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Macroeconomic imbalances and exchange rate regime shifts

Erik Post
MACROECONOMIC IMBALANCES AND EXCHANGE RATE REGIME SHIFTS

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Abstract

This paper uses a dynamic stochastic rational expectations model of a small open economy to shed some light on factors determining exits from a fixed to a flexible exchange rate regime. Exits are in the model determined by a concern for macroeconomic stabilization. If cost-push shocks are important relative to demand shocks exits should occur more likely in times of low consumption and output, high interest rates, negative asset holdings, current account deficits, high inflation and high domestic prices. If the policy maker is more sensitive to negative rather than positive output deviations the probability of exits increases overall and is tilted toward exits with accompanying depreciation.

Keywords: exchange rates, exchange rate regimes, rational expectations model

JEL classification: E42, E44, E47
1 Introduction

According to the *de facto* classification in IMF (2003) the share of countries with pegged exchange rates decreased from about 80 to about 60 percent between 1990 and 1998. What explains exits from a fixed to a flexible exchange rate regime?\(^1\) The literature explains this in two ways:

(i) Economic fundamentals or speculators drive the authorities towards a point of no return where the only option is to let the currency float.

(ii) Parity is at an unacceptable level to the decision makers and this triggers an optimizing decision to exit from the fixed exchange rate regime.

The first explanation involves what is called "first generation" and "second generation" models of currency crises. In the "first generation" model by Krugman (1979) it is fundamentals themselves that bring about the breakdown of the fixed exchange rate regime. The breakdown is inevitable since an exogenous government deficit is financed by borrowing from the central bank. Since the nominal exchange rate is fixed and purchasing power parity holds, the depreciation pressure on domestic currency is offset by the central bank buying domestic currency with international reserves. With limited reserves, there will come a time when speculators realize that the fixed exchange rate regime cannot be sustained and the currency inevitably depreciates. This model of currency crises appeared to be appropriate for the Latin-American countries experiencing sharp currency depreciations following a fixed exchange rate regime in the 1970’s and 1980’s.

For the countries involved in the ERM-crisis in the early 1990’s there seemed to be less of a problem of poor fundamentals and more of a problem of inconsistencies in policy making that lead to more or less "self-fulfilling" currency crises. Obstfeld (1986), stresses the importance of expectations in the collapse of a fixed exchange rate regime and investigates the possibility of multiple equilibria. This model has been called "second generation" as it stresses the importance of the expectations channel for depreciations and fixed exchange rate regime collapses.

However, there is a considerable similarity in that both the Krugman and the Obstfeld models treat the occurrence of the exchange rate regime collapse as more or less inevitable and something that the policy maker only passively observes without taking a stand on what is preferable and on what actions that would be necessary to defend the fixed exchange rate regime. But as Obstfeld and Rogoff (1995) argue a country is always able to resist a speculative attack if it is truly committed. It

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\(^1\)The "fixed" arrangement comes in many flavors: peg to a single currency (such as the USD), peg to a basket of currencies (which was the case in the ERM) or a currency union such as the EMU.
can do so by buying back the entire monetary base and driving up interest rates. Therefore, the policy maker always has the option not to exit the fixed exchange rate regime; it is only a matter of the willingness of the policy maker to bear the costs of staying.\footnote{As was put at a seminar: "if someone comes up to you and asks for your money, there is always the option not to give up the money although doing so might involve a very high cost..."}

The second explanation of fixed regime exits instead emphasizes the optimizing decision of the policy maker. Edwards (1996), Bensaid and Jeanne (1997), Ozkan and Sutherland (1998), Bénassy-Quéré and Coeuré (2002) and Rebelo and Vegh (2006) present stylized models within this category.\footnote{These studies are in turn based partly on Barro and Gordon (1983).} These studies all have in common that they view the choice of exchange rate regime as an optimizing decision involving economic and political elements. Bensaid and Jeanne (1997) and Ozkan and Sutherland (1998) consider an optimizing policy maker who may voluntarily choose to exit from a fixed exchange rate regime. In these models it is concerns about macroeconomic stability that may make the policy maker exit from the fixed exchange rate regime. Obstfeld (1996) and others argue that this type of model appears to offer a more accurate portrayal of the ERM-crisis and aspects of other crises such as the one in Mexico 1994-95. Although it is an oversimplification that countries which exit from a fixed exchange rate regime do so only for stabilization purposes, stabilization motives will most certainly be important. For example, high unemployment could be costly for the incumbent government and trigger the decision to exit from the fixed exchange rate regime to get a temporarily higher output level under a flexible exchange rate regime.\footnote{In Bergvall (2002, 2005) the author shows by counterfactual simulations that a flexible exchange rate regime is more apt at stabilizing fluctuations in output and prices. This result also comes out endogenously in my model.} In Rebelo and Vegh (2006), it is shown that the mechanical rule of the Krugman-type of model, to leave the fixed exchange rate regime when international reserves are depleted, is at odds with many historical episodes. Instead, it as argued that a country will choose to leave a fixed exchange rate regime because of large expected increases in government spending.

In the empirical literature there seems to be some disagreement with regards to why countries choose to exit from a fixed exchange rate regime. Eichengreen, Rose, and Wyplosz (1995) and Duttagupta and Otker-Robe (2003) find little evidence of systematic correlations between macroeconomic fundamentals and exits. Detragiache, Mody, and Okada (2005) find that episodes of exits are characterized by similar circumstances: an overvalued real exchange rate, falling reserves, and high world interest rates. However, the empirical studies usually do not offer a very rigorous justification of the choice of variables that enter the regressions. This paper
is intended to give some guidance as to what variables that should predict exchange rate regime exits and therefore also be the focus of future empirical work.

Following the optimizing approach in the second strand of the literature described above, the purpose of this paper is to use a dynamic stochastic equilibrium model for a small open economy to examine what variables that endogenously should predict exits from a fixed to a flexible exchange rate regime if the policy maker is concerned with stabilization of output and inflation. Simulation of the model is done in DYNARE (see Collard and Juillard (2005)) and simulated data is studied further by graphical analysis to investigate the links between the endogenous probability of exit and observable fundamentals.

The results indicate that consumption, the current account, interest rates and the domestic prices are related to the probability of exits from a fixed exchange rate regime. The relative importance of factors are dependent on the relative importance of cost-push and preference shocks. If cost-push shocks are relatively important, low consumption, a negative current account, high interest rates, and high domestic prices all increase the probability of an exit with accompanying depreciation of the domestic currency.

The paper is organized as follows. Section 2 presents the model and section 3 considers alternative exchange rate regimes. In section 4 parameters are calibrated; section 5 presents the numerical results and section 6 presents some sensitivity analyses. Section 7 concludes.

2 Model

The model presented below is a dynamic stochastic rational expectations model of a small open economy intended to capture the dynamics of exchange rate regime exits from a fixed to a flexible exchange rate regime. The model is stripped down to the bare minimum. The representative agent in the economy care only about consumption; production is constant returns to scale and entirely demand driven; there is no capital and the government minimizes a discounted loss of deviations from steady state output and inflation. The government cannot levy taxes nor make transfers. Staying in a fixed exchange rate regime no stabilization policy is available since monetary policy is restrained by the defense of the currency peg. In the alternative flexible exchange rate regime, monetary policy is carried out by changes in the interest rate. The world is in continuous steady state with constant prices and interest rates. There are two exogenous shocks that create deviations from the steady state. One is a cost-push shock, the other is a preference shock. The
latter can be interpreted as a pure domestic demand shock. The model, however stylized, puts the optimizing decision of the policy maker within a more realistic setting than previous studies of the issue. The dynamic stochastic setting gives us the ability to evaluate the relative importance of different factors for the probability of exit that comes out endogenously from the model.

The policy maker makes a discrete choice between staying in fixed exchange rate regime, at some economic cost in terms of excessive misalignments of fundamentals, and leaving the fixed exchange rate regime. Either the policy maker chooses to stay in the fixed exchange rate regime, retaining the option of leaving the arrangement in the future, or exits to a flexible exchange rate regime today with monetary policy carried out by an independent and perfectly credible monetary authority.\(^5\) Once the economy has left the fixed exchange rate regime it cannot revert to the fixed regime. Opting out involves some loss of benefits from leaving the fixed exchange rate regime. This loss of benefits may involve the inability to participate in a fixed exchange rate system which itself could be an objective due to national pride or commitment to international cooperation. Also, it may include negative effects on trade because of an increase in short term volatilities of the exchange rate. The policy maker will choose to exit if the perceived benefit in terms of macroeconomic stability outweighs the benefits in the fixed exchange rate environment. The benefit of increased stabilization will be a function of the variables and shocks hitting the economy in each period. Purchasing power parity does not hold continuously, not even for tradable goods. Sticky prices will imply that, following a negative demand shock, domestic prices will decrease, which depreciates the real exchange rate, but not enough to offset the demand shock. With the nominal exchange rate fixed, the gross domestic product will decrease in the short run.

Agents will realize that leaving the fixed exchange rate regime is always an option for the policy maker and will require a compensation for holding domestic currency depending on the perceived probability of an exit from the peg. This premium will drive a wedge between interest rates in the small open economy and the world even in the fixed exchange rate regime.\(^6\) By treating the exit probability as an endogenous variable we can study how competitiveness, international indebtedness and the current account and domestic interest rates affect the probability that a country exits from the fixed exchange rate regime. The model is symmetric in

---

\(^5\)This assumption is important in that the policy maker in the fixed exchange rate arrangement might use monetary policy for political reasons whereas monetary policy in the floating regime is solely for the purpose of macroeconomic stability.

\(^6\)Agents are risk neutral in investment so the premium only compensates investors for expected depreciation of the currency.
that it treats the probabilities of de- and appreciations analogously. Calibration of parameters in the model are based partly on micro studies and partly to make the model fit some stylized facts.

2.1 Demand side

The utility, \( U_t \), for the representative consumer is given by a utility function with constant relative risk aversion and with consumption, \( C_t \), as the only argument,

\[
U_t = \frac{1}{1 - \theta} C_t^{1-\theta}. \tag{1}
\]

Consumption is a geometric average of home goods consumption, \( C_{h,t} \), and foreign goods consumption, \( C_{f,t} \),

\[
C_t = C_{h,t} \gamma C_{f,t}^{1-\gamma}. \tag{2}
\]

Assuming that the small economy representative agent can only invest in domestically denominated assets, \( B_{h,t}^h \), the intertemporal budget constraint becomes

\[
\frac{B_{h,t}^h}{1 + i_t^h} = (B_{h,t-1}^h + P_{h,t} Y_t - P_{c,t} C_t). \tag{3}
\]

The representative agent enters period \( t \) with the home currency denominated assets, \( B_{t-1}^h \), gross of interest rate. The agents work in and own all domestic firms so that the representative agent gets income \( P_{h,t} Y_t \), where \( P_{h,t} \) is the price of domestically produced goods and \( Y_t \) is the gross domestic production. Part of the nominal income is consumed, \( P_{c,t} C_t \), where \( P_{c,t} \) is the consumer price index. What is not consumed is then carried over to the next period with interest.

Optimization on the part of the consumer with respect to \( C_t \) and \( B_{h,t}^h \) with a time varying discount factor, \( \beta_t \),

\[
\max_{C_t, B_{h,t}^h} E_t \sum_{\tau=t}^{\infty} \beta_{\tau} \left[ U_\tau - \lambda_\tau \left( \frac{B_{h,\tau}^h}{1 + i_{\tau}^h} - B_{h,\tau-1}^h - P_{h,\tau} Y_\tau + P_{c,\tau} C_\tau \right) \right], \tag{4}
\]
yields the following conditions:

\[
C_t^{-\theta} = \lambda_t P_{c,t} \tag{5}
\]

and

\[
\frac{\lambda_t}{1 + i_t^h} = \beta_t E_t (\lambda_{t+1}). \tag{6}
\]

Combining (5) and (6) yields the Euler equation,
\[
\frac{C^{-\theta}_t}{\bar{P}_{c,t}} = \beta_t (1 + i^h_t) E_t \left( \frac{C^{-\theta}_{t+1}}{P_{c,t+1}} \right),
\]
which determines the intertemporal allocation of consumption. The discount factor, $\beta_t$, evolves according to
\[
\beta_t = \bar{\beta} + e_t,
\]
where $e_t$ is a persistent preference shock. A sudden decrease in the discount factor makes consumers value future consumption lower, and makes consumption increase today.

Assuming the equivalent utility function and budget constraint for the representative world consumer, but allowing foreign consumers to invest in both small open economy denominated assets as well as foreign assets, we can derive the uncovered interest-rate parity condition,
\[
(1 + i^h_t) = (1 + i^f_t) E_t \left( \frac{S_{t+1}}{S_t} \right),
\]
where $S_t$ is defined as domestic currency needed to buy one unit of foreign currency.\(^7\) A default risk premium, $\phi_t$, is appended to the uncovered interest parity condition so that we get
\[
(1 + i^h_t) = (1 + i^f_t) \phi_t E_t \left( \frac{S_{t+1}}{S_t} \right).
\]
The default risk premium takes the functional form
\[
\phi_t = \exp \left( -\psi B^h_t \right).
\]
The risk premium captures the default risk as perceived by investors with the domestic interest rate being higher than the world interest rate if the economy is a net borrower, i.e. $B^h_t < 0$.\(^8\)

The Cobb-Douglas utility function also implies constant expenditure shares on home and foreign goods,\(^9\)
\[
P_{h,t} C_{h,t} = \gamma P_{c,t} C_t,
\]
and
\(^7\)For derivation of the standard UIP condition, see Appendix.
\(^8\)See Benigno (2001) for a version of this risk adjusted formulation. Allowing for the premium is needed for a well defined steady state of the model, but the premium can be made arbitrarily small.
\(^9\)Constant expenditure shares becomes obvious if optimizing the object function in (4) with respect to $C_{h,t}$ and $C_{f,t}$. 

7
\[ S_t P_{f,t} C_{f,t} = (1 - \gamma) P_{c,t} C_t. \]  
(13)

By substitution of (12) and (13) back into (2) we get the relevant consumer price index (CPI),

\[ P_{c,t} = \frac{1}{\gamma_w} P_{h,t}^{\gamma} (S_t P_{f,t})^{1-\gamma}, \]  
(14)

where \( \gamma_w = \gamma^\gamma (1 - \gamma)^{1-\gamma} \). The export function is derived by making analogous assumptions about the world economy:\(^{10}\)

\[ EX_t = \chi Q_t^{-\eta}, \]  
(15)

where

\[ Q_t = \frac{P_{h,t}}{S_t P_{f,t}}. \]  
(16)

\( EX_t \) is exports; \( Q_t \) is a measure of the competitiveness of domestically produced goods in the international market. An increase of \( Q_t \) indicates that the relative price of domestic goods increases, i.e. domestic goods become uncompetitive on the international market.

In the following we set foreign prices and interest rates constant and focus only on domestic variables with \( P_{f,t} = P_f = 1 \) and \( i_f = i^f \).

### 2.2 Supply side

Output is entirely demand driven but also subject to the effects of a cost-push shock. Market clearing in the goods market implies that production is equal to consumption of domestic goods plus exports,

\[ Y_t = C_{h,t} + EX_t. \]  
(17)

It is assumed that inflation follows a purely forward-looking Phillips curve (e.g. Clarida, Gali, and Gertler (1999)), to which a transitory cost-push shock, \( u_t \), is appended

\[ \pi_t = \lambda (Y_t - Y_n) + \bar{\beta} E_t (\pi_{t+1}) + u_t. \]  
(18)

\( \lambda \) captures the effect of excess demand pressure on inflation and \( \bar{\beta} \) is the discount factor. The logic of equation (18) is that as output increases firms will raise prices and overall inflation will increase.

\(^{10}\)For derivation of the export function, see Appendix.
2.3 Summary of the model

Combining key equations and assuming that preference shocks are autoregressive of order one we get

\[
C_t^{-\theta} = \beta_t (1 + i_t^h) E_t \left[ \frac{P_{ct}}{P_{ct+1}} C_{t+1}^{-\theta} \right], \quad (19)
\]

\[
B_t = (1 + i_t^h) (B_{t-1} + P_{ht}Y_t - P_{ct}C_t), \quad (20)
\]

\[
1 + i_t^h = (1 + i_t^f) \phi_t E_t \left( \frac{S_{t+1}}{S_t} \right), \quad (21)
\]

\[
Y_t = \gamma P_{ct} C_t + \chi Q_t - \eta_t, \quad (22)
\]

\[
\pi_t = \lambda (Y_t - Y_n) + \beta E_t \pi_{t+1} + u, \quad (23)
\]

\[
\beta_t = \bar{\beta} + e_t, \quad (24)
\]

\[
e_t = \rho e_{t-1} + v_t, \quad (25)
\]

\[
u_t \sim i.i.d.N(0, \sigma^u), v_t \sim i.i.d.N(0, \sigma^v). \quad (26)
\]

\[Q_t = \frac{P_{ht}}{S_t}, P_{ct} = \frac{1}{\gamma_w} P_{ct,1} S_t^{1-\gamma} \text{ and inflation is given by } \pi_t = \frac{P_{ct}}{P_{ct-1}} - 1. \]

Equation (19) is the Euler equation; (20) the equation governing the evolution of assets and (21) the uncovered interest-rate parity condition. Equation (22) is the aggregate demand relation; (23) is the Phillips curve and (24) the time variant discount factor. (25) is the persistent preference shock that governs the discount factor and (26) are the pure shocks. This system cannot be solved without further assumptions since there are ten equations and eleven variables, \((C, i^h, P_c, B, P_h, S, Y, Q, \pi, \beta, e)\).

3 Alternative exchange rate regimes

So far a model has been presented without taking a stand on how monetary policy is conducted. In this section I first present two possible solutions to the model, one with a perfectly credible fixed exchange rate where monetary policy has the sole purpose of upholding the value of the domestic currency and one with a flexible exchange rate where monetary policy is used for stabilization purposes. Then, in section 3.3 I study a fixed exchange rate regime that is non-credible.

3.1 A credibly fixed exchange rate

Assuming that the fixed exchange rate regime is perfectly credible we have
\[ S_t = S_{t+1} = 1, \quad (27) \]

and equation (21) becomes

\[ 1 + i^h_t = (1 + i^f_t) \phi_t. \quad (28) \]

The steady state solution can then be solved as\(^{11}\)

\[ C = \gamma_w, \quad i^h = i^f, \quad P_c = \frac{1}{\gamma_w}, \quad B = 0, \quad P_h = 1, \quad S = 1, \quad Y = 1, \quad Q = 1, \quad \pi = 0, \quad \beta = \bar{\beta} \quad \text{and} \quad e = 0. \quad (29) \]

Under the credibly fixed exchange rate regime the interest rate is fully tied down by maintenance of the fixed exchange rate regime and cannot help to stabilize fluctuations. For example, if the economy is hit by a sudden preference shock, \( v_t \), the domestic interest rate cannot be increased to offset output deviations and inflation. Instead, it must stay equal to the world interest rate (abstracting from the risk-premium) and there will be an economic downturn.

At each point in time we can compute an expected discounted loss in the credibly fixed regime, \( L_{t}^{\text{fix}} \), in terms of economic destabilization

\[ L_t^{\text{cred}} = \pi_t^2 + \lambda_L (Y_t - Y_n)^2 + \beta L E_t L_t^{\text{cred}}. \quad (30) \]

\( \lambda_L \) denotes the relative weight put on stabilization of output and \( \beta_L \) is the discount factor of the policy maker. The loss is quadratic in inflation and output deviations from natural output. The modelling of the loss belongs to a class of loss functions commonly used in the monetary policy literature; see Walsh (2003) p. 366.

### 3.2 A flexible exchange rate and stabilization

Now, allow for a flexible exchange rate, so that \( S \) is endogenous. First, take the exceptional case of perfect stabilization of both inflation (and prices) and output:

\[ P_{c,t} = P_{c,t-1}, \quad (31) \]

\[ \pi_t = 0, \quad (32) \]

\(^{11}\)Guess \( e = 0, B = 0, P_h = 1 \) and proceed. Obtain \( i^h = i^f \) from the UIP, \( Y = Y_n \) from the PC and \( C = \gamma_w \) from the asset equation. The steady state solution also yields that \( 1 + i^f = \frac{1}{\beta} \) from the Euler and that \( \chi = 1 - \gamma. \)
and

\[ Y_t = Y_n = 1. \]  

Equation (20) becomes

\[ B_t = (1 + i_t^h) (B_{t-1} + P_{h,t} Y_n - P_{c,t} C_t), \]  

and equation (22) becomes

\[ Y_t = \gamma P_{c,t} C_t + \chi Q_t^{-\eta} = Y_n. \]  

The Phillips curve is replaced by equation (31). \( Y_t \) is replaced by \( Y_n \) in the asset equation to yield equation (34) and equation (35) denotes perfect stabilization of output. The steady state solution is identical to the fixed exchange rate regime augmented with the nominal exchange rate being unity and prices and output unity by assumption. Because of the assumption of perfect stabilization the loss will be equal to zero at all times and by definition there will be a positive exit probability in each period.

Obviously, the assumption of perfect stabilization of output and inflation is an oversimplification. For the sake of realism I assume that, under the flexible exchange rate regime, monetary policy is determined by a standard Taylor rule (Taylor (1993)) where interest rate setting is set as

\[ i_t^h = i^f + 0.5(Y - Y_n) + 1.5\pi_t, \]  

so that in steady state the interest rate will be constant and equal to the foreign interest rate but increase when inflation and output are above steady state. The steady state of the model will be left unchanged from the case of a credible fixed regime.\(^{12}\) Relaxing the assumption of perfect stabilization makes the economy go through periods of high and low inflation and output. Although superior stabilization relative to the fixed exchange rate environment some fluctuations of output and inflation occur and the policy maker observes a loss equivalent to equation (30):

\[ L_{t}^{flex} = \pi_t^2 + \lambda L_t(Y_t - Y_n)^2 + \beta L_t E_t L_{t+1}^{flex}. \]  

Appending the Taylor rule equation to the generic model in equations (19)-(26) we are able to solve the model numerically without further assumptions. We

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\(^{12}\)Guess \( e = 0, B = 0, P_h = 1, S = 1 \) and proceed.
then obtain closed form solutions for the endogenous variables as functions of deviations around steady state in the state variables $P_{c,t-1}, B_{t-1}, e_{t-1}$ and in the contemporaneous shocks $u_t$ and $v_t$. The solutions are Taylor approximation of the first order where the vector of deviations from steady state is denoted by $h = \left( h_B, h_e, h_P, h_u, h_v \right)$.

We get the solution, calibrated with parameter values presented below, for the exchange rate:

$$ S_t = f(a + h) = F(a) + DF_a h $$

$$ = F(a) + \frac{\partial F}{\partial B_{t-1}}(a)h_B + \frac{\partial F}{\partial e_{t-1}}(a)h_e + \frac{\partial F}{\partial P_{c,t-1}}(a)h_P + \frac{\partial F}{\partial u_t}(a)h_u + \frac{\partial F}{\partial v_t}(a)h_v $$

$$ = 1 - 0.13h_B + 1.19h_e + \gamma_w h_P + 0.05h_u - 1.32h_v, $$

for consumption:

$$ C_t = g(a + h) = G(a) + DG_a h $$

$$ = G(a) + \frac{\partial G}{\partial B_{t-1}}(a)h_B + \frac{\partial G}{\partial e_{t-1}}(a)h_e + \frac{\partial G}{\partial P_{c,t-1}}(a)h_P + \frac{\partial G}{\partial u_t}(a)h_u + \frac{\partial G}{\partial v_t}(a)h_v $$

$$ = \gamma_w + 0.06h_B - 1.07h_e + 0h_P - 0.22h_u + 1.19h_v, $$

for inflation:

$$ \pi_t = h(a + h) = H(a) + DH_a h $$

$$ = H(a) + \frac{\partial H}{\partial B_{t-1}}(a)h_B + \frac{\partial H}{\partial e_{t-1}}(a)h_e + \frac{\partial H}{\partial P_{c,t-1}}(a)h_P + \frac{\partial H}{\partial u_t}(a)h_u + \frac{\partial H}{\partial v_t}(a)h_v $$

$$ = 0 - 0.005h_B - 0.14h_e + 0h_P + 0.98h_u + 0.16h_v, $$

for consumer prices:

$$ P_{c,t} = k(a + h) = K(a) + DK_a h $$

$$ = K(a) + \frac{\partial K}{\partial B_{t-1}}(a)h_B + \frac{\partial K}{\partial e_{t-1}}(a)h_e + \frac{\partial K}{\partial P_{c,t-1}}(a)h_P + \frac{\partial K}{\partial u_t}(a)h_u + \frac{\partial K}{\partial v_t}(a)h_v $$

$$ = \frac{1}{\gamma_w} - 0.008h_B - 0.25h_e + 1h_P + 1.71h_u + 0.28h_v, $$
and for output:

\[
y_t = j(a + h) = J(a) + DJ_a h
\]

\[
y_t = J(a) + \frac{\partial J}{\partial B_{t-1}}(a)h_B + \frac{\partial J}{\partial e_{t-1}}(a)h_e + \frac{\partial J}{\partial P_{c,t-1}}(a)h_P + \frac{\partial J}{\partial u_t}(a)h_u + \frac{\partial J}{\partial v_t}(a)h_v
\]

(42)

\[
y_t = 1 - 0.01h_B - 0.41h_e + 0h_P - 0.99h_u + 0.45h_v.
\]

The exchange rate moves with the CPI-price level in the previous period to leave the real exchange rate unaffected.\textsuperscript{13} With lower asset holdings the exchange rate depreciates to make domestic goods cheaper and restore asset equilibrium. The preference shock makes the exchange rate appreciate to stabilize output and prices. Under the current parameterization the cost push shock makes the exchange rate \textit{depreciate} to stabilize output at the cost of some inflation.

Note that consumption, inflation and output all are unaffected by the past price level and that inflation increases close to one-to-one to cost-push shocks whereas output decreases by the same magnitude. The preference shock temporarily makes all three variables increase whereas the cost-push shock increases inflation at the expense of lower output and consumption. The dynamics of consumer prices follows that of inflation since \( \pi_t = \frac{P_{c,t}}{P_{c,t-1}} - 1 \).

Note also that we also can derive an expression for losses in the flexible regime, see equation (37), as a function of inflation and output in the flexible regime. This will be used in solution of the full model below.

3.3 A non-credible fixed exchange rate

Having considered the limiting cases of a credibly fixed exchange rate regime and a fully flexible exchange rate regime I now introduce an active policy maker that makes optimizing decisions to stay in or exit from the fixed exchange rate regime in each period. Figure 1 describes the timing of the model. At the beginning of each period, the policy maker alone observes a stochastic positive value of staying in the fixed exchange rate regime compared to being in a flexible regime and decides whether to exit from or stay in the fixed exchange rate regime. \textit{Ex post}, on the basis of the policy maker’s decision, the agents know if the realized stochastic value of staying in the fixed exchange rate regime was below or above the loss of staying in the fixed exchange rate. Prices are set and production occurs. Thereafter the credit market opens, consumers observe the interest rate and make their saving decisions.

\textsuperscript{13}This becomes obvious if using the definition of the the consumer price index combined with the definition of the terms of trade, keeping the latter constant.
Provided that the economy remains in the fixed exchange rate regime, the interest rate on loans in domestic currency from period $t$ to period $t + 1$ will depend on the probability of an exit at the beginning of period $t + 1$. This probability depends on the expected loss in period $t + 1$. Also, the perceived probability of exits affects inflation in the forward looking Phillips curve and consumption today via the forward looking Taylor rule. Agents compute a probability that the policy maker will exit from the regime in period $t + 1$ based on the assumed probability distribution of the benefit of staying in the fixed exchange rate regime.

![Figure 1: Timing of the model](image)

The relative discounted loss, $L_t^r$, of staying in the fixed exchange rate regime is defined as

$$L_t^r = L_t^{non-cred} - L_t^{flex}, \quad (43)$$

where $L_t^{cred}$ in equation (30) is modified to allow for the possibility of leaving the fixed exchange rate regime in the next period so that

$$L_t^{non-cred} = \pi_t^2 + \lambda_L(Y_t - Y_n)^2 + \beta_L E_t \left[ (1 - z_{t+1}) L_{t+1}^{non-cred} + z_{t+1} L_{t+1}^{flex} \right], \quad (44)$$

where $z_{t+1}$ denotes the probability that the policy maker exits in $t + 1$ and $L^{flex}$ is defined in equation (37). The extra loss of staying in a fixed exchange rate environment, with the option to exit in the future, $L_t^r$, is compared to the benefit of staying. As previously described, countries with fixed nominal exchange rates experience some relative benefit of staying in the regime. In my model, such a relative benefit is needed to make the model non-deterministic. If there would not exist any benefits of the fixed exchange rate regime immediate exit would always be optimal. A similar result is reported in Rebelo and Vegh (2006). Under the assumption made by the agents of the economy that the benefit is uniformly distributed over $0 - \bar{\varepsilon}$,
the perceived probability of the policy maker leaving the fixed environment is

$$z_t = P(\varepsilon_t \leq L_t) = \frac{L^{r}_t}{\bar{\varepsilon}}. \quad (45)$$

Now, I merge the non-credible fixed exchange rate regime with the closed form solutions for relevant variables under the alternative, flexible exchange rate regime. These solutions are represented by equations (38)-(41). In the following I will call the flexible solution the "shadow" solution since it can be seen as the shadow alternative at all points in time. Replicating the closed forms from above for those that enter in $t + 1$, equations (38)-(41), and denoting the shadow solution with subscript "s" we get

$$S_{s,t} = f(a + h), \quad (46)$$
$$C_{s,t} = g(a + h), \quad (47)$$
$$\pi_{s,t} = h(a + h), \quad (48)$$
$$P_{cs,t} = k(a + h). \quad (49)$$

These solutions are appended to the general model in equations (19)-(26) when variables are forward-looking. This is the case for the Euler equation, (19), where future consumption and consumer prices enter, the uncovered interest rate parity equation, (21), where future nominal exchange rate enters and the Phillips curve, (23), where future inflation enters. Appending the shadow solutions to the generic model, weighted by the expected probability of an exit in the next period, $E_t(z_{t+1})$, yields
The system constitutes 18 equations in 18 unknowns:

\( (C, C_s, i^h, P_c, P_{cs}, B, P_h, S_s, Y, Q, \pi, \pi_s, \beta, e, z, L^{\text{non-cred}}, L^{\text{flex}}, L^r) \), where \( L^{\text{flex}} \) and \( L^{\text{non-cred}} \) are given by equations (37) and (44) and the other equations are given in equations (43)-(57) and in the definitions for \( P_c, Q \) and \( \pi \). The steady state is solved for in DYNARE and displayed in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>( i^h )</td>
<td>( i^l )</td>
</tr>
<tr>
<td>( P_c )</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>( B )</td>
<td>1</td>
</tr>
<tr>
<td>( P_h )</td>
<td>1</td>
</tr>
<tr>
<td>( S_s )</td>
<td>1</td>
</tr>
<tr>
<td>( Y )</td>
<td>( Y_n )</td>
</tr>
<tr>
<td>( Q )</td>
<td>1</td>
</tr>
<tr>
<td>( \pi )</td>
<td>( \bar{\beta} )</td>
</tr>
<tr>
<td>( e )</td>
<td>0</td>
</tr>
</tbody>
</table>

The steady state is identical to the solution in the credible and flexible cases presented in (29). However, compared to the credible fixed exchange rate regime, under the non-credible fixed exchange rate regime the probability of an exit with accompanying de- or appreciation will make the fluctuations even more severe, \( \text{ceteris paribus} \), due to the rate of return compensation that must be offered investors.
when buying domestic currency. When a negative demand shock hits, the shadow exchange rate depreciates and there is a positive probability that the policy maker will exit from the fixed exchange rate regime. This will make investors require a compensation of higher interest rates when investing in domestic bonds because they expect a depreciation of a certain magnitude and probability in the next period. This in turn makes the economic downturn more protracted.

4 Calibration

For numerical evaluation of the model we need to calibrate the parameters of the model. I employ micro estimates for a representative small open economy to the extent possible and if not available I calibrate the parameter in question to fit some stylized fact. The time period is taken to be a quarter. The parameter values presented in Table 2 are used to solve for the shadow variables’ closed form solutions, presented above, as well as for the model of a fixed exchange rate regime.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>0.006</td>
</tr>
<tr>
<td>( \beta (i) )</td>
<td>0.9940</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.75</td>
</tr>
<tr>
<td>( \gamma_w (\gamma) )</td>
<td>0.5699</td>
</tr>
<tr>
<td>( \chi (\gamma) )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \eta )</td>
<td>1.5</td>
</tr>
<tr>
<td>( \theta )</td>
<td>2.0</td>
</tr>
<tr>
<td>( \rho_e )</td>
<td>0.9</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.025</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.02</td>
</tr>
<tr>
<td>( \sigma^2_u )</td>
<td>0.0001</td>
</tr>
<tr>
<td>( \sigma^2_v )</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Setting \( i = 0.006 \) makes \( \bar{\beta} = 1 / (1 + i) = 0.9940 \).

\( \gamma \) is the parameter that governs the preferences over home and foreign good. Under Cobb-Douglas preferences over home and foreign goods we know that the consumer will consume \( \gamma \) of its income on home goods and \( 1 - \gamma \) on foreign goods. I set \( \gamma = 0.75 \) so that the domestic goods share in consumer price index is 75 percent. \( \gamma_w \) is a function in \( \gamma \) defined in equation (14) and \( \chi = 1 - \gamma \), the level effect of changes in the real exchange rate to exports, can be solved for in steady state as shown in footnote 13.

\( \eta \) is the elasticity of exports with respect to changes in the relative price of
foreign goods ($\eta > 0$). $\eta = 1.5$ suggests that for a one percent depreciation in the real exchange rate exports are expected to increase by 1.5 percent.$^{14}$

$\theta$ is the coefficient of relative risk aversion in the utility function with constant relative risk aversion. In Mehra and Prescott (1985) various studies are cited and estimates are reported between unity and two for macroeconomic applications. I set $\theta = 2$ to get quite risk averse consumers that would like to smooth consumption to a large extent.

$\lambda$ in the Phillips curve is the contemporaneous effect of the output gap on inflation which should be positive. Holmberg (2006) estimates both closed and open economy versions of the Phillips curve and estimates range from negative to 0.064, depending on model, estimation technique and proxy for demand pressure. In this paper I set $\lambda = 0.025$, which is in the upper part of the distribution of estimates.

$\psi$ is the premium that must be offered to investors when the net asset position differs from zero. I assume that if the whole gross domestic income is borrowed, then the domestic interest rate should be higher by about two percentage points, i.e. $\psi = 0.02$.\[^{15}\]

$\rho_e$ is the coefficient of persistence in the preference shocks that I set to $\rho_e = 0.9$.

$\sigma_u^2 = 0.0001$ reflects a standard deviation of of the cost-push shock of 0.01, i.e. a sudden one percent increase in home prices.

$\sigma_v^2 = 0.0001$ reflects a standard deviation of the preference shock of 0.01, i.e. a one percent deviation in the valuation of future consumption relative to current consumption.

For the model of a non-credible fixed exchange rate regime we need to impose another set of calibrated parameters. These values are presented in Table 3.

<table>
<thead>
<tr>
<th>Table 3: Second set of calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>$\lambda_L$</td>
</tr>
<tr>
<td>$\beta_L$</td>
</tr>
<tr>
<td>$\bar{\varepsilon}$</td>
</tr>
</tbody>
</table>

$\lambda_L$ is the relative value that the policy maker attaches to output deviations. I set $\lambda_L = 0.7$ so that the policy maker cares relatively more about inflation than output deviations.

$\beta_L$ reflects the decision making horizon of the policy maker. Setting $\beta_L = 0.9$ makes the policy maker value losses in 16 periods (the usual time in office) less than

\[^{14}\] The short run relative price elasticity is estimated to 0.3 and long-run to 1.3 for Sweden in Johansson (1998).

\[^{15}\] Benigno (2001) uses 0.01 and 0.001 as values for $\psi$. 

18
20 percent of today’s loss.

$\bar{\varepsilon}$ is the highest value that the stochastic benefit of staying in the fixed environment can take. To get an unconditional probability of exit of $\approx 0.025$, reflecting one expected exit in every ten years, I set $\bar{\varepsilon} = 0.35$.

5 Numerical results

Using the calibrated parameter values we can study the dynamics of the model. First, I present the effect on variables from shocks by plotting their impulse response functions. Second, I simulate the model over a number of periods and study the probability of exit relates to other variables.

5.1 Impulse responses

The impulse response functions are presented in Figures 2-3. A cost-push shock makes inflation increase temporarily with a persistent effect on home prices. Output and consumption decrease with a jump and increase back to baseline. With an increased probability of exits with depreciation, reflected in the shadow exchange rate in the next period, the nominal interest rate increases to maintain interest parity. This result closely resembles the argument in Ozkan and Sutherland (1998) that expectations of a sudden depreciation can build up because of the government’s interest in stabilizing the shock. With such expectations of exits with a depreciation of the currency, risk-neutral investors will require a compensation in the form of higher interest rates. The interest rate also increases because of the negative asset position that builds up when output decreases. These high interest rates will then in turn accentuate the business cycle downturn.

A persistent preference shock increases consumption and output today. The increase in output also makes inflation increase through the Phillips curve. The increase of consumption is financed partly by borrowing abroad. On the one hand, the borrowing abroad makes interest rates increase, on the other hand the expectations of an appreciation makes interest rates decrease. In this parameterization the net contemporaneous effect is positive.

5.2 Model simulation and graphical analysis

Can we find a relation between the underlying variables, either in levels or first differences, and the probability of exit? To answer this question I simulate the model over 2100 periods and drop the first 100 observations to reduce the role of initial
conditions. Then I separate those episodes for which \( S_{s,t} > 1 \), representing periods when the policy maker is tempted to let the currency float with accompanying depreciation. The symmetry of the model makes the arguments the same for exits with appreciations. The simulation can be interpreted as follows. Imagine we start out with a large number of representative small open economies as the one described in this paper. Then, in each period some countries exit to flexible regimes and these are thereafter dropped from the analysis. To model when this happens for single economies is not the purpose of this paper, nor to model these economies after the float has taken place. Instead, recognizing the general tendency of economies to float we wish to see under what circumstances the representative small open economy will do so with a high probability.

In interpreting the results from the model I focus on observables to see if these variables can help us understand when an economy with a fixed exchange rate regime will be inclined to exit to a flexible exchange rate regime. All scatterplots display observables on the horizontal axis and the probability of exit on the vertical axis. All observables are displayed as deviations from its steady state value converted to easily interpretable numbers, except for \( S_s \) that is used only to separate depreciation \((S_s > 1)\) and appreciation episodes \((S_s < 1)\). Figure 4 displays the full results for the simulations, without separating episodes of exits with depreciation from exits with appreciation. The symmetry of the model is revealed in the results by observing the non-linear relation between exits overall and output.

As can be seen in Figure 5 the probability of exit with accompanying depreciation is decreasing in consumption and output. If consumption and output is low, the policy maker may use the option to exit from the fixed exchange rate regime with a depreciation of the currency, an increase of exports, an increase of output and consumption. Also, it appears as if a positive current account as percentage of GDP and lower interest rates increase the exit probability. No clear pattern for the domestic prices and exits with depreciation exists.

The empirical distributions of appreciations and depreciation probabilities, using the simulated data displayed in Figure 4, are displayed in Figures 6-7. The results indicate that exits with appreciations or depreciations, on average, should occur equally likely. The mean probability of exits, regardless of following appreciation or depreciation, is about 2.7 percent.
6 Sensitivity analysis

First, I will present the results when the relative importance of shocks are altered. Then I will study what happens to the results if the policy maker in the fixed exchange rate regime consistently aims at a higher than natural output.

6.1 Relative importance of shocks

Shutting down the preference shock and subjecting the model only to cost-push shocks the results in Figure 8 are obtained. These results are clearly more clear-cut than the full model and some of the results are reversed. Exits with accompanying depreciation are more likely in times of low output, low consumption, a deficit in the current account, high and increasing interest rates and high domestic prices. The mean probability of exits, regardless of following appreciation or depreciation, is about 0.6 percent. This small figure is explained by the trade-off in stabilization policy under a flexible regime.

Shutting down the cost-push shock and subjecting the model only to preference shocks the results in Figure 9 are obtained. Exits with accompanying depreciation occurs more likely when output and consumption is low. Also, as in the baseline model, exits are more likely in times of positive current accounts and low interest rates. No clear effect of domestic prices is detected. The mean probability of exits, regardless of following appreciation or depreciation, is about 1.8 percent. This relatively high figure is due to the fact that if only demand shocks are present, then the flexible regime would be attractive since no trade off in balancing effects on output and inflation exists.

The different predictions about the relation between domestic interest rate, the asset position or the current account and the probability of exit is due to the different dynamics of the model when subjected to different shocks as illustrated by the impulse response figures. Following a cost-push shock inflation increases temporarily followed by a few periods of deflation. At the same time output decreases with a jump and slowly reverts to baseline. In such circumstances the economy would unambiguously need lower interest rates after the initial response when inflation is positive, which in turn implies a depreciation of the domestic currency. This happens at the same time as the net asset position decreases, i.e. the current account is negative, since output is not sufficient to cover current consumption. Therefore we have a negative relation between the current account and the exit probability. Following a preference shock, consumption and output increases resulting in higher inflation via the Phillips curve. In such a situation the economy would need higher
interest rates to stabilize the economy, implying a appreciation of the domestic currency. However, part of the increased consumption is financed by a negative current account. Therefore we have a negative relation between the current account and a appreciation of the currency, or equivalently, a positive relation between the current account and the exit probability.

### 6.2 Output bias

So far we have assumed that the policy objective is the same across exchange rate regimes. Now, suppose instead that the policy maker in the non-credible fixed regime has some incentives to push output above steady state whereas the monetary authority in the flexible regime has no such objectives. Such a difference in policy objectives could arise if the monetary authority in the fixed exchange rate regime is political, and the probability of getting elected for the next term of office is increasing in economic activity, whereas policy in the flexible exchange rate regime is governed by an independent central bank. The loss function in equation (44) is appended by an output bias term, $k$, and becomes

$$L_t^{non-cred} = \pi_t^2 + \lambda_L(Y_t - Y_n - k)^2 + \beta_L E_t [(1 - z_{t+1}) L_{t+1}^{non-cred} + z_{t+1} L_{t+1}^{flex}].$$

(58)

The other equations of the model are left unchanged. With a positive output bias, setting $k > 0$ in equation (58), there will be an incentive for the policy maker to stimulate the economy even in steady state so as to attain the desirable output ($Y_n + k$). The steady state values of the model will be left unchanged with the exception of there being a steady state loss different from zero and an associated positive probability of exit. Setting $k = 0.05$ the mean probability of an exit increases to 4.7 percent, which is a reflection of the increased mean probability of exits with depreciations at 7.2 percent.

The scatterplots in Figure 11 indicate that exits with depreciations will occur more likely when output and consumption is low, and when domestic prices are high. Now, based on the simulated data displayed in Figure 10, the exits are heavily tilted toward depreciations as displayed in Figures 12-13. This happens because the policy maker is tempted to boost output above the natural rate output, which can be done temporarily by a depreciation of the domestic currency. This occurs although the effect of the one time depreciation would not be sustainable since natural output remains unchanged.

The output bias can help us understand the strong bias in the data toward re-
alignments with depreciations rather than appreciations. The conventional wisdom is that countries, given that they are to exit from a fixed exchange rate regime, would be better off doing so when the going is good. If countries take this advice seriously then one should observe more exits with appreciation than with depreciation of the exchange rate. However, Detragiache, Mody, and Okada (2005) identify only three out of forty exits during 1980-2001 that were followed by a nominal appreciation. The authors infer that exits often occur when times are bad. With a positive output bias my model produces the same results; realignments are tilted toward cases of realignments with depreciations. My model offers an explanation to this observation. Since the policy maker intends to push the economy above the natural rate of output she is more inclined to act on negative shocks to output rather than positive. In fact, even with no shocks at all the policy maker will have an incentive to exit with a depreciating exchange rate, resulting in the bias of exits towards exits with accompanying depreciation.
7 Conclusions

This paper makes use of a dynamic stochastic rational expectations model of a representative small open economy to study under what circumstances it is likely that there will be an exit from a fixed to a flexible exchange rate regime. In the shadow flexible alternative, monetary policy is guided by a simple Taylor rule that is superior from the point of view of stabilization of economic shocks. The purpose of the paper is to investigate how observables relate to the probability of an exit when the policy maker is concerned with macroeconomic stabilization.

The main results are summarized as follows. Regardless of the relative size of cost-push and demand shocks, low consumption and low output should trigger exits with depreciations to help to stimulate the economy. It is also shown that high domestic prices, making domestic goods uncompetitive at the international market, also make exits with depreciation more likely. If the domestic debt is large and interest rates are high, then exits with depreciations are more likely to occur. These results are stronger the larger the cost-push shocks are. Inflation and the current account have ambiguous effects on exits with depreciations, depending on the relative size of the two types of shocks.

Saxena (2004) describes episodes of currency crises with resulting depreciations. For the Latin American countries during the 1970’s and 1980’s she points out that the episodes of currency crises and the following depreciation were preceded by high current account deficits and an increase in interest rates.\footnote{Bolivia 1982-85, Brazil 1983; 1986; 1989-90, Chile 1971-74, Peru 1976;1987 and Uruguay 1982} For the European countries during the 1990’s, current account deficits also preceded the crises but the reason for exits out of concerns for international competitiveness is pointed out.\footnote{Germany 1992-93, France 1992-93, United Kingdom 1992-93, Spain 1992-93 and Sweden 1992-93} My model is capable of replicating these findings if one assumes that cost-push shocks are large relative to preference (demand) shocks.

Introduction of an output bias, caused by an opportunistic policy maker, makes exits with depreciation more likely than exits with appreciation since a depreciation temporarily boost output above the natural level. My model gives some intuition to why countries exit with depreciations in bad times rather than in good times.

My model can be extended in several directions. The most obvious extension would be to endogenize the world economy. This would introduce more dynamics since world interest rates and prices would matter for how the small open economy evolves over time. Another possible extension would be to endogenously model the benefits of the fixed exchange rate regime.
8 Figures

Figure 2: Responses of key variables to cost-push shock

Figure 3: Responses of key variables to preference shock
Figure 4: Both shocks: all periods
Figure 5: Both shocks: periods for which $S_{s,t} > 1$
Figure 6: Both shocks: histogram of exit probabilities with accompanying depreciation, $S_{s,t} > 1$

Figure 7: Both shocks: histogram of exit probabilities with accompanying appreciation, $S_{s,t} < 1$
Figure 8: Only cost-push shocks: periods for which $S_{s,t} > 1$
Figure 9: Only preference shocks: periods for which $S_{s,t} > 1$
Figure 10: Both shocks with output bias: all periods
Figure 11: Both shocks with output bias: periods for which $S_{s,t} > 1$
Figure 12: Both shocks with output bias: histogram of exit probabilities with accompanying depreciation, \((S_{s,t} > 1)\)

Figure 13: Both shocks with output bias: histogram of exit probabilities with accompanying appreciation, \((S_{s,t} < 1)\)
References


A Appendix

A.1 Derivation of the uncovered interest parity condition

The representative world consumer’s intertemporal budget constraint is

\[
\frac{B^h_t}{1 + i^h_t} + \frac{S_t B^f_t}{1 + i^f_t} \phi_t = B^h_{t-1} + S_t B^f_{t-1} + P_{f,t} Y_{W,t} - P_{C,t} C_{W,t},
\]

where consumption, \(C_{W,t}\), is a geometric average of world produced good consumption, \(C_{f,t}\), and consumption goods from the small economy, \(C_{h,t}\), so that

\[
C_{W,t} = C_{f,t}^\gamma_w C_{h,t}^{1-\gamma_w}.
\]

\(P_{C,t}\) is derived accordingly. The interpretation of the budget constraint in equation (59) is that the world representative agent enters period \(t\) with the small economy denominated assets \(B^h_{t-1}\), gross of interest rate and world denominated assets \(B^f_{t-1}\), gross of interest rate, and denominated in small economy currency at time \(t\). The agent receives work income \((P_{f,t} Y_{W,t})\) and spends some on consumption \((P_{C,t} C_{W,t})\) in the same period. This value equals the discounted value of what is brought over to \(t+1\). Households work in and own all firms in the economy so all income accrues to households.

Optimization on the part of the consumer with respect to \(C_{W,t}, B^h_t\) and \(B^f_t\),

\[
\max_{C_t, B^h_t, B^f_t} E_t \sum_{\tau=\tau}^{\infty} \beta^\tau \left[ U_{\tau} - \lambda_{\tau} \left( \frac{B^h_{\tau}}{1 + i^h_\tau} + \frac{S_{\tau} B^f_{\tau}}{1 + i^f_\tau} \right) - B^h_{t-1} - S_{\tau} B^f_{t-1} - P_{f,\tau} Y_{W,\tau} + P_{C,\tau} C_{W,\tau} \right],
\]

yields the following conditions:

\[
C_{W,t}^{\gamma_w} = \lambda_t P_{C,t},
\]

\[
\frac{\lambda_t}{1 + i^h_t} = \beta E_t (\lambda_{t+1}),
\]

and

\[
\frac{\lambda_t S_t}{1 + i^f_t} = \beta E_t (\lambda_{t+1} S_{t+1}).
\]

Equating marginal utilities intertemporally yields the Euler equation,
\[
\frac{C_{W,t}^{-\theta}}{P_{cW,t}} = \beta(1 + i^h_t) E_t \left( \frac{C_{W,t+1}^{-\theta}}{P_{cW,t+1}} \right).
\]  
(65)

Combining equations (63) and (64) we get the uncovered interest-rate parity condition,

\[
(1 + i^h_t) = (1 + i^f_t) \frac{E_t \left( \lambda_{t+1} S_{t+1} \right)}{S_t E_t \left( \lambda_{t+1} \right)}.
\]  
(66)

Using the expression for the marginal utility of consumption in (62) we can rewrite equation (66) as

\[
(1 + i^h_t) = (1 + i^f_t) \frac{E_t \left( \frac{C_{W,t+1}^{-\theta}}{P_{cW,t+1}} S_{t+1} \right)}{S_t E_t \left( \frac{C_{W,t+1}^{-\theta}}{P_{cW,t+1}} \right)}.
\]  
(67)

which cannot generally be simplified further since \( E_t \left( \frac{C_{W,t+1}^{-\theta}}{P_{cW,t+1}} S_{t+1} \right) \neq E_t \left( \frac{C_{W,t+1}^{-\theta}}{P_{cW,t+1}} \right) E_t (S_{t+1}) \) unless the covariance of real consumption and the bilateral exchange rate is zero.

However, one can argue that the small open economy exchange rate will carry such small weight in the uncovered interest-rate parity condition so that the covariance terms will tend to zero. This yields the uncovered interest-rate parity condition that will bind for investors and equation (67) can be approximated by the most familiar version of the condition,

\[
(1 + i^h_t) = (1 + i^f_t) E_t \left( \frac{S_{t+1}}{S_t} \right).
\]  
(68)

Equation (68) is the uncovered interest-rate parity condition used in solving the model.
A.2 Derivation of the export function

World utility is given by

\[ U_{W,t} = \frac{1}{1 - \theta} C_{W,t}^{1-\theta}, \]  

(69)

where \( C_{W,t} \) is defined as in section A.1. The world budget constraint, abstracting from investments and exports to the small economy, is given by\(^{18}\)

\[ P_{f,t} Y_{W,t} = P_{f,t} C_{f,t} + \frac{P_{h,t} C_{h,t}}{S_t}. \]

(70)

Maximizing (69) subject to (70) and solving for \( C_{h,t} \) gives

\[ C_{h,t} = \frac{P_{f,t} Y_{W,t}}{S_t \left( \frac{1}{1 - \gamma_w} \right)} = \left( 1 - \gamma_w \right) Y_{W,t} \frac{S_t P_{f,t}}{P_{h,t}} \]  

(71)

which then constitutes the world import of goods produced in the small open economy, \( IM_{W,t} \). In equilibrium world imports must equal small open economy exports,

\[ IM_{W,t} = (1 - \gamma_w) Y_{W,t} \frac{S_t P_{f,t}}{P_{h,t}} = EX_t. \]  

(72)

Taking Sweden as an example of a small open economy with about 0.5 percent of world GDP and Sweden’s steady state output normalized to unity we have that the world is about 200 times that of Sweden. Assuming a constant world output at this level and use that from the steady state solution we also know that \( (1 - \gamma_w) Y_{W,t} \) should equal \( 1 - \gamma \). This implies that \( \gamma_w = 0.99625 \), indicating that the small open economy imports accounts for 0.375 percent of world consumption in steady state. Thus,

\[ EX_t = (1 - \gamma) \frac{S_t P_{f,t}}{P_{h,t}} = (1 - \gamma) Q^{-\eta}_t. \]  

(73)

with \( \eta = 1 \). I allow for \( \eta \neq 1 \) to better match empirical evidence.

Equation (73) is the export function used in solving the model.

---

\(^{18}\)Asset holdings are abstracted from for clarity in derivations. The exchange rate is written in small economy currency units needed to buy world currency to comply with notation in the main text.


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