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## *Department of Economics*

Working Paper 2012:7

### Optimal Monetary Policy under Learning in a New Keynesian Model with Cost Channel and Inflation Inertia

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OPTIMAL MONETARY POLICY UNDER LEARNING IN A  
NEW KEYNESIAN MODEL WITH COST CHANNEL AND INFLATION INERTIA

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# OPTIMAL MONETARY POLICY UNDER LEARNING IN A NEW KEYNESIAN MODEL WITH COST CHANNEL AND INFLATION INERTIA\*

Mikael Bask<sup>†</sup>      Christian R. Proaño<sup>‡</sup>

February 29, 2012

## Abstract

We show that a so-called expectations-based optimal monetary policy rule has desirable properties in a standard New Keynesian model augmented with a cost channel and inflation rate expectations that are partly backward-looking. In particular, optimal monetary policy under commitment is associated with a determinate rational expectations equilibrium that is stable under least squares learning for all parameter constellations considered, whereas, under discretion in policy-making, the central bank has to be sufficiently inflation rate averse for the rational expectations equilibrium to have the same properties.

**Keywords:** Commitment; Cost Channel; Determinacy; Discretion; Inflation Inertia; Least Squares Learning; Optimal Monetary Policy.

**JEL Codes:** C62; E52.

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## 1 INTRODUCTION

Over the past years, the extensive research on the design of optimal monetary policy has significantly improved our understanding of the interaction between the central bank's actions and the private sector's decision making. In this respect, the literature on imperfect knowledge and learning has been of particular relevance given that the rational expectations assumption – according to which agents are able to derive the time-path of an often complex economy's future development – is a rather strong assumption. However, as demonstrated in Evans and Honkapohja [11], if agents are able to infer from data the true nature of the mechanisms driving the economy, their expectations may eventually become rational, even if they initially had incomplete knowledge about these mechanisms.<sup>1</sup>

In an economy with imperfectly knowledgeable agents (cf., Simon [29]), a main task of monetary policy is to enforce a determinate and *learnable* rational expectations equilibrium (REE), as stressed, among others, by Bullard and Mitra [6] in their analysis of a variety of instrument rules along the lines of Taylor [33]-[34]. According to Svensson [31]-[32], however, instrument rules are inferior to targeting rules in policy-making, which are derived from the optimization of the central bank's objective function (e.g., a welfare function), because instrument rules are not consistent with optimal behavior on the part of the central bank. Unfortunately, embedding a targeting rule for monetary policy into a forward-looking macroeconomic model does not guarantee determinacy, even if agents have rational expectations (see Clarida et al. [10]).

By constructing an expectations-based targeting rule that comprises the possibly non-rational expectations of agents, instead of imposing rational expectations, Evans and Honkapohja [12]-[15] show how the central bank can enforce a determinate REE that is stable under least squares learning. Specifically, when the central bank uses an expectations-based rule in policy-making, the interest rate is directly influenced by agents' possibly non-rational expectations and this creates a mechanism that the central bank may use to correct their expectations regarding the economy's law of motion, leading the economy to converge over time to the desired REE.<sup>2</sup> In contrast,

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<sup>1</sup>In literature jargon, such a learning process would take place if the parameter values in the agents' perceived law of motion (PLM) of the economy converge to the economy's actual law of motion (ALM). In this case, the rational expectations equilibrium (REE) is characterized by expectational stability (E-stability) and thus, by extension, is least squares learnable, as proved by Marcet and Sargent [23].

<sup>2</sup>See Bullard [5] and Evans and Honkapohja [16]-[17] for reviews of the literature on monetary

if the central bank wrongly assumes that agents already have rational expectations and uses a fundamentals-based rule in policy-making, there is no mechanism that forces agents to correct their expectations regarding the economy's law of motion and the economy may not converge to the desired REE.

The purpose of the present paper is to examine whether a determinate and learnable REE can be enforced by an expectations-based targeting rule after extending a standard New Keynesian model into two directions: (i) incorporating a cost channel into the model; and (ii) allowing for inertia in inflation rate expectations.

Regarding the first extension, Barth and Ramey [2] and Chowdhury et al. [9], among others, provide empirical evidence that firms' marginal costs are directly affected by the interest rate. The reason is that firms have to pay their production factors before they receive revenues from selling their products and therefore need to borrow money from financial intermediaries. The second extension is related to the empirical evidence of persistence in inflation rates (see, e.g., Altissimo et al. [1] and Galí and Gertler [19]), where it has been shown that the incorporation of the lagged inflation rate improves the ability of models to explain observed inflation rate dynamics. A hybrid specification of the New Keynesian Phillips curve, where the expected inflation rate is a weighted average of the lagged inflation rate and the inflation rate under rational expectations, may therefore be used in models that guide monetary policy.

Our paper contributes to the literature in the following way: Evans and Honkapohja [13] study the advantages of an expectations-based rule for the central bank under discretion in policy-making, and Evans and Honkapohja [12] and [14]-[15] do the same exercise under commitment in policy-making. However, these models lack a cost channel and inertia in inflation rate expectations. Llosa and Tuesta [21] study optimal monetary policy in a model that includes a cost channel, but there is no inertia in inflation rate expectations and monetary policy is not implemented as an expectations-based rule as in this paper. Evans and McGough [18] include both the lagged inflation rate and the lagged output gap in their model, but there is no cost channel and monetary policy is not optimal. Finally, Brückner and Schabert [4] and Surico [30] study monetary policy in a model that includes a cost channel, but there is no inertia in inflation rate expectations and monetary policy is not optimal.

Our paper therefore fills a gap in the literature because we scrutinize optimal monetary policy in a New Keynesian model with a cost channel and inertia in inflation rate expectations. 

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policy design in the New Keynesian model from a least squares learning perspective.

flation rate expectations. More precisely, we investigate under which conditions an expectations-based targeting rule can enforce a determinate and learnable REE in such a model. Indeed, optimal monetary policy under commitment is associated with a determinate REE that is stable under least squares learning for all parameter constellations considered, whereas, under discretion in policy-making, the central bank has to be sufficiently inflation rate averse for the REE to have the same properties.

The remainder of this paper is organized as follows: The baseline model is presented in Section 2, whereas expectations-based targeting rules for the central bank are derived in Section 3 – one rule for discretion in policy-making and another rule for commitment in policy-making. In Section 4, we examine whether these rules can deliver a determinate REE that is stable under least squares learning and, in Section 5, we look at the effects of different forms of misapprehensions in policy-making. Section 6 concludes the paper.

## 2 THE BASELINE MODEL

We study a forward-looking macroeconomic model that is described by the following two equations:

$$x_t = x_{t+1}^e - \alpha (r_t - \pi_{t+1}^{e,*}), \quad (1)$$

$$\pi_t = \beta \pi_{t+1}^{e,*} + \gamma x_t + \delta r_t, \quad (2)$$

where  $x_t$  is the output gap,  $r_t$  is the nominal interest rate controlled by the central bank, and  $\pi_t$  is the inflation rate. Moreover,  $x_{t+1}^e$  and  $\pi_{t+1}^{e,*}$  represent the possibly non-rational expected values of the output gap and the inflation rate, respectively. Regarding the latter variable, we assume as is occasionally done in the literature that  $\pi_{t+1}^{e,*}$  is given by (see, e.g., Galí and Gertler [19])

$$\pi_{t+1}^{e,*} = \omega \pi_{t-1} + (1 - \omega) \pi_{t+1}^e, \quad (3)$$

where  $\pi_{t+1}^e$  represents the possibly non-rational expected value of the inflation rate and  $\omega \in [0, 1]$  is the weight given to the lagged inflation rate in the expectations formation process of  $\pi_{t+1}^{e,*}$ .

Eq. (1) represents a forward-looking IS curve as usually specified in the New Keynesian literature, with  $\frac{1}{\alpha}$  being the intertemporal elasticity of substitution in consumption. Eq. (2) represents a forward-looking Phillips curve augmented with a cost channel (see Ravenna and Walsh [27]), with  $\beta$  being the discount factor used

when the representative household maximizes a discounted sum of instantaneous utilities derived from consumption and leisure,  $\delta$  being the size of the cost channel, which is a function of  $\beta$  and the fraction of firms that set profit-maximizing prices in each time period (i.e., Calvo [8] pricing), and  $\gamma$  being a function of  $\alpha$ ,  $\delta$  and the intertemporal elasticity of substitution in labor supply.

The baseline model in eqs. (1)-(3) differs from the standard model in two respects (see Clarida et al. [10] for a presentation of this model). First, there is a cost channel in the AS curve, which is the last term at the right-hand side of eq. (2). Monetary policy therefore affects the inflation rate via two channels: (i) an increased interest rate dampens the output gap (see the IS equation) and lowers the inflation rate (see the second term at the right-hand side of the AS equation); and (ii) an increased interest rate increases the inflation rate directly via higher marginal costs for firms to borrow money from financial intermediaries. The latter channel is the cost channel. The second difference between the baseline model and the standard model is that inflation rate expectations are partly backward-looking (see eq. (3)).

How the inclusion of the cost channel and the lagged inflation rate in the inflation rate expectations process affect the model's behavior depends (i) on how expectations are formed by the private sector and (ii) on how the central bank implements its policy. These issues are in focus in the next section.

### 3 OPTIMAL MONETARY POLICY

#### 3.1 *Optimal monetary policy under rational expectations*

The baseline model in eqs. (1)-(3) is closed by deriving an interest rate rule for the central bank that minimizes a loss function in the output gap and the inflation rate:

$$\mathcal{W}_t = E_t \sum_{i=0}^{\infty} \beta^i (\theta x_{t+i}^2 + \pi_{t+i}^2), \quad (4)$$

where  $\theta$  is the degree of flexibility in inflation rate targeting and  $E_t$  represents the rational expectations operator.

Taking into account the constraint in the optimization problem – the economy's



law of motion given by eqs. (1)-(3) – the corresponding Lagrangian at time  $t = 0$  is<sup>3</sup>

$$\begin{aligned} \mathcal{L}_0 = E_0 \sum_{i=0}^{\infty} \beta^i \{ & \theta x_i^2 + \pi_i^2 \\ & - \lambda_i \left( \pi_i - (\beta + \delta)(\omega \pi_{i-1} + (1 - \omega)\pi_{i+1}) + \left( \frac{\delta}{\alpha} - \gamma \right) \cdot x_i - \frac{\delta}{\alpha} \cdot x_{i+1} \right) \}. \end{aligned} \quad (5)$$

When there is discretion in policy-making, the central bank solves the optimization problem without taking into account that its interest rate setting behavior today affects the outcomes of the optimization problems that are solved in future time periods. In this case, the first-order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial x_t} = 2\theta x_t - \left( \frac{\delta}{\alpha} - \gamma \right) \cdot \lambda_t = 0, \quad (6)$$

and

$$\frac{\partial \mathcal{L}_t}{\partial \pi_t} = 2\pi_t - \lambda_t + \beta(\beta + \delta)\omega E_t[\lambda_{t+1}] = 0. \quad (7)$$

Alternatively, the central bank can do better by solving for the first-order conditions that support a policy that is consistently optimal over time instead of re-optimizing the Lagrangian in each time period. In this case, when there is commitment in policy-making, the first-order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial x_t} = \frac{\delta}{\alpha} \cdot \lambda_{t-1} + 2\beta\theta x_t - \beta \left( \frac{\delta}{\alpha} - \gamma \right) \cdot \lambda_t = 0, \quad (8)$$

and

$$\frac{\partial \mathcal{L}_t}{\partial \pi_t} = (\beta + \delta)(1 - \omega)\lambda_{t-1} + 2\beta\pi_t - \beta\lambda_t + \beta^2(\beta + \delta)\omega E_t[\lambda_{t+1}] = 0. \quad (9)$$

Because the central bank under commitment in policy-making takes into account that its interest rate setting behavior today affects the outcomes of the optimization problems that are solved in future time periods, the central bank designs a time-path for its policy that is superior to a discretionary policy.

When a commitment mechanism is not available in policy-making, the condition for optimal monetary policy is

$$\pi_t = -\frac{\alpha\theta}{\alpha\gamma - \delta} \cdot x_t + \frac{\alpha\beta(\beta + \delta)\theta\omega}{\alpha\gamma - \delta} \cdot E_t[x_{t+1}], \quad (10)$$

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<sup>3</sup>In the Technical Appendix, derivations of several of the equations in the paper can be found.

whereas, when a commitment mechanism is available in policy-making, the condition for optimal monetary policy is

$$\begin{aligned}\pi_t = & -\frac{\delta}{(\alpha\gamma - \delta)\beta} \cdot \pi_{t-1} + \frac{\alpha(\beta + \delta)\theta(1 - \omega)}{(\alpha\gamma - \delta)\beta} \cdot x_{t-1} - \frac{\alpha\theta}{\alpha\gamma - \delta} \cdot x_t \\ & + \frac{\alpha\beta(\beta + \delta)\theta\omega}{\alpha\gamma - \delta} \cdot E_t[x_{t+1}].\end{aligned}\tag{11}$$

Starting with the optimality condition in eq. (10), which is eqs. (6)-(7) combined, the lead output gap is included in the condition because the lagged inflation rate is included in the expectations formation process in eq. (3). Notice that this term also vanishes when  $\omega = 0$ . Continuing with the optimality condition in eq. (11), which is eqs. (8)-(9) combined, terms for the lagged inflation rate and the lagged output gap are now added. The second term is typical in conditions when a commitment mechanism is available in policy-making, whereas the first term is due to the presence of the cost channel. Notice that the latter term also vanishes when  $\delta = 0$ .

The time-inconsistency problem in policy-making is also revealed in the equations above because the condition for optimal monetary policy under discretion (see eq. (10)) is not consistent with the same condition under commitment (see eq. (11)). However, this problem can be solved by assuming a “timeless perspective” (see Woodford [35]), meaning that the optimal monetary policy is assumed to have been implemented long time enough that agents in the economy believe that the central bank is committed to the policy and that the condition for a discretionary policy therefore does not hold.<sup>4</sup>

### 3.2 *Expectations-based targeting rules*

Targeting rules are superior to instrument rules in monetary policy and targeting rules under commitment are superior to targeting rules under discretion. However, an important problem with targeting rules under the assumption of rational expectations (i.e., fundamentals-based rules) is that they may not guarantee determinacy (see Clarida et al. [10]). Evans and Honkapohja [16] review different implementations of targeting rules taking the econometric learning approach (i.e., expectations-based rules), and our aim is to extend the findings in their review to a New Keynesian model with a cost channel and inflation rate expectations that are partly backward-looking.

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<sup>4</sup>See Sauer [28] for the start-up costs when implementing optimal monetary policy in a timeless perspective.

For the sake of analytical simplicity, we assume a contemporaneous data (or time  $t$ -dating) specification of the information set of the central bank despite that it implies an inconsistency between the information sets of the central bank and the private sector. Indeed, as pointed out by McCallum [25], under such a specification, the central bank has superior information compared to the private sector because it reacts to information about the inflation rate and the output gap at time  $t$ , whereas the private sector reacts to information available as of time  $t - 1$ .

A specification of the information sets of the central bank and the private sector that corrects this tension is the contemporaneous expectations (or time  $t - 1$ -dating) specification. Under this specification, the central bank sets the interest rate in response to current expectations, formed using information available as of time  $t - 1$ , of the current inflation rate and the current output gap. However, Bullard and Mitra [6] demonstrate that despite the fact that the contemporaneous expectations specification of the information sets is to prefer to the contemporaneous data specification, because of its consistent treatment of the information sets of the central bank and the private sector, both specifications are equivalent as determinacy is concerned.

Having discussed this issue, the following optimal interest rate rule is derived by taking agents' expectations as given and not imposing rational expectations:

$$r_t = \kappa_0 x_{t-1} + \kappa_1 x_t + \kappa_2 x_{t+1}^e + \kappa_3 \pi_{t-1} + \kappa_4 \pi_{t+1}^e. \quad (12)$$

The rule in eq. (12) is named an expectations-based targeting rule. Under discretion in policy-making, the coefficients in this rule are

$$\begin{cases} \kappa_0 = 0, \\ \kappa_1 = \frac{\alpha\theta}{(\alpha\gamma-\delta)^2}, \\ \kappa_2 = -\frac{1}{\alpha\gamma-\delta} \cdot \left( \frac{\alpha\beta(\beta+\delta)\theta\omega}{\alpha\gamma-\delta} - \gamma \right), \\ \kappa_3 = \frac{(\alpha\gamma+\beta)\omega}{\alpha\gamma-\delta}, \\ \kappa_4 = \frac{(\alpha\gamma+\beta)(1-\omega)}{\alpha\gamma-\delta}, \end{cases} \quad (13)$$

whereas, under commitment in policy-making, the coefficients are

$$\begin{cases} \kappa_0 = -\frac{\alpha(\beta+\delta)\theta(1-\omega)}{(\alpha\gamma-\delta)^2\beta}, \\ \kappa_1 = \frac{\alpha\theta}{(\alpha\gamma-\delta)^2}, \\ \kappa_2 = -\frac{1}{\alpha\gamma-\delta} \cdot \left( \frac{\alpha\beta(\beta+\delta)\theta\omega}{\alpha\gamma-\delta} - \gamma \right), \\ \kappa_3 = \frac{1}{\alpha\gamma-\delta} \cdot \left( (\alpha\gamma + \beta)\omega + \frac{\delta}{(\alpha\gamma-\delta)\beta} \right), \\ \kappa_4 = \frac{(\alpha\gamma+\beta)(1-\omega)}{\alpha\gamma-\delta}. \end{cases} \quad (14)$$

The differences between the coefficients in eqs. (13)-(14) are that a term for the lagged output gap ( $\kappa_0$ ) is included when a commitment mechanism is available in policy-making, which is a typical result in the literature, and that the term for the lagged inflation rate ( $\kappa_3$ ) is smaller when there is discretion in policy-making. The latter difference is due to the presence of the cost channel, which vanishes when  $\delta = 0$ .

Let us now investigate the properties of the macroeconomic model that has been developed in Sections 2 and 3. Specifically, we would like to address the following questions in the analysis: Under which conditions is this model characterized by a determinate REE that is stable under least squares learning? Further, how are these conditions affected by the size of the cost channel and the importance of the lagged inflation rate in inflation rate expectations?

#### 4 DETERMINACY AND LEARNABILITY UNDER DISCRETION AND COMMITMENT

Both under discretion and commitment in policy-making, the complete model in matrix form is

$$\mathbf{\Gamma} \cdot \mathbf{y}_t = \mathbf{\Theta} \cdot \mathbf{y}_{t+1}^e + \mathbf{\Lambda} \cdot \mathbf{y}_{t-1}, \quad (15)$$

where the coefficient matrices are

$$\mathbf{\Gamma} = \begin{bmatrix} 1 + \alpha\kappa_1 & 0 \\ -(\gamma + \delta\kappa_1) & 1 \end{bmatrix}, \quad (16)$$

$$\mathbf{\Theta} = \begin{bmatrix} 1 - \alpha\kappa_2 & \alpha(1 - \omega - \kappa_4) \\ \delta\kappa_2 & \beta(1 - \omega) + \delta\kappa_4 \end{bmatrix}, \quad (17)$$

$$\mathbf{\Lambda} = \begin{bmatrix} -\alpha\kappa_0 & \alpha(\omega - \kappa_3) \\ \delta\kappa_0 & \beta\omega + \delta\kappa_3 \end{bmatrix}, \quad (18)$$

and the variable vector is

$$\mathbf{y}_t = \begin{bmatrix} x_t & \pi_t \end{bmatrix}', \quad (19)$$

where  $\kappa_0 = 0$  under discretion in policy-making and the magnitude of  $\kappa_3$  differs under discretion and commitment in policy-making.

A procedure to determine whether the model in eqs. (15)-(19) has a determinate REE is to rewrite the model into first-order form and then to compare the number of predetermined variables with the number of eigenvalues of a certain matrix (that we

derive below) that are outside the unit circle (see Blanchard and Kahn [3]). Further, if the model has a determinate REE, the solution to the matrix equation in eq. (15) can be represented in the minimum state variable (MSV) form (see McCallum [24]):

$$\mathbf{y}_t = \mathbf{\Pi} \cdot \mathbf{y}_{t-1} + \mathbf{\Phi}, \quad (20)$$

where  $\mathbf{\Pi}$  and  $\mathbf{\Phi}$  solve the following matrix equation system:

$$\begin{cases} \mathbf{\Pi}^2 - \mathbf{\Theta}^{-1} \cdot \mathbf{\Gamma} \cdot \mathbf{\Pi} + \mathbf{\Theta}^{-1} \cdot \mathbf{\Lambda} = \mathbf{0}, \\ (\mathbf{\Gamma} - \mathbf{\Theta} \cdot \mathbf{\Pi} - \mathbf{\Theta}) \cdot \mathbf{\Phi} = \mathbf{0}. \end{cases} \quad (21)$$

Because  $\kappa_0$  and  $\kappa_3$  in the optimal monetary policy rule in eq. (12) have different magnitudes under discretion and commitment in policy-making, the matrices in the matrix equation system in eq. (21) are different under discretion and commitment in policy-making as well.

To have an economy with a determinate REE that is stable under least squares learning, the parameter values in the perceived law of motion (PLM) of the economy, which is the MSV solution in eq. (20), have to converge to the economy's actual law of motion (ALM). Moreover, McCallum [26] shows that for a broad class of linear rational expectations models, which includes the model in eqs. (15)-(19), that a determinate REE is also an expectationally stable (E-stable) REE when the dating of expectations is time  $t$ , which is assumed herein. Thus, because an E-stable REE is a necessary and sufficient condition for a least squares learnable REE (see Marcet and Sargent [23]), all regions for a determinate REE that are found in the figures below are also regions for a least squares learnable REE.

#### 4.1 The case of discretion in policy-making

When there is discretion in policy-making, we make use of the following variable vector when rewriting the model in eqs. (15)-(19):

$$\mathbf{y}_{d,t} = \begin{bmatrix} x_t & \pi_t & \pi_t^L \equiv \pi_{t-1} \end{bmatrix}', \quad (22)$$

meaning that the coefficient matrices are

$$\mathbf{\Gamma}_d = \begin{bmatrix} \mathbf{\Gamma} & -\mathbf{\Lambda}_2 \\ 0 & 1 & 0 \end{bmatrix}, \quad (23)$$

and

$$\Theta_d = \begin{bmatrix} & \Theta & 0 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (24)$$

where  $\Lambda_2$  is the second column in matrix  $\Lambda$ , meaning that the complete model in matrix form is

$$\Gamma_d \cdot \mathbf{y}_{d,t} = \Theta_d \cdot \mathbf{y}_{d,t+1}^e. \quad (25)$$

Because there is one variable in eq. (22) that is predetermined,  $\pi_t^L$ , exactly one eigenvalue of the matrix  $\Gamma_d^{-1} \cdot \Theta_d$  must be outside the unit circle for the model to have a determinate REE that is stable under least squares learning. In contrast, if more than one eigenvalue are outside the unit circle, the model has an indeterminate REE, and if all eigenvalues are inside the unit circle, then there is no stable REE in the model.

Because analytical conditions for determinacy and learnability are too complex and cumbersome to interpret, we illustrate our findings numerically.<sup>5</sup> We use the following values of the structural parameters for this exercise:  $\alpha = \frac{1}{2}$  (see Levin et al. [20] and Lubik and Schorfheide [22] who have estimated this parameter for the U.S. economy to be  $\frac{1}{2.04}$  and  $\frac{1}{1.86}$ , respectively),  $\beta = 0.99$ , and  $\gamma = 0.072$  (which is an estimate for the U.S. economy as well, see Chowdhury et al. [9]).

For different sizes of the cost channel, Figure 1 shows regions in  $(\omega, \theta)$ -space that give rise to a determinate and least squares learnable REE, an indeterminate REE, and no stable REE. Recall that  $\omega$  is the weight given to the lagged inflation rate in inflation rate expectations and that  $\theta$  is the degree of flexibility in inflation rate targeting in policy-making. The latter parameter belongs to the unit interval in the analysis. To have a reference, Chowdhury et al. [9] have estimated the cost channel parameter for the U.S. economy to be  $\delta = 0.03$ .

First, for smaller sizes of the cost channel (i.e.,  $\delta \leq 0.010$ ), the whole region in  $(\omega, \theta)$ -space is associated with a determinate and least squares learnable REE. It is already known that the economy is characterized by such a desirable outcome when backward-looking expectations do not play any role in inflation rate expectations (see Evans and Honkapohja [13]), but it is now clear that this result holds irrespective of the importance of the lagged inflation rate in the expectations formation process.

Second, for larger sizes of the cost channel (i.e.,  $\delta \geq 0.040$ ), there is no region in  $(\omega, \theta)$ -space that is associated with a determinate and least squares learnable

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<sup>5</sup>MATLAB routines for this purpose are available on request from the authors.

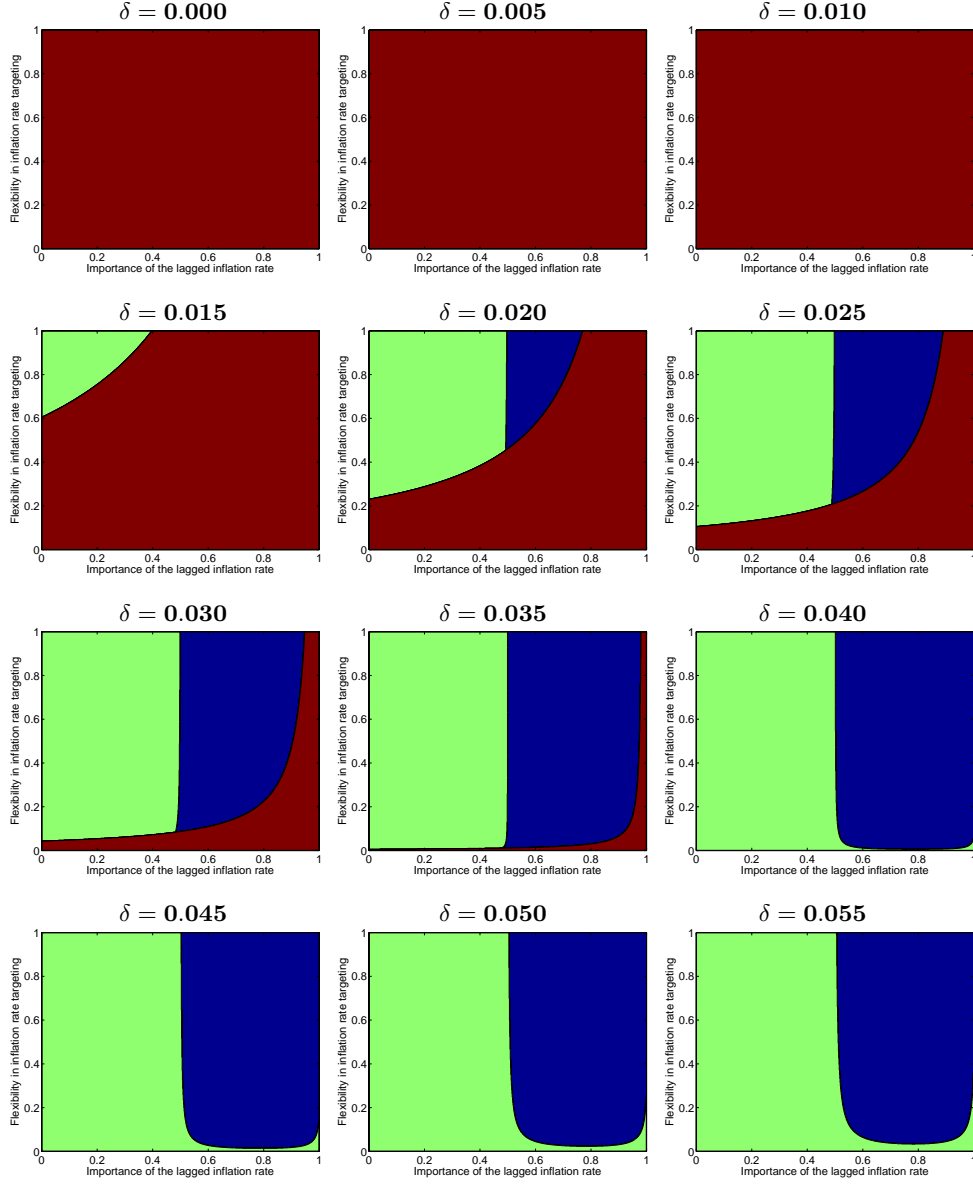


Figure 1: The case of discretion in policy-making with region for a determinate REE that is stable under least squares learning (red area) and regions for an indeterminate REE (green area) and no stable REE (blue area).

REE. The reason is that a large enough cost channel has a perverse effect on the parameters in the optimal monetary policy rule in eqs. (12)-(13). Specifically, when<sup>6</sup>

$$\delta > \alpha\gamma, \quad (26)$$

<sup>6</sup>To be more precise, when  $\delta > \alpha\gamma = \frac{1}{2} \cdot 0.072 = 0.036$ .

the central bank decreases the interest rate when the expected inflation rate increases, which means that monetary policy is stimulating the economy so that the expected inflation rate increases even further. Obviously, in the absence of the cost channel, this perverse situation could never arise.

Finally, for intermediate sizes of the cost channel (i.e.,  $0.015 \leq \delta \leq 0.035$ ), the boundary of the determinacy region in  $(\omega, \theta)$ -space has a positive slope. A greater importance of the lagged inflation rate in inflation rate expectations may therefore turn the economy to equilibrium determinacy, given the degree of inflation rate targeting in policy-making, at the same time as a stricter inflation rate targeting in policy-making may turn the economy to equilibrium determinacy, given the importance of the lagged inflation rate in inflation rate expectations.

Why is there a trade-off between the degree of inertia in inflation rate expectations and the degree of flexibility in inflation rate targeting in monetary policy for equilibrium determinacy? To begin with, a greater importance of the lagged inflation rate in inflation rate expectations increases the degree of inertia in monetary policy because  $\kappa_3$  in eqs. (12)-(13) increases in magnitude and thus, by extension, affects the degree of inertia in the economy because  $\kappa_3$  appear in eq. (18), which determines the impact  $\mathbf{y}_{t-1}$  has on  $\mathbf{y}_t$  (see eq. (15)). That inertia in monetary policy can help alleviate problems of indeterminacy and non-existence of stable equilibria in an economy is demonstrated in Bullard and Mitra [7].

A greater importance of the lagged inflation rate in inflation rate expectations also directly affects the degree of inertia in the economy, and not only indirectly via its effect on monetary policy, because  $\omega$  appear in eq. (18) as well. After scrutinizing the separate contributions of these two effects of the lagged inflation rate in inflation rate expectations for equilibrium determinacy, it is clear that it is the combination of these two effects that creates the positive slope of the boundary of the determinacy region in  $(\omega, \theta)$ -space.<sup>7</sup> To end with, even though the degree of flexibility in inflation rate targeting in monetary policy does not affect the degree of inertia in the economy, it is clear that its impact on the output gap and the expected output gap variables in the interest rate rule may turn the economy to equilibrium determinacy.

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<sup>7</sup>In fact, Figure 3 in Section 5.2 illustrates the direct effect on equilibrium determinacy of inflation rate inertia when the indirect effect via monetary policy is shutdown (i.e.,  $\omega = 0$  in eq. (13)).



#### 4.2 The case of commitment in policy-making

When there is commitment in policy-making, we make use of the following variable vector when rewriting the model in eqs. (15)-(19):

$$\mathbf{y}_{c,t} = \begin{bmatrix} x_t & \pi_t & x_t^L \equiv x_{t-1} & \pi_t^L \equiv \pi_{t-1} \end{bmatrix}', \quad (27)$$

meaning that the coefficient matrices are

$$\mathbf{\Gamma}_c = \begin{bmatrix} \mathbf{\Gamma} & -\mathbf{\Lambda}_1 & -\mathbf{\Lambda}_2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad (28)$$

and

$$\mathbf{\Theta}_c = \begin{bmatrix} & \mathbf{\Theta} & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (29)$$

where  $\mathbf{\Lambda}_1$  and  $\mathbf{\Lambda}_2$  are the first and second columns in matrix  $\mathbf{\Lambda}$ , respectively, meaning that the complete model in matrix form is

$$\mathbf{\Gamma}_c \cdot \mathbf{y}_{c,t} = \mathbf{\Theta}_c \cdot \mathbf{y}_{c,t+1}^e. \quad (30)$$

Because there are two variables in eq. (27) that are predetermined,  $x_t^L$  and  $\pi_t^L$ , exactly two eigenvalues of the matrix  $\mathbf{\Gamma}_c^{-1} \cdot \mathbf{\Theta}_c$  must be outside the unit circle for the model to have a determinate REE that is stable under least squares learning. In contrast, if more than two eigenvalues are outside the unit circle, the model has an indeterminate REE, and if less than two eigenvalues are outside the unit circle, then there is no stable REE in the model.

After extensively examining a large number of parameter constellations, including unrealistic ones, there seems to be no restrictions in  $(\omega, \theta)$ -space on the existence of a determinate REE that is stable under least squares learning. Our *conjecture* is therefore that there is always a determinate and least squares learnable REE when a commitment mechanism is available in policy-making. Recall that  $\kappa_0 < 0$  and that  $\kappa_3$  has a larger magnitude under commitment than under discretion in policy-making (see eqs. (12)-(14)). Thus, because  $\kappa_0$  and  $\kappa_3$  appear in eq. (18), which determines the impact  $\mathbf{y}_{t-1}$  has on  $\mathbf{y}_t$  (see eq. (15)), the degree of inertia in the economy is affected when a commitment mechanism is available in policy-making. Apparently, the degree of inertia in the economy is so strong in this case that we have equilibrium determinacy for all parameter constellations considered.

## 5 MISAPPREHENSIONS IN POLICY-MAKING

In this section, we briefly investigate what happens if the central bank wrongly believes that the economy's law of motion is governed by the standard New Keynesian model when, instead, it is governed by a model that also has a cost channel and/or inflation rate expectations that are partly backward-looking.

### 5.1 *Misapprehension about the cost channel*

As Figure 2 illustrates, under the assumption that the central bank believes that  $\delta = 0$  in eqs. (13)-(14) when, in fact,  $\delta > 0$ , the shape of the region in  $(\omega, \theta)$ -space for a determinate and least squares learnable REE is not affected when there is discretion in policy-making, even though inflation rate targeting can now be more flexible. It is therefore tempting to conclude that the central bank should not care about the cost channel when setting the interest rate because it is then easier to achieve a determinate and least squares learnable REE. However, one must not forget that monetary policy no longer is optimal due to the incorrect belief regarding the presence of the cost channel.

When there is commitment in policy-making, there are no restrictions in  $(\omega, \theta)$ -space to have a determinate and least squares learnable REE.

### 5.2 *Misapprehension about inflation inertia*

In Figure 3, it is assumed that the central bank believes that  $\omega = 0$  in eqs. (13)-(14) when, instead,  $\omega > 0$ . Not surprisingly, when there is discretion in policy-making, the maximum flexibility in inflation rate targeting to have a determinate and least squares learnable REE is not affected by the importance of the lagged inflation rate in inflation rate expectations. Moreover, for intermediate sizes of the cost channel (i.e.,  $0.015 \leq \delta \leq 0.035$ ), the region in  $(\omega, \theta)$ -space for a determinate and least squares learnable REE is now smaller because the boundary of the determinacy region in  $(\omega, \theta)$ -space has a positive slope when there are no misapprehensions in inflation rate expectations (cf., Figure 1 in Section 4.1).

When there is commitment in policy-making, there are almost no restrictions in  $(\omega, \theta)$ -space to have a determinate and least squares learnable REE.

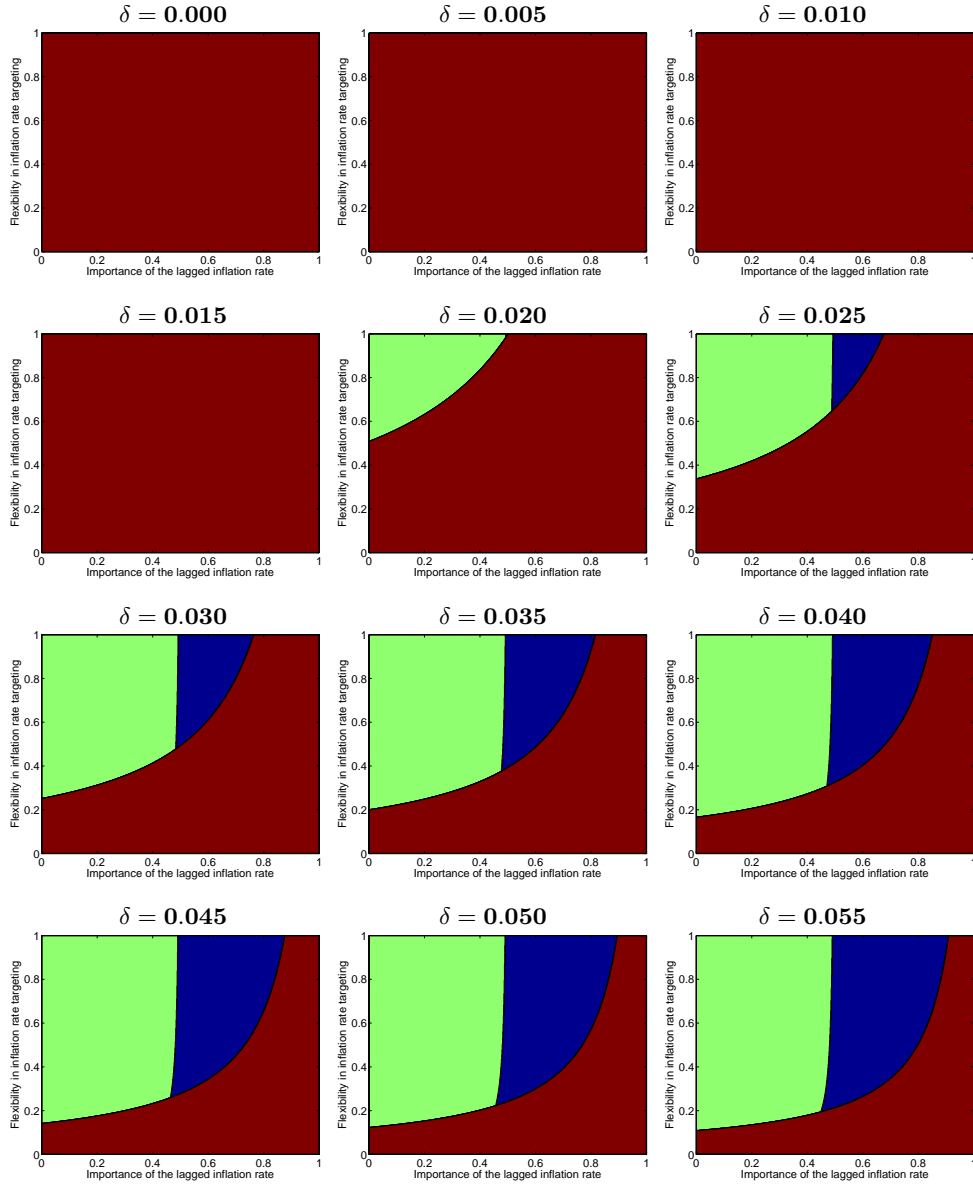


Figure 2: The case of discretion in policy-making with misapprehension about the cost channel and with region for a determinate REE that is stable under least squares learning (red area) and regions for an indeterminate REE (green area) and no stable REE (blue area).

## 6 CONCLUSIONS

Herein, we have scrutinized whether a determinate and least squares learnable REE can be enforced by an optimal monetary policy rule after extending a standard New

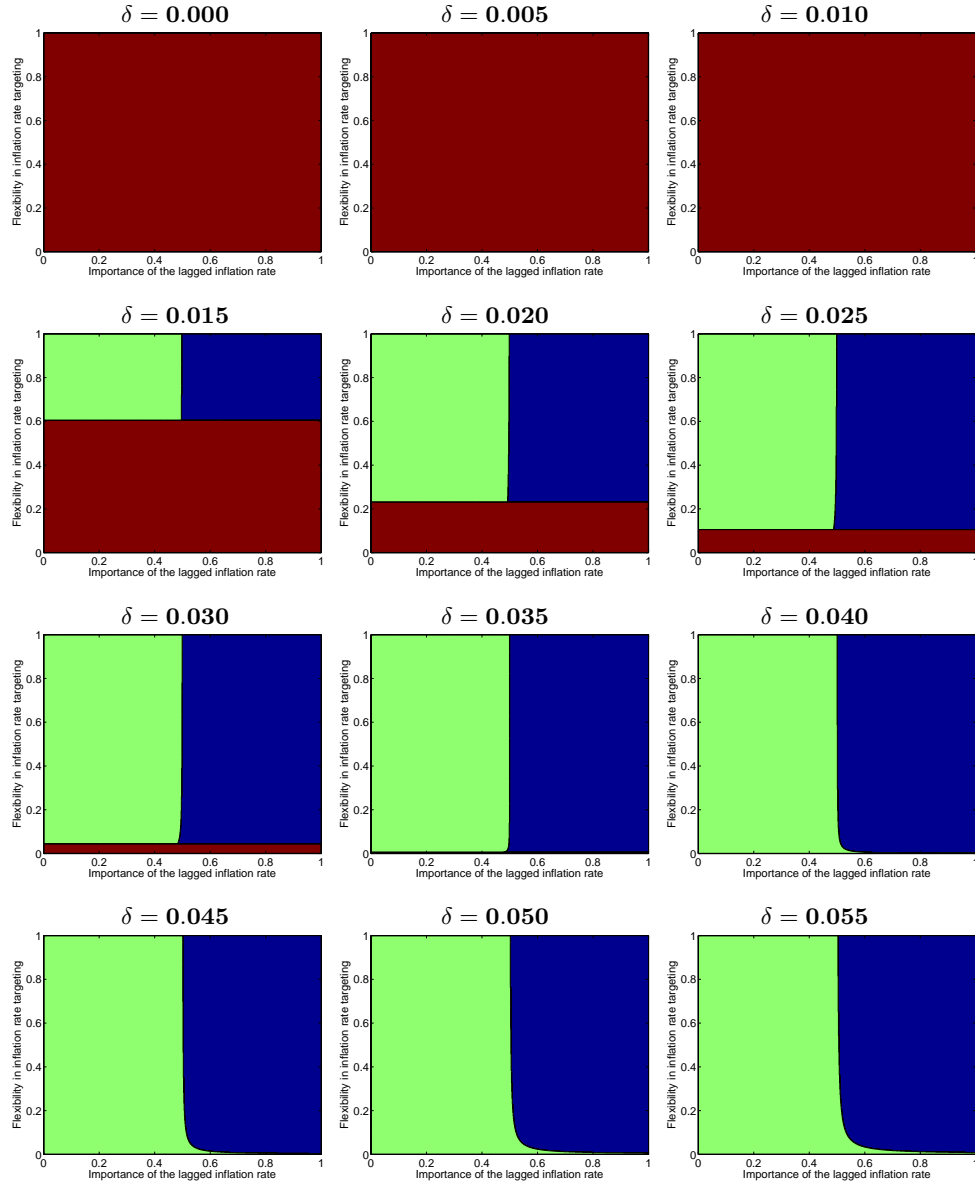


Figure 3: The case of discretion in policy-making with misapprehension about inflation rate inertia and with region for a determinate REE that is stable under least squares learning (red area) and regions for an indeterminate REE (green area) and no stable REE (blue area).

Keynesian model into two directions: (i) incorporating a cost channel into the model; and (ii) allowing for inertia in inflation rate expectations. The background to these extensions is the empirical evidence from real-world economies (see, e.g., Altissimo

et al. [1], Barth and Ramey [2], Galí and Gertler [19], and Chowdhury et al. [9]).

What we have found in this paper is that an expectations-based optimal monetary policy rule, originally proposed by Evans and Honkapohja [12]-[15], still has desirable properties in a New Keynesian model with the aforementioned features. In particular, optimal monetary policy under commitment is associated with a determinate REE that is stable under least squares learning for all parameter constellations considered, whereas, under discretion in policy-making, the central bank has to be sufficiently inflation rate averse for the REE to have the same properties.

Thus, having determinacy and least squares learnability of the REE as important objectives in policy-making, the central bank should implement an expectations-based optimal monetary policy rule under commitment to achieve these goals, especially when inflation rate targeting is forced by the political opinion to be rather flexible, because a discretionary policy may not guarantee a favorable outcome in the economy. Optimal monetary policy under commitment is, of course, also always superior to a discretionary policy from a welfare perspective.

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#### TECHNICAL APPENDIX

##### *Constraint in the Lagrangian*

Substitute the expectations formation process in eq. (3) into the IS and AS equations in eqs. (1)-(2), respectively:

$$\begin{cases} x_t = x_{t+1}^e - \alpha (r_t - (\omega\pi_{t-1} + (1-\omega)\pi_{t+1}^e)), \\ \pi_t = \beta (\omega\pi_{t-1} + (1-\omega)\pi_{t+1}^e) + \gamma x_t + \delta r_t. \end{cases} \quad (\text{A.1})$$

Solve the first equation in eq. (A.1) for  $r_t$ :

$$r_t = \frac{1}{\alpha} \cdot (x_{t+1}^e - x_t) + \omega\pi_{t-1} + (1-\omega)\pi_{t+1}^e,$$

and substitute the resulting equation into the second equation in eq. (A.1):

$$\begin{aligned} \pi_t &= \beta (\omega\pi_{t-1} + (1-\omega)\pi_{t+1}^e) + \gamma x_t \\ &\quad + \delta \cdot \left( \frac{1}{\alpha} \cdot (x_{t+1}^e - x_t) + \omega\pi_{t-1} + (1-\omega)\pi_{t+1}^e \right), \end{aligned}$$

or

$$\pi_t = (\beta + \delta) (\omega\pi_{t-1} + (1-\omega)\pi_{t+1}^e) - \left( \frac{\delta}{\alpha} - \gamma \right) \cdot x_t + \frac{\delta}{\alpha} \cdot x_{t+1}^e,$$

which is the economy's law of motion.



*Optimality condition when discretion in policy-making*

Solve eq. (6) for  $\lambda_t$ :

$$\lambda_t = -\frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot x_t. \quad (\text{A.2})$$

Forward eq. (A.2) one time period:

$$E_t[\lambda_{t+1}] = -\frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot E_t[x_{t+1}]. \quad (\text{A.3})$$

Substitute eqs. (A.2)-(A.3) into eq. (7):

$$2\pi_t - \left( -\frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot x_t \right) + \beta(\beta + \delta)\omega \cdot \left( -\frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot E_t[x_{t+1}] \right) = 0,$$

which gives eq. (10).

*Optimality condition when commitment in policy-making*

Solve eq. (8) for  $\lambda_t$ :

$$\lambda_t = -\frac{\delta}{(\alpha\gamma - \delta)\beta} \cdot \lambda_{t-1} - \frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot x_t. \quad (\text{A.4})$$

Forward eq. (A.4) one time period:

$$E_t[\lambda_{t+1}] = -\frac{\delta}{(\alpha\gamma - \delta)\beta} \cdot \lambda_t - \frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot E_t[x_{t+1}],$$

and substitute the resulting equation into eq. (9):

$$\begin{aligned} & (\beta + \delta)(1 - \omega)\lambda_{t-1} + 2\beta\pi_t - \beta\lambda_t \\ & + \beta^2(\beta + \delta)\omega \cdot \left( -\frac{\delta}{(\alpha\gamma - \delta)\beta} \cdot \lambda_t - \frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot E_t[x_{t+1}] \right) \\ = & 0, \end{aligned}$$

or

$$\begin{aligned} & (\beta + \delta)(1 - \omega)\lambda_{t-1} + 2\beta\pi_t \\ & - \beta \left( 1 + \frac{(\beta + \delta)\delta\omega}{\alpha\gamma - \delta} \right) \cdot \lambda_t - \frac{2\alpha\beta^2(\beta + \delta)\theta\omega}{\alpha\gamma - \delta} \cdot E_t[x_{t+1}] \\ = & 0, \end{aligned}$$

or

$$\begin{aligned} \lambda_t = & \frac{(\alpha\gamma - \delta)(\beta + \delta)(1 - \omega)}{(\alpha\gamma + (\beta + \delta)\delta\omega - \delta)\beta} \cdot \lambda_{t-1} \\ & - \frac{2\alpha\beta(\beta + \delta)\theta\omega}{\alpha\gamma + (\beta + \delta)\delta\omega - \delta} \cdot E_t[x_{t+1}] + \frac{2(\alpha\gamma - \delta)}{\alpha\gamma + (\beta + \delta)\delta\omega - \delta} \cdot \pi_t, \end{aligned}$$

where this equation and eq. (A.4) are two equations in  $\lambda_t$  and  $\lambda_{t-1}$ . Solve these equations for these variables, but *notice that  $\lambda_t$  is  $\lambda_{t-1}$  one time period forward in time*:

$$\begin{cases} \lambda_t = -A\lambda_{t-1} - Bx_t, \\ \lambda_t = C\lambda_{t-1} - DE_t[x_{t+1}] + F\pi_t, \end{cases} \quad (\text{A.5})$$

or

$$-A\lambda_{t-1} - Bx_t = C\lambda_{t-1} - DE_t[x_{t+1}] + F\pi_t,$$

or

$$\lambda_{t-1} = -\frac{B}{A+C} \cdot x_t + \frac{D}{A+C} \cdot E_t[x_{t+1}] - \frac{F}{A+C} \cdot \pi_t. \quad (\text{A.6})$$

Forward eq. (A.6) one time period:

$$\lambda_t = -\frac{B}{A+C} \cdot E_t[x_{t+1}] + \frac{D}{A+C} \cdot E_t[x_{t+2}] - \frac{F}{A+C} \cdot E_t[\pi_{t+1}]. \quad (\text{A.7})$$

Substitute the equations for  $\lambda_{t-1}$  and  $\lambda_t$  in eqs. (A.6)-(A.7) into the first equation in eq. (A.5):

$$\begin{aligned} & -\frac{B}{A+C} \cdot E_t[x_{t+1}] + \frac{D}{A+C} \cdot E_t[x_{t+2}] - \frac{F}{A+C} \cdot E_t[\pi_{t+1}] \\ &= \frac{AB}{A+C} \cdot x_t - \frac{AD}{A+C} \cdot E_t[x_{t+1}] + \frac{AF}{A+C} \cdot \pi_t - Bx_t, \end{aligned}$$

or

$$E_t[\pi_{t+1}] = -A\pi_t + \frac{BC}{F} \cdot x_t + \frac{AD-B}{F} \cdot E_t[x_{t+1}] + \frac{D}{F} \cdot E_t[x_{t+2}].$$

Backward the previous equation one time period:

$$\pi_t = -A\pi_{t-1} + \frac{BC}{F} \cdot x_{t-1} + \frac{AD-B}{F} \cdot x_t + \frac{D}{F} \cdot E_t[x_{t+1}],$$

and substitute back  $A$ ,  $B$ ,  $C$ ,  $D$  and  $F$  into the resulting equation:

$$\begin{aligned} \pi_t &= -\frac{\delta}{(\alpha\gamma - \delta)\beta} \cdot \pi_{t-1} \\ &+ \frac{2\alpha\theta}{\alpha\gamma - \delta} \cdot \frac{(\alpha\gamma - \delta)(\beta + \delta)(1 - \omega)}{(\alpha\gamma + (\beta + \delta)\delta\omega - \delta)\beta} \cdot \frac{\alpha\gamma + (\beta + \delta)\delta\omega - \delta}{2(\alpha\gamma - \delta)} \cdot x_{t-1} \\ &+ \left( \frac{\delta}{(\alpha\gamma - \delta)\beta} \cdot \frac{2\alpha\beta(\beta + \delta)\theta\omega}{\alpha\gamma + (\beta + \delta)\delta\omega - \delta} - \frac{2\alpha\theta}{\alpha\gamma - \delta} \right) \\ &\cdot \frac{\alpha\gamma + (\beta + \delta)\delta\omega - \delta}{2(\alpha\gamma - \delta)} \cdot x_t \\ &+ \frac{2\alpha\beta(\beta + \delta)\theta\omega}{\alpha\gamma + (\beta + \delta)\delta\omega - \delta} \cdot \frac{\alpha\gamma + (\beta + \delta)\delta\omega - \delta}{2(\alpha\gamma - \delta)} \cdot E_t[x_{t+1}], \end{aligned}$$

which gives eq. (11).

*Interest rate rule when discretion in policy-making*

Substitute the first equation in eq. (A.1) into the second equation in eq. (A.1):

$$\begin{aligned}\pi_t &= \beta (\omega \pi_{t-1} + (1 - \omega) \pi_{t+1}^e) \\ &\quad + \gamma (x_{t+1}^e - \alpha (r_t - (\omega \pi_{t-1} + (1 - \omega) \pi_{t+1}^e))) + \delta r_t,\end{aligned}$$

or

$$\pi_t = (\alpha\gamma + \beta) (\omega \pi_{t-1} + (1 - \omega) \pi_{t+1}^e) + \gamma x_{t+1}^e - (\alpha\gamma - \delta) r_t. \quad (\text{A.8})$$

Substitute the optimality condition in eq. (10) into eq. (A.8), but do not assume rational expectations:

$$\begin{aligned}& -\frac{\alpha\theta}{\alpha\gamma - \delta} \cdot x_t + \frac{\alpha\beta(\beta + \delta)\theta\omega}{\alpha\gamma - \delta} \cdot x_{t+1}^e \\ &= (\alpha\gamma + \beta) (\omega \pi_{t-1} + (1 - \omega) \pi_{t+1}^e) + \gamma x_{t+1}^e - (\alpha\gamma - \delta) r_t,\end{aligned}$$

which gives eqs. (12)-(13).

*Interest rate rule when commitment in policy-making*

Substitute the optimality condition in eq. (11) into eq. (A.8), but do not assume rational expectations:

$$\begin{aligned}& -\frac{\delta}{(\alpha\gamma - \delta)\beta} \cdot \pi_{t-1} + \frac{\alpha(\beta + \delta)\theta(1 - \omega)}{(\alpha\gamma - \delta)\beta} \cdot x_{t-1} \\ & -\frac{\alpha\theta}{\alpha\gamma - \delta} \cdot x_t + \frac{\alpha\beta(\beta + \delta)\theta\omega}{\alpha\gamma - \delta} \cdot x_{t+1}^e \\ &= (\alpha\gamma + \beta) (\omega \pi_{t-1} + (1 - \omega) \pi_{t+1}^e) + \gamma x_{t+1}^e - (\alpha\gamma - \delta) r_t,\end{aligned}$$

which gives eq. (12) and eq. (14).

*Complete model*

Substitute the interest rate rule in eq. (12) into eq. (A.1) and the resulting equation system gives eqs. (15)-(19).

*MSV solution to the complete model*

Forward the matrix equation in eq. (20) one time period, but do not assume rational expectations:

$$\mathbf{y}_{t+1}^e = \mathbf{\Pi} \cdot \mathbf{y}_t + \mathbf{\Phi}.$$

Substitute the previous matrix equation into eq. (15):

$$\mathbf{\Gamma} \cdot \mathbf{y}_t = \mathbf{\Theta} \cdot (\mathbf{\Pi} \cdot \mathbf{y}_t + \mathbf{\Phi}) + \mathbf{\Lambda} \cdot \mathbf{y}_{t-1},$$

or

$$\mathbf{y}_t = (\mathbf{\Gamma} - \mathbf{\Theta} \cdot \mathbf{\Pi})^{-1} \cdot \mathbf{\Lambda} \cdot \mathbf{y}_{t-1} + (\mathbf{\Gamma} - \mathbf{\Theta} \cdot \mathbf{\Pi})^{-1} \cdot \mathbf{\Theta} \cdot \mathbf{\Phi}.$$

Then, by solving the following matrix equation system, the elements in the coefficient matrices in eq. (20) can be determined:

$$\begin{cases} \mathbf{\Pi} = (\mathbf{\Gamma} - \mathbf{\Theta} \cdot \mathbf{\Pi})^{-1} \cdot \mathbf{\Lambda}, \\ \mathbf{\Phi} = (\mathbf{\Gamma} - \mathbf{\Theta} \cdot \mathbf{\Pi})^{-1} \cdot \mathbf{\Theta} \cdot \mathbf{\Phi}, \end{cases}$$

which gives eq. (21).

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