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# Myopic Loss Aversion, the Equity Premium Puzzle, and GARCH

Martin Ågren\*

October, 2006

## Abstract

The paper replicates the study of Benartzi and Thaler (1995), who suggest a behavioral explanation to the equity premium puzzle by myopic loss aversion. A technical extension to their methodology is suggested where conditional heteroskedasticity is incorporated when simulating returns, in place of the original temporal independence assumption. Swedish data is considered in addition to U.S. data. First, it is found that myopic loss aversion can explain the U.S. equity premium over bonds, although the obtained evaluation periods are somewhat shorter than a year. For example, over the full U.S. sample period from 1926 to 2003, evaluation periods of seven and ten months are found using the original and the new approach to simulating returns, respectively. Second, myopic loss aversion suggestively explains the Swedish equity premium as well, which is new to the literature. Third, throughout the analysis of both data sets, longer evaluation periods are obtained under conditional heteroskedasticity. The last result indicates that myopic loss-averse and, in turn, cumulative prospect theory investors are sensitive to the distributional assumption made on returns.

*JEL classification:* G12, C22

*Keywords:* prospect theory, loss aversion, equity premium puzzle, conditional heteroskedasticity, GARCH

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# 1 Introduction

Imagine having the opportunity to flip a coin and win either \$50 or \$100, a bet with an expected value of \$75. How much would you be willing to pay for such a bet to take place? Following the results of Mehra and Prescott (1985), an individual is so averse to the uncertainty of this gamble that she would think it is worth only \$51.21. Their study, however, does not concern gambles of this kind, but focuses on the risk-return relationship of stocks to less risky assets, such as bonds. Stocks have outperformed bonds to a large extent over the past century. Using a standard consumption-based general equilibrium model, Mehra and Prescott (1985) find that in order to explain the large equity risk premium a relative risk aversion in excess of 30 is needed. Theoretical arguments and estimates from various studies, such as Arrow (1971) and Friend and Blume (1975), suggest that this parameter should be in the neighborhood of three. Mehra and Prescott (1985), finding the model implied level of risk aversion unreasonably high, announce the existence of an anomaly they call the equity premium puzzle (EPP).

Economists have been struggling to solve the puzzle for more than twenty years now. Plausible explanations have been studied with varying success. As originally stated, Mehra and Prescott's (1985) model relies on three assumptions on individual behavior and asset market structure. First, individual preferences are explained by a power utility function over consumption in an expected utility framework.<sup>1</sup> Second, the market is complete, meaning that every element of risk can be diversified, and, third, the market is free of transaction costs. Although all three assumptions have been debated when trying to solve the puzzle, the literature on the choice of individual preferences is most profound. Attempts in explaining the EPP within this line of research include generalized expected utility of Epstein and Zin (1991), and habit formation of Constantinides (1990).<sup>2</sup>

Behavioral finance presents an alternative explanation from those of the traditional framework. Benartzi and Thaler (1995) (BT henceforth) suggest a clarification of the EPP that incorporates several experimentally observed behavioral concepts, the most important ones being loss aversion and mental accounting. Loss aversion is the individual tendency to be more sensitive to losses than to gains, and is the main ingredient of Kahneman and Tversky's (1979, 1992) prospect theory. This descriptive utility theory can explain some of the economic anomalies that neoclassical expected utility theory cannot. Mental accounting refers to how people

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<sup>1</sup>The most common power utility function is  $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$ , which has a constant relative risk aversion equal to  $\gamma$ .

<sup>2</sup>Kocherlakota (1996) and Siegel and Thaler (1997) present general literature overviews.

are affected by information feedback on their portfolio. BT argue that loss-averse investors will find a risky portfolio even more risky if they evaluate it frequently. Evaluating the portfolio less "myopically" will reduce the risk.<sup>3</sup> The two concepts together form a preference scheme BT call myopic loss aversion (MLA). Using monthly data on aggregate stock returns and five-year bond returns in the United States over the historical period 1926 to 1990, BT derive the prospect theory utility of holding an all-stocks and an all-bonds portfolio at various evaluation periods (information feedback frequencies). The large observed equity premium can be explained by MLA preferences if financial investors have an evaluation period of approximately twelve months, since the two portfolio's have equal utilities at this evaluation period. BT conclude that their result is intuitive, since most individual investors file their taxes on a yearly basis, and receive reports from their brokers, mutual funds and retirement accounts annually.

The present paper takes a closer look at the proposed MLA solution to the EPP. Specifically, the distributional assumption made on stock returns is addressed. When deriving the prospective utility of holding a portfolio at a specific evaluation period, the portfolio returns distribution needs to be simulated. BT use a non-parametric bootstrap technique in this simulation, implying that any existing serial correlation in returns is removed by construction. In fact, their method implicitly assumes that stock returns are independent over time. Fama and French (1988), among others, point out the existence of mean reversion (negative serial correlation) in stock returns, and one might consider whether neglecting this could have affected BT's results. As BT claim though, the findings of mean reversion are only trivial over short periods of evaluation (up to one year), and should not be a concern. The claim that there exists a predictable component in average stock returns over longer time horizons has, also, been criticized by, e.g., Lamoureux and Zhou (1996). However, is the assumption of temporal independence justified? Although the first moment might be uncorrelated, what about the second? In the finance literature it is broadly accepted that stock returns display variations in volatility over time, i.e., return volatility tends to cluster, bringing time periods of frequent large swings, and other periods of calm and low volatility in returns. Such conditional heteroskedasticity affects the unconditional returns distribution, making its shape different from the one obtained under temporal independence. Will a simulation approach incorporating conditional heteroskedasticity still support BT's proposed explanation to the EPP? How is the evaluation period consistent with the historical equity premium affected by conditional heteroskedasticity? The current paper investigates these issues.

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<sup>3</sup>To read more on mental accounting, see Thaler (1985) and Thaler (1999).

Relaxing the temporal independence assumption, the paper introduces a parametric approach to simulating returns distributions in the model of BT. The method takes the stock return's varying volatility-structure into account by estimating a generalized autoregressive conditionally heteroskedastic (GARCH) process from which the distribution of returns is simulated. Since various evaluation periods, i.e., aggregations of data to different frequencies are considered, I follow Drost and Nijman's (1993) work on temporal aggregation of GARCH processes. Drost and Nijman (1993) prove that the set of symmetric (weak) GARCH( $p,q$ ) processes is closed under temporal aggregation. Thus when aggregating high-frequency data generated by, say, a GARCH(1,1) process, the obtained lower frequency data generating process is also GARCH(1,1) but with a new set of parameter values. Drost and Nijman (1993) present formulas for deriving these low-frequency parameters using the corresponding high-frequency ones. Aggregated low-frequency returns can then be simulated from this aggregated GARCH process, and conditional heteroskedasticity is preserved under aggregation.

The study considers the U.S. equity premium, and the Swedish one as well. Campbell (2002) shows that the EPP exists not only in the U.S. but in other economies, among them Sweden. Both data sets consist of monthly aggregate stock and long-term bond returns covering 1926 to 2003 for the U.S., and 1919 to 2003 for Sweden. Following BT, the analysis is carried out by calculating the prospect theory utilities of an all-stocks and an all-bonds portfolio as a function of the evaluation period. Tversky and Kahneman (1992) estimates of the prospect theory parameters are employed, although parameter variations are considered in the subsequent sensitivity analysis. When simulating returns, both the non-parametric bootstrap approach, following BT, and the proposed parametric method incorporating conditional heteroskedasticity are considered.

Over the full U.S. sample of data, evaluation periods of seven and ten months are obtained when using the non-parametric bootstrap and the parametric GARCH approach, respectively. These evaluation periods are smaller than the twelve-month counterpart reported by BT. The twelve-month evaluation period is not replicated over BT's sample period of 1926 to 1990 either. When simulating returns under temporal independence using this subsample, an evaluation period of six months is obtained. One far reached explanation for the difference is the use of different data sets; the current study uses data from Ibbotson Associates, while BT use CRSP data. Perhaps more likely however, the interpretation of BT's non-parametric bootstrap simulation technique could somehow be mistaken, although I have no reason to believe this to be the case. Nonetheless, obtaining a six-month evaluation period

instead of twelve months suggests that the MLA model is sensitive to the method used when simulating portfolio returns distributions.

A problem with the GARCH approach occurs when the full Swedish data sample is considered, and analysis using this approach is made on two subsamples of data instead.<sup>4</sup> The non-parametric approach produces a twelve-month evaluation period over the full sample however, in line with BT, which suggests that MLA can explain the large Swedish equity premium. This result is new to the literature, since previous studies have not applied MLA preferences to the EPPs of non-U.S. economies. Furthermore, longer evaluation periods are obtained under conditional heteroskedasticity when studying the two subsamples of data. For example, over the period from July 1961 to December 2003, evaluation periods of ten and fifteen months are obtained using the bootstrap and GARCH methods, respectively.

Throughout the analysis of both data sets, an overall longer evaluation period is found to be consistent with the observed equity premium when the GARCH approach to simulating returns is used compared with the bootstrap approach. The result suggests that the MLA investor finds stocks to be more risky when return volatility is time-varying. Also, it further indicates that the MLA model is sensitive to the procedure applied when simulating returns, which, in turn, indicates that MLA investors are sensitive to the shape of the returns distribution. Therefore, the two simulation techniques are analyzed concerning how the first four unconditional moments evolve as the evaluation period increases. Plausibly, the skewness of the returns distribution is important for the loss-averse investor, which is intuitive since loss aversion induces an asymmetric preference over gains and losses. Furthermore, although somewhat different evaluation periods are obtained, they can be considered to be of similar magnitude as the twelve-month counterpart reported by BT, thus supporting their result.

The rest of the paper is outlined as follows: Section 2 presents some related research. Section 3 introduces the MLA framework, where prospect theory is central, and discusses the intuition behind its solution to the EPP. In section 4, BT's approach to simulating returns distributions is explained. The section also introduces the paper's contribution to this simulation concerning GARCH processes. Section 5 presents an application to financial data, where results are reported on and discussed. Section 6 concludes.

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<sup>4</sup>See section 5.3.

## 2 Related Research

BT influence a large body of related research. Barberis, Huang, and Santos (2001) introduce loss aversion in a consumption-based general equilibrium model. They find that loss aversion alone does not produce a large enough equity premium however, and, therefore, extend the model to incorporate the importance of prior outcomes, another idea from psychology. A loss is seen as less painful if it comes after a period of gains, while it hurts badly if it follows subsequent losses. Using this extended model, Barberis et al. (2001) report an equity premium of historical magnitude at reasonable parameter values.

Disappointment aversion of Gul (1991) is another set of preferences related to loss aversion and prospect theory that has been applied to the EPP. Similar to loss aversion, a disappointment averse agent has an asymmetric preference for outcomes. One main difference is that the reference point from which gains and losses are derived is endogenous in the model. Loss aversion usually sets current wealth as the (exogenous) reference point. Ang, Bekaert, and Liu (2005) employ disappointment aversion in studying the equity premium, and find that a reasonable level of disappointment aversion is consistent with the historical U.S. equity premium.

Durand, Lloyd and, Wee Tee (2004) investigate BT's methodology, just like I do in this paper. To determine the "equilibrium" evaluation period, BT use a graphical inspection of the crossing-point of two lines, which represent the respective all-stocks and all-bonds portfolio utilities at different evaluation periods. Durand et al. (2004) rely on statistical tests to determine the crossing-point, e.g., the Wilcoxon single-rank test, and argue that BT's analysis is not robust to the modification. However, one can criticize Durand et al. (2004) in the way they interpret BT's method of sampling lower frequency returns. They claim that low-frequency returns are sampled over data clusters, which implies that a number of return observations can overlap, in turn implying that returns are not assumed independent. I believe this is an erroneous interpretation.

Related work concerns experimental studies of MLA preferences among individual investors as well. This research involves both students (Thaler, Tversky, Kahneman, and Schwartz, 1997) and professional traders (Haigh and List, 2004) in an individual setting as well as under market conditions (Gneezy, Kapteyn, and Potters, 2003). The major result is that MLA is an observed behavior among financial investors.

### 3 Myopic Loss Aversion

MLA combines mainly two experimentally observed behavioral concepts, namely loss aversion and mental accounting. Prospect theory of Kahneman and Tversky (1979) and its modified version cumulative prospect theory of Tversky and Kahneman (1992) incorporate loss aversion. Cumulative prospect theory extends the original version by making it possible to derive the utility of gambles of more than two outcomes, and by incorporating first-order stochastic dominance. Before discussing the intuition behind BT's proposed explanation to the EPP, cumulative prospect theory is presented.

#### 3.1 Cumulative Prospect Theory

In line with empirical evidence, cumulative prospect theory has the following key elements: reference dependence; utility is derived through comparing the outcome with a reference level of outcome, loss aversion; losses loom larger than gains do, risk-seeking; while being risk-averse over pure gains individuals are risk-seeking over pure losses, and non-linear probabilities; outcome probabilities are not weighted linearly but are non-linearly transformed.

Following Tversky and Kahneman (1992), the cumulative prospect theory utility (henceforth prospective utility)  $U$  of a lottery  $L$  with outcomes  $\{x_i\}_{i=1}^n$  and corresponding probabilities  $\{p_i\}_{i=1}^n$  is derived as

$$U(L) = \sum_{i=1}^n \pi(p_i) \cdot v(x_i), \quad (1)$$

where  $n$  is the total number of outcomes,  $\pi(\cdot)$  is a function that transforms probabilities, and  $v(\cdot)$  is a value function. This value function depends on changes in outcomes rather than absolute levels, as in traditional expected utility. Assuming a zero reference point, the value function is defined as

$$v(x) = \begin{cases} x^\gamma & \text{if } x \geq 0 \\ -\lambda(-x)^\gamma & \text{if } x < 0 \end{cases}, \quad (2)$$

where  $\gamma$  reflects the degrees of risk aversion over gains and risk-seeking over losses, and  $\lambda$  is the loss aversion parameter.<sup>5</sup> Tversky and Kahneman (1992) estimate the parameters in (2) through laboratory experiments to  $\hat{\gamma} = 0.88$  and  $\hat{\lambda} = 2.25$ , which

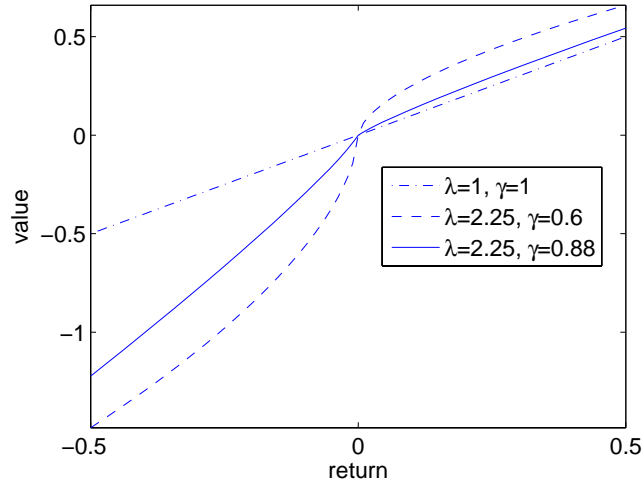
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<sup>5</sup>Tversky and Kahneman (1992) actually consider two separate parameters for risk aversion over gains and risk-seeking over losses, respectively, but as the estimates of the two are equal, I choose to unify the parameters.



will be used throughout the application.<sup>6</sup> These estimates result in an "S-shaped" value function kinked at the origin, as figure 1 shows.

**Figure 1:** The Value Function



The figure displays the cumulative prospect theory value function for varying parameter values.

The probability transformation function  $\pi(\cdot)$  uses the whole cumulative distribution function as an argument. Ranking all outcomes in increasing order from  $-m$  to  $n$ , where  $m$  and  $n + 1$  are the numbers of strictly negative and positive outcomes, respectively, the function has the following form:

$$\pi(p_i) = \begin{cases} w(p_{-m}) & \text{if } i = -m \\ w(p_{-m} + \dots + p_i) - w(p_{-m} + \dots + p_{i-1}) & \text{if } -m < i < 0 \\ w(p_i + \dots + p_n) - w(p_{i+1} + \dots + p_n) & \text{if } 0 \leq i \leq n - 1 \\ w(p_n) & \text{if } i = n \end{cases}, \quad (3)$$

where

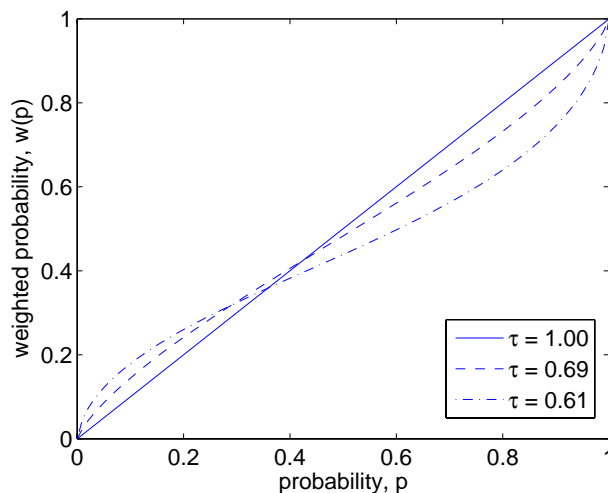
$$w(p) = \frac{p^\tau}{(p^\tau + (1-p)^\tau)^{1/\tau}}. \quad (4)$$

The functional form of  $w(\cdot)$  in (4) is an attempt in describing the Allais-type behavior violating the expected utility theorem.<sup>7</sup> Figure 2 presents the weighting function  $w(\cdot)$ . Instead of weighting probabilities linearly in an objective way, the probability of gains and losses are weighted subjectively. When  $\tau < 1$  in (4), the transformation  $\pi(\cdot)$  over-weights small cumulative probabilities, and under-weights

<sup>6</sup>The reference point will be zero.

<sup>7</sup>Allais (1953) challenges the expectation principle by showing that a change in probabilities from 0.99 to 1 has larger impact on preferences than a change from 0.10 to 0.11.

**Figure 2:** The Probability Weighting Function



The figure displays the cumulative prospect theory probability weighting function at different parameter values.

moderate to large ones. Tversky and Kahneman (1992), conducting individual experiments, estimate the parameter  $\tau$  to 0.61 in the domain of gains, and 0.69 in the domain of losses. An estimate of  $\tau$  equal to one produces linear weights so that probabilities are treated objectively. Moreover, the probability transformation  $\pi(\cdot)$  should not be mistaken for a probability measure, since the weighted probabilities necessarily do not sum up to one.

### 3.2 Explaining the Large Equity Premium with Myopic Loss Aversion

BT use cumulative prospect theory in their proposed MLA solution to the EPP, and emphasize the role of loss aversion for the individual decision-maker. They stress an important implication the preference scheme brings to portfolio evaluation periods, i.e., the mental accounting loss-averse investors perform. To further understand the core of their reasoning, loss aversion and mental accounting are well illustrated by the famous example of Samuelson (1963). In the example, Samuelson offers a colleague of his a fifty-fifty chance of winning \$200 or losing \$100. The colleague turns the bet down, saying that he would feel a \$100 loss more than a \$200 gain, but expresses the willingness to take on one hundred such bets. This exemplifies loss aversion and, also, the kind of mental accounting it can imply. To see why, assume

that Samuelson's colleague has the following piece-wise linear value function over changes in wealth:

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2.5x & \text{if } x < 0 \end{cases} . \quad (5)$$

Considering the value function in (5) and, for simplicity, objective probabilities, the prospective utility of the single bet equals  $0.5 \cdot 200 + 0.5 \cdot 2.5 \cdot (-100) = -25$ . Since its prospective utility is negative, the bet is rejected. But what about a game of two bets? The attractiveness of this gamble will depend heavily on the mental accounting of the problem. If the two bets are treated separately the game has double the unattractiveness. However, if the two bets are compounded into a single bet, it will have positive expected utility, and will be accepted by the colleague.<sup>8</sup> As it turns out, compounding any number of bets greater than one will be favorable for the colleague so long as he does not have to monitor the separate bets being played. Moreover, Samuelson (1963) proves in a theorem that if an individual turns down a bet at every level of wealth, accepting a multiple gamble contradicts expected utility maximization. Thus the behavior of Samuelson's colleague is inconsistent with the traditional theory.

A parallel to the above example is a loss-averse investor choosing between stocks and bonds. In this setup, the evaluation period is crucial for the investor's attitude to the risk of the investment. If the decision-maker evaluates her portfolio on a daily basis a portfolio consisting of stocks will be unattractive, since stock returns go down almost as often as they go up from day to day, and losses are mentally doubled. On the other hand, consider a long evaluation period of, say, ten years. The investor can rest assured that stocks most surely will increase in value every ten years. Hence, a stock portfolio can be an unattractive investment if evaluated often, but an attractive one over longer evaluation periods. Low-risk bond portfolios are not affected by this phenomenon to the same extent, since they do not display as frequent losses. Moreover, it is important to distinguish an investor's evaluation period from her investment horizon. Although the planning horizon may be five years, the time between portfolio evaluations can be twelve months.

The above argument brings two questions to mind. First, how loss-averse are financial investors? Second, assuming a reasonable level of loss aversion, how short an evaluation period will be in accordance with the observed equity risk premium? BT use the estimate of loss aversion provided by Tversky and Kahneman (1992), i.e. 2.25, and find that an investor with cumulative prospect theory preferences

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<sup>8</sup>The compounded lottery has outcome-probability-set  $\{\$400, 0.25; \$100, 0.50; -\$200, 0.25\}$  with prospective utility equal to 25.

will be indifferent between an all-stocks portfolio and an all-bonds portfolio if the evaluation period is twelve months. At this interval in between evaluations there is an equilibrium, where investors are content with the risk-return relationship of stocks and bonds.<sup>9</sup> Thus BT argue that MLA preferences are consistent with the historical magnitude of equity premium, since it compensates the investor for her fear of stock portfolio losses as well as her myopic way of evaluating the portfolio.

## 4 Simulating Returns Distributions

To determine the evaluation period that makes a loss-averse investor indifferent between the returns of stocks and bonds, BT derive the prospective utilities of holding these assets at various lengths in between evaluations. Different frequencies of data are used to reflect these evaluation periods. If the agent evaluates her portfolio every six months her utility of holding a stock portfolio is derived using six-month data on stock returns. To apply equation (1), the possible portfolio outcomes and corresponding probabilities need to be determined at each data frequency. With a historical data set at hand, this involves simulating distributions of returns at different frequencies. Such simulations can be performed in different ways. BT use a non-parametric bootstrap approach.

### 4.1 Non-Parametric Bootstrap Approach

Using the high-frequency monthly data, an  $n$ -month return is simulated by, first, drawing  $n$  returns at random (with replacement), and, second, deriving the low-frequency return as if the  $n$  returns were consecutive. Let us say the three monthly returns  $x_1$ ,  $x_2$  and  $x_3$  are drawn. Using these high-frequency returns, the low-frequency three-month return is calculated as  $(1 + x_1)(1 + x_2)(1 + x_3) - 1$ . The procedure is performed 100,000 times to obtain a smooth  $n$ -month return distribution.<sup>10</sup> A histogram over the data is then derived, using an interval size of choice, so that one can associate the possible returns (midpoint of every histogram interval) with a specific probability (the frequency of returns in each interval divided by the total number of returns).<sup>11</sup> For example, BT use an interval size of twenty, which

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<sup>9</sup>The evaluation period where stocks and bonds are associated with equal utilities is at times referred to as the equilibrium evaluation period. This equilibrium should not be mistaken for a general equilibrium.

<sup>10</sup>When  $n = 1$ , drawing 100,000 returns with replacement will produce a large number of equal returns, since there is a limit to the sample size. As  $n$  increases the number of possible combinations increases, and a smoother (more continuous) set of low-frequency returns is obtained.

<sup>11</sup>The interval size for the histograms is fifty throughout the application. The results are not dependent on the interval size however.

constructs a distribution over twenty portfolio outcomes. Hence, the risky stock portfolio investment is seen as a gamble over twenty predetermined outcomes with specific probabilities. Equation (1) is thus directly applicable.

Using historical data on any portfolio, it is straightforward to derive its prospective utility at an evaluation period of choice. One only needs to decide on the level of loss aversion and how often the investor evaluates her portfolio, i.e., what frequency of data to use. Since the non-parametric bootstrap method constructs low-frequency data by drawing returns at random, any existing serial correlation is removed. Implicitly, the observations are assumed independent. A direct way to produce low-frequency returns without removing a serial dependence would be to derive the actual  $n$ -month returns. However, such a method does not produce sufficiently many observations to obtain a smooth distribution of returns. Close to 100,000 observations are needed for this purpose, motivating the use of simulation techniques. By introducing a parametric approach, this paper relaxes the independence assumption of BT, and produces smooth returns distributions where conditional heteroskedasticity is preserved.

## 4.2 Parametric Approach Using a GARCH Model

In the finance literature, it is broadly accepted that financial time series, e.g., exchange rates and stock market returns display volatilities that vary over time. One way of modeling the fluctuations is by fitting the data to a GARCH model, which parametrizes the conditional variance. Bollerslev (1986) introduces the generalization of ARCH models, originally presented by Engle (1982). Furthermore, Drost and Nijman (1993) study the temporal aggregation of GARCH processes, and prove that the set of symmetric (weak)  $\text{GARCH}(p,q)$  processes is closed under temporal aggregation.<sup>12</sup> Thus when aggregating high-frequency data generated by, say, a  $\text{GARCH}(1,1)$  process, the obtained lower frequency data generating process is also  $\text{GARCH}(1,1)$  but with a new set of parameter values. Formulas for deriving these low-frequency parameters using the corresponding high-frequency ones are presented.

### 4.2.1 Temporal Aggregation of GARCH Processes

In the application, the high-frequency dynamics is modeled using a  $\text{GARCH}(1,1)$  model with Gaussian innovations. This data generating process is the most widely

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<sup>12</sup>Drost and Nijman (1993) point out that in applied work one assumes that the result holds for strong GARCH processes too.

used GARCH model in the finance literature, and captures the conditional heteroskedasticity of financial time series well. The unconditional data distribution can be shown to have fatter tails than the normal distribution, which is a commonly observed feature of empirical stock returns distributions.

Letting  $\{x_t\}_{t \in \mathbb{Z}}$  denote the return series, the GARCH (1,1) model takes on the following form:

$$\begin{aligned} x_t &= c + u_t, \\ u_t &= \sqrt{h_t} \cdot v_t, \\ h_t &= \psi + \beta h_{t-1} + \alpha u_{t-1}^2, \end{aligned} \tag{6}$$

where  $\{v_t\}$  is an independent identically distributed Gaussian sequence with zero mean and unit variance, and  $(c, \psi, \beta, \alpha)'$  gathers the model's parameters. The conditional variance of the innovation  $u_t$  is given by  $h_t$ . The restrictions  $\psi > 0$ ,  $\beta \geq 0$ , and  $\alpha \geq 0$  are sufficient to ensure the non-negativity of  $h_t$ . Also, the restriction  $\alpha + \beta < 1$  is needed for  $h_t$  to be covariance-stationary.<sup>13</sup>

Letting  $h_{(m)t}$  denote the conditional variance of the aggregated (low-frequency) series  $\{u_{(m)t}\}$ , Drost and Nijman (1993) show that  $h_{(m)t}$  has GARCH(1,1) dynamics

$$h_{(m)t} = \psi_{(m)} + \beta_{(m)} h_{(m)t-1} + \alpha_{(m)} u_{(m)t-1}^2, \tag{7}$$

where  $\psi_{(m)}$ ,  $\beta_{(m)}$  and  $\alpha_{(m)}$  are the aggregated GARCH(1,1) parameters. Following Drost and Nijman (1993), these low-frequency parameters are given by

$$\psi_{(m)} = m\psi \frac{1 - (\beta + \alpha)^m}{1 - (\beta + \alpha)}, \tag{8}$$

$$\alpha_{(m)} = (\beta + \alpha)^m - \beta_{(m)}, \tag{9}$$

where  $\beta_{(m)} \in (0, 1)$  is the solution to the quadratic equation

$$\frac{\beta_{(m)}}{1 + \beta_{(m)}^2} = \frac{a(\beta, \alpha, \kappa_u, m)(\beta + \alpha)^m - b(\beta, \alpha, m)}{a(\beta, \alpha, \kappa_u, m)\{1 + (\beta + \alpha)^{2m}\} - 2b(\beta, \alpha, m)}, \tag{10}$$

with

$$\begin{aligned} a(\beta, \alpha, \kappa_u, m) &= m(1 - \beta)^2 + 2m(m - 1) \frac{(1 - \beta - \alpha)^2(1 - \beta^2 - 2\beta\alpha)}{(\kappa_u - 1)\{1 - (\beta + \alpha)^2\}} \\ &\quad + 4 \frac{\{m - 1 - m(\beta + \alpha) + (\beta + \alpha)^m\}\{\alpha - \beta\alpha(\beta + \alpha)\}}{1 - (\beta + \alpha)^2}, \end{aligned} \tag{11}$$

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<sup>13</sup>To read more on GARCH processes, see Hamilton (1994).

and

$$b(\beta, \alpha, m) = \{\alpha - \beta\alpha(\beta + \alpha)\} \frac{1 - (\beta + \alpha)^{2m}}{1 - (\beta + \alpha)^2}. \quad (12)$$

Notice that the unconditional kurtosis,  $\kappa_u$ , is included in equation (11). It is derived as

$$\kappa_u = \kappa_\xi \frac{1 - (\beta + \alpha)^2}{1 - (\beta + \alpha)^2 - (\kappa_\xi - 1)\alpha^2}, \quad (13)$$

where  $\kappa_\xi = 3$  denotes the kurtosis of the standard Gaussian innovations  $\{v_t\}$ .

#### 4.2.2 How Are the Formulas Applied?

Using the temporal aggregation of GARCH processes framework, it is possible to keep the serial dependence in the data variance when aggregating observations. To simulate a distribution for returns at an aggregate level, one starts by estimating the high-frequency GARCH(1,1) model in (6). The aggregated GARCH parameters are derived using the above formulas (8) - (13). Low-frequency innovations,  $u_{(m)t}$ , are simulated using equation (7) together with  $u_{(m)t} = \sqrt{h_{(m)t}} \cdot v_t$ . After adding the aggregate mean return to these innovations, the set of low-frequency returns is complete, and the simulated low-frequency returns distribution can finally be derived using the same histogram procedure as in the non-parametric bootstrap approach, making equation (1) directly applicable.<sup>14</sup> Thus, the GARCH approach is an alternative to the bootstrap approach to simulating returns distributions at various data frequencies. The important difference is that the GARCH procedure takes the conditional heteroskedasticity of financial data into account.

### 4.3 Differences in Simulated Distributions

What do the simulated distributions look like? Figure 3 exemplifies distributions for 3-, 12-, and 24-month returns using both the non-parametric bootstrap (panels A1-A3), as well as the GARCH approach (panels B1-B3). Monthly U.S. stock returns over the sample period from January 1926 to December 2003 are used (see table 1). Quite naturally, the distributions display larger means and become more outspread with a greater aggregation, irrespective of the method used. Notice that the distributions estimated with the non-parametric approach tend to display a larger (positive) skewness with a greater aggregation. With the GARCH approach the distributions are symmetric by construction. By visual inspection it is difficult to detect any differences in kurtosis across the two approaches, although there might

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<sup>14</sup>The aggregate  $m$ -month mean return is derived as  $(1 + \eta)^m - 1$ , where  $\eta$  denotes the mean of the monthly return series.

**Table 1:** Summary Statistics for U.S. and Swedish Monthly Returns

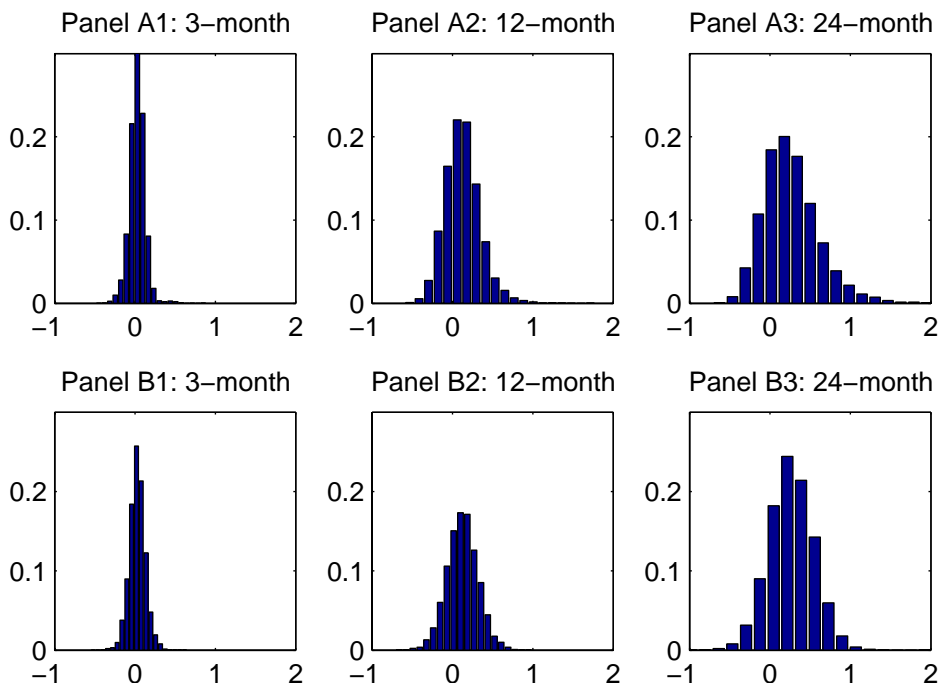
	U.S.		U.S.		Sweden		Sweden		Sweden	
	1926:1 - 2003:12		1926:1 - 1990:12		1919:1 - 2003:12		1919:1 - 1961:6		1961:7 - 2003:12	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
Mean (%)	0.99	0.46	0.97	0.40	0.92	0.56	0.62	0.39	1.23	0.74
Max. (%)	42.56	15.23	42.56	15.23	27.58	16.47	19.39	16.47	27.58	7.51
Min. (%)	-29.73	-9.82	-29.73	-8.41	-27.12	-10.79	-27.12	-10.79	-21.49	-5.93
Std. dev. (%)	5.62	2.27	5.86	2.20	4.87	1.91	4.01	2.09	5.58	1.70
Skewness	0.39	0.68	0.46	1.00	-0.11	0.30	-0.52	0.70	-0.04	-0.30
Kurtosis	12.45	8.10	12.49	9.55	6.19	12.57	8.91	15.34	4.74	5.68
Annual EP (%)	6.55		7.06		4.41		2.80		6.04	
No. obs.	936		780		1020		510		510	

The figure presents summary statistics of nominal returns on the U.S. and the Swedish aggregate stock markets over various sample periods. The U.S. data is provided by Ibbotson Associates, and the Swedish data stems from an updated version of Frennberg and Hansson's (1992) data set. EP stands for equity premium.



be such. Overall, the two methods of generating stock returns distributions show changes in, possibly, all four unconditional moments as the aggregation increases.

**Figure 3:** Estimated Stock Returns Distributions for 3-, 12-, and 24-Month Returns



The figure illustrates approximated distributions of financial stock returns. The non-parametric bootstrap approach is employed in panels A1-A3, while panels B1-B3 use the GARCH approach. U.S. aggregate stock returns covering 1926:1 - 2003:12 are used. The histogram interval size is twenty.

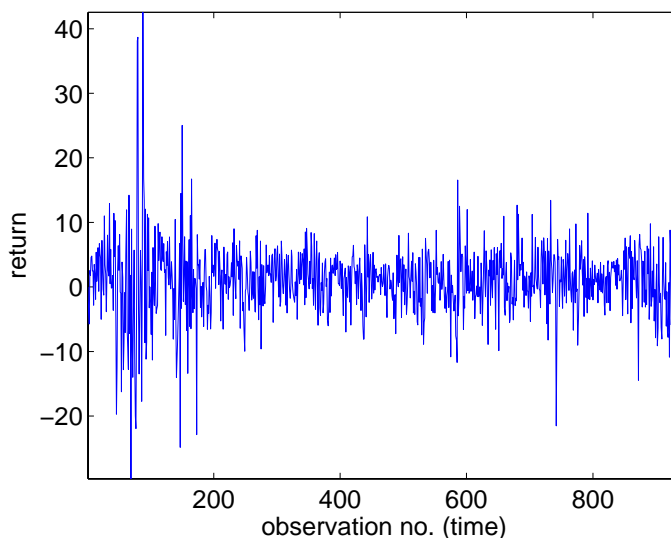
## 5 Application to Financial Data

This section returns to the proposed MLA solution to the puzzling magnitude of historical equity premium. Recall that the investor's preferences have two main factors of risk: loss aversion and portfolio evaluation myopia. Just as BT, the relationship between the levels of these risks and the equity premium is analyzed.

### 5.1 Data Sets

Campbell (2002) shows that the EPP exists not only in the U.S. but in several other economies, among them Sweden. I consider two sets of data, representing the U.S. and Sweden. In this way, the proposed explanation of BT is analyzed not only focusing on the originally investigated U.S. equity premium, but another one as well.

**Figure 4:** U.S. Monthly Stock Returns Covering 1926 to 2003



The table displays U.S. monthly stock returns (%) over the sample period from January 1926 to December 2003. Data source: Ibbotson Associates.

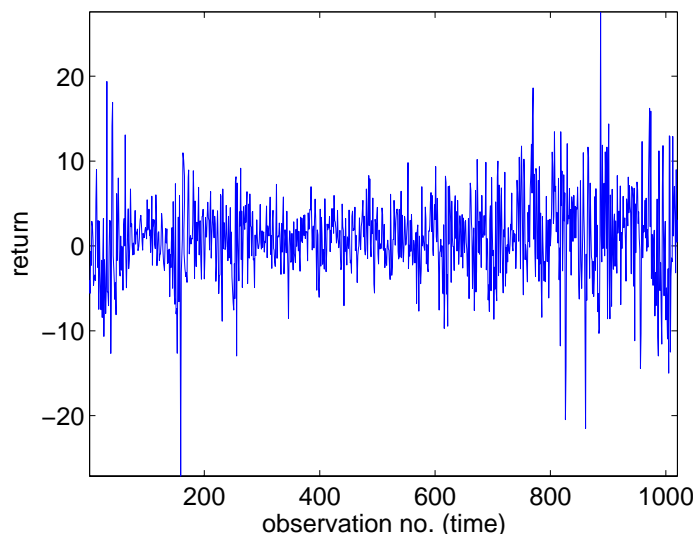
The U.S data is from Ibbotson Associates, while the Swedish data set is an updated version of the one presented by Frennberg and Hansson (1992). Following BT, long-term bond returns are considered when studying the equity premium as opposed to Treasury bills, since bonds are the closest substitutes to stocks for the long-term investor. Both data sets thus consist of monthly returns of a long-term government bond, along with monthly aggregate stock returns.<sup>15</sup> Dividends are included in the stock returns, assumed reinvested at the end of each period, which is important since dividends are a part of the utility of holding a stock portfolio. Furthermore, BT argue that in a descriptive model, nominal returns are the ones that matter for the investor, since they are given most prominence in annual reports. For this reason, nominal returns are studied, although real returns are considered in the subsequent sensitivity analysis. The U.S. data sample period covers January 1926 to December 2003, yielding a total of 936 observations, and the Swedish one covers January 1919 to December 2003, which yields 1020 observations.

Table 1 reports on summary statistics. Over the full sample period, the U.S. aggregate stock market has risen by 0.99 percent per month on average. Considering the monthly average bond return of only 0.46 percent, stocks have outperformed bonds quite substantially. The annual equity premium is 6.55 percent, which is

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<sup>15</sup>Mehra and Prescott (1985) originally analyze the equity premium over Treasury bills, but Campbell (2002), for instance, shows that the EPP is just as severe when bond returns are considered.

**Figure 5:** Swedish Monthly Stock Returns Covering 1919 to 2003



The table displays Swedish monthly stock returns (%) over the sample period from January 1919 to December 2003. Data source: Frennberg and Hansson (1992).

of similar magnitude as in several other economies. The sample period BT use, i.e., January 1926 to December 1990, is studied here as well, and the annual equity premium is slightly larger at 7.06 percent. Furthermore, over the full sample period, larger standard deviations are reported for stock returns compared with bond returns; 5.62 and 2.27 percent, respectively. Observe, also, that stock returns are mildly positively skewed, with a skewness of 0.39. The empirical kurtosis is substantial at 12.45.

Historically, the stock market has dominated the bond market in Sweden too, with average monthly returns of 0.92 and 0.56 percent for the aggregate stock and bond markets, respectively. Over the full sample period, the annual equity premium is 4.41 percent. The stock and bond return standard deviations are 4.87 and 1.91 percent, respectively. Moreover, the empirical stock returns distribution displays only slight negative skewness at -0.11, and a large kurtosis of 6.19.

## 5.2 Are the Stock Returns Conditionally Heteroskedastic?

Since a model of conditional heteroskedasticity is employed, it is natural to analyze the data for this characteristic. Figures 4 and 5 graph the stock return time series for the U.S. and Sweden, respectively, over their full sample periods. The volatility seems to cluster over time, showing periods of frequent large swings, and other

**Table 2:** Results of GARCH(1,1) Estimations and ARCH LM tests

	U.S. 1926:1- 2003:12	U.S. 1926:1- 1990:12	Sweden 1919:1- 2003:12	Sweden 1919:1- 1961:6	Sweden 1961:7- 2003:12
$c$	1.094 ( $<0.001$ )	1.086 ( $<0.001$ )	1.066 ( $<0.001$ )	1.075 ( $<0.001$ )	1.093 ( $<0.001$ )
$\psi$	0.693 ( $<0.001$ )	0.774 ( $<0.001$ )	0.572 ( $<0.001$ )	0.486 (0.005)	1.949 (0.013)
$\beta$	0.860 ( $<0.001$ )	0.857 ( $<0.001$ )	0.812 ( $<0.001$ )	0.803 ( $<0.001$ )	0.761 ( $<0.001$ )
$\alpha$	0.119 ( $<0.001$ )	0.121 ( $<0.001$ )	0.175 ( $<0.001$ )	0.170 ( $<0.001$ )	0.188 ( $<0.001$ )
ARCH	220.1 ( $<0.001$ )	185.2 ( $<0.001$ )	75.9 ( $<0.001$ )	43.5 ( $<0.001$ )	34.4 ( $<0.001$ )

The table presents parameter estimates from a GARCH(1,1) estimation using percentage returns on the U.S. and Swedish aggregate stock markets over different sample periods. ARCH is the ARCH Lagrange multiplier test statistic using 12 lags.  $p$ -values are given in parentheses.

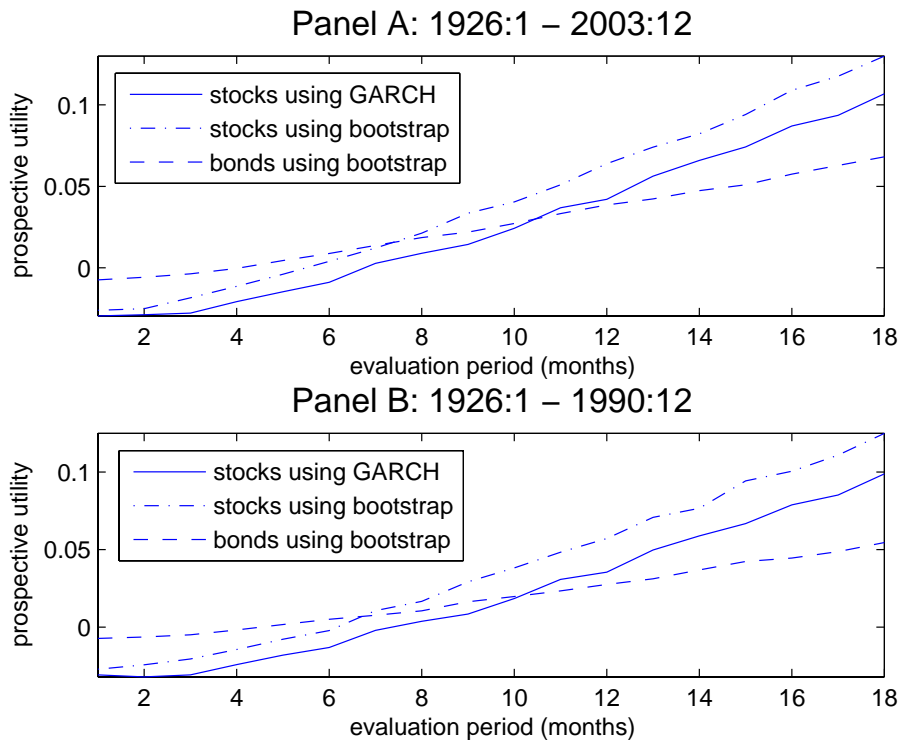
periods of calm and low volatility in returns. Table 2 reports on ARCH Lagrange multiplier test statistics of Engle (1982), which are significant throughout, indicating the presence of time-varying volatility. The table, also, presents estimation outputs from fitting the GARCH(1,1) model in (6) to the stock return series. Indeed, the GARCH(1,1) parameter estimates are significant throughout, which supports the conditional heteroskedasticity of stock returns.

### 5.3 Comparing Evaluation Periods Obtained from the Two Approaches

Figure 6 presents portfolio prospective utility as functions of the evaluation period using U.S. data. The portfolio consist of either one hundred percent stocks or one hundred percent bonds. The point where the respective function lines cross gives the evaluation period at which the investor finds the two portfolios equally attractive. When simulating stock returns distributions at different data frequencies, the non-parametric bootstrap approach as well as the parametric GARCH approach are employed. Thus the stock portfolio utility over an increasing evaluation period is represented by two lines in the figure.

Panel A presents the results using the full U.S. sample period from 1926 to 2003. With the bootstrap procedure, the equilibrium evaluation period is about seven months, while the GARCH simulation approach produces an approximate evaluation period of ten months. Thus the GARCH approach produces a longer

**Figure 6:** Prospective Utility as a Function of the Evaluation Period Using U.S. Data



The figure shows prospective utility as a function of the investor's portfolio evaluation period using U.S. data. The portfolio either consists of one hundred percent stocks or one hundred percent long-term bonds. Two approaches to simulating returns distributions are considered; the non-parametric bootstrap approach, and the parametric approach using a GARCH model. Panels A and B consider different data sample periods.

evaluation period than the bootstrap counterpart, i.e., when temporal independence is set aside and time-varying volatility is incorporated, the equilibrium evaluation period increases.

BT analyze the monthly sample period from 1926 to 1990 for which panel B presents the results. The obtained evaluation periods are six months using the bootstrap method, and ten months using the GARCH approach. Again, the longer evaluation period is produced under conditional heteroskedasticity. The procedure I employ when simulating returns distributions non-parametrically does not replicate the twelve-month evaluation period of BT however. Since the U.S. market is studied over the same sample period as BT consider, an evaluation period of six months is quite unexpected. One far reached explanation for the difference is the use of

different data sets. Although both measure U.S. aggregate stock and long-term bond returns, the data set used in the current study is from Ibbotson Associates, while BT consider CRSP data. Perhaps more likely however, the applied non-parametric bootstrap simulation techniques differ somehow. Although I have no reason to believe that the interpretation of BT's simulation procedure presented in section 4.1 to be incorrect, the possibility exists that some part of the interpreted procedure is different from the one BT use.<sup>16</sup> Obtaining an evaluation period of six instead of twelve months suggests that prospective utility, and the MLA model, is sensitive to the exact method used when simulating returns.

Next, the MLA framework is applied to the Swedish equity premium. A problem with the GARCH method of simulating returns distributions arises when the full data sample is considered however. The estimated GARCH(1,1) model has an implied unconditional kurtosis that is negative at -2.2, which does not make sense, and makes temporal aggregation impossible.<sup>17</sup> This indicates that the GARCH model is misspecified, which could be caused by a structural break in the eighty-five year data sample. To resolve the problem in a simple way, the sample is split into two equally-sized parts covering 1919:1 to 1961:6 and 1961:7 to 2003, and each subsample is studied separately. Over the subsamples, the implied unconditional kurtosis from estimating the GARCH(1,1) model come out positive, causing no problems for temporal aggregation. Moreover, although the GARCH simulation method cannot be applied to the full sample period of data, analysis is still carried out using the non-parametric bootstrap approach.

Table 1 reports on summary statistics over the two subsamples of Swedish data. Over the first half, stocks have outperformed bonds to an extent measured by an annual equity premium of 2.8 percent, while the second half presents a corresponding equity premium of just over six percent. Table 2 presents tests of ARCH structure in the stock returns as well as GARCH(1,1) estimation outputs, which statistically indicate the presence of time-variation in stock return conditional volatility over both subsamples.

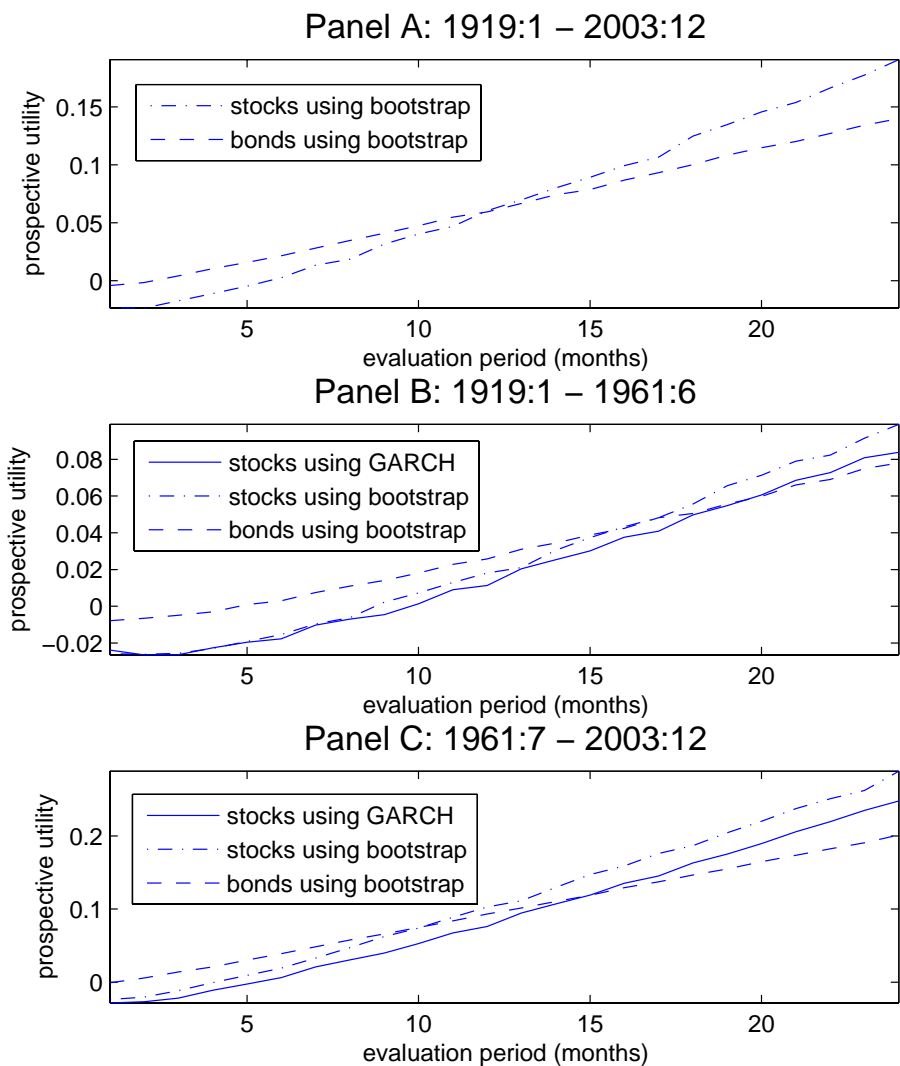
Figure 7 presents portfolio prospective utility as functions of the evaluation period using the Swedish data. Over the full sample, the bootstrap approach produces an evaluation period of twelve months, as panel A shows. The result is consistent with BT, and suggests that the EPP of Sweden can be explained by MLA preferences with a yearly evaluation period. The result is new to the literature, since

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<sup>16</sup>It is unclear whether BT compound returns continuously or not. The results come out the same irrespectively though.

<sup>17</sup>Kurtosis is defined as the fourth moment about the mean divided by the squared variance, and is non-negative by construction.

**Figure 7:** Prospective Utility as a Function of the Evaluation Period Using Swedish Data



The figure shows prospective utility as a function of the investor's portfolio evaluation period using Swedish data. The portfolio either consists of one hundred percent stocks or one hundred percent long-term bonds. Two approaches to simulating returns distributions are considered; the non-parametric bootstrap approach, and the parametric approach using a GARCH model. Panels A-C consider different data sample periods.

previous studies have not applied MLA preferences to non-U.S. equity premiums.

Over the subsample from 1919:1 to 1961:6, panel B displays an equilibrium evaluation period of about sixteen months using the bootstrap method, and nineteen months using the GARCH approach. Furthermore, panel C graphs the functions over the second half of the Swedish full sample period: 1961:7 to 2003:12. Evaluation periods of ten and fifteen months for the respective bootstrap and GARCH approaches can be elicited. Thus the results using the Swedish data show that evaluation periods are quite variable and depend on the data sample considered. They, also, support the previous finding when U.S. data was used, that the GARCH approach to simulating returns produces a longer evaluation period than the corresponding one obtained using the bootstrap method.

To summarize, the results using the U.S. and Swedish data produce varying evaluation periods depending of the sample studied. Nevertheless, the obtained evaluation periods can be considered to be of similar magnitude as the twelve-month counterpart reported by BT, and thus their result is supported. Furthermore, the evaluation periods produced under conditional heteroskedasticity are longer than the ones produced under temporal independence throughout. The prospective utility maximizer needs a longer period between evaluations to be content with the equity premium when returns are simulated with time-varying volatility. Since a longer evaluation period is needed, stocks are perceived as more risky under conditional heteroskedasticity. Interestingly, this result further indicates that prospective utility, and the MLA model, is sensitive to the simulation procedure applied, which relates to the distributional assumption made on stock returns.

## 5.4 Sensitivity Analysis

The previous analysis used Tversky and Kahneman's (1992) estimates of the parameters of (2) and (4), i.e.,  $\lambda = 2.25$ ,  $\gamma = 0.88$ ,  $\tau = 0.61$  over gains, and  $\tau = 0.69$  over losses. Are the obtained results sensitive to changes in these estimates? By varying the parameter values it seems that the parameter of loss aversion, to begin with, has a strong influence on prospective utility. Using the U.S. data over the full sample, a loss aversion equal to three, i.e.  $\lambda = 3$ , raises the obtained evaluation periods considerably from seven and ten months, when  $\lambda = 2.25$ , to thirteen and sixteen months using the non-parametric and parametric methods of simulating returns, respectively. The increase in evaluation period is intuitive, since it compensates for the additional risk perceived at a higher level of loss aversion. The risk-return relationship of stocks and bonds is thus kept at equilibrium.

The parameter of risk aversion over gains and risk-seeking over losses,  $\gamma$ , does not seem to have any strong impacts on prospective utility. Altering the value function



from having  $\gamma = 0.88$  to  $\gamma = 1$ , i.e. the function becomes piece-wise linear, affects the equilibrium evaluation periods by only one month or so.

The weighting function parameter  $\tau$  shows of some importance though. When altered to  $\tau = 1$ , implying that probabilities are treated objectively, the obtained evaluation periods change from seven and ten months to five and five months, using the bootstrap and GARCH approaches, respectively. For one, when  $\tau = 1$ , both approaches to simulating returns result in smaller evaluation periods needed to match the historical equity premium, indicating that stocks are perceived as less risky. For the other, the two approaches produce approximately *equal* evaluation periods under objective probabilities. Interestingly, this is not only the case when  $(\lambda, \gamma) = (2.25, 0.88)$ , but is a general result that holds irrespective of the values of  $\lambda$  and  $\gamma$  so long as probabilities are treated objectively.<sup>18</sup> Thus, when probabilities are objective, the MLA investor becomes less sensitive to the distributional shape of portfolio returns.

Another sensitivity analysis involves the use of real instead of nominal returns. Do the previously obtained results change when using data in real terms? The answer is no. For instance, when considering the U.S. data in real terms over the full sample period, evaluation periods of about five and nine months are obtained, using the bootstrap and GARCH approaches, respectively.<sup>19</sup> These evaluation periods do not differ much from the corresponding ones obtained when nominal returns are employed, i.e., seven and ten months.

BT consider comparing the prospective utility of an all-stocks portfolio with an all-bills portfolio as well, i.e., short-term Treasury bills are considered as the alternative investment to stocks instead of long-term bonds. Such an analysis does change the evaluation periods to some extent, e.g., over the U.S. subsample of 1926 to 1990, eleven- and fourteen-month evaluation periods are obtained when using the non-parametric and parametric approaches, respectively.<sup>20</sup> The eleven-month evaluation period obtained under temporal independence is thus in line with BT. Recall that a six-month evaluation period was produced under temporal independence when using bonds. Since BT's approximate yearly evaluation period is replicated when bills are assumed to be the alternative investment, perhaps the long-term bond data used here differs from the corresponding data used by BT. This could be the cause for the previously reported different result. Furthermore, overall longer evaluation periods

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<sup>18</sup>This result is also found using the Swedish data.

<sup>19</sup>Figures A.1 and A.2 in the appendix present prospective utility as functions of the evaluation period when using real data, for both the U.S. and Sweden.

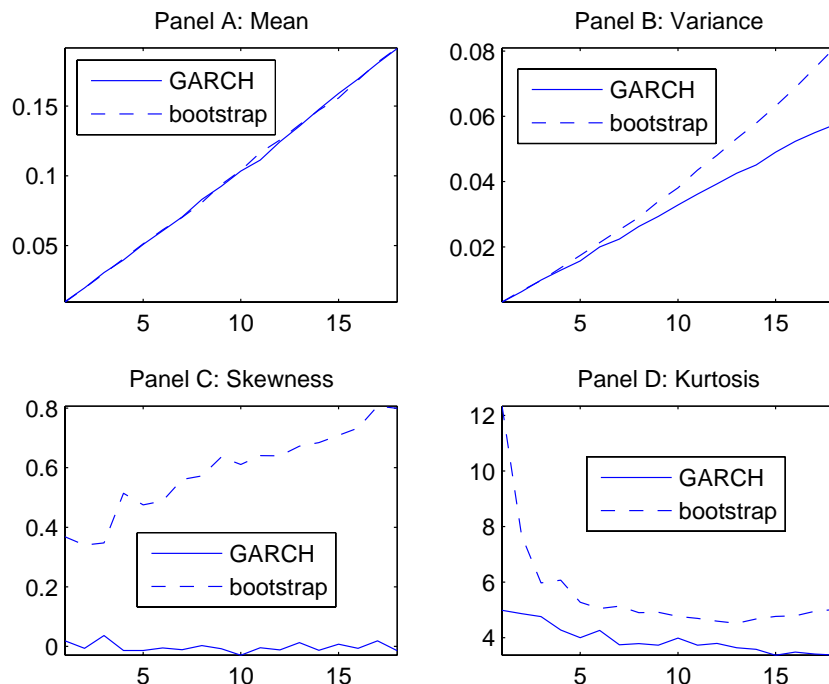
<sup>20</sup>Figures A.3 and A.4 in the appendix show prospective utility as functions of the evaluation period when Treasury bills are considered as the alternative investment to stocks.

are obtained when using bills compared with bonds in the analysis. Bills are thus considered to be more attractive than bonds under MLA preferences. The portfolio evaluation period needs to be less frequent in order to make stocks as appealing as the alternative investment.

## 5.5 What Drives the Results?

One main result of this paper is that the MLA model seems to be sensitive to how returns distributions are simulated. When using a simulation technique that incorporates conditional heteroskedasticity rather than simulating under temporal independence, the evaluation periods consistent with the historical equity premiums in the U.S. and Sweden increase. What are the distributional differences in simulated returns causing for this result? Figure 3 exemplifies simulated distributions using the two approaches. The histograms indicate that the unconditional moments, which describe the distribution's shape, change as the evaluation period increases.

**Figure 8:** Unconditional Moments as a Function of the Evaluation Period



The figure shows the unconditional moments mean, variance, skewness, and kurtosis of simulated returns over an increasing evaluation period. Both simulation methods; the non-parametric bootstrap and the parametric GARCH, are considered. U.S. aggregate stock returns covering 1926:1 to 2003:12 are employed.

Figure 8 makes it more clear what distributional changes actually occur. Panels A-D present the first four unconditional moments of the simulated returns distributions as functions of the evaluation period when using both the bootstrap and the GARCH simulation methods. The full sample of monthly U.S. aggregate stock returns is used. Panel A shows that the mean evolves similarly for the two methods. With a longer evaluation period, the increase in the variance is steeper when the bootstrap approach is considered compared with the GARCH approach, as panel B shows. This would suggest that stocks are perceived as more risky under temporal independence, which was however not found in the previous investigation. Therefore, the changes in the skewness and the kurtosis, shown in panels C and D, must play a part in the way the MLA investor perceives the risk of a stock investment, making stocks more favorable under temporal independence than under conditional heteroskedasticity. Panel C shows that the skewness of the bootstrap-simulated distribution rises with an increasing evaluation period, while the GARCH method leaves the skewness at practically zero everywhere, which is not surprising since the GARCH(1,1) is symmetric. Since the prospective utility maximizer suffers from loss aversion, the increasing skewness is possibly the factor that dominates the effect of a more progressive variance, making the risky investment more attractive under temporal independence than under conditional heteroskedasticity. However, such a reasoning is speculative, and further research on the relationship between prospective utility and the distributional skewness is needed. Moreover, the kurtosis falls when the evaluation period increases across both methods, as panel D shows, with somewhat larger levels of kurtosis when using the bootstrap method overall.

## 6 Conclusions

The paper replicates the study of BT, who suggest an explanation to the EPP by MLA preferences, and, furthermore, considers a technical extension to their methodology. Specifically, the distributional assumption made on returns is addressed. When simulating returns distributions, conditional heteroskedasticity is incorporated through a GARCH model in place of the temporal independence assumption of BT. Moreover, Swedish data is considered in addition to U.S. data, which further extends BT's analysis.

Over the full U.S. sample of data, evaluation periods of seven and ten months are obtained when using the non-parametric bootstrap approach, which assumes temporal independence in returns, and the GARCH approach, respectively. These evaluation periods are smaller than the twelve-month counterpart reported by BT. Furthermore, the twelve-month evaluation period of BT is not replicated when simu-

lating returns under temporal independence, although a U.S. data set covering BT's sample period of 1926 to 1990 is used. Instead, an evaluation period of six months is obtained. The applied non-parametric bootstrap simulation techniques might differ somehow, although I have no reason to believe it to be the case. Nonetheless, obtaining an evaluation period of six instead of twelve months suggests that prospective utility, and MLA, is sensitive to the method used when simulating returns distributions.

A problem with the GARCH approach occurs when the full Swedish data sample is considered, and analysis using this approach is made on two subsamples of data instead. The non-parametric approach produces a twelve-month evaluation period over the full sample though, in line with BT, which suggests that MLA can explain the EPP of Sweden. This result is new to the literature, since previous studies have not applied MLA preferences to the equity premiums of non-U.S. economies. Furthermore, overall longer evaluation periods are obtained under conditional heteroskedasticity when studying the two subsamples of data. For example, over the period from July 1961 to December 2003, evaluation periods of ten and fifteen months are obtained using the bootstrap and GARCH methods, respectively.

Throughout the analysis, longer evaluation periods are produced under conditional heteroskedasticity. Does this result fail BT's explanation to the EPP? Since BT's twelve-month evaluation period is quite approximative, and the evaluation periods obtained here are of similar magnitude, I would say the answer is no. However, the obtained difference between the two approaches further indicates that the MLA model is sensitive to the method of simulating returns. This relates to the returns' distributional assumption, and thus to the shape of the simulated returns distribution. Therefore, the two simulation techniques are analyzed with respect to the how the first four unconditional moments evolve as the evaluation period lengthens. Plausibly, the skewness of the returns distribution is important for the loss-averse investor, which is intuitive since loss aversion induces an asymmetric preference over gains and losses. Further research on prospective utility in relation to the distributional skewness is suggested.

The sensitivity of the paper's results are analyzed with respect to various modifications. One interesting finding is that the probability weighting function seems to be important for MLA. When altering the weighting function parameter so that probabilities are treated linearly, the two approaches to simulating returns produce equal evaluation periods irrespective of the value function's parameter-values and whether U.S. or Swedish data is considered. This indicates that MLA investors become less sensitive to the distributional assumption made on returns when probabilities are objective. Further research investigating this issue is suggested.

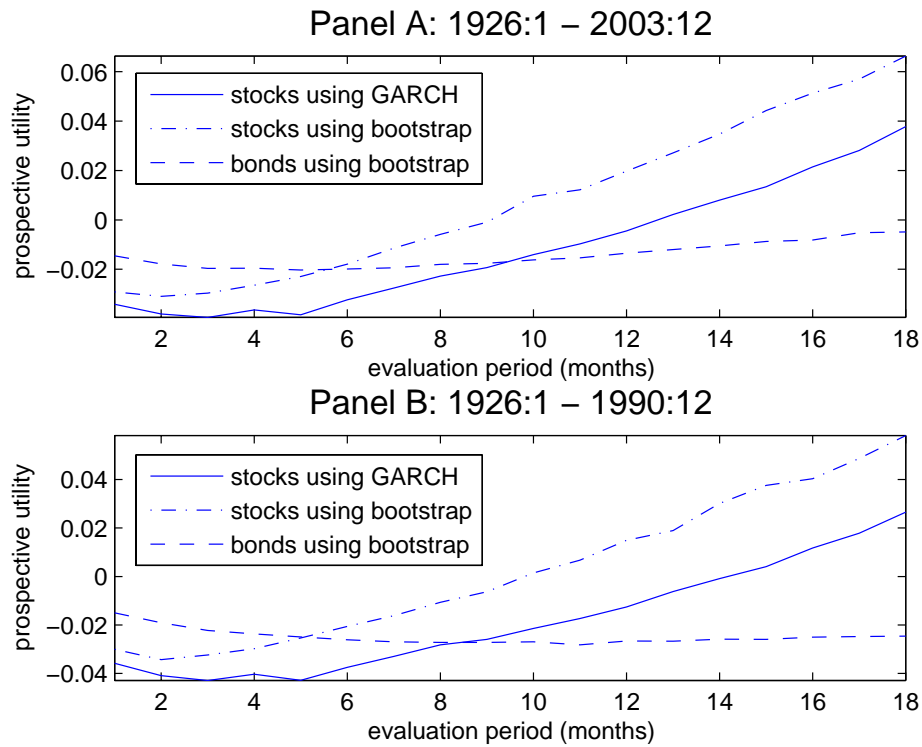
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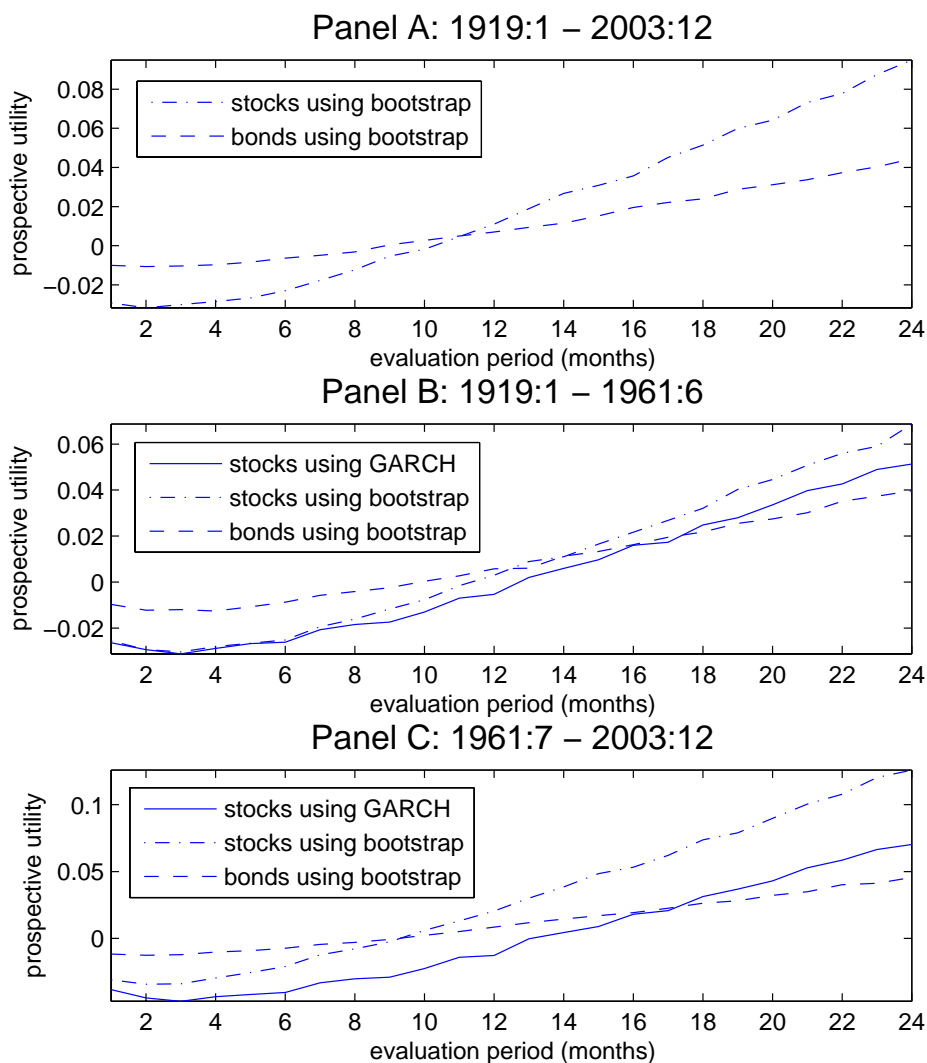
# Appendix

**Figure A.1:** Prospective Utility as a Function of the Evaluation Period Using U.S. Data in Real Terms



The figure shows prospective utility as a function of the investor's portfolio evaluation period using U.S. data in real terms. The portfolio either consists of one hundred percent stocks or one hundred percent long-term bonds. Two approaches to simulating returns distributions are considered; the non-parametric bootstrap approach, and the parametric approach using a GARCH model. Panels A and B consider different data sample periods.

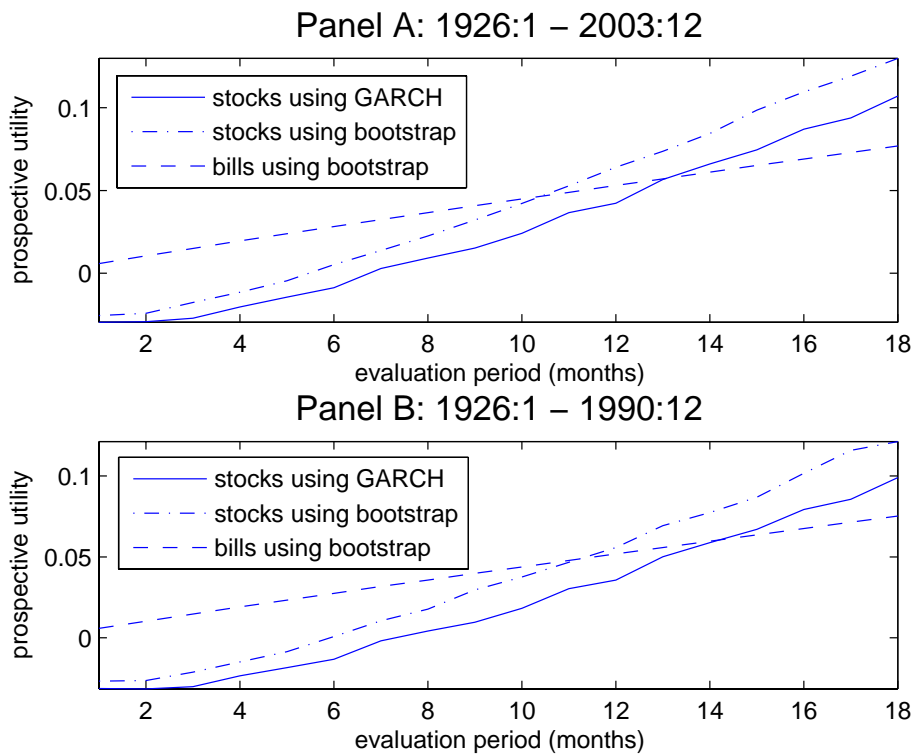
**Figure A.2:** Prospective Utility as a Function of the Evaluation Period Using Swedish Data in Real Terms



The figure shows prospective utility as a function of the investor’s portfolio evaluation period using Swedish data in real terms. The portfolio either consists of one hundred percent stocks or one hundred percent long-term bonds. Two approaches to simulating returns distributions are considered; the non-parametric bootstrap approach, and the parametric approach using a GARCH model. Panels A-C consider different data sample periods.

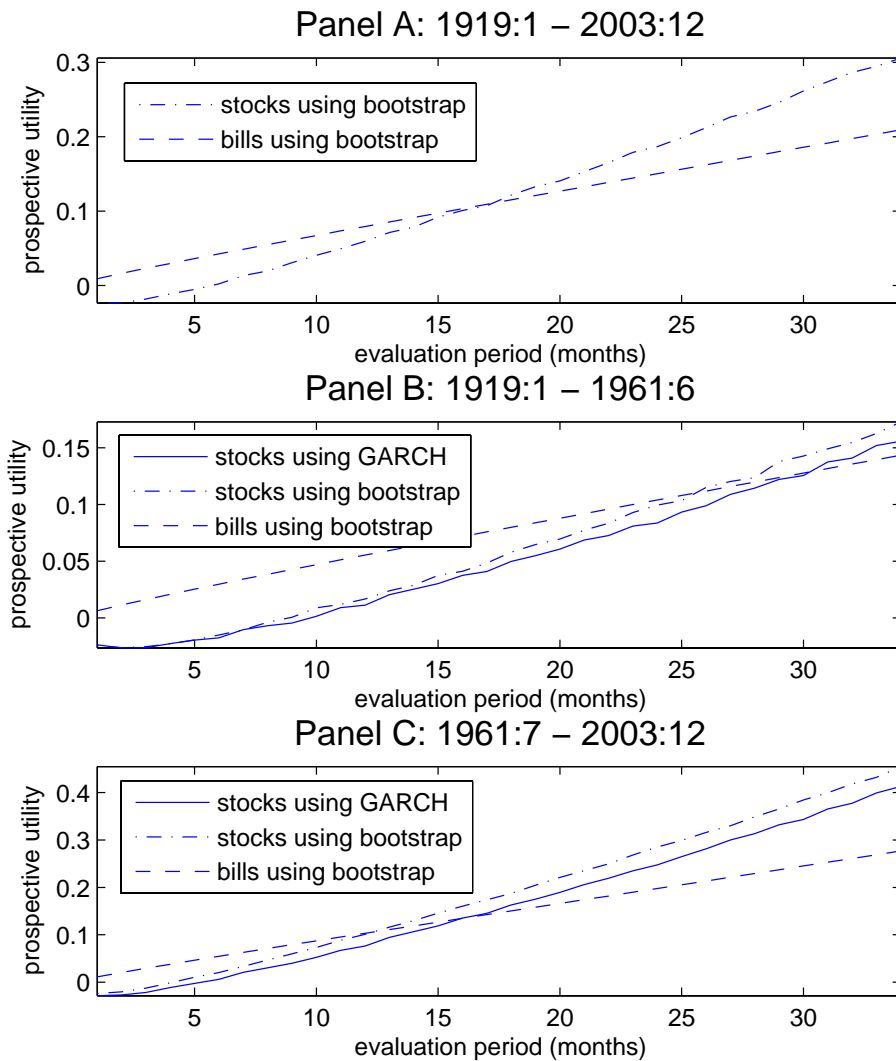


**Figure A.3:** Prospective Utility as a Function of the Evaluation Period Using U.S. Data with Bills as the Alternative Investment



The figure shows prospective utility as a function of the investor's portfolio evaluation period using U.S. data. The portfolio either consists of one hundred percent stocks or one hundred percent short-term Treasury bills. Two approaches to simulating returns distributions are considered; the non-parametric bootstrap approach, and the parametric approach using a GARCH model. Panels A and B consider different data sample periods.

**Figure A.4:** Prospective Utility as a Function of the Evaluation Period Using Swedish data with Bills as the Alternative Investment



The figure shows prospective utility as a function of the investor’s portfolio evaluation period using Swedish data. The portfolio either consists of one hundred percent stocks or one hundred percent short-term Treasury bills. Two approaches to simulating returns distributions are considered; the non-parametric bootstrap approach, and the parametric approach using a GARCH model. Panels A-C consider different data sample periods.