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Structural Breaks in Iron-Ore Prices: The Impact of the 1973 Oil Crisis

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STRUCTURAL BREAKS IN IRON-ORE PRICES: THE IMPACT OF THE 1973 OIL CRISIS*

NIKOLAY ANGELOV[†]

January 20, 2006

Abstract

This paper investigates the time-series properties of the price of iron ore. The focus is on testing a unit-root null hypothesis against a trend-stationary alternative, with a structural break allowed under both hypotheses. We consider unit-root tests with or without structural breaks, applied on historical prices of five different qualities of Swedish and Brazilian iron ore. New and more accurate critical values for the exogenous-break tests are calculated, and several of the asymptotic tests are accompanied by their bootstrap counterparts due to the limited sample sizes.

Using unit-root tests allowing for an exogenous structural break in 1973, the null hypothesis of a unit root is rejected for three of the five series. The sign and nature of the estimated breaks correspond to the state of the iron and steel industry during the first half of the 1970s. The bootstrap tests give results close to those from the asymptotic ones.

Key words: iron-ore prices; structural break; unit-root test; bootstrap

JEL classification: C15; C22; Q30

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1 INTRODUCTION

This paper deals with the time-series properties of Swedish and Brazilian iron ore. We focus on unit-root testing, and more specifically, unit-root tests that allow for structural breaks. Before stating the purpose and delimitations of the paper, a motivation for research in the field will be given, followed by a short survey of the literature on structural-break unit-root tests and an overview of earlier studies of the empirical properties of commodity prices.

Whether prices of mineral commodities are mean-reverting or follow a random walk is important for several reasons. Firstly, as noted in Schwartz (1997), the stochastic behaviour of commodity prices has important implications for the valuation of projects related to them, for instance mines or oil plants. The optimal investment rule, denoted by p^* , is the commodity price above which it is optimal to undertake the project. It depends on the underlying price process of the commodity. Schwartz analyzes an investment project in a copper mine and evaluates it using three different mean-reverting models for the underlying commodity. The results are then compared with a case where the underlying process follows a geometric random walk. The study suggests that the real options approach induces investment too late (i.e., when prices are too high) when mean reversion in prices is neglected.¹

Also Metcalf and Hassett (1995) investigate this matter, but come to the conclusion that although mean reversion has an influence on investment, there are two opposing effects which, on average, offset each other. Mean reversion has the effect of reducing the volatility of project returns, thereby reducing p^* ; this has a positive effect on investment. But there is also a negative effect: the maximum price that will be achieved over a given time period will decrease when mean reversion is introduced, thus decreasing the likelihood of reaching the trigger value.

In a recent paper, Sarkar (2003) acknowledges the two latter effects, but adds a third one, namely what he calls the risk-discounting effect. The intuition is that when mean reversion is introduced, the volatility of project returns falls, hence a lower risk-adjusted discount rate should be used for project evaluation. The lower discount rate works in two opposite directions: It results in a higher project value, and (since the option to invest can be seen as a put option and Sarkar assumes stochastic costs) it increases the option value. Sarkar's conclusion is that using a geometric random walk to approximate a mean-reverting process might have a significant impact on investment. For short-lived projects, mean reversion is more likely to have a negative impact on investment, while the effect on long-lived projects is expected to be positive. In either case, Sarkar asserts that approximating a mean-reverting process with a random walk is not justifiable.

¹See Schwartz (1997, Table XX, p. 967) for the complete set of results.

Secondly, another characteristic of a random walk is of importance, namely the fact that shocks have a permanent effect on the level of the series, rather than dying out with time. The persistence of shocks is important for the design of short- and long-term policies in countries that are largely dependent on export of a certain commodity. For instance, the optimal management of stabilization funds depends on the nature and persistence of shocks in commodity prices.

The rest of this section is organized as follows: After a short history of structural-break unit-root testing for macroeconomic time series, the literature with applications to commodities is surveyed. This is followed by a motivation for and an outline of the present paper.

Ever since the influential article by Nelson and Plosser (1982), the question of whether shocks have a transitory or a permanent effect on macroeconomic time series has been thoroughly studied. Using the augmented Dickey-Fuller test and a data set consisting of 14 macroeconomic time series, Nelson and Plosser accept the null hypothesis for almost all of the investigated series.

Perron (1989) challenges these results by postulating that only the Great Crash in 1929 and the oil-price shock in 1973 have had permanent effects on the series included in the Nelson and Plosser data set. By introducing a new test procedure, allowing for a one-time change in the level, in the slope, or both in the level and in the slope of the trend function, Perron rejects the unit-root hypothesis for 11 out of the 14 series. The oil-price shock and the Great Crash are modelled as exogenous shocks, in the sense that they are not realizations of the data generating processes of the various series. Instead, they are assumed to occur at an a priori known date.

The latter approach has been criticized and new methods have been proposed. One of the objections concerns the assumption of an exogenous break with a known date. Christiano (1992) asserts that in practice, one never selects a break date without prior information about the data. Neglecting this pre-test examination gives p -values which overstate the likelihood of the trend-break alternative hypothesis. Consequently, in Christiano's study pre-testing is taken into account, and bootstrap testing of the null hypothesis of a unit root in GNP results in high observed p -values.

The approach in Zivot and Andrews (1992) is similar to that of Christiano in its search for a break, but instead of solely relying on bootstrap methods, the asymptotic distributions of the test statistics are derived. Besides that, the finite sample distributions of the test statistics are estimated using Monte Carlo methods. Using both the asymptotic critical values and the finite-sample distributions, the study finds less conclusive evidence against the unit-root hypothesis than Perron on the Nelson and Plosser data set.

Another issue that has been discussed is the number of structural breaks in the testing procedures. By applying a model with two endogenous breaks on the Nelson and Plosser

data set, a study by Lumsdaine and Papell (1997) shows results that lie somewhere in between those of Zivot and Andrews, and Perron. The authors emphasize the importance of model selection in determining both the number of breaks and the type of break, rather than a specific preference for two endogenous breaks.

Accordingly, some studies have focused on explicit testing schemes for the number of structural breaks. One of the latest contributions is the one by Bai and Perron (2003), where a general procedure for estimation of the break dates in multiple structural change models is presented. In addition, a procedure that allows one to test the null hypothesis of l changes versus the alternative hypothesis of $l + 1$ changes is proposed. The study is concerned with estimation, and not specifically with unit-root testing.

Mineral prices have received much less attention than macroeconomic series in the empirical literature on unit-root testing, and even less in the case when structural change is allowed. One of the earlier contributions to time-series modelling of non-renewable commodity prices is the one by Slade (1982). She introduces a theoretical model for long-run price movements of mineral commodities, suggesting a U-shaped time path for the price process. An economic intuition is that initially, technological change implies falling prices, while in the long run, scarcity dominates the price process and prices rise. The empirical part of the article consists of fitting the simplest U-shaped time path, the quadratic trend, to eleven non-renewable commodities' deflated prices, among those pig iron. For most of the series, Slade finds the simple equation

$$p_t = \alpha + \beta_1 t + \beta_2 t^2 + v_t$$

suitable. For all commodities, however, the residuals are found to be severely first-order serially correlated. Instead of adding AR-terms in the regression, the OLS-estimates are adjusted for autocorrelation.

Jumping some ten years ahead in time, Agbeyegbe (1993) uses partly the same data set as Slade, but with another focus. He clearly addresses the issue of unit-root testing, applying several unit-root tests to the deflated price series. In those tests, he allows for a quadratic time trend in the data. Agbeyegbe concludes that three out of the four analyzed price series, including pig iron, are adequately represented as processes with a unit root and a quadratic time trend. Several issues not considered in the article but worth investigating are raised by the author, including the possibility of a structural change in the series.

The subsequent study by Ahrens and Sharma (1997) takes the analysis still further, including, among other tests, a unit-root test allowing for a trend break. They choose one of the six types of models introduced in Perron (1989), a model allowing for an instantaneous break both in the level and in the slope of the linear trend function at different globally important events. Those are the Great Crash in 1929, the outbreak of

World War II in 1939, and the end of the war in 1945. For each analyzed non-renewable commodity, the structural breaks unit-root test is applied while assuming a single break at any of the dates. Four out of eight analyzed series, among those pig iron, appear to be trend stationary when allowing for a possible structural break.

Two things should be borne in mind when interpreting Ahrens' and Sharma's results: First, the model by Perron assumes only one exogenous break. If that exogenous break date is changed, and the test still results in a rejection of the null hypothesis of a unit root, some ambiguities may arise as to when the break occurs. Perhaps a model with multiple exogenous breaks would be more suitable to use. Second, the authors do not take into account Perron and Vogelsang (1993), where it is stated that the limiting distribution and critical values of the test statistic from the earlier paper by Perron are incorrect. In order to be able to use the critical values, a dummy variable must be added to the test regression used by Ahrens and Sharma.

Still one relevant study is that by León and Soto (1997). While the previously described article only uses the structural breaks test as one among other unit-root tests, León and Soto explicitly focus on structural breaks testing procedures. Also, the data consist of both non-renewable and renewable commodities. The authors rely on the procedure introduced by Zivot and Andrews (1992), which endogenizes the choice of structural break. The model builds on Perron (1989), but instead of assuming an exogenous break, a data-dependent search routine is introduced, with corresponding critical values for the unit-root test statistic. The only non-renewable commodities analyzed are aluminium, copper, and silver. Allowing for a break in the level, or in the trend function, of the aluminium series rejects the unit-root null hypothesis at the ten percent level. For the copper series, the null is rejected at the ten percent level only when allowing for a break in the level of the series. The silver price series seems to be better parameterized as difference stationary.

One of the latest contributions is due to Howie (2002). The study applies several unit-root tests, with different trend function specifications, to 12 different mineral commodity price series. This study is the only one of those mentioned so far that includes an iron ore series in the analysis. The price series pertains to US-produced iron ore, and is found to be difference stationary; in other words, none of the four different tests reject the null hypothesis of a unit root. For the pig iron price series included, two of the tests (the augmented Dickey-Fuller test with a linear trend and a unit-root test with a quadratic trend) reject the unit-root null hypothesis.

The present study contributes to the previous literature in several ways. Firstly, none of the previously mentioned studies applies structural-break unit-root tests to iron-ore prices. As already mentioned, Howie (2002) analyses iron-ore prices, but excludes the possibility of structural breaks. We argue that any analysis concerning the time-series properties of

iron-ore prices should take into account the state of the iron and steel industry during the first half of the 1970s, as described in OECD (1975) and OECD (1976) (see section 4.2). Relative to other prices in the economy, the unusually high demand for steel during that period ought to have resulted in a positive price shock for iron ore.

Secondly, instead of using only a few structural-break unit-root tests among those available, we apply various tests with a single exogenous break, with a single endogenous break and several ways of selecting the break date, and finally tests with two endogenous breaks. All these are modifications of the well-known augmented Dickey-Fuller (ADF) test, which is also applied to the series.

Finally, on the more technical side, new and more accurate critical values are calculated for the exogenous-break tests. Furthermore, we do not merely rely on asymptotic tests. Several of the exogenous break tests, and also the ADF-test, are bootstrapped, mainly due to the limited sample sizes of the series.

Thus, the purpose of this study is to investigate whether the examined time series exhibit a unit root, if we allow for a structural break at an a priori known or unknown date, or at two unknown dates.

Several delimitations are made:

Perhaps the most important is the choice of data. The global market for iron ore consists of two markets—an Asian market and a European one—and this paper focuses on the latter. That being so, a motivation for the choice of specific price series from the European market is given in section 3.

Furthermore, all tests exhibit linear trend functions. Since the natural logarithm of the series is used in the analysis (see section 3), this implies exponential trends in the levels of the series. A quadratic time trend might be more appropriate (see the passage on earlier studies above), but no structural-break unit-root tests allowing for a quadratic trend are available to the author.

Finally, the focus is on tests allowing for a structural break, and thus only the ADF-test—among an ocean of no-break unit-root tests—is considered in the analysis.

The remaining part of the paper is organized as follows: The next section describes all models used, and is followed by a description of the data. After a section on the empirical results, the main findings are concluded. A description of the bootstrap tests used, the routine for calculating critical values, some of the tables, and all figures, can be found in various appendices.

2 MODELS FOR UNIT-ROOT TESTING

In this section, the models used for unit-root testing are accounted for. We begin with the well known augmented Dickey-Fuller test, followed by a description of a number of tests with exogenous or endogenous structural breaks, and with one or two possible breaks.

2.1 AUGMENTED DICKEY-FULLER (ADF) TEST

There are many different unit-root tests available, many of those surveyed in Maddala and Kim (1998). The ADF-test is chosen here largely because it is the building block of the structural-break tests to be described later on; the main purpose of this paper is to apply the structural-break models, and with that in mind, a comparison with the ADF-test is relevant.

The general testing equation is

$$p_t = \alpha + \rho p_{t-1} + \sum_{j=1}^k \gamma_j \Delta p_{t-j} + \beta t + u_t, \quad (1)$$

where $\Delta p_t = p_t - p_{t-1}$ and $u_t \sim \text{i.i.d.}(0, \sigma^2)$. The null hypothesis is that of a unit root, $\rho = 1$, while the alternative is a trend stationary process with $\rho < 1$.

One way of testing for unit root under this specification is to use the test statistic

$$\tau_{\hat{\rho}} = \frac{\hat{\rho} - 1}{\hat{\sigma}_{\hat{\rho}}}.$$

Under H_0 , $\tau_{\hat{\rho}}$ does not converge to a t -distribution in large samples; critical values can instead be found in many standard books in econometrics, for instance Hamilton (1994, Table B.6, p. 763).

Popular as this τ -test may be, it is somewhat debatable. In fact, under H_0 , $\rho = 1$ implies $\beta = 0$;² thus, we would actually want to test

$$H_0 : \{\rho = 1 \text{ and } \beta = 0\} \text{ against } H_1 : \{\rho < 1 \text{ and } \beta \neq 0\}. \quad (2)$$

If only the τ -statistic is used, there is an ambiguity as to how to interpret the coefficients under the null and under the alternative. For instance, under the alternative hypothesis, α is the level of the series and β is the coefficient of the linear trend. But under the null, α is the coefficient corresponding to t , while β is the coefficient corresponding to t^2 in a quadratic trend specification.

Now, testing (2) is by no means a standard procedure, since the sign of ρ is incorporated in the alternative.³ Abadir and Distaso (2003) propose a new class of statistics for testing similar hypothesis. In our setting, the test statistic would be a sum with of independent statistics: one for testing $\rho = 1$ against $\rho < 1$, and another for testing $\beta = 0$ against $\beta \neq 0$.

But here, we instead rely on an F -statistic for testing

²See for instance Maddala and Kim (1998, p. 38–39)

³The reason for specifying the direction is that, in analogy with the τ -test above, we would like to rule out the explosive case.

$$H_0 : \{\rho = 1 \text{ and } \beta = 0\} \text{ against } H_1 : \{\rho \neq 1 \text{ and } \beta \neq 0\}, \quad (3)$$

i.e., a test not ruling out the explosive case.⁴ The test statistic Φ_3 from Dickey and Fuller (1981) is equivalent to a standard regression F -test for performing (3), so it is well-suited to our needs. Besides critical values based on the asymptotic distribution of Φ_3 , Table VI in that article provides simulated finite sample critical values, not deviating much from the asymptotical quartiles. We use the former for sample sizes of 50.

When the test regression is estimated, a data-dependent procedure is used to find the optimal k , the number of lag parameters included in the regression. The routine is a general-to-specific one and it chooses k so that the coefficient on the last included lagged difference is significant at the 10%-level, while it is insignificant for larger k s, up to a maximum lag length of k^{\max} . For further reference, we call this procedure k -sig.

Ng and Perron (1995) compare the properties of this general-to-specific routine with other ways of selecting the lag length (i.e., information criteria). They conclude that the sequential procedure may result in a certain loss of power due to overparametrization, while the often too parsimonious model chosen with the different information criteria results in more severe size distortions. Thus, the sequential procedure is recommended. A discussion on different methods for choosing k can also be found in Maddala and Kim (1998, pp. 77–78).

The F - and τ -tests based on simulated critical values are accompanied by corresponding model-based bootstraps. For details on the latter, see appendices A.1, A.2, and A.5.

2.2 UNIT-ROOT TESTING WHEN A STRUCTURAL BREAK IS ALLOWED

Perron (1989) suggests a unit-root test, where both under the null and the alternative hypotheses, an exogenous one-time change in the level, in the slope, or both in the level and in the slope of the trend function is allowed for. The study shows how standard tests for unit root against the alternative of trend stationarity fail to reject the null even if the true data generating process is that of stationary fluctuations around a trend function which contains a one-time break.

Perron extends the Dickey-Fuller testing strategy to ensure a consistent testing procedure against shifting trend functions. The tests are applied to a number of macroeconomic time series, and for most of the series, the null is rejected. The only shocks that are found to have permanent effect are the 1929 crash and the 1973 oil-price shock.

Three models under the null hypothesis are considered, denoted by A , B , and C , with

⁴This approximation of the directional test with an ordinary F -test can be expected to be quite accurate for the data used in this paper. In the bootstrap tests described in appendix A.5, the fraction of δ -estimates that were larger than zero were estimated. For none of the five investigated series the fraction was larger than one percent, implying that a double-sided test—for practical purposes and for our data—is almost equivalent to a directional one.

corresponding models under the alternative. Given a series $\{p_t\}_0^T$, the hypotheses are as follows:

Models under the null:

$$\begin{aligned}
(A) \quad p_t &= \mu + dD(TB)_t + p_{t-1} + e_t, \\
(B) \quad p_t &= \mu_1 + p_{t-1} + (\mu_2 - \mu_1)DU_t + e_t, \text{ and} \\
(C) \quad p_t &= \mu_1 + p_{t-1} + dD(TB)_t + (\mu_2 - \mu_1)DU_t + e_t,
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
D(TB)_t &= 1 \text{ if } t = T_B + 1, 0 \text{ otherwise;} \\
DU_t &= 1 \text{ if } t > T_B, 0 \text{ otherwise;} \text{ and} \\
A(L)e_t &= B(L)\xi_t,
\end{aligned}$$

$\xi_t \sim \text{i.i.d.}(0, \sigma^2)$, with $A(L)$ and $B(L)$ p th and q th polynomial, respectively, in the lag operator L . Thus, the innovation series $\{e_t\}$ is taken to be of the ARMA(p, q) type with (p, q) possibly unknown.

Models under the alternative:

$$\begin{aligned}
(A) \quad p_t &= \mu_1 + \beta t + (\mu_2 - \mu_1)DU_t + \nu_t, \\
(B) \quad p_t &= \mu + \beta_1 t + (\beta_2 - \beta_1)DT_t^* + \nu_t, \text{ and} \\
(C) \quad p_t &= \mu_1 + \beta_1 t + (\mu_2 - \mu_1)DU_t + (\beta_2 - \beta_1)DT_t + \nu_t,
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
DT_t^* &= t - T_B \text{ if } t > T_B, 0 \text{ otherwise,} \\
DT_t &= t \text{ if } t > T_B, 0 \text{ otherwise, and} \\
\nu_t &\sim \text{ARMA}(p + 1, q).
\end{aligned}$$

Under both the null and the alternative, T_B refers to the time of the structural break, i.e., the period at which the change in the parameters of the trend function occurs.⁵

Model (A) is called the crash model. Under the unit-root null, the model allows for a one-time shock at T_B . The alternative hypothesis of trend-stationarity allows for a one-time shift in the intercept of the trend function. Model (B) is referred to as the

⁵The relations between the null and the alternative hypotheses might not be obvious, but in fact the

changing growth model. Under the alternative hypothesis, a change in the slope of the trend function without any sudden change in the level at the time of the break is allowed for. Model (C) allows for both effects to take place simultaneously, i.e., a sudden change in the level of the series, followed by a different growth path.

The methods above imply that the structural break occurs instantaneously, which does not need to be the case. Perron suggests an extension, where it is supposed that the time series reacts gradually to a shock function. Consider, for instance, the deterministic part of the alternative of Model (A). From (5) we can write it as

$$\eta_t^A = \mu_1 + \beta t + \gamma DU_t,$$

where $\gamma = (\mu_2 - \mu_1)$. Perron calls this a trend function, and we adhere to that denotation. A plausible adjustment is to define the trend function as

$$\eta_t^A \equiv \mu_1 + \beta t + \gamma\psi(L)DU_t,$$

where $\psi(L)$ is an invertible polynomial in L with $\psi(0) = 1$. The long-run change in the level is given by $\gamma\psi(L)$ while the immediate impact is simply γ . A similar framework holds for models (B) and (C).

One way to incorporate such a gradual change in the trend function is to suppose that the series responds to a shock in the trend function the same way as it reacts to any innovation. This can be achieved by imposing $\psi(L) = B(L)^{-1}A(L)$. Nesting the corresponding models under the null and the alternative hypotheses results in the following regressions:⁶

null is a special case of the alternative. For instance, in the case of Model (A) we can write the null as

$$\begin{aligned} p_t &= \mu + dD(TB)_t + p_{t-1} + e_t \\ &= \mu + dD(TB)_t + (\mu + dD(TB)_{t-1} + p_{t-2} + e_{t-1}) + e_t \\ &= p_0 + \mu t + d \sum_{j=0}^{t-1} D(TB)_{t-j} + \sum_{j=0}^{t-1} e_{t-j} \\ &= p_0 + \mu t + dDU_t + v_t \\ &\equiv \mu_1 + \beta t + (\mu_2 - \mu_1)DU_t + v_t, \end{aligned}$$

where $v_t = (v_{t-1} + e_t) \sim \text{ARIMA}(p, 1, q)$. Now, an $\text{ARIMA}(p, 1, q)$ process can be seen as a special case of an $\text{ARMA}(p+1, q)$ process with the first autoregressive coefficient equal to one. Thus, for Model (A), the null hypothesis is a special case of the alternative, and in a similar way the same holds for Models (B) and (C).

⁶For details on why DU_t is missing from the RHS of the equation for Model (B), see Perron (1989, p. 1381)

$$\begin{aligned}
(A) \ p_t &= \mu^A + \beta^A t + \theta^A DU_t \\
&\quad + d^A D(TB)_t + \alpha^A p_{t-1} + \sum_{i=1}^k c_i^A \Delta p_{t-i} + e_t, \\
(B) \ p_t &= \mu^B + \beta^B t \\
&\quad + \gamma^B DT_t^* + \alpha^B p_{t-1} + \sum_{i=1}^k c_i^B \Delta p_{t-i} + e_t, \text{ and} \\
(C) \ p_t &= \mu^C + \beta^C t + \theta^C DU_t \\
&\quad + \gamma^C DT_t + d^C D(TB)_t + \alpha^C p_{t-1} + \sum_{i=1}^k c_i^C \Delta p_{t-i} + e_t.
\end{aligned} \tag{6}$$

The equations in (6) imply so called innovational outliers (*IO*), while those mentioned earlier exhibit additive outliers (*AO*). We are interested in the limiting distributions of statistics for testing the unit-root hypothesis in both cases, which will be accounted for below.⁷

First, let us consider the case with additive outliers. Let $\{\tilde{p}_t^i\}$, $i = A, B, C$, be the estimated residuals from the following regressions:

$$\begin{aligned}
(A) \ p_t &= \mu^A + \beta^A t + \theta^A DU_t + \tilde{p}_t^A, \\
(B) \ p_t &= \mu^B + \beta^B t + \gamma^B DT_t^* + \tilde{p}_t^B, \text{ and} \\
(C) \ p_t &= \mu^C + \beta^C t + \theta^C DU_t + \gamma^C DT_t + \tilde{p}_t^C.
\end{aligned} \tag{7}$$

Consider the autoregression

$$\tilde{p}_t^i = \alpha^i \tilde{p}_{t-1}^i + \sum_{j=0}^k \omega_j^i D(TB)_{t-j} + \sum_{j=1}^k c_j^i \Delta \tilde{p}_{t-j}^i + \tilde{e}_t. \tag{8}$$

For testing $\alpha = 1$ for $i = A, C$, Theorem 2 in Perron (1989, p. 1373) gives the asymptotic distribution of

$$\tau_{\hat{\alpha}^i} = \frac{\hat{\alpha}^i - 1}{\hat{\sigma}_{\hat{\alpha}^i}},$$

where $\hat{\alpha}^i$ is the least squares estimator of α^i .⁸ The critical values for the test can be found in Tables IV.B and VI.B on pp. 1376–1377 in the same article.

Model (*B*) demands a somewhat different treatment. For the case when $\{e_t\}$ is i.i.d.,

⁷As pointed out Perron and Vogelsang (1992) and Perron and Vogelsang (1993), Perron (1989) contains an error concerning the limiting distribution of the test statistic in the case with additive outliers.

⁸See Perron and Vogelsang (1993, p. 249).

	H_0	H_1
Model (A)	$\alpha^A = 1, \beta^A = 0, \theta^A = 0$	$\alpha^A < 1, \beta^A \neq 0, \theta^A \neq 0$
Model (B)	$\alpha^B = 1, \beta^B = 0, \gamma^B = 0$	$\alpha^B < 1, \beta^B \neq 0, \gamma^B \neq 0$
Model (C)	$\alpha^C = 1, \beta^C = 0, \gamma^C = 0$	$\alpha^C < 1, \beta^C \neq 0, \gamma^C \neq 0$

TABLE 1 The joint hypotheses in the *IO*-setting

one must run the regression

$$\tilde{p}_t^B = \alpha^B \tilde{p}_{t-1}^B + \tilde{e}_t \quad (9)$$

and consult the limiting distribution of $\tau_{\tilde{\alpha}^B}$ in Perron and Vogelsang (1993, p. 248), and the corresponding critical values from Table I in the same article. The same asymptotic distribution corresponds to the case when we are dealing with Model (B) and $\{e_t\}$ follows an ARMA(p, q) process; however, the test regression takes the following form:

$$\tilde{p}_t^B = \alpha^B \tilde{p}_{t-1}^B + \sum_{j=1}^k c_j^B \Delta \tilde{p}_{t-j}^B + \tilde{e}_t. \quad (10)$$

Things are simpler when we are dealing with innovational outliers. The limiting distribution of $\tau_{\hat{\alpha}^i}$, $i = A, B, C$, from (6) is given in Theorem 2 in Perron (1989, p. 1373), with $\hat{\alpha}^i$ being the least squares estimator from each regression. The simulated critical values for the test using $\tau_{\hat{\alpha}^i}$ can be found in Tables IV.B, V.B, and VI.B on pp. 1376–1377 in the same article. But in our paper, new critical values resulting from more replications than in Perron are calculated; more details are given in section 4.2 and in appendix B.

As in the ADF-case, using a τ -statistic to test the null hypothesis of a unit root is somewhat problematic. In Table 1, the full set of restrictions implied by the unit-root null in the *IO*-models are displayed, as stated in Perron (1989, pp. 1380–1381).⁹

Following section 2.1, we use an F -test, i.e., instead of having $\alpha^i < 1$ in the alternative, we have $\alpha^i \neq 1$, for $i = A, B, C$. Furthermore, since the article by Perron only provides τ -tests, we rely on bootstrap procedures of the type described in appendix A.5.

2.3 SPECIAL CASE 1: UNIT-ROOT TEST ALLOWING FOR AN UNKNOWN BREAK DATE

The procedures above have been criticized for the assumption of an exogenous break date. It has been claimed that in practice, a researcher would never choose T_B before pre-examining the data. This being so, not taking this pre-testing into account violates the exogeneity assumption and results in size distortions.

⁹We concentrate on the *IO*-case since it more easily allows for an F -test than the two-step *AO* testing procedure. As will be seen in 4.2, the *AO* and *IO* tests give very similar results when the τ -test is applied, so omitting *AO* in the F -test setting is not expected to alter the results.

On the other hand, in a summary assessment of the literature on endogenizing the structural break, Maddala and Kim (1998, p. 398) claim that searching for a break with data-dependent methods is not completely justifiable. In practice, there is a lot of prior information, and there is no reason why we should not use it. Assuming that we know of a drastic policy change or some major event, it is not very meaningful to search for a break over the entire sample; instead, any potential break search should be concentrated around that event.

The present study examines a case where a break search is conducted over the entire sample period. The analysis is based on the methods in Vogelsang and Perron (1998), where the assumption of a known break date is dropped; instead, the date of possible change is considered as unknown.

Two methods to choose T_B are considered. The first method chooses the T_B such that $\tau_{\hat{\alpha}^i}$, $i = A, B, C$, from running (6), (8), (9), or (10), is minimized. This choice of T_B corresponds to the break date which is most likely to reject the unit-root null hypothesis. The resulting test statistic is $\tau_{\hat{\alpha}^i}^*(m, T_B(\tau_{\hat{\alpha}^i})) = \inf_{T_B \in [0, T]} \tau_{\hat{\alpha}^i}(m, T_B, k)$, where $m = AO$ or $m = IO$.

The second routine involves choosing T_B to maximize some statistic that tests the significance of one or more of the break parameters (θ, γ) . For Model (A), T_B is chosen using the maxima of the test statistics for testing $\theta > 0$ and $|\theta| > 0$, i.e., $t_{\hat{\theta}A}$ or $|t_{\hat{\theta}A}|$, from (6) or (7). The maximum of $|t_{\hat{\theta}A}|$ is used when the direction of the break is not known, while the maximum/minimum of $t_{\hat{\theta}A}$ is used when the direction of the break is a priori known to be positive/negative. If the break is known to have a negative sign, the minimum of $t_{\hat{\theta}A}$ is used.

For Model (B), the break date is chosen to maximize $t_{\hat{\gamma}B}$ or $|t_{\hat{\gamma}B}|$ (or to minimize $t_{\hat{\gamma}B}$ if the break sign is known to be negative) from (6) or (7).

For Model (C), T_B is chosen as the break date that maximizes $t_{\hat{\gamma}C}$, $|t_{\hat{\gamma}C}|$, or $F_{(\hat{\theta}, \hat{\gamma})C}$, from regressions (6) or (7). Once again, if the direction of the break is known to be negative, $t_{\hat{\gamma}C}$ is minimized. $F_{(\hat{\theta}, \hat{\gamma})C}$ is the F -statistic for testing the joint hypothesis $\theta = \gamma = 0$.

To summarize, the second routine results in the following statistics: $\tau_{\hat{\alpha}^A}^*(m, T_B(t_{\hat{\theta}A}))$ or $\tau_{\hat{\alpha}^A}^*(m, T_B(|t_{\hat{\theta}A}|))$; $\tau_{\hat{\alpha}^B}^*(m, T_B(t_{\hat{\gamma}B}))$ or $\tau_{\hat{\alpha}^B}^*(m, T_B(|t_{\hat{\gamma}B}|))$; and $\tau_{\hat{\alpha}^C}^*(m, T_B(t_{\hat{\gamma}C}))$, $\tau_{\hat{\alpha}^C}^*(m, T_B(|t_{\hat{\gamma}C}|))$, or $\tau_{\hat{\alpha}^C}^*(m, T_B(F_{(\hat{\theta}, \hat{\gamma})C}))$.

For any possible choice of T_B , the data-dependent procedure k -sig described in section 2.1 is used to find the optimal k , the number of lag parameters included in the regressions.

Both the asymptotic and the finite sample critical values of the different $\tau_{\hat{\alpha}^i}^*$ -statistics are accounted for in Vogelsang and Perron (1998).

2.4 SPECIAL CASE 2: UNIT-ROOT TEST ALLOWING FOR MULTIPLE UNKNOWN BREAKS

A model following the Perron methodology, but with multiple breaks, is proposed in Morimune and Nakagawa (1997). There, the possible breaks are assumed to be exogenously given. Another article dealing with the issue is Bai and Perron (2003), with a focus on estimation of the break dates and model estimation, rather than tests for a unit root.

In this paper, the model of choice is instead the one proposed by Lumsdaine and Papell (1997). It involves a search for the location of two possible breaks in the following model setting:

$$\begin{aligned}
 (AA) \ p_t &= \mu + \beta t + \theta_1 DU_{1t} + \theta_2 DU_{2t} \\
 &\quad + \alpha p_{t-1} + \sum_{i=1}^k c_i \Delta p_{t-i} + e_t \\
 (CA) \ p_t &= \mu + \beta t + \theta_1 DU_{1t} + \gamma DT_{1t} + \theta_2 DU_{2t} \\
 &\quad + \alpha p_{t-1} + \sum_{i=1}^k c_i \Delta p_{t-i} + e_t \\
 (CC) \ p_t &= \mu + \beta t + \theta_1 DU_{1t} + \gamma DT_{1t} + \theta_2 DU_{2t} + \gamma_2 DT_{2t} \\
 &\quad + \alpha p_{t-1} + \sum_{i=1}^k c_i \Delta p_{t-i} + e_t,
 \end{aligned} \tag{11}$$

where

$$\begin{aligned}
 DU_{it} &= 1 \text{ if } t > T_{Bi}, 0 \text{ otherwise; and} \\
 DT_{it} &= t - T_{Bi} \text{ if } t > T_{Bi}, 0 \text{ otherwise,}
 \end{aligned}$$

where $i = AA, CA,$ or CC . Model (AA) corresponds to Model (A) with two break dates; Model (CA) corresponds to Model (C) at the first break date and Model (A) at the second break date; and Model (CC) corresponds to Model (C) with two break dates.

Lumsdaine and Papell compute the limiting distribution of the estimated test statistic $\tau_{\hat{\alpha}^i}^*(T_{B1}, T_{B2}) = \inf_{T_{B1}, T_{B2}} \tau_{\hat{\alpha}^i}(T_{B1}, T_{B2})$ over distinct pairs of values of (T_{B1}, T_{B2}) for $T_{B1} = k_0, k_0+1, \dots, T-k_0, T_{B2} = k_0, k_0+1, \dots, T-k_0, T_{B1} \neq T_{B2}, T_{B1} \neq (T_{B2} \pm 1)$. Here, $k_0 = T\delta_0$, and δ_0 represents some startup fraction of the sample.

3 DATA

In this section we give a description of the data set used in the paper. But we begin with a short overview of the European market for iron ore, closely following Hellmer (1997, part I).

The world iron-ore market consists of a European and an Asian market. The price of iron ore is set once a year in negotiations between the major supplier and the major user in each market. Negotiating parts in the European market are most commonly steel producers from Germany and the Brazilian iron ore company CVRD. The resulting prices from the negotiations in each market are taken as the market prices for the coming year, used by all other exporters of iron ore; thus, the Swedish company LKAB can be seen as a price-taker and has no influence on the world price. All prices to Europe are set as f.o.b. prices¹⁰, but prices are adjusted so that in principle, all prices to the buyer's port are equalized. Usually this means that for the European market, the f.o.b. price in Brazil is lower than the f.o.b. price in Sweden because of the shipping distance. There are two predominant qualities of iron ore traded on the world market: sinter feed (also called fines), and pellets, which earns a premium relative to fines.

The world trade in iron ore was limited before 1940. Trading volumes began to grow after the second world war, but the most significant increases were during the 1950s and 1960s, when transportation costs fell and more distant iron-ore deposits became profitable to exploit. Thus if one wants to examine the European or the world market for iron ore, data from the period before the second world war is not of any great interest.

Our data set consists of prices from Brazil and Sweden, which suits the purpose of this paper well: Firstly, the data are from a period when world trade was substantial. Secondly, Brazil is one of the most significant players on the European market, and Sweden is a significant one.

Prices are expressed in US cents per metric ton 1% Fe-unit, which is equivalent to US dollars per metric ton pure iron. The data are supplied from the World Bank and from LKAB and consist of the following time series:

- Brazilian fines, 1960 to 2002 (BR f): Companhia Vale do Rio Doce (CVRD) contract price for European markets. Itabira sinter feed f.o.b. Tubarão, up to 1986, and Carajas sinter feed f.o.b. Carajás, thereafter. (source: the World Bank)
- LKAB fines f.o.b. Narvik, 1970 to 2002 (LKAB f N): Luossavaara-Kiirunavaara Aktiebolag (LKAB) contract price for European markets (source: LKAB)
- LKAB fines f.o.b. Luleå, 1970 to 2002 (LKAB f L) (source: LKAB)

¹⁰f.o.b. = free on board: the seller is obliged to ship the ore to the sea port and load it on a vessel, and the buyer covers the overseas costs.

- LKAB pellets f.o.b. Narvik/Luleå, 1970 to 2002 (LKAB p) (source: LKAB), and
- CVRD pellets f.o.b. Tubarão, 1971 to 2002 (CVRD p) (source: LKAB).

Prices are deflated with yearly averages of the Producer Price Index for finished goods from the U.S. Department of Labor, Bureau of Labor Statistics, with 1982 as base year. The deflated series are transformed by taking their natural logarithm. Thus, $p_t = \ln(P_t^N \times 100/PPI_t)$, where P_t^N are nominal \$US-prices and PPI_t is the Producer Price Index ($PPI_{1982} = 100$). In the sequel, 'level' denotes p_t and 'first difference' denotes $\Delta p_t = p_t - p_{t-1}$.

4 RESULTS

Following the structure of section 2, we begin with the empirical results from the ADF-test. After a discussion of the results from the tests with an exogenous structural break, the outcome of the endogenous-break procedures is commented. The section is closed with the two-breaks procedure involving a break search.¹¹

Before continuing, some general comments are in place. To begin with, unit-root testing is a tricky business. Several books have been written on the subject, and among other things, the question of why we should bother to test for unit root at all is raised. This study therefore tries to give a motivation of why the issue is important for mineral commodities, as stated in the introduction.

The main, and perhaps obvious, guideline the author uses for drawing conclusions from the empirical results, is that a rejection of the unit-root null at a low significance level is more valuable information than a non-rejection. In other words, a non-rejected null is by no means regarded as an accepted one. The reason is that for a population autoregressive parameter lying very close to, but not equal to one, few of the available tests can be relied on. Instead of concluding that the process has a unit root, a non-rejection of the null can be seen as either that, or as a sign of a stationary time-series process with a first-order autoregressive parameter close to one.

Another issue concerns the bootstrap tests performed.¹² Obviously, a case where the result from the asymptotic test coincides with the result from the bootstrap is worth more than a case where the different types of tests give different results. As will be seen later in the paper, the bootstrap results often, but not always, agree nicely with the asymptotic ones.

¹¹All results in the paper were generated using Ox version 3.00.

¹²For a description of the bootstrap routines, see Appendix A.

	$p_t = \alpha + \rho p_{t-1} + \sum_{j=1}^k \gamma_j \Delta p_{t-j} + \beta t + u_t$			$\Delta p_t = \theta \Delta p_{t-1} + u_t$
	$\hat{\alpha}$	$\hat{\rho}$	$\hat{\beta}$	$\hat{\theta}$
BR f	1.091 [2.998] ¹	0.681 (-3.028)	-0.003 [-1.890] ¹⁰	0.211 (-5.116) ¹
LKAB f N	2.198 [3.467] ¹	0.409 (-3.492) ¹⁰	-0.013 [-3.084] ¹	0.054 (-5.201) ¹
LKAB f L	2.265 [3.822] ¹	0.385 (-3.825) ^{2.5}	-0.011 [-3.427] ¹	0.087 (-5.019) ¹
LKAB p	1.550 [2.710] ^{2.5}	0.627 (-2.733)	-0.008 [-2.204] ⁵	0.144 (-4.729) ¹
CVRD p	1.439 [2.749] ^{2.5}	0.641 (-2.762)	-0.006 [-2.154] ⁵	0.203 (-4.376) ¹

[t-value]^c/(τ -statistic)^c significant at the $c\%$ level

TABLE 2 Dickey-Fuller tests for a unit root in the level and in the first difference with $k^{\max} = 4$. Critical values at the 0.01, 0.025, 0.05, and 0.1 significance levels: -4.15, -3.80, -3.50, and -3.18. The 0.01 critical value with no trend or intercept is -2.62.

4.1 AUGMENTED DICKEY-FULLER TEST

The test is applied both to each series (equation (1)) and to the first difference of each series ($\Delta p_t = \theta \Delta p_{t-1} + u_t$). Using the modified Box-Pierce Q -statistic, the residuals from the test regressions are checked for serial correlation, and none is found at the 10% significance level.¹³ As stated earlier, k is determined using the k -sig routine described above, with $k^{\max} = 4$. This represents roughly ten percent of the sample sizes, as in Ng and Perron (1995), Perron (1989), Lumsdaine and Papell (1997), and in the empirical part of Perron (1997). If k^{\max} is found to be binding, it is not increased.

The results from the ADF-tests are displayed in Table 2. Starting with the results from the test on the differenced series it can be seen that in all five cases, the null hypothesis is rejected at the one percent significance level. The three fines series have the highest $|\tau|$ -values. The bootstrap results from Table 8 (see appendix A) give a similar picture: p -values close to zero for the three fines series, and between two and three percent for pellets. Note that for the pellets series, the bootstrap gives higher p -values than what is suggested by the asymptotic test. Nevertheless, given all test results, it is very unlikely that any of the differenced series contains a unit root.

The outcome of the test on the level of the series shows a more heterogeneous picture. First consider the results from the τ -test. Beginning with the Brazilian fines series, neither the large sample test nor the bootstrap reject the null. The autoregressive coefficient is estimated at 0.68, a much higher value than for the rest of the fines series. The test value

¹³The modified Box-Pierce Q -statistic is applicable to the case when the regressors cannot be assumed to be exogenous, as is the case with lagged dependent variables. For details about the statistic, see Hayashi (2000, p. 147). The test is performed for $p = 1, 2, \dots, 15$, where p is the order of autocorrelation checked for. For instance, for $p = 4$, the null hypothesis is that the first 4 autocorrelations are simultaneously zero, and the alternative is that at least one is different from zero.

for the LKAB fines f.o.b. Narvik lies very close to the five percent critical value, confirmed by the bootstrap size estimate of 0.06. For the f.o.b. Luleå series, we get a rejection at the 2.5 percent level, and a bootstrap value of 0.03. The estimated trend coefficients are negative and significant, at the one percent level for the latter two series and at the ten percent level for Brazilian fines. The test on the two pellets series results in no rejection of the null at the ten percent level, and the trend coefficient estimate is once again negative.

The Φ_3 -statistics—in the same series order as in Table 2—are 4.58, 6.10, 7.42, 3.74, and 3.84. From Dickey and Fuller (1981, Table VI) we see that the critical values are 9.31 (1%), 7.81 (2.5%), 6.73 (5%), and 5.61 (10%) for $T = 50$. Thus, we reject the null at the ten percent level for LKAB fines f.o.b. Narvik, and at the five percent level for LKAB fines f.o.b. Luleå. Table 8 in appendix A for the Φ_3 -tests reveal estimated p -values of 0.09 and 0.04 for the two series respectively. Thus, the results from applying the Φ_3 -tests are close to those from the τ -tests.

To summarize, we can draw the following conclusions from the ADF-test: None of the differenced series is likely to contain a unit root. Concerning the tests in level, only in two cases we get rejections of the null, namely for the τ/Φ_3 -tests at significance levels of roughly six/nine percent for the LKAB fines f.o.b. Narvik series and three/four percent for LKAB fines f.o.b. Luleå.

4.2 A SINGLE BREAK AT A KNOWN DATE

The tests described in section 2.2 are applied to each series. The year for the first oil crisis is chosen as the time of the instantaneous break, i.e., $T_B = 1973$. This date is chosen for two reasons: October 1973 brought an oil embargo by the Organization of Arab Petroleum Exporting Countries, cutting into the supply of oil and elevating prices to very high levels. This event, known as the Oil Crisis, had an immense impact on the world economy. Thus, it would be natural to assume that the energy crisis had an effect on iron ore prices, and we postulate a structural break in 1973.

A second, perhaps more important motivation for the choice, can be seen by examining the OECD (1975) and OECD (1976) reports. With 1972 as a starting year, there was a steady increase in activity in the world iron and steel industry. In spite of the slowdown in general economic activity and the gathering energy crisis, these strong market conditions continued at least to mid-1974. The demand from steel-using sectors in the industrialized countries was continuously high. Also, many developing countries with greater purchasing power because of the higher prices of agricultural products and raw materials, increased their steel imports considerably. Many of the steel-intensive industries were very active; the shipbuilding industry was bustling in 1973, and demand for capital goods in most OECD countries was higher in 1972 and also in 1973 than the years before.

This picture of the iron and steel industry implies a positive price shock around 1973.

Indeed, as will be seen in the results below, the estimated shock in the examined iron ore prices is positive when significant.

For each test regression, the number of lags is determined by the k -sig procedure, with $k^{\max} = 4$. Also, for each test, the modified Q -statistic is computed for $p = 1, 2, \dots, 15$, with no rejections at the ten percent level for any of the series.

The p -values reported in the result tables are calculated using simulated distributions for $\tau_{\hat{\alpha}^A}$ and $\tau_{\hat{\alpha}^C}$. Critical points for different values of the nuisance parameter λ (the ratio between the break date and the sample size) are displayed in Table 9. One of the reasons for not using the critical values reported in Perron (1989, Tables IV.B and VI.B) is that a direct comparison with the estimated p -values from the bootstrap tests is made much easier if the whole distribution is available. Another slight improvement is achieved by using simulated distributions for more exact values of λ . In Perron's article, critical values for $\lambda = 0.1, 0.2, \dots, 0.9$ are reported, while for the iron ore samples we need values for $\lambda = 0.0909, 0.1212$, and 0.3256 . Perhaps the most important gain has to do with the number of replications used for simulating the distributions of the test statistics: we use 50,000 replications, while Perron uses 5,000. For more details, see appendix B.

Some figures may be helpful before we continue. In Figures 3–7 (see appendix D), the estimated broken trend functions according to the filtering regressions from the equations in (7) are shown. The graphs illustrate what the Perron (AO) tests are all about: the data is filtered by a broken trend function, and the residuals from that equation are tested for unit root by ADF-techniques, with critical values that correspond to the type of filtering.

Not surprisingly, Model (B) does not fit the data. Therefore, the test results from that model are omitted in the sequel. We only consider four possible models: Model (A) with additive or innovation outliers, and Model (C) with additive or innovation outliers.

Before continuing with the results, a note on how to interpret some of the coefficients might be useful. When we are dealing with additive outliers, θ measures the magnitude of the instantaneous break in the level of the trend function in Models (A) and (C), while γ measures the magnitude of the change in the slope of the trend function in Model (C). For the case of innovation outliers, the same descriptions hold, but now the break is not instantaneous; instead, the series reacts gradually to the breaks.

The filtering equations for the two models with additive outliers are presented in Table 3, while the results from the asymptotic and bootstrap tests can be found in Tables 4 and 8, respectively. For all series, the estimated coefficients in the filtering equation of Model (A) are significant, except for the θ -estimate from the two pellets series. Note that the estimates of θ are all positive, complying with the situation in the iron and steel market described above. For Brazilian fines, the coefficient estimates from Model (C) are all significant, but for the rest of the series they generally are not (except for μ). Introducing the γ -coefficient does not seem to fit other series than the Brazilian fines prices. As was

	$\hat{\mu}^i$	$\hat{\beta}^i$	$\hat{\theta}^i$	$\hat{\gamma}$
Model (A):	$p_t = \mu^A + \beta^A t + \theta^A DU_t + \tilde{p}_t^A$			
BR f	3.4579 [123.0160] ¹	-0.0175 [-9.2015] ¹	0.3049 [6.0398] ¹	
LKAB f N	3.6395 [62.6146] ¹	-0.0231 [-9.0323] ¹	0.1716 [2.2959] ⁵	
LKAB f L	3.5467 [70.4802] ¹	-0.0207 [-9.3330] ¹	0.2295 [3.5480] ¹	
LKAB p	4.1029 [53.3282] ¹	-0.0215 [-6.3294] ¹	0.1379 [1.3941]	
CVRD p	3.9201 [48.3026] ¹	-0.0181 [-5.8157] ¹	0.1455 [1.4775]	
Model (C):	$p_t = \mu^C + \beta^C t + \theta^C DU_t + \gamma DT_t + \tilde{p}_t^C$			
BR f	3.5626 [74.6537] ¹	-0.0315 [-5.6207] ¹	0.1547 [2.0896] ⁵	0.0155 [2.6275] ²
LKAB f N	3.6453 [25.3272] ¹	-0.0254 [-0.4838]	0.1657 [1.0778]	0.0023 [0.0436]
LKAB f L	3.5276 [28.3234] ¹	-0.0130 [-0.2869]	0.2490 [1.8713] ¹⁰	-0.0077 [-0.1684]
LKAB p	4.0632 [21.3473] ¹	-0.0056 [-0.0805]	0.1783 [0.8767]	-0.0159 [-0.2285]
CVRD p	4.0025 [18.4255] ¹	-0.0593 [-0.5897]	0.0623 [0.2757]	0.0413 [0.4101]
[t-value] ^c significant at the c% level				

TABLE 3 Results from filtering regressions, additive outlier; $T_B = 1973$.

the case in the ADF-setting, both for the (A) and for the (C) model, the estimated trend is negative when significant.

For the Brazilian iron ore series, the p -values from the asymptotic test for the (A) and the (C) models are 0.02 and 0.003, respectively. The bootstrap tests give values of 0.03 and 0.006, which are close to the asymptotic ones.

In analogue with the results from the ADF-test, the two pellets series differ from the rest. Neither series gives any rejection: the asymptotic/bootstrap p -values are 0.63/0.65 (A) and 0.66/0.67 (C) (LKAB pellets); and 0.37/0.21 (A) and 0.39/0.20 (C) (CVRD pellets). The latter series is the first case thus far when the asymptotic and bootstrap tests return significantly different p -values, although all well above ten percent.

The two types of fines series from LKAB give two rejections of the null for Model (A). Using the asymptotic/bootstrap tests, the p -values are 0.08/0.07 for the f.o.b. Narvik series, and somewhat lower for the f.o.b. Luleå fines lying at 0.02/0.02. The results from applying the Model (C) tests are similar with 0.09/0.04 and 0.03/0.01. As in with CVRD pellets above, the bootstrap tests give significantly lower p -value estimate than the asymptotic tests.

Now let us turn to the case with innovational outliers displayed in Tables 5 (the test

	k	$\hat{\alpha}$	$\hat{\sigma}_{\hat{\alpha}}$	$\tau_{\hat{\alpha}}$	p -value
Model (A):					
BR f	1	0.381	0.151	-4.099	0.023
LKAB f N	1	0.464	0.152	-3.534	0.076
LKAB f L	1	0.369	0.155	-4.066	0.021
LKAB p	0	0.736	0.123	-2.151	0.629
CVRD p	1	0.678	0.123	-2.619	0.365
Model (C):					
BR f	1	0.184	0.158	-5.150	0.003
LKAB f N	1	0.464	0.151	-3.541	0.091
LKAB f L	1	0.371	0.156	-4.029	0.029
LKAB p	0	0.738	0.123	-2.127	0.663
CVRD p	1	0.679	0.123	-2.601	0.389

TABLE 4 Unit-root tests allowing for a single exogenous break, additive outlier, with $k^{max} = 4$. Test equation: (8) with $T_B = 1973$. For critical values see Table 9.

equations), 6 (the test results), and 8 (bootstrap τ - and F -tests). As with AO , the test equations of Model (A) generally give coefficient estimates that are significant, while the ones of from Model (C) are not completely so. Note also that the only significant estimate of θ in Model (C) is positive, complying with the economic intuition. Furthermore, the pellets series once again differ from the rest: the coefficient estimates of θ from Model (A) are not significant for those series, while they are significant for the rest.

Let us begin with the results of the τ -tests. For Brazilian ore, we get rejections at the one percent level for both Models (A) and (C), and both from the asymptotic and from the bootstrap tests, with all estimated p -values close to zero. Applying the tests to LKAB fines f.o.b. Luleå also results in rejections of the null hypotheses. The outcome of the test for the f.o.b. Narvik series gives higher p -values, and the bootstrap tests give lower p -values than the asymptotic ones, the difference being largest for Model (C). Also with innovation outliers the pellets series stand out, with none of the tests giving any rejections.

Recall that the article by Perron does not give any F -tests of joint hypotheses; thus, our paper relies solely on bootstrap tests for the Models (A) and (C) with innovational outliers, as displayed in Table 8. For Model (C) IO , the estimated p -values are close to those from the τ -tests. For Model (A) IO , all F -tests give p -values that are higher than the corresponding τ -tests. But the only important difference concerns LKAB fines f.o.b. Narvik: for that series, with an F -test, we are able to reject the null at the ten percent level, while with a τ -test, we get a rejection at the five percent level. To summarize, the F -tests give approximately the same results as the τ -tests. Notice, though, that we have only used bootstrap methods to perform the first-mentioned.

	$\hat{\mu}^i$	$\hat{\beta}^i$	$\hat{\theta}^i$	$\hat{\gamma}$	\hat{d}^i	\hat{a}	k
Model (A): $p_t = \hat{\mu}^A + \hat{\beta}^A t + \hat{\theta}^A DU_t + \hat{d}^A D(TB)_t + \hat{a}^A p_{t-1} + \sum_{i=1}^k \hat{c}_i \Delta p_{t-i} + \hat{e}_t$							
BR f	2.1258 [5.2191] ¹	-0.0115 [-4.7119] ¹	0.2172 [4.3148] ¹		-0.1014 [-1.2460]	0.3837	1
LKAB f N	2.1745 [3.5052] ¹	-0.0137 [-3.0992] ¹	0.1447 [1.8152] ¹⁰		0.1547 [1.4259]	0.3889	1
LKAB f L	2.6167 [4.3698] ¹	-0.0153 [-3.9431] ¹	0.1949 [2.6269] ²		0.0640 [0.6557]	0.2555	1
LKAB p	1.4464 [2.6233] ²	-0.0069 [-1.8571] ¹⁰	0.0631 [0.7343]		0.2258 [2.0111] ¹⁰	0.6377	1
CVRD p	1.3407 [2.4846] ²	-0.0065 [-1.9808] ¹⁰	0.1095 [1.0560]		0.0782 [0.7448]	0.6422	1
Model (C): $p_t = \hat{\mu}^C + \hat{\beta}^C t + \hat{\theta}^C DU_t + \hat{\gamma}^C DT_t + \hat{d}^C D(TB)_t + \hat{a}^C p_{t-1} + \sum_{i=1}^k \hat{c}_i \Delta p_{t-i} + \hat{e}_t$							
BR f	2.5022 [5.7317] ¹	-0.0231 [-3.6115] ¹	0.1265 [1.8881] ¹⁰	0.0112 [1.9552] ¹⁰	-0.1164 [-1.4808]	0.3007	1
LKAB f N	2.7927 [3.8832] ¹	-0.2152 [-1.6824]	-0.5666 [-1.2371]	0.2020 [1.5758]	0.1587 [1.5053]	0.4129	1
LKAB f L	3.1677 [4.7778] ¹	-0.2030 [-1.8302] ¹⁰	-0.4706 [-1.1779]	0.1883 [1.6931]	0.0728 [0.7719]	0.2854	1
LKAB p	1.9109 [2.6306] ²	-0.1490 [-1.0304]	-0.4368 [-0.8469]	0.1423 [0.9830]	0.2271 [2.0206] ¹⁰	0.6459	1
CVRD p	1.1377 [1.7524] ¹⁰	-0.0638 [-0.4465]	-0.0646 [-0.1759]	0.0587 [0.4108]	0.0619 [0.5550]	0.7352	0

[t-value]^c significant at the c% level

TABLE 5 Results from unit-root test regressions, specifications with innovational outliers; $T_B = 1973$ and $k^{max} = 4$.

4.3 SPECIAL CASE 1: A SINGLE BREAK AT AN UNKNOWN DATE

Ideally, a researcher with an idea of when a break should occur would like the search procedure to pick out that particular date, or any date close to it. The results for Brazilian iron ore displayed in Table 10 are quite satisfactory in that respect:¹⁴ All procedures resulting in a rejection of the null hypothesis at the 2.5 or five percent level pick out 1972 (in one out of six cases) or 1973 as the break date. In the exogenous tests 1973 is selected as the date of the break, but the year before in the *IO*-setting, as is the case here, can also be justified. A gradual change starting in 1972 is not implausible if one consults the OECD reports mentioned earlier.

Unfortunately, none of the results for the rest of the series is easy to interpret. For LKAB fines f.o.b. Narvik we get quite many rejections with the years 1980–1982 as break dates, and some with a break in 1987; and for LKAB fines f.o.b. Luleå we get three rejections at the ten percent level, with 1972, 1980, and 1982 as break dates (see Tables 11 and 12). The search on LKAB pellets in Table 13 results in some rejections, some of

¹⁴Tables 10–14 can be found on pp. 35–37.

	k	$\hat{\alpha}$	$\hat{\sigma}_{\hat{\alpha}}$	$\tau_{\hat{\alpha}}$	p -value
Model (A):					
BR f	1	0.384	0.117	-5.265	0.001
LKAB f N	1	0.389	0.166	-3.672	0.056
LKAB f L	1	0.256	0.166	-4.494	0.006
LKAB p	1	0.638	0.132	-2.749	0.324
CVRD p	1	0.642	0.134	-2.675	0.339
Model (C):					
BR f	1	0.301	0.120	-5.811	0.0004
LKAB f N	1	0.413	0.162	-3.615	0.078
LKAB f L	1	0.285	0.161	-4.445	0.009
LKAB p	1	0.646	0.132	-2.680	0.383
CVRD p	0	0.735	0.134	-1.969	0.711

TABLE 6 Unit-root tests allowing for a single exogenous break, innovational outlier, with $k^{max} = 4$. Test equations: (6) with $T_B = 1973$. For critical values see Table 9.

the dates being around 1980, and some in 1988. Finally, as can be seen in Table 14, the procedure chooses dates close to 1980 when rejections occur for CVRD pellets.

Naturally, even for dates far from 1973, the rejections of the null above are valid in the statistical sense, but it is beyond the scope of this paper to try to find the economic intuition behind the break dates. It is possible that either the second oil crisis, or the world recession in the beginning of the 1980s, is responsible for a break in the series.

4.4 SPECIAL CASE 2: TWO BREAKS AT UNKNOWN DATES

The results from the test can be found in Table 7, and for Brazilian fines they are not very convincing: we get one rejection of the null at the ten percent level. The model giving the rejection is *AA*; one of the breaks is found to be in 1974 and the second in 1989. Models *CA* for Narvik fines and *CC* for Luleå fines choose 1973 and 1989 as the break dates (with rejections at the 2.5 percent level). For both Swedish fines series we get rejections using each type of model at a maximum significance level of ten percent.

Both pellets series behave similarly, with rejections for model *AA* (with 1980 and 1982 as dates), and for model *CA* (1988 and 1982).

One interesting feature of the results above is that dates around 1980/1982 are often chosen as one of the break dates in rejections at significance level five percent or less. The reasons why a break is postulated in 1973 is given earlier in this paper. Concerning a possible break soon after 1980 one reason might be the global recession around that date, although a more thorough investigation is left for future research.

5 CONCLUSIONS

Using unit-root tests allowing for an exogenous structural break, the null hypothesis of a unit root is rejected for Brazilian fines and LKAB fines f.o.b. Luleå. It is interesting to

i	$T_{B1}(\tau_{\hat{\alpha}^i})$	<i>Test results</i>		
		$T_{B2}(\tau_{\hat{\alpha}^i})$	k	$\tau_{\hat{\alpha}^i}^*(T_{B1}, T_{B2})$
		BR f		
AA	1989	1974	1	-6.089 ^{10%}
CC	1982	1973	1	-6.298
CA	1973	1982	1	-6.296
		LKAB f N		
AA	1982	1973	1	-6.089 ^{10%}
CC	1986	1992	4	-7.162 ^{2.5%}
CA	1989	1973	1	-7.099 ^{2.5%}
		LKAB f L		
AA	1982	1973	1	-7.002 ^{1%}
CC	1973	1989	1	-7.057 ^{2.5%}
CA	1973	1982	1	-7.168 ^{2.5%}
		LKAB p		
AA	1980	1982	3	-7.191 ^{1%}
CC	1987	1973	1	-6.174
CA	1988	1982	3	-7.142 ^{2.5%}
		CVRD p		
AA	1980	1982	3	-6.862 ^{2.5%}
CC	1987	1974	1	-5.715
CA	1988	1982	3	-7.341 ^{1%}
		<i>Critical values</i> [‡]		
i	1%	2.5%	5%	10%
AA	-6.94	-6.53	-6.24	-5.96
CC	-7.34	-7.02	-6.82	-6.49
CA	-7.24	-7.02	-6.65	-6.33

[‡]Source: Lumsdaine and Papell (1997). $\tau_{\hat{\alpha}^i}^*(\cdot)^{c\%}$ significant at the $c\%$ level.

TABLE 7 Unit-root tests involving a search for two breaks, $k^{max} = 4$.

note that the LKAB fines f.o.b. Narvik series gives rejections at higher significance levels than the f.o.b. Luleå series. The two pellets series stand out, since none of the exogenous-break test equations seems to fit the data, and we get no rejections of the null. This is an interesting result, since the price formation should in theory give the same properties for pellets as for fines.

If all models with exogenous trend breaks are to be summarized, the model of choice for the three fines series would be Model (A) from Perron (1989), either with additive or with innovational outliers. That model assumes a break in the level, but no change in the slope of the trend function. With innovational outliers the break is not instantaneous; instead, the series reacts gradually to a structural change.

The results from the tests with endogenous break(s) are more difficult to interpret, except for Brazilian fines with one break, where we get many rejections of the null involving breaks in 1973 (the first oil crisis). For the rest of the series, a number of rejections occur around 1980–1982 (the world recession following the second oil embargo), but also other break dates are estimated. The routine involving a search for one break must be handled with care: depending on the criteria used, we may get different break dates. The search

routine involving two breaks gives quite many rejections of the null hypothesis, but in many cases it is difficult to give an economic explanation for the break dates chosen.

To summarize the empirical findings, a time-series analysis of the five iron-ore prices must take into consideration the structural break that seems to have taken place during the first half of the 1970s. The state of the steel and iron ore industry seems to have caused a structural break in three of the five examined series. Evidently, it is possible that neglecting the break, as is done in the simple ADF-test, results in a false non-rejection of the unit-root null hypothesis. This is the case for Brazilian fines. For LKAB fines f.o.b. Narvik and Luleå, we get a rejection of the null hypothesis also by using the ADF-test, but the estimated first autoregressive coefficient is too high when the break is neglected.

Finally, more accurate critical values for Models (A) and (C) with an exogenous break are given and used, compared to the ones reported in Perron (1989). Instead of relying on 5,000 replications to simulate the distributions of the test statistics, we use 50,000. Perron (1989, p. 1377) suggests that the limiting distributions are symmetric around $\lambda = 0.5$. This is more clearly seen with 50,000 than with 5,000 replications.

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A BOOTSTRAP PROCEDURES

	ADF, level	ADF, diff	(A) AO	(A) IO	(C) AO	(C) IO
<i>Tests using τ-statistics</i>						
BR f	0.138	0.002	0.032	0.002	0.006	0.001
LKAB f N	0.061	0.001	0.069	0.048	0.042	0.049
LKAB f L	0.030	0.001	0.021	0.009	0.012	0.009
LKAB p	0.238	0.023	0.655	0.248	0.667	0.252
CVRD p	0.226	0.028	0.211	0.262	0.202	
<i>Tests using F/Φ_3-statistics</i>						
BR f	0.202			0.003		0.003
LKAB f N	0.086			0.095		0.047
LKAB f L	0.041			0.024		0.008
LKAB p	0.330			0.449		0.351
CVRD p	0.313			0.449		

TABLE 8 Empirical size from bootstrap tests, 50,000 replications. The first two columns correspond to an ADF-test in level or first difference. The rest can be read as follows: for instance, (A) AO is Model (A) with an additive outlier from Perron (1989). The number of lags is the same as in the corresponding asymptotic tests presented earlier in the paper, $T_B = 1973$ where applicable. For detailed information on the bootstrap procedures, see the corresponding appendix.

In this section, the bootstrap procedures used in the paper are described. Two different classes of tests are used: bootstrap tests based on τ - or F -statistics. For each series, five different bootstrap versions of various asymptotic τ -tests are applied: the ADF-test with both a constant and a trend; the Model (A) Perron test with additive (AO) and innovation outliers (IO); and the Model (C) Perron test with AO and IO. In addition, the ADF-test with no constant or trend is applied to each differenced series.

Three bootstraps based on F -statistics are applied to each series: the ADF-test with both a constant and a trend, the Model (A) Perron test with innovation outliers (IO), and the Model (C) Perron test with IO.

The results from the various bootstrap tests are displayed in Table 8. The number of replications R is set to 50,000 in each bootstrap.

A.1 BOOTSTRAP DICKEY-FULLER TEST WITH A CONSTANT, NO TREND, AND $k = 0$ (τ -TEST)

This test is only applied to the difference of the series; for simplicity, we write ' p_t ' instead of ' Δp_t '. The example in Davison and Hinkley (1997, pp. 391–393) and the recommendations in Maddala and Kim (1998, ch. 10) are followed, and the test involves the following steps:

1. Using the sample $\{p_1\}_1^T$, estimate the model

$$p_t = \alpha + \rho p_{t-1} + u_t \quad (12)$$

by OLS and construct the test statistic $\tau = (\hat{\rho} - 1)/\hat{\sigma}_{\hat{\rho}}$, where $\hat{\sigma}_{\hat{\rho}}$ is the estimated standard error of $\hat{\rho}$. The null hypothesis is $\rho = 1$, while the alternative is $\rho < 1$.

2. Construct $T - 1$ residuals e_t under H_0 ,

$$e_t = \Delta p_t = \alpha + u_t.$$

3. Create R bootstrap samples by sampling with replacement and with equal probability from $\{e_t\}_2^T$ to obtain simulated innovations $\{e_t^*\}_2^T$.
4. For each bootstrap sample $r = 1, 2, \dots, R$, create a $\{p_t^*\}$ sequence using recursion: set $p_1^* = p_1$, and then

$$p_t^* = p_{t-1}^* + e_t^*, \quad t = 2, 3, \dots, T.$$

5. For each r , run the regression

$$p_t^* = \alpha^* + \rho^* p_{t-1}^* + u_t,$$

where $t = 2, 3, \dots, T$, and calculate the test statistic $\tau^* = (\hat{\rho}^* - 1)/\hat{\sigma}_{\hat{\rho}^*}$.

6. Estimate the size of the test by $\frac{(\sum_{r=1}^R I_r) + 1}{R + 1}$, where $I_r = 1$ if $\tau^* < \tau$ and $I_r = 0$ otherwise.

As is evident from above, no centering of the residuals is performed, and that applies to all results in Table 8. In a preliminary study, all results were calculated with or without centering for $R = 1,000$, and the differences obtained were insignificant. Thus centering the residuals is not expected to change the results.

A.2 BOOTSTRAP DICKEY-FULLER TEST WITH TREND AND $k > 0$ (τ -TEST)

1. Estimate the regression

$$\Delta p_t = \alpha + \delta p_{t-1} + \beta t + \sum_{j=1}^k \gamma^j \Delta p_{t-j} + e_t, \quad (13)$$

by OLS and construct the test statistic $\tau_{\hat{\delta}} = \hat{\delta}/\hat{\sigma}_{\hat{\delta}}$. The null hypothesis is $\delta = 0$, and the alternative is $\delta < 0$.

2. Generate residuals from (13) under H_0 , i.e.,

$$\hat{e}_t = \Delta p_t - \hat{\alpha} - \hat{\beta}t - \sum_{j=1}^k \hat{\gamma}^j \Delta p_{t-j},$$

where the coefficients are estimated with H_0 imposed, i.e., from running (13) without p_{t-1} as regressor.

3. Create R bootstrap samples by sampling with replacement and with equal probability from $\{\hat{e}_t\}_{k+2}^T$ to obtain simulated innovations $\{\hat{e}_t^*\}_{k+2}^T$.
4. For each bootstrap sample $r = 1, 2, \dots, R$, create the sequences $\{\Delta p_t^*\}$ and $\{p_t^*\}$: set $\Delta p_t^* = \Delta p_t$ and $p_t^* = p_t$ for $t = 2, 3, \dots, k+1$, and use the recursive equations

$$\begin{aligned} \Delta p_t^* &= \hat{\alpha} + \hat{\beta}t + \sum_{j=1}^k \hat{\gamma}^j \Delta p_{t-j}^* + \hat{e}_t^* \text{ and} \\ p_t^* &= \Delta p_t^* + p_{t-1}^* \end{aligned}$$

for $t = k+2, 3, \dots, T$. Calculate the test statistic $\tau_{\hat{\delta}^*} = \hat{\delta}^*/\hat{\sigma}_{\hat{\delta}^*}$ using step (1).

5. Estimate the size of the test by $(1 + \sum_{r=1}^R I_r)/(1 + R)$, where $I_r = 1$ if $\tau_{\hat{\delta}^*} < \tau_{\hat{\delta}}$ and $I_r = 0$ otherwise.

A.3 BOOTSTRAP PERRON TEST OF MODEL (A) WITH ADDITIVE OUTLIERS (τ -TEST)

The description of the bootstrap for Model (C) is omitted due to its similarity: the only difference is the inclusion of one more dummy regressor.

1. Run the regression

$$p_t = \mu^A + \beta^A t + \theta^A D U_t + \tilde{p}_t$$

to obtain the OLS-estimates $(\hat{\mu}^A, \hat{\beta}^A, \hat{\theta}^A)$.

2. Estimate the model

$$\tilde{p}_t = \alpha \tilde{p}_{t-1} + \sum_{j=0}^k \omega_j D(TB)_{t-j} + \sum_{j=1}^k c_j \Delta \tilde{p}_{t-j} + u_t \quad (14)$$

by OLS and construct the test statistic $\tau_{\hat{\alpha}} = (\hat{\alpha} - 1)/\hat{\sigma}_{\hat{\alpha}}$. The null hypothesis is $\alpha = 1$, while the alternative is $\alpha < 1$.

3. Generate residuals from (14) under H_0 ,

$$u_t = \Delta \tilde{p}_t - \sum_{j=0}^k \hat{\omega}_j D(TB)_{t-j} - \sum_{j=1}^k \hat{c}_j \Delta \tilde{p}_{t-j},$$

where $\hat{\omega}_j$ and \hat{c}_j are estimated with H_0 imposed.

4. Create R bootstrap samples by sampling with replacement and with equal probability from $\{u_t\}_{k+2}^T$ to obtain simulated innovations $\{u_t^*\}_{k+2}^T$.
5. For each bootstrap sample $r = 1, 2, \dots, R$, create $\{\Delta\tilde{p}_t^*\}$ and $\{\tilde{p}_t^*\}$ sequences using recursion: set $\Delta\tilde{p}_t^* = \Delta\tilde{p}_t$ and $\tilde{p}_t^* = \tilde{p}_t$ for $t = 2, 3, \dots, k+1$, and use the recursive equations

$$\begin{aligned}\Delta\tilde{p}_t^* &= \sum_{j=0}^k \hat{\omega}_j D(TB)_{t-j} + \sum_{j=1}^k \hat{c}_j \Delta\tilde{p}_{t-j}^* + u_t^* \text{ and} \\ \tilde{p}_t^* &= \Delta\tilde{p}_t^* + \tilde{p}_{t-1}^*\end{aligned}$$

for $t = k+2, 3, \dots, T$.

6. For each r , create

$$p_t^* = \hat{\mu}^A + \hat{\beta}^A t + \hat{\theta}^A DU_t + \tilde{p}_t^*,$$

where $t = 2, 3, \dots, T$, and calculate the test statistic $\tau_{\hat{\alpha}^*} = (\hat{\alpha}^* - 1)/\hat{\sigma}_{\hat{\alpha}^*}$ using steps (1) and (2) above.

7. Estimate the size of the test by $(1 + \sum_{r=1}^R I_r)/(1 + R)$, where $I_r = 1$ if $\tau_{\hat{\alpha}^*} < \tau_{\hat{\alpha}}$ and $I_r = 0$ otherwise.

A.4 BOOTSTRAP PERRON TEST OF MODEL (A) WITH INNOVATION OUTLIERS (τ -TEST)

The description of the bootstrap for Model (C) is omitted due to its similarity.

1. Estimate the regression

$$\Delta p_t = \mu^A + \beta^A t + \theta^A DU_t + d^A D(TB)_t + \delta p_{t-1} + \sum_{i=1}^k c_i^A \Delta p_{t-i} + e_t \quad (15)$$

by OLS and construct the test statistic $\tau_{\hat{\delta}} = \hat{\delta}/\hat{\sigma}_{\hat{\delta}}$. The null hypothesis is $\delta = 0$, and the alternative is $\delta < 0$.

2. Generate residuals from (15) under H_0 , i.e.,

$$\hat{e}_t = \Delta p_t - \hat{\mu}^A - \hat{\beta}^A t - \hat{\theta}^A DU_t - \hat{d}^A D(TB)_t - \sum_{i=1}^k \hat{c}_i^A \Delta p_{t-i},$$

where the coefficients are estimated with H_0 imposed, i.e., from running (15) with no p_{t-1} on the right hand side.

3. Create R bootstrap samples by sampling with replacement and with equal probability from $\{\hat{e}_t\}_{k+2}^T$ to obtain simulated innovations $\{\hat{e}_t^*\}_{k+2}^T$.

4. For each bootstrap sample $r = 1, 2, \dots, R$, create the sequences $\{\Delta p_t^*\}$ and $\{p_t^*\}$: set $\Delta p_t^* = \Delta p_t$ and $p_t^* = p_t$ for $t = 2, 3, \dots, k+1$, and then for $t = k+2, 3, \dots, T$, use the recursive equations

$$\begin{aligned}\Delta p_t^* &= \hat{\mu}^A + \hat{\beta}^A t + \hat{\theta}^A D U_t + \hat{d}^A D(TB)_t + \sum_{i=1}^k \hat{c}_i^A \Delta p_{t-i}^* + \hat{e}_t^* \text{ and} \\ p_t^* &= \Delta p_t^* + p_{t-1}^*.\end{aligned}$$

Calculate the test statistic $\tau_{\hat{\delta}^*} = \hat{\delta}^* / \hat{\sigma}_{\hat{\delta}^*}$ using step (1).

5. Estimate the size of the test by $(1 + \sum_{r=1}^R I_r) / (1 + R)$, where $I_r = 1$ if $\tau_{\hat{\delta}^*} < \tau_{\hat{\delta}}$ and $I_r = 0$ otherwise.

A.5 BOOTSTRAP DICKEY-FULLER TEST WITH TREND AND $k > 0$ (Φ_3 -TEST)

The F -statistic below corresponds to the Φ_3 -statistic from Dickey and Fuller (1981). From the F -statistics class, only this procedure is shown, since the A (IO) and C (IO) versions are similar. (To perform the structural-break bootstraps, simply add the relevant dummy variables to the regressions.)

1. Estimate the regression

$$\Delta p_t = \alpha + \delta p_{t-1} + \beta t + \sum_{j=1}^k \gamma^j \Delta p_{t-j} + e_t, \quad (16)$$

by OLS and construct the test statistic $F_{(\hat{\delta}, \hat{\beta})} = (\mathbf{R}\mathbf{b} - \mathbf{q})' [s^2 \mathbf{R}(\mathbf{X}'\mathbf{X})^{-1} \mathbf{R}']^{-1} (\mathbf{R}\mathbf{b} - \mathbf{q}) / 2$, where

$$\begin{aligned}\mathbf{R}_{2 \times (3+k)} &= \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \end{pmatrix}, \\ \mathbf{b}_{(3+k) \times 1} &= (\hat{\alpha} \ \hat{\delta} \ \hat{\beta} \ \hat{c}_1 \ \cdots \ \hat{c}_k)', \quad \mathbf{q}_{2 \times 1} = \mathbf{0}, \\ \mathbf{X}_{T \times (3+k)} &= (\mathbf{x}_1 \ \mathbf{x}_2 \ \cdots \ \mathbf{x}_T)', \quad s^2 = \mathbf{e}'\mathbf{e} / (T - 3 - k), \\ \mathbf{e}_{T \times 1} &= (e_1 \ e_2 \ \cdots \ e_T)', \text{ and} \\ \mathbf{x}_t_{(3+k) \times 1} &= (1 \ p_{t-1} \ t \ \Delta p_{t-1} \ \cdots \ \Delta p_{t-k})'.\end{aligned}$$

$F_{(\hat{\delta}, \hat{\beta})}$ is used to test $H_0 : \boldsymbol{\theta} = \mathbf{0}$ against $H_0 : \boldsymbol{\theta} \neq \mathbf{0}$, where $\boldsymbol{\theta} = (\hat{\delta} \ \hat{\beta})'$.

2. Generate residuals from (16) under H_0 , i.e.,

$$\hat{e}_t = \Delta p_t - \hat{\alpha} - \sum_{j=1}^k \hat{\gamma}^j \Delta p_{t-j},$$

where the coefficients are estimated with H_0 imposed, i.e., from running (16) without p_{t-1} and t as regressors.

3. Create R bootstrap samples by sampling with replacement and with equal probability from $\{\hat{e}_t\}_{k+2}^T$ to obtain simulated innovations $\{\hat{e}_t^*\}_{k+2}^T$.
4. For each bootstrap sample $r = 1, 2, \dots, R$, create the sequences $\{\Delta p_t^*\}$ and $\{p_t^*\}$: set $\Delta p_t^* = \Delta p_t$ and $p_t^* = p_t$ for $t = 2, 3, \dots, k+1$, and use the recursive equations

$$\begin{aligned} \Delta p_t^* &= \hat{\alpha} + \sum_{j=1}^k \hat{\gamma}^j \Delta p_{t-j}^* + \hat{e}_t^* \text{ and} \\ p_t^* &= \Delta p_t^* + p_{t-1}^* \end{aligned}$$

for $t = k+2, 3, \dots, T$. Calculate the test statistic $F_{(\hat{\delta}^*, \hat{\beta}^*)}$ using step 1.

5. Estimate the size of the test by $(1 + \sum_{r=1}^R I_r)/(1+R)$, where $I_r = 1$ if $F_{(\hat{\delta}^*, \hat{\beta}^*)} > F_{(\hat{\delta}, \hat{\beta})}$ and $I_r = 0$ otherwise.

B SIMULATED DISTRIBUTIONS FOR $\tau_{\hat{\alpha}}$

Assume that $W(t)$ is a standard Wiener process. In order to simulate the distributions of the test statistics $\tau_{\hat{\alpha}^i}$, $i = A, C$, from Theorem 2 in Perron (1989, p. 1373) the following functions of $W(t)$ need to be approximated: $W(1)$, $W(\lambda)$, $\int_0^\lambda W(r)dr$, $\int_0^\lambda rW(r)dr$, $\int_\lambda^1 W(r)dr$, $\int_\lambda^1 rW(r)dr$, $\int_0^1 W(r)dr$, $\int_0^1 rW(r)dr$, and $\int_0^1 W(r)^2dr$. Below we show how these functions are approximated and the expressions are accompanied by (very) informal proofs.

Let $\{e_t\}_{t=1}^T$ be a sequence of pseudo-random i.i.d. $\mathbf{N}(0, 1)$ variables. Let ' \xrightarrow{L} ' denote convergence in distribution, and ' $\stackrel{d}{=}$ ' equality in distribution. Then for $\lambda \in [0, 1]$ and $x_t = \sum_{i=1}^t e_i$ we get:

1.

$$T^{-\frac{1}{2}}x_T \stackrel{d}{=} W(1) \sim \mathbf{N}(0, 1)$$

Proof: Being a sum of normally distributed variables, the expression is normally distributed with mean zero and variance

$$\begin{aligned} \text{Var}(T^{-\frac{1}{2}}x_T) &= T^{-1}\text{Var}(x_T) \\ &= T^{-1}T = 1. \end{aligned}$$

2.

$$T^{-\frac{1}{2}}x_{\lfloor \lambda T \rfloor} \xrightarrow{L} W(\lambda) \sim \mathbf{N}(0, \lambda),$$

where $[\cdot]$ denotes the integer-part function.

Proof: Obviously, $T^{-\frac{1}{2}}x_{[\lambda T]}$ is normally distributed with mean zero and with variance

$$\text{Var}(T^{-\frac{1}{2}}x_{[\lambda T]}) = T^{-1}\text{Var}(x_{[\lambda T]}) = T^{-1}[\lambda T] \xrightarrow{L} \lambda.$$

3.

$$T^{-\frac{3}{2}} \sum_{t=1}^{[\lambda T]} x_t \xrightarrow{L} \int_0^\lambda W(r)dr.$$

Proof: We have

$$T^{-\frac{1}{2}}x_t = W\left(\frac{t}{T}\right), \quad (17)$$

which can be proved in a similar manner as 1. above. Then we can write

$$\begin{aligned} T^{-\frac{3}{2}} \sum_{t=1}^{[\lambda T]} x_t &= \sum_{t=1}^{[\lambda T]} T^{-\frac{1}{2}}x_t \frac{t - (t-1)}{T} = \sum_{t=1}^{[\lambda T]} T^{-\frac{1}{2}}x_t \Delta\left(\frac{t}{T}\right) \\ &\stackrel{d}{=} \sum_{t=1}^{[\lambda T]} W\left(\frac{t}{T}\right) \Delta\left(\frac{t}{T}\right) \xrightarrow{L} \int_0^\lambda W(r)dr, \end{aligned}$$

where $r \equiv \frac{t}{T}$.

Equation (17) and the last identity are used in the proofs of 4-6 and of 9 below.

4.

$$T^{-\frac{5}{2}} \sum_{t=1}^{[\lambda T]} tx_t \xrightarrow{L} \int_0^\lambda rW(r)dr.$$

Proof:

$$\begin{aligned} T^{-\frac{5}{2}} \sum_{t=1}^{[\lambda T]} tx_t &= \sum_{t=1}^{[\lambda T]} \left(\frac{t}{T}\right) T^{-\frac{1}{2}}x_t \Delta\left(\frac{t}{T}\right) \\ &\stackrel{d}{=} \sum_{t=1}^{[\lambda T]} \left(\frac{t}{T}\right) W\left(\frac{t}{T}\right) \Delta\left(\frac{t}{T}\right) \xrightarrow{L} \int_0^\lambda rW(r)dr. \end{aligned}$$

5.

$$T^{-\frac{3}{2}} \sum_{t=[\lambda T]}^T x_t \xrightarrow{L} \int_\lambda^1 W(r)dr.$$

Proof:

$$\begin{aligned}
T^{-\frac{3}{2}} \sum_{t=[\lambda T]}^T x_t &= \sum_{t=[\lambda T]}^T T^{-\frac{1}{2}} x_t \Delta\left(\frac{t}{T}\right) \\
&\stackrel{d}{=} \sum_{t=[\lambda T]}^T W\left(\frac{t}{T}\right) \Delta\left(\frac{t}{T}\right) \xrightarrow{L} \int_{\lambda}^1 W(r) dr,
\end{aligned}$$

6.

$$T^{-\frac{5}{2}} \sum_{t=[\lambda T]}^T tx_t \xrightarrow{L} \int_{\lambda}^1 rW(r) dr.$$

Proof:

$$\begin{aligned}
T^{-\frac{5}{2}} \sum_{t=[\lambda T]}^T tx_t &= \sum_{t=[\lambda T]}^T \left(\frac{t}{T}\right) T^{-\frac{1}{2}} x_t \Delta\left(\frac{t}{T}\right) \\
&\stackrel{d}{=} \sum_{t=[\lambda T]}^T \left(\frac{t}{T}\right) W\left(\frac{t}{T}\right) \Delta\left(\frac{t}{T}\right) \xrightarrow{L} \int_{\lambda}^1 rW(r) dr.
\end{aligned}$$

7.

$$T^{-\frac{3}{2}} \sum_{t=1}^T x_t \xrightarrow{L} \int_0^1 W(r) dr.$$

Proof: This is 3 with $\lambda = 1$.

8.

$$T^{-\frac{5}{2}} \sum_{t=1}^T tx_t \xrightarrow{L} \int_0^1 rW(r) dr.$$

Proof: This is 4 with $\lambda = 1$.

9.

$$T^{-2} \sum_{t=1}^T x_t^2 \stackrel{d}{=} \int_0^1 W(r)^2 dr.$$

Proof:

$$T^{-2} \sum_{t=1}^T x_t^2 = \sum_{t=1}^T (T^{-\frac{1}{2}} x_t) (T^{-\frac{1}{2}} x_t) \Delta\left(\frac{t}{T}\right) \stackrel{d}{=} \int_0^1 W(r)^2 dr.$$

Following Perron, for $\{e_t\}_{t=1}^{1,000}$, the sums 1–9 are used to calculate one simulated value of $\tau_{\hat{\alpha}A}$ or $\tau_{\hat{\alpha}C}$. The latter is done with the help of the expressions in Perron (1989, pp. 1373–1374). (Note that there are several misprints, as pointed out in Perron and Vogelsang (1993).) This is replicated 5,000 times, thereby simulating the distributions of

the statistics. For illustration, the density functions for $\lambda = 0.3256$ are plotted in Figure 1. This break ratio corresponds to the Brazilian fines sample range with $T_B = 1973$, while for the CVRD pellets sample we have $\lambda = 0.0909$, and for the rest of the series, $\lambda = 0.1212$.

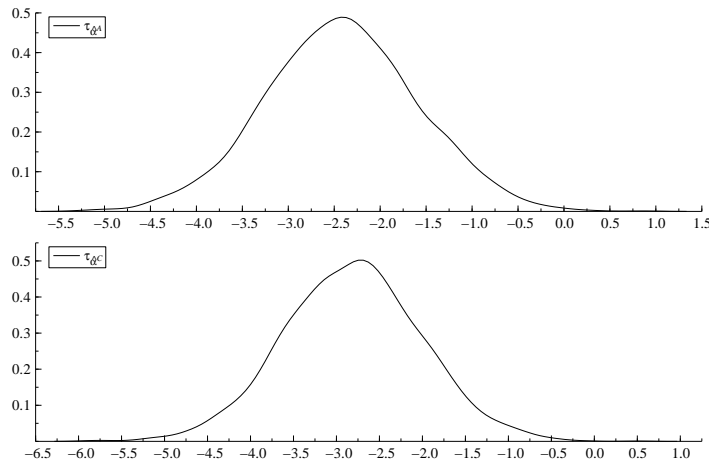


FIGURE 1 Simulated distributions of $\tau_{\hat{\alpha}^A}$ and $\tau_{\hat{\alpha}^C}$ using 5,000 replications and $\lambda = 0.3256$.

A comparison with the percentage points in Perron (1989, Tables IV.B and VI.B) reveals some slight differences. In order to see how robust the results from the simulation are with $T = 1,000$ and number of replications $R = 5,000$, the experiment is repeated once. The difference between the first and the second simulation (as measured by the sum of squared differences between corresponding quartiles) is of the same order as the difference between any of them and Perron's critical tables, and can thus be attributed to chance.

In order to increase the reliability of the critical values, a simulation with $R = 50,000$ is performed twice; since the differences between quartiles from the two sets of simulations is much smaller than for $R = 5,000$, in this paper we use 50,000 replications.

The percentage points of the simulated distributions of $\tau_{\hat{\alpha}^A}$ and $\tau_{\hat{\alpha}^C}$ are displayed in Table 9, and the new simulated distributions are plotted in Figure 2.

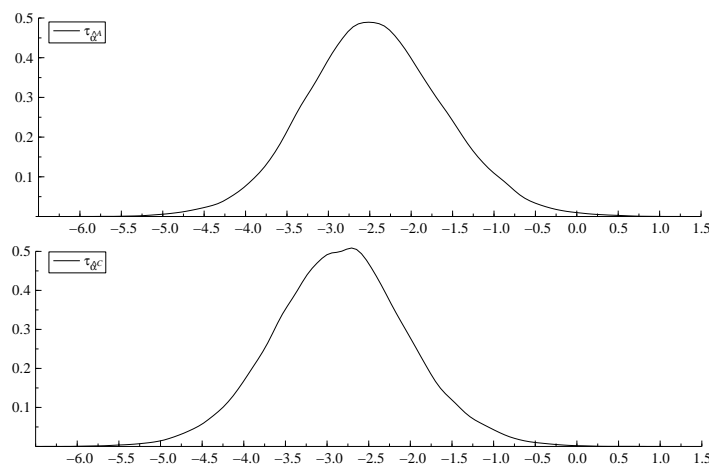


FIGURE 2 Simulated distributions of $\tau_{\hat{\alpha}^A}$ and $\tau_{\hat{\alpha}^C}$ using 50,000 replications and $\lambda = 0.3256$.

$\lambda =$	0.09	0.1	0.12	0.2	0.3	0.33	0.4	0.5	0.6	0.7	0.8	0.9
	$\tau_{\hat{\alpha}^A}$											
1%	-4.29	-4.28	-4.33	-4.36	-4.38	-4.41	-4.35	-4.34	-4.36	-4.37	-4.38	-4.27
2.5%	-3.95	-3.96	-3.98	-4.04	-4.05	-4.06	-4.04	-4.02	-4.04	-4.05	-4.05	-3.95
5%	-3.67	-3.69	-3.72	-3.79	-3.79	-3.78	-3.77	-3.75	-3.77	-3.79	-3.79	-3.69
10%	-3.37	-3.39	-3.41	-3.49	-3.49	-3.48	-3.47	-3.45	-3.46	-3.49	-3.49	-3.39
90%	-1.39	-1.39	-1.42	-1.46	-1.39	-1.37	-1.28	-1.22	-1.27	-1.40	-1.47	-1.39
95%	-1.09	-1.07	-1.11	-1.16	-1.06	-1.04	-0.93	-0.85	-0.91	-1.08	-1.16	-1.07
97.5%	-0.81	-0.79	-0.82	-0.87	-0.77	-0.77	-0.63	-0.52	-0.61	-0.78	-0.88	-0.78
99%	-0.46	-0.48	-0.49	-0.53	-0.44	-0.41	-0.27	-0.13	-0.24	-0.43	-0.55	-0.44
	$\tau_{\hat{\alpha}^C}$											
1%	-4.33	-4.35	-4.43	-4.59	-4.69	-4.76	-4.81	-4.81	-4.80	-4.73	-4.60	-4.34
2.5%	-4.01	-4.02	-4.08	-4.27	-4.38	-4.43	-4.48	-4.49	-4.48	-4.39	-4.28	-4.01
5%	-3.72	-3.75	-3.80	-3.99	-4.12	-4.16	-4.21	-4.23	-4.22	-4.12	-3.99	-3.75
10%	-3.41	-3.43	-3.50	-3.68	-3.83	-3.86	-3.91	-3.94	-3.92	-3.82	-3.68	-3.43
90%	-1.41	-1.41	-1.46	-1.61	-1.77	-1.81	-1.91	-1.96	-1.90	-1.77	-1.60	-1.41
95%	-1.09	-1.09	-1.13	-1.29	-1.45	-1.49	-1.59	-1.65	-1.59	-1.44	-1.28	-1.09
97.5%	-0.81	-0.80	-0.85	-0.99	-1.14	-1.20	-1.31	-1.39	-1.30	-1.15	-0.99	-0.79
99%	-0.46	-0.49	-0.50	-0.64	-0.80	-0.90	-0.98	-1.07	-0.97	-0.82	-0.62	-0.46

TABLE 9 Percentage points of the simulated distributions of $\tau_{\hat{\alpha}^A}$ and $\tau_{\hat{\alpha}^C}$, 50,000 replications.

C RESULTS FROM SEARCH FOR ONE BREAK

i	m	$T_B(s)$	Test results			Critical values [‡]			
			s	k	$\tau_{\hat{\alpha}^i}^*(m, T_B(s))$	1%	2.5%	5%	10%
A	AO	1977	$-\tau_{\hat{\alpha}^i}$	1	-4.5062	-5.92	-5.49	-5.20	-4.83
A	IO	1972	$-\tau_{\hat{\alpha}^i}$	1	-5.4409 ^{5%}	-5.92	-5.58	-5.23	-4.92
C	AO	1973	$-\tau_{\hat{\alpha}^i}$	1	-5.1505	-6.17	-5.87	-5.56	-5.21
C	IO	1973	$-\tau_{\hat{\alpha}^i}$	1	-5.8114 ^{5%}	-6.32	-5.90	-5.59	-5.29
A	AO	1974	$t_{\hat{\theta}^i}$	0	-4.1345	-5.24	-4.86	-4.53	-4.14
A	IO	1973	$t_{\hat{\theta}^i}$	1	-5.2654 ^{5%}	-5.70	-5.21	-4.92	-4.53
A	AO	1974	$t_{ \hat{\theta}^i }$	0	-4.1345	-5.35	-4.95	-4.73	-4.34
A	IO	1973	$t_{ \hat{\theta}^i }$	1	-5.2654	-5.85	-5.51	-5.18	-4.83
C	AO	1980	$-t_{\hat{\gamma}^i}$	1	-3.9648	-5.62	-5.30	-4.98	-4.60
C	IO	1980	$-t_{\hat{\gamma}^i}$	1	-3.2880	-5.77	-5.38	-4.98	-4.55
C	AO	1973	$t_{ \hat{\gamma}^i }$	1	-5.1505 ^{5%}	-5.64	-5.31	-5.01	-4.65
C	IO	1973	$t_{ \hat{\gamma}^i }$	1	-5.8114 ^{2.5%}	-6.07	-5.61	-5.33	-4.94
C	AO	1974	$F_{(\hat{\theta}, \hat{\gamma})^C}$	0	-4.1592	-5.71	-5.42	-5.11	-4.77
C	IO	1973	$F_{(\hat{\theta}, \hat{\gamma})^C}$	1	-5.8114 ^{5%}	-6.29	-5.95	-5.59	-5.23

[‡]Source: Perron (1997) and Vogelsang and Perron (1998).
 $\tau_{\hat{\alpha}^i}^*(\cdot)^{c\%}$ significant at the $c\%$ level.

TABLE 10 Unit-root test involving a search for a single break, BR f; $k^{max} = 4$.

i	m	$T_B(s)$	Test results			Critical values [‡]			
			s	k	$\tau_{\hat{\alpha}^i}^*(m, T_B(s))$	1%	2.5%	5%	10%
A	AO	1980	$-\tau_{\hat{\alpha}^i}$	1	-5.0214 ^{10%}	-5.92	-5.49	-5.20	-4.83
A	IO	1981	$-\tau_{\hat{\alpha}^i}$	3	-5.8173 ^{2.5%}	-5.92	-5.58	-5.23	-4.92
C	AO	1980	$-\tau_{\hat{\alpha}^i}$	1	-5.4626 ^{10%}	-6.17	-5.87	-5.56	-5.21
C	IO	1988	$-\tau_{\hat{\alpha}^i}$	4	-5.8053 ^{5%}	-6.32	-5.90	-5.59	-5.29
A	AO	1973	$t_{\hat{\theta}^i}$	0	-2.9049	-5.24	-4.86	-4.53	-4.14
A	IO	1973	$t_{\hat{\theta}^i}$	1	-3.6722	-5.70	-5.21	-4.92	-4.53
A	AO	1982	$t_{ \hat{\theta}^i }$	3	-4.9100 ^{5%}	-5.35	-4.95	-4.73	-4.34
A	IO	1981	$t_{ \hat{\theta}^i }$	3	-5.8173 ^{2.5%}	-5.85	-5.51	-5.18	-4.83
C	AO	1977	$-t_{\hat{\gamma}^i}$	1	-4.1391	-5.62	-5.30	-4.98	-4.60
C	IO	1975	$-t_{\hat{\gamma}^i}$	0	-2.9520	-5.77	-5.38	-4.98	-4.55
C	AO	1977	$t_{ \hat{\gamma}^i }$	1	-4.1391	-5.64	-5.31	-5.01	-4.65
C	IO	1988	$t_{ \hat{\gamma}^i }$	4	-5.8053 ^{2.5%}	-6.07	-5.61	-5.33	-4.94
C	AO	1977	$F_{(\hat{\theta}, \hat{\gamma})^C}$	1	-4.1391	-5.71	-5.42	-5.11	-4.77
C	IO	1988	$F_{(\hat{\theta}, \hat{\gamma})^C}$	4	-5.8053 ^{5%}	-6.29	-5.95	-5.59	-5.23

[‡]Source: Perron (1997) and Vogelsang and Perron (1998).
 $\tau_{\hat{\alpha}^i}^*(\cdot)^{c\%}$ significant at the $c\%$ level.

TABLE 11 Unit-root test involving a search for a single break, LKAB f N; $k^{max} = 4$.

i	m	$T_B(s)$	Test results			Critical values [‡]			
			s	k	$\tau_{\hat{\alpha}^i}^*(m, T_B(s))$	1%	2.5%	5%	10%
A	AO	1982	$-\tau_{\hat{\alpha}^i}$	3	-5.0140 ^{10%}	-5.92	-5.49	-5.20	-4.83
A	IO	1972	$-\tau_{\hat{\alpha}^i}$	1	-5.1241 ^{10%}	-5.92	-5.58	-5.23	-4.92
C	AO	1980	$-\tau_{\hat{\alpha}^i}$	1	-5.5135 ^{10%}	-6.17	-5.87	-5.56	-5.21
C	IO	1981	$-\tau_{\hat{\alpha}^i}$	1	-4.7768	-6.32	-5.90	-5.59	-5.29
A	AO	1973	$t_{\hat{\theta}^i}$	1	-4.0661	-5.24	-4.86	-4.53	-4.14
A	IO	1973	$t_{\hat{\theta}^i}$	1	-4.4945	-5.70	-5.21	-4.92	-4.53
A	AO	1973	$t_{ \hat{\theta}^i }$	1	-4.0661	-5.35	-4.95	-4.73	-4.34
A	IO	1973	$t_{ \hat{\theta}^i }$	1	-4.4945	-5.85	-5.51	-5.18	-4.83
C	AO	1976	$-t_{\hat{\gamma}^i}$	0	-3.6190	-5.62	-5.30	-4.98	-4.60
C	IO	1975	$-t_{\hat{\gamma}^i}$	1	-3.3771	-5.77	-5.38	-4.98	-4.55
C	AO	1976	$t_{ \hat{\gamma}^i }$	0	-3.6190	-5.64	-5.31	-5.01	-4.65
C	IO	1987	$t_{ \hat{\gamma}^i }$	4	-3.1619	-6.07	-5.61	-5.33	-4.94
C	AO	1976	$F_{(\hat{\theta}, \hat{\gamma})^C}$	0	-3.6190	-5.71	-5.42	-5.11	-4.77
C	IO	1988	$F_{(\hat{\theta}, \hat{\gamma})^C}$	4	-4.2320	-6.29	-5.95	-5.59	-5.23

[‡]Source: Perron (1997) and Vogelsang and Perron (1998).
 $\tau_{\hat{\alpha}^i}^*(\cdot)^{c\%}$ significant at the $c\%$ level.

TABLE 12 Unit-root test involving a search for a single break, LKAB f L; $k^{max} = 4$.

i	m	$T_B(s)$	Test results			Critical values [‡]			
			s	k	$\tau_{\hat{\alpha}^i}^*(m, T_B(s))$	1%	2.5%	5%	10%
A	AO	1981	$-\tau_{\hat{\alpha}^i}$	3	-4.9952 ^{10%}	-5.92	-5.49	-5.20	-4.83
A	IO	1981	$-\tau_{\hat{\alpha}^i}$	3	-5.5740 ^{5%}	-5.92	-5.58	-5.23	-4.92
C	AO	1981	$-\tau_{\hat{\alpha}^i}$	3	-4.9965	-6.17	-5.87	-5.56	-5.21
C	IO	1987	$-\tau_{\hat{\alpha}^i}$	4	-5.7444 ^{5%}	-6.32	-5.90	-5.59	-5.29
A	AO	1988	$t_{\hat{\theta}^i}$	0	-2.7007	-5.24	-4.86	-4.53	-4.14
A	IO	1986	$t_{\hat{\theta}^i}$	0	-2.5291	-5.70	-5.21	-4.92	-4.53
A	AO	1980	$t_{ \hat{\theta}^i }$	1	-4.7183 ^{10%}	-5.35	-4.95	-4.73	-4.34
A	IO	1981	$t_{ \hat{\theta}^i }$	3	-5.5740 ^{2.5%}	-5.85	-5.51	-5.18	-4.83
C	AO	1977	$-t_{\hat{\gamma}^i}$	3	-3.4449	-5.62	-5.30	-4.98	-4.60
C	IO	1975	$-t_{\hat{\gamma}^i}$	0	-2.7267	-5.77	-5.38	-4.98	-4.55
C	AO	1977	$t_{ \hat{\gamma}^i }$	3	-3.4449	-5.64	-5.31	-5.01	-4.65
C	IO	1987	$t_{ \hat{\gamma}^i }$	4	-5.7444 ^{2.5%}	-6.07	-5.61	-5.33	-4.94
C	AO	1977	$F_{(\hat{\theta}, \hat{\gamma})^C}$	3	-3.4449	-5.71	-5.42	-5.11	-4.77
C	IO	1987	$F_{(\hat{\theta}, \hat{\gamma})^C}$	4	-5.7444 ^{5%}	-6.29	-5.95	-5.59	-5.23

[‡]Source: Perron (1997) and Vogelsang and Perron (1998).
 $\tau_{\hat{\alpha}^i}^*(\cdot)^{c\%}$ significant at the $c\%$ level.

TABLE 13 Unit-root test involving a search for a single break, LKAB p; $k^{max} = 4$.

i	m	$T_B(s)$	Test results			Critical values [‡]			
			s	k	$\tau_{\hat{\alpha}^i}^*(m, T_B(s))$	1%	2.5%	5%	10%
A	AO	1980	$-\tau_{\hat{\alpha}^i}$	1	-5.4422 ^{5%}	-5.92	-5.49	-5.20	-4.83
A	IO	1981	$-\tau_{\hat{\alpha}^i}$	3	-5.3910 ^{5%}	-5.92	-5.58	-5.23	-4.92
C	AO	1980	$-\tau_{\hat{\alpha}^i}$	1	-5.9371 ^{2.5%}	-6.17	-5.87	-5.56	-5.21
C	IO	1982	$-\tau_{\hat{\alpha}^i}$	3	-4.7500	-6.32	-5.90	-5.59	-5.29
A	AO	1988	$t_{\hat{\theta}^i}$	1	-2.8362	-5.24	-4.86	-4.53	-4.14
A	IO	1987	$t_{\hat{\theta}^i}$	0	-2.2996	-5.70	-5.21	-4.92	-4.53
A	AO	1982	$t_{ \hat{\theta}^i }$	3	-4.0663	-5.35	-4.95	-4.73	-4.34
A	IO	1981	$t_{ \hat{\theta}^i }$	3	-5.3910 ^{5%}	-5.85	-5.51	-5.18	-4.83
C	AO	1977	$-t_{\hat{\gamma}^i}$	2	-3.5232	-5.62	-5.30	-4.98	-4.60
C	IO	1976	$-t_{\hat{\gamma}^i}$	0	-3.1546	-5.77	-5.38	-4.98	-4.55
C	AO	1977	$t_{ \hat{\gamma}^i }$	2	-3.5232	-5.64	-5.31	-5.01	-4.65
C	IO	1976	$t_{ \hat{\gamma}^i }$	0	-3.1546	-6.07	-5.61	-5.33	-4.94
C	AO	1980	$F_{(\hat{\theta}, \hat{\gamma})^C}$	1	-5.9371	-5.71	-5.42	-5.11	-4.77
C	IO	1976	$F_{(\hat{\theta}, \hat{\gamma})^C}$	0	-3.1546	-6.29	-5.95	-5.59	-5.23

[‡]Source: Perron (1997) and Vogelsang and Perron (1998).
 $\tau_{\hat{\alpha}^i}^*(\cdot)^{c\%}$ significant at the $c\%$ level.

TABLE 14 Unit-root test involving a search for a single break, CVRD p; $k^{max} = 4$.

D FIGURES

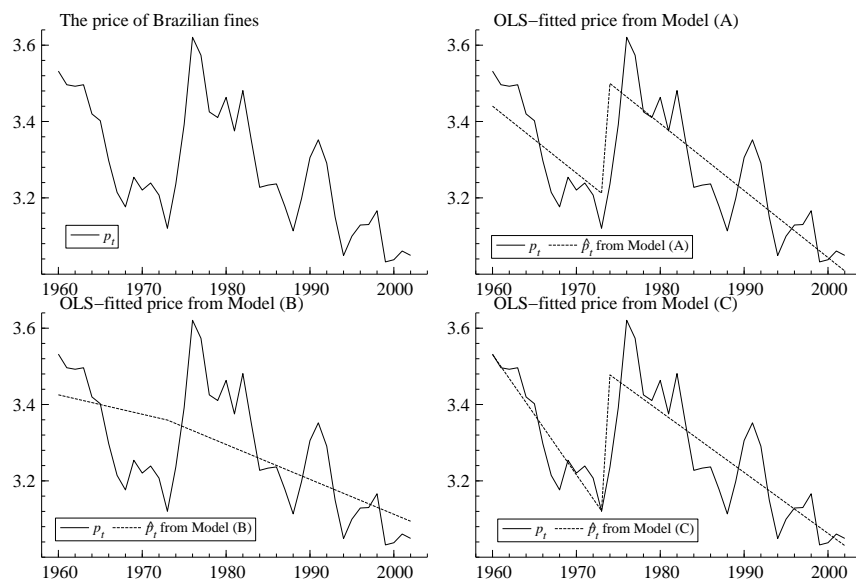


FIGURE 3 \hat{p}_t under the alternative hypotheses of Models (A), (B), and (C), with AO ; $T_B = 1973$.

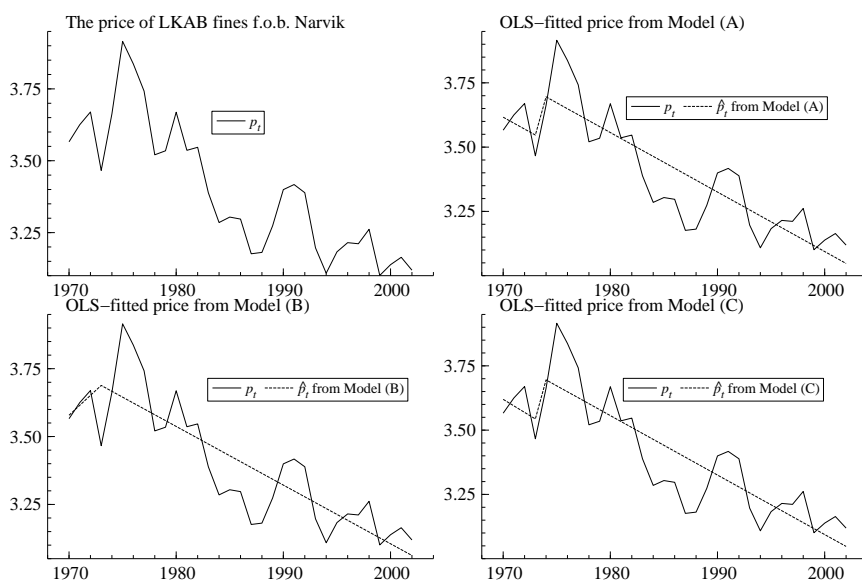


FIGURE 4 \hat{p}_t under the alternative hypotheses of Models (A), (B), and (C), with AO ; $T_B = 1973$.

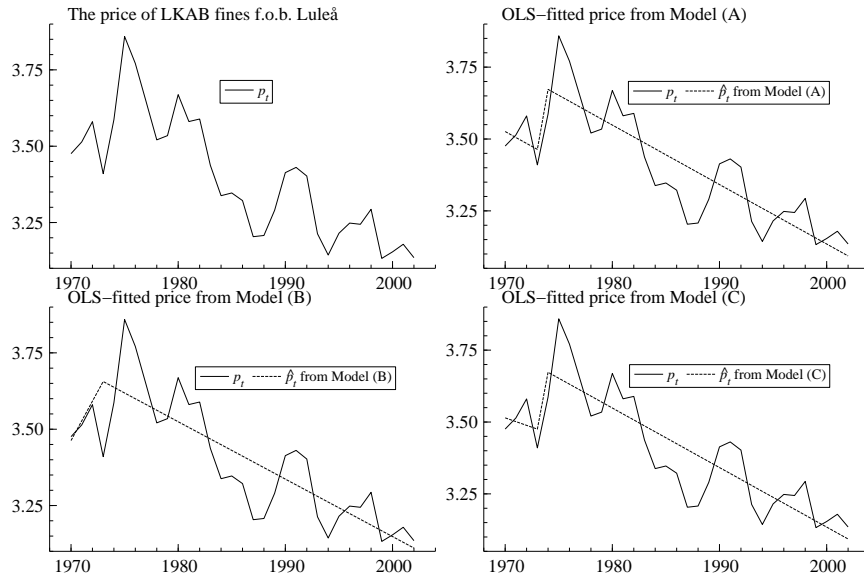


FIGURE 5 \hat{p}_t under the alternative hypotheses of Models (A), (B), and (C), with AO ; $T_B = 1973$.

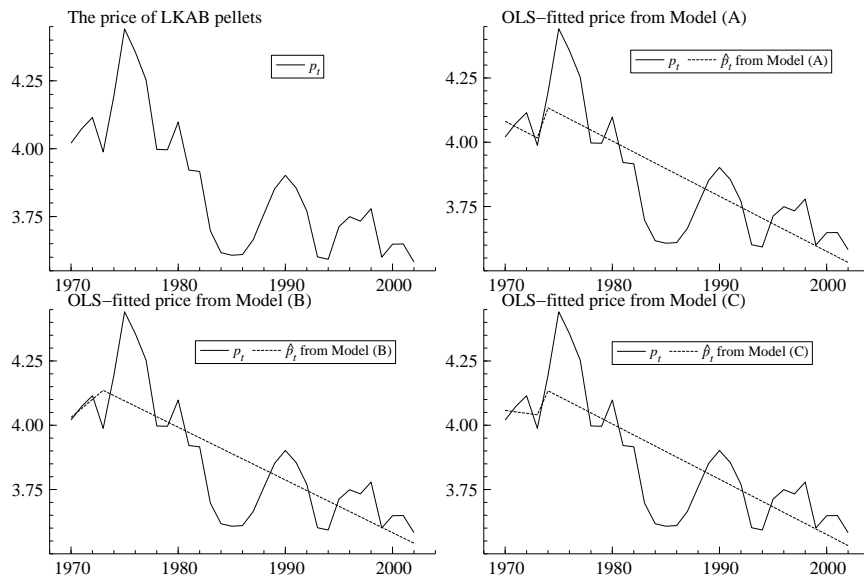


FIGURE 6 \hat{p}_t under the alternative hypotheses of Models (A), (B), and (C), with AO ; $T_B = 1973$.

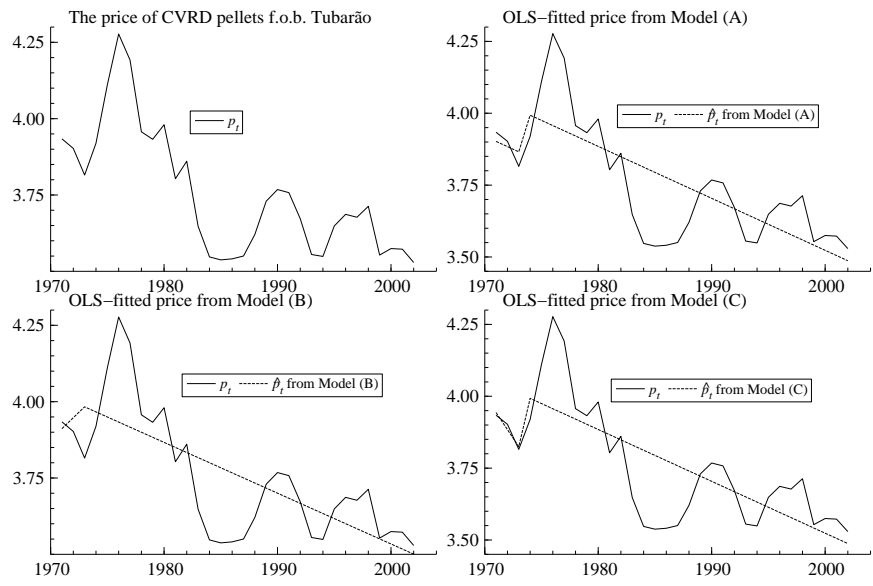


FIGURE 7 \hat{p}_t under the alternative hypotheses of Models (A), (B), and (C), with AO ; $T_B = 1973$.

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