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# A Firm Bargaining with Many Workers.<sup>α</sup>

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## Abstract

The purpose of this paper is to analyze bargaining between a firm and a finite set of workers. In particular employment choice and the payoffs in equilibrium are studied. In the model, the firm first selects the workers it wants to hire. The selected workers then decide whether they want to proceed in bargaining with the firm. Finally, bargaining takes place.

In contrast to Stole & Zweibel (1996), we assume that contracts are binding. The payoff for a worker is given by a share of the contribution to production, treating all other workers as employed, in addition to the worker's outside option.

**Keywords :** Bargaining, Labor demand.

**JEL Classification :** C78, D21, J23.

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# 1 Introduction

During the last two decades, noncooperative bargaining models have been studied excessively. One of the earliest articles is that of Ståhl (1972) ; and an overview can be found in Osborne & Rubinstein (1990). In the firm-worker bargaining models there is usually one worker bargaining with the firm. Horn & Wolinsky (1988) and Jun (1989) study the situation in which the firm is bargaining with two workers. However, with the exception of Stole & Zweibel (1996), there are few models describing the situation when a firm is bargaining with several workers.

Stole & Zweibel assume that labor contracts can be renegotiated at any time before production starts. This assumption leads to wages and profits corresponding to the Shapley value. One objection to the Stole & Zweibel approach is that it is normally not that simple to renegotiate contracts. For example, a contract usually is legally binding and might stipulate that a specific time must elapse before a contract can be terminated. In addition, breaking of contracts might lead to a legal conflict that could be costly. In our model, the contracts are binding.

This paper has as a main purpose to study a model of worker-firm bargaining where the firm is bargaining with several workers. We assume that signed contracts cannot be renegotiated.

The theoretical model of the present paper builds primarily on two papers. One is the paper by Horn & Wolinsky (1988) where a bargaining situation between a firm and two workers is analyzed. The other is by Gül (1989) in which noncooperative foundations for the Shapley value are analyzed.

In the paper by Gül there are a finite number of agents who each owns some asset. In each period two agents are matched. One of the agents is selected to make a proposal, which is either accepted or rejected by the respondent. In the model presented below, one worker is chosen at random to bargain with the firm in each period, a model that is different from Gül's, where two agents are chosen at random in each period. The motivation for always selecting the firm is that the owner of the firm owns some specific asset. Without this asset, two workers joined together cannot produce any surplus. Hence, it is not meaningful for two workers to bargain with each other. This change in how bargainers are selected gives us a different solution than the Shapley value, which is the solution derived in Gül's paper.

One difference of this paper compared with Horn & Wolinsky is that we allow for an arbitrary number of workers. The bargaining structure is also slightly different. In Horn & Wolinsky, the first and second workers alternate bargaining with the firm. Furthermore, we allow for heterogeneous workers.

The analysis starts by developing the model. First, the firm chooses some

workers it wants to hire. These workers subsequently decide whether they want to bargain with the firm. Lastly, the selected workers who accepted start the bargaining process with the firm. The bargaining takes place as follows. One of the workers without an agreement is chosen at random to bargain with the firm. The choice is made with the same probability for each worker. Subsequently, bargaining between the chosen worker and the firm takes place. The proposer is chosen to be either the firm or the worker with equal probabilities. The payoffs in equilibrium are computed. Each employed worker receives half of the surplus he/she and the firm can produce, given that all other workers are treated as employed. The reason is that the firm or worker can always threaten the counterpart to wait with an agreement until the worker and the firm are the only agents without an agreement. The unemployed workers get their outside option.

The model is fully developed in the next section. Extensions of the model are examined in section 3 and concluding comments are provided in Section 4.

## 2 The model

The agents consist of a set of workers, denoted  $W$ , with generic element  $i$ , and a firm, denoted  $F$ . There are two goods, leisure and a consumption good. Let  $u_i$  denote the utility for worker  $i$  when not working. We assume that a worker either works some fixed amount of time or does not work at all.

Let  $\mathcal{W}$  be the set of subsets of  $W$ , including the empty set. Let  $f : \mathcal{W} \rightarrow \mathbb{R}^2$  denote the production function.

We restrict attention to a class of economies that satisfy the following condition. Consider two arbitrary disjoint sets  $C$  and  $D$  of workers. Suppose the firm employs the workers in  $C$ . Then, adding the set  $D$  to the firm leads to an increase in output larger than the sum of the losses of utility for the workers in  $D$ .

**Definition 1** The production function  $f$  and  $(u_i)_{i \in W}$  satisfy restricted strict superadditivity<sup>1</sup> if

$$\text{for all } C, D \subseteq W \text{ such that } C \cap D = \emptyset, f(C \cup D) \geq f(C) + \sum_{i \in D} u_i;$$

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<sup>1</sup>Superadditivity is a commonly used concept in coalition form games. See for example Osborne & Rubinstein (1995).

## 2.1 Step 1. Employment game

In the first step, the firm chooses the workers it wants to employ. Thereafter, the selected workers simultaneously decide whether to bargain with the firm or not. Thus, the strategy space of the firm is  $S_F = \{C, \mu, W, g\}$ ; with generic element  $s_F$ . The strategy space for worker  $i$  is

$$S_i = \begin{cases} \{Y, E, S; N, O, g\} & \text{if } i \in S_F; \\ \{?\} & \text{otherwise;} \end{cases}$$

with generic element  $s_i$ . Let  $S = \prod_{i \in W} S_i$ . Let  $v_i : S \rightarrow \mathbb{R}$  be the payoff function for agent  $i$ . A worker that chooses not to bargain with the firm gets  $u_i$ . The payoffs for the set of selected workers that chose to bargain with the firm are determined in the bargaining game described below. Let  $W^0(s_F) = \{i \in S_F \mid s_i = Y, E, S, g\}$  be the set of workers that accepts to bargain with the firm.

## 2.2 Step 2. The Bargaining game

The bargaining game is denoted  $\Gamma_i$ . Consider  $W^0(s_F) \cap W$ . In the remainder of this section we write  $W^0 = W^0(s_F)$ . In the bargaining game, the players consist of the workers in  $W^0$  and the firm.

When the firm reaches an agreement at some time  $t$  with worker  $i$ , the worker leaves the bargaining game. The firm can use the labor for worker  $i$  for the remainder of the bargaining game. Moreover, let  $\delta \in [0, 1)$  be the common discount factor for all the players. As long as no agreement is reached, worker  $i$  gets  $(1 - \delta)u_i$  per period. We also assume that at each round of bargaining, each of the players selected to bargain has to pay a cost  $\frac{c}{2}$ . The total bargaining cost in each period is then  $c$ .

Consider an outcome of the bargaining game, assuming  $c = 0$ . Suppose an agreement between worker  $i$  and the firm is reached at time  $T$ . Let  $w_{iT} \geq 0$  denote the wage payment to worker  $i$  at time  $T$ . The payoff of worker  $i$  is given by

$$(1 - \delta)^{T-1} \left( \sum_{t=1}^{T-1} \delta^{t-1} u_i + \delta^{T-1} w_{iT} \right)$$

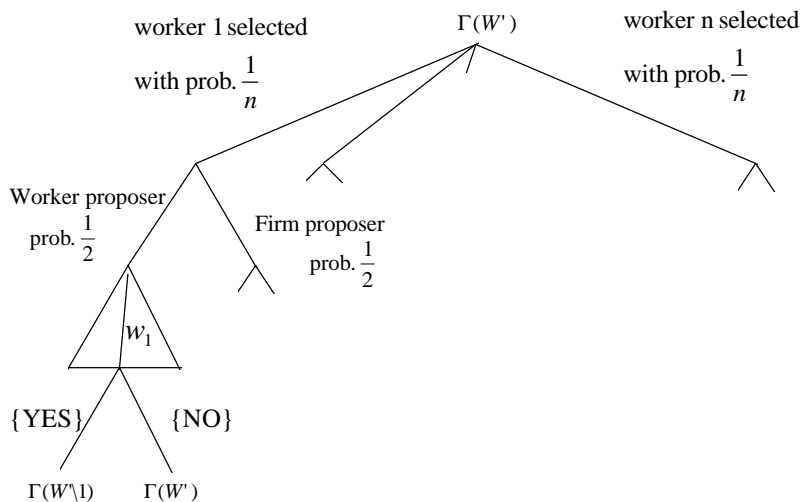
Let  $w_t \geq 0$  be the wage payment by the firm at time  $t$ . Let  $G_t$  denote the set of workers the firm has reached an agreement with at time  $t$ . The payoff of the firm is given by

$$\sum_{t=1}^T \delta^{t-1} \left( (1 - \delta) f(G_t) - w_t \right)$$

The firm bargains with one worker at a time. The worker is randomly chosen from among those workers without an agreement. The choice is made with equal probabilities for all workers. Next, bargaining begins between the chosen worker and the firm. The proposer is chosen to be either the firm or the worker with equal probabilities.

The bargaining game is depicted in Figure 1. Let  $\Gamma_i(W^0)$  denote the subgame when no worker has reached an agreement with the firm and  $\Gamma_i(W^0ni)$  denote the subgame when worker  $i$  has reached an agreement with the firm.

Figure 1: A description of the bargaining game



Let  $H_t$  be the set of all possible histories up to period  $t$ , with generic element  $h_t$ . A strategy for a player  $i$  in time period  $t$  is a function

$$\sigma_{it} : H_{t-1} \rightarrow \{ \text{YES}, \text{NO} \} \quad \text{if } i \text{ is selected to bargain and to be the proposer.}$$

$$\sigma_{it} : H_{t-1} \rightarrow \{ \text{YES}, \text{NO} \} \quad \text{if } i \text{ is selected to bargain but not to be the proposer.}$$

The strategy for the firm is similarly defined but with the difference that the firm is always selected to bargain. A strategy in the bargaining game for

player  $i$  is denoted  $\frac{3}{4}_i$ . Let  $\frac{3}{4} = (\frac{3}{4}_i)_{i \in W^0 \cup F}$  and let the set of possible strategy profiles in the bargaining game be denoted  $\mathcal{S}$ .

Let  $U_i(h_t; \frac{3}{4})$  denote the expected payoff for player  $i$ , given the strategy profile  $\frac{3}{4}$  after the history  $h_t$ .

**Definition 2** A strategy profile  $\frac{3}{4} \in \mathcal{S}$  is efficient if

$$\sum_{i \in W \cup F} U_i(h_0; \frac{3}{4}) \geq \sum_{i \in W \cup F} U_i(h_0; \frac{3}{4}^0) \quad \text{for all } \frac{3}{4}^0 \in \mathcal{S}:$$

Thus, a strategy profile is efficient if there is no other strategy profile that yields a higher sum of expected utilities to the players.

In general, the strategies at any time are a function of all possible histories up to period  $t - 1$ . However, we restrict attention to stationary strategies.

**Definition 3** A strategy  $\frac{3}{4}_i$  for player  $i$  is stationary if for any  $t \geq 1$ , any two histories  $h_{t-1} \in H_{t-1}$  and  $h_{t^0-1} \in H_{t^0-1}$  such that the firm has reached an agreement with the same set of workers, we have  $\frac{3}{4}_{it}(h_{t-1}) = \frac{3}{4}_{it^0}(h_{t^0-1})$ .

Thus, for any two histories leading up to agreements with the same set of workers, a player is restricted to play the same strategy. A strategy profile  $\frac{3}{4}$  is stationary if the strategy for each player is stationary.

Stationarity implies that for any two histories  $h_t$  and  $h_{t^0}$  such that the firm has reached an agreement with the same set of workers, we have  $U_i(h_t; \frac{3}{4}) = U_i(h_{t^0}; \frac{3}{4})$ . Let  $E$  denote a set of workers without an agreement. Consider any stationary strategy profile  $\frac{3}{4}$ . For any  $h_t$  such that the firm has reached an agreement with  $W^0 \cap E$  we define  $U_i(E; \frac{3}{4}) = U_i(h_t; \frac{3}{4})$ .

Let  $\frac{3}{4}(\pm)$  denote the stationary strategy profile which is constructed as follows. For any  $E$  and  $i \in E$  we have the following. Recall that  $E$  denotes the workers without an agreement. Presume a worker  $i$  and the firm bargain. Then, if the worker is the proposer he/she offers the firm  $\pm U_F(E; \frac{3}{4}(\pm)) + (1 - \pm)f(W^0 \cap E)$ . This bid is accepted by the firm. Similarly, if the firm is the proposer it offers the worker  $\pm U_i(E; \frac{3}{4}(\pm)) + (1 - \pm)U_i$ . This bid is accepted by the worker. Thus, when any two players are bargaining the proposer offers the other player his continuation payoff. Any player presented with a bid equal to his continuation payoff accepts the bid. This strategy is well defined for subgames with only one worker remaining. It follows by induction that it is well defined for any subgame with more than one worker remaining. A formal analysis is found in the proof of Theorem 1.

We have the following lemma.

**Lemma 1** For all  $i \in W^0 \cup F$ , the expected utilities  $U_i(W^0; \frac{3}{4}(\pm))$  are continuous functions of  $\pm$  on  $[0; 1]$ :

Proof. See Appendix. ■

Given some discount factor  $\delta \in [0; 1)$ , let  $\text{SPES}(\delta)$  denote the set of efficient subgame perfect equilibria in stationary strategies.

Lemma 2 Suppose restricted strict superadditivity is satisfied. For all  $\delta$ , if  $\text{SPES}(\delta) \neq \emptyset$ , then  $\text{SPES}(\delta) = \text{F}^{\infty}(\delta)$ .

Proof. See Appendix. ■

By using Lemma 1 and Lemma 2 we can show the following.

Theorem 1 Let  $\{\delta_k\}_{k=1}^{\infty}$  be a sequence such that  $\delta_k \in [0; 1)$  for all  $k$ . If  $\lim_{k \rightarrow \infty} \delta_k = 1$  then, for all  $C_i \in W^0$ ,

$$\lim_{k \rightarrow \infty} U_i(W^0; \mathcal{M}(\delta_k)) = \frac{f(W^0)_i - f(W^0_{ni}) + \psi_i}{2} \cdot \frac{c}{2}$$

Also,

$$\lim_{k \rightarrow \infty} U_F(W^0; \mathcal{M}(\delta_k)) = f(W^0)_i \times \prod_{i \in W^0} \frac{f(W^0)_i - f(W^0_{ni}) + \psi_i}{2} \cdot \frac{c}{2}$$

Proof. See Appendix. ■

The intuition is that if a worker makes unacceptable demands when bargaining with the firm, then ultimately other workers will strike deals with the firm, leaving the worker as the only one without an agreement.

Theorem 2 Suppose restricted strict superadditivity is satisfied and  $c > 0$ . Then, there exists  $\underline{\delta} < 1$  such that for all  $\delta \in [\underline{\delta}; 1)$ ,  $\text{SPES}(\delta)$  is nonempty.

Proof. See Appendix.<sup>2</sup> ■

Remark 1 A potential problem is that the firm can make negative profits. The reason why the firm still reaches an agreement with all the workers is that otherwise it has to pay the bargaining cost ad infinitum. This makes it profitable for the firm to agree. The way we choose to circumvent this weakness of the model is to allow the firm to choose employment before bargaining starts. Then negative profits can be avoided.

<sup>2</sup>The bargaining cost could be a function of  $\delta$ . If the result in Theorem 2 is to be true we need  $\lim_{\delta \rightarrow 1} c(\delta) > 0$ .



## 2.3 Equilibrium in the employment game

Now consider the equilibrium in the employment game. Let

$$\hat{c} = \min_{\hat{W} \in \mathcal{W}} \min_{i \in \mathcal{W}} \left[ \frac{1}{2} f(\hat{W})_i + \frac{1}{2} f(\hat{W}ni)_i + \frac{1}{2} \hat{u}_i \right] : \quad (1)$$

By restricted strict superadditivity it follows that  $\hat{c} > 0$ . We have the following result.

**Theorem 3** Let  $s^*$  be a subgame perfect equilibrium strategy profile in the employment game. For  $c < \hat{c}$  and  $\pm > \frac{1}{2}$ , for all  $i \in \mathcal{W}$ , we have  $s_i^* = fYESg$ . Also,

$$s_F^* = \arg \max_{\hat{W} \in \mathcal{W}} \left[ \frac{1}{2} f(\hat{W})_i + \frac{1}{2} f(\hat{W}ni)_i + \frac{1}{2} \hat{u}_i \right] : \quad (1)$$

**Proof.** First, consider any  $s_F \in \mathcal{S}_F$ . The payoffs for any worker  $i \in \mathcal{S}_F$  that accepts to bargain with the firm is given by the payoffs in Theorem 1 (where  $W^0 = f_j \in \mathcal{S}_F \mid s_j = fYESg$ ). From the definition of  $\hat{c}$ , restricted strict superadditivity and  $c < \hat{c}$  the payoff when accepting is strictly higher than is the payoff when rejecting bargaining with the firm, for any  $W^0 \in \mathcal{S}_F$ . Thus, any worker selected to bargain with the firm accepts. From  $s_i^*$  being a best response in the subgame following  $s_F$  it follows that, for all  $i \in \mathcal{W}$  we have  $s_i^* = fYESg$ . The firm then chooses  $s_F^*$  as in expression (1). ■

By restricted strict superadditivity any worker that is selected by the firm strictly gains when  $c$  is small. Thus, in a subgame perfect equilibrium, all workers selected by the firm accept. The firm's optimal labor choice is then given by expression (1).

The payoffs for the firm and workers selected by the firm is given by Theorem 1 where  $W^0 = s_F^*$ . For any worker  $i \in \mathcal{W} \cap s_F^*$  the payoff is  $\hat{u}_i$ .

## 3 Extensions

### 3.1 Arbitrary selection probabilities

In the bargaining game above, each worker is selected with the same probability to bargain with the firm. Now suppose that, after a history  $h_t$ , a worker  $i$  without an agreement is selected to bargain with the firm with probability  $p_i(h_t)$ . Further, assume that the worker is selected to be the proposer with probability  $p_i^p(h_t)$ .

We assume that the selection probabilities are stationary in the following sense. Consider two histories  $h_t$  and  $h_{t^0}$  such that the firm is without an agreement with the same set of workers. Then  $p_i(h_t) = p_i(h_{t^0})$  and  $p_i^p(h_t) = p_i^p(h_{t^0})$ . Recall that  $E$  denotes the set of workers without an agreement. Then, for all  $E \subseteq W^0$ , we can write  $p_i(E) = p_i(h_t)$  and  $p_i^p(E) = p_i^p(h_t)$  for any  $h_t$  such that the firm is without an agreement with the workers in  $E$ . We assume that the bargaining cost is split according to the proposer selection probabilities.

Let  $MP_i(W^0) = f(W^0) - f(W^0 - i) - \psi_i$ . In general, the payoff for a worker is a linear combination of the payoffs in two player games. This follows from stationarity. In two player games a worker  $i$  gets a fraction of  $MP_i(W^0)$ , in addition to his outside option and a share of the bargaining cost, for  $\pm$  close to one.

We have the following result.

**Theorem 4** Suppose that  $\prod_{i \in E} \frac{p_i(E)p_i^p(E)}{(1-p_i(E))(1-p_i^p(E))} < 1$  holds for all  $E \subseteq W^0$ . Let  $f_{\pm^k}^1$  be a sequence such that  $\pm^k \in [0, 1)$  for all  $k$ . If  $\lim_{k \rightarrow \infty} \pm^k = 1$  then, for all  $i \in W^0$ ,

$$\lim_{k \rightarrow \infty} U_i(W^0; \mathbb{A}(\pm^k)) = p_i^p(i)MP_i(W^0) + \psi_i - p_i^p(i)c$$

Also,

$$\lim_{k \rightarrow \infty} U_F(W^0; \mathbb{A}(\pm^k)) = f(W^0) - \sum_{i \in W^0} (p_i^p(i)MP_i(W^0) + \psi_i) - \sum_{i \in W^0} (1 - p_i^p(i))c$$

**Proof.** See Appendix. ■

The condition on the selection probabilities is needed to guarantee continuity of the expected utilities. The payoff for a worker depends only on the worker's own marginal productivity. The reason is similar to the explanation given previously in this paper. If the worker demands a higher wage than the firm is willing to agree to, the firm can threaten the worker with agreeing with all other workers first, leaving the worker alone without an agreement. An interesting fact is that the probability that a worker is selected to bargain with the firm does not affect the payoffs. Even if a worker is selected with a high probability to bargain with the firm he cannot ask for a higher wage. This follows because the firm can reject the worker's offer and agree with the other workers first. Because  $\pm$  is close to one the cost of carrying out this action is small. Some of the other results also change. For example, for step 1 in Lemma 2 to be true, we need conditions on the selection probabilities.

**Theorem 5** Suppose  $c > 0$ . Then there exists  $\frac{1}{2} < \alpha < 1$  such that for all  $\alpha \in [\frac{1}{2}, 1)$  the strategy profile  $\mathcal{A}(\alpha)$  is a stationary subgame perfect Nash Equilibrium.

**Proof.** This follows from a modification of the proof in Theorem 2. ■

In this case, we cannot necessarily show that the strategy profile  $\mathcal{A}(\alpha)$  is efficient. To do this we need conditions on the selection probabilities. The reason we need these conditions can be readily understood with reference to the following example. Suppose we have three workers. Furthermore, presume one worker, say worker 1, is much more productive than are the other two workers. Suppose that if one of the low productive workers is selected, the selection probabilities in the next period are as follows. If the firm agrees with the selected worker, then in the next period the firm meets the other low-productive worker with a very high probability; otherwise, it meets the high-productive worker with a high probability. Depending on the production function, efficiency might require that the firm disagrees with the selected low-productive worker.

## 4 Conclusions

In this paper we derive payoffs for a firm bargaining with a finite set of workers. First, the firm selects the workers it wishes to hire. The selected workers then decide whether they want to bargain with the firm or not. Finally, bargaining takes place.

In the basic model, all workers are selected to bargain with the firm with equal probability. In the bargaining game, we study efficient, stationary subgame perfect equilibria. In equilibrium, it is found that the payoff for an employed worker is given by half of the contribution to production, in addition to the worker's outside option, treating all other workers as employed. This holds because if a worker's demands are unacceptable to the firm, the firm can threaten the worker with agreeing with all other workers before it agrees with the worker that demands too much. On the other hand, if the firm demands too much, the worker can wait until the firm has agreed with all the other workers, as prescribed by the firm's strategy.

We also consider the case when the workers are selected with different probabilities. We study a strategy profile that has the property whereby an agreement is reached with each worker the first time the worker is selected to bargain with the firm. It is shown that this profile is a stationary subgame

perfect equilibrium. The payoff for an employed worker is a share of the workers contribution to production, treating all other workers as employed, in addition to the workers outside option.

This result is in contrast to that of Stole & Zweibel where the workers get the Shapley value. In the model of Stole & Zweibel, the worker can get a share of the inframarginal units. The reason is that if a worker disagrees with the firm, this leads to a series of recontracting with the remainder of the workers. In the model presented here, this is not the case.

## A Appendix

Let us assume that the firm is without an agreement with the workers in  $E \cup W^0$ . The expected payoff for the firm, given the strategy profile  $\mathcal{A}(\pm)$ , is

$$U_F(E; \mathcal{A}(\pm)) = \sum_{i \in E} \frac{1}{2jEj} \left[ \pm U_F(E \setminus i; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 \setminus [i, nE]) \right] + \frac{c}{2} \sum_{i \in E} \frac{1}{2jEj} \left[ \pm U_i(E; \mathcal{A}(\pm)) + (1 - \pm) u_i \right]$$

The term  $\pm U_F(E \setminus i; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 \setminus [i, nE])$  is the utility for the firm when it is chosen as bidder when bargaining with worker  $i$ , and the term  $\pm U_i(E; \mathcal{A}(\pm)) + (1 - \pm) u_i$  is the bid offered by worker  $i$  when the worker is bidder. The expression  $\pm U_F(E \setminus i; \mathcal{A}(\pm))$  is the discounted expected utility of the firm in the beginning of the next period, after agreeing with worker  $i$ . The expression  $(1 - \pm) f(W^0 \setminus [i, nE])$  is the production the firm obtains in the current period by employing worker  $i$ . The expression  $\pm U_i(E; \mathcal{A}(\pm)) + (1 - \pm) u_i$  is the wage paid to the worker. The firm makes a proposal such that the worker is indifferent regarding accepting or rejecting the offer. The expression  $\pm U_i(E; \mathcal{A}(\pm))$  is the discounted expected utility in the beginning of the next period for the worker if no agreement is reached. The expression  $(1 - \pm) u_i$  is the utility for the worker in the current period. Each worker is selected with probability  $\frac{1}{jEj}$  and the proposer is selected with probability  $\frac{1}{2}$  from the selected worker and the firm.

Analogously, the expected payoff for worker  $i$ , given the strategy profile  $\mathcal{A}(\pm)$ , is given by

$$U_i(E; \mathcal{A}(\pm)) = \frac{1}{2jEj} \left[ \pm U_F(E \setminus i; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 \setminus [i, nE]) \right] + \frac{c}{2} \left[ \pm U_F(E; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 \setminus [i, nE]) \right] + \frac{1}{jEj} \sum_{\substack{j \in E \\ j \neq i}} \left[ \pm U_i(E \setminus j; \mathcal{A}(\pm)) + (1 - \pm) u_i \right]$$

The first term is the payoff when worker  $i$  is selected to bargain with the firm and the worker is bidder. The second term is the payoff for worker  $i$  when selected to bargain with the firm and the firm is bidder. The remaining

terms involves the payoff when some other worker is selected to bargain with the firm.

These expressions can be written in the form (when  $j \in E$ ,  $i \neq j$ )

$$\begin{pmatrix} 0 & a & b & b & \dots & b & b & 1 \\ \vdots & b & d & 0 & \dots & 0 & 0 & 0 \\ \vdots & b & 0 & d & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \dots & 0 & 0 & \vdots \\ \vdots & b & 0 & 0 & \dots & 0 & d & 0 \\ \vdots & b & 0 & 0 & \dots & 0 & 0 & d \end{pmatrix} \begin{pmatrix} U_F(E; \mathcal{M}(\pm)) \\ U_1(E; \mathcal{M}(\pm)) \\ \vdots \\ \vdots \\ \vdots \\ U_{j \in E}(E; \mathcal{M}(\pm)) \end{pmatrix} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} D_F^\pm(E) \\ D_1^\pm(E) \\ \vdots \\ \vdots \\ \vdots \\ D_{j \in E}^\pm(E) \end{pmatrix}; \quad (2)$$

where  $a = 1 - \frac{\pm}{2}$ ,  $b = \frac{\pm}{2j \in E}$  and  $d = 1 - \frac{\pm}{2j \in E}$ . This follows from rearranging the expected utilities for the players.

Here,

$$D_F^\pm(E) = \frac{1}{2j \in E} \sum_{i \in E} \pm U_F(E; \mathcal{M}(\pm)) + (1 - \pm) f(W^0 | \text{in} E) + c_i (1 - \pm) u_i + \frac{1}{2} (1 - \pm) f(W^0 | nE);$$

and

$$D_i^\pm(E) = \frac{1}{2j \in E} \sum_{i \in E} \pm U_F(E; \mathcal{M}(\pm)) + (1 - \pm) f(W^0 | \text{in} E) + c_i (1 - \pm) f(W^0 | nE) + \frac{1}{j \in E} \sum_{\substack{j \in E \\ j \neq i}} [\pm U_i(E; \mathcal{M}(\pm)) + (1 - \pm) u_i] + \frac{1 - \pm}{2j \in E} u_i;$$

### A.1 Proof of Lemma 1

**Lemma 1** For all  $E \in \mathcal{W}^0$ , for all  $i \in F \setminus E$ , the expected utilities  $U_i(E; \mathcal{M}(\pm))$  are continuous functions of  $\pm$  on  $[0; 1]$ :

**Proof.**

The inverse of the matrix in ?? is

$$\begin{pmatrix} 0 & a & b & b & \dots & b & b & 1 \\ \vdots & b & d & e & \dots & e & e & 0 \\ \vdots & b & e & d & \dots & e & e & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \dots & e & e & 0 \\ \vdots & b & e & e & \dots & e & d & e \\ \vdots & b & e & e & \dots & e & e & d \end{pmatrix}; \quad (3)$$

where  $\hat{a} = \frac{2jE_j \pm}{2jE_j \pm (jE_j + 1)}$ ,  $\hat{b} = \frac{\pm}{\pm(jE_j + 1)}$ ,  $\hat{d} = \frac{1}{2jE_j \pm} + \frac{\pm^2}{2jE_j \pm (jE_j + 1)}$  and  $\hat{e} = \frac{\pm}{2jE_j \pm} + \frac{\pm}{2jE_j \pm (jE_j + 1)}$ . Since  $jE_j \geq 2$  all coefficients are continuous in  $\pm$  for  $\pm \in [0; 1]$  and well defined.

When only one worker,  $i \in W^0$ , remains ( $jE_j = 1$ ), it follows from Lemma 1 in Gül (1989) that

$$U_F(i; \mathcal{A}(\pm)) = \frac{f(W^0) + f(W^0 n_i) + \hat{u}_i}{2} + \frac{c}{2}$$

and

$$U_i(i; \mathcal{A}(\pm)) = \frac{f(W^0) + f(W^0 n_i) + \hat{u}_i}{2} + \frac{c}{2}$$

Since  $U_i(E; \mathcal{A}(\pm))$  is continuous in  $\pm$  for  $F$  and any  $i \in W^0$  when  $jE_j = 1$ , it follows by induction<sup>3</sup> and continuity of the inverse in  $\pm$  that the vector of  $U_i(E; \mathcal{A}(\pm))$  terms is continuous in  $\pm$  for any finite player game. ■

## A.2 Proof of Lemma 2

**Lemma 2** Suppose restricted strict superadditivity is satisfied. If  $SPES(\pm)$  is nonempty for some  $\pm$ , then  $SPES(\pm) = \mathcal{A}(\pm)g$ .

**Proof.** **Step 1.** We claim that restricted strict superadditivity implies that any efficient strategy profile requires that an agreement is reached when a firm and worker meet.

Suppose that this is not the case. Then there is a strategy profile  $\mathcal{A}$  that is efficient, where after some history, the firm disagrees with some worker  $i$ .

**Case 1.** Given  $\mathcal{A}$  an agreement is ultimately reached with all players. Pick a history  $h_t$  such that one of the last disagreements is just after  $h_t$ . Then construct a new strategy profile  $\mathcal{A}^0$  such that an agreement is reached at all meetings in the subtree following  $h_t$ , and identical to  $\mathcal{A}$  otherwise. Let  $S \subseteq I$  be the players without an agreement immediately after  $h_t$ .

Let  $S^k$  denote the players who have reached an agreement with the firm in period  $k$  after  $h_t$ . Let  $\Pr(S^k \subseteq S)$  denote the probability that an agreement has been reached with the players in  $S^k$  in the  $k$  periods following  $h_t$ , given that an agreement initially was reached with the players in  $W^0 \cap S$ . Let  $\Phi(T) = (1 - \pm) f(T) + \sum_{i \in W^0 \cap T} \hat{u}_i$ . By restricted strict superadditivity,  $\Phi(T) > \Phi(T^0)$  for any  $T \subseteq \mathcal{A}(T^0)$ .

<sup>3</sup>If  $U_i(E; \mathcal{A}(\pm))$  are continuous in  $\pm$  when  $jE_j = 1$ ; this implies that  $D_i^\pm(E^0)$  is continuous in  $\pm$  for  $jE_j = 2$  and implies that  $U_i(E^0; \mathcal{A}(\pm))$  are continuous in  $\pm$ , and so on.

Subcase 1.  $k = 1$ . The sum of expected utility in period 1 in the subtree following  $h_t$  is then under  $\frac{3}{4}^0$

$$\sum_{j \in \mu_S} \Pr(j \in S) [\Phi(S \setminus j \setminus i)] = \sum_{j \in \mu_S} \frac{1}{jS_j} [\Phi(S \setminus j \setminus i)]:$$

The sum of expected utility in period 1 in the subtree following  $h_t$  is under  $\frac{3}{4}$

$$\sum_{j \in \mu_S \setminus i} \frac{1}{jS_{j+1}} [\Phi(S \setminus j)]:$$

The difference in expected utility between  $\frac{3}{4}^0$  and  $\frac{3}{4}$  in period 1 in the subtree following  $h_t$  is then

$$\begin{aligned} & \sum_{j \in \mu_S} \frac{1}{jS_j} [\Phi(S \setminus j \setminus i)] - \sum_{j \in \mu_S \setminus i} \frac{1}{jS_{j+1}} [\Phi(S \setminus j)] \\ &= \sum_{j \in \mu_S} \frac{1}{jS_{j+1}} [\Phi(S \setminus j \setminus i) - \Phi(S \setminus j)] \\ &+ \sum_{j \in \mu_S} \frac{1}{jS_j} - \frac{1}{jS_{j+1}} [\Phi(S \setminus j \setminus i)] - \frac{1}{jS_{j+1}} [\Phi(S \setminus i)] \\ &> \sum_{j \in \mu_S} \frac{1}{jS_{j+1}} [\Phi(S \setminus j \setminus i) - \Phi(S \setminus j)] \\ &+ \sum_{j \in \mu_S} \frac{1}{jS_j} - \frac{1}{jS_{j+1}} [\Phi(S \setminus i)] - \frac{1}{jS_{j+1}} [\Phi(S \setminus i)] > 0: \end{aligned}$$

The first inequality follows by restricted strict superadditivity. In the last expression, the first row is positive by restricted strict superadditivity and the last row zero since  $\sum_{j \in \mu_S} \frac{1}{jS_j} - \frac{1}{jS_{j+1}} = \frac{1}{jS_{j+1}}$ . Then, the last inequality follows.

Thus,  $\frac{3}{4}^0$  yields a higher sum of expected utility compared with  $\frac{3}{4}$  in period 1 in the subtree following  $h_t$ .

Subcase 2.  $2 \leq k \leq jS_j - 1$ . The sum of expected utility in period  $k$  in the subtree following  $h_t$  is under  $\frac{3}{4}^0$

$$\sum_{S^k \in \mu_S} \Pr(S^k \in S) \Phi(S \setminus S^k \setminus i):$$

The sum of expected utility in period  $k$  in the subtree following  $h_t$  is under  $\frac{3}{4}$

$$\sum_{S^k \in \mu_S \setminus i} \Pr(S^k \in S \setminus i) \Phi(S \setminus S^k):$$



Consider the difference in expected production.

$$\begin{aligned}
 & \sum_{S^k \mu S} \Pr(S^k j S) \mathbb{E} \Phi(S [ S^k [ i ] )_i - \sum_{S^k \mu S [ i ]} \Pr(S^k j S [ i ] ) \mathbb{E} \Phi(S [ S^k )_i \\
 = & \sum_{S^k \mu S} \Pr(S^k j S [ i ] ) \mathbb{E} \Phi(S [ S^k [ i ] )_i - \sum_{S^k \mu S [ i ]} \Pr(S^k j S [ i ] ) \mathbb{E} \Phi(S [ S^k )_i \\
 & + \sum_{S^k \mu S} \Pr(S^k j S)_i - \sum_{S^k \mu S [ i ]} \Pr(S^k j S [ i ] ) \mathbb{E} \Phi(S [ S^k [ i ] )_i \\
 = & \sum_{S^k \mu S} \Pr(S^k j S [ i ] ) \mathbb{E} \Phi(S [ S^k [ i ] )_i - \sum_{S^k \mu S [ i ]} \Pr(S^k j S [ i ] ) \mathbb{E} \Phi(S [ S^k )_i \\
 & + \sum_{S^{k-1} \mu S} \sum_{\substack{j \in S \\ j \notin S^{k-1}}} \frac{\Pr(S^{k-1} [ j j S )}{j S^k} - \sum_{S^{k-1} \mu S} \frac{\Pr(S^{k-1} [ j j S [ i ] )}{j S^k} \mathbb{E} \Phi(S [ S^{k-1} [ i [ j ] )_i :
 \end{aligned}$$

The terms  $\Pr(S^{k-1} [ j j S )$  have to be divided by  $|S^k|$  since  $\Pr(S^k j S)$  occur  $|S^k|$  times in the double summation.<sup>4</sup> Symmetry implies, for all  $S$ , that  $\Pr(S^{k-1} [ j j S ) > \Pr(S^{k-1} [ j j S [ i ] )$  for any  $S^{k-1} \mu S$ . By restricted strict superadditivity, it follows that the last expression is strictly bigger than the following expression

$$\begin{aligned}
 & \sum_{S^k \mu S} \Pr(S^k j S [ i ] ) \mathbb{E} \Phi(S [ S^k [ i ] )_i - \sum_{S^{k-1} \mu S} \Pr(S^{k-1} [ i j S [ i ] ) \mathbb{E} \Phi(S [ S^{k-1} [ i ] )_i \\
 & + \sum_{\substack{j \in S \\ j \notin S^{k-1}}} \frac{\Pr(S^{k-1} [ j j S )}{j S^k} - \sum_{S^{k-1} \mu S} \frac{\Pr(S^{k-1} [ j j S [ i ] )}{j S^k} \mathbb{E} \Phi(S [ S^{k-1} [ i ] )_i :
 \end{aligned}$$

The first term is positive by restricted strict superadditivity. For the sum of expected utility to be larger in period  $k$  after  $h_t$  we need for all  $2 \leq k \leq j S_j$

<sup>4</sup>This follows since there are  $|S^k|$  different ways of partitioning the set  $S^k$  into two sets  $S^{k-1}$  and  $j$ .

that the following expression holds for each  $S^{k_i-1} \in S$

$$\sum_{\substack{j \in S \\ j \neq S^{k_i-1}}} \frac{\Pr(S^{k_i-1} [j] j S)}{j S^k} + \sum_{\substack{j \in S \\ j \neq S^{k_i-1}}} \frac{\Pr(S^{k_i-1} [j] j S [i])}{j S^k} + \Pr(S^{k_i-1} [i] j S [i]) = 0 \quad (4)$$

Since the selection probabilities in any period must sum up to one we also know that

$$\sum_{S^{k_i-1} \in S} \sum_{\substack{j \in S \\ j \neq S^{k_i-1}}} \frac{\Pr(S^{k_i-1} [j] j S)}{j S^k} = 1$$

and

$$\sum_{S^{k_i-1} \in S} \sum_{\substack{j \in S \\ j \neq S^{k_i-1}}} \frac{\Pr(S^{k_i-1} [j] j S [i])}{j S^k} + \Pr(S^{k_i-1} [i] j S [i]) = 1:$$

This guarantees that condition (??) holds with equality for all  $S^{k_i-1} \in S$ .

Thus,  $\frac{3}{4}^0$  yields a higher sum of expected utility compared with  $\frac{3}{4}$  in period  $2 \cdot k \cdot j S_j$  in the subtree following  $h_t$ .

Subcase 3. For period  $k$  where  $k > j S_j$  in the subtree following  $h_t$ , all workers are employed under  $\frac{3}{4}^0$ . By restricted strict superadditivity, the expected production is at least as high as under the profile  $\frac{3}{4}$  in period  $k$  in the subtree following  $h_t$ .

Since the expected production is higher in all periods in the subtree following  $h_t$ , and identical elsewhere, the sum of expected utility is higher under  $\frac{3}{4}^0$  compared with  $\frac{3}{4}$ .

Case 2. After some history  $h_t$  all meetings end in disagreement. Let  $T$  denote the remaining players. The sum of expected utility in each of the following periods is  $(1 - \alpha) f(W^0 n T) + \sum_{j \in T} \alpha_j$ . Now consider the profile  $\frac{3}{4}^0$  where an agreement is reached at any meeting after  $h_t$ , and identical to  $\frac{3}{4}$  otherwise. In any period an agreement is reached with more players than  $W^0 n T$ . Then it follows by restricted strict superadditivity that the sum of expected utility is larger.

Since the sum of expected utility is higher in all periods in the subtree following  $h_t$ , and identical elsewhere, the sum of expected utility is higher under  $\frac{3}{4}^0$  compared with  $\frac{3}{4}$ .

Step 2. Given Step 1, the strategy profile  $\frac{3}{4}(\pm)$  is efficient. Let  $\frac{3}{4}^2$  be a SPES( $\pm$ ). Suppose that after the history  $h_{t-1}$  the firm is without an agreement with the workers in  $E$ . Consider player  $i \in E$ . The expected utility for

$i$  in the beginning of period  $t$  is  $U_i(h_{t-1}; \mathcal{A}^n)$ , given  $h_{t-1}$ : Suppose  $i$  is selected to bargain, but not to be the proposer. Let  $h_t$  be the history after  $i$  has rejected a proposal. Then, any bid strictly larger than  $\pm U_i(h_t; \mathcal{A}^n) + (1 - \pm) \hat{u}_i$  is accepted. Also, any bid strictly smaller is rejected. By efficiency, a bid equal to  $\pm U_i(h_t; \mathcal{A}^n) + (1 - \pm) \hat{u}_i$  is accepted. Stationarity implies that  $U_i(h_{t-1}; \mathcal{A}^n) = U_i(h_t; \mathcal{A}^n) = U_i(E; \mathcal{A}^n)$ .

Suppose that  $|E_j| = 1$ . Then, from the analysis above, the strategy profiles  $\mathcal{A}^n$  and  $\mathcal{A}(\pm)$  coincide (see Lemma 1 in Gül (1989)). Then,  $U_i(E; \mathcal{A}^n) = U_i(E; \mathcal{A}(\pm))$ .

Now suppose  $|E_j| \geq 2$ . Assume that the strategies  $\mathcal{A}^n$  and  $\mathcal{A}(\pm)$  coincide for games with less than  $|E_j|$  players. Then, for all  $G \supseteq E$ ,  $U_i(G; \mathcal{A}(\pm)) = U_i(G; \mathcal{A}^n)$ , for all  $i \in F \setminus G$ . It therefore follows from (??) and the definition of  $D_i^\pm(E)$  that  $U_i(E; \mathcal{A}(\pm)) = U_i(E; \mathcal{A}^n)$ , for all  $i \in F \setminus E$ . This implies that the strategies  $\mathcal{A}^n$  and  $\mathcal{A}(\pm)$  coincide for games with  $|E_j|$  players. ■

### A.3 Proof of Theorem 1

For the proof we need the following lemma.

**Lemma** Let  $f_{i=1}^k g_{i=1}^1$  be a sequence such that  $\pm^k \in [0; 1)$  for all  $k$ .

If  $\lim_{k \rightarrow \infty} \pm^k = 1$ , if for any  $G \supseteq E$  such that  $|G_j| = |E_j| - 1$  for all  $i \in G$

$$\lim_{k \rightarrow \infty} U_i(G; \mathcal{A}(\pm^k)) = \frac{f(W^0)_i - f(W^0)_{ni} + \hat{u}_i}{2} \quad i \in G$$

and

$$\lim_{k \rightarrow \infty} U_F(G; \mathcal{A}(\pm^k)) = f(W^0)_i \times \frac{f(W^0)_i - f(W^0)_{ni} + \hat{u}_i}{2} \quad i \in G$$

then, for all  $i \in E$ ,

$$\lim_{k \rightarrow \infty} U_i(E; \mathcal{A}(\pm^k)) = \frac{f(W^0)_i - f(W^0)_{ni} + \hat{u}_i}{2} \quad i \in E$$

and

$$\lim_{k \rightarrow \infty} U_F(E; \mathcal{A}(\pm^k)) = f(W^0)_i \times \frac{f(W^0)_i - f(W^0)_{ni} + \hat{u}_i}{2} \quad i \in E$$

Proof. We have (when  $\pm = 1$ )

$$\begin{pmatrix} 0 \\ \vdots \\ U_F^1(E; \mathfrak{A}(1)) \\ U_1^1(E; \mathfrak{A}(1)) \\ \vdots \\ U_{jEj}^1(E; \mathfrak{A}(1)) \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ A \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ a^0 & b^0 & b^0 & \vdots & \vdots & b^0 & b^0 \\ b^0 & d^0 & e^0 & \vdots & \vdots & e^0 & e^0 \\ b^0 & e^0 & d^0 & \vdots & \vdots & e^0 & e^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & e^0 & e^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & e^0 & e^0 \\ b^0 & e^0 & e^0 & \vdots & e^0 & d^0 & e^0 \\ b^0 & e^0 & e^0 & \vdots & e^0 & e^0 & d^0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ \vdots \\ D_F^1(E) \\ D_1^1(E) \\ \vdots \\ D_{jEj}^1(E) \end{pmatrix}$$

where  $a^0 = \frac{1_i 2jEj}{1_i jEj}$ ,  $b^0 = \frac{1}{1_i jEj}$ ,  $d^0 = \frac{2jEj(1_i jEj)_i 1}{(2jEj_i 1)(1_i jEj)}$  and  $e^0 = \frac{1}{(2jEj_i 1)(1_i jEj)}$ . Also, we know that

$$D_F^1(E) = \frac{1}{2jEj} \sum_{i \in E} U_F(E \setminus i; \mathfrak{A}(1))_i \quad c:$$

From the hypothesis in the lemma we know that

$$U_F(E \setminus i; \mathfrak{A}(1)) = f(W^0)_i \sum_{\substack{j \in E \\ j \neq i}} \frac{f(W^0)_i f(W^0 \setminus nj) + \hat{u}_j}{2} \quad (jEj_i 1) \frac{c}{2} \quad c:$$

Therefore,

$$D_F^1(E) = \frac{1}{2jEj} \sum_{i \in E} f(W^0)_i (jEj_i 1) \sum_{\substack{j \in E \\ j \neq i}} \frac{f(W^0)_i f(W^0 \setminus ni) + \hat{u}_i}{2} \quad (jEj_i 1) \frac{c}{2} \sum_{i \in E} f(W^0)_i \quad c: \quad \#$$

Also, we know that

$$D_i^1(E) = \frac{1}{2jEj} U_F(E \setminus i; \mathfrak{A}(1)) + \frac{1}{jEj} \sum_{\substack{j \in E \\ j \neq i}} U_i(E \setminus j; \mathfrak{A}(1)):$$

The expression for  $U_i(E \setminus j; \mathfrak{A}(1))$  is given by

$$U_i(E \setminus j; \mathfrak{A}(1)) = \frac{f(W^0)_i f(W^0 \setminus ni) + \hat{u}_i}{2} \quad (jEj_i 1) \frac{c}{2};$$

from the hypothesis in the lemma.

By plugging the expressions  $U_i(E \setminus j; \mathfrak{A}(1))$  and  $U_F(E \setminus i; \mathfrak{A}(1))$  into  $D_i^1(E)$  we get

$$D_i^1(E) = \frac{1}{2jEj} \sum_{\substack{j \in E \\ j \neq i}} f(W^0)_i \sum_{\substack{j \in E \\ j \neq i}} \frac{f(W^0)_i f(W^0 \setminus nj) + \hat{u}_j}{2} \quad (jEj_i 1) \frac{c}{2}$$

$$+ \frac{jEj - 1}{jEj} \cdot \frac{f(W^0_i) - f(W^0_{ni}) + \psi_i}{2} \cdot \frac{c^2}{2} :$$

The expression for  $U_F^1(E; \mathfrak{A}(1))$  is given by

$$\frac{1 - 2jEj}{1 - jEj} D_F^1(E) + \frac{1}{1 - jEj} \sum_{i \in E} D_i^1(E) :$$

The expression for  $U_i^1(E; \mathfrak{A}(1))$  is given by

$$\frac{1}{1 - jEj} D_F^1(E) + \frac{2jEj(1 - jEj) - 1}{(2jEj - 1)(1 - jEj)} D_i^1(E) + \frac{1}{(2jEj - 1)(1 - jEj)} \sum_{j \in E, j \neq i} D_j^1(E) :$$

By using the expressions for the  $D$ s, it can be shown that the payoffs are as stated in the theorem by using some algebra.<sup>5</sup> ■

From the proof in Lemma 1, the payoffs, when only one worker remains, are as in the lemma. The conclusion in Theorem 1 therefore follows by induction and from setting  $E = W^0$ . ■

## A.4 Proof of Theorem 2

Before the proof of this theorem, we introduce some additional notation. Consider any set of workers  $E$  without an agreement. For the firm to (weakly) gain by making an acceptable proposal to a worker  $i \in E$  the following must hold, given that the players adhere to  $\mathfrak{A}(\pm)$  in future periods

$$\pm U_F(E; \mathfrak{A}(\pm)) + (1 - \pm) f(W^0_{inE}) \geq (\pm U_i(E; \mathfrak{A}(\pm)) + (1 - \pm) \psi_i) \pm U_F(E; \mathfrak{A}(\pm)) + (1 - \pm) f(W^0_{nE}) : \quad (5)$$

A similar condition holds if worker  $i$  makes a proposal. The first two terms on the left hand side (LHS) of the inequality is the payoff for the firm, if the firm is allowed to use the labor of worker  $i$ ; the last two terms is the wage sum paid to the worker. The wage sum paid to the worker is equal to what the worker would acquire by rejecting the firm's bid. The LHS is the firm's utility when making the lowest acceptable offer to the worker. The right

<sup>5</sup>The possibly easiest way of doing this is to compute the coefficient in front of each  $f(\cdot)$  and  $c$  term.

hand side (RHS) is the firm's utility when making an unacceptable offer to the worker. If an agreement is to be reached, the LHS must be larger than the payoff for the firm when making an unacceptable offer to the worker. Now we define

$$\begin{aligned} \epsilon_i(E; \alpha) &= \alpha U_F(E; \alpha) + (1 - \alpha) f(W^0 | i \in E) \\ &= \alpha U_F(E; \alpha) + (1 - \alpha) f(W^0 | i \in E) + \alpha (U_i(E; \alpha) + (1 - \alpha) u_i); \end{aligned}$$

for all  $i \in W^0$ .

Clearly, by the argument in Lemma 1, it follows that  $\epsilon_i(E; \alpha)$  is continuous for any  $i \in E$  and for any  $E \in W^0$ .

**Theorem 2** Suppose restricted strict superadditivity is satisfied and  $c > 0$ , then there exists  $\frac{1}{2} < \alpha < 1$  such that, for all  $\alpha \in [\frac{1}{2}, 1)$ , SPES( $\alpha$ ) is nonempty.

**Proof.** Let  $E \in W^0$  be arbitrary. Suppose that the firm has not agreed with the workers in  $E$ . By using the method in the proof of Theorem 1, the solution to the game where an agreement has been reached for any  $W^0 \cap E$  can be computed by induction.

In the limit, the expression in LHS of (??) converges to

$$\begin{aligned} & U_F(E; \alpha(1)) + U_i(E; \alpha(1)) \\ &= f(W^0 | i \in E) + \sum_{j \in E} \frac{f(W^0 | i \in E) + f(W^0 | j \in E) + u_j}{2} + (j \in E | i \in E) \frac{c}{2} + \frac{c}{2}; \end{aligned}$$

Also, the RHS converges to

$$U_F(E; \alpha(1)) = f(W^0 | i \in E) + \sum_{j \in E} \frac{f(W^0 | i \in E) + f(W^0 | j \in E) + u_j}{2} + (j \in E) \frac{c}{2};$$

This implies that

$$\epsilon_i(E; 1) = c;$$

Let

$$\alpha_i(E) = \min_{\alpha \in [0, 1]} \arg \min_{\alpha \in [0, 1]} f \epsilon_i(E; \alpha) - \epsilon_i(E; \alpha) \leq 0 \text{ for all } \alpha \in [0, 1];$$

which exists and is unique by continuity of  $\epsilon_i(E; \alpha)$ . Also,  $\alpha_i(E) < 1$  for all  $i \in E$  since  $\epsilon_i(E; 1) = c$ .

Let

$$\pm(E) = \max_{i \in E} \pm_i(E):$$

Clearly,  $\pm(E) < 1$  for any  $E \in W^0$ . Then, let

$$\pm^1 = \max_{E \in W^0} \pm(E):$$

Clearly, it follows that  $\pm^1 < 1$ .

Consider the strategy profile  $\mathfrak{A}(\pm)$  where the firm offers worker  $i$

$$\pm U_i(E; \mathfrak{A}(\pm)) + (1 - \pm) \psi_i$$

if the firm is the proposer and the worker offers the firm

$$\pm U_F(E; \mathfrak{A}(\pm)) + (1 - \pm) f(W^0 \setminus E)$$

if worker  $i$  is the proposer and where these offers (and higher) are accepted. Then, we claim that, for any  $\pm \in [\pm^1, 1)$ , this profile is an equilibrium. To deviate by making an unacceptable proposal is clearly not a profitable deviation since  $U_i(E; \pm) \geq 0$ . Also, to reject an offer equal to the continuation payoff cannot be a profitable deviation. ■

## A.5 Proof of Theorem 4

First, we need to introduce some notation. Suppose the firm is without an agreement with the workers in  $E$ . Let  $K_f^i(E)$  be the coefficient in front of  $f(W^0)$ . Let  $K_{MP_i}^i(E)$  be the coefficient in front of  $MP_i(W^0)$ . Let  $K_{\psi_i}^i(E)$  be the coefficient in front of  $\psi_i$ . Also, let  $K_c^i(E)$  be the coefficient in front of  $c$  for worker  $i$ . Thus, for  $\pm$  close to one, the payoff for any worker  $i \in E$  is approximately

$$K_f^i(E) f(W^0) + \sum_{i \in E} K_{MP_i}^i(E) MP_i(W^0) + \sum_{i \in E} K_{\psi_i}^i(E) \psi_i + K_c^i(E) c:$$

We need to show that  $K_f^i(W^0) = 0$ ,  $K_{\psi_i}^i(W^0) = 0$  for  $i \notin E$ ,  $K_{MP_i}^i(W^0) = p_i^p(i)$ ,  $K_c^i(W^0) = (1 - p_i^p(i))$  and  $K_{\psi_i}^i(W^0) = 1$  for all  $i \in W^0$ .

Consider a subgame where the workers in  $E \subset W^0$  have not agreed with the firm. Let  $p_i(E) = p_i$  and  $p_i^p(E) = p_i^p$ . Then, the expected payoff for the firm is

$$U_F(E; \mathfrak{A}(\pm)) = \sum_{i \in E} p_i (1 - p_i^p) \pm U_F(E \setminus i; \mathfrak{A}(\pm)) + (1 - \pm) f(W^0 \setminus E) + \sum_{i \in E} p_i^p c$$

$$\sum_{i \in E} p_i p_i^p [\pm U_i(E; \mathcal{A}(\pm)) + (1 - \pm) \dot{u}_i] g$$

$$+ \sum_{i \in E} p_i p_i^p \pm U_F(E; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 nE) :$$

Analogously, the expected payoff for worker  $i \in E$  is given by

$$U_i(E; \mathcal{A}(\pm)) = p_i p_i^p \pm U_F(E; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 [inE]) + \sum_{j \in E} p_j [\pm U_i(E; \mathcal{A}(\pm)) + (1 - \pm) \dot{u}_i]$$

**Lemma** Suppose  $\sum_{i \in E} \frac{p_i(E) p_i^p(E)}{(1 - p_i(E))(1 - p_i^p(E))} < 1$ . Suppose the firm is without an agreement with the workers in  $E$ . Suppose, for all  $G \supseteq E$ ,  $U_i(G; \mathcal{A}(\pm))$  is continuous in  $\pm$  on  $[0, 1]$  and well defined for all  $i \in F \subseteq G$ . Then, the expected utilities  $U_i(E; \mathcal{A}(\pm))$  are continuous functions of  $\pm$  for all  $i \in E$  on  $[0, 1]$ :

**Proof.** The expected payoffs are given by

$$\begin{matrix} \text{O} & a & b_1 & b_2 & \dots & \dots & b_{j \in E} & \text{O} & U_F(E; \mathcal{A}(\pm)) & \text{1} & \text{O} & D_F^\pm(E) & \text{1} \\ \text{A} & b_1^0 & d_1 & 0 & \dots & \dots & 0 & \text{A} & U_1(E; \mathcal{A}(\pm)) & \text{A} & \text{A} & D_1^\pm(E) & \text{A} \\ & b_2^0 & 0 & d_2 & \dots & \dots & 0 & & \vdots & & & \vdots & \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & & & \vdots & \\ & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & \vdots & & & \vdots & \\ & 0 & & & & & 0 & 0 & \vdots & & & \vdots & \\ \text{A} & b_{j \in E}^0 & 0 & 0 & \dots & \dots & 0 & 0 & U_{j \in E}(E; \mathcal{A}(\pm)) & \text{A} & \text{A} & D_{j \in E}^\pm(E) & \text{A} \end{matrix} \quad (6)$$

where  $a = 1 - \sum_{i \in E} p_i p_i^p$ ,  $b_i = \pm p_i (1 - p_i^p)$ ,  $b_i^0 = \pm p_i p_i^p$  and  $d_i = 1 - \pm p_i (1 - p_i^p)$ . This follows from rearranging the expected utilities for the players.

Here,

$$D_F^\pm(E) = \sum_{i \in E} p_i (1 - p_i^p) \pm U_F(E; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 [inE]) + \sum_{i \in E} p_i p_i^p (1 - \pm) f(W^0 nE)$$

and

$$D_i^\pm(E) = p_i p_i^p \pm U_F(E; \mathcal{A}(\pm)) + (1 - \pm) f(W^0 [inE]) + (1 - \pm) f(W^0 nE)$$



$$+ \sum_{\substack{j \in E \\ j \neq i}} p_j [\pm U_i(E \setminus j; \mathcal{A}(\pm)) + (1 \mp \pm) \psi_i] + p_i(1 \mp p_i^p)(1 \mp \pm) \psi_i$$

By hypothesis in the lemma, these expressions are continuous in  $\pm$ . Consider the inverse of the matrix in ??.

$$\begin{array}{ccccccc} \mathbf{0} & & & & & & \mathbf{1} \\ a_{00} & a_{10} & \vdots & \vdots & \vdots & \vdots & a_{j \in E j 0} \\ a_{01} & a_{11} & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{02} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{0j \in E j} & a_{1j \in E j} & \vdots & \vdots & \vdots & \vdots & a_{j \in E j j} \end{array}$$

We can show that

$$a_{00} = \frac{1}{1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}}$$

$$a_{0j} = \frac{\pm p_j p_j^p}{1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}} a_{00}$$

for  $j \geq 1$ ,

$$a_{k0} = \frac{\pm p_k(1 \mp p_k^p)}{(1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}} a_{00}$$

for  $k \geq 1$ ,

$$a_{kj} = \frac{\pm p_k(1 \mp p_k^p)}{(1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}} \frac{\pm p_j p_j^p}{1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}} a_{00}$$

for  $k \neq j, k, j \geq 1$  and

$$a_{kk} = \frac{1}{(1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}} + \frac{\pm p_k p_k^p}{(1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}} \frac{\pm p_k(1 \mp p_k^p)}{(1 \mp \sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))}} a_{00} \text{ for } k \geq 1:$$

Suppose that

$$\sum_{i \in E} \frac{p_i p_i^p}{(1 \mp p_i(1 \mp p_i^p))} < 1:$$

Then, for any  $\pm \in [0, 1)$  it is the case that

$$\sum_{i \in E} \frac{p_i p_i^p}{(1 \mp \pm p_i(1 \mp p_i^p))} < 1:$$

Thus, for any  $\pm \in [0, 1]$  the inverse is well defined. Since each  $a_{ij}$  is continuous in  $\pm$ , continuity of the inverse follows. Also, since each  $D_i^\pm(E)$  is continuous

in  $\pm$ , continuity of  $U_i(E; \mathfrak{A}(\pm))$  follows. ■

Suppose only one worker  $i$  remains. The probability that worker  $i$  is selected to be the proposer is  $p_i^p(i)$ . It follows from a slight modification of Lemma 1 in Gül(1989) that

$$U_F(i; \mathfrak{A}(\pm)) = f(W) \prod_{i \in E} p_i^p(i) f(W^0) \prod_{i \in E} (1 - p_i^p(i)) c$$

and

$$U_i(i; \mathfrak{A}(\pm)) = p_i^p(i) f(W^0) \prod_{i \in E} (1 - p_i^p(i)) c$$

Clearly, these expressions are continuous in  $\pm$ .

By this fact, the Lemma and an induction argument, we have the following corollary.

**Corollary 1** Suppose  $\prod_{i \in E} \frac{p_i(E)p_i^p(E)}{(1 - p_i(E))(1 - p_i^p(E))} < 1$  holds for all  $E \subseteq W^0$ . Then, for all  $E \subseteq W^0$ , for all  $i \in F \setminus E$ ,  $U_i(E; \mathfrak{A}(\pm))$  is continuous in  $\pm$  on  $[0; 1]$ .

Furthermore, the payoff in a game with  $|E| \geq 2$  is a linear combination of payoffs in games with only one worker remaining. This follows from repeatedly applying the matrix operation above for all games with less than  $|E|$  players.

Now consider the case when  $\pm = 1$ . The payoff for a worker  $k$  is then

$$\sum_{j \in F \setminus E} a_{jk} D_j^1(E)$$

Suppose the theorem is true for games with less than  $|E|$  players. Thus, for all  $G \subseteq E$ ,  $K_f^i(G) = 0$ ,  $K_{\cup_i}^i(G) = 0$  for  $l \notin i$  and  $K_{\cup_i}^i(G) = 1$  for all  $i \in G$ . Consider  $K_f^i(E)$ . The payoff for each worker can depend only on  $f(W^0)$  through the  $U_F(E \setminus i; \mathfrak{A}(1))$  terms. In any of these terms the coefficient in front of  $f(W^0)$  is one, by efficiency. Now consider the coefficients in front of the  $U_F(E \setminus i; \mathfrak{A}(1))$  terms. The sum of the coefficients in front of the  $U_F(E \setminus i; \mathfrak{A}(1))$  terms can be shown to be zero by some algebra. This implies that the expected utility for any worker  $i \in E$  in a game with  $|E|$  workers only is a function of the payoffs for the workers expected utilities in  $|E| - 1$  worker games. Thus,  $K_f^i(E) = 0$ .

By similar arguments and some algebra, it can be shown that  $K_{\cup_i}^i(E) = 0$  for  $l \notin i$ ,  $K_{\cup_i}^i(E) = 1$ ,  $K_{MP_i}^i(E) = p_i^p(i)$ ,  $K_c^i(E) = \prod_{i \in E} p_i^p(i)$  and  $K_{MP_i}^i(E) = 0$  for  $l \notin i$ .

The result then follows by setting  $E = W^0$ . ■

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