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*Department of Economics*

# Can a time-varying equilibrium real interest rate explain the excess sensitivity puzzle?

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PUZZLE?

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# Can a time-varying equilibrium real interest rate explain the excess sensitivity puzzle?\*

Annika Alexius<sup>†</sup> and Peter Welz<sup>‡</sup>

September 2006

## Abstract

The strong response of long-term interest rates to macroeconomic shocks has typically been explained in terms of informational asymmetries between the central bank and private agents. The standard models assume that the equilibrium real interest rate is constant over time and independent of structural shocks. We incorporate time-variation in the equilibrium real interest rate as function of structural shocks to e.g. productivity and demand. This extended model implies that forward interest rates at long horizons move about 40 basis points as the short-term interest rate increases one percentage point. In terms of regressions of changes in long-term interest rates on changes in the short-term interest rate, including a time-varying equilibrium real interest rate explains about half of the puzzle.

Key words: Term structure, equilibrium real interest rate, unobserved components model

JEL classification: E43, E52, C51

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# 1 Introduction

Long-term interest rates display large movements in response to macroeconomic shocks, typically in the same direction as the policy controlled short-term interest rate (Gürkaynak, Sack and Swanson (2005), Ellingsen and Soderstrom (2005)). Hence the entire yield curve tends to shift in a parallel manner. Standard models of monetary policy however imply that long-term interest rates should remain stable when the economy is hit by shocks. This behavior has been labelled *excess sensitivity* and/or *excess volatility puzzle*. Excess volatility denotes the finding that the variance of long-term interest rates is far greater than what can be explained within the standard models used in this literature<sup>1</sup>, whereas excess sensitivity concerns the observed large movements in long-term interest rates in the same direction as the short-term interest rate in response to macroeconomic shocks. Since we are interested in explaining the prevalence of parallel shifts of the yield curve we adhere to the excess sensitivity version of the puzzle.

Gürkaynak et al. (2005), Ellingsen and Soderstrom (2005), and Beechey (2004) amongst others construct different models of why shocks could have stronger and more persistent effects on expected future short-term rates. In Gürkaynak et al. (2005) the private sector adjusts its expectations of the long-run inflation target in response to macroeconomic shocks in a manner that generates the observed movements of long-term interest rates. Ellingsen and Soderstrom (2005) show that a model with private central bank information about future inflation generates moderate parallel shifts in the yield curve

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<sup>1</sup>See e.g. Shiller (1979) for an early discussion.

when the economy is hit by shocks to supply and demand. They estimate that the ten-year interest rate rises on average by 25 basis points in response to an unexpected one percentage point increase in the policy-controlled short rate. In Beechey (2004) long-term interest rate movements in response to shocks arise from a non-stationary inflation target combined with adaptive learning.

Existing explanations of the excess sensitivity puzzle provide different mechanisms through which shocks today affect expected inflation several years into the future. The second component of a nominal interest rate is the real interest rate. In this paper we relax the assumption of a constant equilibrium real interest rates used in the models discussed above and allow macroeconomic shocks to have persistent effects on the equilibrium real interest rate, which in turn generates movements in future nominal short-term interest rates. We are not trying to argue that expected inflation does not play an important role in the determination of nominal interest rates. It is nevertheless relevant to investigate to what extent movements in real interest rate can explain the puzzle, especially as the assumption of a constant equilibrium real interest rate is neither theoretically nor empirically plausible. Shocks to technology or the time preference of consumers alter the equilibrium real interest rate in general equilibrium models. Several empirical studies find that the equilibrium real rate can be modelled as non-stationary as in Laubach and Williams (2003) or near unit root process as in Mesonnier and Renne (2004), implying that shocks have permanent effects or at least highly persistent effects.<sup>2</sup>

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<sup>2</sup>Other contributions in the same vein are Andres, Lopez-Salido and Nelson (2004) and

We use an unobserved components model estimated with the Kalman filter to extract a measure of the equilibrium real interest rate and show that the theoretical model augmented with the estimated process explains a significant fraction of the excess sensitivity puzzle. Forward interest rates at long horizons display much more movements in response to shocks when shocks have persistent effects on the equilibrium real interest rate than when the natural real rate of interest is constant. An alternative empirical formulation of the excess sensitivity puzzle is that long-term interest rates appear to react to changes in the short-term interest rate. We show that the slope coefficients from regressions of changes in long-term interest rates on changes in short-term interest rates are about 20% higher when the equilibrium real interest rate is time-varying than when it is constant. Because we do not investigate whether a time-varying equilibrium real interest rate explains the puzzle better or worse than alternative explanations, we do not claim that our solution is the correct one. We do, however, show that equilibrium real interest rates display sufficient variation and dependence on structural shocks to produce the observed co-movements of long-term interest rates and short-term interest rates.

Because long-term interest rates consist of expected future short-term interest rates (plus possible term premia), the excess sensitivity puzzle can be illustrated in terms of movements in expected future short-term interest rates or forward rates in response to shocks. To generate persistent effects on forward rates, either the stochastic process of the shock itself or the effects of the shock on other variables in the model need to be highly persistent.

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Manrique and Marqués (2004).

Gürkaynak et al. (2005) conclude that the degree of persistence of nominal shocks required to solve the excess sensitivity puzzle is unreasonably high. We demonstrate that the effects of real shocks propagated through the equilibrium real rate is sufficiently persistent to explain up to half the puzzle.

The paper is organized as follows. Section 2 presents a semi-structural general equilibrium model with a time-varying equilibrium real rate. In Section 3 we estimate this time-varying real rate. Section 4 analyses the implications for the excess sensitivity puzzle in two ways: first by examining the effect of shocks on long-horizon forward rates and second through the coefficients of regressions of long-rate changes on short rate movements. Section 5 concludes.

## 2 A stylized model

The macroeconomic models used to analyze the excess sensitivity puzzle in Gürkaynak et al. (2005), Orphanides and Williams (2005), and Ellingsen and Soderstrom (2005) are similar. They are chosen to be empirically relevant and typically include both forward looking and backward looking terms in the Phillips curve and Euler equation. The specification we use to analyze to which extent a time-varying equilibrium real interest rate can explain the puzzle is taken from Rudebusch (2002a). This model shares the main features from the stylized Orphanides and Williams (2005) model but features richer dynamics. A version of it is also used in Ellingsen and Soderstrom (2005) and Gürkaynak et al. (2005).

Let  $y_t$  denote the output gap measured as the percent deviation of output from its potential,  $\pi_t$  is the annualized quarterly inflation rate, and



$r_t = i_t - E_{t-1}\bar{\pi}_{t+3}$  is the real interest rate measured as the difference of the nominal interest rate and four quarter inflation expectations calculated as  $E_{t-1}\bar{\pi}_{t+3} \equiv E_{t-1}\frac{1}{4}\sum_{j=0}^3\pi_{t+j}$ . Aggregate demand follows an AR(2) process and is affected by monetary policy or the deviation of the real interest rate from the equilibrium real interest rate  $r^*$ . The latter is defined as the real interest rate that has a neutral effect on demand or the real interest rate that keeps the output gap equal to zero given that the economy is in equilibrium, i.e. that output is at its potential. Finally, aggregate demand is affected by an i.i.d. demand shock  $\varepsilon_t^y$ . The Phillips curve allows for both backward looking and forward looking behavior, where  $\phi_\pi$  is the weight of expected inflation and  $(1 - \phi_\pi)$  is the weight of lags one to four of inflation.  $\varepsilon_t^\pi$  denotes the cost push shock:

$$y_t = \phi_\pi E_{t-1}y_{t+1} + (1 - \phi_\pi) \sum_{s=1}^2 \alpha_{yi} y_{t-s} - \alpha_r (r_{t-1} - r_{t-1}^*) + \varepsilon_t^y \quad (1)$$

$$\pi_t = \phi_\pi E_{t-1}\bar{\pi}_{t+3} + \beta_{y1} y_{t-1} + (1 - \phi_\pi) \sum_{s=1}^4 \beta_{\pi s} \pi_{t-s} + \varepsilon_t^\pi. \quad (2)$$

In the standard models used to analyze the excess sensitivity puzzle as well as in the original Rudebusch model, the equilibrium real interest rate or the interest rate that has zero effect on the output gap is assumed to be constant over time and unaffected by structural shocks, i.e.  $r_t^* = r$  for all  $t$ . There is however abundant empirical evidence that the natural real interest rate shifts over time and with the shocks hitting the economy. We assume that the equilibrium real interest rate follows a first order autoregressive process and that it is affected by structural shocks to the economy:

$$r_t^* = \rho_r r_{t-1}^* + \rho_e \varepsilon_t^y + \rho_\pi \varepsilon_t^\pi + \varepsilon_t^* \quad (3)$$

Our structural model is a stylized version of more elaborate models in which

one would e.g. account for an explicit link between the marginal product of capital and the real interest rate. Also, in some DSGE-models that feature endogenous investment decisions the equilibrium real interest rate depends on the capital stock (Woodford, 2003), a persistent and slowly moving variable. These considerations motivate our choice of equation (3). The shock  $\varepsilon_t^*$  could thus be seen as a proxy for effects on the equilibrium real interest rate that do not have a contemporaneous effect on the output gap or inflation. It is clear that allowing for a highly persistent time-varying equilibrium real interest rate in the aggregate demand relation will increase persistence in the nominal interest rate if the central bank responds to it.

The model is closed by adding a Taylor rule, possibly including interest rate smoothing as is often found in empirical studies<sup>3</sup>

$$i_t = f_i i_{t-1} + (1 - f_i)(r_t^* + f_\pi \pi_t + f_y y_t) + \varepsilon_t^i. \quad (4)$$

We will consider  $f_i = 0$  as well as  $f_i \neq 0$ .

Most parameter values can be taken from Rudebusch (2003a, b). However, we have to estimate values for the parameters pertaining to the time-varying equilibrium real interest rate in order to calibrate the model and analyze the response of the short-term and long-term interest rates to shocks.

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<sup>3</sup>We assume that the inflation target is constant and equal to zero. Gürkaynak et al. (2005) and Ellingsen and Soderstrom (2005) discuss the potential of a time-varying inflation target to explain the excess sensitivity puzzle.

### 3 Estimating a time-varying equilibrium interest rate

The extent to which a time-varying equilibrium interest rate can explain the excess sensitivity puzzle depends on the size of the parameters  $\rho_r, \rho_\pi, \rho_e$  and the standard deviations of the shocks. If the equilibrium real interest rate is highly autocorrelated or close to a random walk, only small shocks are required to create large co-movements between long-term interest rates and short-term interest rates. In this section we estimate a time-varying equilibrium real interest rate using an unobserved components model.

#### 3.1 Data

All data is measured in quarterly frequency and obtained from the Federal Reserve Bank of St. Louis database (FRED) covering the period 1959Q1-2004Q3. For the potential level of output ( $y^{pot}$ ) we use the series provided by the Congressional Budget Office (CBO), and output is measured as 3 decimal real GDP in billions of chained year 2000 U.S. dollars. We construct the output gap as  $\tilde{y}_t = 100 \log(y_t/y_t^{pot})$ . The nominal interest rate is measured as quarterly averages of the monthly Federal Funds rate and the real interest rate is defined as the difference between the nominal interest rate and the year-on-year inflation rate measured by the percentage change in the GDP chain-type price index.

#### 3.2 Empirical Specification

In this section we focus on estimating the parameters in the dynamic specification for the equilibrium real interest rate as the remaining parameters

are calibrated.<sup>4</sup> Our specification is closely related to the estimated model by Rudebusch and Svensson (1999) and Rudebusch (2002a) and also used in Laubach and Williams (2003) and Mesonnier and Renne (2004). This model has been quite successful in fitting the data and summarizing features of large scale macroeconomic models.

The empirical specification is similar to those of Laubach and Williams (2003) and Mesonnier and Renne (2004). However, focus on estimating the equilibrium real interest rate as an AR(1)-process that depends on the demand shock and the additional natural real rate shock as in the theoretical model and treat potential output as observable variable unlike the aforementioned authors.<sup>5</sup> We formulate the following empirical model.

$$y_t = \alpha_{y1}y_{t-1} + \alpha_{y2}y_{t-2} - \alpha_r\tilde{r}_{t-1} + \varepsilon_t^y \quad (5)$$

$$r_t^* = \rho_r r_{t-1}^* + \rho_e \varepsilon_t^y + \varepsilon_t^* \quad (6)$$

$$\tilde{r}_t = d_1\tilde{r}_{t-1} + d_2\tilde{r}_{t-2} + \varepsilon_t^r \quad (7)$$

$$\tilde{r}_t = r_t - r_t^* \quad (8)$$

The first equation is the empirical counterpart of our aggregate demand equation (1), and the second equation describes the dynamics of the equilibrium real interest rate. We postulate a stationary AR(2)-process for the interest rate gap in equation (7), assuming that the real rate gap follows similar dy-

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<sup>4</sup>This is obviously a second or even third best strategy. We have attempted and failed to estimate the full model in equation (1) to (4). The main problem appears to be the monetary policy equation as the long sample period covers several policy regimes. In addition, we have only obtained insignificant responses of the equilibrium real interest rate to the supply shock in equation (2) (i.e.  $\rho_\pi = 0$ ), which renders this equation superfluous to the estimation of (3).

<sup>5</sup>See Clark and Kozicki (2004) for a comparison of models that estimate jointly the natural rate of interest and potential output with models where only the natural rate of interest is estimated and potential output is given by the CBO-measure.

namics as the output gap.<sup>6</sup> Note in particular that our model allows the equilibrium real interest rate to be either stationary or nonstationary, the latter case implying that  $r_t$  and  $r_t^*$  must be cointegrated.

Our specification differs from the ones by Laubach and Williams (2003) and Mesonnier and Renne (2004) with respect to equation (6) and (7). The former authors postulate  $r_t^*$  to be nonstationary because in their model the equilibrium real rate depends on the trend growth rate of potential output which itself is driven by a random walk. In line with our definition of the equilibrium real interest rate in the previous section, note that in the absence of shocks equation (5) implies that in the long run a zero output gap should go in hand with a zero interest rate gap.

We do not take a particular stance on monetary policy here because the estimated time interval spans several different monetary policy regimes. Equation (7) only postulates that the Federal Reserve has conducted monetary policy such that the deviation of the real interest rate from its equilibrium remained stationary over the estimation period. This is a fairly general assumption that encompasses a dynamic Taylor rule of the type that we are using in the theoretical model.

We assume that all shocks in the model are independent of each other implying a diagonal variance-covariance matrix of the transition equation

$$\Sigma = \begin{bmatrix} \sigma_y^2 & & \\ & \sigma_{\tilde{r}}^2 & \\ & & \sigma_{r^*}^2 \end{bmatrix}.$$

The model is written in state space form (see Appendix A for a detailed representation) and the value of the likelihood function can then be calculated

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<sup>6</sup>A similar approach has earlier been used for estimations of potential output. See e.g. Clark (1987).

with the Kalman filter. We maximize the likelihood function by standard procedures and calculate the negative inverse Hessian in order to find the standard errors of the estimates.<sup>7</sup> Next we discuss the estimation results.

### 3.3 Empirical results

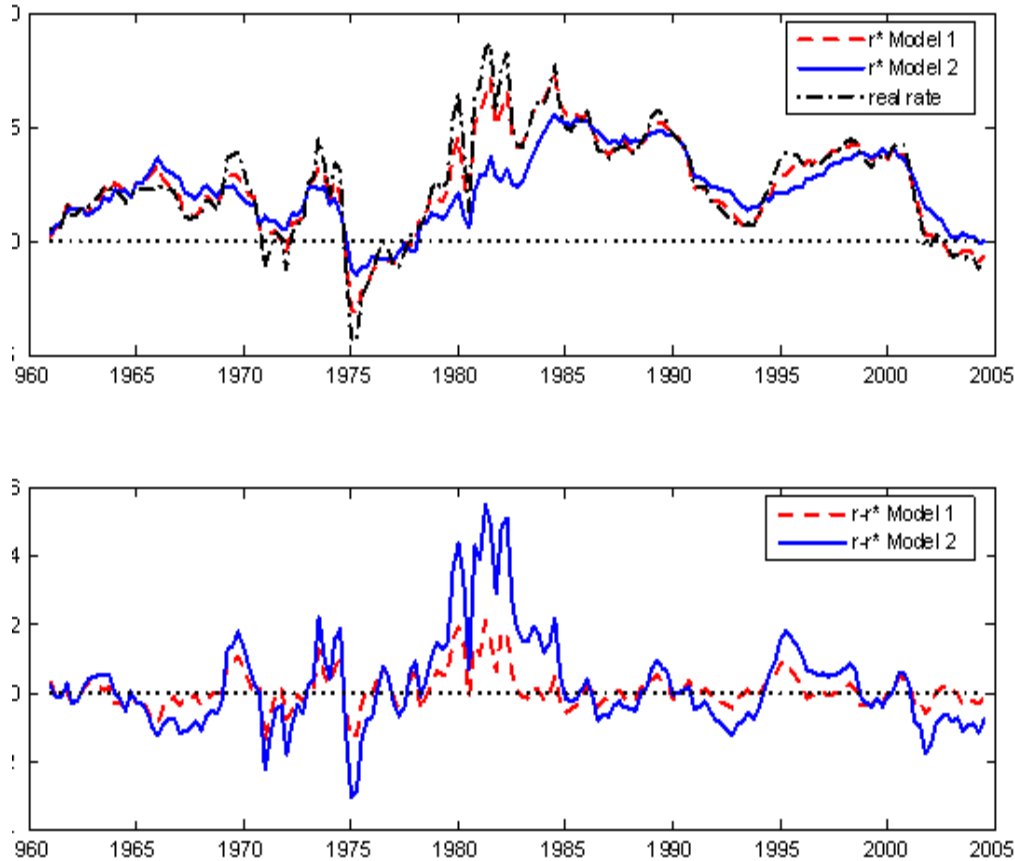
Orphanides and Williams (2002) compare six different methods of measuring the natural real rate and find considerably different estimates depending on the method used. We estimate two different models. In model 1 all parameters are estimated freely whereas in model 2 the standard deviation of the equilibrium real interest rate is fixed to  $\sigma_{r^*} = 0.322$ . This value is taken from Clark and Kozicki (2004). The smaller standard deviation leads to a smoother estimate of the equilibrium real interest rate.

Figure (2) displays the graphs for the two models. The top panel plots the estimates of the equilibrium real interest rate from the one-sided Kalman filter together with the observed real interest rate. The bottom panel presents the interest rate gaps from the two models. We notice that in model 1 the estimated equilibrium real interest rate seems to follow the real rate quite closely. Nevertheless the bottom panel of figure (1) shows that there is sizeable variation in the real rate gap with up to 2 percentage points. Note that this measure gives information about the stance of monetary policy as well. As expected, the estimates from model 2 produce a flatter estimate of

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<sup>7</sup>Since we assume that the equilibrium real interest rate is stationary we can calculate a proper prior distribution of the state vector. The initial covariance matrix is calculated by  $vec(P_{1|0}) = (I - T \otimes T)^{-1}vec(RQR')$ , the means of the real rate gap and the output gap are set to zero and the mean of  $r_t^*$  is set equal to the mean of the real interest rate over the sample period (see Appendix A for details of the state space model and the notation). For calculation of the log-likelihood we use Paul Söderlind's Kalman filter Gauss code adapted to Matlab.

the equilibrium real interest rate.



**Figure 1:** Estimated equilibrium real interest rates and real interest rate gaps.

Both our two estimated models produce an estimate of the natural real rate that is time-varying, affected by shocks to the economy, and highly persistent. The coefficient estimates for the two models are shown in table 2. The equilibrium real interest rate appears to be highly persistent with an estimated coefficient of about 0.98 and 0.99, respectively.<sup>8</sup> The other coefficient estimates are rather similar with the exception of  $\alpha_r$ , the sensitivity

<sup>8</sup>In Appendix B we show plots of the conditional likelihoods around the optimum for each parameter. These plots confirm that  $\rho_r$  should be smaller than 1 and thus the equilibrium real rate should be stationary.

of the output gap with respect to the real rate gap, which is plausibly much smaller in the second model, and closer to estimates from models that assume a constant equilibrium real interest rate (e.g. Rudebusch (2002a)) but also with the results found by Laubach and Williams (2003) who estimate a time-varying equilibrium real interest rate and report values for  $\alpha_r$  in the range from 0.088 to 0.122.<sup>9</sup>

**Table 1: Estimation results**

Parameter	Model 1	Model 2
$a_{y1}$	1.061 (11.26)	1.111 (14.12)
$\alpha_{y2}$	-0.118 (-1.30)	-0.161 (-2.00)
$\alpha_r$	0.366 (2.47)	0.168 (2.46)
$d_1$	0.965 (5.65)	0.923 (8.89)
$d_2$	-0.277 (-2.03)	-0.116 (-1.14)
$\rho_r$	0.977 (54.25)	0.987 (89.91)
$\rho_e$	0.257 (2.43)	0.316 (2.82)
$\sigma_y$	0.731 (14.26)	0.786 (18.16)
$\sigma_{\tilde{r}}$	0.595 (4.01)	0.827 (15.86)
$\sigma_{r^*}$	0.662 (4.60)	0.322 (-)
Log-likelihood	-440.58	-444.69
AIC / BIC	-4.978 / -4.796	-5.037 / -4.873

Notes: In model 2  $\sigma_{r^*} = 0.322$  is fixed.

AIC (BIC) = Akaike (Bayesian) information criterion

<sup>9</sup>It is however to clear why the equilibrium real interest rate should be smoother than the real interest rate. Smets and Wouters (2003) find for instance that the natural real rate, that in their model is defined as the real interest rate that obtains when all prices are flexible and nominal shocks are absent, varies *more* than the real interest rate.



## 4 Does a time-varying interest rate explain the puzzle?

We use two different kinds of output from the model to analyze the extent to which a time-varying equilibrium real interest rate can solve the excess sensitivity puzzle. First, the impulse response functions of the short-term interest rate to different shocks are derived. They demonstrate that the model is capable of generating larger movements at 20-40 quarter horizons with a time-varying interest rate than when it is assumed to be constant. Second, time series data on the term structure are generated from the model and changes in these artificial long-term interest rates are regressed on changes in the short-term rate. This is in a sense the original version of the excess sensitivity puzzle. If the long-term (five-year) interest rate reacts with a coefficient of 0.3 or more we conclude that our explanation is a potential solution to it. Again we compare the results with and without a time-varying equilibrium real interest rate.

The model is calibrated using the estimated values from Rudebusch (2002 a, b) given in table (1), and our estimated parameters concerning the time-varying equilibrium real interest rate.<sup>10</sup>

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<sup>10</sup>Because this model cannot be solved analytically we use algorithms provided in the Dynare-package for Matlab to obtain numerical simulated responses of the nominal interest rate  $i_t$  to shocks for specific sets of parameter values.

**Table 2: Parameter values**

Aggregate Supply	Aggregate Demand	Monetary Policy	Equilibrium real interest rate
$\phi_\pi = 0.29$	$\alpha_{y1} = 1.15$	$f_\pi = 1.53$	$\rho_r = 0.987$
$\beta_{\pi1} = 0.67$	$\alpha_{y2} = -0.27$	$f_y = 0.93$	$\rho_e = 0.316$
$\beta_{\pi2} = -0.14$	$\alpha_r = 0.09$	$f_i = 0$ (0.5)	$\sigma_{r^*} = 0.322$
$\beta_{\pi3} = 0.40$	$\sigma(\varepsilon^y) = 0.833$	$\sigma(\varepsilon^i) = 1$	
$\beta_{\pi4} = 0.07$			
$\beta_y = 0.13$			
$\sigma(\varepsilon^\pi) = 1.012$			

Sources: Rudebusch (2002a) (aggregate demand and supply), Rudebusch (2002b) (monetary policy), own estimates (equilibrium real interest rate).

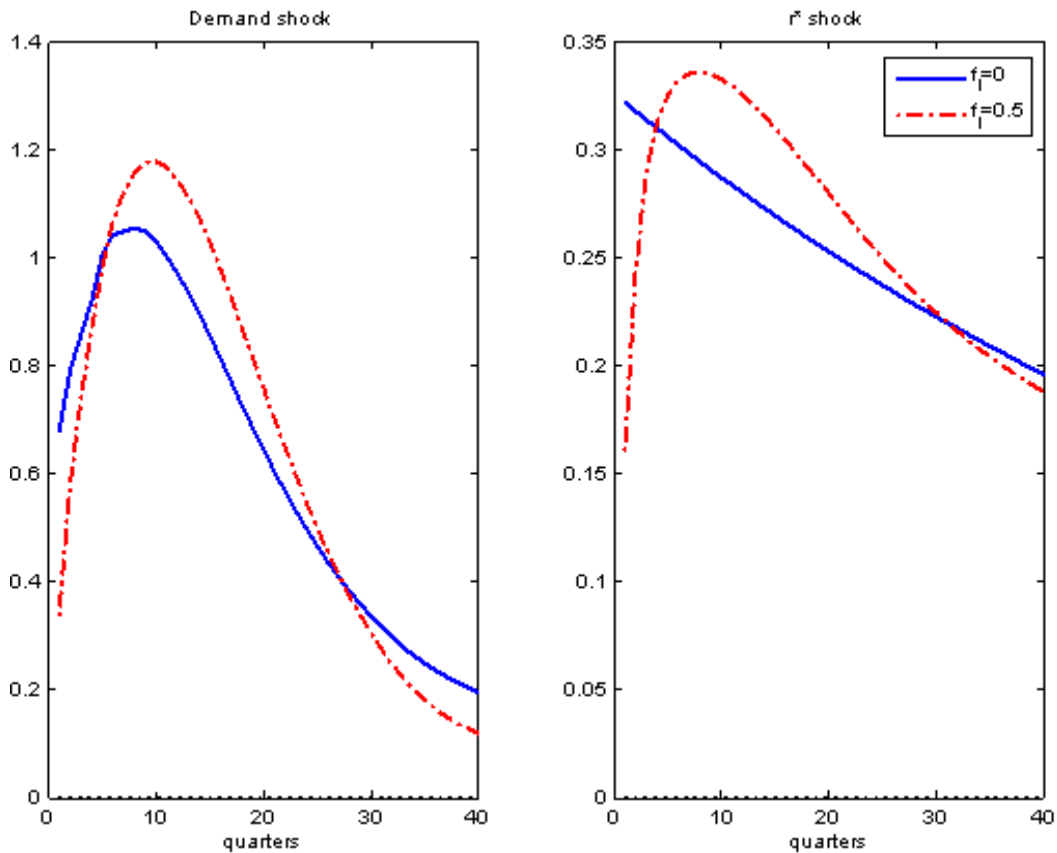
#### 4.1 Impulse responses of nominal interest rates

According to the rational expectations hypothesis of the term structure, long-term interest rates equal average expected future short-term interest rates over the horizon in question (plus possible term premia). The excess sensitivity puzzle can be illustrated in terms of movements in expected future short-term interest rates or forward rates in response to shocks. Standard models imply that expected short-term interest rates five or ten years into the future do not move in as the economy is hit by a shock. Including a time varying equilibrium real interest rate explains the excess sensitivity to the extent that expected future short-term interest rates react to shocks when this feature is added to the model.

Given a constant equilibrium real interest rate, our model implies that expected future short-term interest rates display small movements five years after a shock and no movements at all after ten years.<sup>11</sup> When the equilibrium real interest rate is allowed to react to shocks, the effects on forward rates

<sup>11</sup>See appendix C for an illustration.

are larger and more persistent. Figure (3) below shows in the left panel the response of the nominal short term interest rate to an aggregate demand shock, and the right panel shows the response to a natural real rate shock for the case without interest rate smoothing ( $f_i = 0$ ) and for the case with interest rate smoothing ( $f_i = 0.5$ ), respectively.



**Figure 2:** Responses of forward rates to shocks

Under the assumed parametrization of the model there is a considerable effect on forward interest rates of about 16 (interest rate smoothing) to 28 (no interest rate smoothing) basis points even at the ten year horizon in response to a demand shock. Even stronger results are obtained for a real interest rate shock with movements of about 36 (interest rate smoothing) to

38 (no interest rate smoothing) basis points at the ten year horizon. These effects are of the same magnitude as those found by Ellingsen and Soderstrom (2005). However, long-term interest rates move more in our model because the effects one to five year horizons are larger. In a sense, the puzzle is overexplained in Figure 2 because long-term interest rates or the average forward rate over e.g. ten years actually display larger movements than the short-term interest rate or period zero forward rate.

## 4.2 Regression evidence from interest rate changes

The original formulation of the excess sensitivity puzzle concerns the high correlation between changes in long-term interest rates and changes in the short-term interest rates used as monetary policy instrument. Some authors distinguish between expected and unexpected changes in monetary policy, where only unexpected changes are expected to affect long-term interest rates since expected changes are already discounted for. For instance, Ellingsen and Soderstrom (2005) find that the U.S. ten-year interest rates increases by 0.25 percentage points as the federal funds rate is unexpectedly increased by one percentage point.

We generate time series data (10 000 observations) on the short-term interest rate controlled by the central bank and the five-year interest rate from the model. The first differences of the long-term rate are then regressed on the short-term interest rate. Because we construct the long-term interest rate according to the expectations hypothesis, it is by definition only affected by unexpected changes in the short-term rate. Hence it is not necessary to distinguish between expected and unexpected monetary policy changes.

While impulse responses of forward rates show how much long-term interest rates move in response to specific shocks, this exercise demonstrates the extent to which the complete simulated model including all shocks (also those to which the equilibrium real interest rate does not respond) generates the observed co-movements of long-term and short-term interest rates.

Four different cases are examined in order to determine the effects of a time-varying equilibrium real interest rate and separate them from the effects of interest rate smoothing: With smoothing but a constant equilibrium real interest rate, without smoothing but still a constant equilibrium real interest rate, with smoothing and a time-varying equilibrium real interest rate, and finally without smoothing but with a time-varying equilibrium real interest rate.

**Table 3: Regression results**

	time-varying $r^*$	constant $r^*$
smoothing	0.4082	0.1904
no smoothing	0.2155	0.0312

Slope coefficients from regressing changes in long-term interest rates on changes in short-term interest rates  $\Delta i_t^{20} = \alpha + \beta \Delta i_t^1 + \epsilon_t$

Table 3 reports the results from regressing changes in long-term (five-year) interest rates on changes in the short-term (monthly) interest rate controlled by the central bank. The  $\beta$ -coefficients when the equilibrium real interest rate is affected by the same shocks as monetary policy are 0.41 with interest rate smoothing and 0.22 without interest rate smoothing. The corresponding numbers with a constant equilibrium real rate are 0.19 and 0.03, respectively. Hence, the  $\beta$ -coefficient is about 0.2 percentage points larger with a time-varying equilibrium real interest rate than without it. Quantitatively, including a time-varying equilibrium real interest rate can

hence explain about half the excess sensitivity puzzle.

The regression evidence conveys a slightly different picture of the effects of adding a time-varying equilibrium real interest rate and/or interest rate smoothing than the forward rate graphs. There are considerably more movements in the forward rates at long horizons with smoothing than without, which per se implies that smoothing helps to explain the excess sensitivity puzzle. However, regressing long-term rates on changes in short-term rates yields the opposite result - the slope coefficients are higher without smoothing than with. Clearly, long-term interest rates move more with the short-term interest rate in the same period when there is no smoothing and the shocks have larger effects on both interest rates on impact. Nevertheless, both types of results indicate that allowing the equilibrium real interest rate to vary over time and react to structural shocks helps to explain the excess sensitivity puzzle.

## **5 Conclusions**

Previous studies of the excess sensitivity of long-term interest rates to macroeconomic shocks have focused on generating persistent movements in expected inflation, for instance by introducing asymmetric information and learning about the preferences of the central bank. This paper explores an alternative explanation to the puzzle, namely that macroeconomic shocks cause persistent movements in the equilibrium real interest rate. The standard models of the monetary transmission mechanism used to analyze the excess sensitivity puzzle assume that the equilibrium real interest rate is constant over time and unrelated to macroeconomic shocks. However, empirical

evidence has shown that equilibrium real interest rates do vary considerably over time and is affected by shocks to e.g. preferences and productivity. We incorporate this notion into one of the standard models frequently employed in this literature and investigate to what extent it generates movements in long-term interest rates in response to shocks.

In order to obtain parameter values for the natural real interest rate equation we estimate a time varying equilibrium real interest rate for the U.S. over roughly the last 50 years using an unobserved components model. We show that the equilibrium real interest rate displays considerable time variation and is affected by structural shocks. Furthermore, movements in the equilibrium real interest rate are highly persistent.

Given the estimated parameters we use a stylized model to show that a time-varying equilibrium real interest rate that is influenced by structural shocks has the potential to account for co-movements of short term and long term interest rates. Slope coefficients from regressing changes in long-term rates on changes in short-term rates are about 0.2 percentage points higher when the natural rate is allowed to vary than when it is assumed to be constant. The intuition for the result is that due to the persistence in the equilibrium real interest rate, shocks have long lasting effects which transmit to forward rates and thereby also long-term interest rates.

Our approach focuses on the real side of the economy and we are thus not able to address nominal shocks. We do not argue that a time-varying equilibrium real interest rate that is affected by structural shocks is the only or even a more important factor behind the excess sensitivity puzzle than for instance informational asymmetries and imperfect knowledge as discussed

by Gürkaynak et al. (2005) and Ellingsen and Soderstrom (2005). However, allowing real shocks to affect the equilibrium real interest rate creates an additional mechanism through which the effects of shocks become more persistent. It would be desirable to develop models that incorporate several of these approaches and ideally also compare the explanatory power of each approach.

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# Appendices

## A State space representation

Our state space model has the following general representation:

$$Y_t = Z\alpha_t \quad (\text{Measurement equations})$$

$$\alpha_t = T\alpha_{t-1} + R\varepsilon_t \quad (\text{Transition equations})$$

where  $E(\varepsilon_t) = 0$ ,  $E(\varepsilon_t\varepsilon_s) = 0$  for all  $t \neq s$  and  $E(\varepsilon_t\varepsilon_t') = Q$  and

state vector  $\alpha_t = (\tilde{y}_t, \tilde{y}_{t-1}, \tilde{r}_t, \tilde{r}_{t-1}, r_t^*)'$

residual vector  $\varepsilon_t = (\varepsilon_t^y, \varepsilon_t^{\tilde{r}}, \varepsilon_t^{r^*})'$

### Measurement equations

$$\tilde{y}_t = \tilde{y}_t,$$

$$r_t = \tilde{r}_t + r_t^*$$

### Transition equations

$$\tilde{y}_t = a_{y1}\tilde{y}_{t-1} + a_{y2}\tilde{y}_{t-2} - a_r\tilde{r}_{t-1} + \varepsilon_t^y$$

$$\tilde{r}_t = d_1\tilde{r}_{t-1} + d_2\tilde{r}_{t-2} + \varepsilon_t^{\tilde{r}}$$

$$r_t^* = \rho_r r_{t-1}^* + \rho_e \varepsilon_t^y + \varepsilon_t^{r^*}$$

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} a_{y1} & a_{y2} & -a_r & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_1 & d_2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_r \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ \rho_e & 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} \sigma_y^2 & & \\ & \sigma_{\tilde{r}}^2 & \\ & & \sigma_{r^*}^2 \end{bmatrix}$$

Prediction equations

$$\hat{\alpha}_{t|t-1} = T\hat{\alpha}_{t-1} \quad (\text{B1})$$

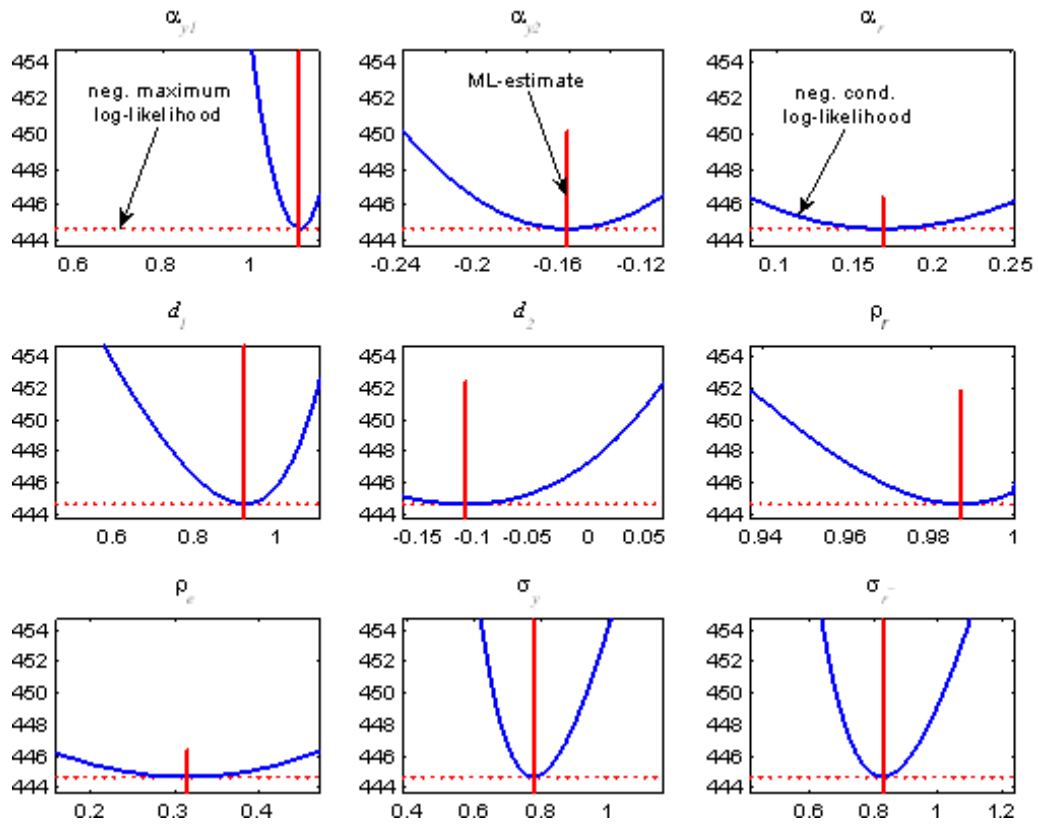
$$P_{t|t-1} = TP_{t-1}T' + RQR' \quad (\text{B2})$$

Updating equations where  $F_t = ZP_{t|t-1}Z'$ .

The likelihood can be computed conditional upon the initial observation  $Y_0$  using a prediction-error decomposition (Harvey, 1989, p. 125). The prediction error is defined as  $\nu_t = Y_t - Z\hat{\alpha}_{t|t-1}$ , and assuming that  $\alpha_t$  is Gaussian,  $\hat{\alpha}_{t|t-1}$  is also Gaussian with covariance matrix  $P_{t|t-1}$ . It follows that the log-likelihood can be written as

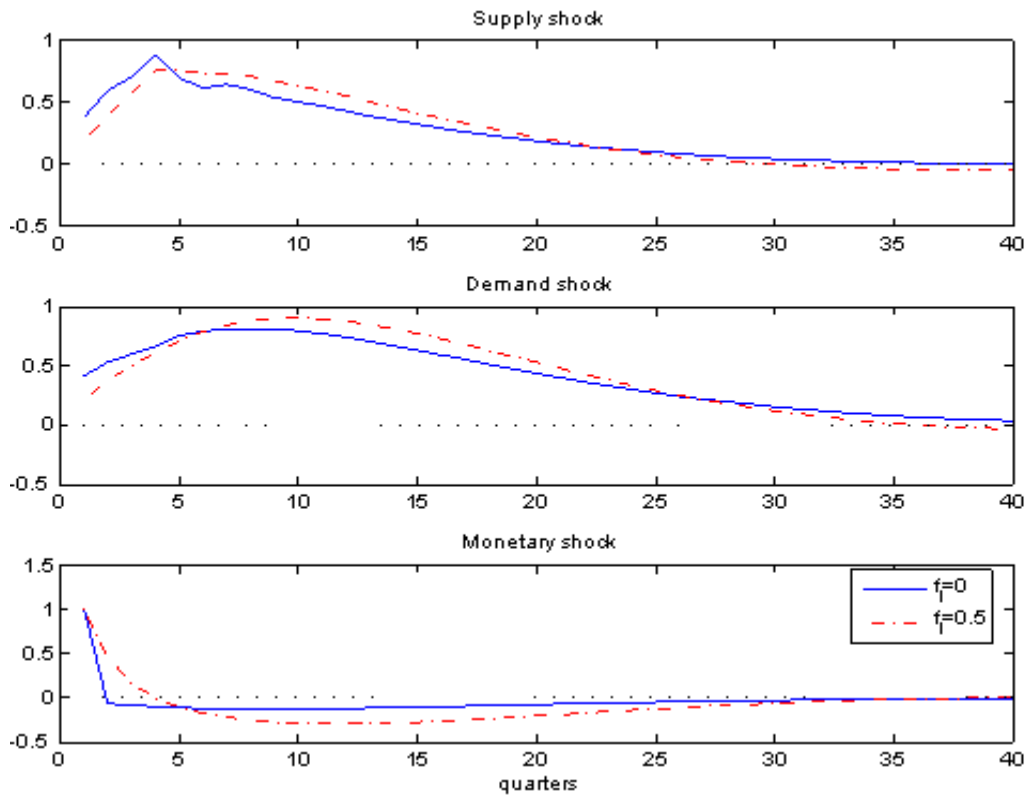
$$\log L(Y | \xi) = -\frac{NT}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log |F_t| - \frac{1}{2} \sum_{t=1}^T \nu_t' F_t^{-1} \nu_t. \quad (\text{B5})$$

## B Conditional likelihoods



**Figure 3:** Negative conditional log-likelihoods around the optimum for model 2.

### C Impulse responses of forward rates given a constant real interest rate



Impulse responses of forward rates to shocks given a constant equilibrium real interest rate

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