieved by reducing the marginal taxes.

The work hour effect is present also for the low ability worker, but in that case there are other forces at work as well. In order to get a more profound understanding of what drives the result for the low skilled, we need to take a closer look at the basic Stiglitz (1982) model.

The result that the optimal marginal tax on high skilled is zero in the basic Stiglitz (1982) model is not very surprising (see Stiglitz, 1987, page 1004, for a brief rationalization of this result). But why should the marginal tax on low skilled be positive? The answer is that increasing the marginal tax on the low skilled is the only way the government can obtain more money from the high skilled and redistribute to the low skilled, provided that the SSC

binds. Think of an initial situation where the "promised" utility to the low skilled ( $\bar{V}_L$ ) is sufficiently large to make the SSC bind. What happens when we increase the "promise"? Without changing the low skilled's marginal tax, the mimicker will then be better off than the non mimicking high skilled worker. The only way to offset this is to increase the marginal tax for the low skilled group. Increased marginal tax on the low skilled reduces the mimicker's utility more than the low skilled's utility. This is formally shown in Appendix B but it is easily made plausible from an intuitive point of view. We know that the mimicker is always better off than the low skilled worker, since she has a higher wage. But in the limit where the marginal tax for the low skilled individuals approaches one, neither the low skilled worker nor the mimicker will work at all. This eliminates the utility difference between the mimicker and the low skilled worker; there is no point in having a high wage if you do not work. This intuitive experiment thus suggests that increasing the low skilled's marginal tax has more severe effects on the mimicker's utility than on the low skilled's utility.

We can now summarize the intuition for the basic Stiglitz (1982) model as follows. In a situation where the "promised" utility to the low skilled group is sufficiently low for the SSC not to bind, we have a first best solution with zero marginal tax for both groups. Increasing the "promise" makes the SSC bind. By the above intuition we then need to raise the low skilled's marginal tax in order to redistribute more income from the high skilled to the low skilled, and thereby keep the "promise". The mechanism works because the higher marginal tax hits the mimicker harder than it hits the low skilled worker.

In our model with search frictions both of the above effects are present. We have the positive effect on the low skilled's marginal rate stemming from the same qualitative source as in the basic Stiglitz (1982) model. But we also have a negative effect on the low skilled's marginal tax stemming from the work hour effect discussed above. It is ambiguous which effect that dominates; the optimal marginal tax rate on the low ability group could be both positive and negative.

## 2.5 Numerical Results

In this section we present numerical results. This gives us the possibility to study the total redistribution rendered by the optimal tax design, as well as the optimal marginal tax rates. The approach taken here is not to present the whole set of Pareto optimal solutions to the tax problem; instead we choose one single point in the Pareto set. This is done through the maximization of a strictly concave welfare function ( $W$ ). The welfare function is given by:

$$W = \frac{1}{\chi} [V_{eH}^\chi + V_{eL}^\chi], \quad (32)$$

where  $\chi \in (-\infty, 1)$  is a parameter that determines the magnitude of the government's egalitarian ambition. The incentives for equality is decreasing in  $\chi$ .<sup>10</sup>

The matching function is taken to be of Cobb-Douglas form, i.e.:

$$M_j = m (s_j u_j)^\sigma v_j^{1-\sigma},$$

where  $m \in (0, \infty)$  and  $\sigma \in (0, 1)$  are exogenous parameters.

The cost of working is given by:

$$g(L_j) = \frac{1}{\rho} L_j^\rho \rightarrow g\left(\frac{Y_j}{w_j}\right) = \frac{1}{\rho} \left[\frac{Y_j}{w_j}\right]^\rho.$$

The uncompensated labor supply elasticity is then given by  $\epsilon_j = \frac{1}{\rho-1}$ . We set  $\rho = 4$  which gives  $\epsilon_H = \epsilon_L = 1/3$ .<sup>11</sup>

The parameters are chosen to roughly represent the United States economy. When calibrating the model we use a linear tax structure, instead of the optimal non-linear one. This is because a linear tax system is probably a better approximation of the existing tax system, than an optimal non-linear

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<sup>10</sup>In the limit when  $\chi \rightarrow -\infty$  we get the maxi-min welfare function; it is also known as Rawlsian welfare function. In the other limit, when  $\chi \rightarrow 1$  we arrive at the utilitarian welfare.

<sup>11</sup>This is in the middle range of the labor supply elasticities reported in the survey by Blundell and MaCurdy (1999). Simulations have been made with both higher and lower values of  $\epsilon_j$  and this does not change the qualitative results concerning the optimal tax design and the generated outcome.



system would be. We set the linear tax to  $t = 0.3$  to represent a total tax revenue at 30% of GDP.<sup>12</sup>

High skilled and low skilled are represented by collage education and high school education ,respectively, as defined in Katz and Autor (1999). For 1990 the wage differential between these two categories are 1.65 , and the two groups can roughly be approximated to be of the same size (see tables 8 and 12 in Katz and Autor, 1999). We therefore set  $y_H = 1.65y_L$ .

We set  $\sigma = 0.5$  which is higher than the estimates reported in Blanchard and Diamond (1989) but lower than the estimate in Pissarides (1986). We let the separation rate be given by  $\phi = 0.000828$  which gives an annual separation rate around 30%, when taking the day as basic time unit.<sup>13</sup> The wages are given by  $w_j = \delta y_j$ , where  $\delta \in (0, 1)$  is an exogenous parameter that needs to be calibrated.  $\delta$ ,  $\kappa$ ,  $y_L$  and  $m$  are chosen to obtain reasonable values of the unemployment rates ( $u_j$ ), the expected time of an unemployment spell in weeks ( $\tau_j$ ) and the total cost of filling a vacancy (measured in weekly labour cost) ( $c_j$ ). We set  $\delta = 0.96$ ,  $\kappa = 1.00$ ,  $m = 0.021$ . This gives  $u_H = 0.055$ ,  $u_L = 0.076$ ,  $\tau_H = 8.2$ ,  $\tau_L = 9.6$ ,  $c_H = 7.2$  ,  $c_L = 6.1$ . According to OECD (1997) the average unemployment rate in the United States was 6.5% for the years 1983-1996. The duration of completed unemployment spells was 11.4 weeks during 1984-1989 (see Layard et al, 1991).<sup>14</sup>

Table 1 shows the simulation result for some different values of  $\chi$ . In the table, "S" stands for the presented search model, "Opt" stands for the optimal tax design, "LS" stands for laissez faire – hence the no tax solution – and "W" stands for a corresponding Walrasian model with no unemployment.<sup>15</sup> Furthermore we define the relative tax paid (or transfer received) by a worker of type  $j$  according to  $\overline{T}_j \equiv \frac{Y_j - C_j}{Y_j}$ .

<sup>12</sup>OECD (2002) reports that total tax revenues as percentage of GDP amount to 29.6% for the United States in the year 2000.

<sup>13</sup>Layard et al (1991) report an average inflow rate into unemployment of 30.8% for 1984-1989.

<sup>14</sup>For the whole set of endogenous variables in the calibrated model, see appendix A.

<sup>15</sup>This is the solution that the search/matching model would generate if we let, e.g.  $\phi \rightarrow 0$  or  $m \rightarrow \infty$ . It belongs to the category "the simplest case" in Stiglitz (1982).

Table 1. Optimal tax design in search equilibrium and Walrasian equilibrium with increasingly egalitarian preferences.

$\chi$		0.9	0.5	-1	-100
$T'_H$	S,opt	-0.042	-0.041	-0.036	-0.008
$T'_L$	S,opt	-0.052	-0.017	0.074	0.370
$\overline{T}_H$	S,opt	0.12	0.12	0.13	0.16
$\overline{T}_L$	S,opt	-0.24	-0.25	-0.27	-0.37
$s_H$	S,opt	0.871	0.870	0.868	0.858
$s_L$	S,opt	0.824	0.825	0.827	0.830
$u_H$	S,opt	0.045	0.045	0.046	0.047
$u_L$	S,opt	0.056	0.056	0.058	0.065
$\frac{V_{eH}}{V_{eL}}$	S,opt	1.25	1.24	1.21	1.14
$u_H$	S,LF	0.044	0.044	0.044	0.044
$u_L$	S,LF	0.061	0.061	0.061	0.061
$s_H$	S,LF	0.912	0.912	0.912	0.912
$s_L$	S,LF	0.767	0.767	0.767	0.767
$\frac{V_{eH}}{V_{eL}}$	S,LF	2.00	2.00	2.00	2.00
$T'_H$	W,opt	0.000	0.000	0.000	0.000
$T'_L$	W,opt	0.008	0.051	0.124	0.464
$\overline{T}_H$	W,opt	0.13	0.13	0.14	0.17
$\overline{T}_L$	W,opt	-0.25	-0.26	-0.28	-0.41
$\frac{V_{eH}}{V_{eL}}$	W,opt	1.21	1.20	1.18	1.09
$\frac{V_{eH}}{V_{eL}}$	W,LF	1.95	1.95	1.95	1.95

Parameter settings:  $y_H = 0.825, y_L = 0.5, w_H = 0.792, w_L = 0.48, \sigma = 0.5, \phi = 0.000828, \kappa = 1, r = 0, m = 0.021$

As expected, more egalitarian preferences give higher redistribution. But it might seem somewhat surprising that the redistribution is quite large even for a high value of  $\chi$ . This can be explained as follows. When being in a situation where the redistribution is so low that the SSC does not bind, increased redistribution is very "cheap", since it can be implemented using

differentiated lump sum tax/transfers.<sup>16</sup> But once the SSC binds, further redistribution becomes expensive, because it can not be attained – for the reasons described above – without raising the low skilleds’ marginal tax.<sup>17</sup>

From the simulation we see that the presence of unemployment makes it harder to redistribute; the relative tax paid by the high skilled workers is lower in the unemployment case than in the Walrasian case. This is not surprising, since now the tax system does not only distort the employed workers’ consumption choice, but also the unemployed workers’ search behavior. The individual search behavior is socially efficient in the laissez faire case, so any distortion of the workers’ incentive to search must imply a dead weight loss.

The efficiency loss from higher redistribution is also captured by the unemployment rates; higher redistribution generates higher unemployment rates for both skill groups. Increasing the marginal tax gives shorter work hours, which through the work hour effect generates lower tightness and higher unemployment. In the case for low skilled this is partially compensated by an increased search intensity resulting from a higher value of being employed. But for the high skilled the search intensity decreases with increased redistribution, which gives an additional push to the unemployment rate. The change in the high skilleds’ unemployment rate is however very small due to the quite modest increase in the marginal tax rate.

One final interesting property of the simulation result needs to be commented on. Why does the optimal tax design consist of a higher high skilled marginal tax for low values of  $\chi$  than for high values of  $\chi$ ? This effect stems

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<sup>16</sup>From a strictly technical point of view one might argue that differentiated lump sum tax/transfer is not a feasible tax instrument, at the given information set. But any tax structure that implies zero marginal tax at both types optimal income choice, has in practice the exact same implication as a differentiated lump sum tax/transfer. It is in this sense we will use the concept ”lump sum tax/transfer” in the subsequent text.

<sup>17</sup>This intuitive argument seem to assume that the welfare function always gives us incentive to redistribute enough to make the SSC bind. This is true in the basic Stiglitz model for any concave welfare function and separable utility functions; see Arnott, Hosios and Stiglitz (1988). But in our model with unemployment this needs to be checked for every choice of  $\chi$ . It turns out that for all the cases presented in the table the SSC does bind. This is also true for the numerical exercise in section 3.

from the fact that the high skilled workers have longer work hours than they would choose in laissez faire equilibrium. This means that raising the marginal tax on high skilled workers gives them higher instantaneous utility, if the total tax paid is held constant.<sup>18</sup> In order to redistribute more to the low skilled individuals, the government thus needs to raise the marginal tax on high skilled and thereby make the non mimicking worker better off.

### 3 Endogenous Wages

We now turn to an extension of the model presented in the previous section. So far wages have been fixed. In this section we will allow for the wages to be determined endogenously. The most frequent practice, in this class of labor market models, is to let the wages be determined by generalized Nash bargaining (see e.g. Pissarides, 2000).

#### 3.1 The Model

The model is identical to the one presented above, apart from the wage determination. When a firm and a worker are matched they bargain over wages. While bargaining, both parts recognize the tax function  $T(wL)$ ; the tax function is exogenous to both parts in the bargaining process. The generalized Nash bargaining solution is given by the following maximization problem:

$$\max_{w_{ji}} [\beta \ln (E_{ji} - U_j) + (1 - \beta) \ln (J_{ji} - V_j^{Vac})], \quad (33)$$

where the sub index  $i$  indicates the specific match i.e. only  $E_{ji}$  and  $J_{ji}$  are functions of  $w_{ji}$ .  $\beta \in (0, 1)$  is a measure of the workers' bargaining power.

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<sup>18</sup>If the functional form of  $T(Y)$  was specified, we would not be able to think of  $T'$  and  $T$  as unrelated, since  $T'$  and  $T$  then would then be linked. But since the tax function can take on any shape, we can treat  $T'$  and  $T$  as parameters. A change in  $T'$  is assumed to be made with a corresponding change in the whole tax function, leaving  $T$  unaffected, and vice versa. In the subsequent text we will treat  $T'$  and  $T$  as distinct parameters.

The first order condition of (33) is given by:<sup>1920</sup>

$$E_j - U_j = \frac{\beta}{1 - \beta} (1 - T'_j) \frac{1}{1 - \epsilon_j \left( \frac{y_j}{w_j} - 1 \right)} J_j \quad (34)$$

Where  $\epsilon_j = \frac{\partial L_j}{\partial w_j} \frac{w_j}{L_j}$  is the labor supply elasticity. By making use of (9), (13) and the free entry condition we can derive the following wage equation from (34):

$$\frac{C_j - g(L_j) + g(s_j)}{r + \phi + s\alpha(\theta_j)} = \frac{\beta}{1 - \beta} (1 - T'_j) \frac{1}{1 - \epsilon_j \left( \frac{y_j}{w_j} - 1 \right)} \frac{\kappa y_j}{q(\theta_j)}. \quad (35)$$

The full equilibrium – in the  $\theta_j$ ,  $s_j$  and  $w_j$  space – is determined by the wage equation above, the first order condition for search (11) and the tightness equation resulting from the free entry condition (14). The comparative static properties of this system of equations are ambiguous; it is unclear how a marginal increase in e.g.  $T'_j$  affects the endogenous variables in equilibrium. There are different mechanisms working in opposite directions. We know that the "work hour effect" indicates a positive relation between  $T'_j$  and  $u_j$ , but this may now be offset by a new wage effect working through the bargaining. However, in the toy-model described below we will work out some useful comparative static properties of a special case of the model described above. This will serve as an illustration of one particular mechanism; it will emphasize the "new" effect that the tax system has on the equilibrium unemployment rate and bargained wages. The "old" effect is the work hour effect: a lower marginal tax, gives longer work hours, which lowers the vacancy cost per work hour. The work hour effect is present in the endogenous wage model as well. But now there is an additional effect at work, and as we will see, this new effect counteracts the work hour effect in some aspects.

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<sup>19</sup>After the derivation we have imposed symmetry and thereby eliminated the subindex  $i$ .

<sup>20</sup>We have here made the assumption that the tax function is locally linear, hence  $T''_j = 0$ .

### 3.1.1 A Toy-Model – Exogenous Labor Supply

The toy-model is a one-type version of the above endogenous wage model.<sup>21</sup> Furthermore we make the assumption that the labor supply and the search intensity are fixed:  $L = \bar{L}$  and  $s = \bar{s}$ .<sup>22</sup> The equations that describe the equilibrium in  $\theta$  and  $w$  space are given by:

$$\frac{\kappa y}{q(\theta)} = \frac{\bar{L}(y - w)}{r + \phi}, \quad (36)$$

and

$$\frac{w\bar{L} - g(\bar{L}) + g(\bar{s})}{r + \phi + \bar{s}\alpha(\theta)} = \frac{\beta}{1 - \beta} (1 - T') \frac{\kappa y}{q(\theta)}, \quad (37)$$

where (36) is the free entry condition, and (37) is the bargained wages.

Once we know  $\theta$  we can calculate  $u$  from the flow equilibrium, which is given by:

$$u = \frac{\phi}{\phi + \bar{s}\alpha(\theta)}. \quad (38)$$

We will now analyze the comparative statics of this model with respect to  $T'$  and  $T$ . Starting with  $T'$  we can derive:

$$\frac{dw}{dT'} < 0, \quad (39)$$

$$\frac{d\theta}{dT'} > 0 \text{ and} \quad (40)$$

$$\frac{du}{dT'} < 0. \quad (41)$$

Raising the marginal tax makes it more costly to give the workers' utility in the form of high wage. The firms will therefore seize a larger part of the

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<sup>21</sup> In this model we do not focus on the optimal tax design; we focus purely on the comparative static properties. These properties are easily illustrated in a model with homogenous workers, so we do not need a second type of skill. This means that we drop the sub index  $j$  and  $\theta_j$  becomes  $\theta$  etc.

<sup>22</sup> This assumption eliminates what we have called the "work hour effect", since the labor supply is now unaffected by changes in the marginal tax.

”cake” in the wage bargaining. Lower wage raises tightness, which through the flow equilibrium gives lower unemployment. This result is consistent with Sørensen (1999) and Hansen (1999) who find, in closely related models, that increased tax progressivity gives lower unemployment.

For  $T$  we arrive at:

$$\frac{dw}{dT} > 0, \quad (42)$$

$$\frac{d\theta}{dT} < 0 \text{ and} \quad (43)$$

$$\frac{du}{dT} > 0. \quad (44)$$

A worker who is exposed to a lump sum tax will thus partially be compensated for this through a rise in the bargained wage. The increase in the bargained wage lowers the feasible tightness, which raises unemployment.

## 3.2 Numerical Results

We now return to the full model with endogenous labor supply and search intensity, keeping the results from the toy-model in mind. Since the model is rather complex, we do not derive any analytical results concerning the optimal tax design. Instead we present some numerical simulations. The simulations are made with the same welfare function and the same parameter settings as for the exogenous wage model (see table 1). We consider the symmetric case  $\beta = 0.5$ , i.e. the worker and the firm share the same bargaining power. The results are presented in table 2.

Table 2. Optimal tax design in search equilibrium with increasingly egalitarian preferences.

$\chi$		0.9	0.5	-1	-100
$T'_H$	S,opt	-0.027	-0.029	-0.034	-0.072
$T'_L$	S,opt	-0.060	-0.027	0.061	0.415
$\overline{T}_H$	S,opt	0.13	0.13	0.14	0.18
$\overline{T}_L$	S,opt	-0.25	-0.26	-0.28	-0.44
$s_H$	S,opt	0.869	0.868	0.865	0.850
$s_L$	S,opt	0.826	0.827	0.830	0.835
$u_H$	S,opt	0.049	0.049	0.050	0.053
$u_L$	S,opt	0.045	0.044	0.042	0.033
$w_H$	S,opt	0.795	0.795	0.795	0.797
$w_L$	S,opt	0.475	0.474	0.472	0.457
$\frac{V_{eH}}{V_{eL}}$	S,opt	1.23	1.22	1.18	1.07
$s_H$	FM	0.912	0.912	0.912	0.912
$s_L$	FM	0.767	0.767	0.767	0.767
$u_H$	FM	0.043	0.043	0.043	0.043
$u_L$	FM	0.052	0.052	0.052	0.052
$w_H$	FM	0.792	0.795	0.795	0.795
$w_L$	FM	0.476	0.476	0.476	0.476
$\frac{V_{eH}}{V_{eL}}$	FM	1.99	1.99	1.99	1.99

Parameter settings:  $\beta = 0.5$  other parameters as in table 1.

In contrast to the exogenous wage model, the optimal marginal tax on high skilled is now increasing in the egalitarian preference parameter  $\chi$ ; increased preference for equality gives lower optimal marginal tax on the high skilled. This is the most striking difference of the optimal tax structure in the endogenous wage model relative to the exogenous wage model. For low skilled, however, the optimal tax structure follows the same qualitative pattern in both models.

When looking at the total tax transfers we see that the redistribution from high skilled to low skilled is higher in the present model than in the previous.



Endogenizing the wages thus makes it easier to redistribute from high to low skilled. This is also captured by the utility ratio  $\frac{V_{eH}}{V_{eL}}$ ; the ratio is – at a given value of  $\chi$  – lower in the present model. The redistribution from high skilled to low skilled is not fully assessed by only looking at the tax system itself. This is since some of the redistribution works through the unemployment rates. We see that the tax system raises the high skilled’s unemployment and lowers the low skilled’s unemployment; the unemployment rate for the high skilled is actually higher than the unemployment for the low skilled for all values of  $\chi$ .

### 3.3 Intuitive Discussion of the Results

First of all, we know from (42) that the redistribution in itself (think of a lump sum tax/transfer from high to low skilled) raises the high skilled’s wage and lowers the low skilled’s wage. The effect of this is higher unemployment for high skilled and lower for low skilled. Even at  $\chi = 0.9$  the redistribution is substantial, but the government has no problem making the SSC hold. This means that the government can use the marginal tax instrument to reach efficiency, rather than keeping the high skilled workers from mimicking. The marginal tax instruments are therefore used to correct for some of the wage adjustments caused by the redistribution. The optimal high skilled marginal tax is relatively high, which restrains the wage increase, and the optimal low skilled marginal tax is relatively low, which restrains the wage decrease.<sup>23</sup> When  $\chi$  decreases the SSC becomes increasingly important. As in the previous fixed wage model, the government reacts to this by increasing the marginal tax on low skilled. But the effect this has on the SSC is now twofold. As in the fixed wage model it has a larger negative labor supply effect on the mimicker than on the low skilled worker. This means that the mimicker is hit harder by the increase in the marginal tax rate, than the

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<sup>23</sup>The marginal tax on the high skilled is negative for all values of  $\chi$  in table 2. This however is not a general result. It may well be that the marginal tax for the high skilled is positive for some other set of parameters.

low skilled worker is.<sup>24</sup> However, raising the marginal tax for the low skilled now also affect the bargained wages. If we believe that the toy-model effect dominates, this implies a decrease in the bargained wages, which we also can observe in table 2. At a first glance this may seem as something that would be bad for the low skilled and therefore would give the government problems to increase the redistribution. But surprisingly and interestingly, it is exactly the opposite way around; this wage effect makes it easier to redistribute in the endogenous wage model. The reason is that the decrease in the wage is partially redistributed back to the low skilled workers in the form of lower unemployment, which is something that the mimicker does not benefit from. Hence both the mimicker and the low skilled worker suffer from the wage decrease – the mimicker suffers because the wage decrease lowers his/her labor supply – but only the low skilled worker benefits from the induced fall in the unemployment rate. The government can therefore use the marginal tax instrument to perform a very intricate form of redistribution through the low skilled unemployment rate. The higher the preference for equality is, the more the unemployment rate is used as a tool for redistribution.

For the high skilled group, the marginal tax is decreased, which contrasts the result from the previous model. The reason is – as in the fixed wage model – that the government needs to make the non mimicking workers better off, in order to redistribute more to the low skilled workers. But when the wages are determined by bargaining, the non mimicking worker’s instantaneous utility is no longer increasing in the marginal tax.<sup>25</sup> Instead – as the toy model suggests – lower marginal tax for high skilled workers raises the wage and thereby the instantaneous utility.<sup>26</sup> However, the previous effect – that

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<sup>24</sup>See the intuitive discussion of the numerical results for the exogenous wage model.

<sup>25</sup>As before we here consider the total tax paid by the high skilled worker as fixed; hence the change in the marginal tax rate is compensated by alteration of the total tax function, in a way that keeps the total tax paid constant.

<sup>26</sup>This intuitive explanation is a little hazy. What matters in the SSC is not only the instantaneous utility of the high skilled worker in itself, but rather the difference between the mimicker’s utility and the non mimicker’s utility. An effect working through the wage does thus affect both the mimicker and the non mimicking worker, which makes the effect on the SSC ambiguous. However, it is straight forward to show that a rise in the high

increased work hours lowers the instantaneous utility for high skilled workers – is still there, but the simulation suggests that the effect of increased wage is larger.

The unemployment rate for the high skilled can be explained analogously. If this new wage effect on unemployment (the toy model effect) outweighs the work hour effect, a decrease in the marginal tax rate would give higher unemployment instead of lower. As we see in table 2, lower marginal tax is accompanied by higher unemployment, which indicates that the wage effect does indeed outweigh the work hour effect. Lower marginal tax on high skilled thus raises the high skilled’s wage, lowers the tightness and increases the unemployment rate.

## 4 Concluding Remarks

The paper has extended the basic Stiglitz (1982) model of optimal income taxation into general search equilibrium. In the model with fixed wages we find that the ”work hour effect” gives the government incentives to lower the marginal tax rate for both high and low skilled workers. The result from the basic Stiglitz (1982) model – zero marginal tax on high skilled and positive marginal tax on low skilled – is modified to negative marginal tax on high skilled and ambiguous sign for low skilled, in the exogenous wage model. From simulations we find that the optimal marginal tax on high skilled depends positively on the level of redistribution. The reason for this is that raising the marginal tax on high skilled workers gives them higher instantaneous utility, if the total tax paid is held constant. In order to redistribute more to the low skilled individuals, the government thus needs to raise the marginal tax on high skilled and thereby make the non mimicking worker better off.

When the wages are determined endogenously by bargaining between the firm and the worker, simulations show that the results for high skilled are 

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skilled wage has a larger positive effect on the non mimicking worker’s utility than it has on the mimicker’s. The wage effect from lowering the high skilled’s marginal tax does thus also make the non mimicking worker better off relative to the mimicker.

dramatically changed. We can no longer rule out positive marginal tax on high skilled and the optimal marginal tax on high skilled now depends negatively on the level of redistribution. The reason for this is that the bargained wage may depend negatively on the marginal tax. In order to increase the instantaneous utility of the non mimicking worker the government needs also to take this new wage effect into consideration.

Endogenizing the wages opens up for the government to perform redistribution through the unemployment rate. Raising the marginal tax on low skilled has – at least in the numerical examples we have looked at – a negative effect on the low skilleds’ unemployment rate. Giving the low skilled workers utility in the form of low unemployment is a very elegant way to redistribute, since the mimicker does not benefit from a decrease in unemployment for low skilled.

When looking at the optimal redistribution we find that the presence of unemployment makes it harder for the government to redistribute from high to low skilled. This result holds for both the exogenous wage model and the endogenous wage model, but there is least redistribution in the exogenous wage model.

In future work it would be interesting to explore the optimal tax transfer regime when the government utilizes the full information set. This means that the government would not only set an optimal non-linear tax function, but also an optimal non-linear unemployment benefit function. The benefits would thus be a function of the income. When the firm and the worker meet to bargain over the wage, they will recognize that the future value of being unemployed depends on the bargained wage. This may give the government a powerful instrument to monitor the wage formation.

# Appendix A

Table 2. Calibrated model, linear tax  $t = 0.3$ .

Model	Exogenous wages	Endogenous wages $\beta = 0.5$
$u_H$	0.055	0.058
$u_L$	0.076	0.069
$L_H$	0.82	0.82
$L_L$	0.70	0.69
$s_H$	0.81	0.81
$s_L$	0.68	0.68
$w_H$	0.792	0.794
$w_L$	0.480	0.478
$\frac{V_{eH}}{V_{eL}}$	2.01	2.00
$\theta_H$	0.69	0.63
$\theta_L$	0.50	0.61
$\tau_H$	8.16	8.60
$\tau_L$	9.65	8.69
$c_H$	7.19	6.81
$c_L$	6.08	6.78
$q(\theta_H)$	0.025	0.027
$q(\theta_L)$	0.030	0.027
$\alpha(\theta_H)$	0.018	0.017
$\alpha(\theta_L)$	0.015	0.016

## Appendix B

We want to prove:

$$\frac{dv\left(C_L, \frac{Y_L}{w_L}\right)}{dT'_L} > \frac{dv\left(C_L, \frac{Y_L}{w_H}\right)}{dT'_L}$$

Proof

Straightforward derivation gives:

$$\begin{aligned} & \frac{dv\left(C_L, \frac{Y_L}{w_L}\right)}{dT'_L} - \frac{dv\left(C_L, \frac{Y_L}{w_H}\right)}{dT'_L} \\ &= \frac{dC_L}{dT'_L} \left[ v_1\left(C_L, \frac{Y_L}{w_L}\right) - v_1\left(C_L, \frac{Y_L}{w_H}\right) \right] + \\ & \quad \frac{dY_L}{dT'_L} \left[ v_2\left(C_L, \frac{Y_L}{w_L}\right) \frac{1}{w_L} - v_2\left(C_L, \frac{Y_L}{w_H}\right) \frac{1}{w_H} \right] \\ & > 0, \end{aligned}$$

since:

$$\begin{aligned} w_H > w_L &\rightarrow \frac{Y_L}{w_H} < \frac{Y_L}{w_L} \rightarrow v_2\left(C_L, \frac{Y_L}{w_H}\right) > v_2\left(C_L, \frac{Y_L}{w_L}\right) \rightarrow \\ v_2\left(C_L, \frac{Y_L}{w_H}\right) \frac{1}{w_H} &> v_2\left(C_L, \frac{Y_L}{w_L}\right) \frac{1}{w_L}. \end{aligned}$$

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