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Procurement Bidding in First-Price and Second-Price Sealed Bid Common Value Auctions

Anders Lunander^{*}

Abstract

Within the framework of the common value model, we examine the magnitude of the difference in expected outcome between first-price and second-price sealed bid auctions. The study is limited to two empirical specifications of bidders' signals: Weibull and normal distribution. The optimal bid functions and the expected procurer's cost under both auction formats are derived. Simulations are undertaken to analyze the impact that random draws of signals have on the differences in outcome from the two auction formats. Using estimates from structural estimation in previous empirical work on first-price auction data, where Weibull and normal distributions of signals have been applied, the hypothetical expected gain from switching from a first-price sealed bid auction to a second-price sealed bid auction mechanism is computed.

Keywords: Common value auctions, procurement, Vickrey auction

JEL Classification: D44 (auctions)

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1. Introduction

Government procurement, which represents an important share of total government expenditure (typically 10–15% of GDP), is regulated by internationally agreed trade rules under the World Trade Organization. Members of the European Community (EC) must also follow EC procurement directives. A cautious interpretation of these agreements and rules is that they restrict bidding to a sealed-bid procedure and do not allow the procurer to use open or sequential bidding mechanisms, such as the English auction.¹ The dominant bidding mechanism overall seems to be the first-price sealed bid auction (first-price auction henceforth). From auction theory, however, we know that in some environments the second-price sealed bid auction (second-price auction henceforth) is at least as good or even better than the first-price auction in terms of the procurer's expected cost. Milgrom and Weber (1982) show that if an affiliated auction model, such as the symmetric common value model, is used to describe a particular bidding environment, then the second-price auction yields a lower expected procurement cost than the first-price auction mechanism.

In view of this theoretical result, the infrequent use of the second-price auction procedure in procurement auctions is somewhat puzzling. It is true that if we remove the assumptions of risk-neutral bidders (firms), symmetry, and non-cooperative behavior, the first-price auction may be preferable to the second-price auction, but it is unlikely that this alone explains why second-price auctions are rare. Rothkopf et al. (1990) consider various possible reasons for the scarcity of the second-price auction, some of which they find to be plausible. One such reason is that a bidder may be reluctant to bid his reservation cost if he fears that the procurer in a second-price auction will cheat by putting in imaginary bids to force the price below the

¹ Article XIII:1–3 of the Agreement on Government Procurement (WTO) states that "the opportunities that may be given to tenderers to correct unintentional errors of form between the opening of tenders and the awarding of the contract shall not be permitted to give rise to any discriminatory practice," and that "all tenders solicited . . . shall be received and opened under procedures and conditions guaranteeing the regularity of the openings."

second-lowest bid. Another is that by bidding its true cost a firm reveals valuable information which it might be important to keep secret if the firm anticipates that, in the event of winning the auction, it will have to negotiate with subcontractors, labor unions, or financial institutions. The authors find the argument that the rarity of the second-price auction could be due to inertia unpersuasive. Although institutions are slow to implement innovations, it seems unlikely that Vickrey's result from the sixties could have passed unnoticed, without any measures being taken.

Assuming that institutions have long been familiar with the revenue-ranking of first- and second-price auctions, this paper sets out to explain part of the passivity of institutions in regard to adopting the second-price auction by examining the cardinal difference between the auction formats. It is important to consider the magnitude of the expected gain from switching auction procedures and not focus solely on the ordinal ranking of auction procedures. Although a switch from a first-price to a second-price auction yields an expected lower procurement cost, this gain may be regarded as too small to cover the implementation cost. The benefits have to be substantial in order to give up a well-tested and reliable procurement method for an untried mechanism.

A number of theoretical and empirical studies have focused on bidding behavior in the symmetric first-price common value auction, given different empirical specifications of the distribution of bidders' signals and the distribution of the true cost [e.g. Rothkopf (1969), Smiley (1979), Thiel (1988), Levin and Smith (1991), Paarsch (1992), Wilson (1992)]. Little work has, however, been devoted to deriving the explicit bid functions for the second-price auction and comparing the predicted outcome with that from the first-price auction, given the same underlying structure of cost and signals.

In this paper we map the general symmetric bid functions of the first-price and second-price auctions onto two different assumptions regarding the distribution of the bidders' signals: a

normal distribution and a Weibull distribution. These are the only distributions that to our knowledge have been applied to and estimated for first-price auction data with fairly good fit (Paarsch, 1992). Given the assumptions of the common value auction model, we can use these structural estimates to derive hypothetically the predicted gain of using the second-price auction instead.

The paper is organized in the following manner: Section 2 presents the bidding model for the first-price and second-price auction. In Section 3 and 4 the theoretical models are mapped onto the empirical specifications, and the optimal bid functions and the expected procurer's cost are derived. In Section 5 we use estimates from the structural estimation in Paarsch (1992) on first-price auction data to illustrate the procurer's expected cost reduction by switching from a first-price auction to a second-price auction procedure. Finally, Section 6 summarizes and concludes the paper.

2. The Model

It is assumed that there are n risk-neutral bidders bidding for a particular contract, where the cost of carrying out the contract, c , is identical but unknown to all bidders prior to bidding. The bidder who submits the lowest bid is selected as the winner and awarded the contract. Before the auction, each bidder receives a private signal, z_i , concerning the cost of the contract, which he uses to form an unbiased estimate of c , that is $E(c/z_i)$. No bidder knows the estimate of any other bidder. The bidders' signals, z_i , are positively correlated (affiliated) with a cumulative distribution function, $F(z|c)$. The bidders have prior beliefs about the true cost, c , which is characterized by the cumulative distribution function, $G(c)$. The number of bidders and the distribution functions of z_i and c are assumed to be common knowledge. In deriving the equilibrium strategies in both auctions, I focus on bidder i .

2.1 First-Price Sealed Bid Auction²

In the first-price auction, where the winning bidder is paid the amount of his bid to perform the task, the bidder, given his signal z_i , sets his bid b_i to maximize the expected payoff

$$\Pi(b, z) = \int_{-\infty}^{\infty} (b_i - c) \left(1 - F(\mathbf{b}^{-1}(b_i)|c)\right)^{n-1} g(c|z_i) dc \quad (1)$$

where

$$g(c|z_i) = \frac{f(z_i|c)g(c)}{\int_{-\infty}^{\infty} f(z_i|c)g(c)dc} \quad (2)$$

is the posterior distribution of c ; $\mathbf{b}^{-1}(b_i)$ is the inverse of the equilibrium strategy function of the $n-1$ other bidders; and $\left(1 - F(\mathbf{b}^{-1}(b_i)|c)\right)^{n-1}$ is the probability that bidder i wins the contract.

Substituting (2) into (1), the maximization problem can be written as

$$\max_{b_i} \int_{-\infty}^{\infty} (b_i - c) \left(1 - F(\mathbf{b}^{-1}(b_i)|c)\right)^{n-1} f(z_i|c)g(c)dc \quad (3)$$

which yields the first-order condition

$$\begin{aligned} & \int_{-\infty}^{\infty} \left(1 - F(\mathbf{b}^{-1}(b_i^*)|c)\right)^{n-1} f(z_i|c)g(c)dc \\ & - \int_{-\infty}^{\infty} (b_i^* - c)(n-1) \left(1 - F(\mathbf{b}^{-1}(b_i^*)|c)\right)^{n-2} f(\mathbf{b}^{-1}(b_i^*)|c) \frac{d\mathbf{b}^{-1}(b_i^*)}{db_i} f(z_i|c)g(c)dc = 0. \end{aligned} \quad (4)$$

Imposing symmetry among bidders, $b^* = \mathbf{b}(z_i)$ and rearranging the terms gives the first-order differential equation

$$\mathbf{b}(z) = e^{-\int_{-\infty}^z p(u)du} \left(\int_{-\infty}^z e^{\int_{-\infty}^u p(t)dt} q(u)du + C \right) \quad (5)$$

where the constant, C , is determined by the appropriate boundary conditions.

2.2 Second-Price Sealed Bid Auction³

² The derivation in this section is based on Paarsch (1992). See also Wilson (1977).

³ The derivation is based on Milgrom and Weber (1982).

In the second-price auction, where the winning bidder is paid the amount of the second-lowest bid to perform the task, the bidder's expected payoff when its signal is z_i and it bids an amount b is

$$\begin{aligned}\Pi(b, z) &= \int_{b^{-1}(b)}^{\infty} (\mathbf{b}(y) - c(z, y)) f_{Y_1}(y|z) dy \\ &= \int_{b^{-1}(b)}^{\infty} (c(y, y) - c(z, y)) f_{Y_1}(y|z) dy\end{aligned}\tag{6}$$

where $c(z, y) = E[C_i | Z_i = z, Y_1 = y]$, which is the expectation of the cost to bidder i when the signal received by him is z and the lowest signal among the other bidders is y . The second-last term in equation (6) denotes the conditional probability distribution function of the first-order statistics, that is, the smallest order statistics among the signals $(Z_j | j \neq i)$. Since c is increasing in its first arguments [for all $y < z$, $c(z, y) - c(y, y) > 0$ and for all $y > z$, $c(z, y) - c(y, y) < 0$] the integral is maximized by choosing b so that $\mathbf{b}^{-1}(b) = z$, or equivalently by choosing $b = \mathbf{b}(z)$. Thus, symmetric equilibrium in a second-price auction is defined as

$$\mathbf{b}(z) = c(z, z).\tag{7}$$

Milgrom (1981) shows that this can be written as

$$\mathbf{b}(z) = \frac{\int c(n-1)f(z|c)^2[1-F(z)]^{n-2}g(c)dc}{\int (n-1)f(z|c)^2[1-F(z)]^{n-2}g(c)dc}.\tag{8}$$

3. Bidding under Weibull Distributed Signals

The bidder's signal is drawn from a Weibull distribution, with the probability density function

$$\begin{aligned} f(z_i|c) &= \mathbf{g}_1 \mathbf{g}_2 z_i^{\mathbf{g}_2-1} e^{-\mathbf{g}_1 z_i^{\mathbf{g}_2}} & 0 < \mathbf{g}_1, \mathbf{g}_2 \text{ and } 0 \leq z_i \\ g(c) &= \begin{cases} U[0, \infty] = k \\ \propto 1/c^l \end{cases} \end{aligned} \quad (9)$$

where $E[z_i] = c$ implying that $\mathbf{g}_1 = \left(\frac{\Gamma(1+1/\mathbf{g}_2)}{c} \right)^{\mathbf{g}_2}$.

The dispersion of $f(z_i|c)$ is related to the inverse of \mathbf{g}_2 , that is, the higher the values of \mathbf{g}_2 , the more concentrated the distribution. Further, it is assumed that

$$g(c) \propto 1/c^2. \quad (10)$$

Smiley (1979) shows that if $g(c)$ is proportional to $1/c^l$ where l is a real number, then the symmetric equilibrium bid function is proportional to the signal; that is, $\mathbf{b}(z_i) = \mathbf{r}z_i$, where $\mathbf{r} (> 1)$ is a constant of proportionality.

Substituting (9) and (10) into Bayes's rule, we obtain

$$E[c|z_i] = \frac{\int_0^\infty cf(z_i|c)g(c)dc}{\int_0^\infty f(z_i|c)g(c)dc} = z_i \quad (11)$$

Equation (11) shows that the posterior expected value is just the signal itself, that is, non-informative. The bidder's prior expectation about c does not shift the posterior expected value of c away from its signal z_i .⁴

⁴ Making the assumption that $g(c) \propto 1/c$ gives $E[c|z_i] = \Gamma(1+1/\mathbf{g}_2)\Gamma(1-1/\mathbf{g}_2)z_i$

3.1 Optimal Bid Functions

Substituting the distributional specifications into the general bid functions for the first-price and second-price sealed bid auctions, (5) and (8), the optimal bid functions are computed as

$$\begin{aligned} \text{(first-price auction)} \quad b(z_i) &= \frac{g_2(n-1)n^{1/g_2}}{g_2(n-1)-1} z_i \\ (12) \end{aligned}$$

$$\begin{aligned} \text{(second-price auction)} \quad b_{2nd}(z_i) &= \frac{n^{1/g_2} g_2}{g_2 + 1} z_i. \\ (13) \end{aligned}$$

Figures 1a and 1b plot the optimal bid functions against the number of bidders, given $g_2 = 5$, and against different values of g_2 , given $n = 8$. The figures illustrate two main predictions within the symmetric common value auction model: (i) the optimal bid in the first-price auction initially decreases with the number of bidders due to competition, but then, as the fear of the winner's curse dominates the competitive effect, the bid increases with the number of bidders; (ii) an increased dispersion of signals (lower value of g_2) gives rise to a higher adverse selection bias, which causes the bidder to adjust his bid upwards.

Figure 1a: Optimal Bid versus Number of Bidders ($z=1, g_2 = 5$)

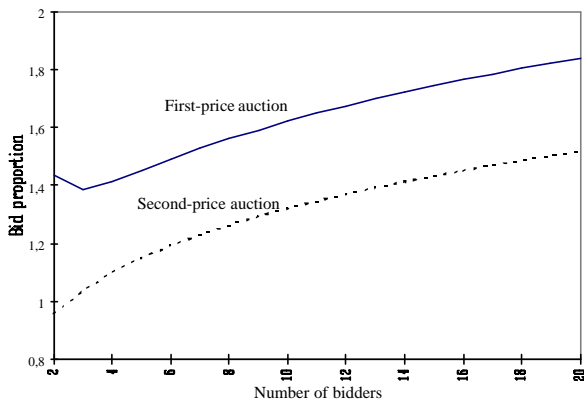
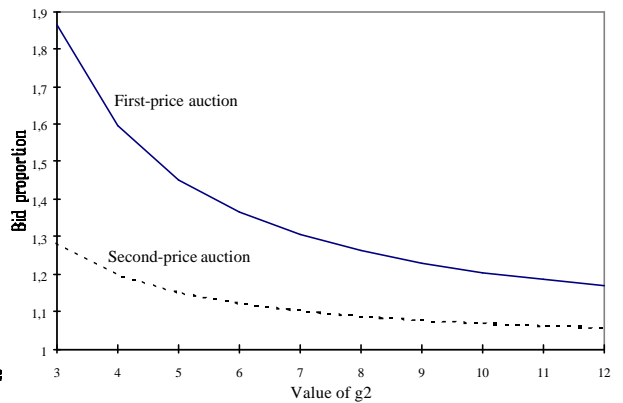


Figure 1b: Optimal Bid versus Dispersion of Signals ($z=1, n=5$)



3.2 Expected and Simulated Procurement Cost

In order to rank the two auction formats in terms of lowest procurement cost, the expected winning bid $E[w]$ in the first-price auction is compared with the expected second-lowest bid $E[b_{(2:n)}]$ in the second-price auction. Making use of

$$h(b_{(i:n)}) = f(z_{(i:n)}) \frac{dz_{(i:n)}}{db_{(i:n)}} \text{ where } f(z_{(i:n)}) = \frac{n!}{(i-1)!(n-1)!} \{F(z)\}^{i-1} \{1-F(z)\}^{n-1} f(z), \text{ the}$$

procurer's expected payment under both auction regimes is derived as

$$\text{(first-price auction)} \quad E[w] = \frac{g_2(n-1)}{g_2(n-1)-1} c, \quad (14)$$

$$\text{(second-price auction)} \quad E[b_{(2:n)}] = \frac{n^{1/g_2} g_2}{g_2 + 1} \left(\frac{n}{(n-1)^{1/g_2}} - \frac{n-1}{n^{1/g_2}} \right) c. \quad (15)$$

In table 1 the difference between (14) and (15) is computed for various number of bidders and two sizes of the dispersion of signals. By letting $c=1$, the procurer's expected payment equals the constant of proportionality.

Table 1: The procurer's expected cost ($c=1$)

(1) n	$g_2 = 5$			$g_2 = 10$		
	(2) $E(w)$	(3) $E(b_{(2:n)})$	(4) Diff.	(5) $E(w)$	(6) $E(b_{(2:n)})$	(7) Diff.
2	1,25	1,081	0,169	1,111	1,040	0,072
3	1,111	1,045	0,067	1,053	1,022	0,031
4	1,071	1,031	0,041	1,034	1,015	0,019
5	1,053	1,023	0,029	1,026	1,012	0,014
10	1,023	1,011	0,012	1,011	1,005	0,006
15	1,014	1,007	0,007	1,007	1,003	0,004
20	1,011	1,005	0,005	1,005	1,003	0,003

The table shows that the difference between the constant of proportionality of the bid the procurer pays diminishes rapidly as the number of bidders increases. Increasing the dispersion

of signals, i.e., lowering g_2 , drives up the difference in the procurer's expected payment between the auction formats, but the magnitude of this difference is reduced to 1–3% when there are more than four bidders.

To complete the picture of the differences in outcome between the two auction formats, a number of simulations are carried out. For a given number of n bidders, n signals z are randomly drawn from the Weibull distribution and evaluated in the optimal bid functions of the two auction formats (equations 12 and 13). For each draw of n signals, we then compute the difference between the lowest bid in the first-price auction and the second-lowest bid in the second-price auction, that is

$$b_{1^{st}}(z_{(1:n)}) - b_{2^{nd}}(z_{(2:n)}). \quad (16)$$

Setting $E[c] = 1$ throughout the simulations implies that g_1 is determined for a given value of g_2 . I focus on two cases of dispersion of signals, $g_2 = 5$ ($\rightarrow g_1 = 0.6525$) and $g_2 = 10$ ($\rightarrow g_1 = 0.6073$). The number of bidders ranges from 2 to 20. Using the specifications above, each auction is simulated 10 000 times and the mean, the median and a 90% confidence interval are computed. The results from the simulations are presented in figures 2a and 2b. The computed mean values of the 10 000 simulated differences for a given number of bidders reflect the results in column four and column seven in table 1. Positive values show that the procurer's cost is higher under the first-price auction. The median value of the difference in procurement cost is located above the mean value, indicating that the probability density of differences is concentrated well above zero.

Figures 2a-2b: Simulated Differences in Procurement Costs under Weibull Distributed Signals

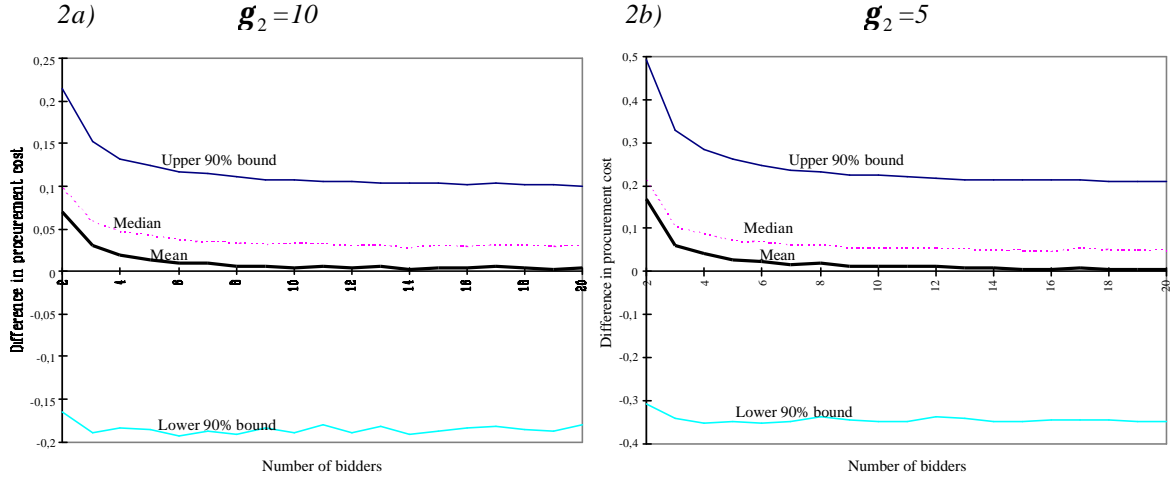


Table 2 illustrates the share of positive outcomes in the simulations, that is, the cases when the second-lowest bid in the second-price auction is lower than the lowest bid in the first-price auction. The decreasing ratio of positive outcomes supports our previous finding that the second-price auction is most advantageous with few bidders.

Table 2: The probability of obtaining a lower cost under the second-price auction than under the first-price auction for various number of bidders.

Number of bidders	2	3	4	5	10	15	20
$\mathbf{b}_{1st}(z_{(1n)}) > \mathbf{b}_{2nd}(z_{(2n)})$	76%	69%	66%	64%	62%	62%	62%

Removing 5% of the simulated differences in procurement costs in each tail of the distribution for every number of bidders, gives a 90% confidence interval with an upper and a lower bound.

4. Bidding under Normally Distributed Signals

An alternative approach to deriving a computationally tractable solution for $\mathbf{b}(z)$ is to assume that the signals are normally distributed with mean c and variance \mathbf{s}^2 , and that the bidders have identical and diffuse prior distribution for the unknown cost c .

Formally, the specifications of the distributions are denoted

$$f(z_i|c) = \frac{1}{\mathbf{s}} f\left(\frac{z_i - c}{\mathbf{s}}\right)$$

$$c \sim U[-\infty, \infty] \rightarrow g(c) = k. \quad \forall c$$

(17)

Again, using Bayes's rule, we see that the bidder's prior expectation about c does not shift the posterior expected value of c away from its signal z_i .

$$E[c|z_i] = \frac{\int_{-\infty}^{\infty} c \frac{1}{\mathbf{s}\sqrt{2\mathbf{p}}} e^{-\left(\frac{z_i - c}{2\mathbf{s}}\right)^2} k dc}{\int_{-\infty}^{\infty} \frac{1}{\mathbf{s}\sqrt{2\mathbf{p}}} e^{-\left(\frac{z_i - c}{2\mathbf{s}}\right)^2} k dc} = z_i. \quad (18)$$

4.1 Optimal Bid Functions

Given the assumptions above and, additionally, the assumption that the estimation errors are normal and independent of the true cost, then Levin and Smith (1991), in a comment on Thiel (1988), show that the optimal bid function in the first-price auction can be derived as

$$b = \mathbf{b}(z) = z - \mathbf{a}_n \mathbf{s} + \mathbf{g} \exp\left(-z \mathbf{x}_{(1:n)} / \mathbf{s}\right) \quad (19)$$

where

$$\mathbf{x}_{(1:n)} = \int_{-\infty}^{\infty} u n(1 - \Phi(u))^{n-1} f(u) du < 0 \quad (20)$$

and

$$\mathbf{a}_n = \frac{\int_{-\infty}^{\infty} nu^2(1-\Phi(u))^{n-1}f(u)du}{\mathbf{x}_{(1:n)}} < 0. \quad (21)$$

Levin and Smith show that individual rationality implies that $\gamma \geq 0$. Note that equation (20) is the expectation of the first order statistic in a sample of size n of the standard normal distribution. The γ is a parameter indexing the family of Nash equilibria, where one ($\mathbf{g} = 0$) is linear in the estimate. In the case ($\mathbf{g} = 0$), equation (19) turns into the linear form

$$b = \mathbf{b}(z) = z - \mathbf{a}_n \mathbf{s}. \quad (22)$$

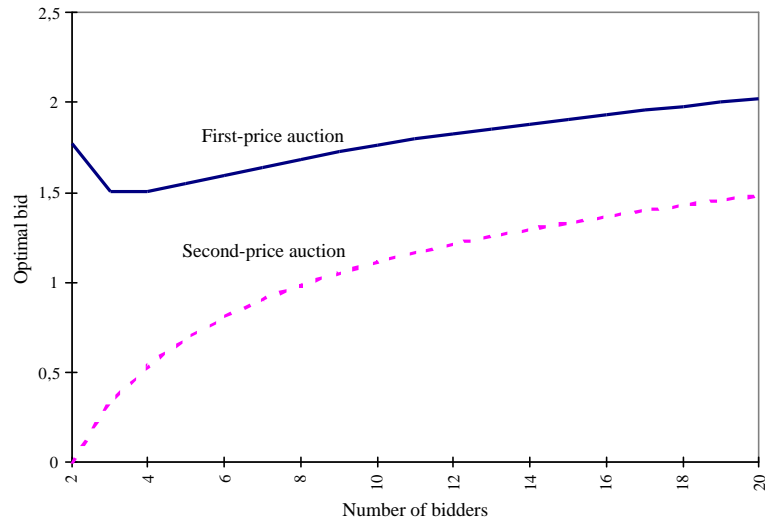
The corresponding optimal bid function in the second-price auction can be derived as⁵

$$\mathbf{b}_{2^{nd}}(z_i) = z_i - \mathbf{a}_n \mathbf{s} + \mathbf{s} \frac{1}{\mathbf{x}_{(1:n)}}.$$

(23)

In figure 3 the optimal bids in both auction formats are plotted against the number of bidders.

Figure 3: Optimal Bid versus Number of Bidders ($z=0$, $\mathbf{s}=1$)



Again, the optimal bid in the first-price auction initially decreases with the number of bidders due to competition, but then, as the fear of the winner's curse dominates the competitive effect,

⁵ See appendix for proof.

the bid increases with the number of bidders. From (22) and (23), it is also obvious that the optimal bid rises as the dispersion of signals increases.⁶

4.2 Expected and Simulated Procurement Cost

As in the previous section, the difference in procurement cost between the first-price sealed bid and the second-price sealed bid auction is examined by comparing the expected winning bid in the first-price auction with the expected second-lowest bid in the second-price auction. The expected winning bid in the first-price auction and the second-lowest bid in the second-price auction are

$$\text{(first-price auction)} \quad E[w] = c + \mathbf{s}(\mathbf{x}_{(1:n)} - \mathbf{a}_n) \quad (24)$$

$$\text{(second-price auction)} \quad E[b_{(2:n)}] = c + \mathbf{s} \left(\frac{1}{\mathbf{x}_{(1:n)}} + \mathbf{x}_{(2:n)} - \mathbf{a}_n \right) \quad (25)$$

where

$$\mathbf{x}_{(2:n)} = \int_{-\infty}^{\infty} n(n-1)x(1-F(x))^{n-2}F(x)f(x)dx \quad (26)$$

and $\mathbf{x}_{(1:n)} - \mathbf{a}_n \geq 0$, $\frac{1}{\mathbf{x}_{(1:n)}} + \mathbf{x}_{(2:n)} - \mathbf{a}_n \geq 0$ for $n \geq 2$.

Thus, the difference between the two expected values can be expressed as a positive linear function in the dispersion of signals where the coefficient is determined by the number of bidders. In table 3 we have computed the difference between (24) and (25) for various number of bidders, where $c = 0$ and $\mathbf{s} = 1$.

⁶ Note that $|\alpha| > 1/\xi_{(1:n)}$ for $n > 2$

For small numbers of bidders the difference is significant, but the difference decreases rapidly as the number of bidders increases.

Table 3: The procurer's expected cost ($c=0$ and $s=1$)

n	$E(w)$	$E(b_{(2:n)})$	Diff.
2	1,208	0,564	0,644
3	0,661	0,326	0,335
4	0,478	0,239	0,239
5	0,385	0,193	0,192
10	0,224	0,111	0,113
15	0,173	0,085	0,088
20	0,149	0,071	0,078

Again, we complete the picture of the difference in outcome by randomizing the signals and evaluating the same set of signals in the optimal bid functions (22) and (23). For each given number of bidders, the corresponding number of signals are randomly drawn from the standardized normal distribution. In each simulated auction with the same number of bidders, the lowest bid in the first-price auction is compared with the second-lowest bid in the second-price auction. Every auction for a given number of bidders is simulated 10 000 times.

Figure 4: Simulated Differences in Procurement Costs under Normally Distributed Signals

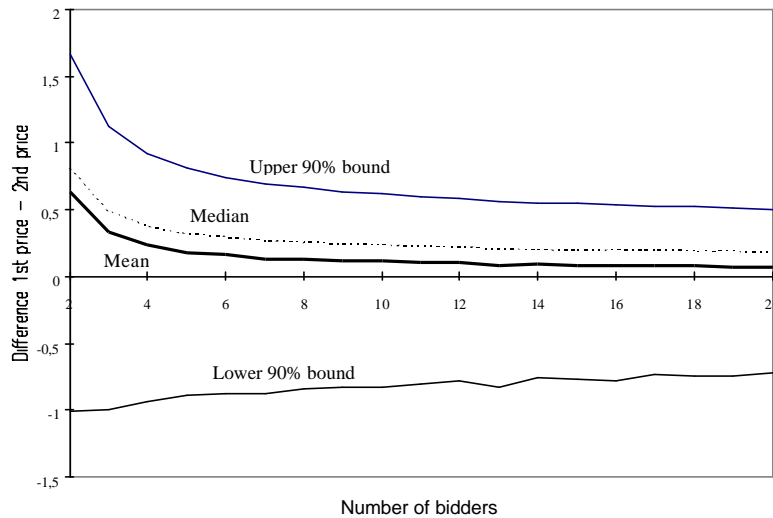


Figure 4 illustrates the mean, the median and a 90% confidence interval from the simulations. The procurer's expected difference in cost between the auction formats converges to zero as the number of bidders increases.⁷ For each number of bidders, we also examine the procurer's probability of getting a lower cost with the second-price auction by counting the number of positive outcomes from the 10 000 simulations.

Table 4: The probability of obtaining a lower cost under the second-price auction than under the first-price auction for various number of bidders

Number of bidders	2	3	4	5	10	15	20
$\mathbf{b}_{1^{st}}(z_{(1:n)}) > \mathbf{b}_{2^{nd}}(z_{(2:n)})$	79%	74%	71%	69%	68%	67%	67%

Again, where there are very few bidder the second-price auction is very likely to generate a lower procurement cost than the first-price auction does, but with increased competition the superiority of the second-price auction decreases somewhat.

5. A Numerical Example

Paarsch (1992) examines whether bidding behavior in tree-planting contract auctions held in the province of British Columbia, Canada, can be explained by either the private values or the common value model. Using various empirical specifications, Paarsch rejects the private values model and finds evidence consistent with rational behavior within the common value model, where both the Weibull distribution and the normal distribution specification of signals seem to fit the data fairly well.

Given that bidders' actions are consistent with theory when facing a second-price sealed bid auction, we may use the estimated structural parameters of the distribution of the signals in

⁷ Given the large number of simulations, one may regard $\mathbf{b}_{1^{st}}(z_{(1:n)})$ and $\mathbf{b}_{2^{nd}}(z_{(2:n)})$ as independent random variables. The difference in expected value then breaks down to $\mathbf{S}(\mathbf{x}_{(1:n)} - 1 / \mathbf{x}_{(1:n)} - \mathbf{x}_{(2:n)})$, which converges to zero as $n \rightarrow \infty$.

Paarsch (1992) to illustrate hypothetically what could be predicted to have been gained by the procurer by using the second-price sealed bid auction instead of the first-price sealed bid auction. The maximum likelihood estimates of the structural parameters for the Weibull distribution and the normal distribution c, g_2 and s in Paarsch are reproduced in table 5. The estimates are based on the winning bids from 144 auctions.

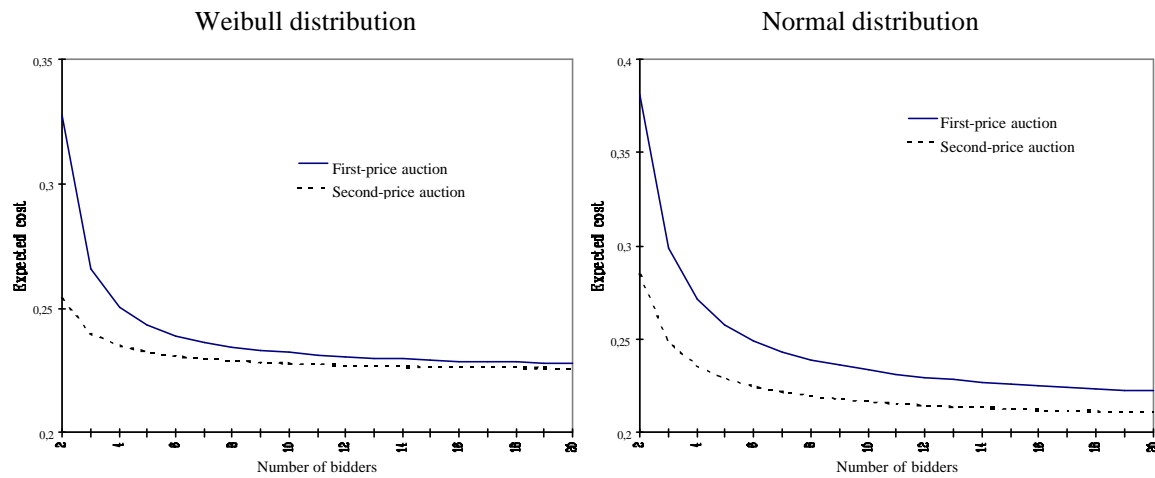
Table 5: Maximum Likelihood Estimates of Structural Parameters^a

Parameter	<u>Distribution</u>	
	Weibull	Normal
c	0,224	0,20
g_2	3,162	–
s	–	0,150
	<u>Mean</u>	<u>Min.</u>
n	15.57	3
		<u>Max.</u>
		28

^a The parameters are for costs measured in dollars per tree planted

Making use of equations (14), (15), (22), and (23) the difference in expected costs between the first-price and the second-price auction for various numbers of bidders is presented in figure 5. Given the Weibull distribution, the estimated gain from a switch is relatively low, about 2–4% for auctions with few bidders and diminishing rapidly as the number of bidders increases.

Figure 5: Expected Cost under the First-Price and Second-Price Auctions (dollars per tree planted)



The difference in expected costs under the normal distribution exhibits a similar pattern, although the expected gains from a switch of auction format are somewhat higher.

6. Summary

This analysis of the magnitude of the expected difference between a first-price sealed bid auction and a second-price sealed bid auction is based on only two distributions of signals and costs, which of course limits the general value of this study. We have shown that the superiority of the second-price auction over the first-price auction in terms of expected procurer's cost is related to changes in the number of bidders and the dispersion of signals. Given our distributions of signals and cost, the second-price auction is most beneficial when there are very few bidders or when there is high dispersion of bidders' signals. The simulations also indicate that the probability of getting a better outcome when using the second-price auction, given a moderate number of bidders (>5), is about two-thirds. Making use of the results from Paarsch (1992), who finds the common value model consistent with observed behavior, we estimate that the predicted hypothetical gain of switching to the second-price auction may be significant for a small number of bidders (<5) but is relatively low for a larger number of bidders.

The first-price sealed bid auction is a well-established mechanism in governmental procurement bidding. This mechanism was surely considered when the national acts and the international agreements on public procurement were formulated. Changing a procedure that is regarded as more or less formal requires resources. If the estimates of the costs and benefits of altering the procedure indicate that there is very little gain from switching methods, an institution may very well refrain from implementing the second-price auction. This argument seems especially to hold when the procurer expects that the number of potential bidders will be relatively large and that the dispersion of their signals will be small.

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Appendix

Optimal bid function with normally distributed signals⁸

(In what follows, the bidder's signal is denoted x)

$$b(x) = \frac{\int (n-1)cf(x|c)^2[1-F(x|c)]^{n-2}g(c)dc}{\int (n-1)f(x|c)^2[1-F(x|c)]^{n-2}g(c)dc}$$

Restrictions that guarantee the existence of linear strategies:

- 1) Each bidder's prior distribution of value is diffuse: $g(c)$ is constant for all c .
- 2) Estimation errors are statistically independent of the contract's true cost:

$$F'(X_i - c|c) = f(X_i - c|c) = f(X_i - c).$$

- 3) Each bidder's estimate of the cost is unbiased: $E(X_i) = c$.

Note that

$$F(x|c) = \int_{-\infty}^x f(t|c)dt = \int_{-\infty}^{x-c} f(t|c)dt = \int_{-\infty}^{x-c} f(t)dt = F(x-c)$$

where $t = t - c$. The third equality is due to restriction 2.

$$b(x) = \frac{k(n-1) \int - (x-c-x) \frac{1}{s} (sf(x|c))^2 [1-F(x-c)]^{n-2} \frac{1}{s} dc}{k(n-1) \int (sf(x|c))^2 [1-F(x-c)]^{n-2} s^{-2} dc}$$

Let $z = \frac{x-c}{s}$ and $\frac{dx}{dz} = s$. Then, $f(z) = sf(x|c)$ and $dc = -s dz$.

$$\begin{aligned} b(x) &= \frac{\int (x/s - z) f(z)^2 [1-F(z)]^{n-2} dz}{1/s \int f(z)^2 [1-F(z)]^{n-2} dz} \\ &= \frac{x/s \int f(z)^2 [1-F(z)]^{n-2} dz}{1/s \int f(z)^2 [1-F(z)]^{n-2} dz} - \frac{\int z f(z)^2 [1-F(z)]^{n-2} dz}{1/s \int f(z)^2 [1-F(z)]^{n-2} dz} \\ &= x - \frac{\int z f(z)^2 [1-F(z)]^{n-2} dz}{1/s \int f(z)^2 [1-F(z)]^{n-2} dz} \end{aligned}$$

⁸ The derivation is based on the derivation of the corresponding bid function under the first-price auction (see Levin and Smith, 1991).

$$= x - \frac{\mathbf{s}n(n-1) \int_{-\infty}^{\infty} zf(z)^2 [1-F(z)]^{n-2} dz}{n(n-1) \int_{-\infty}^{\infty} f(z)^2 [1-F(z)]^{n-2} dz}$$

Assume that x_i is distributed normally, with mean c and variance \mathbf{s}^2 .

$$\begin{aligned} \mathbf{b}(z) &= x - \frac{\int_{-\infty}^{\infty} zf(z)^2 [1-F(z)]^{n-2} dz}{1/\mathbf{s} \int_{-\infty}^{\infty} f(z)^2 [1-F(z)]^{n-2} dz} \\ &= x - \frac{\mathbf{s}n(n-1) \int_{-\infty}^{\infty} zf(z)^2 [1-F(z)]^{n-2} dz}{n(n-1) \int_{-\infty}^{\infty} f(z)^2 [1-F(z)]^{n-2} dz} \\ &= x + \frac{\mathbf{s}n(n-1) \int_{-\infty}^{\infty} zf(z)^2 [1-F(z)]^{n-2} dz}{\mathbf{X}_{(1:n)}} \quad \text{where } \mathbf{X}_{(1:n)} < 0 \\ &= x + \frac{\mathbf{s}n \left\{ 1/n - \int_{-\infty}^{\infty} z^2 f(z)^2 [1-F(z)]^{n-1} dz \right\}}{\mathbf{X}_{(1:n)}} \\ &= x - \mathbf{a}\mathbf{s} + \mathbf{s} \frac{1}{\mathbf{X}_{(1:n)}} \end{aligned}$$

Q.E.D.