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## Redistribution and Provision of Public Goods in an Economic Federation

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*Department of Economics*

# Redistribution and Provision of Public Goods in an Economic Federation

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REDISTRIBUTION AND PROVISION OF PUBLIC GOODS  
IN AN ECONOMIC FEDERATION

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# **Redistribution and Provision of Public Goods in an Economic Federation<sup>\*</sup>**

Thomas Aronsson<sup>α</sup> and Sören Blomquist<sup>β</sup>

## **Abstract**

This paper concerns redistribution and provision of public goods in an economic federation with two levels of government: a local government in each locality and a central government for the economic federation as a whole. We assume that each locality is characterized by two ability-types (high and low), and that their distribution differs between localities. The set of policy instruments facing the central government consists of a nonlinear income tax and a lump-sum transfer to each local government, while the local governments use proportional income taxes and the transfers from the central government to finance the provision of local public goods. The purpose is to characterize the tax and expenditure structure in a decentralized setting, where the central and local governments have distinct roles to play, and also compare this tax and expenditure structure with the second best resource allocation. We show how the redistributive role of taxation is combined with a corrective role, since tax base sharing among the central and local governments gives rise to a vertical fiscal external effect. In addition, the central government does not in general implement the second best resource allocation with the instruments at its disposal.

Keywords: Fiscal external effects, nonlinear taxation.

JEL codes: D60, H21, H23, H77.

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## 1. Introduction

Ever since the seminal article by Mirrlees (1971), there has been a steady development of our understanding of how redistribution via nonlinear income taxation can be obtained in an efficient way. Part of this literature also addresses how an efficient nonlinear income tax interacts with commodity taxes and public provision of public and private goods. Meanwhile, most previous studies have dealt with ‘unified’ economies, in which there is no distinction between different levels in the public sector. This is somewhat surprising considering that countries are often characterized by geographical localities and/or regions that are allowed to collect local taxes and provide local public services. The idea of extending the optimal income tax problem to an economic federation (with a distinction between central and local governments) is interesting from a theoretical point of view, since it opens up for the use of additional policy instruments in comparison with the traditional optimal income tax model. It is also interesting as a complement to previous studies on optimal public policies in economic federations, which typically use proportional tax instruments and/or disregard the possibility of asymmetric information. The purpose of this paper is to extend the theory of optimal nonlinear income taxation and provision of public goods to a framework, where part of the decisions in the public sector are made by local governments.

An important resource allocation problem that often characterizes economic federations is vertical fiscal external effects<sup>1</sup>, which arise from tax base sharing among different levels in the public sector. Typically, local governments neglect that increases in the local income tax rates reduce the tax base of the central government, implying a tendency to underestimate the marginal cost of public funds<sup>2</sup>. Therefore, to reach the socially optimal resource allocation within the given fiscal structure, it is necessary for the central government to try to influence the decisions made by the local governments. This idea was brought to attention by Hansson and Stuart (1987) and Johnson (1988). Several authors have addressed the policy options available to the central government,

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<sup>1</sup> Another important resource allocation problem is horizontal fiscal external effects, which are associated with direct interaction among different localities (e.g. via labor mobility and spillover effects of local public goods). The standard reference here is Oates (1972).

<sup>2</sup> Dahlby and Wilson (2003) extend the analysis to situations where the vertical fiscal external effect is not necessarily negative: their contribution is to study how an increase in the tax rates imposed by the lower level of government may actually increase the tax base of the federal government. The mechanisms

in case vertical fiscal external effects influence the resource allocation. Boadway and Keen (1996) assume that both the central and local governments use proportional income taxes, and that the central government can transfer resources lump-sum between the two levels in the public sector. They also assume that the localities are identical, and that each locality can be characterized by a representative agent<sup>3</sup>. Their results show that the central government can implement the second best resource allocation by choosing its own income tax rate to be equal to zero, i.e. only the local level of government collects tax revenues by means of distortionary taxes, whereas the central government collects resources lump-sum from the local governments in order to finance its own expenditures. The latter means, in turn, that the optimal fiscal gap is negative. Other studies have focused the attention on the potential role of transfer schemes as well as on other tax instruments. For instance, Aronsson and Wikström (2001, 2003) show that proportional income taxation at each level of government can, in certain situations, be combined with an intergovernmental transfer scheme designed to induce the correct incentives<sup>4</sup>. Similarly, in the context of commodity taxation, Dahlby (1996) proposes a matching arrangement in order to internalize a vertical fiscal external effect.

Following Boadway and Keen (1996) and Boadway et al. (1998), our paper addresses an economic federation where both levels of government use income taxes, and the central government is able to transfer resources lump-sum between the two levels of government. The main difference is that the central government, in our case, has access to a (general) nonlinear income tax and solves its optimization problem subject to self-selection constraints. We consider an extension of the two-type model developed by Stern (1982) and Stiglitz (1982), where the distribution of ability-types differs between localities. To be more specific, we assume that the central government uses a nonlinear income tax to redistribute income from high income earners to low income earners, whereas the local governments use proportional income taxes to finance the provision of local public goods. Each local government also receives a lump-sum transfer (positive or negative) from the central government. This setting is interpretable in

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emphasized by Dahlby and Wilson are the wage elasticity of the labor demand and whether or not public goods provided by the lower level of government affect the productivity.

<sup>3</sup> Boadway et al. (1998) extend the analysis by assuming that the agents in each locality differ in ability. In their framework, each level of government uses a proportional income tax in combination with a lump-sum transfer to the private sector, while the central government is also able to reallocate resources lump-sum between the two levels of government.

several different ways. One is in terms of a federal structure such as U.S., whereas another is that the local governments represent municipal or regional governments of the type characterizing the Nordic countries.

In comparison with earlier studies, our paper contributes to the literature in primarily two ways. First, by introducing asymmetric information and allowing the central government to use a nonlinear income tax, we are able to extend the self-selection approach to optimal taxation into a policy problem for an economic federation. In our case, the decision by the central government to use distortionary taxation will follow from the structure of the model and not by assumption. Our framework also recognizes how the use of inflexible policy instruments at the local level may restrict the policy options of the central government. Second, contrary to the previous studies based on the self-selection approach to optimal taxation that we are aware of, our paper addresses heterogeneity both within and between local jurisdictions.

The paper focuses on income redistribution, as well as on how the central government may modify its use of income taxation in order to correct for externalities associated with tax base sharing. To simplify the analysis as much as possible, we disregard horizontal interaction among the localities such as spillover effects of local public goods and labor mobility. In section 2, we describe the model. Section 3 analyzes the second best policy in a benchmark version of the model, where all policy decisions are made by the central government, whereas section 4 concerns the public policies in a decentralized setting where a distinction is made between the central and local governments. Section 5 summarizes the results.

## 2. The Model

Consider an economy with  $K$  localities. We adopt a two-type version of the optimal income taxation model, implying that each locality is characterized by high-ability individuals and low-ability individuals, and that the distribution of ability-types differs between localities. Individuals have identical preferences. This means that the utility

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<sup>4</sup> The first of these two papers considers a policy problem with more than two levels of government, whereas the second addresses vertical external effects and risk-sharing simultaneously.

function neither differs among ability-types nor among localities. The utility facing an individual of ability-type  $i$  in locality  $k$  is given by

$$U_k^i = U(C_k^i, l_k^i, g_k)$$

where  $C$  is private consumption,  $l$  hours of work and  $g$  a local public good. We assume that the function  $U(\cdot)$  is increasing in  $C$  and  $g$ , decreasing in  $l$  and strictly quasiconcave, as well as that all goods are normal. We also assume that the utility function is additively separable in the local public good. This assumption simplifies the analysis. It is also in line with several previous studies on optimal taxation and public expenditures in economic federations referred to in the introduction.

The productivity of each ability-type does not depend on location, meaning that  $w^1$  and  $w^2$  (where  $w^2 > w^1$ ) are the wage rates facing the two ability-types in all  $K$  localities. The gross income of each ability-type may, nevertheless, differ between localities, since the income tax and, therefore, the hours of work may differ. In this paper, we would like to distinguish between the tax parameters of the central and local governments in a simple way, and we follow Marceau and Boadway (1994) by writing the individual budget constraints in their virtual form by linearizing them at the equilibrium. Furthermore, since the distribution of ability-types differs between the localities, we do not want to restrict the central government to tax all localities according to the same tax schedule. Therefore, instead of assuming that all individuals face the same national income tax schedule independently of location, it follows that possible differences or similarities between the localities with respect to the national income tax is a result of optimization. The national tax system facing ability-type  $i$  in locality  $k$  is summarized by two parameters: the marginal income tax rate,  $\tau_k^i$ , and an intercept term,  $-T_k^i$ . This means that the budget constraint facing an individual of ability-type  $i$  in locality  $k$  can be written

$$w_k^i l_k^i (1 - \tau_k^i - t_k) - T_k^i = C_k^i$$



where  $t_k$  is the income tax rate decided upon by the local government. The consumers choose private consumption and hours of work to maximize utility subject to the budget constraint. By defining the hours chosen by ability-type  $i$  in locality  $k$  as follows;

$$l_k^i = l(w^i, \tau_k^i, T_k^i, t_k)$$

the indirect utility function can be written as

$$\begin{aligned} V_k^i &= V(w^i, \tau_k^i, T_k^i, t_k, g_k) \\ &= U(w^i l(w^i, \tau_k^i, T_k^i, t_k)(1 - \tau_k^i - t_k) - T_k^i, l(w^i, \tau_k^i, T_k^i, t_k), g_k) \end{aligned}$$

The properties of the indirect utility function are (applying the envelope theorem)

$$\frac{\partial V_k^i}{\partial \tau_k^i} = \frac{\partial V_k^i}{\partial t_k} = -\frac{\partial U_k^i}{\partial C_k^i} w^i l_k^i \quad (1)$$

$$\frac{\partial V_k^i}{\partial T_k^i} = -\frac{\partial U_k^i}{\partial C_k^i} \quad (2)$$

$$\frac{\partial V_k^i}{\partial g_k} = \frac{\partial U_k^i}{\partial g_k} \quad (3)$$

To simplify the notations, we assume that the number of inhabitants is the same in all localities and normalize the population in each locality to one. However, the proportions of high-ability and low-ability types differ across localities. We denote the proportion of low-ability types in locality  $k$  by  $\pi_k$  and the proportion of high-ability types by  $(1 - \pi_k)$ . By following the convention in much of the earlier literature on optimal nonlinear income taxation, we assume that the purpose of redistribution is to redistribute from high income earners to low income earners, implying that the most interesting aspect of self-selection will be to prevent the high-ability type from mimicking the low-ability type. The indirect utility function of the mimicker is written

$$\begin{aligned}
\hat{V}_k^2 &= \hat{V}^2(w^1, w^2, \tau_k^1, T_k^1, t_k, g_k) \\
&= U(w^1 l(w^1, \tau_k^1, T_k^1, t_k)(1 - \tau_k^1 - t_k) - T_k^1, \frac{w^1}{w^2} l(w^1, \tau_k^1, T_k^1, t_k), g_k) \\
&= \hat{U}_k^2
\end{aligned}$$

with properties

$$\frac{\partial \hat{V}_k^2}{\partial \tau_k^1} = \frac{\partial \hat{V}_k^2}{\partial t_k} = -\frac{\partial \hat{U}_k^2}{\partial C_k^1} w^1 l_k^1 + \left[ \frac{\partial \hat{U}_k^2}{\partial C_k^1} w^1 (1 - \tau_k^1 - t_k) + \frac{\partial \hat{U}_k^2}{\partial \hat{l}_k^2} \frac{w^1}{w^2} \right] \frac{\partial l_k^1}{\partial \tau_k^1} \quad (4)$$

$$\frac{\partial \hat{V}_k^2}{\partial T_k^1} = -\frac{\partial \hat{U}_k^2}{\partial C_k^1} + \left[ \frac{\partial \hat{U}_k^2}{\partial C_k^1} w^1 (1 - \tau_k^1 - t_k) + \frac{\partial \hat{U}_k^2}{\partial \hat{l}_k^2} \frac{w^1}{w^2} \right] \frac{\partial l_k^1}{\partial T_k^1} \quad (5)$$

$$\frac{\partial \hat{V}_k^2}{\partial g_k} = -\frac{\partial \hat{U}_k^2}{\partial g_k} \quad (6)$$

since  $\partial l_k^1 / \partial \tau_k^1 = \partial l_k^1 / \partial t_k$ , and where  $\hat{l}_k^2 = l_k^1 (w^1 / w^2)$ .

In sections 3 and 4 below, we consider two versions of the taxation-provision problem;

(i) **Second best.** This is basically a command optimum problem, where all decisions are made by the central government. The only informational constraint is that the government does not know whether a given individual is a high-ability or low-ability type. On the other hand, the government knows the proportions of high-ability and low-ability types in each locality. The policy instruments facing the government are the parameters of the income tax function as well as locality specific public goods.

(ii) **Decentralized solution.** This is intended to represent a federal structure with two levels of government. It is important to emphasize that the federal structure as such and the set of policy instruments will be taken as given in the analysis. Our concern is, instead, to study how the central government uses its policy instruments, when each local government is allowed to make independent decisions about taxation and expenditures. The policy instruments facing the central government are the parameters of the income tax function and lump-sum transfers to the local governments. Each local government provides a local public good, which is financed by a proportional income

tax and the transfer payment from the central government. The federal government will be assumed to act as a Stackelberg leader, whereas the local governments act as followers. This seems reasonable in an economy with many small localities, where the consequences for the central government of the actions of a single local government are small, whereas the decisions made by the central government are important for each local government.

### 3. Second Best; centralized solution with locality specific public expenditures

We assume that the central government faces a (generalized) Utilitarian social welfare function with different weights attached to the high-ability type and low-ability type, respectively. In addition, since all policies are decided upon by the central government, there is no need to use local income taxes and intergovernmental transfer payments. In terms of the model presented above, this means that the local income tax rates and the transfers from the central to the local governments are equal to zero. Accordingly, the second best model is formulated as if the central government chooses the levels of local public goods.

The optimal tax and expenditure problem is given by<sup>5</sup>;

$$\underset{\tau_k^1, T_k^1, \tau_k^2, T_k^2, g_k}{Max} \sum_k [\pi_k \alpha^1 V_k^1 + (1 - \pi_k) \alpha^2 V_k^2]$$

$$\text{s.t.} \quad V_k^2 \geq \hat{V}_k^2 \quad k = 1, \dots, K$$

$$\sum_k [\pi_k (\tau_k^1 w^1 l_k^1 + T_k^1) + (1 - \pi_k) (\tau_k^2 w^2 l_k^2 + T_k^2) - g_k] \geq 0$$

where  $V_k^1$ ,  $V_k^2$ ,  $\hat{V}_k^2$ ,  $l_k^1$  and  $l_k^2$  were defined above. The first set of restrictions above constitute self-selection constraints, implying that the high-ability type in each locality

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<sup>5</sup> An alternative formulation would be to assume that the government maximizes the utility of one of the ability-types subject to a minimum utility constraint for the other. We have chosen to use a social welfare function defined as the sum of the social welfare functions for the local governments (see below), which makes it easy to address the consequences of interaction between the two levels in the public sector. This assumption is also in accordance with several previous studies on public policy in economic federations.

is (weakly) better off by behaving as a high-ability type than by being a mimicker. Note also that, since the population in each locality is immobile, there is no need for other self-selection constraints than those referring to the incentives of the high-ability type to mimic the low-ability type in the same locality. The second restriction is the budget constraint of the government. Since the budget constraint is defined in terms of a sum of differences between the locality specific revenues and expenditures, it follows that the government is able to redistribute across the localities. As we mentioned above, another important feature of the optimization problem is that the distribution of ability-types differs between localities. Therefore, we do not want to restrict the government to impose the same tax schedule in all localities.

The Lagrangean becomes

$$L = \sum_k [\pi_k \alpha^1 V_k^1 + (1 - \pi_k) \alpha^2 V_k^2] + \sum_k \lambda_k [V_k^2 - \hat{V}_k^2] \\ + \gamma \sum_k [\pi_k (\tau_k^1 w^1 l_k^1 + T_k^1) + (1 - \pi_k) (\tau_k^2 w^2 l_k^2 + T_k^2) - g_k]$$

The first order conditions can be written as (for  $k = 1, \dots, K$ )

$$\frac{\partial L}{\partial \tau_k^1} = \pi_k \alpha^1 \frac{\partial V_k^1}{\partial \tau_k^1} - \lambda_k \frac{\partial \hat{V}_k^2}{\partial \tau_k^1} + \gamma \pi_k [w^1 l_k^1 + \tau_k^1 w^1 \frac{\partial l_k^1}{\partial \tau_k^1}] = 0 \quad (7)$$

$$\frac{\partial L}{\partial T_k^1} = \pi_k \alpha^1 \frac{\partial V_k^1}{\partial T_k^1} - \lambda_k \frac{\partial \hat{V}_k^2}{\partial T_k^1} + \gamma \pi_k [\tau_k^1 w^1 \frac{\partial l_k^1}{\partial T_k^1} + 1] = 0 \quad (8)$$

$$\frac{\partial L}{\partial \tau_k^2} = [(1 - \pi_k) \alpha^2 + \lambda_k] \frac{\partial V_k^2}{\partial \tau_k^2} + \gamma (1 - \pi_k) [w^2 l_k^2 + \tau_k^2 w^2 \frac{\partial l_k^2}{\partial \tau_k^2}] = 0 \quad (9)$$

$$\frac{\partial L}{\partial T_k^2} = [(1 - \pi_k) \alpha^2 + \lambda_k] \frac{\partial V_k^2}{\partial T_k^2} + \gamma (1 - \pi_k) [\tau_k^2 w^2 \frac{\partial l_k^2}{\partial T_k^2} + 1] = 0 \quad (10)$$

$$\frac{\partial L}{\partial g_k} = \pi_k \alpha^1 \frac{\partial V_k^1}{\partial g_k} + (1 - \pi_k) \alpha^2 \frac{\partial V_k^2}{\partial g_k} - \gamma = 0 \quad (11)$$

To derive the marginal income tax rate characterizing each ability-type in each locality, we use equations (1)-(10) together with the first order condition for the hours of work facing each individual and the Slutsky condition, i.e.

$$\begin{aligned} \frac{\partial U_k^i}{\partial C_k^i} w^i (1 - \tau_k^i) + \frac{\partial U_k^i}{\partial l_k^i} &= 0 \\ \frac{\partial l_k^i}{\partial \tau_k^i} &= - \left[ \frac{\partial \tilde{l}_k^i}{\partial \omega_k^i} - \frac{\partial l_k^1}{\partial T_k^i} l_k^i \right] w^i, \end{aligned}$$

where  $\tilde{l}_k^i$  is the compensated labor supply of ability-type  $i$  in locality  $k$  and  $\omega_k^i = w^i (1 - \tau_k^i)$ . Consider Proposition 1;

**Proposition 1:** *In a unified framework, where all policy decisions are made by the central government, the marginal income tax rates of the two ability-types are characterized by*

$$\begin{aligned} \tau_k^1 &= \frac{1}{\pi_k w^1} \lambda_k^* \left[ \frac{\partial \hat{U}_k^2 / \partial C_k^1}{\partial \hat{U}_k^2 / \partial \hat{l}_k^2} \frac{w^1}{w^2} - \frac{\partial U_k^1 / \partial C_k^1}{\partial U_k^1 / \partial l_k^1} \right] \\ \tau_k^2 &= 0 \end{aligned}$$

for  $k = 1, \dots, K$ , where  $\hat{l}_k^2 = l_k^1 (w^1 / w^2)$  and  $\lambda_k^* = \lambda_k (\partial \hat{U}_k^2 / \partial C_k^1) / \gamma$ .

Although Proposition 1 is derived in the context of an economic federation, within which the income distribution differs between the localities, the marginal income tax structure resembles that of a framework in which there is no distinction between localities. It is, nevertheless, important to emphasize that the tax structure has a local dimension. We can interpret Proposition 1 such that each locality has its own tax structure, with the marginal tax rate being positive for the low-ability type (since the mimicker has flatter indifference curves in consumption-income space than the low-ability type) and zero for the high-ability type. The result that the localities have different tax structures is due to the assumptions that the income distribution differs

between localities, and that the population in each locality is immobile. Therefore, there is no mechanism that ensures that the utility of each ability-type is independent of locality at the optimum, implying that the tax function will generally differ between the localities. Note that the localities would continue to differ with respect to tax schedules even if we were to introduce mobility across localities, as long as the mobility is not perfect.

Note finally that the simple structure of equation (11) depends on the assumption that  $g_k$  is additively separable in terms of the utility function. We will return to the condition for the provision of the local public good below, where equation (11) is compared to the corresponding condition in a decentralized framework.

#### 4. The decentralized solution

We begin by describing the optimization problems facing the local governments and the central government, respectively. Having done that, we continue by examining the optimal policy for the central government.

##### *The optimization problems of the local governments*

Each local government decides on the rate of a proportional income tax,  $t_k$ , and the level of a local public good,  $g_k$ . Each local government also receives a lump-sum transfer,  $R_k$ , from the central government. The local governments act as Nash competitors to one another as well as towards the central government. The latter means that each local government treats the decision variables of the central government as exogenous.

In accordance with the assumptions made above, each local government faces a generalized Utilitarian objective function. We can write the optimization problem for local government  $k$  as follows;

$$\underset{t_k, g_k}{Max} \quad \pi_k \alpha^1 V_k^1 + (1 - \pi_k) \alpha^2 V_k^2 \quad (12)$$

$$\text{s.t.} \quad t_k [\pi_k w^1 l_k^1 + (1 - \pi_k) w^2 l_k^2] + R_k - g_k = 0 \quad (13)$$

where the price of the public good has been normalized to one. We also add the nonnegativity constraints  $t_k \geq 0$  and  $g_k \geq 0$ . By substituting equation (13) into equation (12), we obtain a utility maximization problem in  $t_k$  subject to the constraint  $t_k \geq 0$ . The first order condition is presented in the Appendix. If the nonnegativity constraint does not bind, we can use the first order condition to solve for the local income tax rate

$$t_k = t(\tau_k^1, T_k^1, \tau_k^2, T_k^2, R_k, \pi_k) \quad (14)$$

where the two wage rates and the parameters  $\alpha^1$  and  $\alpha^2$  have been suppressed for notational convenience. Finally, substituting equation (14) into equation (13), we obtain the equilibrium provision of the local public good.

#### *The central government*

The central government maximizes the social welfare function described in section 3 subject to its budget constraint and the self-selection constraints, as well as subject to the restrictions that each local government obeys equations (13) and (14). The latter restrictions represent the reaction function for each local income tax rate and the budget constraint of each local government, respectively. In principle, therefore, the central government faces a classical optimal nonlinear income tax problem, with the exception that it must also recognize how the local governments respond to its policy. We can formulate the problem for the central government as

$$\text{Max}_{\tau_k^1, T_k^1, \tau_k^2, T_k^2, R_k} \sum_k [\pi_k \alpha^1 V_k^1 + (1 - \pi_k) \alpha^2 V_k^2]$$

$$\begin{aligned} \text{s.t.} \quad & V_k^2 \geq \hat{V}_k^2 \quad k = 1, \dots, K \\ & \sum_k [\pi_k (\tau_k^1 w^1 l_k^1 + T_k^1) + (1 - \pi_k) (\tau_k^2 w^2 l_k^2 + T_k^2) - R_k] \geq 0 \\ & t_k = t(\tau_k^1, T_k^1, \tau_k^2, T_k^2, R_k, \pi_k) \quad k = 1, \dots, K \end{aligned}$$

$$g_k = t_k [\pi_k w^1 l_k^1 + (1 - \pi_k) w^2 l_k^2] + R_k \quad k = 1, \dots, K$$

where  $V_k^1$ ,  $V_k^2$ ,  $\hat{V}_k^2$ ,  $l_k^1$  and  $l_k^2$  are defined as above. The Lagrangean is given by

$$\begin{aligned} L = & \sum_k [\pi_k \alpha^1 V_k^1 + (1 - \pi_k) \alpha^2 V_k^2] + \sum_k \lambda_k [V_k^2 - \hat{V}_k^2] \\ & + \gamma \sum_k [\pi_k (\tau_k^1 w^1 l_k^1 + T_k^1) + (1 - \pi_k) (\tau_k^2 w^2 l_k^2 + T_k^2) - R_k] \end{aligned}$$

By collecting the terms that reflect the indirect effects of each policy instrument via  $t_k$  and  $g_k$ , the first order conditions can be written (for  $k = 1, \dots, K$ )

$$\frac{\partial L}{\partial \tau_k^1} = \pi_k \alpha^1 \frac{\partial V_k^1}{\partial \tau_k^1} - \lambda_k \frac{\partial \hat{V}_k^2}{\partial \tau_k^1} + \gamma \pi_k [w^1 l_k^1 + \tau_k^1 w^1 \frac{\partial l_k^1}{\partial \tau_k^1}] + \delta_{\tau_k^1} = 0 \quad (15)$$

$$\frac{\partial L}{\partial T_k^1} = \pi_k \alpha^1 \frac{\partial V_k^1}{\partial T_k^1} - \lambda_k \frac{\partial \hat{V}_k^2}{\partial T_k^1} + \gamma \pi_k [\tau_k^1 w^1 \frac{\partial l_k^1}{\partial T_k^1} + 1] + \delta_{T_k^1} = 0 \quad (16)$$

$$\frac{\partial L}{\partial \tau_k^2} = [(1 - \pi_k) \alpha^2 + \lambda_k] \frac{\partial V_k^2}{\partial \tau_k^2} + \gamma (1 - \pi_k) [w^2 l_k^2 + \tau_k^2 w^2 \frac{\partial l_k^2}{\partial \tau_k^2}] + \delta_{\tau_k^2} = 0 \quad (17)$$

$$\frac{\partial L}{\partial T_k^2} = [(1 - \pi_k) \alpha^2 + \lambda_k] \frac{\partial V_k^2}{\partial T_k^2} + \gamma (1 - \pi_k) [\tau_k^2 w^2 \frac{\partial l_k^2}{\partial T_k^2} + 1] + \delta_{T_k^2} = 0 \quad (18)$$

$$\frac{\partial L}{\partial R_k} = \delta_{R_k} - \gamma = 0 \quad (19)$$

where  $\delta_{\tau_k^1}$ ,  $\delta_{T_k^1}$ ,  $\delta_{\tau_k^2}$ ,  $\delta_{T_k^2}$  and  $\delta_{R_k}$  represent the indirect effects of the central government's decision variables via the reaction function for the local income tax rate and the local public budget constraint. These terms are defined in the Appendix.

It is instructive to begin by analyzing the income tax structure without requiring that the transfer payments from the central government to the local governments must be



optimally chosen. This enables us to study how the tax structure decided upon by the central government must be modified in order to recognize the decisions made by the local governments. It also simplifies the analysis of the intergovernmental transfer payments to be carried out below. By using equations (15)-(18) together with the properties of the indirect utility function discussed in section 2, we are able to characterize the marginal income tax rates associated with the policy of the central government. Consider Proposition 2;

**Proposition 2:** *In a decentralized setting, the marginal income tax rates decided upon by the central government are characterized by*

$$\begin{aligned}\tau_k^1 &= \frac{1}{\pi_k w^1} \lambda_k^* \left[ \frac{\partial \hat{U}_k^2 / \partial C_k^1}{\partial \hat{U}_k^2 / \partial \hat{l}_k^2} \frac{w^1}{w^2} - \frac{\partial U_k^1 / \partial C_k^1}{\partial U_k^1 / \partial l_k^1} \right] \\ &\quad + \frac{1}{\pi_k (w^1)^2 (\partial \tilde{l}_k^1 / \partial \omega_k^1)} [\delta_{\tau_k^1} - \delta_{T_k^1} w^1 l_k^1] \\ \tau_k^2 &= \frac{1}{(1 - \pi_k) (w^2)^2 (\partial \tilde{l}_k^2 / \partial \omega_k^2)} [\delta_{\tau_k^2} - \delta_{T_k^2} w^2 l_k^2]\end{aligned}$$

for  $k = 1, \dots, K$ .

The tax policy implicit in Proposition 2 seems to differ from the second best policy. The reason is that the tax structure, in this case, reflects a mixture of self-selection motives for taxation and correction for the vertical fiscal external effect. In comparison with the tax structure that applies in the second best, which was discussed in connection to Proposition 1, each tax formula in Proposition 2 contains an additional term, which arises because the central government acts as a leader and recognizes how each local government responds to its policy.

To provide some basic intuition, note that if  $\delta_{\tau_k^i} > 0$  ( $< 0$ ), this means that a higher marginal income tax rate imposed by the central government on ability-type  $i$  leads to higher (lower) welfare via the reaction function for the local income tax rate and/or the local public budget constraint. This provides an incentive for the central government to choose a higher (lower) marginal income tax rate for ability-type  $i$  than it would

otherwise have done. Similarly, if  $\delta_{T_k^i} > 0$  ( $< 0$ ), ceteris paribus, a higher lump-sum component increases (decreases) the welfare via the reaction function for the local income tax rate and/or the local public budget constraint. Given the revenues to be used for the transfer payment, this means that the central government will have an incentive to choose a higher (lower) lump-sum component and, therefore, a lower (higher) marginal income tax rate than it would otherwise have done.

To go further, let us turn to the optimal transfer payments to the local governments as well as their implications for the marginal income tax rates. Our concern will be to analyze the additional terms in the marginal income tax formulas that are due to the reaction function for the local income tax rate and the local public budget constraint. Let us use the short notation

$$\mu_k = \pi_k \alpha^1 \frac{\partial V_k^1}{\partial g_k} + (1 - \pi_k) \alpha^2 \frac{\partial V_k^2}{\partial g_k} \quad (20)$$

where  $\mu_k$  is interpretable in terms of a Lagrange multiplier associated with the policy problem of local government  $k$ ; as such, it represents the (perceived) marginal cost in utility terms of providing the public good in locality  $k$ . Consider Proposition 3;

**Proposition 3:** *If the central government is able to implement optimal lump-sum transfers to the local governments, then*

$$\begin{aligned} \delta_{\tau_k^1} - \delta_{T_k^1} w^1 l_k^1 &= -\mu_k t_k \pi_k (w^1)^2 \frac{\partial \tilde{l}_k^1}{\partial \omega_k^1} - [\mu_k - \gamma] \left[ \frac{\partial t_k}{\partial \tau_k^1} - \frac{\partial t_k}{\partial T_k^1} w^1 l_k^1 \right] t_{k,R} \\ \delta_{\tau_k^2} - \delta_{T_k^2} w^2 l_k^2 &= -\mu_k t_k (1 - \pi_k) (w^2)^2 \frac{\partial \tilde{l}_k^2}{\partial \omega_k^2} - [\mu_k - \gamma] \left[ \frac{\partial t_k}{\partial \tau_k^2} - \frac{\partial t_k}{\partial T_k^2} w^2 l_k^2 \right] t_{k,R} \end{aligned}$$

for  $k = 1, \dots, K$ , where  $t_{k,R} = 1/(\partial t_k / \partial R_k)$ .

Proof: See the Appendix.

Since the two formulas in the proposition are analogous, we concentrate on the interpretation of the formula referring to the low-ability type. The first term on the right hand side,

$$-\mu_k t_k \pi_k (w^1)^2 \frac{\partial \tilde{l}_k^1}{\partial \omega_k^1},$$

is negative and contributes, therefore, to decrease the marginal income tax rate decided upon by the central government. The intuition is that tax distortions associated with the local public policy are exacerbated by the distortions imposed by the tax policy of the central government. This is seen by observing that increases in the local utility cost of providing the public good, the local income tax rate and the compensated labor supply derivative all contribute to make this expression larger in absolute value. As such, there is an incentive for the central government to choose a lower marginal income tax rate than it would have done in the absence of local income taxation.

To interpret the second term on the right hand side of the first formula in Proposition 3,

$$-[\mu_k - \gamma] \left[ \frac{\partial t_k}{\partial \tau_k^1} - \frac{\partial t_k}{\partial T_k^1} w^1 l_k^1 \right] t_{k,R},$$

let us combine the first order condition for the local income tax problem with the first order condition for the central government's choice of lump-sum transfer to the local government. In this case, we can derive

$$\begin{aligned} -[\mu_k - \gamma] t_{k,R} &= \gamma [\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k}] \\ &\quad + \lambda_k \left[ \frac{\partial V_k^1}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k} \right] \end{aligned} \tag{21}$$

Note that the first order condition for the local income tax problem implies  $t_{k,R} < 0$ . As a consequence, the right hand side of equation (21) is negative, if (i) the labor supply curves are upward sloping, and (ii) the direct utility loss of the low-ability type following a higher local income tax rate exceeds the direct utility loss of the mimicker. In this case,  $\mu_k - \gamma < 0$ , which means that local government  $k$  overprovides the public good relative to the provision associated with using the second best formula. This means, in turn, that the vertical fiscal external effect is negative.

Suppose that  $\mu_k - \gamma < 0$ . Then, if  $\partial t_k / \partial \tau_k^1 < 0$  ( $> 0$ ), it follows that

$$-[\mu_k - \gamma]t_{k,R} \frac{\partial t_k}{\partial \tau_k^1}$$

contributes to increase (decrease) the national marginal income tax rate facing the low-ability type. The intuition is, of course, that the central government has an incentive to reduce the provision of the local public good. Similarly, if  $\partial t_k / \partial T_k^1 < 0$  ( $> 0$ ), it follows that

$$[\mu_k - \gamma]t_{k,R} \frac{\partial t_k}{\partial T_k^1} w^1 l_k^1$$

contributes to decrease (increase) the national marginal income tax rate facing the low-ability type. This is so because, if an increase intercept part of the national marginal tax schedule works to decrease (increase) the local income tax rate, *ceteris paribus*, the national government will use more (less) of the intercept part than it would otherwise have done, in order to reduce the local provision of the public good, and then implement a lower (higher) marginal tax rate to meet the revenue requirement.

It is interesting to compare the results derived here with those of previous studies. Boadway and Keen (1996) and Boadway et al. (1998) also analyze optimal taxation in an economic federation, where the central government can transfer resources lump-sum between the two levels of government. As in our study, they also assume that the central government acts as a Stackelberg leader, whereas the local governments act as followers. The main difference between these studies and our study is that, while our study is based on the assumptions that the central government is able to vary the income tax schedule between localities and faces a self-selection constraint for each locality, the other two studies assume that the central government uses a proportional income tax that is not allowed to vary between the localities. An interesting result derived by Boadway and Keen (1996) is that the central government can implement the second best resource allocation by choosing its own income tax rate to be equal to zero. This means that the local governments collect all tax revenues that are associated with the use of distortionary labor income taxation. As such, the vertical external effect disappears. The central government may, in turn, impose a lump-sum fee on the local government in order to finance its own expenditures (if any).

In our model, the central government is not in general able to implement the second best resource allocation by using income taxation in combination with lump-sum transfers to the local governments. Note first that it is not an optimal strategy for the central government to choose its own marginal income tax rates to be equal to zero: such a policy does not implement the second best resource allocation derived in section 3. The reason is that the nonlinear income tax is superior to proportional income taxes from the point of view of redistribution. Furthermore, in the second best model analyzed in section 3, the central government is able to control the consumption and hours of work for each ability-type as well as the provision of local public goods. In the decentralized setting, on the other hand, the central government must, in addition, try to control the local income tax rate, meaning that the set of policy instruments is not, in general, comprehensive enough to implement the second best resource allocation. Therefore, there is a need for an additional policy instrument: for instance, a tax or subsidy imposed by the central government that is proportional to the local income tax rate.

So far, we have concentrated on the situation where  $\mu_k - \gamma < 0$ . However, since the local governments (by assumption) are not allowed to subsidize labor, there is a special case in which the central government is able to implement the second best. If each local government would prefer to underprovide the public good relative to the second best formula, and if the central government chooses the size of the lump-sum transfer to each local government to exactly correspond to the resources spent on the public good in the second best optimum, then each local government may choose a zero income tax rate. As such, both the expenditure side and the tax structure implemented by the central government will be those derived in section 3. Interestingly, this situation would also imply a positive fiscal gap. In the context of optimal taxation under vertical fiscal external effects, the optimal fiscal gap has previously been addressed by Boadway and Keen (1996), who for reasons described above found that the optimal fiscal gap is negative. Here, the opposite applies, since the central government is able to force the local governments into a corner solution, where the local income tax rates are equal to zero.

## 5. Discussion

This paper concerns redistribution and provision of public goods in an economic federation. Contrary to previous studies dealing with similar issues, our analysis is based on an extended version of the two-type optimal nonlinear tax problem. The set of policy instruments facing the central government consists of a nonlinear income tax and a lump-sum transfer to each local government. The informational constraints are similar to those characterizing previous studies on nonlinear taxation in economies without a federal structure: the governments are able to observe the gross income, while they do not observe whether a given individual is a high-ability type or a low-ability type. The local governments, on the other hand, use proportional income taxes and the transfer payment from the central government to finance a local public good. We also assume that the policy is decided upon in such a way that the central government acts as Stackelberg leader, and the local governments are followers.

We would like to emphasize two conclusions;

- In the second best resource allocation, where all taxes and expenditures are decided upon by the central government, the national tax schedule will generally differ between the localities. This result also remains in a decentralized framework, where both the central and the local governments have distinct roles to play. The reason is that the income distribution and, therefore, the costs of financing the local public good differ between the localities. Although our model is simplified in the sense that we disregard labor mobility, it is worth emphasizing that this result will remain, as long as perfect mobility is not feasible.
- In a decentralized framework, the results do not necessarily imply that the marginal income tax rate of the low-ability type is positive, or that the marginal income tax rate of the high-ability type is zero (as they would be in the absence of local governments). The reason is that the redistributive role of taxation is combined with a corrective role. In addition, the set of policy instruments is not comprehensive enough to implement the second best in general: the nonlinear income tax and the transfer payment cannot be used in order to perfectly control the consumption and hours of work of both ability-

types as well as the public good, since the central government also must correct the resource allocation problem associated with the vertical fiscal external effect.

## Appendix

*The first order condition for the local income tax rate:*

Let us denote the objective function of local government  $k$  as

$$V_k = \pi_k \alpha^1 V_k^1 + (1 - \pi_k) \alpha^2 V_k^2$$

By substituting the budget constraint of the local government, given by equation (13), into the objective function, the first order condition for the local income tax rate can be written as

$$\begin{aligned} \frac{\partial V_k}{\partial t_k} &= \pi_k \alpha^1 \left[ \frac{\partial V_k^1}{\partial t_k} + \frac{\partial V_k^1}{\partial g_k} \frac{\partial g_k}{\partial t_k} \right] + (1 - \pi_k) \alpha^2 \left[ \frac{\partial V_k^2}{\partial t_k} + \frac{\partial V_k^2}{\partial g_k} \frac{\partial g_k}{\partial t_k} \right] \leq 0 \\ \frac{\partial V_k}{\partial t_k} t_k &= 0 \end{aligned} \quad (A1)$$

where  $g_k = t_k [\pi_k w^1 l_k^1 + (1 - \pi_k) w^2 l_k^2] + R_k$  and  $l_k^i = l(w^i, \tau_k^i, T_k^i, t_k)$  for  $i = 1, 2$ .

*The Structure of Indirect Responses to the Policy of the Central Government:*

$$\begin{aligned} \delta_{\tau_k^1} &= \pi_k \alpha^1 \left[ \frac{\partial V_k^1}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^1} + \frac{\partial V_k^1}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^1} + \frac{\partial g_k}{\partial \tau_k^1} \right) \right] \\ &+ (1 - \pi_k) \alpha^2 \left[ \frac{\partial V_k^2}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^1} + \frac{\partial V_k^2}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^1} + \frac{\partial g_k}{\partial \tau_k^1} \right) \right] \\ &+ \lambda_k \left[ \frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k} \right] \frac{\partial t_k}{\partial \tau_k^1} + \gamma [\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^1} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^1}] \end{aligned} \quad (A2)$$

$$\begin{aligned}
\delta_{T_k^1} = & \pi_k \alpha^1 \left[ \frac{\partial V_k^1}{\partial t_k} \frac{\partial t_k}{\partial T_k^1} + \frac{\partial V_k^1}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial T_k^1} + \frac{\partial g_k}{\partial T_k^1} \right) \right] \\
& + (1 - \pi_k) \alpha^2 \left[ \frac{\partial V_k^2}{\partial t_k} \frac{\partial t_k}{\partial T_k^1} + \frac{\partial V_k^2}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial T_k^1} + \frac{\partial g_k}{\partial T_k^1} \right) \right] \\
& + \lambda_k \left[ \frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k} \right] \frac{\partial t_k}{\partial T_k^1} + \gamma [\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} \frac{\partial t_k}{\partial T_k^1} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k} \frac{\partial t_k}{\partial T_k^1}]
\end{aligned} \tag{A3}$$

$$\begin{aligned}
\delta_{\tau_k^2} = & \pi_k \alpha^1 \left[ \frac{\partial V_k^1}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^2} + \frac{\partial V_k^1}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^2} + \frac{\partial g_k}{\partial \tau_k^2} \right) \right] \\
& + (1 - \pi_k) \alpha^2 \left[ \frac{\partial V_k^2}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^2} + \frac{\partial V_k^2}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^2} + \frac{\partial g_k}{\partial \tau_k^2} \right) \right] \\
& + \lambda_k \left[ \frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k} \right] \frac{\partial t_k}{\partial \tau_k^2} + \gamma [\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^2} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k} \frac{\partial t_k}{\partial \tau_k^2}]
\end{aligned} \tag{A4}$$

$$\begin{aligned}
\delta_{T_k^2} = & \pi_k \alpha^1 \left[ \frac{\partial V_k^1}{\partial t_k} \frac{\partial t_k}{\partial T_k^2} + \frac{\partial V_k^1}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial T_k^2} + \frac{\partial g_k}{\partial T_k^2} \right) \right] \\
& + (1 - \pi_k) \alpha^2 \left[ \frac{\partial V_k^2}{\partial t_k} \frac{\partial t_k}{\partial T_k^2} + \frac{\partial V_k^2}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial T_k^2} + \frac{\partial g_k}{\partial T_k^2} \right) \right] \\
& + \lambda_k \left[ \frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k} \right] \frac{\partial t_k}{\partial T_k^2} + \gamma [\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} \frac{\partial t_k}{\partial T_k^2} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k} \frac{\partial t_k}{\partial T_k^2}]
\end{aligned} \tag{A5}$$

$$\begin{aligned}
\delta_{R_k} = & \pi_k \alpha^1 \left[ \frac{\partial V_k^1}{\partial t_k} \frac{\partial t_k}{\partial R_k} + \frac{\partial V_k^1}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial R_k} + \frac{\partial g_k}{\partial R_k} \right) \right] \\
& + (1 - \pi_k) \alpha^2 \left[ \frac{\partial V_k^2}{\partial t_k} \frac{\partial t_k}{\partial R_k} + \frac{\partial V_k^2}{\partial g_k} \left( \frac{\partial g_k}{\partial t_k} \frac{\partial t_k}{\partial R_k} + \frac{\partial g_k}{\partial R_k} \right) \right] \\
& + \lambda_k \left[ \frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k} \right] \frac{\partial t_k}{\partial R_k} + \gamma [\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} \frac{\partial t_k}{\partial R_k} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k} \frac{\partial t_k}{\partial R_k}]
\end{aligned} \tag{A6}$$

*Proof of Proposition 3:*

Consider the part of the proposition that refers to the low-ability type. Taking the difference between  $\delta_{\tau_k^1}$  and  $\delta_{T_k^1} w^1 l_k^1$ , while using equations (A2) and (A3) together with the first order condition for the local income tax rate (assuming  $t_k > 0$  for all  $k$ ) gives



$$\begin{aligned}
\delta_{\tau_k^1} - \delta_{T_k^1} w^1 l_k^1 &= [\pi_k \alpha^1 \frac{\partial V_k^1}{\partial g_k} + (1 - \pi_k) \alpha^2 \frac{\partial V_k^2}{\partial g_k}] [\frac{\partial g_k}{\partial \tau_k^1} - \frac{\partial g_k}{\partial T_k^1} w^1 l_k^1] \\
&+ [\lambda_k (\frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k}) + \gamma (\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k})] \\
&\times [\frac{\partial t_k}{\partial \tau_k^1} - \frac{\partial t_k}{\partial T_k^1} w^1 l_k^1] \quad (A7)
\end{aligned}$$

Since

$$\begin{aligned}
\frac{\partial g_k}{\partial \tau_k^1} &= t_k \pi_k w^1 \frac{\partial l_k^1}{\partial \tau_k^1} \\
\frac{\partial l_k^1}{\partial \tau_k^1} &= -[\frac{\partial \tilde{l}_k^1}{\partial \omega_k^1} - \frac{\partial l_k^1}{\partial T_k^1} l_k^1] w^1
\end{aligned}$$

we have

$$\frac{\partial g_k}{\partial \tau_k^1} - \frac{\partial g_k}{\partial T_k^1} w^1 l_k^1 = -t_k \pi_k (w^1)^2 \frac{\partial \tilde{l}_k^1}{\partial \omega_k^1} \quad (A8)$$

By substituting equation (A8) into equation (A7), we obtain

$$\begin{aligned}
\delta_{\tau_k^1} - \delta_{T_k^1} w^1 l_k^1 &= [\pi_k \alpha^1 \frac{\partial V_k^1}{\partial g_k} + (1 - \pi_k) \alpha^2 \frac{\partial V_k^2}{\partial g_k}] [-t_k \pi_k (w^1)^2 \frac{\partial \tilde{l}_k^1}{\partial \omega_k^1}] \\
&+ [\lambda_k (\frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k}) + \gamma (\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k})] \\
&\times [\frac{\partial t_k}{\partial \tau_k^1} - \frac{\partial t_k}{\partial T_k^1} w^1 l_k^1] \quad (A9)
\end{aligned}$$

Finally, use equation (A6) to derive

$$\begin{aligned}
&- \{ [\pi_k \alpha^1 \frac{\partial V_k^1}{\partial g_k} + (1 - \pi_k) \alpha^2 \frac{\partial V_k^2}{\partial g_k}] \frac{\partial g_k}{\partial R_k} - \gamma \} \\
&= [\lambda_k (\frac{\partial V_k^2}{\partial t_k} - \frac{\partial \hat{V}_k^2}{\partial t_k}) + \gamma (\pi_k \tau_k^1 w^1 \frac{\partial l_k^1}{\partial t_k} + (1 - \pi_k) \tau_k^2 w^2 \frac{\partial l_k^2}{\partial t_k})] \frac{\partial t_k}{\partial R_k}
\end{aligned}$$

and substitute into equation (A9). By observing that  $\partial g_k / \partial R_k = 1$  conditional on  $t_k$ , we obtain the first formula in the proposition. The second formula can be derived in a similar way.v

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