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Optimal Unemployment Insurance in Search Equilibrium*

by

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Abstract

Should unemployment compensation be paid indefinitely at a fixed rate or should it decline (or increase) over a worker's unemployment spell? We examine these issues using an equilibrium model of search unemployment. The model features worker-firm bargaining over wages, free entry of new jobs, and endogenous search effort among the unemployed. The main result is that an optimal insurance program implies a declining sequence of unemployment compensation over the spell of unemployment. Numerical calibrations of the model suggest that there are non-trivial welfare gains associated with switching from an optimal uniform benefit structure to an optimally differentiated system.

Keywords: Job search; Unemployment; Unemployment insurance

JEL classification: H21; J65; D8

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1. Introduction

The economics of unemployment insurance (UI) has attracted considerable attention over the past couple of decades. The research has primarily been concerned with positive analysis, such as the effects of UI benefits on the duration of unemployment. Much less interest has been devoted to the normative issues: what does an *optimal* UI system look like?[1] The ultimate rationale for public UI is, after all, to provide income insurance for risk averse workers. A welfare analysis of UI policies thus requires a unified treatment of the insurance benefits provided by UI as well as the adverse incentive effects induced by the usual moral hazard problems. The purpose of this paper is to contribute to the normative analysis of UI by means of an equilibrium model of search unemployment.

The seminal papers on optimal UI appeared in the late 1970s (Baily, 1978; Flemming, 1978; Shavell and Weiss, 1979). These papers analyzed the problem of UI design in an optimal taxation framework. The moral hazard problem entered through the unemployed worker's search effort; more generous benefits caused lower search intensity and hence longer spells of unemployment. Shavell and Weiss focused in particular on the optimal sequencing of benefits. Their analysis, based on a partial equilibrium search model, suggested that benefits should decline over the spell of unemployment, provided that the unemployed can influence their job-finding probability. Baily's two-period analysis analogously suggested a case for a redundancy payment, i.e., a lump-sum transfer to the worker at the start of the unemployment spell.

A recent paper by Davidson and Woodbury (1997) examines whether benefits should be paid indefinitely or for a fixed number of weeks. The analysis is cast in a search and matching framework, albeit with a fixed number of jobs and exogenous wages. Davidson and Woodbury conclude that the optimal UI program should offer risk-averse workers *indefinite* benefit payments, a conclusion that seems to suggest that most existing UI programs with finite benefit

[1] See Atkinson and Micklewright (1991) and Holmlund (1997) for two recent surveys and assessments of the literature on UI and unemployment.

periods are sub-optimal.[2] Cahuc and Lehmann (1997) also investigate the consequences of the time sequence of UI benefits. Their model ignores job search but allows for endogenous wage determination through union-firm bargaining. They find that a constant time sequence is likely to yield a lower unemployment rate than a program with a declining time profile – the reason being that a decreasing benefit schedule increases the welfare of the short-term unemployed at the expense of the long-term unemployed, which in turn causes an increase in wage pressure.

Our paper reexamines the question of the optimal sequencing of benefits using an equilibrium model of search unemployment. The model, drawing on Pissarides (1990), features worker-firm bargaining over wages and free entry of new jobs. We allow for endogenous search effort, which in general will influence the rate at which unemployed workers escape to employment. Search effort as well as the wage bargains are affected by the parameters of the UI program. For ease of exposition, and without loss of generality, we mainly focus on a two-tiered UI program, i.e., a program with two benefit levels. Workers who lose their jobs are entitled to UI benefits, so a spell of employment qualifies for insurance payments in the event of subsequent unemployment. UI benefits may not be paid indefinitely, however; some workers lose their benefits and are thereafter entitled to "social assistance". Social assistance payments have infinite duration but are lower than regular benefits. We ask whether a two-tiered system dominates, in welfare terms, a program with indefinite payments of a constant wage replacement rate. The answer to this question turns out to be an unambiguous yes. Thus, the result of Shavell and Weiss (1979) extends to a general equilibrium setting with endogenous wages and unemployment. We also show that the result generalizes to the case with a multi-tiered benefit structure; the level of unemployment compensation should decline monotonically over the spell of unemployment.

Our analysis ignores the possibility of smoothing consumption through borrowing and saving. The optimal replacement rate would presumably be lower if workers had access to the capital market compared to a situation where consumption is forced to equal income in each

[2] Davidson and Woodbury do not offer a formal proof of their conclusion, however. We conjecture that their result is driven by the fact that they are assuming that compensation after benefit exhaustion is exogenous and arbitrarily set to zero. Thus, they are comparing two extremes, i.e., indefinite payments of a fixed benefit level vs. a program with fixed potential duration after which benefits drop to zero. It is to be expected that risk aversion would preclude a system which forces consumption to drop to zero in some states (see Shavell, 1979).

period. A few recent papers have addressed the welfare implications of UI using general equilibrium search models that allow for capital markets. For instance, Costain (1997) develops a model with endogenous search effort and precautionary savings. The set up of the wage bargain is greatly simplified by ignoring that the wage in general will depend on outside opportunities; hence, the benefit level will not directly affect the outcome of the bargain. Despite this simplification, the model is much too complicated to solve analytically, so the results are based on numerical calibrations. According to Costain: "Optimal (replacement) ratios in the range of 30 % to 40 % seem to arise very easily". Valdivia (1996) reports similar results using a calibrated matching model with precautionary savings, but with a somewhat different wage setting rule than the one adopted in Costain's papers. Neither of these papers consider the optimal sequencing of benefits over time, however.

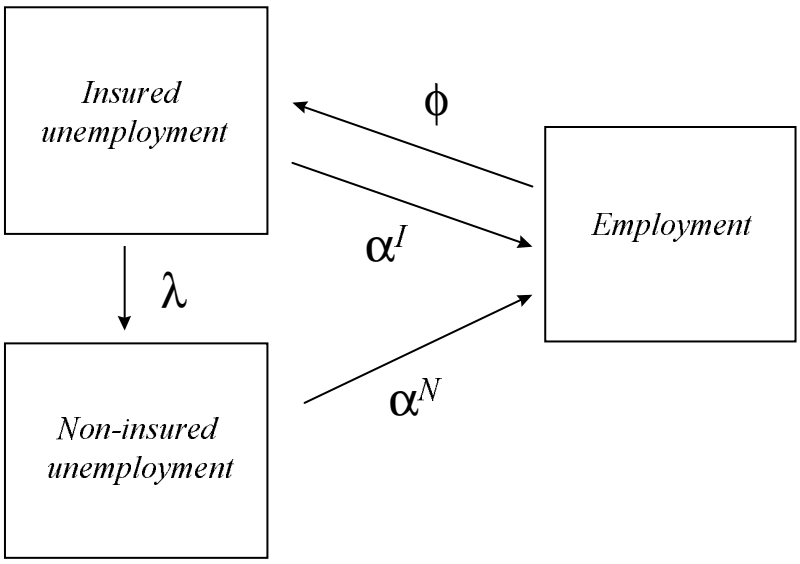
The next section of the paper presents the basic model and some of its comparative statics properties. Some of the results from this positive analysis are well known from the search literature, whereas others are new. Section 3 turns to the normative analysis and shows that search effort is too low in market equilibrium, the reason being that workers do not internalize the tax burden they impose on others by reducing their search effort. We also derive the key result that a two-tiered benefit structure dominates a system with indefinite payment of a constant replacement rate. The result is driven by a feature known from partial equilibrium search models, which implies that the effect of higher benefits on the individual worker's search behavior depends on whether he is presently qualified for UI or not. A rise in benefits will in general increase search effort among those not insured, as this will bring them quicker to employment which results in eligibility for future UI payments. A two-tiered UI system exploits this "entitlement effect" by providing incentives for active search among workers not currently entitled to benefits. Section 3 also offers a numerical calibration of the model with some calculations of the optimal replacement rates. The numerical experiments suggest that the optimal degree of differentiation should be substantial. The welfare gains of switching from a uniform benefit structure to an optimal two-tired one are non-trivial.

2. The Model

2.1 Job Matching and Labor Market Flows

Consider an economy with a fixed labor force, without loss of generality normalized to unity. Workers are either employed or unemployed; individuals have infinite horizons and time is continuous. Employed workers are separated from their jobs and enter unemployment at the exogenous rate ϕ. A worker who enters unemployment is immediately eligible for UI benefits; the unemployed worker is *insured* as long as he receives UI benefits. Benefits are time-limited, however. We assume for simplicity that benefits expire at an exogenous rate λ, so the expected *potential* duration of benefit receipt is $1/\lambda$. An unemployed worker whose benefits have expired is referred to as *non-insured*. The insured worker escapes from unemployment and enters employment at the rate α^I, whereas the non-insured worker enters employment at the rate α^N. *Figure* 1 illustrates the labor market flows.

Figure 1: Labor market flows



The assumption that benefits have a stochastic rather than a fixed duration is made for tractability.[3] It is not restrictive, however, as the analysis generalizes to the case with an arbitrary number of insured unemployment states, each characterized by a given benefit level (see Appendix A.4). Search effort varies by unemployment state, but is constant within a state. The possibility of time-varying search within a given benefit period can safely be ignored as the number of steps in the "benefit ladder" becomes sufficiently large. In addition, one might argue that the practical implementation of the work test in existing UI systems is bound to involve a degree of randomness in benefit receipt from the job searcher's perspective, since the authorities cannot monitor the job acceptance behavior of all unemployed workers.

Unemployed individuals can affect the rate at which they enter employment. Let s^I denote search intensity of a representative insured worker and s^N the corresponding intensity for a non-insured worker. The effective number of searchers in the economy is then given as $S \equiv s^I u^I + s^N u^N$, where u^I and u^N denote the number of unemployed in the two categories. The matching process is summarized by an aggregate matching function that relates the flow of hirings (H) to the number of effective searchers and the number of vacancies (v), i.e.,

$$H = H(S, v). \tag{1}$$

The probability per unit time that individual i gets an acceptable offer is given by $\alpha_i^j \equiv s_i^j H(S,v)/S = s_i^j \alpha(\theta)$, $j = I, N$, assuming constant returns to scale; $\theta \equiv v/S$ is a measure labor market tightness and $\alpha(\theta) = H(S,v)/S = H(1,\theta)$. Firms fill vacancies at the rate $q(\theta) = H(S,v)/v = H(1/\theta, 1)$. Clearly, $\alpha(\theta) = \theta q(\theta)$. Differentiating $\alpha(\theta)$ with respect to θ we have $\alpha'(\theta) = q(\theta)(1-\eta) > 0$, where η is the elasticity of the expected duration of a vacancy with respect to θ; due to constant returns we have $\eta \in (0,1)$. Also, $q'(\theta) < 0$; thus, the tighter the labor market, the more difficult it is to fill a vacancy.

The flow equilibrium conditions for this economy are as follows:

[3] If benefit duration is fixed and perfectly predictable one would in general have to deal with the issue of how search effort changes over the spell of insured unemployment. Such time dependence is due to the fact that the value of insured unemployment declines as the worker approaches the date of benefit exhaustion; see Mortensen (1977).

$$\phi e = \alpha^I u^I + \alpha^N u^N \tag{2}$$

$$\alpha^N u^N = \lambda u^I . \tag{3}$$

The first condition pertains to employment (e) and the second to non-insured unemployment. Equations (2) and (3) imply a flow equilibrium condition for insured unemployment as well. The solution for the employment rate takes the form:

$$e = \frac{\mu^I(\lambda + \alpha^I)}{\phi + \mu^I(\lambda + \alpha^I)}, \tag{4}$$

where we define μ^j as the number of unemployed in state j relative to total unemployment: $\mu^j \equiv u^j / (u^I + u^N)$. In particular, the solution for μ^I is given by:

$$\mu^I = \frac{\alpha^N}{\lambda + \alpha^N} . \tag{5}$$

2.2 Worker Behavior

Workers do not have access to a capital market, so individuals consume all of their income in each instant. The employed worker's income is given by his wage, , and the insured unemployed worker's income by UI benefits, B. Non-insured unemployed workers receive a transfer, Z, from the government; Z can be thought of as "social assistance", available for workers who have run out of benefits. We proceed under the assumption that $B \geq Z$; an objective of the normative analysis will be to determine whether this inequality is socially optimal.

The utility of unemployed workers is decreasing in search effort, as search reduces available leisure time. The utility functions can accordingly be written as $\upsilon(B, s^I)$ and $\upsilon(Z, s^N)$. An employed individual works a fixed amount of hours ($\bar{h}$) and does not search; hence we write his utility function as $\upsilon(w, \bar{h}) = \upsilon(w)$. The utility functions are assumed to have the following properties:

$$\upsilon_w > 0,\ \upsilon_{ww} \le 0;\ \upsilon_X > 0,\ \upsilon_{XX} \le 0,\ X \in \{B \text{ if } j = I, Z \text{ if } j = N\};\ \upsilon_s < 0,\ \upsilon_{ss} < 0.$$

Let U^I, U^N and denote the expected present values of being in insured unemployment, non-insured unemployment, and employment, respectively. The relevant value functions for worker can be written as asset equations of the form:

$$rU_i^I = \upsilon(B, s_i^I) + \alpha_i^I (E - U_i^I) - \lambda(U_i^I - U^N) \tag{6}$$

$$rU_i^N = \upsilon(Z, s_i^N) + \alpha_i^N (E - U_i^N) \tag{7}$$

$$rE_i = \upsilon(w_i) - \phi(E_i - U^I), \tag{8}$$

where is the subjective rate of time preference. These present values can be solved in terms of the utilities pertaining to each state and the transition rates. For many purposes it is the in present values between employment and unemployment that matter; these are given in Appendix A.1.

An unemployed worker in state chooses search intensity, s_i^j, to maximize the value of unemployment, $U_i^j(\cdot)$. Using $\partial\alpha_i^j / \partial s_i^j = \alpha(\theta)$ and the fact that all workers in state choose an identical search intensity, the first-order conditions have the following structure:

$$\Lambda_s^j \equiv \upsilon_s^j + \alpha(\theta)(E - U^j) = 0, \quad j = I, N, \tag{9}$$

where $\upsilon_s^j = \partial\upsilon(X, s^j) / \partial s^j$. Thus, in equilibrium, the marginal cost of increasing search effort is equated to the expected marginal gain of doing so. Assuming that the second-order condition holds ($\upsilon_{ss} < 0$), we can state the following partial equilibrium results:

PROPOSITION 1: ( ) an increase in UI benefits ( ) reduces s^I (if $\upsilon_{sB}^I = \partial^2\upsilon(B, s^I) / \partial B \partial s^I \le 0$) but increases s^N; (*ii*) an increase in the potential duration of UI benefit receipt ($1/\lambda$), reduces s but increases s^N; (*iii*) an increase in social assistance (Z) reduces s^N (if $\upsilon_{sZ}^N \le 0$) and s .

Proof: Differentiate (9) implicitly with respect to B, λ, and Z, using (A.1): $\Lambda^I_{sB} < 0$ (if $\upsilon^I_{sB} \leq 0$), $\Lambda^N_{sB} > 0$; $\Lambda^I_{s\lambda} > 0$, $\Lambda^N_{s\lambda} < 0$; $\Lambda^I_{sZ} < 0$, $\Lambda^N_{sZ} < 0$ (if $\upsilon^N_{sZ} \leq 0$). Note that the "if-statements" are ones of sufficiency.

The result that non-insured search may rise when the benefit level is increased is known in the literature as an "entitlement effect" and it is due to Mortensen (1977). The entitlement effect arises because higher benefits make employment more attractive relative to non-insured unemployment as a spell of employment is a prerequisite for benefit eligibility, i.e., $E - U^N$ is increasing in B. For the exact same reason, non-insured search is increasing in benefit duration.

In the remainder of the paper we will assume that the utility function takes the following specific form:

$$\upsilon = \begin{cases} \dfrac{(c\ell^{\delta})^{\sigma} - 1}{\sigma}, & \sigma \leq 1,\ \sigma \neq 0,\ \delta \in (0,1) \\ \ln c + \delta \ln \ell, & \sigma = 0,\ \delta \in (0,1) \end{cases}, \tag{10}$$

where c denotes consumption and ℓ leisure. The absence of a capital market implies that consumption equals income in each state. Consumption and leisure in the three states are given by: $c = w$ and $\ell = T - \bar{h}$ if employed; $c = bw$ and $\ell = T - s^I$ if insured unemployed; and finally $c = zw$ and $\ell = T - s^N$ if non-insured unemployed, where T denotes the available time.[4]

and are thus proportional to the *aggregate* wage, where b and z are the wage replacement rates in the two states.

2.3 Firm Behavior and Wage Determination

The model of firms and wage determination follows Pissarides (1990) closely. Let J and V denote the expected present values of an occupied job and a vacant job respectively. Labor productivity is constant and denoted y; the cost of holding a vacancy open is denoted k_0. Wages are taxed at the proportional rate τ. The flow values of having an occupied and a vacant job are then given by

[4] If individuals choose their working time, taking wages as given, hours during employment will be independent of the wage and given as $\bar{h} = T/(1+\delta)$. The specific utility function is commonly used in the growth literature as it is consistent with a balanced growth rate assumption; see King *et al.* (1988).

$$rJ = y - \omega - \phi(J - V) \tag{11}$$

$$rV = -k_0 + q(\theta)(J - V)\,, \tag{12}$$

where $\omega \equiv w(1+\tau)$ is the real labor cost, inclusive of the payroll tax. For simplicity we assume that firms discount the future at the same rate as workers. There are no costs associated with opening a vacancy. Therefore, free entry ensures that $V = 0$. From (11) and (12), then, we get:

$$J = \frac{y - \omega}{r + \phi} = \frac{k_0}{q(\theta)}\,. \tag{13}$$

We will refer to (13) as a zero-profit condition. We take the cost of holding a vacancy to be proportional to labor productivity, i.e., $k_0 = ky$, which is akin to the conventional idea that vacancy costs are proportional to real wage costs (see Pissarides, 1990).[5] Under this assumption wage costs are proportional to the marginal product of labor:

$$\omega = [1 - (r + \phi)k/q(\theta)]y = \omega(\theta), \quad \omega'(\theta) < 0\,. \tag{14}$$

According to equation (14), firms react by posting fewer vacancies (θ falls) when ω increases.

Wages are set in decentralized Nash-bargains between individual workers and firms. Wages can be renegotiated at any time, in which case the relevant fallback position for the worker is the state of insured unemployment, irrespective of whether he entered employment from insured or non-insured unemployment. TheNash-bargain thus solves the problem:

$$\max_{w_i} \Omega(w_i) = \left[E_i(w_i) - U^I\right]^{\beta}\left[J_i(w_i) - V\right]^{1-\beta},\ \beta \in (0,1)\,,$$

where the definition of J_i is analogous to (11). The first-order condition for the maximization of the Nash-product can be written as:

[5] To rationalize this assumption, think of a world where firms allocate its workforce between production and recruitment activities. In such a set up the cost of recruiting – the vacancy cost – consists of the alternative cost, i.e., the marginal product of labor.

$$\frac{E-U^I}{\upsilon_w w}=\frac{\beta}{1-\beta}\frac{J}{\omega}, \tag{15}$$

where the free entry condition and symmetry across bargaining units has been imposed. Note that $E-U^I$ depends on labor market tightness, the various transition rates and compensation in the three states (see Appendix A.1).

2.4 Equilibrium

The equilibrium of this model may in principle be very complex. However, the model has a convenient recursive structure which simplifies the analysis considerably:

LEMMA 1: The zero-profit condition (13) and the wage setting equation (15) determine ω and θ, independently of the remaining endogenous variables. With θ determined, we get search behavior from (9). Having determined θ and s^j, we get e and μ^I from (4) and (5). Finally, the tax rate is given by a balanced budget requirement and the wage by $w \equiv \omega/(1+\tau)$.

Proof: According to (13) and (15) the crucial question is whether the left-hand side of (15) depends on the remaining endogenous variables. We first show that $-$ is independent of ^{j}. From (8) we have that E and U^I are linearly related; consequently, $\partial U^I/\partial s^I = \partial E/\partial s^I = 0$ by the Envelope theorem. According to (6) and (8) we have $U^I = f(U^N)$, where the function $f(\cdot)$ is independent of s^N. Differentiate with respect to s^N: $\partial U^I/\partial s^N = f'(\cdot)\partial U^N/\partial s^N$. Hence, $\partial U/\partial s^N = \partial E/\partial s^N = 0$. Next, note that the left-hand side of (15) does not depend on w given our utility function, $B = bw$ and $Z = zw$. These assumptions also imply that s is independent of w.

The assumption of constant wage replacement rates and the specific utility function thus render search and unemployment independent of the payroll tax. Together with $k_0 = ky$, these assumptions also imply that changes in labor productivity have no effect on search and unemployment.

We arrive at a single equation determining θ by combining the free-entry condition ($J = ky/q(\theta)$) with the solution to the wage bargain:

$$\Psi(\theta,\lambda,b,z) \equiv \frac{1-\sum_j m^j \rho^j}{\sigma} - (r+\phi+\sum_j m^j \alpha^j)\frac{\beta}{1-\beta}\frac{k/q(\theta)}{1-(r+\phi)k/q(\theta)} = 0, \qquad (16)$$

where $m^I = (r+\alpha^N)/(r+\alpha^N+\lambda)$, $m^N = 1-m^I$ and

$$\rho^j = x^\sigma \left[\frac{T-s^j}{T-\bar{h}}\right]^{\delta\sigma}, \quad x \in \{b \text{ if } j=I, z \text{ if } j=N\}.$$

Equation (16) defines $\theta = \theta(\lambda,b,z)$. The properties of this relationship are as follows:

PROPOSITION 2: Equilibrium labor market tightness increases if benefit duration, the benefit level or social assistance is reduced.

Proof: By implicit differentiation: $\theta_\lambda > 0,\ \theta_b < 0,\ \theta_z < 0$.

The intuition for Proposition 2 is straightforward: every change that reduces workers' threat point in the wage bargain, produces more moderate wage setting behavior on the part of workers and, consequently, increases equilibrium market tightness.

To derive the equations for equilibrium search intensity, we substitute the Nash-bargaining solution and the free-entry condition into the first-order conditions for optimal search:[6]

$$\rho^I \frac{\delta}{T-s^I} = \left[\frac{\beta}{1-\beta}\right]\frac{\theta k}{1-(r+\phi)k/q(\theta)} \qquad (17a)$$

$$\rho^N \frac{\delta}{T-s^N} = \frac{\alpha(\theta)}{r+\lambda+\alpha^N}\frac{\rho^I-\rho^N}{\sigma} + \left[\frac{r+\lambda+\alpha^I}{r+\lambda+\alpha^N}\right]\left[\frac{\beta}{1-\beta}\right]\frac{\theta k}{1-(r+\phi)k/q(\theta)} = 0. \qquad (17b)$$

[6] Equation (17b) is derived as follows: first decompose $(E-U^{iN})$ into $E-U^{iN} = (E-U^i)+(U^i-U^{iN})$; then use $U^i - U^{iN} = (r+\lambda+\alpha^{iN})^{-1}\{\upsilon^i - \upsilon^{iN} + (\alpha^i-\alpha^{iN})(E-U^i)\}$ and eliminate $(E-U^i)$ using (15) and (13).

Equations (17) define the following "semi-reduced forms": $s^I = s^I(\theta, b)$ and $s^N = s^N(\theta, \lambda, b, z)$. Lemma 2 summarizes the properties of equilibrium search:

LEMMA 2: Equilibrium search effort has the following properties: $s^I_\theta > 0$, $s^N_\theta > 0$, $s^N_b > 0$, and $s^N_\lambda < 0 \ \forall\sigma$; $s^I_b \leq 0$ and $s_z < 0$ if $\sigma \geq 0$; $s^I_b > 0$ and $s^N_z \gtrless 0$ if $\sigma < 0$.

Proof: Implicit differentiation of (17), recognizing that the right-hand side of (17b) is independent of s^j by the Envelope theorem.

There are three things to note about Lemma 2. First and foremost, there is an *equilibrium entitlement effect*. Second, it illustrates the importance of treating wages as endogenous. In equilibrium, the only remaining direct effect of the benefit system on s^I stems from the fact that utility may be non-separable in consumption and leisure; if $\sigma = 0$ we have $s^I = s^I(\theta)$ and the UI parameters have an effect only insofar as they affect market tightness. Third, we noted in Proposition 1 that the effect on, e.g., insured search of increasing UI benefits depended on whether consumption and leisure are substitutes or complements in the production sense. With our specific utility function, this provision is translated into a condition on the degree of relative risk aversion; if $\sigma < 0$, insured search increases with *b*; moreover, it is not possible to sign the effects on non-insured search in response to an increase in *z*. This being said, we will mostly discuss the *positive* results in terms of $\sigma > 0$ from here on.[7] The sign of σ will not matter for our *normative* results, however.

To find the general equilibrium effects of changing *b*, *z*, or λ, we invoke $\theta = \theta(\lambda, b, z)$:

[7] The crucial importance of the cross-derivative of utility with respect to leisure and consumption for search behavior is common in the literature; see, e.g., Mortensen (1977). Risk aversion will influence whether two goods are complements or substitutes; as Samuelson (1974, p. 1277) notes: "…(risk aversion) pushes all complementarity coefficients towards substitutability". Empirical work on search intensity is scarce, but Jones (1989) finds some support for the hypothesis that higher benefits reduce the search effort among benefit recipients. On the other hand, Blau and Robins (1990) and Wadsworth (1991) find that unemployed workers eligible for unemployment compensation search more actively than those not eligible. As suggested by Wadsworth (1992), the latter results may reflect that benefit claimants enter into an environment favorable to search, involving regular contacts with officials at employment exchange offices, for example. Our analysis ignores these aspects of the search process.

PROPOSITION 3: Equilibrium search intensity among insured workers increases when the UI benefit level (if $\sigma \geq 0$), potential benefit duration, or social assistance is reduced.

Proof: The general equilibrium effects on insured search are given by:

$$\frac{ds^I}{db} = s^I_\theta \theta_b + s^I_b < 0; \quad \frac{ds^I}{d\lambda} = s^I_\theta \theta_\lambda > 0; \quad \frac{ds^I}{dz} = s^I_\theta \theta_z < 0 .$$

The signs of these total effects follow from Proposition 2 and Lemma 2.

The comparative static effects on the search behavior of the non-insured are slightly more complicated. Consider an increase in insured income (b). With θ constant, such a change increases non-insured search because of the entitlement effect; however, a rise in b improves workers' position in the wage bargain, increasing ω and reducing θ. A reduction of θ in turn lowers the expected gain from search so this effect decreases search effort; in sum, the total effect is ambiguous. Performing the required calculations we have:

$$\frac{ds^N}{db} = s^N_b + s^N_\theta \theta_b = -\frac{\theta_b q(\theta)}{(r+\alpha^N)\upsilon^N_{ss}} \left\{ r(1-\eta)(E-U^N) - \frac{\phi\eta}{1-(r+\phi)\,k/q(\theta)} (E-U^I) \right\}.$$

For analogous reasons we get:

$$\frac{ds^N}{d\lambda} = s^N_\lambda + s^N_\theta \theta_\lambda = -\frac{\theta_\lambda q(\theta)}{(r+\alpha^N)\upsilon^N_{ss}} \left\{ r(1-\eta)(E-U^N) - \frac{\phi\eta}{1-(r+\phi)\,k/q(\theta)} (E-U^I) \right\}.$$

Although the signs of these equilibrium effects are ambiguous in general, we can sign the expressions provided that the discount rate is small enough:

PROPOSITION 4: Equilibrium search intensity among non-insured workers increases in the UI benefit level and in benefit duration as the discount rate approaches zero. Moreover, equilibrium search intensity among the non-insured is decreasing in social assistance (if $\sigma \geq 0$).

Proof: By inspection: $\lim_{r\to 0}\{ds^N/db\}>0$ and $\lim_{r\to 0}\{ds^N/d\lambda\}<0$. Also, from Proposition 2 and Lemma 2:

$$\frac{ds^N}{dz}=s_z^N+s_\theta^N\theta_z<0.$$

The reason for the importance of the discount rate is the following. In order to enjoy the direct utility gain from an increase in *b*, a presently non-insured unemployed worker has to pass through employment. By contrast, the indirect costs (i.e., the reduction in θ) accrues immediately since the outflow rate from non-insured unemployment is reduced. The smaller the discount rate, the greater the weight assigned to events that happen later on in life.

The effects of changes in *b* or λ on employment and unemployment are in general ambiguous; the reason is that these parameters of the UI system may have opposite effects on the search behavior of the insured and non-insured. The equilibrium employment effects of changes in *b* or λ will depend on the number of unemployed individuals who are covered by UI. In fact, it is possible to sign the employment effects only in the extremes, i.e., when we start from a situation where all of the unemployed are either insured ($\mu^I=1$) or non-insured. When $\mu^I=1$, we obtain the conventional result: employment increases with reductions of the generosity of the UI system, i.e., when *b* is lowered or the duration of UI is reduced. Generally, however, it is true that raising*z* reduces equilibrium employment.

It may also be of interest to look at budget neutral reforms of the benefit system. A special case of our model implies the key result in Cahuc and Lehmann (1997). Suppose – as Cahuc and Lehmann – that search effort is exogenous and equal across the two unemployment states. Using (16), it is easy to show that an increase in *b* offset by a decline of *z*, so as to hold (pre-reform) unemployment expenditures constant, will reduce labor market tightness provided that $r>0$. "Front-loading" of the benefit system has this effect since the welfare of the insured increases at the expense of the non-insured; wage claims rise as it is the welfare of the insured that is of direct relevance for wage setting.

3. Optimal Unemployment Insurance

3.1 The Optimal Policy

We are now ready to address the welfare economics of unemployment insurance. What does a socially optimal UI system look like? Determining the welfare optimizing policy involves an optimum choice of the policy variables $\{b, z, \lambda\}$. We take the welfare objective to be utilitarian, i.e.,[8]

$$W = erE + u^I rU^I + u^N rU^N + erJ + vrV \tag{18}$$

Substituting the explicit expression for the value functions into (18), invoking the flow equilibrium conditions, and taking the limit of the resulting expression as $r \to 0$, yields the following simple expression:

$$W = e\upsilon(w) + u^I \upsilon(bw, s^I) + u^N \upsilon(zw, s^N)\,. \tag{19}$$

Thus, the flow of steady state welfare simplifies to a weighted average of instantaneous utilities. We ignore discounting in order to be able to compare alternative steady states without considering the adjustment process. The general case of a positive rate of discount is more complicated and we are restricted to numerical analysis on this account; as we show in the next section, allowing for > 0 has no implication for our key result.

The tax rate is determined through the budget constraint, which simply states that taxes on the total wage bill are used to finance UI benefits and social assistance transfers. Since $Z = zw$ and $B = bw$, the budget restriction can be written as:

$$\tau e = bu^I + zu^N\,. \tag{20}$$

[8] Here, we implicitly assume that firms are owned by a group of pure "rentiers" who do not work.

Before we characterize the optimal policy, let us consider the derivative changes of (19) with respect to s^j. This yields the following result:

PROPOSITION 5: Search intensity is too low in market equilibrium.

Proof: Differentiate (19) with respect to s^j, while recognizing that $w \equiv \omega(\theta)/(1+\tau)$, $e + u^I + u^N \equiv 1$, $e = e(\theta, \lambda, s^I, s^N)$, $u^N = u^N(\theta, \lambda, s^I, s^N)$, and $\tau = \tau(b, z, e, u^N)$:

$$\frac{\partial W}{\partial s^j} = u^j \left[\upsilon_s^j + \alpha(\theta)(E - U^j) \right] - \upsilon_w w \left[e + \sum_j u^j \rho^j \right] \frac{d\tau}{ds^j} (1+\tau)^{-1}.$$

According to the first-order conditions for optimal search, (9), we get:

$$\frac{\partial W}{\partial s^j} = -\upsilon_w w \left[e + \sum_j u^j \rho^j \right] \frac{d\tau}{ds^j} (1+\tau)^{-1} > 0, \quad j = I, N, \tag{21}$$

as $d\tau / ds^j = (\partial \tau / \partial e)(\partial e / \partial s^j) + (\partial \tau / \partial u^N)(\partial u^N / \partial s^j) < 0$.

Increases in search reduce equilibrium unemployment and, hence, required unemployment expenditures. Since these gains are external to the unemployed, search intensity is too low in market equilibrium.

Now, let us proceed to the optimal choice of the parameters of the benefit system: b, z, and λ. Maximizing (19) taking into account that $\theta = \theta(b, z, \lambda)$, $s^I = s^I(\theta, b)$, $s^N = s^N(\theta, b, z, \lambda)$ and the rest of the constraints listed above we have:

$$\frac{dW}{db} = \frac{\partial W}{\partial b} + \sum_j \frac{\partial W}{\partial s^j} s_b^j + \theta_b \frac{dW}{d\theta} = 0 \tag{22a}$$

$$\frac{dW}{dz} = \frac{\partial W}{\partial z} + \frac{\partial W}{\partial s^N} s_z^N + \theta_z \frac{dW}{d\theta} = 0 \tag{22b}$$

$$\frac{dW}{d\lambda} = \frac{\partial W}{\partial \lambda} + \frac{\partial W}{\partial s^N} s_\lambda^N + \theta_\lambda \frac{dW}{d\theta} = 0, \tag{22c}$$

where $dW/d\theta = \partial W/\partial\theta + \sum_j (\partial W/\partial s^j) s_\theta^j$. We can establish the following result:

PROPOSITION 6: The optimal benefit system involves $b > z$, provided that an interior solution to (22b) exists.

Proof (sketch; see Appendix A.2 for details): Pick an arbitrary $\lambda^0 \in (0, \infty)$ and consider the trial solution $b = z$. A uniform benefit structure cannot be optimal if $dW/db > 0$ at $b = z$. Making use of (16), (22a) and (22b) it is possible to derive

$$\frac{dW}{db} = \frac{1}{\mu^N} \frac{\partial W}{\partial s^N} s_b^N > 0 .$$

According to Lemma 2, we have $s_b^N > 0$ ($\forall\sigma$). So, at = , $dW/db > 0$.

COROLLARY: The potential duration of UI benefit receipt is finite and positive: $\lambda \in (0, \infty)$.

Proof: By construction the policies $\{\lambda^0, b^0 = z^0\}$, zero potential duration $\{\lambda \to \infty, z^0\}$, and infinite potential duration of UI benefit receipt $\{\lambda = 0, b^0\}$ all yield identical welfare, since they effectively pay the same uniform wage replacement rate. According to Proposition 6, however, $W(\lambda^0, b > z^0) > W(\lambda^0, b^0 = z^0)$; therefore, an interior λ is optimal.

The result that a two-tiered benefit structure dominates a uniform one is driven by the entitlement effect, $_b^N > 0$, in conjunction with fact the search effort is too low in equilibrium, $\partial W / \partial s^N > 0$. A rise b has less adverse effects on aggregate search effort than a rise in z as it induces increased search among the non-insured. To illustrate the mechanisms at work further, we take a look at the case where utility is logarithmic.[9]

[9] The provision of the proposition translates into a restriction on σ. Provided that agents are sufficiently risk averse, they will demand some insurance and, if such demand exists, it will always be optimal to differentiate the benefit system. An alternative interpretation of Proposition 6 might be useful. By invoking (22b) on (22a), we evaluate (22a) at the optimal uniform insurance under moral hazard. Hence, the costs associated with an unequal flow of income are of the second order for marginal differences between b and z. The gains associated with restoring search incentives is of the first order, however.

AnExample:TheCaseofLogarithmicUtility

A logarithmic utility function introduces additive separability between consumption and leisure, which simplifies the expressions somewhat; it is noteworthy, however, that the basic properties of the problem remain the same. With log-utility: $s^I = s^I(\theta)$ and $s^N = s^N(\theta, b/z, \lambda)$; thus, the *relative* compensation in the two unemployment states determines non-insured search.

Combining (22a) and (22b) we get an equation for the relative size of b and z:

$$\mu^I\left[\frac{b}{z}-1\right]=\frac{s_b^N b}{s^N}>0\,. \tag{23}$$

Because of the entitlement effect there is a relative tax saving of increasing b rather than z, which implies that $b>z$ is optimal. Again, it worth noting that if we would ignore search effort, a uniform benefit structure would be optimal.

Introducing the explicit expression for the derivative of non-insured search we have:

$$\frac{b}{z}-1=\left[\frac{T-s^N}{s^N}\right]^2\frac{1}{\delta}\,. \tag{23′}$$

Although neat looking, (23′) is only an implicit function in (b/z) since $^N = {}^N(\theta, b/z, \lambda)$ and $\theta = \theta(z, b/z, \lambda)$. Nevertheless, it illustrates the claim of Proposition 6.

3.2 Numerical Results

To provide some rough indications of the numbers involved we have calibrated the model numerically. The matching function is taken to be Cobb-Douglas, i.e., $H = aS^{\eta}V^{1-\eta}$, where $S = s^I u^I + s^N u^N$. The utility function exhibits constant relative risk aversion and is specified in (10). The day is taken as the basic time unit.

We impose the "Hosios-condition", $\beta = \eta$, implying that equilibrium labor market tightness

is efficient absent policy interventions.[10] The rationale for doing so is that we do not want our results to be influenced by particular assumptions about whether market tightness is efficient or not; moreover, Moen (1997) finds that the efficient outcome arises endogenously in a framework where employers can announce the pay associated with vacant jobs. We set $\eta = 0.5$, which is in the upper range of the estimates in Blanchard and Diamond (1989). Other parameters imposed are $a = 0.023$ and $y = 1$. Hours of work are set to $\bar{h} = T/(1+\delta)$, which is the optimal working time if workers were free to choose.

For the remaining parameters, T, δ, ϕ and k, we calibrated the model assuming that utility is logarithmic, the benefit structure uniform, and $z = b = 0.3$. A wage replacement rate of 30 percent approximately corresponds to a uniform characterization of the present generosity of UI and welfare benefits in the US;[11] also, it is reasonably close to the OECD average replacement ratio in 1995 (see Martin, 1996). T, δ and k were then chosen such that we got an unemployment duration of 12 weeks, $s^I = s^N = s = 1$, and a partial equilibrium elasticity of unemployment duration with respect to benefits of 0.5 – which is in the middle range of the available estimates (see Layard ., 1991). This procedure resulted in values of $T = 1.60$, $\delta = 0.72$, and $k = 4.13$. The value of k implies that the expected vacancy cost ($k/q(\theta)$) amounts to around 14 weeks of employers' labor cost (ω).[12] The separation rate, finally, was set at $\phi = 0.000828$, which implies an annual separation rate of around 30 percent and an unemployment rate of 6.5 percent (given the above value of unemployment duration).[13]

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[10] As in all models of search and matching equilibrium, there are externalities associated with firm and worker entry into the market; hence, market tightness (θ) is not necessarily efficient. Provided that the matching function is constant returns to scale, however, there exists a rule for sharing the total surplus of a match which yields the efficient θ; see Hosios (1990). The efficient outcome occurs if workers' share of the total surplus (β) equals the elasticity of the expected vacancy duration with respect to tightness (η). When there are no policy interventions, we also arrive at the conclusion that θ is efficient if and only if $\beta=\eta$.

[11] A uniform characterization of the US benefit system is given by: $\gamma b(1-\exp(-s\alpha/\lambda))+(1-\gamma)z$, where γ denotes the fraction of unemployed workers eligible for UI (according to Blank and Card, 1991, γ is roughly equal to 0.5). The value of UI (the first term) is corrected to take its finite nature into account. Setting the expected duration of unemployment to 12 weeks, the potential duration of UI to 26 weeks, $\gamma = 0.5$, $b = 0.5$, and $z = 0.17$ (as suggested by Wang and Williamson, 1996), yields a replacement rate of around 0.3.

[12] Admittedly, expected vacancy costs are on the high side compared to the few estimates available. A benevolent interpretation is that vacancy costs also reflect training costs, but these two types of costs are distinct and should probably not be treated as synonymous. Note, however, that studies similar in spirit to ours (e.g., Costain, 1997 and Valdivia, 1996) encounter the same problem: recruitment costs are much too high to be consistent with the data. An extreme example is Valdivia (1996) where expected vacancy costs equal 4.5 times the *quarterly* producer wage.

[13] The average rate of unemployment in the US 1983-96 was 6.5 %; see OECD (1997). The inflow rate into unemployment averaged 30.8 % per year and the average duration of completed unemployment spells was 11.4 weeks during 1984-89; see Layard *et al*. (1991). Our calibration corresponds to the one in Mortensen (1994).

T 1 presents the outcome in terms of some key variables. We measure the welfare gain associated with a particular policy in the following way. Let the welfare associated with two kinds of policies be denoted by W^1 and W^0. A measure of the welfare gain of policy 1 relative to policy 0 is, then, given by the value ofξ that solves

$$W^1((1-\xi)c\ell^{\delta},\ \cdot) = W^0. \tag{24}$$

ξ has the interpretation of the consumption tax that would make a representative individual indifferent between living in policy regime 1 and 0 respectively. ξ_{base} denotes the welfare gain relative to the base run, which has a replacement rate of 30 percent. We have conducted two types of experiments: first, choosing the optimal uniform benefit system; and second, choosing the optimally differentiated benefit system. The welfare gain of moving from an optimal uniform to an optimally differentiated system is approximately given by the difference between the two entries for ξ_{base}.

The simulations reveal, unsurprisingly, that optimal benefit generosity increases in the degree of relative risk aversion $(1-\sigma)$.[14] With a uniform benefit structure, the optimal wage replacement rate varies between 27 percent ($\sigma = 0.5$) and 42 percent ($\sigma = -1$). These numbers are of the same order of magnitude as results obtained by others using related models (e.g., Valdivia, 1996). Although the computed replacement rates may seem low compared to most real-world UI systems, it is important to remember that effective replacement rates typically are much lower than the statutory ones because of various restrictions on eligibility (Martin, 1996). We also note that the implied employment rates decline from 94 to 89.5 percent as risk

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We have also conducted simulations for low values of ϕ (an annual separation rate of around 10 %) and for high values (an annual rate of 50 %); the results are only marginally changed.

[14] Note that we apply a broad definition of "consumption", since it refers to goods as well as leisure; hence, risk aversion implies that individuals would like to smooth the flow of the consumption index: $c\ell^{\circ}$; see Hansen and Imrohoroglu (1992) for a similar procedure.

Table 1: Optimal UI for various values of risk aversion

	$\sigma = 0.5$			$\sigma = 0$			$\sigma = -0.5$			$\sigma = -1$		
	$b = z = 0.3$ (base run)	$b = z$ optimal	$\{b, z, \lambda\}$ optimal	$b = z = 0.3$ (base run)	$b = z$ optimal	$\{b, z, \lambda\}$ optimal	$b = z = 0.3$ (base run)	$b = z$ optimal	$\{b, z, \lambda\}$ optimal	$b = z = 0.3$ (base run)	$b = z$ optimal	$\{b, z, \lambda\}$ optimal
Welfare and spread												
ξ_{base} (%)	--	0.02	0.03	--	0.43	0.65	--	1.47	2.10	--	2.91	4.08
ξ_{US} (%)	--	--	0.36	--	--	0.38	--	--	1.54	--	--	3.55
CV	0.560	0.591	0.659	0.503	0.402	0.415	0.465	0.325	0.338	0.439	0.279	0.304
Policy variables												
z (%)	30	26.67	24.06	30	37.91	33.84	30	40.49	35.59	30	41.81	36.16
b/z	1	1	1.45	1	1	1.69	1	1	1.88	1	1	2.04
$1/\lambda$ (weeks)	--	--	5.6	--	--	8.4	--	--	12.2	--	--	17.4
Market variables												
θ	0.192	0.200	0.198	0.268	0.216	0.199	0.364	0.239	0.202	0.489	0.263	0.203
e (%)	93.71	94.09	94.04	93.50	91.64	91.61	93.38	89.98	90.31	93.35	88.61	89.54
μ (%)	--	--	34.55	--	--	36.27	--	--	40.45	--	--	45.37
s^{N}/T (%)	76	80	82	63	53	59	53	41	48	45	34	41
s^{I}/T (%)	s/T	s/T	75	s/T	s/T	49	s/T	s/T	45	s/T	s/T	45

Notes: $CV = \text{std}(c\ell^{o})/\text{E}(c\ell^{o})$ denotes the coefficient of variation of $c\ell^{\delta}$. Parameters: $a = 0.023$, $\beta = \eta = 0.5$, $\phi = 0.000827604$, $T = 1.5998$, $\delta = 0.719514$, $y = 1$, and $k = 4.13297$.

aversion increases, an implication of the fact that higher risk aversion calls for a more generous unemployment compensation.

The potential duration of unemployment benefits is increasing in risk aversion as insurance arguments would suggest. More surprising, perhaps, is another result. It turns out that the optimal degree of benefit *differentiation* increases in risk aversion. With "low" risk aversion we have $b/z = 1.45$, and with "high" risk aversion $b/z = 2.04$, numbers that suggest that the degree of differentiation should be substantial. The key to understanding this result is to note that the time devoted to search declines with risk aversion, partly because of risk aversion *per se*,[15] but also because of the adjustments of the benefit system induced by risk aversion. When risk aversion is high, the optimal uniform system calls for increasing benefit generosity which has adverse effects on search incentives. When the unemployed devote a small fraction of their time to search there is great scope for increasing non-insured search via the entitlement effect (c.f. equation 23′). Thus, increasing the wedge between regular benefits and social assistance is an efficient way of restoring search incentives when risk aversion is high; agents are willing to pay the price of increased dispersion in order to increase search and reduce taxes.

The welfare gains implied by moving from an optimal uniform benefit system to an optimal two-tiered system appear to be fairly substantial for realistic degrees of risk aversion: agents would be willing to pay between 0.2 ($\sigma = 0$) to 1.2 ($\sigma = -1$) percent of consumption to live in the optimal two-tiered system as opposed to the optimal uniform one.[16] The gains of designing the benefit system optimally *per se* are of course even greater. The effects on employment of moving from the optimal uniform to the differentiated system, on the other hand, seem to be negligible and can go in either direction.

We have also asked the following question: What are the welfare effects of moving from the current US benefit system to the optimal two-tiered one? The US unemployment insurance is crudely characterized by: $b = 0.5$ and a potential duration of 26 weeks; finally, we set $z = 0.17$, as average social assistance payments per recipient amounted 17 percent of average earnings in

[15] To see this, hold θ constant and differentiate (17) with respect to σ.
[16] Empirical studies based union wage setting models usually find a coefficient of relative risk aversion ($1-\sigma$) that ranges between 1 and 4; see, for example, Carruth and Oswald (1985) and Farber (1978). It should be noticed, though, that risk aversion refers to income only in these studies.

1991 (see Wang and Williamson, 1996). The welfare gains (ξ_{US}) range from 0.4 to 3.6 percent for realistic degrees of risk aversion. In general, these gains are obtained through more generous levels of compensation combined with shorter potential duration of regular benefits.

Discounting and the Political Economy of UI

We conclude this section by analyzing the case with discounting. We do this for two reasons. First, we want to examine whether our results depend on the absence of discounting; as shown in Section 2.4, front-loading the benefit system may have adverse employment effects when $r > 0$ and search effort is *exogenous* (Cahuc and Lehmann, 1997). Second, when $r > 0$ distributional issues are relevant; thus, we can ask whether a proposal to move to a differentiated benefit system is politically viable, in the sense that it pleases the (presumably) employed majority.

Table 2 presents the results of varying the annual discount rate from 0 to 20 percent. We look only at the case of log-utility and use the same parameter values as in *Table* 1. The formal basis for the conducted experiments is briefly outlined in Appendix A.3.

Table 2: Optimal UI and discounting ($\sigma = 0$)

	Annual discount rate			
	0	0.05	0.10	0.20
b/z	1.69	1.70	1.71	1.74
z (%)	33.84	33.74	33.65	33.49
$1/\lambda$ (weeks)	8.45	8.27	8.09	7.79
ξ^e (%)	0.22	0.27	0.32	0.44
ξ^u (%)	0.22	0.15	0.08	–0.06

Notes: See *Table* 1 for parameter values. ξ^k, $k = e,u$ are measures of the welfare gain associated with a move from the optimal uniform to the optimal two-tiered benefit system. The definition of ξ^k is analogous to equation (24); explicit formulas are given in Appendix A.3. ξ^e refers to the employed and ξ^u to the unemployed.

As individuals become more myopic, the value of unemployment relative to employment increases. Therefore, workers opt for higher wages in the wage bargain; moreover, the present value of an occupied job decreases, given the wage cost. For these two reasons, labor market tightness declines, given the benefit system. Aggregate search effort falls with the discount rate – directly (since $E - U^j$ is decreasing in *r*) as well as indirectly (since θ falls). The decrease in search suggests that the case for exploiting the entitlement effect is strengthened. As shown in *Table* 2, the benefit system becomes *more* differentiated when the discount rate increases. It seems that the adverse effects on tightness of front-loading the benefit system is compensated by reducing the period of UI benefit receipt.

We have also calculated the consumption taxes that would make different categories of individuals indifferent between the optimal uniform system to the optimal two-tiered one. ξ^e shows the implied welfare gain for the employed and ξ^u gives the welfare change for the unemployed. ξ^u is the consumption tax that would make an unemployed individual indifferent between, on the one hand, a weighted average of the value of insured and non-insured unemployment in the two-tiered system and, on the other hand, the value of unemployment in the uniform system. These welfare measures are evaluated at the policy that maximizes welfare for society as a whole. The move to a differentiated benefit system always represents a gain for the employed. Thus, a proposal to introduce an optimal two-tiered (or multi-tiered) benefit system is a politically viable one. The political support received from the unemployed, on the other hand, varies with the discount rate: with low rates of time preference they are in favor of the policy, but when they discount the future heavily (20 percent annually) they actually oppose the proposal.

4. Concluding Remarks

The main result of this paper is that the socially optimal UI program is characterized by a declining time profile of benefit payments over the job searcher's spell of unemployment. A two-tiered benefit structure is optimal because it exploits the differential impact of higher benefits on search incentives among insured and non-insured unemployed workers: raising the compensation offered to the insured induces additional search effort among the non-insured

(the so called entitlement effect). The numerical calibration of the model suggests that the welfare gains obtained from optimal differentiation may well be substantial if individuals are sufficiently risk averse. The numerical results also indicate that a proposal to move from a uniform to a differentiated benefit structure has the virtue of being politically viable, assuming that the employed comprise the majority of the electorate.

Our main result is in sharp contrast to the claim of Davidson and Woodbury (1997), who argue that the potential duration of UI benefits should be unlimited. Their model differs from ours in certain respects, one being the treatment of wages and the number of jobs. Davidson and Woodbury have a model where wages and the number of jobs are exogenously fixed, whereas our analysis treats wages as well as the number of jobs as endogenously determined. More importantly, their policy analysis runs contrary to ours. In fact, their main argument seems to be driven by a comparison between two extremes, i.e., indefinite payments of a fixed compensation level vs. a program with fixed potential duration after which compensation drops to zero. We would argue that our analysis is less restrictive than the one presented by Davidson and Woodbury. Although many real-world UI systems involve fixed potential duration of benefits, this does not imply that there is no income available when regular benefits have been exhausted.

The absence of savings is an unrealistic feature of our analysis, although it should be kept in mind that an analytical treatment of equilibrium search models with endogenous savings has proven to be extremely difficult. Would the optimality of the two-tiered benefit structure carry over to a model where workers were allowed to save? It is not obvious why it shouldn't. The driving forces, in particular the entitlement effect, would still be present and motivate some differentiation of benefits.

A complete welfare analysis of UI policies would also have to consider the eligibility rules and how these affect behavior on both sides of the labor market. Existing systems require that workers have demonstrated some attachment to the labor force, e.g., in the form of a minimum number of weeks in employment over the past year. A normative analysis of these issues in an equilibrium framework remains a topic for future research.

Appendix

A.1 Present value differences

In a symmetric equilibrium, the differences between the expected present values are:

$$\begin{aligned} E - U^I &= A^{-1}\left\{(r+\alpha^N)(\upsilon(w)-\upsilon^I)+\lambda(\upsilon(w)-\upsilon^N)\right\} \\ E - U^N &= A^{-1}\left\{(r+\lambda+\alpha^I)(\upsilon(w)-\upsilon^N)+\phi(\upsilon^I-\upsilon^N)\right\} \\ U^I - U^N &= A^{-1}\left\{(r+\phi+\alpha^I)(\upsilon^I-\upsilon^N)+(\alpha^I-\alpha^N)(\upsilon(w)-\upsilon^I)\right\}, \end{aligned} \tag{A.1}$$

where $A \equiv (r+\lambda+\alpha^N)(r+\phi+m^I\alpha^I+m^N\alpha^N)$.

A.2 Proof of Proposition 6

The proof is by contradiction. A uniform benefit structure cannot be optimal if $dW/db > 0$ at $b = z$. Pick an arbitrary $\lambda^0 \in (0,\infty)$ and consider the trial solution $b = z$ (At $b = z$, (22c) is irrelevant since there is no real difference between insured and non-insured unemployment. For the trial solution to be well-defined, the value of λ must be interior, however; otherwise either or is not determined.). First, we want to eliminate the term $\theta_b\, dW/d\theta$ in (22a). From (16): $\theta_b = (\mu^I\rho^I)/(b\Psi_\theta)$ and $\theta_z = (\mu^N\rho^N)/(z\Psi_\theta)$ as $r\to 0$, where $\Psi_\theta = \partial\Psi/\partial\theta < 0$. Consequently, $\theta_b\, dW/d\theta = \left[(\mu^I\rho^I z)/(\mu^N\rho^N b)\right]\theta_z\, dW/d\theta$. Next, suppose that (22b) holds at $b = z$. Solving (22b) for $\theta_z\, dW/d\theta$ and substituting into the expression for dW/db we get:

$$\frac{dW}{db} = \frac{\partial W}{\partial b} - \frac{\mu^I}{\mu^N}\frac{\partial W}{\partial z} + \frac{\partial W}{\partial s^I}s_b^I - \frac{\mu^I}{\mu^N}\frac{\partial W}{\partial s^N}s_z^N + \frac{\partial W}{\partial s^N}s_b^N,$$

where we have used $\rho^I/\rho^N = 1$ at $b = z$. Also, $\partial W/\partial b = \left[\mu^I/\mu^N\right]\partial W/\partial z$ and $\partial W/\partial s^I = \left[\mu^I/\mu^N\right]\partial W/\partial s^N$. We then have

$$\frac{dW}{db} = \frac{\partial W}{\partial s^N}\left[\frac{\mu^I}{\mu^N}\left(s_b^I - s_z^N\right) + s_b^N\right].$$

To determine the sign of $(s_b^I - s_z^N)$ we make use of (17). Implicit differentiation yields:

$$s_z^N \frac{z}{T - s^N} = s_b^I \frac{b}{T - s^I} - \frac{\rho^N}{\rho^I} s_b^N \frac{b}{T - s^N}.$$

So, $s_b^I\big|_{b=z} - s_z^N\big|_{b=z} = s_b^N$ and we obtain the desired result:

$$\frac{dW}{db} = \frac{1}{\mu^N} \frac{\partial W}{\partial s^N} s_b^N > 0,$$

since $s_b^N > 0$ ($\forall\sigma$), according to Lemma 2. Hence, at $b =$, $dW/db > 0$.

A.3 Allowing for Discounting

When $r > 0$ proper policy analysis requires that we consider the comparative statics on the actual path of the economy. In this section, we briefly outline the basis for the simulation results in *Table* 2.

Starting from equation (18), we can derive the steady state flow of per capita utility:

$$W = e\upsilon(w) + u^I \upsilon(bw, s^I) + u^N \upsilon(zw, s^N) + er \frac{ky}{q(\theta)}. \tag{A.2}$$

If $r \to 0$, this expression simplifies to (19). Equation (A.2) depends on time (*t*) because of the matching technology. In particular, the economy is governed by the following equations of motion:

$$\dot{e} = \alpha^I u^I(t) + \alpha^N u^N(t) - \phi e(t) \tag{A.3}$$

$$\dot{u}^N = \lambda u^I(t) - \alpha^N u^N(t). \tag{A.4}$$

In order to find the welfare effects of the benefit scheme we calculate the effects of *permanent* differential changes in *b*, *z*, and λ on the flow value of welfare, starting from a steady state equilibrium given by (2) and (3). We do this using the methodology in Diamond (1980).

When $r > 0$, the timing of gains and losses are relevant. Therefore, the proper objective is: $\hat{W}(t) = r\int_0^{\infty} \exp(-rt)W(t)\,\mathrm{dt}$. Substituting from (A.2) we get:

$$\hat{W}(t) = r\int_0^{\infty} \exp(-rt)\left\{e(t)\upsilon(w(t)) + (1-e(t)-u^N(t))\upsilon(bw(t),s^I) + u^N(t)\upsilon(zw(t),s^N) + e(t)r\frac{ky}{q(\theta)}\right\}\mathrm{dt},$$

where $e(t)$ and $u^N(t)$ satisfy (A.3) and (A.4) with initial conditions given by (2) and (3). Notice that we take the policy change to be unanticipated and permanent. Under these assumptions, we have $\dot{\theta} = 0$ and (16) holds at all times; the reason why the model has this property is that it is free to open and close vacancies (Pissarides, 1990). Unemployment expenditure, however, is determined by the evolution of (A.3) and (A.4); hence, the wage is time-dependent as it is a function of the tax rate. We derive the policy that would be optimal in steady state by setting the derivatives of $\hat{W}$ with respect to b, z, and λ equal to zero. Budget balance is assumed to hold in every period.

To illustrate the calculation of the consumption taxes (ξ^k, $k = e,u$) in *Table* 2, let us define $\hat{W}^e = r\int_0^{\infty} \exp(-rt)rE(t)\,\mathrm{dt}$ and $\hat{W}^u = r\int_0^{\infty} \exp(-rt)\left[\mu^I(t)rU^I(t) + \mu^N(t)rU^N(t)\right]\mathrm{dt}$. $\hat{W}^e$ is the flow of welfare accruing to an employed. $\hat{W}^u$ is a weighted average of the flow of welfare accruing to individuals in insured and non-insured unemployment respectively; since we use μ^j ($j = I,N$) as weights, $\hat{}^u$ is a welfare measure for a representative unemployed individual. Now, consider the consumption taxes that would make the two categories indifferent between the optimally differentiated system and the optimal uniform system. Clearly, ξ^e must satisfy $\hat{W}^e(\xi^e) = rE$ and ξ^u, $\hat{W}^u(\xi^u) = rU$, where the left-hand side is evaluated at the optimal two-tiered system and the right-hand side at the optimal uniform system. Assuming logarithmic utility functions we have $\hat{W}^k(\xi^k) = \ln(1-\xi^k) + \hat{W}^k$ and hence

$$\xi^e = 1 - \exp\left[rE - \hat{W}^e\right]$$
$$\xi^u = 1 - \exp\left[rU - \hat{W}^u\right].$$

A.4 Several Unemployment States

Let us introduce the following notation: $X_j = x_j w$, $j = 1,\ldots,J$, denotes unemployment compensation in state j, s_j search effort, $\alpha_j = s_j \alpha(\theta)$ the outflow rate from unemployment, u_j unemployment, and $\mu_j = u_j/(1-e)$ the fraction of the unemployed in state .

The flow equilibrium conditions take the form:

$$\begin{aligned} \phi e &= \alpha(\theta)\textstyle\sum_j s_j u_j \\ (\lambda + \alpha_j)u_j &= \lambda u_{j-1}, \quad j = 2,\ldots,J-1 \\ \alpha_J u_J &= \lambda u_{J-1}. \end{aligned} \tag{A.5}$$

The flow values of being in each of the possible labor market states are given by:

$$\begin{aligned} rE &= \upsilon(w) - \phi(E - U_1) \\ rU_j &= \upsilon(X_j, s_j) + \alpha_j(\theta)(E - U_j) - \lambda(U_j - U_{j+1}), \quad j = 1,\ldots,J-1 \\ rU_J &= \upsilon(X_J, s_J) + \alpha_J(\theta)(E - U_J). \end{aligned}$$

Thus, we restrict attention to the case where the potential duration of benefit receipt is identical across unemployment states and given by $1/\lambda$.

The equilibrium conditions are straightforward generalizations of the case of two unemployment states. Equilibrium labor market tightness is determined by

$$\Psi(\theta, x_1, \cdots, x_J, \lambda) \equiv \frac{1}{\sigma}\left[1 - \textstyle\sum_j \mu_j \rho_j\right] - (\phi + \textstyle\sum_j \mu_j \alpha_j)\left[\frac{\beta}{1-\beta}\right]\frac{k/q(\theta)}{1 - \phi\, k/q(\theta)} = 0, \tag{A.6}$$

where $\rho_j = x_j^{\sigma}\left[(T - s_j)/(T - \bar{h})\right]^{\delta\sigma}$ and $r = 0$ for simplicity. From (A.6) we have that market tightness is decreasing in x_j:

$$\frac{\partial\theta}{\partial x_j} = \frac{\mu_j \rho_j}{\Psi_\theta x_j} < 0 \ (\text{as } \Psi_\theta < 0). \tag{A.7}$$

In equilibrium, search is given by

$$\rho_1 \frac{\delta}{T - s_1} = \left[\frac{\beta}{1-\beta}\right] \frac{\theta k}{1 - \phi\, k/q(\theta)}$$

$$\rho_j \frac{\delta}{T - s_j} = \frac{D_j}{\mu_j}\left\{\mu_1(\lambda + \alpha_1)\left[\frac{\beta}{1-\beta}\right]\frac{\theta k}{1 - \phi\, k/q(\theta)} \sum_{l=j}^{J} \mu_l\right\} + \frac{D_j}{\mu_j}\alpha(\theta)\left[\sum_{k=1}^{j-1}\mu_k \sum_{l=j}^{J}\mu_l \frac{\rho_k - \rho_l}{\sigma}\right], \quad j = 2, \ldots, J, \tag{A.8}$$

where D_j denotes the duration of state j: $D_j = 1/(\lambda + \alpha_j)$, $j = 1, \ldots, J-1$; $D_J = 1/\alpha_J$.

LEMMA A1: Equilibrium search intensity in state $j \geq 2$ is increasing in x_{j-1}, decreasing in x_j (sufficient condition: $\sigma \geq 0$), and decreasing in x_{j+1}.

Proof: Implicit differentiation of (A.8), recognizing that the right-hand side of (A.8) is independent of s_j.

The welfare objective and the budget constraint are given by: $W = e\upsilon(w) + \sum_j u_j \upsilon(x_j w, s_j)$ and $\tau e = \sum_j x_j u_j$ respectively. Consider the derivative of W with respect to $_j$, taking (A.5) and the budget constraint into account:

$$\frac{\partial W}{\partial s_j} = -\upsilon_w w\left[e + \sum_j u_j \rho_j\right]\frac{d\tau}{ds_j}(1+\tau)^{-1} > 0, \quad j = 1, \ldots, J. \tag{A.9}$$

Thus search intensity is too low in market equilibrium, because of the taxation externality alluded in the main text.

Consider the fully optimal policy, i.e., a choice of $\{x_1, \ldots, x_J, \lambda\}$. To simplify the exposition we consider only a logarithmic utility function. As argued in the main text, this does not change any of the fundamental properties of the problem. Thus, maximizing $W(\cdot)$ subject to (A.5), the budget constraint, $\theta = \theta(x_1, \ldots, x_J, \lambda)$, $s_1 = s_1(\theta)$, and $s_j = s_j(x_1, \ldots, x_J, \lambda, \theta)$, $j \geq 2$, yields the optimality conditions:

$$\frac{dW}{dx_j} = \frac{\partial W}{\partial x_j} + \sum_{k=2}^{J} \frac{\partial W}{\partial s_k}\frac{\partial s_k}{\partial x_j} + \frac{dW}{d\theta}\frac{\partial \theta}{\partial x_j} = 0, \;\; j = 1, \ldots, J \tag{A.10}$$

$$\frac{dW}{d\lambda} = \frac{\partial W}{\partial \lambda} + \sum_{k=2}^{J} \frac{\partial W}{\partial s_k}\frac{\partial s_k}{\partial \lambda} + \frac{dW}{d\theta}\frac{\partial \theta}{\partial \lambda} = 0, \tag{A.11}$$

where $dW/d\theta = \partial W/\partial\theta + \sum_j (\partial W/\partial s_j)(\partial s_j/\partial\theta)$.

PROPOSITION A1: Given $\lambda \in (0, \infty)$, the optimal benefit policy involves $x_1 > x_2 > \ldots > x_J$, provided that $x_J > 0$.

Proof: Suppose that the J–th condition in (A.10) holds with equality and let us consider the $(J-1)$–th. Using (A.7) we can rewrite the latter condition as follows:

$$\frac{dW}{dx_{J-1}} x_{J-1} = \frac{\partial W}{\partial x_{J-1}} x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial W}{\partial x_J} x_J + \sum_{k=2}^{J} \frac{\partial W}{\partial s_k}\left\{\frac{\partial s_k}{\partial x_{J-1}} x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial s_k}{\partial x_J} x_J\right\} = 0.$$

Now, $(\partial s_k/\partial x_{J-1})x_{J-1} - (\mu_{J-1}/\mu_J)(\partial s_k/\partial x_J)x_J = 0$ for $k = 2, \ldots, J-2$. Equation (A.9) and $(\partial W/\partial x_j)x_j = u_j\left[1 - x_j/(e(1+\tau))\right]$, $\forall j$, yields:

$$\frac{dW}{dx_j} x_j = (1+\tau)^{-1}\left\{\frac{u_{J-1}}{e}\left[x_J - x_{J-1}\right] - \sum_{k=J-1}^{J} \frac{d\tau}{ds_k}\left[\frac{\partial s_k}{\partial x_{J-1}} x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial s_k}{\partial x_J} x_J\right]\right\} = 0.$$

Since $d\tau/ds_j < 0$, $\forall j$, and s_J is increasing in x_{J-1} but decreasing in x_J, we know that $d\tau/ds_J\left[(\partial s_J/\partial x_{J-1})x_{J-1} - (\mu_{J-1}/\mu_J)(\partial s_J/\partial x_J)x_J\right] < 0$. There is left to show that raising x_J has more adverse effects on s_{J-1} than raising $_{J-1}$. Differentiation of (A.8) gives:

$$\frac{\partial s_{J-1}}{\partial x_{J-1}} x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial s_{J-1}}{\partial x_J} x_J = D_{J-1}\alpha(\theta)\frac{(T - s_{J-1})^2}{\delta}\mu_{J-1} > 0.$$

Therefore, $x_{J-1} > x_J$. Proceeding analogously one can establish that $x_{J-2} > x_{J-1}$; indeed for any $j = 2,\ldots,J$, we have $x_{j-1} > x_j$.

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