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Who takes care of the children? The quantity–quality model revisited*

Michael Lundholm and Henry Ohlsson

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Abstract

We study the Becker and Lewis (1973) quantity–quality model of children adding an explicit child care time constraint for parents. Parents can take care of the children themselves or purchase day care. Our results are: (i) If there only is own care, a quantity–quality trade–off, different from that of Becker and Lewis (1973), arises. The income effect on fertility is positive if child quantity is a closer complement than child quality to the consumption of goods. (ii) If, instead, there is a combination of purchased and own care, the effect of income on fertility is ambiguous, even if quantity of children is a normal good in the standard sense. This is the Becker and Lewis (1973) result extended to a situation with a binding child care time constraint. The conclusion is that the Becker and Lewis (1973) result holds as long as at least some child care is purchased.

JEL: D1, J13

Keywords: fertility, child care, time constraint, quantity–quality trade–off

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1 Introduction

How does increased family income affect fertility? The standard answer is that fertility increases with income if the quantity of children is a normal good. The seminal contribution of Becker and Lewis (1973) (henceforth denoted BL) shows that this answer is seriously misleading.[1] A *ceteris paribus* increase in quality implies an increase in the marginal rate of substitution between quantity and quality, if quantity is a normal good. But such an increase in quality also increases the relative price of quantity in the BL model unlike standard models where prices are constant. The direction of the change in quantity when income increases is, therefore, indeterminate.

BL use a single period model that includes all phases of life for parents. Sometimes, see Hotz et al. (1997), the perspective of a newly married couple is emphasised. Recent empirical studies, e.g., Connelly (1992), Powell (1997) and Blau and Hagy (1998), adopts this perspective. They also recognise that small children require child care, which BL do not.[2]

Our purpose is to study how changes in income affect fertility in the quantity–quality model when parents face an explicit child care time constraint. We assume that the quality of children depends on the type of child care provided. In addition to taking care of the children themselves (own care) parents can also purchase care (day care).[3]

In some cases we replicate the BL results, in other cases we do not. Our main results are: If parents exclusively take care of the children themselves, a quantity–quality trade–off, of a different kind than that of Becker and Lewis (1973), arises. The income effect on fertility is positive if the quantity of children is a closer complement to consumption goods than the quality of children.

If there is a combination of own and purchased care, we find that the effect of income on fertility still is ambiguous when the quantity of children is a normal good. Necessary conditions for a solution with both own and purchased care are, however, that the marginal utility of spending time with the children is low and that the marginal utility of an additional child is high. This combination of conditions is not impossible but is somewhat odd.[4] But

[1] For an early discussion see Becker (1960) and for further development Becker and Tomes (1976). For policy discussions see, e.g., Batina (1986), Cigno (1983, 1986), Ermisch (1989), and Nerlove et al. (1984, 1986).

[2] See also Lundholm and Ohlsson (1998) who apply the same perspective to analyse wage determination and female labour force participation.

[3] We also assume that parents without constraints can choose how much day care they want to purchase.

[4] Odd since parents on the margin like having more children but have, in a sense, a low preference for spending more time with them.

this is the Becker and Lewis (1973) result extended to a situation with a binding child care time constraint. The main conclusion is, therefore, that the Becker and Lewis (1973) result holds as long as at least some child care is purchased.

In section 2 we describe our generalisation of the BL model. Section 3 derives the results and section 4 concludes the paper.

2 Model

A parent reproduces asexually and chooses the quantity of children $n \in N = \{n \in \mathbb{R}_+ : n \geq 1\}$. Child quality $q \in \mathbb{R}_+$ is only acquired through child care, which is produced by the parent herself or purchased. Producing the care herself, the parent spends $c \in \mathbb{R}_+$ of her own time to take care of her n children. The number of purchased hours of day care are $d \in \mathbb{R}_+$ during which the child gets the full attention of a care taker. The total care time during the childhood of each child is $d + c$.[5] This must not be less than the total childhood time D during which *each* child needs care; $D \leq c + d$.

The quality of own care for each child equals the average time during which a child gets the full attention of the care taker; i.e., $\frac{c}{n}$. Purchased quality per child is proportional to the number of purchased hours of day care. As a matter of convenience we choose units so that an hour of day care yields a unit of quality. Own and purchased quality are perfect substitutes and the parent treats all children identically.

Average quality is $q = \frac{c}{n} + d$. We assume that the child care time constraint is binding, i.e., $D = c + d$. This implies that $q = D - \frac{(n-1)}{n}c$ and $\partial q/\partial c = n^{-1} - 1 \leq 0$. Substitution of day care for own care reduces average quality when the child care time constraint is binding.[6]

Working hours $\overline{h} \in \mathbb{R}_{++}$ are fixed and paid the wage rate w. Lifetime income is spent on own lifetime consumption ($x \in \mathbb{R}_+$), the price of which is numeraire and normalised to unity, or on purchased quality pnd, where p is the unit price of purchased day care, n is the number of children, and d is the quantity of purchased care per child. The parent's budget constraint is $w\overline{h} = x + pnd$.

The total time endowment during the lifetime T is spent on market work, taking care of one's children and leisure time ($\ell \in \mathbb{R}_+$): $T - \overline{h} = c + \ell$. Nat-

[5] We abstract from parents' infrequent and short–time purchases of other peoples time to take care of children, e.g., baby sitting.

[6] If the child care time constraint is not binding, $D < c + d$, which occurs if the parent purchases a lot of quality, then the model becomes analogous to the BL-model. Their results are also replicated.

urally, total childhood time is less than total time, $D \leq T$. The assumption that time in market work is fixed, leaves the parent with an own child care–leisure choice in the time dimension.

Parents have preferences represented by the quasi–concave utility function $U^* : \mathbb{R}^4_+ \times \mathbb{R}_{++} \times N \to \mathbb{R}$ defined by $U^*(x, \overline{h}, c, \ell, q, n)$. We use the notation $U^*_1 = \frac{\partial U^*(\cdot)}{\partial x}$ etc to denote the partial derivatives and assume that $U^*_1 > 0$, $U^*_2 < 0$, $U^*_3 \gtreqless 0$, $U^*_4 > 0$, $U^*_5 > 0$ and $U^*_6 > 0$. Hence, we do not make any particular assumption about how the parent values the time spent with her own children.

Combining the time constraints yields

$$T - \overline{h} - D = \ell - d. \tag{1}$$

Substituting for c, ℓ, and q, the problem of a parent can be written as

$$\max_{x,d,n} \quad U(x, d, n) \text{ s.t. } w\overline{h} = x + pnd \text{ and } d \geq 0, \tag{2}$$

where $U(x, d, n) = U^*\left(x, \overline{h}, D - d, T - \overline{h} - D + d, d + \frac{D-d}{n}, n\right)$. This problem has the following first order conditions for $x^* > 0$, $d^* \geq 0$ and $n^* \geq 1$

$$U_x - \lambda^* = 0, \tag{3a}$$

$$U_d - \lambda^* p n^* - \mu^* \leq 0 \quad d^* \geq 0 \quad \mu^* d^* = 0, \tag{3b}$$

$$U_n - \lambda^* p d^* = 0, \text{ and} \tag{3c}$$

$$w\overline{h} - x^* - pn^* d^* = 0 \tag{3d}$$

where $\lambda^* > 0$ and μ^* are the Lagrange multipliers in the optimal point associated with the constraints. Alternatively, we can express the derivatives of U in terms of derivatives of U^*, i.e.,

$$U_x = U^*_1, \tag{4a}$$

$$U_d = -U^*_3 + U^*_4 + \frac{n-1}{n} U^*_5 \text{ and} \tag{4b}$$

$$U_n = -\frac{D-d}{n^2} U^*_5 + U^*_6. \tag{4c}$$

3 Quality vs. quantity

We now revisit the problem of quantity versus quality of children and ask how fertility is affected by income changes when there is an explicit child care time constraint. We make the analysis in two steps: In subsection 3.1 we consider the corner solution where the parent produces all child care herself ($d^* = 0$). The interior solution when child care is arranged through a combination of purchased care ($d^* > 0$) and own care is discussed in subsection 3.2.

3.1 Only own care

Suppose that there is only own care; i.e., $d^* = 0$. Then (3b) implies $U_d - \lambda^* p n^* \leq 0$, possibly with a strict inequality. This situation may occur when the parent loves staying home to take care of the children and, therefore, $U_3^* > 0$ and also relatively high. Consumption is given by equation (3d) and equals income, which can be defined as $\overline{y} = w\overline{h}$. The main issue is how fertility is affected by income changes; i.e., what is the sign of $\frac{\partial n}{\partial \overline{y}}$?

When we consider the effect on an income increase the first order condition describing individual behaviour simplifies to $U_n = 0$, with the second order condition $\Delta_2 := U_{nn} < 0$. In the notation of the general model we have

$$U_n = -\frac{D}{n^2}U_5^* + U_6^* = 0 \text{ and} \tag{5a}$$

$$U_{nn} = \frac{D}{n^3}U_5^* + \frac{D^2}{n^4}U_{55}^* - \frac{2D}{n^2}U_{56}^* + U_{66}^* < 0. \tag{5b}$$

If $U_{nn} < 0$ we can continue with the comparative static analysis. However, this is not necessarily the case. Because quality is non–linear in quantity a solution satisfying the first order condition may be a local optimum only, giving lower or the same utility as the global optimum. Also, the global optimum may be the corner solution $n = 1$. In the following we disregard these problems and assume that the second order condition is satisfied so that there exists a unique interior solution ($n^* > 1$) for the quantity of children.

Total differentiation the quantity of children n with respect exogenous income $\overline{y}$ yields

$$\frac{\partial n}{\partial \overline{y}} = \frac{1}{\Delta_2}\left[\frac{D}{n^2}U_{51}^* - U_{61}^*\right], \tag{6}$$

where the denominator is negative by the second order condition.

The equation (6) shows the quantity–quality trade–off when there is no purchased care. This condition states that if the quantity of children is a sufficiently closer complement to the consumption of goods than quality in the sense of $\frac{D}{n^2}U_{51}^* - U_{61}^* < 0$, then increased exogenous income will increase the quantity of children. This would be the case, for example, if quality is a substitute and quantity is a complement to the consumption of goods.[7] Since $\frac{\partial q}{\partial n} = -Dn^{-2} < 0$ the quality of children will be reduced. However, if the quality of children is a closer complement to consumption of goods than quantity in the sense of $\frac{D}{n^2}U_{51}^* - U_{61}^* > 0$, then increased exogenous income will reduce the quantity of children and also increase the quality of children.

[7] Note that this definition of complementarity, the Pareto–Georgescu criterion, may deviate from the standard definition that the compensated cross elasticity should be positive. See e.g., Samuelson (1974).

3.2 Purchased and own care

In a completely interior solution the parent is using purchased as well as own care. Then (3b) is strictly binding and $\mu^* = 0$ so that $d^* > 0$. Let the (assumed) unique solution satisfying these first order conditions (3a)–(3c) be denoted $(x(\overline{y}), d(\overline{y}), n(\overline{y}))$. Once again the issue is how fertility is affected by income changes; i.e., what is the sign of $\frac{\partial n}{\partial \overline{y}}$?

Consider now the optimal non–linear solution evaluated in a linear model. In such a model we can write the budget as $I = p_n \tilde{n} + p_d \tilde{d} + x$, where $I = \overline{y} + pd(\overline{y})n(\overline{y})$ is full income, $p_n = pd(\overline{y})$ and $p_d = pn(\overline{y})$. Let $\tilde{S}_{ij}$ denote the substitution effect in the linear model where $i, j = p, d$. Standard symmetry gives $\tilde{S}_{ij} = \tilde{S}_{ji}$. Then we have

$$\frac{\partial n}{\partial \overline{y}} = \frac{\frac{\partial \tilde{n}(I)}{\partial I}\left(p\tilde{S}_{dn} - 1\right) - \frac{\partial \tilde{d}(I)}{\partial I} p\tilde{S}_{nn}}{p^2 \tilde{S}_{dd}\tilde{S}_{nn} - \left(p\tilde{S}_{dn} - 1\right)^2}, \tag{7}$$

where $\frac{\partial \tilde{n}(I)}{\partial I}$ and $\frac{\partial \tilde{d}(I)}{\partial I}$ are standard income effects. Normality of the quantity of children in the standard sense implies $\frac{\partial \tilde{n}(I)}{\partial I} > 0$, but this is not sufficient to sign equation (7); see Razin and Sadka (1995, p. 20f) for a discussion about various conditions signing (7). This is essentially the BL result. The difference is that 'total quality' in their model corresponds to 'purchased quality', i.e., day care, in our model.

Necessary conditions for an interior solution are that the marginal utilities of purchased care and quantity are positive; i.e., $U_d > 0$ and $U_n > 0$. Although these marginal utilities look similar to the marginal utilities in BL we see the difference clearly if we study the signs of (4b) and (4c) rather than simply the signs of the derivatives of U. Then we see that given our assumptions $U_d \gtreqless 0$ and $U_n \gtreqless 0$.

Utility is affected through three different channels when the parent purchases an additional unit of day care. *First*, by the child care time constraint, the amount of time spent with children is reduced. This reduces utility if the parent likes to be with the children. *Second*, by the time constraint, more leisure time becomes available since working hours are fixed, which increases utility. *Third*, the quality per child is affected. As a direct effect, quality increases with one unit while the reduction of own care with one unit only reduces quality with n^{-1} units. Therefore, the quality effect is non–negative since we assume that $n^* \geq 1$. This means that the marginal utility of purchased care is positive if the second and third effects dominate the first effect.

When the quantity of children is increased marginally there will be a direct positive effect on utility and an indirect negative effect through reduced

quality. If the direct effect dominates the indirect effect, then additional children will increase utility.

We can note that the necessary condition for purchased day care is met if the parent dislikes spending own time to take care of the children. In other words, $U_3^* < 0$ in the optimal point is sufficient for $U_d > 0$. Moreover, the necessary condition for the quantity of children is met if all care is purchased. In other words, $d^* = D$ is sufficient for $U_n \geq 0$. But this would move us from an interior solution to a corner solution or even beyond that. When the parent purchases more day care than necessary the child care time constraint is no longer binding and we are back to the model and the results of BL.

4 Conclusions

Becker and Lewis (1973) show that the effect of income on fertility is ambiguous, even if the quantity of children is a normal good in the standard sense. In this paper we have shown that this result extends to a situation where parents face an explicit child care time constraint and choose a combination of purchased day care and child care produced by themselves.[8]

On the other hand, if parents exclusively care for the children themselves, a different kind of quantity–quality trade–off arises. More children reduce the quality of an hour of the parent's time spent on child care. The income effect of fertility now is positive if the quantity of children is a closer complement than quality to the consumption of goods.

Consequently, the conclusion is that Becker and Lewis (1973) result extends to a situation with a binding child care time constraint as long as at least some child care is purchased.

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[8] We assume that the parent has a time choice between own care and leisure while time in market work is exogenous. A natural extension of this analysis is to instead allow for an own care – labor supply choice.

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