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Working Paper Measuring the stability of a dynamic system: The case of the stock market turmoil 2007-2008

Working Paper, No. 2010:25

Provided in Cooperation with: Department of Economics, Uppsala University

Suggested Citation: Bask, Mikael; Widerberg, Anna (2010) : Measuring the stability of a dynamic system: The case of the stock market turmoil 2007-2008, Working Paper, No. 2010:25, Uppsala University, Department of Economics, Uppsala, https://nbn-resolving.de/urn:nbn:se:uu:diva-161741

This Version is available at: https://hdl.handle.net/10419/82571

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Working Paper 2010:25 Department of Economics

Measuring the Stability of a Dynamic System: The Case of the Stock Market Turmoil 2007-2008

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MEASURING THE STABILITY OF A DYNAMIC SYSTEM: THE CASE OF THE STOCK MARKET TURMOIL 2007-2008

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Measuring the Stability of a Dynamic System: The Case of the Stock Market Turmoil 2007-2008¹

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Version: December 28, 2010

Abstract: The aim of this paper is to demonstrate how the change in actual and potential market risks in the Dow Jones Industrial Average (DJIA) during the two-year period 2007-2008 can be analyzed with the help of (λ, σ^2) -analysis. In the empirical analysis, the average of the Lyapunov exponents for the dynamic system generating DJIA returns is used as the stability measure, λ , whereas the squared DJIA return is used as the variability measure, σ^2 . The main findings are as follows: (i) the potential market risk in the DJIA did not fluctuate that much during 2007, with the exceptions of early fall and near the end of the year; (ii) the potential market risk fluctuated a lot during 2008, especially in early August and in the middle of September; and (iii) the actual market risk in the DJIA was considerably higher near the end of 2008, especially in October, compared with the rest of the period.

Keywords: Dow Jones; Financial Crisis; Lyapunov Exponents; Market Risk; Potential Market Risk; Stability; Volatility.

¹ We are grateful to Hannu Kahra for his comments on an earlier draft of this paper. The usual disclaimer applies.

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1. Actual and potential market risks and (λ, σ^2) -analysis

It is without doubt of uttermost importance to understand the mechanisms behind the variability of asset returns since they are associated with market risk. Indeed, the essence of market risk is that the actual return on a portfolio of assets may be very different than the expected return. It is also for this reason a measure of market risk is crucial for a successful risk management (see Dowd, 2005, for a collection of market risk measures).³

Further, which is the point of departure of this paper, there is an important difference between (actual) market risk and potential market risk in a portfolio of assets (see Bask, 2010).

To comprehend this difference, let σ^2 and λ denote the conditional variance of portfolio returns and the stability of the dynamic system generating these returns, respectively, and let

(1)
$$\sigma^2 = \sigma^2(\lambda, \varepsilon)$$

illustrate the relationship between these two variables, where ε denotes shocks to the dynamic system. Because of the shocks (ε), there is no one-to-one correspondence between the conditional variance of portfolio returns (σ^2) and the stability of the dynamic system generating these returns (λ).

The aforementioned means that λ is not a measure of market risk. Instead, λ is a measure of *potential* market risk, whereas σ^2 is a measure of actual market risk (see Bask, 2010).

Specifically, a change in a portfolio's potential market risk may or may not change the portfolio's actual market risk since it depends on how much the variance of the shocks to the dynamic system generating portfolio returns has changed, if there has been any change at all. The variability of portfolio returns should therefore be contrasted with the stability of the dynamic system generating these returns and this task is accomplished with the help of (λ, σ^2) -analysis (see Bask, 2010).

The aim of this paper is to demonstrate how the change in actual and potential market risks in the Dow Jones Industrial Average (DJIA), using daily data from 1 January 2005 to 31 December 2008, can be analyzed with the help of (λ, σ^2) -analysis.⁴

Specifically, the potential market risk in the DJIA is estimated using a rolling window, comprising of two years of data, resulting in a time series of λ for the period from 1 January 2007 to 31 December 2008. A time series of σ^2 is also estimated that shows how the actual market risk in the DJIA has developed over the same period of time. Thus, the period covers the financial turmoil that started in the U.S. and thereafter was spread to the rest of the industrialized world with devastating effects.

The main findings are as follows: (i) the point estimates of the potential market risk in the DJIA did not fluctuate that much during 2007, with the exceptions of early fall and near the end of the year; (ii) the point estimates of the potential market risk fluctuated a lot during 2008, especially in early August and

³ We do not restrict our interpretation of market risk to the interpretation given within the capital asset pricing model.

⁴ The DJIA is a portfolio of the stocks in 30 of the largest and most widely held public companies in the U.S. Of course, one can always argue whether the DJIA or some other index is the more appropriate stock market index to use in a study of portfolio risk. However, since the aim of this paper is to demonstrate with an example how actual and potential market risks may be related to each other, the specific choice of stock market index is of secondary importance.

in the middle of September; and (iii) the point estimates of the actual market risk in the DJIA was considerably higher near the end of 2008, especially in October, compared with the rest of the period.

The rest of this paper is organized as follows: The methodology is outlined in Section 2, where the focus is on measuring the potential market risk in a portfolio of assets since this measure is new in the literature. The empirical findings are presented in Section 3, and the paper is concluded in Section 4. The latter section also contains suggestions for further research.

2. Methodology

This section consists of two parts. In Section 2.1, we discuss how to measure the potential market risk in a portfolio of assets, and, in Section 2.2, we show how the actual market risk in the same portfolio of assets can be measured.

2.1. Measuring potential market risk

2.1.1. Intuition behind λ ⁵

To understand why λ provides a measure of the stability of a dynamic system, assume for the moment that portfolio returns, s_t , is determined by a linear autoregression of order one:

$$(2) s_t = \beta s_{t-1} + \varepsilon_t,$$

where ε_t is independently and identically distributed shocks with zero mean and finite variance. If we also assume that the autoregression is stable, $|\beta| < 1$, we can decompose the variability of portfolio returns as follows:

(3)
$$\sigma^2 \equiv var(s_t) = \frac{var(\varepsilon_t)}{1-\beta^2} \equiv \frac{var(\varepsilon_t)}{1-(exp(\lambda))^2}.$$

Thus, portfolio returns are more variable when the shocks are more variable, but also when the autoregression is less stable. Observe that for a stable autoregression, $\lambda < 0$ since $\lambda \equiv log |\beta|$. Consequently, λ approaches zero from below when the autoregression decreases in stability.

This line of reasoning can in a straightforward way be extended to linear autoregressions of higher orders than one:

If the portfolio return model instead is a linear autoregression of order n, the stability of the model can be measured by the product of the modulus of the n eigenvalues that solve the model's eigenvalue problem since this measure describes the rate of contraction of an n-dimensional volume in ndimensional space. Apparently, an autoregression with a faster contraction than another autoregression is the more stable autoregression.

However, since we cannot escape from the fact that linearity is a special case of non-linearity, a tool is needed that is able to determine the stability of a non-linear system and not only of a linear system

⁵ The arguments herein can in various degrees also be found in Bask (2010), Bask and de Luna (2002, 2005) and Bask and Widerberg (2009).

such as a linear autoregression. But, unfortunately, the eigenvalues discussed above cannot be used in such a stability analysis and the reason is that these eigenvalues are defined for a linear system.

What we therefore need is another set of eigenvalues that can be used in the stability analysis of a non-linear system:

Indeed, in the same manner as the product of the modulus of the n eigenvalues of an n-dimensional linear system describes the convergence speed of an n-dimensional volume in n-dimensional space, the average of the n Lyapunov exponents for an n-dimensional non-linear system,

(4)
$$\lambda \equiv \frac{1}{n} \sum_{i=1}^{n} \lambda_i,$$

describes the convergence speed of an *n*-dimensional volume in *n*-dimensional space. The reason is that the Lyapunov exponents for a non-linear system correspond to the (natural logarithms of the) eigenvalues of a linear system.

We will return below to the definition of the Lyapunov exponents, but we can already now give an intuitive explanation of why λ in (4) provides information on the stability of a non-linear dynamic system:

Consider two initial values of a dynamic system, where the difference in initial values can be view as a shock to the system. Then, the largest Lyapunov exponent, λ_1 , measures the slowest exponential rate of convergence of two trajectories that start at the aforesaid two initial values. To be more precise, λ_1 measures the convergence of the shock in the direction defined by the eigenvector corresponding to this eigenvalue. If the difference in initial values lies in another direction, then the converge is faster. Thus, λ_1 describes a "worst case scenario" (see Bask and de Luna, 2005).

Potter (2000) should also be mentioned in this context since he too argues that λ_1 describes the convergence speed of a non-linear dynamic system to its "equilibrium" (see Shintani, 2006, who examines convergence speeds of exchange rates toward purchasing power parity using λ_1).

Furthermore, λ in (4) describes an "average scenario" (see Bask and de Luna, 2005) since λ measures the average exponential rate of convergence of the aforementioned two trajectories of the dynamic system. In fact, λ measures the convergence of the shock in an average direction defined by the eigenvectors corresponding to the different eigenvalues, λ_i , $i \in [1, 2, ..., n]$. To be more precise, a shock has a smaller effect on a dynamic system with a smaller λ than it has on a system with a larger λ .

2.1.2. Definition of λ

How are then the Lyapunov exponents for a dynamic system defined?

Assume that the dynamic system $f: \mathbb{R}^n \to \mathbb{R}^n$ generates

(5)
$$S_{t+1} = f(S_t) + \varepsilon_{t+1}^s$$
,

where S_t and ε_t^s are the state of the system and the shock to the system, respectively. Then, for an *n*-dimensional system as in (5), there are *n* Lyapunov exponents that are ranked from the largest to the smallest value:

(6)
$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n.$$

Assume temporarily that there are no shocks and consider how the dynamic system f amplifies the distance between the neighboring states S_0 and S_0 :

(7)
$$S_j - S_j = f^j(S_0) - f^j(S_0) \cong Df^j(S_0)(S_0 - S_0),$$

where $f^{j}(S_{0}) = f(\cdots f(f(S_{0})) \cdots)$ is *j* successive iterations of the system starting at state S_{0} and Df is the Jacobian of the system:

(8)
$$Df^{j}(S_{0}) = Df(S_{j-1})Df(S_{j-2})\cdots Df(S_{0}).$$

Associated with each Lyapunov exponent, λ_i , $i \in [1, 2, ..., n]$, there are nested subspaces $U^i \subset \mathbb{R}^n$ of dimension n + 1 - i with the property that

(9)
$$\lambda_i \equiv \lim_{j \to \infty} \frac{\log_e \|Df^j(S_0)\|}{j} = \lim_{j \to \infty} \frac{1}{j} \sum_{k=0}^{j-1} \log_e \|Df(S_k)\|,$$

for all $S_0 \in U^i - U^{i+1}$. Based on an ergodic theorem, known as Oseledec's multiplicative ergodic theorem, the limits in (9) exist and are independent of S_0 almost surely with respect to the measure induced by the deterministic process $\{S_t\}_{t=1}^{\infty}$ (see Guckenheimer and Holmes, 1983, for a careful discussion of the Lyapunov exponents and their properties).

Then, allow for shocks to the dynamic system, which means that the aforementioned measure is induced by the stochastic process $\{S_t\}_{t=1}^{\infty}$. In this case, the Lyapunov exponents have been named smooth Lyapunov exponents in the literature (see Bask and de Luna, 2002, 2005, but also McCaffrey et al., 1992, and Nychka et al., 1992, even though the latter two papers do not use this term).

2.1.3. Estimation of λ

Because the actual form of the dynamic system f is not known, it may seem like an impossible task to determine the stability of this system. Fortunately, it is possible to reconstruct the dynamics using only a scalar time series and thereafter measure the stability of the reconstructed dynamic system.

Associate the dynamic system f with an observer function $g: \mathbb{R}^n \to \mathbb{R}$ that generates portfolio returns:

(10)
$$s_t = g(S_t) + \varepsilon_t^m,$$

where $s_t \in S_t$ and ε_t^m are the portfolio return and the measurement error, respectively. Thus, (10) means that the portfolio return series

(11)
$$\{s_t\}_{t=1}^N$$

is observed, where N is the number of consecutive returns in the time series.

The observations in the scalar time series in (11) contain information on unobserved state variables that can be utilized to define a state in present time. Specifically, let

(12)
$$T = (T_1, T_2, \cdots, T_M)^{tr}$$

be the reconstructed trajectory, where T_t is the reconstructed state and M is the number of states on the reconstructed trajectory. Moreover, the reconstructed state in present time is

(13)
$$T_t = \{s_t, s_{t+1}, \cdots, s_{t+m-1}\},\$$

where *m* is the embedding dimension. Thus, *T* is an $M \times m$ matrix and the constants *M*, *m* and *N* are related as M = N - m + 1.

Takens (1981) proved that the function

(14)
$$\Phi(S_t) = \{g(f^0(S_t)), g(f^1(S_t)), \cdots, g(f^{m-1}(S_t))\},$$

which maps the *n*-dimensional state S_t onto (and not only into) the *m*-dimensional state T_t , is an embedding when m > 2n.⁶ This means that the function is a smooth function that performs a one-to-one coordinate transformation and has a smooth inverse.

A function that is an embedding preserves topological information about the unknown dynamic system f such as the Lyapunov exponents. In particular, such a function induces another function $h: \mathbb{R}^m \to \mathbb{R}^m$ on the reconstructed trajectory,

(15)
$$T_{t+1} = h(T_t),$$

which is topologically conjugate to the unknown dynamic system f:

(16)
$$h^{j}(T_{t}) = \Phi \circ f^{j} \circ \Phi^{-1}(T_{t}).$$

h is therefore a reconstructed dynamic system that has the same Lyapunov exponents as the unknown dynamic system f.

To be able to estimate the Lyapunov exponents for the unknown dynamic system f generating portfolio returns, one must first estimate the reconstructed dynamic system h. However, because

(17)
$$h: \begin{pmatrix} s_t \\ s_{t+1} \\ \vdots \\ s_{t+m-1} \end{pmatrix} \rightarrow \begin{pmatrix} s_{t+1} \\ s_{t+2} \\ \vdots \\ h(s_t, s_{t+1}, \cdots, s_{t+m-1}) \end{pmatrix},$$

the estimation of the reconstructed dynamic system h reduces to the estimation of h:

(18)
$$s_{t+m} = h(s_t, s_{t+1}, \cdots, s_{t+m-1});$$

this is essentially a non-linear autoregression of order m (without an error term). Moreover, because the Jacobian Dh on the reconstructed state T_t is

(19)
$$Dh(T_t) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ 0 & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ \frac{\partial h}{\partial s_t} & \frac{\partial h}{\partial s_{t+1}} & \frac{\partial h}{\partial s_{t+2}} & \cdots & \frac{\partial h}{\partial s_{t+m-1}} \end{pmatrix},$$

a feed-forward neural network can be used to estimate the above derivatives and thus to consistently estimate the Lyapunov exponents (see Dechert and Gencay, 1992, Gencay and Dechert, 1992, McCaff-

⁶ Be aware that the condition m > 2n is a sufficient but not a necessary condition to have an embedding (see Sauer et al., 1991, for an introduction to embedology).

rey et al., 1992, and Nychka et al., 1992), and this is because Hornik et al. (1990) have shown that a function and its derivatives of any unknown functional form can be approximated arbitrarily accurately by such a network.⁷

We have used NETLE (see <u>tliu.iweb.bsu.edu/research/#programs</u>), a software program developed by R. Gencay, C.-M. Kuan and T. Liu, when estimating the Lyapunov exponents for the dynamic system h in (17) to be able to calculate λ in (4) (see Gencay and Dechert, 1992, and Kuan and Liu, 1995, for details).

Finally, Shintani and Linton (2004) show that a neural network estimator of the Lyapunov exponents, like the one in Gencay and Dechert (1992), is asymptotically normal. Our conjecture is therefore that asymptotic normality also holds for a neural network estimator of λ in (4) since the eigenvectors corresponding to the Lyapunov exponents are pairwise orthogonal.

2.1.4. Estimation of a time series of λ

The specific approach in this paper, when measuring how the potential market risk in the DJIA has developed over time, consists of the following three steps:

- 1. Estimate l_{max} in $\lambda = \frac{1}{l_{max}} \sum_{i=1}^{l_{max}} \lambda_i$:
 - 1.1. Estimate λ using data from 1 January 2005 to 31 December 2008, where l_{max} runs from 2 to 20 Lyapunov exponents that also is the number of inputs in the neural network. The number of hidden units in the neural network for each l_{max} runs from 2 to 10 units.
 - 1.2. \hat{l}_{max} is the l_{max} that minimizes the Schwarz Information Criterion.
- 2. Estimate a time series of $\lambda = \frac{1}{\hat{l}_{max}} \sum_{i=1}^{\hat{l}_{max}} \lambda_i$ for the period 1 January 2007 31 December 2008:
 - 2.1. Estimate λ for 1 January 2007 using data from 2 January 2005 to 1 January 2007.
 - 2.2. Estimate λ for date d in year y (i.e., estimate $\lambda_{(d,y)}$) using data from (d + 1, y 2) to (d, y).
 - 2.3. Estimate λ for 31 December 2008 using data from 1 January 2007 to 31 December 2008.
- 3. Estimate trends in λ :
 - 3.1. Carry out the least squares regression $\hat{\lambda}_{(d,y)} = \beta_0 + \beta_1 \cdot t + \varepsilon_t$, where d and y belong to month m and t is time. Thus, the regression is based on point estimates of λ that all belong to the same month. When $\hat{\beta}_1 < 0$ ($\hat{\beta}_1 > 0$), the dynamic system has become more (less) stable during month m.
 - 3.2. Carry out the least squares regression $\hat{\lambda}_{(d,y)} = \beta_0 + \beta_1 \cdot t + \beta_2 \cdot dummy_m + \varepsilon_t$, where d and y belong to the two consecutive months m 1 and m, and t is time. Thus, the regression is based on point estimates of λ that all belong to the same two months. When $\hat{\beta}_2 \neq 0$, the change in stability during month m differs from the change in stability during month m 1.

As a complement to these three steps, we will also plot and look at the time series of $\Delta\lambda_{(d,y)}$.

⁷ The estimation of the Lyapunov exponents dates back to Wolf et al. (1985) and several other estimation methods have been proposed in the literature since then. However, even though Wolf et al. (1985) have been cited by more than 3 000 papers in peer-reviewed journals (in December 2010), the Lyapunov exponents for a dynamic system are still almost unheard of within the economics and finance communities.

2.2. Measuring actual market risk

Because we associate the actual market risk in a portfolio of assets with the variability of portfolio returns, we need a variability measure. Moreover, to make it as simple as possible in this paper, we use realized volatility as our variability measure. To be more precise, the squared portfolio return,

$$\sigma_t^2 = s_t^2,$$

is our measure of the actual market risk in a portfolio of assets, which here is the DJIA. To have a correspondence with step 2.2 in Section 2.1.4, be aware that (20) also can be written as follows:

(21)
$$\sigma_{(d,y)}^2 = s_{(d,y)}^2.$$

We will also plot and look at the time series of $\Delta \sigma^2_{(d,v)}$.

3. Empirical findings

Let us start the analysis with a plot of the DJIA – also known as the Dow Jones Index – for the period from 1 January 2007 to 31 December 2008. See Figure 1 for this plot.

[Figure 1 about here.]

We will not give a detailed account of the development of the stock market in the U.S. during the twoyear period 2007-2008. We only note that after the stock market reached a high in 12 October 2007 (with DJIA equal to 14 093.08), the stock market had fallen with more than 46 percent when it reached a low in 20 November 2008 (with DJIA equal to 7 552.29).

Moreover, after inspection of Figure 2, plotting the realized volatility over the period, it is clear that the actual market risk in the DJIA – measured by σ^2 – was considerably higher near the end of 2008, especially in October, compared with the rest of the two-year period. See also Figure 3 for the first difference in the actual market risk for the two-year period 2007-2008, which shows how the actual market risk fluctuated over the period.

[Figures 2-3 about here.]

What about the potential market risk in the DJIA – measured by λ ? In Figures 4-5, the level and change in the potential market risk in the DJIA are presented for the two-year period 2007-2008.⁸ Two findings are visible: (i) the potential market risk did not fluctuate that much during 2007, with the exceptions of early fall and near the end of the year; and (ii) the potential market risk fluctuated a lot during 2008, especially in early August and in the middle of September.

[Figures 4-5 about here.]

If we focus on the fluctuations in the actual and potential market risks in the DJIA, which are found in Figures 3 and 5, it seems to be the case that the potential market risk was a leading indicator of the actual market risk in the DJIA. This finding is easier to see if we magnify the figures and only present

⁸ The number of inputs in the neural network that minimized the Schwarz Information Criterion was $\hat{l}_{max} = 6$, and the number of hidden units was 10 in this case.

the first differences in the actual and potential market risks in the DJIA during the three-month period August-October 2008. See Figures 6-7 for these plots.

[Figures 6-7 about here.]

First, 4-5 August and 10-11 September 2008 are the two episodes in which the *potential* market risk in the DJIA fluctuated the most. The first episode is not connected with any special event in the U.S. or elsewhere in the world, whereas the second episode occurred a few days after the federal takeover of Fannie Mae and Freddie Mac, and a few days before the bankruptcy of Lehman Brothers.

Second, 13-14 October and 28-29 October 2008 are the two episodes in which the *actual* market risk in the DJIA fluctuated the most. This, of course, is due to the facts that realized volatility is our definition of the actual market risk and that the stock market fell by more than 10 percent in 13 October 2008 and that it rose by more than 10 percent in 28 October 2008.

What about trends, both within as well as between months, in the potential market risk in the DJIA during the two-year period 2007-2008? See Table 1 for the results.

[Table 1 about here.]

Even though the potential market risk in the DJIA did not fluctuate that much during 2007, with the exceptions of early fall and near the end of the year, there were nonetheless significant changes in the potential market risk within 7 of the 12 months. Also, 4 of these months belong to the first half of 2007. However, when it comes to significant changes in the potential market risk between months, we only have one such case.

If we turn to the more turbulent 2008, it is clear that there were significant changes in the potential market risk within 5 of the 12 months. Note also that all these changes are significant at the 1 %-level. In 2007, 4 of the 7 changes are significant at the 1 %-level. The contrast with 2007 is mainly about the number of cases in which there are significant changes in the potential market risk between months since there were 4 such cases in 2008.

The most turbulent period at the stock market in the U.S., at least according to the preceding analysis, was the four-month period July-October 2008.

4. Conclusions and suggestions for further research

The method in Bask and de Luna (2002) – which is the origin of (λ, σ^2) -analysis – was first used in Bask and de Luna (2005) in a large-scale analysis of the European monetary integration with the creation of the Economic and Monetary Union. (λ, σ^2) -analysis has also been used in Bask and Widerberg (2009), where the focus is on the step-wise integration of the Nordic power market, Nord Pool.

Specifically, Bask and de Luna (2005) show that when most of the currencies became more stable, a majority of them also became less volatile. For example, following the agreement of the Maastricht Treaty in December 1991, most currencies became more stable and less volatile, whereas they became less stable and more volatile when the Danish public in June 1992 voted against the treaty.

Further on, Bask and Widerberg (2009) show that the step-wise integration of the Nordic power market most of the time was associated with more stable and less volatile electricity prices. These findings should be put side by side with Bask et al. (2008) who show that the degree of competition at Nord Pool has increased during the same period of time.

The aim of the present paper was to demonstrate how the change in actual and potential market risks in the DJIA during the two-year period 2007-2008 can be analyzed with the help of (λ, σ^2) -analysis. In the empirical analysis, the average of the Lyapunov exponents for the dynamic system generating portfolio returns was used as the stability measure, λ , whereas the squared portfolio return was used as the variability measure, σ^2 . Daily data for the four-year period 2005-2008 were used in the analysis.

Our main findings were as follows: (i) the point estimates of the potential market risk in the DJIA did not fluctuate that much during 2007, with the exceptions of early fall and near the end of the year; (ii) the point estimates of the potential market risk fluctuated a lot during 2008, especially in early August and in the middle of September; and (iii) the point estimates of the actual market risk in the DJIA was considerably higher near the end of 2008, especially in October, compared with the rest of the period.

Thus, it seems to be the case that the potential market risk was a leading indicator of the actual market risk in the DJIA during the two-year period 2007-2008. This is an attention-grabbing finding that should be examined more carefully in further research. In particular, is it the case that large fluctuations in the actual market risk in a portfolio of assets often is preceded by pronounced fluctuations in the potential market risk in the same portfolio of assets?

Part of further research is also the derivation of a distributional theory for our stability measure. Recall that our conjecture is that asymptotic normality holds for a neural network estimator of λ in (4) since the eigenvectors corresponding to the Lyapunov exponents are pairwise orthogonal, and that Shintani and Linton (2004) show that a neural network estimator of the Lyapunov exponents, like the one in Gencay and Dechert (1992), is asymptotically normal.

We should also point out that the Lyapunov exponents we estimated for our stability measure were the global ones. One could therefore argue that we instead should have estimated the local Lyapunov exponents or the Lyapunov-like index (see Yao and Tong, 1994). Still, we believe that the use of the global Lyapunov exponents can be a good approximation when the stability of the dynamics over a longer time span is under study. Anyhow, the use of the local Lyapunov exponents and the Lyapunovlike index when measuring the stability of a dynamic system should be part of further research.

Last of all, even though we are the first to admit that (λ, σ^2) -analysis still is an incomplete tool from a statistical point of view, we anyway believe that the example provided in this paper illustrates well the potential of (λ, σ^2) -analysis. This is also how this paper should be read; we argue that one should make a distinction between actual and potential market risks in portfolio analysis, and we also present a method on how to empirically distinguish between these two risks using time series data, but since the statistical toolbox not yet is complete, part of further research is the completion of such a toolbox.

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Figure 1 Dow Jones Index (DJIA) for the period 1 January 2007 – 31 December 2008



Figure 2 Realized volatility (σ^2) for the period 1 January 2007 – 31 December 2008







Figure 4 Lambda (λ) for the period 1 January 2007 – 31 December 2008







Figure 7 First difference in lambda ($\Delta\lambda$) for the period 1 August 2008 – 31 October 2008

Month	Stability change	Significant change within month	Significant change between months
February 2007	Less stable	Yes (1 %)	No
March 2007	Less stable	Yes (1 %)	No
April 2007	More stable	Yes (5 %)	No
May 2007	Less stable	No	No
June 2007	More stable	Yes (1 %)	No
July 2007	Less stable	No	No
August 2007	Less stable	Yes (1 %)	No
September 2007	More stable	No	No
October 2007	More stable	Yes (5 %)	Yes (1 %)
November 2007	More stable	Yes (5 %)	No
December 2007	Less stable	No	No
January 2008	More stable	No	No
February 2008	Less stable	Yes (1 %)	No
March 2008	More stable	Yes (1 %)	No
April 2008	More stable	Yes (1 %)	Yes (1 %)
May 2008	Less stable	No	No
June 2008	More stable	No	No
July 2008	Less stable	Yes (1 %)	Yes (5 %)
August 2008	More stable	No	Yes (10 %)
September 2008	Less stable	Yes (1 %)	Yes (10 %)
October 2008	More stable	No	No
November 2008	More stable	No	No
December 2008	Less stable	No	No

Table 1 Trends in lambda (λ) during the period January 2007 – December 2008

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