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Wage Formation and Redistribution

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WAGE FORMATION AND REDISTRIBUTION

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Wage Formation and Redistribution*

Per Engström[†]

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Abstract

The paper extends the basic Stiglitz (1982) model of optimal non-linear income taxation into a model featuring endogenous unemployment and wages. This means that the government needs to consider the effects on wages and unemployment when designing the optimal tax function. The tax systems' effects on the wage formation and the unemployment rates result in new intricate redistribution channels. A key result of the paper is that the government may, in order to redistribute, use the marginal tax rates to raise the unemployment rate for the high-skilled and lower it for the low-skilled workers.

JEL-classification: H21, J22, J41, J64

Keywords: Optimal Non-Linear Income Taxation, Wage Formation, Tax Progressivity, Unemployment

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1 Introduction

It is safe to say that the Stiglitz (1982) model of optimal taxation is the main workhorse of modern studies of the equity-efficiency tradeoff. In a reasonably simply way, compared to its "big brother" Mirrlees (1971), it highlights the informational problem facing a government trying to redistribute resources from the high-skilled (high-waged) workers to the low-skilled (low-waged) workers. The key assumption is that a worker's skill-type is private information that may not be used by the government when designing the tax-transfer structure.¹ The optimal solution must therefore be consistent with self-selection; i.e. it must be in each worker's own interest to reveal her true skill-type to the government.

Much literature in this strand of research has focused on finding intricate policy instruments that may slacken the constraint imposed by compatibility with self-selection. Prominent examples include the economics of "tagging" and the inclusion of commodity taxation.² However, so far the treatment of the labor market has, with some exceptions, been Walrasian. And that is where this paper departs from previous research.

The treatment of the labor market will be fairly general and based on a reduced form approach; the source of unemployment could be thought of either as search frictions (see e.g. Pissarides 2000) or labor union influence over wages (see e.g. Layard and Nickell, 1990). These two classes of labor market models share some important features in terms of how the tax parameters – the marginal taxes and the total tax/transfer – affect the wage formation. Both types of models generally assume that the wages are determined by Nash bargaining; in the search unemployment literature the bargaining is typically between a single worker and a single firm, while in the labor union models the bargaining takes place between a firm (or many firms) and a labor

¹Note that the problem would indeed be trivial without this key assumption. If skill-type was observable the government could, without restriction, engage in lump-sum redistribution.

²For seminal paper on tagging see Akerlof (1978) and on the inclusion of commodity taxation Atkinson and Stiglitz (1976) and Mirrlees (1976).

union. In any model with Nash bargaining over wages one can derive the result that increased *tax progressivity* (reasonably defined) leads to a reduction in the bargained wages. This result is in the literature referred to as *wage moderation* caused by more progressive taxation, and it can be separated into two different mechanisms: i) an increase in the marginal tax rate will lower the bargained wages (we will refer to this as *wage restrain*) and ii) an increase in the total taxes paid will increase the bargained wage (referred to as *wage compensation*).³ The models also share a natural trade-off between wages and unemployment; higher wages have a cost in terms of higher unemployment. A large number of empirical studies have found support for that increased tax progressivity leads to wage moderation, e.g. Malcomson and Sartor (1987), Lockwood and Manning (1993) and Holmlund and Kolm (1995), just to mention a few.

Optimal redistribution in economies with unemployment is not a well explored field, but there are some previous studies. Examples are Saez (2002) and Boone and Bovenberg (2004). The main value added from our analysis is to introduce endogenous wage formation with a trade-off between employment and wages. To my knowledge there are only two previous studies that capture this trade-off in an optimal non-linear income taxation setup. These are Aronsson and Sjögren (2003) and Hungerbüler et al (2005). Our paper is more closely related to Hungerbüler et al (2006).⁴ They develop a continuous

³In the literature in this field one usually define a measure of tax progressivity. In any bargaining setting, increased wage progressivity will generally lead to a reduction in the bargained wage. In our model, however, we will separate the effect of an increase in the marginal tax and the effect of an increase in the total tax. We therefor need not introduce a measure of tax progressivity.

⁴In Aronsson and Sjögren (2003) the utility of an unemployed worker is independent of whether the worker is high-skilled or low-skilled. This is because their model entails ex post utilities in a one-shot setting. In the present paper, however, the workers care only about expected ex ante utilities. Our setting can be justified in an infinitely repeated setting in which discounting is ignored. Since this feature – i.e. ex ante expected utility instead of ex post utilities – is crucial to the results derived, we cannot relate further to the results in Aronsson and Sjögren (2003). Their model also differs in that they consider both commodity taxation and unemployment benefits as part of the social planner’s policy instruments.

skill-type model featuring search unemployment. In contrast to the existing studies on optimal income taxation, they derive the optimal tax structure holding the labor supply fixed.⁵ An important result in their paper is that the marginal tax rates should be relatively high on the top of the income distribution in order to reduce the increase in wages that the redistribution itself causes (*wage compensation*). This is an interesting result as it is in sharp contrast to the classical result, that follows from both Mirrlees (1971) and Stiglitz (1982), with zero marginal tax on top of the income distribution. We will discuss how these issues relate to our analysis later on in the paper.

As we will see, the endogenous wage structure in our setting will open up for new ways for the government to perform redistribution. The marginal tax instruments can be used to affect the wage formation as well as the traditional effects on labor supply. It is tempting to conjecture that this structure of the model would give the government incentive to raise the wages for the low-skilled and lower the wages for the high-skilled in order to reduce the income inequality between the two groups. This is, however, not the case; in fact the government does the exact opposite – it raises the wage for the high-skilled and lowers the wage for the low-skilled. It does so by reducing the marginal tax for the high-skilled and by raising the marginal tax for the low-skilled.⁶ This is the main result of the paper, and to our knowledge it is new to the literature.⁷

To understand the result one must consider the informational constraint facing the government. The optimal tax structure will raise the unemployment rate for the high-skilled and lower it for the low-skilled, which can be seen as a form of redistribution. The advantage of using the unemployment rates as a means to redistribute is that a high-skilled worker will never benefit from a reduction in the unemployment rate facing the low-skilled workers. In this way a social planner may circumvent some of the redistributive limi-

⁵We will, however, stick to the usual setup with endogenous labor supply.

⁶The result refers to when we abstract from "labor market externalities", which are explained in the section 3.

⁷The mechanism is briefly touched upon, however, in the numerical part of Engström (2002).

tations the asymmetric information causes.

The paper is outlined as follows. We present the model in chapter 2. The general features of unemployment determination and wage determination are presented as well as the workers' and the government's problems. In section 3 we derive some results and discuss them. Section 4 concludes.

2 The Model

2.1 General Structure

The structure of the labor market is not explicitly modelled. Instead we assume a wage function and an unemployment function for the two separate skill-groups, consisting of high-skilled (H) and low-skilled (L) workers. These functions will be allowed to have very general characteristics, but their overall features will be consistent with modern labor market models. The wage functions capture the tax parameters' effects on the bargained wages while the unemployment functions capture how the unemployment levels depend on the bargained wages.

More specifically we define the wage functions as:

$$w_j = f_j(T'_j, T_j), \quad (1)$$

where w_j is the wage for the workers of skill-level j ($j = H, L$), while T'_j and T_j are the marginal tax and the total tax (or transfer when $j = L$) intended for a j -type worker.⁸ The signs of the partial derivatives are $f_{T'_j}^j < 0$, in order to capture the *wage restrain* effect, and $f_{T_j}^j > 0$, which captures the *wage compensation* effect. A wage function with these properties can be derived from a wide range of labor market models – almost any model with wage bargaining will produce these comparative static properties.⁹ Even though the wages are endogenous to the tax parameters, we make the traditional

⁸We write "intended for" since the government cannot directly observe each worker's skill-level. This is explained in more detail in the sub-section that discusses the government's problem.

⁹See e.g. Hansen (1999) for a survey.

assumption that $w_H > w_L$ since this inequality is fundamental in this strand of literature. The assumption is reasonable if the underlying productivity differences between the two groups are large compared to the tax-induced wage adjustments.

The unemployment functions, with u_j representing the type-specific unemployment rate, are defined as:

$$u_j(w_j), \tag{2}$$

where $u'_j > 0$ in order to capture the trade-off between employment and wages. The unemployment functions can be thought of in terms of an inverse measure of aggregate labor demand; in any type of labor market model there will be a negative relation between aggregate labor demand and the price of labor, i.e. the wage.

For simplicity we let the number of high-skilled workers be equal to the number of low-skilled workers and we normalize this number to unity. This means that u_j also represents the number of unemployed workers of type j .

2.2 The Workers

The employed workers derive utility from consumption (C_j) and disutility from work (l_j). We ignore unemployment benefits, so the unemployed individuals have no income.¹⁰ We let the workers' utility function be linear in consumption and for simplicity we normalize the marginal utility of consumption to unity. Hence:

$$v(C_j, l_j) = C_j - g(l_j), \tag{3}$$

¹⁰The optimal use of unemployment benefits in an optimal taxation framework is indeed a subject that deserves attention. However, for simplicity reasons we exclude this from the present analysis. In Engström (2003) unemployment benefits are introduced in a model featuring optimal taxation. The main result in that paper is that high unemployment benefits targeted at the low-skilled workers may be an effective way to relax the self-selection-constraint. This is interesting because it gives a redistributive flavor to unemployment benefits, which otherwise mostly (at least in economic research) has been seen as a pure insurance tool.

if employed and

$$v(0, 0) \equiv 0, \tag{4}$$

if unemployed. For $g(\cdot)$ holds that $g'(\cdot) > 0$ and $g''(\cdot) < 0$. Note also that we have normalized $g(0)$ to zero.

Let $T(Y_j)$ be the total tax paid by a worker with gross income $Y_j = w_j l_j$. A representative worker's maximization problem then takes the following form:

$$\max_{l_j} v(l_j w_j - T(l_j w_j), l_j).$$

The first-order condition for this problem can be written as:

$$T'_j = 1 - \frac{g'(l_j)}{w_j}. \tag{5}$$

This representation of the first-order condition is convenient for the subsequent analysis but may not be very intuitive. We can, however, rewrite eq. (5) in a more intuitive form by solving for $g'(l_j)$ and obtain:

$$g'(l_j) = w_j (1 - T'_j), \tag{6}$$

which simply says that the marginal cost of work should equal the net marginal income.

2.3 The Government

The government's only objective is redistribution. Hence, all the tax that is collected is redistributed back as transfers. The government seeks to find the optimal solutions to the tax problem. As noted above, we do not consider unemployment benefits; only the employed workers are exposed to taxes and transfers. As in the basic Stiglitz (1982) model, we assume that the government can only tax income; each worker's skill is not revealed to the government.

We assume that the government cares about the workers' expected utility, which is given by:

$$v_j^e = (1 - u_j(w_j))v_j = (1 - u_j(w_j))(C_j - g(l_j)).$$

In order to solve the government's problem we follow the usual practice and replace the unobservable labor supply (l_j) using the identity $Y_j = w_j l_j$. The Pareto-efficient tax regimes are then given by maximizing v_H^e w.r.t. C_H, Y_H, w_H, C_L, Y_L and w_L , subject to $v_L^e = \bar{v}_L$, where \bar{v}_L is a "promised" expected utility to the low-skilled workers, plus four additional constraints that we discuss below.

The self-selection-constraint (SSC)

Since the government's information set is limited, the tax function can only be conditioned on income and not on the type of worker. The government does not observe each worker's wage or work-hours separately, it can only observe each worker's gross earnings. This opens up for workers to engage in mimicking. Specifically a high-skilled worker can pretend to be a low-skilled worker, by working less in order to earn the same gross income as a low-skilled worker.¹¹ The government needs thus to ensure that a high-skilled worker prefers the income that was intended for her before any other income. One needs only consider one critical point on the gross income scale, namely the income that was intended for the low-skilled worker. To ensure that a high-skilled worker will have no incentive to engage in mimicking, the following must hold:

$$C_H - g\left(\frac{Y_H}{w_H}\right) \geq C_L - g\left(\frac{Y_L}{w_H}\right), \quad (7)$$

which simply means that the utility of a high-skilled worker must be at least as high as the utility of a high-skilled worker pretending to be low-skilled. The gain from switching to mimicking is that you pay less taxes (you get a transfer instead) but the cost is that you need to reduce your labor supply below the otherwise optimal level.

¹¹In principle it is possible also for the low-skilled worker to mimic the high-skilled worker. In this paper however, we choose to ignore this possibility and focus on the "normal" case, which also most of the existing literature has focused on. If the government wants to redistribute from the high-skilled to the low-skilled, the low-skilled will never have incentive to mimic the high-skilled. We capture this by assuming that \bar{v}_L is always so high that the low-skilled receive a transfer.

The government's budget restriction

The second constraint is simply the government's budget restriction. We assume that the government has no other goal than redistribution, so total consumption should equal total income in the economy. That is:

$$(1 - u_H(w_j)) Y_H + (1 - u_L(w_j)) Y_L = (1 - u_H(w_j)) C_H + (1 - u_L(w_j)) C_L. \quad (8)$$

Wage determination

The last two constraints concern the wage determination. The wage function is expressed in terms of the tax parameters (9). The tax parameters, however, are left out of the maximization problem and identified recursively. The total tax (transfer) targeted at the j -type group is simply identified as:

$$T_j = Y_j - C_j, \quad (9)$$

and the marginal tax rates are identified through the first order conditions of labor supply, eq. (5). When determining the optimal tax function(s) we must take the taxes effects on unemployment and wages into consideration. In order to do this we need to find how the wages relate to the other choice variables (C_H, Y_H, C_L and Y_L). We use eq. (5) and eq. (9) to replace the tax parameters in the wage functions, eq. (1), and derive the relation:

$$w_j = f_j(Y_j - C_j, 1 - \frac{g'(\frac{Y_j}{w_j})}{w_j}), \quad (10)$$

which gives w_j as an implicit function of C_j and Y_j , denoted $w_j(C_j, Y_j)$. Differentiation gives:

$$w_C^j \equiv \frac{\partial w_j(C_j, Y_j)}{\partial C_j} = -\frac{f_{T_j}^j}{1 - \frac{f_{T_j}^j}{w_j^2} \left(g_j' + \frac{g_j''}{w_j} Y_j \right)} < 0 \quad (11)$$

and:

$$w_Y^j \equiv \frac{\partial w_j(C_j, Y_j)}{\partial Y_j} = \frac{f_{T_j}^j - f_{T_j}^j \frac{g'}{w_j^2}}{1 - \frac{f_{T_j}^j}{w_j^2} \left(g_j' + \frac{g_j''}{w_j} Y_j \right)} > 0. \quad (12)$$

The signs of the partial derivatives are quite natural. We can think of an increase in consumption as a reduction in the total tax (increase in the transfer), which lowers the wage (the wage compensation effect). And an increase in gross income can be thought of as a reduction in the marginal tax – since this would induce longer work-hours and thereby raise the gross income – which increases the wage (the wage restrain effect). We will subsequently express these last two constraints as $w_j = w_j(C_j, Y_j)$.

2.4 Finding the Pareto-optimal Tax Structure

We are now at the point when we can specify the government's maximization problem formally. The problem can be written:

$$\max_{C_H, Y_H, w_H, C_L, Y_L, w_L} (1 - u_H(w_H))(C_H - g(\frac{Y_H}{w_H})) \quad (13)$$

s.t.

$$(1 - u_L(w_L))(C_L - g(\frac{Y_L}{w_L})) = \bar{v}_L, \quad (14)$$

$$C_H - g\left(\frac{Y_H}{w_H}\right) = C_L + g\left(\frac{Y_L}{w_H}\right), \quad (15)$$

$$(1 - u_H(w_H))Y_H + (1 - u_L(w_L))Y_L = (1 - u_H(w_H))C_H + (1 - u_L(w_L))C_L, \quad (16)$$

$$w_H = w_H(C_H, Y_H) \quad (17)$$

and

$$w_L = w_L(C_L, Y_L). \quad (18)$$

The Lagrange function for the problem can be written:

$$\begin{aligned} \Psi = & (1 - u_H)(C_H - g_H) \quad (19) \\ & + \mu [(1 - u_L)(C_L - g_L) - \bar{v}_L] + \lambda_H [C_H - g_H - C_L + g_m] \\ & + \gamma [(1 - u_H)Y_H + (1 - u_L)Y_L - (1 - u_H)C_H - (1 - u_L)C_L] \\ & + \chi_H(w_H - w_H(C_H, Y_H)) + \chi_L(w_L - w_L(C_L, Y_L)), \end{aligned}$$

where μ , λ_H , γ , χ_H and χ_L are Lagrange multipliers. It holds that $\mu > 0$, $\lambda_H \geq 0$ and $\gamma > 0$. Note also that we have defined g_m by $g_m \equiv g(\frac{Y_L}{w_H})$.

The first order conditions are given by:

$$\frac{\partial \Psi}{\partial C_H} = (1 - u_H) + \lambda_H - \gamma(1 - u_H) - \chi_H w_C^H = 0, \quad (20)$$

$$\frac{\partial \Psi}{\partial Y_H} = -(1 - u_H) \frac{g'_H}{w_H} - \lambda_H \frac{g'_H}{w_H} + \gamma(1 - u_H) - \chi_H w_Y^H = 0, \quad (21)$$

$$\frac{\partial \Psi}{\partial w_H} = -u'_H(C_H - g_H) + (1 - u_H) \frac{g'_H}{w_H^2} Y_H + \lambda_H \left(\frac{g'_H}{w_H^2} Y_H - \frac{g'_m}{w_H^2} Y_L \right) - \gamma u'_H(Y_H - C_H) + \chi_H = 0, \quad (22)$$

$$\frac{\partial \Psi}{\partial C_L} = \mu(1 - u_L) - \lambda_H - \gamma(1 - u_L) - \chi_L w_C^L = 0, \quad (23)$$

$$\frac{\partial \Psi}{\partial Y_L} = -\mu(1 - u_L) \frac{g'_L}{w_L} + \lambda_H \frac{g'_m}{w_H} + \gamma(1 - u_L) - \chi_L w_Y^L = 0 \quad (24)$$

and

$$\frac{\partial \Psi}{\partial w_L} = \mu \left[-u'_L(C_L - g_L) + (1 - u_L) g'_L \frac{Y_L}{w_L^2} \right] - \gamma u'_L(Y_L - C_L) + \chi_L = 0. \quad (25)$$

These first order conditions (FOC) follow the same logic as in the basic Stiglitz (1982) model. There are, however, some important differences. The presence of unemployment modifies the FOC for C_j and Y_j to some extent; the last terms, which include χ_j , are new. This captures the fact that the choices of C_j and Y_j affect the bargained wage. An important difference is also the two FOC with respect to wages, which are naturally not present in a model with fixed wages. The first two terms in eq. (22) capture the direct wage effects on the expected utility, one positive effect on the utility when employed and one negative effect on the chance of being employed; the third term is the direct effect of a wage increase (for the high-skilled) on the SSC, which is straightforward to show is positive; the fourth term captures the budget effect. Due to the interdependence between the bargained wages and the other choice variables there are also secondary effects stemming from the wage constraint. This is captured by the last term in eq. (22), which has indeterminate sign. The same logic holds for the low-skill FOC with respect

to the wage, see eq. (25). Note, however, that w_L has no direct effect on the SSC since, given the other choice variables, it is only w_H that enters the relevant utility expressions.

3 Results and Discussion

By making use of the FOC for the high-skilled, eq. (20), (21) and (22), and for the low-skilled, eq. (23), (24) and (25), we can derive expressions for the optimal marginal tax rates for high- and low-skilled, respectively. After some tedious algebra we reach the following expressions for the optimal marginal tax rates:

$$T'_H = (w_C^H + w_Y^H) [\Omega_1^H (\epsilon_{ew}^H - \epsilon_{vw}^H) - \Omega_2^H \lambda_H] \quad (26)$$

and

$$T'_L = \left(1 - \frac{g'_m}{w_H}\right) \lambda_H + (w_Y^L + w_C^L) [\Omega_1^L \mu (\epsilon_{ew}^L - \epsilon_{vw}^L) + \Omega_2^L \lambda_H], \quad (27)$$

where we have introduced some simplifying variables Ω_i^j ($j = H, L$ and $i = 1, 2$). The exact definitions of the Ω_i^j are given in Appendix A. All the Ω 's are positive; in Appendix B we show that $\Omega_1^j > 0$ hold for ($j = H, L$), while that $\Omega_2^j > 0$ ($j = H, L$) are shown in Appendix C. Furthermore we have defined the variables ϵ_{ew}^j and ϵ_{vw}^j ($j = H, L$) as:

$$\epsilon_{ew}^j \equiv -\frac{\partial(1 - u_j)}{\partial w_j} \frac{w_j}{1 - u_j} = \frac{u'_j w_j}{1 - u_j} > 0 \quad (28)$$

and

$$\epsilon_{vw}^j \equiv \frac{\frac{Y_j}{w_j} g'_j}{Y_j - g_j} > 0. \quad (29)$$

The interpretations of these variables are straightforward; ϵ_{ew}^j is the employment elasticity w.r.t. the wage, which can be thought of as the labor demand elasticity, while ϵ_{vw}^j can be shown to be the wage-elasticity of utility, evaluated at the point where $T_j = 0$.

A final thing that needs to be commented on in eq. (26) and eq. (27) is that $w_Y^j + w_C^j > 0$ ($j = H, L$) hold. This follows directly from (11) and

(12).¹²

Proposition 1 *When the SSC does not bind (i.e. $\lambda_H = 0$) the optimal marginal tax rates are*

$$\begin{aligned} T'_j &> 0 \text{ if } \epsilon_{ew}^j > \epsilon_{vw}^j, \\ T'_j &< 0 \text{ if } \epsilon_{ew}^j < \epsilon_{vw}^j \\ &\text{and} \\ T'_j &= 0 \text{ if } \epsilon_{ew}^j = \epsilon_{vw}^j \end{aligned}$$

for $j = H, L$.

Proof The proof follows directly from (26) and (27). ■

Proposition 1 corresponds to the result in Stiglitz (1982) that the marginal taxes should be zero in the case when the SSC does not bind. However, due to the presence of unemployment in our model, we need an additional criterion for this to hold. The labor market outcome needs not be constrained efficient; unemployment may be too high or too low relative to a socially efficient outcome. Since the government has the possibility to affect the wage formation through the tax parameters, there may be reasons for both positive and negative marginal tax rates. In the case when $\epsilon_{ew}^j > \epsilon_{vw}^j$, hence when the social value of increased employment is higher than the social value of increased general wage level, the induced unemployment is too high and the government will raise the marginal tax T'_j in order to restrain the wages and thereby increase employment. The cost of increasing efficiency in terms of unemployment is reduced efficiency in the workers' labor supply decision. This trade-off has been analyzed in different settings by e.g. Holmlund and Kolm (1994) and Sørensen (1999). The result could also be seen in analogy with the literature on externalities and optimal taxation; see seminal paper by Sandmo (1975) and for a more recent survey see Cremer et al (1998). In the case when $\epsilon_{ew}^j \neq \epsilon_{vw}^j$ there is a labor market externality that the government can adjust for by making use of the marginal tax rates. Hungerbühler et

¹²Note that an equal raise in Y_j and C_j could be thought of as a reduction in T'_j while holding T_j constant, hence $w_Y^j + w_C^j > 0$.

al (2006) derive a related result in a rather different setting. In their model there is a continuum of heterogeneous workers that differ in their productivities. The government has egalitarian ambitions and wishes to redistribute from the highly productive workers to the workers with low productivity. In *laissez faire* the labor markets are efficient, but when the government takes from the high-skilled their wages will rise to an inefficiently high level, due to the compensation effect. The government balances this wage increase with an increase in the high-skill marginal tax, which restrains the high-skill wages. The optimal marginal tax rate on the top of the skill-distribution will therefore be positive. However, their model abstracts, at least in the analytical part, from endogenous labor supply, which leaves out an important efficiency cost of positive marginal tax rates.

So far the results could have been derived in a one-type model; when $\lambda_H = 0$ there is no cost of keeping the high-skilled workers from mimicking. We now turn to the main subject of the paper. From now on we will focus on how the SSC affects the outcome and we therefore assume that $\epsilon_{ew}^j = \epsilon_{vw}^j$ hold, which means that there is no labor market externality, in the sense discussed above, of changes in the marginal tax rate. Proposition 2 below shows the optimal marginal tax rates corresponding to the case when the government suffers from its informational constraints, hence when $\lambda_H > 0$.

Proposition 2 *When $\epsilon_{ew}^j = \epsilon_{vw}^j$ ($j = H, L$) hold and $\lambda_H > 0$ the optimal marginal tax rates are given by:*

$$T'_H = - (w_C^H + w_Y^H) \Omega_2^H \lambda_H < 0 \quad (30)$$

and

$$T'_L = \left(1 - \frac{g'_m}{w_H}\right) \lambda_H + (w_Y^L + w_C^L) \Omega_2^L \lambda_H > 0. \quad (31)$$

Proof The expressions follow directly from eq. (26) and eq. (27). $T'_H < 0$ follows from $\Omega_2^H > 0$ which is shown in Appendix C.

We show below that $T'_L > 0$ and also that both the terms in eq. (31) are positive. That the second term is positive follows from $\Omega_2^L > 0$ (shown

in Appendix C). In order to show that the first term also is positive we first note that the following must hold:

$$1 - \frac{g'_m}{w_H} > 1 - \frac{g'_L}{w_L} = T'_L, \quad (32)$$

since $w_H > w_L$ and $g'_m = g'(\frac{Y_L}{w_H}) < g'_L = g'(\frac{Y_L}{w_L})$. We use this in eq. (31) to derive:

$$T'_L > \frac{(w_Y^L + w_C^L) \Omega_2^L \lambda_H}{1 - \lambda_H} > 0, \quad (33)$$

for $\lambda_H < 1$. When λ_H approaches 1 (from below) the model breaks down, since eq. (33) gives $\lim_{\lambda_H \rightarrow 1^-} T'_L = \infty$. It is therefore no restriction to assume that $\lambda_H < 1$ holds. We thus have that $T'_L > 0$ which in eq. (32) implies that $1 - \frac{g'_m}{w_H} > 0$ and thus that the first term in eq. (31) is positive. Q.E.D. ■

Proposition 2 shows that when we abstract from the need of using the marginal tax tools to correct for inefficiencies in the labor markets, the marginal tax for high-skilled should be negative and the marginal tax for low-skilled positive. This result is derived when $\lambda_H > 0$, hence when the government suffers from its informational constraint. A corresponding Walrasian model (e.g. the baseline model in Stiglitz, 1982) predicts that the marginal tax rates should be zero for the high-skilled and positive for the low-skilled. The positive marginal tax on low-skilled (in a Walrasian model) can be seen as the efficiency price we need to pay in order to redistribute. When the SSC binds the government must reduce the attractiveness of mimicking. Raising the marginal tax on the representative low-skilled worker reduces her labor supply, but it also reduces the mimicker's labor supply. Starting from a situation with zero marginal tax on low-skilled, one realizes that an increase in the low-skill marginal tax has no first order effect on the low-skilled's utility, but a first order negative effect on the mimicker's utility. An increase in the low-skill marginal tax therefore hits a mimicker harder than it hits a low-skilled worker. This is what makes the optimal marginal tax on low-skilled positive when the SSC binds; raising the marginal tax is the measure the government must take in order to reduce the attractiveness of mimicking.

This effect of an increase in the low-skill marginal tax is also present in our model; it is captured by the first term in eq. (31). However, the government

now has more redistributive tools at hand. It has the possibility to affect the wage-formation. At a first glance it may be tempting to assume that the government would want to use the marginal taxes in order to increase the low-skilled's wages and reduce the high-skilled's wages; after all, the difference in wages is what causes inequality in the first place. This would mean positive (wage restraining) marginal tax on the high-skilled and negative marginal tax on the low-skilled. As we see in Proposition 2 the government does the exact opposite; the optimal marginal tax on high-skilled is negative and the second term in eq. (31) – hence the term that captures the effect working through the wage formation – is positive. This means that the government raises the wage for the high-skilled and lowers it for the low-skilled. And in unemployment terms it means an increase in the high-skilled's unemployment rate and a reduction in the low-skilled's unemployment rate. The government thus performs a quite intricate form of redistribution; it gives the low-skilled utility in the form of a lower risk of unemployment and takes from the high-skilled by increasing their risk of unemployment. One realizes that this is in accordance with the government's wish to relax the SSC. Consider what happens when the government lowers the wage for the low-skilled. The utility loss from lower wages is compensated for by a reduction in the risk of unemployment. Both the low-skilled and the mimicker suffer from the low-skill wage reduction but only the low-skilled benefit from a reduction in the low-skill unemployment rate. An increase in the marginal tax on low-skilled now punishes a mimicker in two ways: i) given a fixed wage, an increase in the low-skill marginal tax lowers the low-skill labor supply and thereby also the mimicker's labor supply ii) an increase in the low-skill marginal tax restrains the low-skill wage which decreases the low-skill labor supply further. The analogous logic follows through when considering an increase in the high-skill wage.

4 Concluding Remarks

The paper has extended the basic Stiglitz (1982) model of optimal income taxation into a model featuring involuntary unemployment. The labor markets consist of two skill-specific wage functions and two corresponding unemployment functions. The wage function specifies how the tax parameters influence the bargained wage and the unemployment function captures the trade-off between employment and wages.

In any type of labor market model with wage bargaining one can derive a negative relation between some measure of tax progressivity and the bargained wage. The literature on how tax progressivity induces wage moderation is substantial. Empirical research lends support for that the link between tax progressivity and wage setting is in fact real and not just an esoteric theoretical construct. We have, in this paper, captured the effects on wages by assuming that the wage-function depends positively on the total tax one type of worker pays (*wage compensation*) and negatively on the marginal tax facing this skill-type (*wage restrain*).

The main result of the paper is that the tax instruments' influence on the wage setting, and thereby unemployment, allow the government to perform redistribution through the different unemployment rates facing the two skill-groups. When the informational constraint binds, the government reduces the high-skill marginal tax and increases the low-skill marginal tax. This tax structure increases the high-skill unemployment and reduces the low-skill unemployment. Giving the low-skilled workers utility in the form of low unemployment is an elegant way to redistribute, since the mimicker does not benefit from a decrease in unemployment for the low-skilled. Lowering the low-skill wage and raising the high-skill wage will in this way work as means to relax the informational constraint facing the government.

This way of relaxing the informational constraint by making use of the marginal tax rates influence on unemployment is, to my knowledge, new to the literature. And finding intricate ways to relax this constraint is indeed the focal point of any economics paper analyzing the equity/efficiency tradeoff in models with imperfect information.

The model has left out unemployment benefits from the analysis. There may indeed be interesting interactions between the optimal use of the tax instruments and the optimal use of unemployment benefits in a model featuring endogenous wages. However, due to simplicity and clarity, this has been left for future research.

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Appendix A

Definitions of Ω_i^j ($i = 1, 2$ and $j = H, L$)

$$\Omega_1^H \equiv \frac{\frac{1}{w_H} (Y_H - g_H)}{\left(1 + \epsilon_{ew}^H \epsilon_{wc}^H \frac{T_H}{C_H}\right) (1 + \lambda_H)} \quad (\text{A1})$$

$$\Omega_2^H \equiv \frac{\frac{1}{w_H} \left(\frac{g'_H}{w_H} Y_H - \frac{g'_m}{w_H} Y_L - \epsilon_{ew}^H (Y_H - C_H)\right)}{\left(1 + \epsilon_{ew}^H \epsilon_{wc}^H \frac{T_H}{C_H}\right) (1 - u_H) (1 + \lambda_H)} \quad (\text{A2})$$

$$\Omega_1^L \equiv \frac{\frac{u'_L}{1-u_L} (C_L - Y_L) C_L}{1 + \epsilon_{ew}^L \epsilon_{wc}^L \frac{T_L}{C_L}} \quad (\text{A3})$$

$$\Omega_2^L \equiv \frac{1}{w_L} \frac{(1 - u_L)}{1 + \epsilon_{ew}^L \epsilon_{wc}^L \frac{T_L}{C_L}} \quad (\text{A4})$$

where

$$\epsilon_{wc}^j \equiv \frac{w_C}{w} C < 0, \quad j = H, L.$$

Appendix B

Proof of $\Omega_1^H > 0$ and $\Omega_2^L > 0$.

$\Omega_1^L > 0$ follows directly from the definition (A3) since $C_L > Y_L$, $w_C^L < 0$ and $u'_L > 0$. For $\Omega_1^H > 0$ to hold we technically need $1 + \epsilon_{ew}^H \epsilon_{wc}^H \frac{T_H}{C_H} > 0$, but since the model breaks down as T_H gets as high that the term $\epsilon_{ew}^H \epsilon_{wc}^H \frac{T_H}{C_H}$ approaches -1 , this can be ruled out.

QED.

Appendix C

Proof of $\Omega_2^H > 0$ and $\Omega_2^L > 0$ when $\epsilon_{ew}^H = \epsilon_{vw}^H$ holds.

$\Omega_2^L > 0$ follows directly from the definition (A4). For Ω_2^H , however, the case is more complex. The sign of Ω_2^H is then equal to the sign of $\zeta \equiv \frac{g'_H}{w_H} Y_H - \frac{g'_m}{w_H} Y_L - \epsilon_{ew}^H (Y_H - C_H)$. We use $\epsilon_{ew}^H = \epsilon_{vw}^H$ to rewrite ζ as:

$$\zeta = \frac{1}{(Y_H - g_H) w_H} [g'_H Y_H (C_H - g_H) - (Y_H - g_H) g'_m Y_L]. \quad (\text{C1})$$

Since $g'_H > g'_m$ we have:

$$\zeta > \frac{g'_m}{(Y_H - g_H) w_H} [Y_H (C_L - Y_L) + g_H Y_L - Y_H g_m], \quad (\text{C2})$$

where we also have used the SSC, i.e. $C_H - g_H = C_L - g_m$. Now note that convexity of $g(\cdot)$ and $g(0) = 0$ give:

$$g_m = g\left(\frac{Y_L}{w_H}\right) = g\left(\frac{Y_L w_L}{w_H w_L}\right) = g\left(\frac{Y_L w_L}{w_L w_H}\right) < g_L \frac{w_L}{w_H} \quad (\text{C3})$$

and

$$g_L = g\left(\frac{Y_L}{w_L}\right) = g\left(l_H \frac{l_L}{L_H}\right) < g_H \frac{l_L}{l_H}. \quad (\text{C4})$$

Using (C3) and (C4) in (C2) finally gives:

$$\zeta > \frac{g'_m Y_H}{(Y_H - g_H) w_H} (C_L - Y_L) > 0,$$

and thus $\Omega_2^H > 0$.

QED.

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