

Kolm, Ann-Sofie; Larsen, Birthe

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# Does Tax Evasion Affect Unemployment and Educational Choice?\*

Ann-Sofie Kolm<sup>†</sup> and Birthe Larsen<sup>‡</sup>

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## Abstract

While examining the macroeconomic effects of government tax and punishment policies, this paper develops a three-sector general equilibrium model featuring matching frictions and worker-firm wage bargaining. Workers are assumed to differ in ability, and the choice of education is determined endogenously. Job opportunities in an informal sector are available only to workers who choose not to acquire higher education. We find that increased punishment of informal activities increases the number of educated workers and reduces the number of unemployed workers. Considering welfare, we show it is optimal to choose punishment rates so to more than fully counteract the distortion created by the government's inability to tax the informal sector.

JEL-codes: H26, I21, J64

Keywords: Tax evasion, underground economy, education, matching, unemployment.

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<sup>†</sup>Department of Economics, Uppsala University, Box 513, S-751 20 Uppsala, and Institute for Labour Market Policy Evaluation (IFAU), Box 513, S-751 20 Uppsala. Ph. +46 18 471 7081, Fax +46 18 471 7071. E-mail address: ann-sofie.kolm@nek.uu.se

<sup>‡</sup>Centre for Research in Social Integration and Marginalization (CIM) and Copenhagen Business School, Department of Economics, Solbjerg Plads 3, DK-2000 Copenhagen F. Ph. +45 38 15 25 44, E-mail address: bl.eco@cbs.dk

# 1 Introduction

The last couple of years have witnessed a surge of academic as well as journalistic writings on tax evading activities.<sup>1</sup> One reason for this interest is that tax evasion opportunities create various distortions in the behaviour of economic agents. For example, too much work may be carried out in the informal sector if consumer wages are relatively higher in this sector.<sup>2</sup>

In this paper we argue that also the choice to acquire higher education may be distorted by tax evasion opportunities. If workers with a lower level of education to a larger extent face job opportunities in an informal sector, there are less incentives to acquire higher education. More informal job opportunities, higher taxes or lower punishment fees, reduces the relative pay-off from education; less workers will educate themselves. This hypotheses is consistent with data. For example, while using survey data of the shadow sector in Sicily, Boeri and Garibaldi (2002) show that mainly workers at the lower end of the skill distribution engage in informal activities. In addition Smith and Pedersen (1998), using comprehensive survey data, show that almost half of the informal sector activities in Denmark are carried out within the construction sector. They also find that around 70 percent of the total hours performed in the informal sector is carried out within the service sector or construction sector.

The aim of this paper is threefold. *First*, we want to examine how the government tax and punishment policies of the informal sector affects the incentives to acquire higher education. *Second*, we want to examine how labour market performance, including wage formation, unemployment, wage dispersion, and the size of the informal sector is affected by tax and punishment policies. *Third*, we aim at characterizing the optimal tax and punishment system.

To that end, we develop a three-sector general equilibrium model featuring matching frictions and worker-firm wage bargains. Workers differ in ability and decide whether or not to acquire higher education. We assume that it is costly in terms of effort to acquire higher education, implying that only high ability workers find this worthwhile. Workers who choose not to acquire higher education, whom we refer to as manual workers, face job opportunities in both the formal and the informal sector. Unemployed manual

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<sup>1</sup>See Slemrod and Yitzhaki (2002) and Schneider and Eneste (2000) for two recent surveys of tax avoidance and tax evasion.

<sup>2</sup>See Pedersen and Smith (1998) for evidence for Denmark.

workers search for jobs in both sectors by allocating their search effort optimally between them.

We find that increased government punishment of the informal sector reduces the size of this sector. Moreover, the number of unemployed workers is likely to fall, formal wage dispersion increases, and the number of highly educated workers increases. While considering welfare, we find that it is optimal to choose the punishment rates so to more than fully counteract the distortion created by the government's inability to tax the informal sector.

Early theoretical analyses of tax evasion are provided by Allingham and Sandmo (1972) and Srinivasan (1973), where underreporting of income is modeled as a decision made under uncertainty. Subsequent papers have enhanced the basic model of individual behaviour by, for example, incorporating endogenous labour supply decisions.<sup>3</sup> Also general equilibrium models with tax evasion have been developed (for an excellent example see Cremer and Gahvari (1993)). Several theoretical papers have also recognized that the opportunities for tax evasion differ across occupations. See for example Watson (1985), Pestieu and Possen (1991), and Jung et al. (1994). Occupational choice in this literature is usually thought of as a choice between self-employment, where underreporting is possible, and regular employment, where underreporting is not an option. Pestieu and Possen (1991) assume that workers choose either to be entrepreneurs or regular employees. Workers differ in risk aversion and underreporting is only available for entrepreneurs. They argue that tax evasion should be allowed to some degree in order to maintain a large stock of productive entrepreneurs.

The principal contribution of the analyses in this paper is that we shed light on that the presence of work opportunities in an informal sector can affect the educational attainment in an economy. This has, to our knowledge, not been explored in the previous literature. Another important contribution is that we incorporate an imperfectly competitive labour market. This facilitates an analysis of how tax and punishment policies affect wage setting and unemployment. Previous research is mainly conducted within the public finance tradition. In this literature wages are either assumed to be fixed or determined by market clearing, and by definition, such framework is unable to examine how involuntary unemployment is affected by tax and punishment policies. There has, however, been some recent studies of how tax and

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<sup>3</sup>See for example Andersen (1977) and Sandmo (1981) for early contributions of endogenous labour supply and underreporting of income.

punishment policies affect involuntary unemployment; see Kolm and Larsen (2001,2002), Cavalcanti (2002), Boeri and Garibaldi (2002) and Fugazza and Jacques (2003). This paper differs from these previous studies in that it does not rely on relative price adjustments (as Kolm and Larsen (2001, 2002) and Cavalcanti (2002)) or heterogeneity in moral and a fixed number of vacancies as Fugazzi and Jacques (2003), to generate a stable equilibrium where both formal and informal jobs coexist. Neither does it rely on that informal jobs only comes about as matched formal jobs are hit by a bad productivity shock, as Boeri and Garibaldi (2002), in order to generate coexistence of formal and informal jobs. Rather this paper assumes directed search where the unemployed workers are optimally allocating search effort into both the formal and the informal sector. We assume that the effectiveness of search falls with search time into a sector; see the parallel with a production function. This could capture that different search methods are used when searching for a job in a market. The more time that is used in order to search in a market, the less efficient search methods have to be used.

This paper is organized as follows. Section 2 describes the model. Section 3, examines how the equilibrium variables (e.g. tightness, wages, wage dispersion, employment and unemployment rates, unemployment stock, and the stock of educated workers) are affected by a fully financed change in the punishment system. Section 4 concerns the optimal design of tax and punishment policies. Section 5 concludes.

## 2 The Model<sup>4</sup>

The economy consists of a labour force which differs in ability to acquire education. Abilities,  $e$ , are uniformly distributed between 0 and 1,  $e \in [0, 1]$ . Based on ability, the workers decide whether or not to educate themselves. We assume that it is costless to become a manual worker, but that workers who educate themselves find it costly to do so.<sup>5</sup> The costs of education,  $c(e)$  is decreasing in ability,  $c'(e) < 0$ .

Manual workers face job opportunities in both a formal and an informal sector, whereas highly educated workers only face employment opportunities

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<sup>4</sup>The model is along the line of Pissarides 2000.

<sup>5</sup>This is a simplifying assumption, allowing us to focus on the choice inbetween two skill levels.

in a formal sector.<sup>6</sup> The economy thus consists of three sectors; the formal manual sector (denoted the formal sector  $F$ ), the informal manual sector (denoted the informal sector  $I$ ), and the highly educated sector (denoted  $h$ ). The formal and the informal sector employ manual workers, whereas employment in sector  $h$  requires higher education. Manual workers have productivity  $y_m$  and highly educated workers have productivity  $y_h > y_m$ .

## 2.1 Manual Workers

### 2.1.1 Matching

The manual workers search for jobs in the formal and the informal sector. For simplicity, we assume that only unemployed workers search for jobs. This is a simplification, i.e. we do not acknowledge that the connection to the labour market given by working in the formal sector, brings about job opportunities not available while unemployed. Workers accept job offers as long as the expected payoff exceeds their reservation wage. We disregard any moral considerations the worker could have preventing the worker from applying for a job in the informal sector. The matching functions for the manual sectors are given by

$$X_m^j = (v_m^j)^{1-\eta} \left( (\sigma^j)^\beta u_m \right)^\eta, j = F, I,$$

where  $u_m$  is the unemployment rate facing manual workers, and  $v_m^j$ , is the sectorial vacancy rate. The rates are defined as the numbers relatively to the manual labour force. Each worker's total search intensity is exogenously given and normalised to unity. Manual workers will allocate search effort optimally between the formal and the informal sector.  $\sigma^F$  denotes search effort directed towards the formal sector, and  $\sigma^I$  denotes search effort directed towards the informal sector. To simplify notation, we let  $\sigma^I = \sigma$  and  $\sigma^F = 1 - \sigma$ . The parameter  $\beta < 1$ , captures that the effectiveness of search falls with search effort, i.e., the first unit of search in one sector is more effective than the subsequent units of search.<sup>7</sup> The transition rates into formal and informal

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<sup>6</sup>This is, of course, a simplification. But it is a simplifying assumption consistent with the facts about in which sector informal work takes place wherefore we provided empirical evidence in the introduction.

<sup>7</sup>This could capture that various search methods is used when searching for a job in a market. The more time that is used in order to search, the less efficient search methods has to be used.

sector employment for a particular manual worker  $i$ , are  $\lambda_{mi}^I = \sigma_i^\beta (\theta_m^I)^{1-\eta}$  and  $\lambda_{mi}^F = (1 - \sigma_i)^\beta (\theta_m^F)^{1-\eta}$ , where  $\theta_m^F = \frac{v_m^F}{(1-\sigma)^\beta u_m}$  and  $\theta_m^I = \frac{v_m^I}{\sigma^\beta u_m}$  are labour market tightness. Labour market tightness is measured in effective search units.<sup>8</sup> The rates at which vacant jobs become filled are  $q_m^j = (\theta_m^j)^{-\eta}$ ,  $j = F, I$ .

### 2.1.2 Value Equations

Let  $U_m$ ,  $E_m^F$ , and  $E_m^I$  denote the expected present values of unemployment, and employment in the two sectors. The value functions for worker  $i$  then reads:

$$rU_{mi} = R + \lambda_{mi}^F(E_m^F - U_{mi}) + \lambda_{mi}^I(E_m^I - U_{mi}), \quad (1)$$

$$rE_{mi}^F = R + w_{mi}^F(1 - t) + s(U_m - E_{mi}^F), \quad (2)$$

$$rE_{mi}^I = R + w_{mi}^I(1 - p\delta) + (s + p)(U_m - E_{mi}^I), \quad (3)$$

where  $R$  is a lump sum transfer that all individuals receive from the government,  $r$  is the exogenous discount rate,  $w_m^j$  is the sector wage, and  $s$  is the exogenous separation rate.<sup>9</sup> For simplicity, we assume that the unemployment benefits are equal to zero. The parameter  $t$  is the proportional income tax rate,  $p$  is the probability of being detected working in the informal sector, and  $\delta$  is the proportion of the evaded income the worker has to pay as a punishment fee if detected. The match is dissolved when detected which implies that the separation rate in the informal sector exceeds the formal sector separation rate.

Let  $J_m^F$  and  $V_m^F$  represent the expected present values of an occupied job and a vacant job in the formal sector, respectively. The arbitrage equations for a job paying the wage  $w_{mi}^F$  and a vacant job in the formal sector are then

$$rJ_{mi}^F = y_m - w_{mi}^F(1 + z) + s(V_m^F - J_{mi}^F), \quad (4)$$

$$rV_m^F = q_m^F(J_m^F - V_m^F) - k, \quad (5)$$

where  $z$  is the payroll tax rate. Vacancy costs are denoted  $k$ .

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<sup>8</sup>One could, of course, choose a different definition of tightness, for example  $\theta_m^F = v_m^F/u_m$  and  $\theta_m^I = v_m^I/u_m$ . This is, however, of no importance for the results.

<sup>9</sup> $R$  reflects that the government has some positive revenue requirements. For simplicity, we assume that this revenue is distributed equally across the population in terms of a lump-sum transfer or some public good.

Let  $J_m^I$  and  $V_m^I$  represent the expected present values of an occupied job and a vacant job in the informal sector. The arbitrage equations for a job paying the wage  $w_{mi}^I$  and a vacant job in the informal sector are then

$$rJ_{mi}^I = y_m - w_{mi}^I(1 + p\alpha) + (s + p)(V_m^I - J_{mi}^I), \quad (6)$$

$$rV_m^I = q_m^I(J_m^I - V_m^I) - k, \quad (7)$$

where  $\alpha$  is the proportion of the evaded wage the firm has to pay as a punishment fee if detected.

The unemployed worker  $i$  allocates search between the two sectors,  $\sigma_i$ , in order to maximise the value of unemployment,  $rU_{mi}$ . A necessary condition for an interior solution is satisfied as  $\beta < 1$ . The first order condition can be written as:

$$\frac{(1 - \sigma_i)^{1-\beta}}{(\sigma_i)^{1-\beta}} = \left( \frac{\theta_m^I}{\theta_m^F} \right)^{1-\eta} \frac{E_m^I - U_{mi}}{E_m^F - U_{mi}}. \quad (8)$$

Workers allocate their search between sectors so that the returns to search effort is equal across the two sectors. The left-hand side of equation (8) captures the relative effectiveness of search in the two sectors, whereas the right-hand side is the relative returns to search. The relatively tighter the formal sector is, that is, the higher  $\frac{\theta_m^F}{\theta_m^I}$  is, and the larger the relative utility gain of leaving unemployment for a formal sector job is, the larger is formal sector search.

### 2.1.3 Wage Determination

When a worker and firm meet they bargain over the wage,  $w_{mi}^j$ , taking economy wide variables as given. The first order conditions from the Nash bargaining solutions, with the worker's bargaining power being equal to  $\gamma$ , can be written as:

$$\frac{\gamma}{1 - \gamma} \frac{1}{\phi^t} J_m^F = E_m^F - U_m, \quad (9)$$

$$\frac{\gamma}{1 - \gamma} \frac{1}{\phi^p} J_m^I = E_m^I - U_m, \quad (10)$$

where  $\phi^t = \frac{1+z}{1-t}$  and  $\phi^p = \frac{1+p\alpha}{1-p\delta}$  are the tax- and punishment wedges, and where we have imposed symmetry and the free entry condition,  $V_m^j = 0$ ,  $j = F, I$ .



We can now derive an equation determining how search is allocated between the two sectors in a symmetric equilibrium by substituting (9) and (10) into (8) and using that  $J_m^F = \frac{k}{q_m^F}$  and  $J_m^I = \frac{k}{q_m^I}$  from (5) and (7) together with free entry. This yields:

$$\frac{(1 - \sigma)^{1-\beta}}{(\sigma)^{1-\beta}} = \frac{\theta_m^F}{\theta_m^I} \psi, \quad (11)$$

where

$$\psi = \frac{\phi^p}{\phi^t} = \frac{1 + p\alpha}{1 - p\delta} \frac{1 + z}{1 - t}, \quad (12)$$

is the wedge between the informal sector and the formal sector. We can interpret a  $\psi < 1$ , as if the informal sector is punished to a lesser extent than the formal sector is taxed.<sup>10</sup>

Equation (11) is the core equilibrium equation. Recall from (8) that workers allocate their search between sectors so that the marginal returns to search effort in the two sectors are equal. With wages being endogenously determined in equilibrium, this amounts to account for the wedge,  $\psi$ , and for differences in sectorial labour market tightness,  $\frac{\theta_m^F}{\theta_m^I}$ , when deciding where to allocate search. For example, if formal sector tightness exceeds informal sector tightness, this corresponds to that relatively more open jobs are available in the formal sector and unemployed workers tend to direct their search into the formal sector to a larger extent. If, on the other hand, it is possible to evade taxes by working in the informal sector,  $\psi < 1$ , unemployed workers tend to direct their search into the informal sector to a larger extent.

By use of equation (1)-(7) and (11) in equations (9) and (10), equilibrium producer wages,  $\omega_m^j$ ,  $j = F, I$ , are given by:

$$\omega_m^F = w_m^F (1 + z) = \gamma \left( y_m + k \frac{\theta_m^F}{(1 - \sigma)^{1-\beta}} \right), \quad (13)$$

$$\omega_m^I = w_m^I (1 + p\alpha) = \gamma \left( y_m + k \frac{\theta_m^I}{\sigma^{1-\beta}} \right). \quad (14)$$

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<sup>10</sup>In contrast, if  $\psi = 1$ , the informal sector is punished equally hard as the formal sector is taxed. With risk neutral individuals there is, in one sense, no substantial difference between the tax system and the punishment system other than that the punishment system is a randomized tax system.

Wages increase with labour market tightness and decrease with search intensity in each sector. This follows as a higher labour market tightness and a smaller search intensity improve the worker's bargaining position. An increase in tightness makes it more easy for an unemployed worker to find a job, and at the same time more difficult for a firm to fill a vacancy. This improves the worker's bargaining position, resulting in higher wage demands. An increase in search, will instead reduce the worker's bargaining position. This follows as the effectiveness of search falls with more search, i.e.,  $\beta < 1$ . Sectorial wage demands are hence moderated with increased search in the sector.

>From (13), (14) together with (11) it is clear that producer wages in the formal sector exceed informal sector producer wages when  $\psi < 1$ , and vice versa when  $\psi > 1$ . Moreover, rewriting (13) and (14) in terms of consumer wages we have that consumer wages in the formal sector subceed informal sector consumer wages when  $\psi < 1$ , and vice versa when  $\psi > 1$ . More specifically we have  $\omega^F - \omega^I = \frac{\gamma k \theta_m^F}{(1-\sigma)^{1-\beta}} (1 - \psi)$  and  $w_m^F (1 - t) - w_m^I (1 - p\delta) = \frac{\gamma y_m}{\phi^p} (\psi - 1)$ .

When conducting comparative statics analyses, it will become clear that the results regarding how increased punishment of the informal sector affects unemployment hinges on the size of  $\psi$ . We then focus on the case when  $\psi < 1$ , as this seems to be the most relevant case. Recall that  $\psi < 1$  corresponds to the case where the informal sector is punished to a lesser extent than the formal sector is taxed. Moreover, when  $\psi < 1$ , producer wages are lower and consumer wages are higher in the informal sector, rather than the opposite.

#### 2.1.4 Labour Market Tightness

Labour market tightness for the formal sector and the informal sector are determined by equation (4),(5), (6) and (7) using the free entry condition and the wage equations (13) and (14):

$$k(r + s) (\theta_m^F)^\eta = (1 - \gamma) y_m - \frac{\gamma k \theta_m^F}{(1 - \sigma)^{1-\beta}}, \quad (15)$$

$$k(r + s + p) (\theta_m^I)^\eta = (1 - \gamma) y_m - \frac{\gamma k \theta_m^I}{\sigma^{1-\beta}}. \quad (16)$$

Firms open vacancies in each sector until the discounted profits equal the expected vacancy costs. With  $\psi < 1$ , informal producer wages are lower than

formal producer wages and hence the expected instantaneous profits in the informal sector exceed the instantaneous profits in the formal sector.<sup>11</sup> This makes it more attractive for firms to enter the informal sector which tends to make informal tightness exceed formal tightness. However, as the separation rate is higher in the informal sector,  $s+p > s$ , expected job duration is longer in the formal sector. This makes the formal sector more attractive to enter. Consequently, it is possible to have  $\theta_m^F > \theta_m^I$ , although  $\psi < 1$ .

## 2.2 Sector $h$ , highly educated workers

Highly educated workers applying for jobs in sector  $h$  do not have the opportunity to obtain employment in an informal sector. The total time unit of search is allocated into search in sector  $h$ , i.e.,  $\sigma_h = 1$ . The matching function is given by a standard matching function,  $X_h = (v_h)^{1-\eta} (u_h)^\eta$ , where  $v_h$  is the vacancy rate and  $u_h$  is the unemployment rate for educated workers. Worker and firm transition rates then become  $\lambda_h = (\theta_h)^{1-\eta}$ ,  $\theta_h = \frac{v_h}{u_h}$ , and  $q_h = \theta_h^{-\eta}$ . Educated workers pay the individual educational costs  $c(e_i)$ , where  $e_i$  is the worker's ability,  $e_i \in [0, 1]$  and  $c'(e_i) < 0$ . Let  $U_h$  and  $E_h$  denote the expected present values of unemployment and employment. The value functions for worker  $i$  then reads:<sup>12</sup>

$$rE_{hi} = R + w_{hi}(1-t) + s(U_{hi} - E_{hi}) - c(e_i), \quad (17)$$

$$rU_{hi} = R + \lambda_h(E_h - U_h) - c(e_i). \quad (18)$$

where  $w_h$  denotes the wage for educated workers.

Sector  $h$  firms employ workers with the marginal productivity  $y_h$  and have the time unit probability  $q_h$  of filling a vacancy. Let  $J_h$  and  $V_h$  represent the expected present values of an occupied job and a vacant job in the educated sector. The arbitrage equations for a job paying  $w_{hi}$  and a vacant job in the educated sector are then:

$$rJ_{hi} = y_h - w_{hi}(1+z) + s(V_h - J_{hi}), \quad (19)$$

$$rV_h = q_h(J_h - V_h) - k. \quad (20)$$

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<sup>11</sup>The right-hand side of equations (15) and (16), are simply the instantaneous profits, i.e.,  $\pi^F = y_m - \omega_m^F$  and  $\pi^I = y_m - \omega_m^I$ , where we from the previous section know that  $\omega^F - \omega^I = \frac{\gamma k \theta^F}{(1-\sigma)^{1-\beta}} (1-\psi)$ .

<sup>12</sup>We assume that educational costs is costs to acquire and maintain skill. This is a simplifying assumption and is not important for the results. This simplifying assumption enables us to use a model without having workers being born and die.

Wages,  $w_{hi}$ , are determined by Nash Bargaining with the workers' bargaining power equal to  $\gamma$ . The first order condition can be written analogous to equation (9). We can solve for the bargained wage by using this first order condition and equations (17)-(20), assuming free entry,  $V_h = 0$ , and a symmetric equilibrium:

$$\omega_h = w_h (1 + z) = \gamma (y_h + \theta_h k). \quad (21)$$

Labour market tightness in sector  $h$ ,  $\theta_h$ , can be derived from equations (19) and (20), using the free entry condition and the symmetry assumption together with the wage rule in (21):

$$k(r + s)\theta_h^\eta = (1 - \gamma)y_h - \gamma\theta_h k. \quad (22)$$

It is important to note that the tax rates  $t$  and  $z$  will have no impact on the producer wage, and tightness in sector  $h$ .<sup>13</sup> Tax changes will always be absorbed by changes in the consumer wage, leaving producer wages and tightness unaffected in the sector.

## 2.3 Education

When workers decide whether to acquire higher education or remain a manual worker, they compare the value of unemployment as an educated worker to the value of unemployment as a manual worker. Workers with low ability find it too costly in terms of effort to acquire higher education, whereas high ability workers find it more than worthwhile to do so since they face lower costs of education. The marginal worker has an ability level,  $\hat{e}$ , which makes him just indifferent between acquiring higher education and remaining a manual worker. We can write the condition determining the ability level of the marginal worker as:

$$rU_h = rU_m. \quad (23)$$

We can rewrite this expression in a number of ways using earlier equations. For example, by using equations (1)-(3), it is clear that the value of unemployment for a manual worker is captured by  $rU_m = R + \mu_m^F w_m^F (1 - t) +$

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<sup>13</sup>This follows because we have iso-elastic preferences and a constant replacement rate of unemployment benefits. With unemployment insurance being equal to zero, we have both fixed benefits in real terms as well as a fixed replacement rate.

$\mu_m^I w_m^I (1 - p\delta)$ . This expression reveals that it is the consumer wages in the formal and informal sector, weighted by the employment opportunities in each sector that is important for the unemployment value of manual workers.<sup>14</sup> A similar expression can be derived for the value of unemployment as a educated worker;  $rU_h = R + \mu_h w_h (1 - t) - c(e)$  from (17) and (18), where  $\mu_h = \lambda_h / (r + s + \lambda_h)$ .

As wages are endogenous we can use equations (23), (1) and (18) together with the first order conditions for wages, and equations (5), (7), (11), (20) together with the free entry condition which gives the following simplified condition:

$$c(\hat{e}) = \frac{\gamma}{1 - \gamma} \frac{k}{\phi^t} \left( \theta_h - \frac{\theta_m^F}{(1 - \sigma)^{1-\beta}} \right). \quad (24)$$

Equation (24) gives  $\hat{e}$  as a function of the endogenous variables  $\theta_h$ ,  $\theta_m^F$ , and  $\sigma$ . Workers with  $e \leq \hat{e}$ , choose not to acquire education, whereas workers with  $e > \hat{e}$  acquire education. Hence,  $\hat{e}$  and  $1 - \hat{e}$  resolve the manual and educated labour forces, respectively. Moreover, we note that the tax wedge,  $\phi^t$ , has a direct effect on the number of educated workers, as well as an indirect effect as  $\psi = \phi^p / \phi^t$  influences  $\theta^F$  and  $\sigma$ . The punishment wedge,  $\phi^p$ , only influences  $\hat{e}$  through  $\psi$ .

## 2.4 Unemployment

Steady state employment and unemployment rates for manual and educated workers are derived by considering the flows into and out of employment. The equations determining the employment rates in the formal sector and the informal sector,  $n_m^F, n_m^I$ , and the manual unemployment rate,  $u_m$ , are given by:  $\lambda_m^I u_m \hat{e} = (s + p) n_m^I \hat{e}$ ,  $\lambda_m^F u_m \hat{e} = s n_m^F \hat{e}$ , and  $n_m^F + n_m^I = 1 - u_m$ . The employment and unemployment rates, are the number of employed and unemployed manual workers relative to the manual labour force. The unemployment rate for manual workers that is observable by the government,  $u_m^o$ , is given by  $u_m^o = u_m + n_m^I$ .

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<sup>14</sup>The weights,  $\mu_m^F = \lambda_m^F / (r + s + \lambda_m^F + \frac{r+s}{r+s+p} \lambda_m^I)$  and  $\mu_m^I = \lambda_m^I / (r + s + p + \lambda_m^I + \frac{r+s+p}{r+s} \lambda_m^F)$ , reduce down to  $\mu_m^F = n_m^F$  and  $\mu_m^I = n_m^I$  when the discount rate approaches zero. Consequently, with full discounting, the weights are given by the expected time a worker will be employed in each sector.

Solving for the employment rates, the actual and the observed unemployment rates for manual workers, we obtain:

$$n_m^I = \frac{\frac{\lambda_m^I}{s+p}}{1 + \frac{\lambda_m^I}{s+p} + \frac{\lambda_m^F}{s}}, n_m^F = \frac{\frac{\lambda_m^F}{s}}{1 + \frac{\lambda_m^I}{s+p} + \frac{\lambda_m^F}{s}}, \quad (25)$$

$$u_m = \frac{1}{1 + \frac{\lambda_m^I}{s+p} + \frac{\lambda_m^F}{s}}, u_m^o = \frac{1 + \frac{\lambda_m^I}{s+p}}{1 + \frac{\lambda_m^I}{s+p} + \frac{\lambda_m^F}{s}}. \quad (26)$$

For sector  $h$ , flows into unemployment and out of unemployment are also equalised, whereby the unemployment rate among educated workers is

$$u_h = \frac{1}{1 + \frac{\lambda_h}{s}}. \quad (27)$$

Comparing equations (26) and (27), note that we cannot immediately determine whether unemployment facing manual workers is higher or lower than unemployment for educated workers. Higher productivity for educated workers,  $y_h > y_m$  together with the higher average separation rate for manual workers, tends to make  $u_m$  exceed  $u_h$ . However, as manual workers can apply for jobs in two sectors, which the educated worker cannot, this tends to make  $u_h$  exceed  $u_m$ .

The actual and official total number of unemployed workers are therefore given by the following expressions:

$$\begin{aligned} U_{TOT} &= \hat{e}u_m + (1 - \hat{e})u_h, \\ U_{TOT}^o &= \hat{e}u_m^o + (1 - \hat{e})u_h. \end{aligned}$$

## 2.5 Equilibrium

Equilibrium equations determining labour market tightness in the three sectors,  $\theta_m^F, \theta_m^I, \theta_h$ , search intensity,  $\sigma$ , and the number of educated workers,

$1 - \hat{e}$ , are summarized here for convenience

$$k(r + s)(\theta_m^F)^\eta = (1 - \gamma)y_m - \frac{\gamma k \theta_m^F}{(1 - \sigma)^{1-\beta}}, \quad (28)$$

$$k(r + s + p)(\theta_m^I)^\eta = (1 - \gamma)y_m - \frac{\gamma k \theta_m^I}{\sigma^{1-\beta}}, \quad (29)$$

$$\left(\frac{1 - \sigma}{\sigma}\right)^{1-\beta} = \frac{\theta_m^F}{\theta_m^I} \psi, \quad (30)$$

$$k(r + s)(\theta_h)^\eta = (1 - \gamma)y_h - \gamma k \theta_h, \quad (31)$$

$$c(\hat{e}) = \frac{\gamma}{1 - \gamma} k \frac{1}{\phi^t} \left( \theta_h - \frac{\theta_m^F}{(1 - \sigma)^{1-\beta}} \right). \quad (32)$$

Labour market tightness in the two sectors employing manual workers are determined together with search intensity in (28), (29), and (30). Independently, we solve for labour market tightness in sector  $h$  from (31). Finally, the labour force allocation into manual workers and educated workers is determined by (32) given the solutions for  $\theta_m^F, \theta_h$ , and  $\sigma$ . See the Appendix for details about stability and uniqueness.

### 3 Comparative Statics

This section is concerned with the impact of the tax and punishment system on tightness, search intensity, producer wages, employment and unemployment rates, wage dispersion, the number of unemployed workers, and the number of educated workers. Proofs of all Propositions are given in the Appendix.

We only consider fully financed changes in the punishment rates. Hence, changes in the punishment rates,  $\alpha$  or  $\delta$ , is always followed by adjustments in the tax rates,  $z$  or  $t$ , so as to balance the government budget restriction. The government budget restriction is given by:

$$\hat{e} \left( n_m^F \omega_m^F \left( 1 - \frac{1}{\phi^t} \right) + n_m^I \omega_m^I \left( 1 - \frac{1}{\phi^p} \right) \right) + (1 - \hat{e}) (1 - u_h) \omega_h \left( 1 - \frac{1}{\phi^t} \right) - \xi(p) = R, \quad (33)$$

where  $R$  is the exogenous government revenue requirements and  $\xi(p)$  is auditing costs.<sup>15</sup>

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<sup>15</sup>We note that the tax and punishment rates,  $t$  and  $z$  and  $\delta$  and  $\alpha$ , will not appear in

### 3.1 Tightness, Search Intensity, and Producer Wages

The effects on tightness and allocation of search is summarized in the following proposition.<sup>16</sup>

**Proposition 1** *A fully financed increase in the punishment rates,  $\delta$  or  $\alpha$ , will reallocate search intensity towards the formal sector,  $\sigma$  falls, increase tightness in the formal sector,  $\theta_m^F$ , and reduce tightness in the informal sector,  $\theta_m^I$ . Tightness in sector  $h$  is left unaffected by a fully financed change in the punishment system.*

When tax evasion is punished more severely, unemployed workers will find it optimal to reallocate their search towards the formal sector. However, when search effort is reallocated towards the formal sector, wage demands in the formal sector fall whereas wage demands in the informal sector increase. This follows as the effectiveness of search in the formal sector falls as search increases, which weakens the worker-firm bargaining position. In contrast, the reduced search in the informal sector increases the effectiveness of informal sector search. This improves the bargaining position for workers in the informal sector. As the producer wage facing informal sector firms tends to increase, and the formal sector producer wage tends to fall, firms will exit the informal sector and enter the formal sector; formal sector tightness increases whereas informal sector tightness falls. As formal sector tightness raises relative to tightness in the informal sector, search is further reallocated towards the formal sector. This effect becomes smaller and smaller until the new equilibrium is reached.

We summarize the effects on producer wages in the following proposition:

**Proposition 2** *A fully financed increase in the punishment rates,  $\delta$  or  $\alpha$ , will increase the producer wage in the informal sector,  $\omega_m^I$ , and reduce the producer wage in the formal sector,  $\omega_m^F$ . The producer wage in sector  $h$  is unaffected by changes in the tax and punishment rates.*

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the government budget restriction other than through  $\phi^t$  and  $\phi^p$ . This reflects that it does not matter if we tax (punish) the firm side or the worker side.

<sup>16</sup>For an intuitive interpretation, propositions 1-4 are expressed as if an increase in  $\phi^p$  financed by adjustments in  $\phi^t$  implies that  $\psi = \phi^p/\phi^t$  increases. Other, although perhaps less plausible cases, are of course also incorporated. The propositions simply capture fully financed changes in the tax and punishment systems that affect the relative tax and punishment rates,  $\psi$ , between the formal and informal economy.



Formal sector wage demands are reduced because the worker-firm bargaining position falls as search in the sector becomes less effective. In contrast, wage demands increase in the informal sector as informal sector search becomes more effective with the reduces search effort in the sector. However, there will be a dampening effect on wages due to that formal sector tightness increases and informal sector tightness falls.

### 3.2 Employment and Unemployment Rates

We summarize the results of the impact of the tax and punishment rates on the employment and unemployment rates in the following proposition:

**Proposition 3** *A fully financed increase in the punishment rates,  $\delta$  or  $\alpha$ , will increase the employment rate in the formal sector,  $n_m^F$ , and reduce the employment rate in the informal sector,  $n_m^I$ . The unemployment rate for manual workers that are observable by the government,  $u_m^o$ , falls with the reform. The actual unemployment rate for manual workers,  $u_m$ , falls with the reform if  $\psi < 1$ . The unemployment rate for educated workers,  $u_h$ , is unaffected by the reform.*

It comes as no surprise that increased punishment of the informal sector induces a reallocation of manual workers from the informal towards the formal sector. The formal sector employment rate increases as the transition rate into formal sector employment increases. This, in turn, is caused by the increase in formal sector tightness and the reallocation of search effort towards the formal sector. In contrast, as tightness and search effort in the informal sector fall, the transition rate into informal employment, and consequently the sectorial employment rate, falls.

The unemployment rate for manual workers falls as workers are reallocated towards the formal sector where jobs last on average a longer time.<sup>17</sup> In addition, the official unemployment rate falls both because the actual unemployment rate falls, and because workers are reallocated towards the formal sector.

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<sup>17</sup>In case  $\psi > 1$ , the number of vacancies relatively to unemployment is reduced which will have a counteracting effect on unemployment. Consequently, the unemployment rate may increase in this case.

### 3.3 Wage Dispersion

Wage dispersion in terms of wages received by formal sector manual workers and educated workers is equal to:

$$WD = \frac{w_h - w_m^F}{w_m^F} = \frac{\frac{\omega_h^F}{1+z} - \frac{\omega_m^F}{1+z}}{\frac{\omega_m^F}{1+z}} = \frac{\omega_h^F}{\omega_m^F} - 1.$$

As wage dispersion is increased whenever  $\omega_m^F$  is reduced, we have the following result:

**Proposition 4** *A fully financed increase in the punishment rates,  $\delta$  or  $\alpha$ , will increase wage dispersion in-between formal sector manual workers and educated workers.*

Increased punishment reduces formal sector wages for manual workers and hence increases the wage dispersion in-between educated workers and formal sector manual workers.

### 3.4 Education

A closer examination of (24) reveals that changes in the punishment rates,  $\phi^p = \frac{1+p\alpha}{1-p\delta}$ , affects  $\hat{e}$  through  $\psi$  only, whereas changes in the tax rates,  $\phi^t = \frac{1+z}{1-t}$ , have a direct effect on  $\hat{e}$  in addition to the effects working through  $\psi$ .<sup>18</sup> Therefore, in order to consider the effects of a fully financed change in the punishment rates on the number of educated workers, we have to account for repercussions on  $\hat{e}$  following adjustments in the tax rates; the government budget restriction has to be incorporated explicitly. However, let us first consider the impact on  $1 - \hat{e}$  of a change in the tax and punishment rates separately:

$$\frac{\partial(1 - \hat{e})}{\partial\phi^p}\bigg|_{\phi^t} = -\frac{\frac{\gamma(1-\beta)}{1-\gamma}k^2\eta}{(\phi^t)^2 D_1 c'(\hat{e})} \left( \frac{\eta(r+s+p)}{(\theta_m^I)^{1-\eta}} + \frac{\gamma}{\sigma^{1-\beta}} \right) \frac{(r+s)(\theta_m^F)^\eta}{(1-\sigma)^{2-\beta}} \quad (34)$$

$$\frac{\partial(1 - \hat{e})}{\partial\phi^t}\bigg|_{\phi^p} = -\psi \frac{\partial(1 - \hat{e})}{\partial\phi^p}\bigg|_{\phi^t} + \frac{c(\hat{e})}{c'(\hat{e})\phi^t} < 0, \quad (35)$$

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<sup>18</sup>Recall that the producer wage and the transition rate into employment in sector  $h$  is independent of the tax and punishment rates. Moreover, the producer wages and the transition rates into employment in sector  $F$  and  $I$ , only depend on the tax and punishment rates through the wedge,  $\psi$ .

Equation (34) shows that the number of educated workers increases with higher punishment rates for a given tax system. This summarises the impact on several variables. First, there is a direct effect which induces more workers to educate themselves as increased punishment for a given tax system reduces the informal sector consumer wage. However, increased punishment rates also affect consumer wages through the wage negotiations; wages are reduced in the formal sector but increased in the informal sector. Moreover, employment opportunities increases in the formal sector but falls in the informal sector. This is a consequence of the fact that tightness and search in the formal sector increases whereas the opposite movements take place in the informal sector. The negative impact on the value of being a manual worker,  $rU_m$ , dominates. Hence more workers will educate themselves as increased punishment of the informal sector makes it less attractive to remain a manual worker.

Equation (35) gives the impact on the number of educated workers as the tax rates increases, for a given punishment system. The *first* term captures the effect on the educated labour force of an increase in the tax rates working through  $\psi$ . Increased taxation induces the opposite movements as was described in connection to equation (34) and analogous reasoning can be conducted. The *second* term in (35) captures the impact of increased taxation for a given wedge,  $\psi$ . Increased taxation reduces the consumer wages for both educated and manual workers, but it will not affect the costs of higher education. Consequently, the value of being an educated worker falls by relative more than the value of being a manual worker; the number of educated workers falls. As the effects work in the same direction, we have that the number of educated workers in the economy tends to decrease with increased taxation.

In order to consider fully financed increases in the punishment rates, we need to consider the impact of the tax and punishment rates on the government revenues. The tax and punishment rates affect the government revenues in a number of ways (details are given in an appendix). Considering that we are located on the positively sloped side of the Laffer curves, and hence dynamic adjustments in equilibrium wages, employment rates and labour forces are not dominating the direct effects, government revenue increases with increased tax rates and punishment rates.<sup>19</sup> An increase in the punishment

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<sup>19</sup>Or we could equally well allow for the case where we are located on the negatively sloped side of the Laffer curve, and hence the direct effects are dominated by the dynamic effects. See appendix for details.

rate accordingly calls for reductions in the tax rates in order to maintain a balanced budget. We can then rewrite the government budget restriction in (33) as  $\phi^t = h(\phi^p)$ , where  $\frac{\partial \phi^t}{\partial \phi^p} < 0$ . We have the following result.

**Proposition 5** *A fully financed increase in the punishment rates,  $\delta$  or  $\alpha$ , will increase the number of educated workers.*

### 3.5 Unemployment

This section is concerned with how the number of unemployed workers is affected by changes in the tax and punishment systems. As is clear from section 2.4, the total number of unemployed workers, as well as the number of unemployed workers observable by the government, depends on the division of labour across sectors. Since the division of labour across sectors depends on the tax rates,  $\phi^t$ , separate from the wedge,  $\psi$ , we have to account for the government budget restriction explicitly. Again we assume that we are located on the positively sloped side of the Laffer curves.<sup>20</sup>

**Proposition 6** *A fully financed increase in the punishment rates,  $\delta$  or  $\alpha$ , will reduce total unemployment,  $\frac{\partial U_{TOT}}{\partial \phi^p} < 0$  if  $\psi < 1$  and the rate of unemployed manual workers is higher than the rate of unemployment of higher educated workers,  $u_m > u_h$ .*

In the empirically relevant case where unemployment for manual workers is higher than unemployment of educated workers, higher punishment rates reduce total unemployment. There is both a direct impact from a reduction in the unemployment rate for manual workers and an indirect impact through the shift of workers from the manual labour force towards the educated labour force.

## 4 Welfare

This section is concerned with welfare analysis and optimal design of tax and punishment policies. As we showed in Section 3.4, a reduction in the attractiveness of the informal sector, makes it relatively less attractive to be a manual worker. This induces more workers to educate themselves. We

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<sup>20</sup>see previous footnote.

concluded that increasing the wedge in-between the informal and the formal sector increases the number of educated workers by reducing the relative attractiveness to be a manual worker. Consequently, the size of the wedge is an important determinant for the size of the educated labour force.

This is essential when considering welfare. If economic policy is conducted so as to fight the underground economy, increasing the punishment rates will at the same time increase the number of educated workers. In case the educated labour force is inefficiently low, this provides an additional argument for fighting the informal sector. If, on the other hand, the educated labour force is inefficiently high, combatting the informal sector induces a welfare cost in the sense that the educated labour force becomes even higher.

This suggests that punishment rates are potential instruments for increasing the number of educated workers and thereby welfare. There may, of course, be others more direct instruments if the pure aim is to correct for inefficiencies in the educational level in an economy, but nevertheless, it should be acknowledged that the wedge actually has an impact on the number of educated workers and thereby potentially has an impact on welfare in the economy. This becomes clear if we take a closer look at the welfare effects of the model.

We make use of a utilitarian welfare function, which is obtained by adding all individuals' and firms' steady state flow values of welfare. In order to disregard from congestion externalities we assume that  $\gamma = \eta$ . The social welfare function is written as:

$$W = \hat{e}\tilde{W}_m + \int_{\hat{e}}^1 \tilde{W}_h de,$$

where

$$\begin{aligned}\tilde{W}_m &= u_m r U_m + n_m^F r E_m^F + n_m^I r E_m^I + n_m^F r J_m^F + n_m^I r J_m^I + v_m^F r V_m^F + v_m^I r V_m^I, \\ \tilde{W}_h &= u_h r U_h + (1 - u_h) r E_h + (1 - u_h) r J_h + v_h r V_h.\end{aligned}$$

We assume that firms are owned by "rentiers" who do not work.<sup>21</sup> By making use of the asset equations for workers and firms in the three sectors, imposing the flow equilibrium conditions as well as the government budget

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<sup>21</sup>Having workers owning equal shares across all existing firms, result in identical equilibrium outcomes as well as an identical definition of the welfare function in (36). This can be verified by adding dividends to the worker's asset functions.

restriction in (33), and considering the case of no discounting, i.e.,  $r \rightarrow 0$ , we can write the welfare function as:

$$W = W_m \hat{e} + \int_{\hat{e}}^1 W_h de - \xi(p), \quad (36)$$

where

$$W_m = (1 - u_m) y_m - u_m \Theta k, \quad (37)$$

$$W_h = (1 - u_h) y_h - u_h \theta_h k - c(e), \quad (38)$$

where  $\Theta = (\theta_m^I (\sigma)^\beta + \theta_m^F (1 - \sigma)^\beta)$ .<sup>22</sup> With the assumption of risk neutral individuals, we ignore distributional issues and hence wages will not feature in the welfare function.

Examining the welfare measure in (36) reveals that we have to consider both how the tax and punishment system affects the welfare of the manual and highly educated sector,  $W_m$  and  $W_h$ , but also how it affects the stock of manual and highly educated workers,  $1 - \hat{e}$ . We see from the welfare specification in (36), (37), and (38) that changes in the punishment rates,  $\delta$  or  $\alpha$ , only affects welfare through its affect on the wedge,  $\psi$ . The wedge, in turn, affects total welfare,  $W$ , through its impact on welfare for manual workers,  $W_m$ , and through its impact on the number workers acquiring education,  $1 - \hat{e}$ . As is clear from (38),  $W_h$  is independent of the tax and punishment rates. Changes in the tax rates,  $t$  and  $z$ , on the other hand, affects  $W$  through the wedge,  $\psi$ , but it will also have a direct effect on the number of workers acquiring education.

Let us first consider how a change in the number of workers acquiring education affects the social welfare measure. We can conclude the following:

$$\frac{\partial W}{\partial (1 - \hat{e})} = W_h(\hat{e}) - W_m = (1 - u_h) \omega_h - c(\hat{e}) - n_m^F \omega_m^F - n_m^I \omega_m^I. \quad (39)$$

Welfare raises when more workers acquire education whenever the number of educated workers are too low from a welfare point of view. Similarly,

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<sup>22</sup>This welfare measure is analogous to the welfare measure described in, for example, Pissarides (2000) as it gives the aggregate production minus total vacancy costs, i.e. we note that  $u_m \Theta k = (v^F + v^I) k$ . The educational cost is of course added as a cost to the welfare measure.

welfare falls as more workers acquire education when too many workers are educated from a welfare perspective.<sup>23</sup>

By looking at the market solution for the stock of educated workers, it is clear that it is exclusively the tax and punishment system that distorts educational choice. The equation for educational choice (32) can be written as  $\frac{1}{\phi^t} \left( (1 - u_h) \omega^h - n^F \omega^F - \frac{1}{\psi} n^I \omega^I \right) = c(\hat{e})$  by using the expressions for  $rU_m$  and  $rU_h$  from sections 2.1 and 2.2. We note that there are two sources through which the tax and punishment system distort the educational choice, namely through the punishment tax wedge,  $\psi$ , and through the tax rates,  $\phi^t$ .

The *first* distortion on educational choice appears when  $\psi$  departs from unity. When  $\psi < 1$ , being a manual worker is rather pleasant as then attractive employment opportunities in an informal sector are available. This reduces the incentives of education, which tends to imply that too few workers educate themselves. For  $\psi > 1$ , on the other hand, employment opportunities in an informal sector are not very attractive as then the expected punishment rates are higher than the tax rates. This increases the incentives of education, tending to make too many people educate themselves. The *second* distortion on educational choice follows as tax rates are positive,  $\phi^t > 1$ . Taxation hits educated workers more severely than manual workers, which reduces the incentives of education. Hence, positive tax rates distort the educational choice such that too few workers tend to educate themselves.

From this we can conclude that welfare always increases when more workers educate themselves when  $\psi \leq 1$ . When  $\psi$  is smaller than unity there is an inefficiently low stock of educated workers both because positive tax rates hit educated workers more severely and because manual workers face attractive informal sector employment opportunities. When  $\psi = 1$ , the fact that tax rates are positive, implies that it is still welfare improving that more workers educate themselves. When  $\psi > 1$ , the punishment of the informal sector works as an instrument to counteract the distortion created by the tax rates. Further increases in  $\psi$  above unity eventually fully counteracts the distortion of the tax system, making the market solution stock of educated workers being socially optimal.

Let us now consider the optimal design of tax and punishment systems. Since welfare depends on the division of labour across sectors, and the labour division depends on the tax rates separate from the wedge, we choose the

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<sup>23</sup>This clearly follows by definition as  $W$  is concave in  $(1 - \hat{e})$ , and reaches its maximum when  $(1 - u_h) \omega^h - c(\hat{e}) - n^F \omega^F - n^I \omega^I = 0$ .

punishment rates to maximize welfare accounting for that the tax rates adjust to balance the government budget restriction. We use (33) to write the tax rates as a function of the punishment rates as  $\phi^t = h(\phi^p)$ , where  $\frac{\partial \phi^t}{\partial \phi^p} < 0$ .<sup>24</sup>

Maximizing  $W$  in equation (36), (37), and (38) with respect to  $\phi^p$  enables us to write the first order condition as:

$$\frac{\partial W}{\partial \phi^p} = \frac{\partial W}{\partial (1 - \hat{e})} \left( \frac{\partial (1 - \hat{e})}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial (1 - \hat{e})}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p} \right) + \frac{\partial W_m}{\partial \psi} \left( \frac{\partial \psi}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial \psi}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p} \right) = 0 \quad (40)$$

We can conclude that:

**Proposition 7** *Welfare is maximized when the punishment rate is chosen such that the informal sector is punished to a larger extent than the formal sector is taxed, i.e.,  $\psi^* > 1$ .*

The Proof is given in the Appendix. In case the government has a positive revenue requirement,  $R > 0$ , tax rates has to be positive, and the optimal wedge is larger than unity.<sup>25</sup>

The intuition for the result in proposition 7 follows from that the educational stock is inefficiently low in case  $\psi \leq 1$ . The first term in (40) captures how welfare is affected when more workers acquire education. Recall from Section 3.4 that the number of educated workers increases with increased punishment of the informal sector. Hence as welfare always increases with additional punishment as long as  $\psi \leq 1$ , it is optimal to increase the punishment rates above unity to create incentives for education. However, the second term in (40) becomes negative when  $\psi$  exceeds unity. This term captures how the value of being a manual worker is affected by  $\psi$ . When  $\psi$

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<sup>24</sup>The tax and punishment rates affect the government revenues in a number of ways (details are given in an appendix). Considering that we are located on the positively sloped side of the Laffer curves, and hence dynamic adjustments in equilibrium wages, employment rates and labour forces are not dominating the direct effects, government revenue increases with increased tax rates and punishment rates. An increase in the punishment rate accordingly calls for reductions in the tax rates in order to maintain a balanced budget. Or we could equally well allow for the case where we are located on the negatively sloped side of the Laffer curve, and hence the direct effects are dominated by the dynamic effects. See appendix for details.

<sup>25</sup>A zero tax rate also implies that there are no informal sector that can be punished. Hence it is not possible to obtain the revenue requirements,  $R$  unless  $\phi^t > 1$ .



deviates from unity there is a distortion between the informal and the formal sector. Regarding the second term, if  $\psi < 1$ , too much work from a welfare point of view is carried out in the informal sector, and vice versa when  $\psi > 1$ .<sup>26</sup> Hence, when  $\psi < 1$ , welfare improves when increasing  $\psi$  both because it increases the stock of educated workers and because it reduces the distortion between the informal and formal sector. However, when  $\psi > 1$ , additional punishment of the informal sector brings about a negative effect on welfare as it increases the distortion between the formal and the informal sector. Eventually, further increases in the punishment rates will induce welfare to fall.

From proposition (7), we can conclude that:

**Corollary 8** *The number of educated workers are below what is socially optimal when the punishment rate is chosen optimally, i.e., when  $\psi = \psi^*$ .*

This follows from the first order condition determining the optimal punishment rate (40) and equation (39), where the socially optimal number of educated workers are determined by  $\frac{\partial W}{\partial(1-\hat{e})} = 0$ . From (40) we observe that the welfare maximizing punishment rate is set so that  $\frac{\partial W}{\partial(1-\hat{e})} > 0$ , fulfilling the first order condition as the second term is negative for  $\psi > 1$ , which implies that the educated stock is below what is socially optimal. This intuitively follows as it is not optimal to set the punishment so to fully correct for the inefficiently low stock of educated workers, as such a choice implies a distortion between the formal and the informal sector. As it is a trade-off between the two distortions, it is never optimal to fully eliminate one of the them.

The results that the punishment rates should be set such that the informal sector is punished to a larger extent than the formal sector is taxed, i.e.,  $\psi > 1$ , should clearly not be taken literally. Merely it points at that it is potentially important for welfare to acknowledge that employment opportunities in an informal sector may reduce the incentive to acquire education.

## 5 Conclusion

This paper has examined the consequences of the fact that work opportunities in the informal sector mainly face workers at the lower end of the skill

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<sup>26</sup>This is further made clear by noting that total vacancies per unemployed workers are maximized, and the manual unemployment rate, is minimized when  $\psi = 1$ .

distribution. When considering the individual choice of higher education, we usually think that the wage premium of education and the relative employment probabilities are the important factors to weight against the cost of education. However, acknowledging that mainly manual or service sector workers face work opportunities in the informal sector, these workers may have much better employment perspectives than what is revealed by just considering formal sector wages and unemployment rates. Hence, informal sector employment opportunities may reduce the incentives to acquire higher education and thereby reduce the educated labour force.

This paper developed a three-sector general equilibrium model featuring matching frictions and worker-firm wage bargains. Workers differed with respect to ability, and the choice of education was endogenously determined. Job opportunities in an informal sector were only available to manual workers. We asked if increased punishment of the informal sector and/or reduced taxation induced more workers to educate themselves. The answer to that question was yes, and the story just told provides the intuition behind this result.

The second aim of this paper was to study how labour market performance, and in particular unemployment, were affected by increased punishment of the informal sector and/or reduced taxation. Although some recent studies have shed light on how punishment policies affect unemployment, this paper uses a framework which enabled us to study the impact of tax and punishment policies when workers are searching in both a formal and an informal sector.

Finally, the paper characterized the optimal tax and punishment system. We showed that it was optimal to punish the informal sector to a larger extent than the formal sector is taxed. The results show that it potentially is important to take into account the impact of the employment opportunities in the informal sector on education, as it in turn, may have consequences for welfare. Punishing the informal sector less severely may reduce the educated labour force and welfare.

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## 6 Appendix

### 6.1 Equilibrium

We will draw the equilibrium for the manual sector in a diagram with search intensity,  $\sigma$ , on the first axis and relative tightness on the second axis,  $\frac{\theta^F}{\theta^I}$ . We draw two curves, one representing the two first equilibrium equations, (28) and (29), denoted  $\frac{\theta^F}{\theta^I}(\sigma)$ , that is we have relative labour market tightness as a function of search intensity,  $\sigma$ . The other curve represents equation (30), where we have search intensity as a function of relative tightness,  $\sigma\left(\frac{\theta^F}{\theta^I}\right)$ .

Consider the case when  $\eta = \frac{1}{2}$ . In this case we can solve explicitly for labour market tightness in the two sectors, and then deriving  $\frac{\theta^F}{\theta^I}(\sigma)$  we can show that the slope is negative,  $\frac{\partial \frac{\theta^F}{\theta^I}}{\partial \sigma} \Big|_{\frac{\theta^F}{\theta^I}(\sigma)} < 0$ . That is, a higher search intensity into the informal sector reduces relative labour market tightness. Furthermore, the curve is independent of the wedge,  $\psi$ . The curve representing equation (30),  $\sigma\left(\frac{\theta^F}{\theta^I}\right)$ , also has a negative slope,  $\frac{\partial \sigma}{\partial \left(\frac{\theta^F}{\theta^I}\right)} \Big|_{\sigma\left(\frac{\theta^F}{\theta^I}\right)} < 0$ , corresponding to that search intensity in the informal sector decreases with relative labour market tightness as job opportunities becomes relatively better in the formal sector. An increase in the wedge shifts the curve,  $\sigma\left(\frac{\theta^F}{\theta^I}\right)$ , inwards. Considering  $\frac{\partial \sigma}{\partial \left(\frac{\theta^F}{\theta^I}\right)} \Big|_{\sigma\left(\frac{\theta^F}{\theta^I}\right)}$  and  $\frac{\partial \frac{\theta^F}{\theta^I}}{\partial \sigma} \Big|_{\frac{\theta^F}{\theta^I}(\sigma)}$  we can show that the slope  $\sigma\left(\frac{\theta^F}{\theta^I}\right)$  is steeper than the slope of  $\frac{\theta^F}{\theta^I}(\sigma)$ . Given that an equilibrium exists it will therefore be unique and stable. See the next section for intuition for stability. We illustrate this in Figure 1 for  $\psi < 1$ .

—Figure 1 about here—

The steeper curve represents  $\sigma\left(\frac{\theta^F}{\theta^I}\right)$  and  $\frac{\theta^F}{\theta^I}(\sigma)$  is the flatter curve.

## 6.2 The impact from an increase in the wedge

When the wedge increases, the search intensity curve,  $\sigma\left(\frac{\theta^F}{\theta^I}\right)$ , that is, the steeper curve, shifts inwards to  $\sigma'\left(\frac{\theta^F}{\theta^I}\right)$ . The relative labour market tightness,  $\frac{\theta^F}{\theta^I}(\sigma)$ , curve is not affected. The new equilibrium is where the new search intensity curve,  $\sigma'\left(\frac{\theta^F}{\theta^I}\right)$ , and the relative labour market tightness curve,  $\frac{\theta^F}{\theta^I}(\sigma)$ , intersect. See Figure 2.

—Figure 2 about here—

When the punishment rates increase relative to the tax rates,  $\psi$  increases, search intensity falls for given relative labour market tightness,  $\frac{\theta^F}{\theta^I}$ . We move from the initial equilibrium horizontally to the left till the new search intensity curve is reached. A lower search intensity increases relative tightness as it induces a reduction in formal producer wages and an increase in informal producer wages. In the diagram, this corresponds to a vertical movement until we reach the relative labour market tightness curve. This, in turn, causes another reduction in search intensity, however the impact is now smaller than before. And again, the decrease in  $\sigma$  implies that relative tightness increases. The impacts become smaller and smaller until the new equilibrium is reached.

## 6.3 Proofs of Propositions and Lemmas

### Proof of proposition 1:

**Proof.** Differentiating equations (28), (29) and (30) with respect to  $\psi$  gives:

$$\begin{aligned}\frac{\partial \theta_m^F}{\partial \psi} &= \frac{(1-\beta) \gamma k \theta_m^F \left( \eta k (r+s+p) (\theta_m^I)^{\eta-1} + \gamma k \sigma^{\beta-1} \right)}{D_1 (1-\sigma)^{2-\beta}} > 0, \\ \frac{\partial \theta_m^I}{\partial \psi} &= - \frac{(1-\beta) \gamma k \theta_m^I \left( \eta k (r+s) (\theta_m^F)^{\eta-1} + \gamma k (1-\sigma)^{\beta-1} \right)}{D_1 \sigma^{2-\beta}} < 0, \\ \frac{\partial \sigma}{\partial \psi} &= - \frac{\left( \eta k \frac{r+s+p}{(\theta_m^I)^{1-\eta}} + \gamma k \sigma^{\beta-1} \right) \left( \eta k \frac{r+s}{(\theta_m^F)^{1-\eta}} + \gamma k (1-\sigma)^{\beta-1} \right)}{D_1} < 0,\end{aligned}$$

where  $D_1 = k^2 (1 - \beta) \frac{\theta_m^I}{\theta_m^F} \eta \frac{\frac{(r+s)}{(\theta_m^F)^{1-\eta}} \gamma \sigma^\beta + \frac{r+s+p}{(\theta_m^I)^{1-\eta}} \left( (1-\sigma)^\beta + \frac{\eta(r+s)}{(\theta_m^F)^{1-\eta}} \right)}{\sigma^{2-\beta} (1-\sigma)^\beta} > 0$ . From equation (31) it is apparent that tightness in sector  $h$  is not affected by changes in the tax or punishment rates. A closer look at (33) reveals that the government can choose  $\phi^t$  and  $\phi^p = \psi \phi^t$  for a given  $\psi$  so to increase or reduce revenues in order to balance its budget as long as  $R$  is a feasible revenue requirement. We do not need not have to assume that we are located on the positively sloped side of the Laffer curves. ■

**Proof of Proposition 2:**

**Proof.** Differentiate (13) and (14) with respect to  $\psi$ , considering that  $\theta_m^F, \theta_m^I$ , and  $\sigma$  is affected according to (28), (29), and (30) gives:

$$\begin{aligned} \frac{\partial \omega_m^F}{\partial \psi} &= (1 - \sigma)^{\beta-1} \gamma k \left( \frac{\partial \theta_m^F}{\partial \psi} - \frac{(\beta - 1) \theta_m^F}{1 - \sigma} \frac{\partial \sigma}{\partial \psi} \right), \\ \frac{\partial \omega_m^I}{\partial \psi} &= \gamma k \sigma^{\beta-1} \left( \frac{\partial \theta_m^I}{\partial \psi} + (\beta - 1) \frac{\theta_m^I}{\sigma} \frac{\partial \sigma}{\partial \psi} \right), \end{aligned}$$

which can be rewritten making use of the proof to proposition 1 as

$$\begin{aligned} \frac{\partial \omega_m^F}{\partial \psi} &= - \frac{\gamma k^2 (1 - \beta)}{D_1 (1 - \sigma)^{2-\beta}} \left( \frac{\eta (r + s + p)}{(\theta_m^I)^{1-\eta}} + \gamma \sigma^{\beta-1} \right) \eta (r + s) (\theta_m^F)^\eta < 0, \\ \frac{\partial \omega_m^I}{\partial \psi} &= \frac{\gamma k^2 (1 - \beta)}{D_1 \sigma^{2-\beta}} \left( \frac{\eta (r + s)}{(\theta_m^F)^{1-\eta}} + \gamma (1 - \sigma)^{\beta-1} \right) \eta (r + s + p) (\theta_m^I)^\eta > 0. \end{aligned}$$

From (21) and (22) we observe that wages in sector  $h$  is not affected by changes in the tax and punishment rates. The reform is fully financed, see proof of proposition (1). ■

**Proof of Proposition 3:**

**Proof.** Differentiating the employment equation, (25), with respect to  $\psi$  gives:

$$\frac{\partial n_m^I}{\partial \psi} = \frac{\frac{\partial \lambda_m^I}{\partial \psi} (s + \lambda_m^F) - \lambda_m^I \frac{\partial \lambda_m^F}{\partial \psi}}{s (s + p) \left( 1 + \frac{\lambda_m^I}{s+p} + \frac{\lambda_m^F}{s} \right)^2} < 0, \quad \frac{\partial n_m^F}{\partial \psi} = \frac{\frac{\partial \lambda_m^F}{\partial \psi} (s + p + \lambda_m^I) - \lambda_m^F \frac{\partial \lambda_m^I}{\partial \psi}}{s (s + p) \left( 1 + \frac{\lambda_m^I}{s+p} + \frac{\lambda_m^F}{s} \right)^2} > 0,$$

as the transition rate into the formal manual sector unambiguously increases and the transition rate into the informal sector decreases. Furthermore, the

actual unemployment rate and the official unemployment rate are affected in the following way:

$$\begin{aligned}
\frac{\partial u_m}{\partial \psi} &= - \frac{\frac{\sigma\beta}{s+p} (\theta_m^I)^{1-\eta} \left( \frac{(1-\eta)}{\theta_m^I} \frac{\partial \theta_m^I}{\partial \psi} + \frac{\beta}{\sigma} \frac{\partial \sigma}{\partial \psi} \right) + \frac{(1-\sigma)^\beta}{s} (\theta_m^F)^{1-\eta} \left( \frac{(1-\eta)}{\theta_m^F} \frac{\partial \theta_m^F}{\partial \psi} - \frac{\beta}{1-\sigma} \frac{\partial \sigma}{\partial \psi} \right)}{\left( 1 + \frac{\lambda_m^I}{s+p} + \frac{\lambda_m^F}{s} \right)^2} \\
&= -u_m^2 \frac{(1-\sigma)^{\beta-1}}{s (\theta_m^F)^\eta} \left( -\beta \frac{\partial \sigma}{\partial \psi} \theta_m^F \left( 1 - \frac{\psi}{\rho} \right) + \frac{(1-\eta)(1-\beta)\gamma k}{D_1 \sigma^{1-\beta}} \Lambda \right) < 0 \text{ if } \psi < 1, \\
\frac{\partial u_m^o}{\partial \psi} &= -\frac{\partial n_m^F}{\partial \psi} < 0,
\end{aligned} \tag{41}$$

where  $\rho = \frac{s+p}{s} \left( \frac{\theta_m^I}{\theta_m^F} \right)^\eta$  and  $\Lambda = \eta \left( \frac{1}{\psi} - \frac{\psi}{\rho} \right) (1-\gamma) y_m + \left( 1 - \frac{\psi}{\rho} \right) (1-\eta) \frac{\gamma k \theta_m^F}{(1-\sigma)^{1-\beta}}$ .

For  $r = 0$  we have that  $\rho = \frac{(1-\gamma)y_m - \psi \frac{\gamma k \theta_m^F}{(1-\sigma)^{1-\beta}}}{(1-\gamma)y_m - \frac{\gamma k \theta_m^F}{(1-\sigma)^{1-\beta}}} \geq 1$  for  $\psi \leq 1$  using the labour

market tightness equations. Now, for  $r > 0$ ,  $\frac{(1-\gamma)y_m - \psi \frac{\gamma k \theta_m^F}{(1-\sigma)^{1-\beta}}}{(1-\gamma)y_m - \frac{\gamma k \theta_m^F}{(1-\sigma)^{1-\beta}}} < \rho$ , hence

for  $\psi \leq 1$  is  $\frac{\psi}{\rho} < 1$ . The unemployment rate in sector  $h$  is unaffected by changes in the tax and punishment system as can be seen from (27), (22) and  $\lambda_h = (\theta_h)^{1-\eta}$ . The reform is fully financed, see proof of proposition 1. ■

#### **Proof of Proposition 4:**

**Proof.** Wages in sector  $h$  are unaffected by the change in  $\psi$  and hence wage dispersion  $WD$  increases as  $\frac{\partial \omega_m^F}{\partial \psi} < 0$  from the proof of proposition 2. ■

#### **Proof of Proposition 5:**

**Proof.** The impact on the number of educated workers of a fully financed change in the punishment rates,  $\phi^p$ , is given by

$$\frac{\partial (1-\hat{e})}{\partial \phi^p} = \frac{\partial (1-\hat{e})}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial (1-\hat{e})}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p}. \tag{42}$$

Using equation (34), (35) and  $\frac{\partial \phi^t}{\partial \phi^p} < 0$ , we obtain that  $\frac{\partial (1-\hat{e})}{\partial \phi^p} > 0$ . ■

#### **Proof of Proposition 6:**

**Proof.** The impact on the total number of unemployed workers of a fully financed increase in  $\phi^p$  is given by

$$\frac{\partial U_{TOT}}{\partial \phi^p} = \frac{\partial \hat{e}}{\partial \phi^p} (u_m - u_h) + \hat{e} \frac{\partial u_m}{\partial \psi} \left( \frac{\partial \psi}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial \psi}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p} \right), \tag{43}$$

which is smaller than zero for  $u_m > u_h$  as  $\frac{\partial \hat{e}}{\partial \phi^p} < 0$  from (42) and  $\frac{\partial u_m}{\partial \psi} < 0$  if  $\psi < 1$  and  $\frac{\partial \phi^t}{\partial \phi^p} < 0$ . The total number of officially unemployed workers is affected in the following way

$$\frac{\partial U_{TOT}^o}{\partial \phi^p} = \frac{\partial \hat{e}}{\partial \phi^p} (u_m^o - u_h) + \hat{e} \frac{\partial u_m^o}{\partial \psi} \frac{\partial \psi}{\partial \phi^p}, \quad (44)$$

which has negative sign as  $u_m^o > u_m$  and  $\frac{\partial u_m^o}{\partial \psi} < 0$ . ■

**Proof of Proposition 7:**

**Proof.** The first order condition is given by:

$$\frac{\partial W}{\partial \phi^p} = \frac{\partial W}{\partial (1 - \hat{e})} \left( \frac{\partial (1 - \hat{e})}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial (1 - \hat{e})}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p} \right) + \frac{\partial W_m}{\partial \psi} \left( \frac{\partial \psi}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial \psi}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p} \right) = 0. \quad (45)$$

Consider the first term, where

$$\frac{\partial W}{\partial (1 - \hat{e})} = (1 - u_h) \omega_h - c(\hat{e}) - n_m^F \omega_m^F - n_m^I \omega_m^I,$$

and  $\left( \frac{\partial (1 - \hat{e})}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial (1 - \hat{e})}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p} \right) > 0$ . Comparing to the equation for educational choice (32), which can be written as  $\frac{1}{\phi^t} \left( (1 - u_h) \omega^h - n^F \omega^F - \frac{1}{\psi} n^I \omega^I \right) - c(\hat{e}) = 0$ , we infer that  $\frac{\partial W}{\partial (1 - \hat{e})} > 0$  for  $\psi < \tilde{\psi}$ ,  $\frac{\partial W}{\partial (1 - \hat{e})} = 0$  for  $\psi = \tilde{\psi}$ ,  $\frac{\partial W}{\partial (1 - \hat{e})} < 0$  for  $\psi > \tilde{\psi}$  where  $\tilde{\psi} > 1$  is the wedge that implies the socially optimal educational stock. Hence, considering the first term, welfare increases in  $\phi^p$  until  $\psi = \tilde{\psi}$  and thereafter it decreases with  $\phi^p$ .

Consider the last term where

$$\frac{\partial W_m}{\partial \psi} = -\frac{\partial u_m}{\partial \psi} A - u_m B k, \quad (46)$$

where  $A = (y_m + \Theta k)$ ,  $B = \left( \frac{\partial \theta_m^I}{\partial \psi} \sigma^\beta + \frac{\partial \theta_m^F}{\partial \psi} (1 - \sigma)^\beta + \beta \frac{\partial \theta_m^I}{\partial \sigma^{1-\beta}} \left( 1 - \frac{1}{\psi} \right) \frac{\partial \sigma}{\partial \psi} \right)$  and  $\left( \frac{\partial \psi}{\partial \phi^p} \Big|_{\phi^t} + \frac{\partial \psi}{\partial \phi^t} \Big|_{\phi^p} \frac{\partial \phi^t}{\partial \phi^p} \right) > 0$ . We can show that  $\frac{\partial W_m}{\partial \psi}$  is concave in  $\psi$ . From the proofs of proposition 1 and proposition 3 and using equation (30) we can show that the term is positive for  $\psi < 1$ , zero for  $\psi = 1$ , and negative for  $\psi > 1$ . Recall here that  $s (\theta_m^F)^\eta = (s + p) (\theta_m^I)^\eta$  if  $\psi = 1$  and  $r = 0$ .

Hence, welfare increases with  $\phi^p$  when  $\psi \leq 1$ , and falls with  $\phi^p$  when  $\psi \geq \tilde{\psi}$ . For  $1 < \psi < \tilde{\psi}$ , welfare most likely increases with  $\phi^p$  in order to



reach a maximum at  $\psi^*$  and then falls again. This most likely holds as  $\frac{\partial W}{\partial(1-\hat{e})}$  becomes smaller and smaller when  $\phi^p$  increases within this interval, and  $\frac{\partial W_m}{\partial\psi}$  becomes more and more negative within this interval. However, in the general case we can not rule out that  $\left(\frac{\partial(1-\hat{e})}{\partial\phi^p}|_{\phi^t} + \frac{\partial(1-\hat{e})}{\partial\phi^t}|_{\phi^p} \frac{\partial\phi^t}{\partial\phi^p}\right)$  and  $\left(\frac{\partial\psi}{\partial\phi^p}|_{\phi^t} + \frac{\partial\psi}{\partial\phi^t}|_{\phi^p} \frac{\partial\phi^t}{\partial\phi^p}\right)$  take on values such that the sign of  $\frac{\partial W}{\partial\phi^p}$  takes on both positive and negative values in this interval, although it seems quite unlikely. Although we cannot rule out this case, it is of no importance for Proposition 7. This follows as the maximum is found within this interval, although one may have more than one stationary point. It is then just a matter of choosing the stationary point which yields the highest welfare. ■

## 6.4 Impact on government Revenue

The left hand side (LHS) of (33) is the government revenue, whereas the right hand side is the government revenue requirement. Differentiating the LHS of (33) with respect to  $\phi^p$  and  $\phi^t$  gives the following expressions:

$$\begin{aligned} \frac{\partial LHS}{\partial\phi^p}|_{\phi^t} &= \frac{\partial\hat{e}}{\partial\psi} \frac{\partial\psi}{\partial\phi^p} \left( n_m^F \omega_m^F \left(1 - \frac{1}{\phi^t}\right) + n_m^I \omega_m^I \left(1 - \frac{1}{\phi^p}\right) - n_h \omega_h \left(1 - \frac{1}{\phi^t}\right) \right) \\ &\quad + \hat{e} \frac{\partial\psi}{\partial\phi^p} \left( \frac{\partial n_m^F}{\partial\psi} \omega_m^F \left(1 - \frac{1}{\phi^t}\right) + \frac{\partial n_m^I}{\partial\psi} \omega_m^I \left(1 - \frac{1}{\phi^p}\right) \right) \\ &\quad + \hat{e} \frac{\partial\psi}{\partial\phi^p} \left( \frac{\partial \omega_m^F}{\partial\psi} n_m^F \left(1 - \frac{1}{\phi^t}\right) + \frac{\partial \omega_m^I}{\partial\psi} n_m^I \left(1 - \frac{1}{\phi^p}\right) \right) \\ &\quad + \hat{e} n_m^I \omega_m^I \frac{1}{(\phi^p)^2} \end{aligned}$$

$$\begin{aligned} \frac{\partial LHS}{\partial\phi^t}|_{\phi^p} &= \left( \frac{\partial\hat{e}}{\partial\psi} \frac{\partial\psi}{\partial\phi^t} + \frac{\partial\hat{e}}{\partial\phi^t}|_{\psi} \right) \left( n_m^F \omega_m^F \left(1 - \frac{1}{\phi^t}\right) + n_m^I \omega_m^I \left(1 - \frac{1}{\phi^p}\right) - n_h \omega_h \left(1 - \frac{1}{\phi^t}\right) \right) \\ &\quad + \hat{e} \frac{\partial\psi}{\partial\phi^t} \left( \frac{\partial n_m^F}{\partial\psi} \omega_m^F \left(1 - \frac{1}{\phi^t}\right) + \frac{\partial n_m^I}{\partial\psi} \omega_m^I \left(1 - \frac{1}{\phi^p}\right) \right) \\ &\quad + \hat{e} \frac{\partial\psi}{\partial\phi^t} \left( \frac{\partial \omega_m^F}{\partial\psi} n_m^F \left(1 - \frac{1}{\phi^t}\right) + \frac{\partial \omega_m^I}{\partial\psi} n_m^I \left(1 - \frac{1}{\phi^p}\right) \right) \\ &\quad + \hat{e} \frac{n_m^F \omega_m^F}{(\phi^t)^2} + (1 - \hat{e}) \frac{(1 - u_h) \omega_h^2}{(\phi^t)^2} \end{aligned}$$

We can divide the influences on the government revenue into four categories characterized by each row in the two equations. The *first* row in each equation captures how revenues are altered by the change in the number of educated workers in the economy. The *second* row in each equation gives the impact on revenues as employment in the formal and informal sector is altered. The *third* row in each equation captures how revenues are influenced by changes in the equilibrium producer wage facing manual workers. Finally, the *fourth* row gives the direct effect. We have assumed that the auditing costs are independent of  $\psi$ . However, it is straightforward to assume that is a function of, for example,  $\psi$ . This simply adds an additional effect on the government revenue.

Since the dynamic effects move in different directions it is difficult to determine whether they reinforce or weaken the direct effect. If we say that we are located on the upward sloping part of the Laffer curves, we assume that  $\frac{\partial LHS}{\partial \phi^p}|_{\phi^t} > 0$  and  $\frac{\partial LHS}{\partial \phi^t}|_{\phi^p} > 0$ . That is, the dynamic effects will never dominate the direct effect. Rewrite (33) as  $G(\phi^p, \phi^t) = \hat{e} \left( n_m^F \omega_m^F \left( 1 - \frac{1}{\phi^t} \right) + n_m^I \omega_m^I \left( 1 - \frac{1}{\phi^p} \right) \right) + (1 - \hat{e}) (1 - u_h) \omega_h \left( 1 - \frac{1}{\phi^t} \right) - \xi(p) - R = 0$ . Differentiating  $G(\phi^p, \phi^t)$  with respect to  $\phi^p$  and  $\phi^t$  yields  $\frac{\partial \phi^t}{\partial \phi^p} = -\frac{\frac{\partial G(\cdot)}{\partial \phi^p}}{\frac{\partial G(\cdot)}{\partial \phi^t}} = -\frac{\frac{\partial LHS}{\partial \phi^p}}{\frac{\partial LHS}{\partial \phi^t}} < 0$ , which is used in the proofs of propositions (5), (6), (10), (11) and Lemmas (8), (9).

If we instead assume that the direct effect is dominated by indirect negative effects, we assume that  $\frac{\partial LHS}{\partial \phi^p} < 0$  and  $\frac{\partial LHS}{\partial \phi^t} < 0$ . This is referred to as that we are located on the downward sloping side of the Laffer curves. This implies that  $\frac{\partial \phi^t}{\partial \phi^p} = -\frac{\frac{\partial G(\cdot)}{\partial \phi^p}}{\frac{\partial G(\cdot)}{\partial \phi^t}} = -\frac{\frac{\partial LHS}{\partial \phi^p}}{\frac{\partial LHS}{\partial \phi^t}} < 0$ . Hence propositions (5), (6),

(10), (11) and Lemmas (8), (9) holds also in this case. Propositions (5), (6), (10), (11) and Lemmas (8), (9), however, do not hold if we have  $sign \frac{\partial LHS}{\partial \phi^p}|_{\phi^t} \neq sign \frac{\partial LHS}{\partial \phi^t}|_{\phi^p}$ .