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## Generic NDC: Equilibrium, valuation and risk sharing with and without NDC bonds

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## *Department of Economics*

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Generic NDC - Equilibrium, Valuation and  
Risk Sharing with and without DC Bonds

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GENERIC NDC - EQUILIBRIUM, VALUATION AND RISK SHARING WITH AND WITHOUT NDC  
BONDS

EDWARD PALMER

# **Generic NDC - Equilibrium, Valuation and Risk Sharing with and without NDC Bonds**

By

Edward Palmer<sup>\*</sup>

February 2011

## **Abstract**

The Non-financial (Notional) Defined Contribution (NDC) PAYGO pension scheme is a recent innovation and its generic dimensions have not previously been explored in a coherent context. This paper does this. It derives and analyzes the demographic, economic and distributional properties of NDC. The residual (systematic) longevity risk creates a special problem, solved with an NDC bond, the asset that closes the system financially, transferring residual risk to the government (taxpayers). This guarantees a fixed NDC contribution rate and, thus, intergenerational commitment, with transparent distributional policy pursued through targeted transfers from general tax revenues and taxation of *overall* personal income.

Keywords: Pensions; NDC; Notional Defined Contribution; Non-financial Defined Contribution; NDC Bond; Risk sharing; Pension Policy; Pay-as-you-go; PAYG

Jel-codes: H55; I38; J14; O23; P16

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## 1. Introduction

Until non-financial (notional) defined contribution (NDC) pension schemes first emerged in the mid-1990s, it was generally thought that a *defined contribution* pay-as-you-go pension (PAYG) scheme was an impossible construction. A decade later NDC had been introduced in a number of countries<sup>1</sup> and, in the view of the World Bank (*e.g.* Holzmann & Hinz 2005) it is one of the standard alternatives countries should consider in reforming mandatory public pension systems. Despite its increasing popularity there is still no coherent exposition of the conceptual framework for NDC that answers the question: What are the design properties of “generic NDC” and what are the ramifications of these for dealing with the demographic and economic risks inherent in pay-as-you-go pension schemes, and what are the inherent distributional properties. That is the first goal of this paper.<sup>2</sup> The second is to introduce and define an NDC bond and discuss its role in distributing “residual” risk, *i.e.*, remaining uncovered risk, random or systematic, which arises in practice due to limitations in designing the internal rate of return to be used in practice and uncertain statistical projections of cohort life expectancy used in computing NDC life annuities.

What are the main underpinnings of NDC? NDC is a pay-as-you-go pension scheme in which a fixed contribution rate is the *sine qua non*. This distinguishes NDC from all other pay-as-you-go pension schemes. Every monetary unit of contributions paid by or on behalf of the individual gives an equivalent right and claim on the earnings of future workers. Contributions for the individual are noted on the individual’s personal account. All individual workers over all generations pay the same fixed percentage of earnings into an NDC scheme.

If the NDC scheme is the only mandatory national scheme, a reasonable goal would be for the policy maker to fix the contribution rate at a level that yield adequate earnings replacement for the average worker and over all future generations. NDC can be viewed as a government instrument for creating lifetime saving for retirement, indexed to the growth of the contribution base, that satisfies a basic individual need to transfer lifetime income from years of work to years of retirement. The worker’s contribution is similar in this sense to a premium payment made into a private insurance scheme, the money on the individual’s account is illiquid until retirement. In the pay-as-you-go context of NDC the annuity must take the form of a stream

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<sup>1</sup> By 2002 countries as diverse as Italy, Latvia, the Kyrgyz Republic, Poland, Russia and Sweden had adopted versions of NDC, albeit with varying degrees of sophistication and transition speeds.

<sup>2</sup> The paper builds on a previous paper, *What Is NDC?* (Palmer 2006a).

of lifetime payments. Money paid into an NDC scheme is thus not channeled into the financial market within the framework of the pension scheme, as in a financial pension scheme. The contributions to NDC go to pay the pensions of current pensioners and the payout is in the form of a life annuity.

Accounts in an NDC scheme are accredited with a rate of return, which is driven by the same factors that create growth in the contribution base, productivity and labor force growth. Just as financial accounts adjust automatically to changes in the financial rate of return NDC accounts adjust to the economic rate of return. System liabilities continuously adjust to changing economic and demographic conditions insuring that future contributions will generally be sufficient to cover future pensions. This paper shows how this comes about in practice and identifies and discusses the distributional consequences for individuals.

In NDC, in a static demographic steady state where wages only change with changes in productivity and where accounts of all workers and pensioners are indexed with wage growth, the macro ratio of an average benefit to an average wage would be constant over all generations. In a *demographic non-steady-state*, changes in demography and their repercussions on the size of the labor force will affect the intergenerational distribution of outcomes. If the NDC scheme is a financially closed scheme, as generic NDC is, there is no other option than that workers and pensioners bear the risk inherent in a changing labor market, reflecting the development of fertility, net immigration and determinants of participation. This means that accounts are adjusted by an internal rate of return that reflects both the growth of the covered per capita wage and the covered labor force. The latter, exogenous force, may be positive or negative. The paper discusses the distribution of this risk between workers and pensioners.

The requirement that the system continuously adjust to the internal rate of return is necessary for financial equilibrium, but it is not sufficient. Why is this? In its simplest form the NDC annuity is created by dividing the individual's capital balance with life expectancy at retirement. This covers the cohort idiosyncratic risk, that is the individual risk of living a long rather than a short life, but not the systematic risk associated with chronic systematic over or underestimation of mortality, and as a consequence life expectancy. The systematic life expectancy risk must be borne either by the participants or the insurer. In keeping with the main principle of generic NDC the risk should be borne by the individual – or more generally the (birth) cohort insurance pool.

The principle that the participant should bear the risk would be indisputable if the risk were known in advance and workers could adjust their supply of labor and personal savings decisions accordingly. However, there is considerable uncertainty in forecasting longevity and the extent of the risk cannot be known until the books are closed on the complete life of a birth cohort. This puts extreme burden on older workers whose capacity for adjusting labor supply and personal saving is much worse than a worker's. We argue here that the system should be designed so as to systematically pass off this risk from older to younger generations. This transfer can be accomplished through an NDC bond, which performs the function of a longevity bond in the financial market. This mechanism picks up the *residual* longevity risk, *i.e.*, the difference between the life expectancy factor used in computing the life annuity and the actual outcome. In more mature populations, this risk might be 5-10 percent of the value of annuities (Alho *et al.* 2011).

NDC liabilities in any period are the account values of workers and pensioners. Liabilities are known exactly at present time  $t$ , but assets are not. Given a definition and method of estimating assets, one can construct a ratio of assets to liabilities, *i.e.* a solvency ratio. A financial surplus yields a ratio value above unity; a deficit yields a ratio value below unity. Sweden has designed a balancing mechanism based on this idea. In the Swedish model assets are defined as the average time contributions paid will remain in the system, that is, the distance between the average age of a payment and the average age of a payout of a unit of money contributed, calculated in practice using current cross-sectional data. This yields a duration time for a monetary unit of contributions, and this number multiplied by current contributions provides an estimate of assets.<sup>3</sup>

A potentially serious drawback of the measure of assets applied in Sweden is that it does not yield a *unique* level of assets. The paper demonstrates how the definition of assets is constructed and then discusses the consequences of this construction. Different compositions of the labor market that can lead to the same amount of contributions can nevertheless yield different estimates of the NDC asset. The NDC bond proposed here does not have this deficiency, while it serves the same purpose of defining the NDC asset. The NDC bond is a non-tradable perpetuity in the spirit of Buchanan (1968). The rate of return of the NDC bond is the

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<sup>3</sup> Described beginning in 2001 in the appendix of the *Orange Report*, the annual report of the Swedish pension system: <http://www.forsakringskassan.se/nav/54729d013258be5eea6f5c097dd184d3>.

NDC internal rate of return, in principle the rate of growth of the contribution base. As this paper shows, the individual shares in the bond cover every unit of the total NDC liability to workers and pensioners. In this sense, it is the perfect longevity asset.

The NDC bond is defined and discussed in depth in the paper. What is important in the conceptual context of NDC is that the NDC bond emulates the market contract underlying bond financing of government tax deficits. It creates individual entitlements to future revenues equivalent to the entitlement inherent without the bond. The purpose of creating issuing bond shares is to bind the government to its commitment. By definition, then, the NDC bond is a government issued mortality bond in the sense discussed in the recent literature on financial pension systems (*e.g.*, Blake and Burrows 2001; Blake *et al.* 2006a; Brown & Orszag 2006). If the NDC provider were a private insurance provider, this bond would constitute the perfect asset to match the private pension provider's liabilities.

A salient feature of NDC is that it scales down the pension commitment to a direct link between contributions and benefits. In this sense, it is simply a vehicle for creating lifetime saving. Conceptually, workers lend money to the government in return for bond shares and this money is used to pay the pensions of current pensioners. The individual's assets on balance at retirement are converted into a life annuity and bond shares are cashed in each year of retirement. Given this simple framework, the government can create explicit transfers (contributions) to individual accounts for policy objectives, *e.g.*, to finance child care rights in conjunction with childbirth, to finance pension rights in conjunction with unemployment covered by insurance, disability, etc.,

At first glance one might lament the fact that the NDC bond ties the hands of a government that would prefer to leave open the option of ad hoc adjustments in pensions as a means of regulating or preventing deficits. What NDC does is to even make balancing the government budget a more transparent process. If government needs to cut back on overall expenditures, instead of cutting pensions, which is a frequently pursued trimming device around the world, government would be forced to regulate the budget through the income tax rate, where income taxes are based on an individual's *overall income* from all sources, not just pensions. This is a clear advantage since persons with higher income, regardless of source, would pay more with either a flat or progressive tax than persons with lower incomes. In fact applying the same tax rate to earnings and pensions is an accommodating policy measure where



workers over the minimum age to draw on a public pension may choose to combine work and an NDC pension, where additional contributions from earnings continue to enhance account values. Also, with flat or progressive tax rates, it leads to more money being collected from the better-off compared with the less well-off.

The paper is organized as follows. The next section sets up the NDC accounts. Section 3 derives the NDC equilibrium condition, discusses equilibrium characteristics and valuation of rights. Section 4 introduces the NDC bond. Section 5 discusses the how different approaches to dealing with the risk inherent in the current procedure of computing fixed life annuities at the time of retirement.

## 2. Basic framework of NDC

During the accumulation period in NDC worker,  $i$ , with individual earnings  $w_i$  pays contributions,  $c w_i$ , based on a contribution rate. The individual's contributions are noted on a personal account. They are paid into the NDC "fund", however, which pays the pensions of current pensioners. The fund is a construction for administering payments into and from the system, but is also a real fund in the sense it holds demographic reserves, as well as money that may be transferred from the general budget to finance non-contributory rights, etc. Note that the account set up enables transfers from the general state budget to personal accounts to finance extra pension rights, e.g., in conjunction with child birth, unemployment and disability, as has already been mentioned in the introduction.

At the end of each accounting period  $t$  the accounts of all participants are indexed with  $I_t$ , based on an internal rate of return,  $\alpha_t$ . The internal rate of return, to be derived and discussed below, maintains long-term financial balance for a given contribution rate.

The capital on an individual's account at time,  $T$ , is,

$$(1) \quad K_{i,T} = \sum_{t=1}^T c w_{i,t} I_t,$$

Where  $I$  is an index based on the internal rate of return  $\alpha$

$$(2) \quad I_t = \prod_{i+1}^{T-1} (1 + \alpha_i),$$

$$I_T = 1.$$

Retirement is possible at any time after the participant has reached the *minimum age* to claim a

benefit. Worker  $i$  becomes pensioner  $j$  in period  $\tau$ . To calculate the annuity,  $P_{j,\tau}$ , total capital in the individual's account,  $K^*$ , which also includes the inheritance gains from deceased account holders in the birth cohort, is divided by an annuity divisor,

$$(3) \quad P_{j,\tau} = \frac{K^*_j}{\sum_{t=0}^{\omega-\tau-1} (1+\alpha)^{-t} \cdot {}_tP_x}$$

where the denominator consists of two components, the internal rate of return,  $\alpha$ , and the unit value of a whole life annuity as determined by the projected survival probability of being alive at age  $x+t$  at retirement age  $\tau$ . Denoting mortality of a birth cohort age  $x$  at time  $t$  as  $\mu_x(t)$  the survival probability is  ${}_tP_x = f\{t, \mu(x, t)\} = \exp \int_0^t \mu(x + \Delta t, \Delta t) d\Delta t$ . In principle, life expectancy can be single sex or unisex.<sup>4</sup>

Account balances of participants who do not survive to retirement are distributed as mortality (sometimes called inheritance) credits to the accounts of the survivors on a birth cohort basis, up to the minimum retirement age. Thereafter, the account balances of the newly deceased fulfill the classic function of insurance: to redistribute money from those who live a shorter to those who live a longer life as an annuitant. The account set-up makes it possible for spouses and legal co-habitants to share account balances prior to, at and after retirement, with the latter two constituting a joint life annuity.<sup>5</sup> The first of two spouses to retire could begin to take out a single annuity, which could later be converted into a joint annuity when the second spouse retires, either partially or fully.

The rate of return during the life of the annuitant can be frontloaded into the *initial* annuity. Viewed *ex post*, the frontloading factor can at most be equal to the internal rate of return for persons living the life of cohort  $\kappa$  from period  $\tau$ , which, to maintain financial equilibrium, requires periodic adjustment to correct for deviations (positive or negative) of the actual rate of return from the value used to compute the initial annuity.

The alternative to front-loading the annuity is the more straight-forward policy of periodic (*e.g.*, yearly) indexation with the internal rate of return. The choice between

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<sup>4</sup> Court decisions in Europe and the US regarding the choice of life expectancy in mandatory public schemes, require unisex life expectancy, which entails a overt redistribution from men to women, given the historical and present considerable difference in gender life expectancy.

<sup>5</sup> James (2010) and Klerby, Larsson & Palmer (2010) discuss sharing of NDC account balances and possibilities for contracting joint annuities.

frontloading or straight-forward indexation of the annuity has obvious distributional consequences. For a given cohort life expectancy, frontloading distributes relatively more of a given sum of aggregate cohort benefits to individuals with shorter lives, whereas, for the same *ex post* outcome for the entire cohort, pure indexation rewards persons with longer than average lives.

Given that individual longevity is *not* known to individuals *ex ante*, the choice between straight-forward indexation from a lower initial grant or frontloading is one of time preference: Persons who discount future consumption strongly would prefer frontloading and vice versa. People are given no choice in a mandatory scheme, however, and with some or all of indexation potential indexation already embedded in the initial annuity amount, annual indexation will be less than the rate of growth of per capita wages. This means wages of contemporary wage earners increase faster than pensions and as a consequence the ratio of an average pension of a birth cohort compared to the average overall wage of the country will fall as the pensioner cohort ages. This will put older pensioners into relative poverty, compared with straight-forward indexation. If there is a defined minimum pension level with a top-up (financed from general tax revenues) for pensioners whose personal accounts are not sufficient to reach this level, frontloading can be expected to trigger the guarantee top-up for an increasing number of pensioners as the cohort ages. Seen in this perspective, frontloading transfers some small portion of the financing the overall benefit package to future workers, implicitly breaking with the general principle of NDC that individuals pay for themselves.

Financial equilibrium is maintained in NDC through the internal rate of return, to be discussed in the next section, and the use of life expectancy in determining the pensioner's life annuity. Let  $LE_{\kappa}$  denote the *ex ante* estimate of life expectancy used in computing the NDC annuity and  $LE_{\kappa}^*$  denote *ex post* life expectancy, based on the outcome of all individuals in cohort  $\kappa$ . The condition for system equilibrium is thus:

$$(4) \quad E(LE_{\kappa} - LE_{\kappa}^*) = 0.$$

Life expectancy projections are not an exact science. Wide-spread country experience from the past decades suggests that (4) cannot be expected to hold in practice. As we will discuss below, even best-practice modeling is likely to leave an error within the range of five to ten percent. This risk is discussed in greater depth in a separate section below.

Financial balance in NDC requires that assets,  $A$ , cover liabilities,  $\Lambda$ . The liability to worker  $i$  is the individual's personal account balance at time  $t$ ,  $K_{i,t}$ . The liability to pensioner  $j$  is the stream of future pension income to pensioner  $j$ ,  $\sum P_{j,t}$ , discounted at the internal rate of return. Summing over all workers and pensioners, the present value of liabilities at time  $t$  is:

$$(5) \quad PV(\Lambda_t) = \sum K_{i,t} + \sum P_{j,t} .$$

Assets are a claim on future contributions valued at time  $t$ ,  $C_t$ , and funded assets from previous contributions,  $F$ , also valued at time  $t$ :

$$(6) \quad PV(A_t) = A_t(C_t) + F_t .$$

Financial balance requires:

$$(7) \quad PV(A_t) = PV(\Lambda_t).$$

Expressions (5) - (7) apply in principle to any PAYG system. Three features are unique to NDC, however. First, there is a one-to-one relationship between individual contributions and pension rights. This is generally not true of pay-as-you-go defined benefit (DB) schemes. Second, because of the first feature, in NDC the liabilities to workers and pensioners are always *uniquely* defined in terms of individual personal accounts. Third, with the internal rate of return as the NDC gyro mechanism (see the next section), the contribution rate is held constant over all time, *i.e.*,

$$(8) \quad c_t = \text{constant for all } t.$$

Finally, the individual's account balance at any time  $t$ , discounted with the internal rate of return. In generic NDC, liabilities are always covered at a constant contribution rate, as a result of *instantaneous* automatic adjustment to the internal rate of return. In sum, generic NDC is a self-contained system.

We note in closing this discussion that in NDC, there is no within-scheme distribution of contributions between participants other than (a) the pure insurance distribution of the account balances of the deceased, *i.e.* mortality credits, to the survivors within the insurance pool – or possibly external, family-related survivors; (b) (if used) a redistribution through the use of unisex life expectancy; and (c) (if used) through frontloading of the annuity with an assumed growth rate, if the counterfactual is taken to be lifetime yearly indexation with the internal rate of return. Finally, it is clear now that NDC has no need of “the” pension age, as does a typical pay-as-you-go defined benefit (DB) pension scheme. A minimum age at which an NDC benefit can be claimed is necessary, however, as it is not reasonable to allow any pension age. The age should

be set at an age appropriate for the average worker. This age will differ depending on country specific demographic profiles and would ideally be adjusted with life expectancy.

### 3. Equilibrium, the Rate of Return and Valuation of NDC Rights

This section derives the internal rate of return and discusses how it steers the system towards a constant contribution rate and long-term financial stability. We begin by noting that the difference between assets and liabilities in any pension system is the net pension liability of that system (*e.g.*, Iyer 1999). The concept of the net pension liability provides a point of departure for deriving and understanding the internal rate of return in an NDC pension scheme and the difference between NDC, defined benefit PAYG and financial defined benefit and financial defined contribution schemes. The net pension liability is in a continuous process of change, driven by the flow of new contributions and the payment of benefits:

$$(9) \quad dV(t) = V(t)\pi(t) dt + c\bar{W}(a,t)g(t)L(a,t)\lambda(t)dt - \bar{P}(a,t)R(a,t)\pi(t)dt$$

where

$a$  = age, from the minimum age for coverage in the pension scheme to the age of the oldest person

$\pi$  = internal real rate of return

$\bar{W}(a)$  = average real wage, cohort  $a$

$\bar{P}(a)$  = average pension, cohort  $a$

$L(a)$  = number of covered workers in age  $a$

$P(a)$  = number of pensioners in age  $a$

$g$  = rate of growth of real wages

$\lambda$  = rate of growth of the labor force

$V$  = the net pension liability

$c$  = contribution rate

$t$  = discrete period in time

Note that expression (9) includes all present and future cohorts of workers and pensioners. Note also that for any cross-section in time, there is equivalence between “age” and “cohort,” in the sense that persons who are at a specific age at calendar time  $t$  will also belong to

a specific birth cohort. Although there is a minimum pension age in practice, here we have no need to specify an age. The worker's supply of labor past the minimum age to claim a benefit, whatever it is, depends thus on individual preferences for work and leisure, health, work capacity and demand for the individual's labor.

The difference between a financial and a non-financial scheme in terms of expression (9) is determined by pre-funding, the first term on the right hand side, which, except for demographic funding, would be zero in a paygo scheme. In a *pre-funded* pension scheme, every liability is matched by a financial market asset, captured by the first term on the right-hand-side of (9), and the internal rate of return is the financial market rate of return,  $r$ . Such a pension scheme is a purely *financial* scheme and the systematic risk is financial. We can imagine a financial scheme just starting up requiring several decades to become so mature and begin to pay out pensions. Note, however, that, starting NDC from scratch, for example in a society characterized by a multigenerational families living in the same household could be used as a vehicle to start up a paygo scheme with, for example demogrants being paid out to all persons from a specific "pension" age, creating a flat-rate DB scheme that with time transcends *pro rata* into a full-fledged NDC scheme.

When assets consist *solely* of future contributions, with no pre-funding whatsoever, the scheme is purely *non-financial*. The rate of return is non-financial,  $g(t)\lambda(t)$ , the ramifications of which are discussed in the paragraphs below. The "funding" or assets in a non-financial scheme consists of a flow of income from future contributions, as we have established in the preceding section. A non-financial scheme can nevertheless have a financial position, *i.e.*, positive or negative reserves, due to, for example, shifts from large to small cohorts of workers (and eventually pensioners). An NDC scheme can also hold reserves transferred from the general government budget to finance, for example, non-contributory rights.

The risks of non-financial pension schemes are economic, demographic and political, as opposed to the financial and political risks of financial schemes. This said, financial schemes holding assets in domestic equities can nevertheless be affected an ageing demography if the demographic weight shifts from ages in which the propensity to save is high to ages in which it is low or negative. In other words, home-biased financial portfolios will reflect ageing. In addition, we are all these days very familiar with the repercussions of domestic and international economic risks on domestic financial markets. Nevertheless, it is safe to say that the mechanisms for

transferring domestic economic and demographic risks to financial pension schemes are much more obscure than those linking them with non-financial schemes.

The economic risk in non-financial schemes is that the rate of growth in the fund of future contributions will not be sufficient to cover system liabilities. The economic risk has two components: the first associated with the rate of growth of labor productivity (the average wage) and the second associated with the rate of growth of the labor force, driven by fertility, net migration and individual preferences towards work and leisure. By design NDC deals with this package of effects, as we shall demonstrate, through the internal rate of return, just as a financial defined contribution (FDC) scheme does this through the financial market rate of return,  $r(t)$ .

The remaining demographic risk is the systematic longevity risk, *i.e.*, the risk that pensioners will outlive the funding that is the basis for their annuity contract, referred to in the preceding section. It is worth pointing out that both financial and non-financial schemes are exposed to this risk. The questions raised in conjunction with this risk are, first, how can the risk be minimized technically, and second, how should the remaining residual risk be shared? These questions are addressed in a separate section below. The political risk is the risk that politicians will renege on the terms of the funding contract underlying the payout to pensioners, and is also discussed separately below. This section focuses on issues arising in financial balancing with the internal rate of return in NDC.

In NDC, pension rights are defined uniquely by expressions (1)-(3), *i.e.*, accounts of workers and pensioners, which for pensioners are converted to annuities with a life expectancy factor. Expressions (1)-(3) clearly distinguish NDC from non-financial DB (NDB) scheme, in which there can be any number of definitions of the benefit, depending on the specific rules regulating the benefit. Of course, one possible definition of the defined benefit is precisely that provided by NDC, in which case this specific definition of a DB scheme is equivalent to NDC. Also, as opposed to NDC, the traditional paygo defined-benefit (NDB) scheme has no built-in mechanisms to deal with the economic risk, which is an important distinguishing characteristic between NDC and NDB.

We note also that if the NDC pay-as-you-go scheme has a demographic buffer fund, the internal rate of return is a weighted average of the financial rate of return on the fund and the economic rate of return. In terms of (9), where  $\beta_r$  is the percent of financial assets in total assets,

A, in period  $t$   $\pi = \beta_r(t) r(t) + (1 - \beta_r(t)) g(t)\lambda(t)$ . In a fully pre-funded scheme  $\beta_r = 1$  and  $\pi(t) = r(t)$  and in a non-financial scheme with no reserve fund,  $\pi(t) = g(t)\lambda(t)$ .

Integration of (9) over time,  $t = \mu, \dots, \tau$ , for individuals age  $a = \alpha \dots \kappa$  gives:

$$(10) \quad V(t) e^{-\pi t} = \int_{\mu}^{\theta} \int_{\alpha}^{\kappa} V(t) e^{-\pi t} dt + c \bar{W}(a)L(a) e^{-(g+\lambda)t} dt - \bar{P}(a)R(a) e^{-(g+\lambda)t} dt$$

This expression can be used to examine the characteristics of equilibrium for an NDC pension scheme. In the infinite time horizon the expected value of contributions exactly matches the stream of benefit payments, from  $t = \mu \dots \theta$  and for ages  $a = \alpha \dots \kappa$ . Hence:

$$(11) \quad c = E \left\{ \frac{\int_{\mu}^{\theta} \int_{\alpha}^{\kappa} \bar{P}(a)R(a) e^{-(g+\lambda)t}}{\int_{\mu}^{\theta} \int_{\alpha}^{\kappa} \bar{W}(a)L(a) e^{-(g+\lambda)t}} \right\}$$

In NDC the contribution rate is constant and the rate in (11) can easily be shown to be the same rate that applies for all individuals.<sup>6</sup> Expression (11) depicts all rights, present and future, as converted into pensions (annuities), based on exogenously given and true life expectancy annuity factors in the computation of annuities. From (11) it is clear that the rate of return required to maintain the NDC fixed contribution rate, that is, NDC equilibrium, is the rate of growth of the contribution base,  $(g+\lambda)$ .<sup>7</sup> This is the internal rate of return. We note that both  $g$ , the rate of growth of productivity, and  $\lambda$ , the rate of covered growth of the labor force, are determined exogenously.

For  $(g+\lambda) > 0$ , indexation with less than  $g+\lambda$  creates an undistributed surplus. For  $(g+\lambda) < 0$ , accounts of workers (future pensions) and current pension payments must be adjusted downwards to maintain a constant contribution rate, which in turn guarantees equality of assets and liabilities in expression (5a). Through  $\lambda$ , NDC covers, first, the demographic risk of a fertility rate lower than the 2.1 children per woman needed to reproduce the population, second, the impact on the covered labor force of population migration, and, third, the effect of net changes in time spent in the labor force with labor market remuneration, for any given

<sup>6</sup> This is demonstrated in Appendix 1.

<sup>7</sup> It will not necessarily exactly maintain equilibrium in discrete time because in discrete time the values of  $g$  and  $\lambda$  must necessarily be derived from *ex post* data and are not the *contemporaneous* values appearing in (11).



population. The growth factor  $g$  in the internal rate of return covers the risk associated with wage growth, *i.e.*, productivity growth, where  $g$  is expected to have a long-term positive value.

Since  $g$  is the rate of growth of the (covered) per capital wage, as long as pensions grow at the rate  $g$  the ratio of the average pension to an average wage, which can be called the macro replacement rate, is constant. This is an attractive feature of NDC that holds as long as  $\lambda \geq 0$ . In a demographic environment where  $\lambda > 0$  over generations successive cohorts of pensioners will benefit even more from the demographic dividend, *i.e.*, Samuelson's (1958) biological rate of return. This could be the case where a country experiences decades of continuous net immigration, such as the U.S. has during much of its history. It is also the case in any economy in a transition from the informal to the formal labor force.

If  $\lambda < 0$ , generic NDC pensions will grow at the rate  $(1+g)(1-\lambda)$ , which means that the ratio of an average pension to an average wage, *i.e.*, the macro replacement rate, will be declining. With persistent decline in the working-age population and the covered labor force the growth the above equation together with equations (1)-(3) tell us that both the accounts of workers and benefits of pensioners reflect the "drag" of negative population growth on overall indexation. Most of the world is facing the prospect of  $E(\lambda) < 0$  during the coming century, with rates of decline of a few tenths of a percent per year, according to present UN demographic forecasts (United Nations 2010). There is really no alternative to this procedure, however. One can, and should, discuss the distribution of such a result between older pensioners and younger workers, since the latter will normally have a higher level of human capital and hence income, whereas the human capital and capacity to adapt of the elderly is very limited.

Hence, when  $\lambda < 0$ , maintenance of a constant ratio of an average benefit to an average wage, which is what characterizes an economy where  $\lambda = 0$  always holds, is impossible without injecting "outside money," *i.e.*, taxes from the general budget. NDC by no means precludes external tax subsidization to achieve this result. A country facing the circumstance of long-term declining  $\lambda$  could pursue a policy of taxing workers to subsidize pensioners, *e.g.*, through a wage tax that gives no pension rights, in order to maintain a fixed annuity. Whether taxing workers to maintain a given ratio of the average pension in the face of a declining population is a desirable long-run policy would depend *inter alia* on the overall tax-benefit distributional profile of a country.

Subsidization aimed at maintaining a constant macro replacement rate also opens the door to moral hazard. A tax and transfer policy aimed at maintaining a constant macro replacement rate has the indirect effect of subsidizing too early exit from the labor force, where the alternative is that workers compensate for lower expected future pensions through increased labor supply. This is the main argument against subsidizing pensions in a regime with chronic  $\lambda < 0$ .

What's more, the policymaker has a choice in principle between subsidizing pensions at the expense of workers and using the resources for other purposes, for example to promote childbearing, *e.g.*, pursued by subsidizing childcare, compensated absence from work for a period of care of younger children, for care of sick children etc., all expected to support increased female labor force participation. The long-term effect of this alternative can to move  $\lambda$  closer to a long-run value of  $\lambda \geq 0$ , in principle increasing the internal rate of return and long-run discount rate. This is obviously a more efficient policy than the alternative of taxing workers to subsidize pensions, which embodies the negative efficiency consequences just discussed.

The fact that generic NDC adjusts for changes in  $\lambda$  has other distributional consequences that are important to acknowledge. Consider a long-run demographic steady state where the population is simply reproducing itself in all ages that is suddenly "shocked" by a once and for all baby boom. This leads to extra positive indexation of worker accounts and benefits of pensioners when the baby-boom generation enters the workforce. As the baby boomers exit at retirement  $\lambda$  will turn negative for a period. Accounts of workers and benefits of pensioners who are currently in the insurance pool will be reduced as the baby-boom exit continues to produce negative values of  $\lambda$ . This example helps us to understand that different demographic events can lead to different distributional outcomes.

This example illustrates the possible impact of single events. A general feature of the economic environment is, however, that it is not normally characterized by steady state interrupted by a single shock. Instead, the time line through a country's economic history will be an intricate series of events that together produce continuous changes in both  $g$  and  $\lambda$  with many origins, duration times and intensities. These are all *exogenous* to the NDC pension system. Together, they constitute *the* exogenous rate of return.

Expression (11) does not include a term for inflation. In nominal values the exponential expression in the denominator of (11) is  $e^{(g+\lambda+\zeta)t}$ , where  $\zeta$  is the rate of inflation. Since the

numerator is indexed with the same factor, NDC is inflation neutral. Note also that if all income from work gives rise to contributions at the given contribution rate then the pension system grows at the same rate as the economy's wage base, and  $g + \lambda$  in (11) is the real rate of growth of a Golden Rule economy, where the share of profits in gross national income (and balance in international transfer payments) is constant. Of course, the Golden Rule conditions are not likely to hold in practice because the profit rate may increase or decrease over long periods of time. In addition, the internationalization of capital flows breaks the band between the rate of return on domestic capital investment and the domestic rate of growth. Hence, there is no reason to believe that NDC equilibrium would be maintained with indexation based on growth of national income, instead of the covered wage base.

We note also that there is a less obvious but important equilibrium result inherent in (11), demonstrated in Appendix 2. In equilibrium, the system dependency ratio in (11) has a counterpart in the average number of years of work and years in retirement, evaluated with a de facto macro average ratio of an average benefit to an average pension. In addition, it is shown that given the contribution rate and average wage, the average pension is determined by the average individual's lifetime years of work and years of retirement. This information can (and should) be used to set the contribution rate for a country's NDC scheme.

By way of conclusion we should mention that there are transition issues that must be considered in implementing an NDC scheme. We have already discussed a transition possibility from a setting with primarily traditional intergenerational families. For a country making a conversion from a paygo defined benefit scheme to NDC, there are two important issues: first, how to transform rights of workers in the old system into rights in the newly introduced NDC system, and, second, quantifying the unfinanced legacy from the old system, which should be financed by general revenues. These have been discussed elsewhere.<sup>8</sup>

#### **4. The Systematic Longevity Risk and the NDC Annuity**

Even when steered by the internal rate of return, financial equilibrium in NDC is contingent on equation (4) holding, that is, that the difference between projected and actual longevity is random with expected zero value. In practice independent experts and national statistical agencies have notoriously tended to underestimate the force of increasing longevity,

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<sup>8</sup> These issues are discussed in Palmer (2006b) and Holzmann and Jousten (2011).

regardless of whether they use pure statistical modeling or some combination of a statistical and judgmental strategy. By definition longevity (mortality) projections must be uncertain. As a result there is a substantial risk for the insurer. Private insurers will attempt to construct contracts so as to pass this risk off to the insuree. This is also possible within NDC. The question is, is this a desirable feature of a public system. More fundamentally we ask, how can this risk be distributed given the principle of intergenerational equity underlying NDC?

To begin with, we note that there are two demographic risks involved in annuity provision; an idiosyncratic and a systematic risk. The life annuity, which is fixed at the time of retirement, deals with the idiosyncratic risk by redistributing the remaining account values of those who live shorter lives after retirement to those who live longer lives. In the absence of systematic risk, this procedure is sufficient to yield a zero net result for the entire pool of annuitants (a birth cohort). The question we are addressing now is how to deal with the systematic risk within the context of NDC.

The obvious first step is to improve longevity forecasts. This is easier said than done, however. If it weren't we would observe much better forecasts. Statisticians and actuaries have turned more and more to the use of time series models to project life expectancy in countries where there are sufficient data. This is only a handful of countries. In most countries of the world data is insufficient or of poor quality. Lee and Carter (1992) have presented a time series model that has gained increasing acceptance in settings where reliable data are available. It projects mortality with two components, one that captures the overall trend in population mortality and another that distributes this trend by age. A whole genre of models has emerged developing this general idea even further (*e.g.*, Lee & Miller 2001; Booth et al. 2002; DeJong & Tickle 2006; Haberman and Renshaw 2006; Girosi & King 2007; Cairns, Blake and Dowd 2010). Nevertheless even after applying this state of the art technology considerable systematic error remains (Booth 2006).

A recent study of some variants of the Lee Carter model (Alho *et al.* 2011) shows that, for a group of eight developed countries, this sort of modeling is likely to give a systematic cohort error of around 5 percent, with considerable variation in accuracy among countries, even when estimated for age 65 (instead of from birth). The source of the systematic error for countries studied turns out to be, not so surprisingly, *accelerating* improvements in mortality in older ages. A process that extends over decades.

Given that it is difficult, if not impossible, to eliminate the systematic error, what can be done if its existence is acknowledged? A simple approach would be to adjust annuities continuously as new estimates of life expectancy become available. This puts all the risk entirely on the pensioner and each cohort ends up paying for itself freeing the insurer, in the present context the government, from the systematic risk. Piggott *et al.* (2005) and Valdez *et al.* (2006) have suggested a procedure of this type for private financial insurance schemes. Their proposal is to calculate the uncovered risk on a cohort basis, as the ratio of the actual to the projected life expectancy at each age

$$(25) \quad \psi = \frac{K_{R,t-1}}{K_{R,t}}$$

and distribute it among survivors using as weights the individual weights in the total pool of survivors. The obvious drawback of this procedure is that those who live longer pay for the financial consequences of this. In view of this a simpler approach would be to choose the outer statistical confidence interval for the initial annuity calculation, a practice not unknown to financial schemes, which can lead to an excess of money ending up with the pension provider and pensions that are lower than what turned out to be possible. This would be one way to share the burden of the risk among all annuitants, which seems fairer than continuously distributing it among the survivors.

Another possibility would be to create variable annuities up to a higher age and then fixing the annuity. Using the long time-series data for the four Scandinavian countries (Denmark, Finland, Norway and Sweden) Alho *et al.* show that adjusting annuities up to age 85 would have reduced considerably the life income of older pensioners – while still leaving systematic error (!). In fact, the force of acceleration was so strong in the period covered for Finland that the insurer would still not have come out even with a variable annuity up to age 85. This study shows a clear loss of utility of consumption of older compared to younger persons using this method. Clearly this is an undesirable feature for a public pension system.

Yet another approach would be to allow this error to become embedded in the revision of liabilities in an NDC construction with a balancing mechanism (discussed in more detail in a separate section below) which means that the error is distributed among workers and pensioners as/if it reveals itself in the form of liabilities that exceed assets. This approach distributes the correction for the systematic longevity within the overall insurance collective.

In the next section we present the case for the government to bear the cost of the systematic longevity risk and other possible residual risks through the creation of an NDC bond. The cost of the residual error would be distributed thus to contemporary taxpayers. This is consistent with the fact that it is a systematic and not an idiosyncratic error. The outcome may be similar to applying a balancing mechanism in the sense that the correction process is distributed among workers and pensioners alike, but has the advantage that income taxes are based on overall income, not just pension rights. The general tax base is also broader, incorporating taxes on profits and dividends. This shifts more of the payment burden to taxpayers with higher incomes and encompasses a broad revenue base.

In the end the question of who should bear the remaining systematic longevity risk is a political as much as a technical question, and, once, again depends on the overall tax-transfer context. The device of an NDC bond utilizes the idea that the government is the insurer of last resort. With an NDC bond, life annuities would be fixed at the time of retirement using life expectancy estimates based on the best technology available, produced by an independent agency. The government would then assume its role of the insurer and absorb both losses and profits, with the goal of breaking even.

## **5. The NDC Bond: the Putty that Closes the NDC Scheme?**

In this section we develop the details of the NDC bond. We begin by noting that the liabilities in an NDC pension scheme constitute the government commitment to the participants. If the (nominal) internal real rate of return,  $(1+g)(1+\lambda)(1+\xi)$ , where  $\xi$  is the rate of inflation, is sufficient to maintain equilibrium and if liabilities are indexed accordingly, in principle, future contributions would always be sufficient to pay for liabilities, if the expected deficit or surplus from annuity payments at the end of cohort lives was in fact the expected zero value. In other words, the NDC system as presented here is in equilibrium if these two conditions are fulfilled, albeit with white noise around an expected outcome. In practice, the index constructed to measure the internal rate of return will must be based on information that becomes available only with a lag – whereas the theoretical model presupposes instantaneous information. This can also lead to small system generated errors. We argue here that the role of the government as insurer of last resort is to cover these residual risks. Since in the every-day operating sense the system is in near equilibrium, the NDC system would then be closed by a government commitment to assume

remaining risks. The NDC Bond would be the vehicle used for achieving this result. Here we discuss the implications of this approach to closing the NDC scheme.

The underlying NDC asset is the future contributions of workers, which is the basis of expression (11). By closing the system with the NDC bond, the NDC accounting system ascribes to every unit of liability a corresponding unit of an NDC bond. The point of departure is, first, that the equilibrium rate of return,  $(1+g)(1+\lambda)(1+\xi)$ , is the rate employed, which means that the system will be self-adjusting with regard to the inflow of revenues.

The individual contracts between the government and each and every one of the NDC participants are similar to those created by issuance of *marketed* bonds to back other forms of government debt. For this reason, it can be claimed that the NDC bond also manages *the political risk*. Individual shares in the NDC bond make clear the relative costs of default to different cohorts. This introduces accounting that makes the impact of default transparent for individuals and decision makers. It doesn't mean that the government cannot default, because, as we all know, it can. What it means is that all the cards are on the table for everyone to see. This probably means that, if absolutely necessary, commitments would be cut proportionally. Of course, the underlying principle is that default is out of the question. As we have already argued, this construction of the commitment can also have the effect of steering the any necessary correction in the direction of taxes.

We can view the NDC commitment as a loan from the NDC scheme participants to the government, secured by the government with NDC bonds. With this perspective, the government borrows money from current workers to finance the benefits of current pensioners. The rate of return on this bond is the NDC internal rate of return; the rate that determines account values and maintains financial equilibrium.

We define the NDC bond as a government issued perpetuity bond that can be likened with a marketed bond that offers a real rate of return equivalent to the rate of growth of the covered wage sum. Using Fisher's equation, the return on a marketed government bond can be expressed as  $(1+r) = (1+i)\{1+E(p)\}$ , where here  $r$  is the nominal rate of return and  $p$  the relevant price index, here the consumer price index. Then the NDC bond, with return  $1+r = (1+g)(1+\lambda)(1+p)$  is the perfect inflation hedge for a pension provider, in this case the government.

Seen in this light, NDC provides an annuity at retirement backed by the refuted missing asset for financial pension systems recently proposed and discussed in the literature on financial pensions (*e.g.*, Blake and Burrows 2001; Blake *et al.* 2006a; Brown & Orszag 2006). In fact, in the absence of any other correction mechanism, such as a solvency ratio (to be discussed below), with the NDC bond the government covers the residual systematic longevity risk but without marketing the bond. In this sense, the (unissued) NDC bond is the perfect financial longevity instrument and the NDC bond is the embodiment of the risk free asset, barring the ultimate risk of government “bankruptcy”.

Let us reflect a moment on how bond shares could be created in practice and then return to a discussion of their functionality. Workers are granted shares in the NDC bond that exactly match their account values liabilities and pensioners are granted shares that exactly match their contracted annuity payments. The government creates bond shares to be held by the insurer (the NDC insurance administration) on behalf of the participants that exactly cover these liabilities. All workers and pensioners hold unit bond shares equivalent to their earned rights. The insurer’s (government’s) liability is thus backed by NDC bond shares, which in practice can be held on behalf of the insured by the insurer.

This can be formalized. In terms of the apparatus developed above for the account balances of workers and pensioners,  $K_L$  and  $K_R$ . These are exactly matched by NDC bond equivalents,  $B_L$  and  $B_R$ , such that:

$$(13) \quad PV(\Lambda_t) = K_L + K_R = B_L + B_R = B_t$$

where  $K_L$  and  $K_R$  are the stocks of notional capital of workers and pensioners in expression (7) and  $B_t$  is the total stock of NDC bonds. We specify two expressions for these two groups.

The capital of workers and the equivalent bond holdings is specified as:

$$(14) \quad \begin{aligned} K_{L,t} &= \sum \left\{ \left[ K_{L,a,t-1} + T_{a,t-1} \right] (1 + g_t)(1 + \lambda_t) + \Omega_{LD,a,t} \right\} + \sum \left\{ c W_{a,t} L_{a,t} + T_{a,t} \right\} \\ &\quad - \omega_{a,t} K_{L,t-1} (1 + g_t)(1 + \lambda_t) - \Omega_{LD,a,t} \\ &= \sum K_{L,a,t-1} (1 + g_t)(1 + \lambda_t) + \sum \left\{ c W_{a,t} L_{a,t} + T_{a,t} \right\} \\ &= B_{L,t} \end{aligned}$$

where  $\omega_{a,t}$  is the share of a cohort that exits the labor force for retirement.

The first term within brackets on the r.h.s. of (14) is the previous period’s capital indexed to current values at the end of the preceding period, plus a capital transfer from the accounts of workers who die during the period,  $\Omega_{LD,a,t}$ . The second set of brackets encompasses new



contributions on earnings, cWL. In addition to earnings-based notional capital, the government may also pay money into the system from general revenues to finance non-contributory rights,<sup>9</sup> denoted by  $T_{a,T}$ . The third term denotes capital from workers that exit into retirement. The final term is attrition due to deaths in all cohorts of workers, *i.e.*  $\Omega_{LD,\alpha,t}$ , which represents the redistribution of capital from the accounts of the deceased, *i.e.*, mortality credits, to the survivors in cohort  $a$  in the first set of brackets. Overall liabilities are unaffected by these internal transfers.

Expression (14) says that bond shares in any period  $t$  for workers equal bond shares at the start of the period, indexed with the rate of return  $(1+g)(1+\lambda)$ , plus bond shares for new contributions in period  $t$ , minus bond shares for retiring workers. The share of workers from cohorts exiting for retirement is included in expression (15) below for retirees.

The expression for retirees is:

$$(15) \quad K_{R,t} = \sum \left\{ \rho_{R,\alpha,t} \bar{P}_{R,\alpha,t-1} R_{\alpha,t} (1+g_t)(1+\lambda_t) + \sum \Omega_{RD,\alpha,t} \right\} + \sum K_{RN,t-1} (1+g_t)(1+\lambda_t) - \sum \Omega_{RD,\alpha,t}$$

$$= \sum \rho_{R,\alpha,t} \bar{P}_{R,\alpha,t-1} R_{\alpha,t} (1+g_t)(1+\lambda_t)$$

$$= B_{R,t}$$

where  $\rho_{R,\alpha,t}$  is life expectancy from age  $a$  at time  $t$ .

Expression (15) says that bonds for retired members of cohort  $a$  in period  $t$  equal the cohort's average pension in the preceding period, indexed with the rate of return  $(1+g)(1+\lambda)$  to get the present value, multiplied by the number of pensioners and number of years benefits are expected to be paid, *i.e.*,  $\rho_{R,\alpha,t}$ , which is an estimate that can be revised on a yearly basis until the birth cohort has expired completely. The second set of new terms is the capital of new retirees in year  $t$ . The capital (present value of expected pension payments) of those who die in period  $t$ ,  $\Omega_{RD}$  is redistributed, to surviving retirees and does not affect overall liabilities.

Summing over financial commitments to workers and retirees, the stock of NDC bonds in period  $t$  is

$$(16) \quad B_t = \sum K_{L,\alpha,t-1} (1+g_t)(1+\lambda_t) + \sum \left\{ cW_{\alpha,t} L_{\alpha,t} + T_{\alpha,T} \right\} + \sum \rho_{R,\alpha,t} \bar{P}_{R,\alpha,t-1} R_{\alpha,t} (1+g_t)(1+\lambda_t)$$

Indexation with  $(1+g)(1+\lambda)(1+\zeta)$  harmonizes the nominal rate of return on the stock of NDC bonds with the rate of return on the stock of NDC liabilities.

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<sup>9</sup> Examples are money transferred into the system to cover some specified time in conjunction with child birth, money to cover pension rights of the disabled, persons on unemployment benefits, etc.

With this formulation of the NDC asset, the entire fund, referring back to expression (8), consists of the single asset, the NDC bond, albeit with individual shares. All bond shares are valued with the nominal rate of return  $(1+g_t)(1+\lambda_t)(1+\xi)$ . The “fund” earns exactly the rate of return on the bond shares needed to fulfill the NDC commitment and this is the rate of return required to maintain financial equilibrium.

It would seem that the creation of the NDC bond – and bond shares – renders an explicit financial reserve fund unnecessary; with the creation of the NDC bond the government has already committed itself to covering the entire commitment. The reserve fund still has a purpose, however. It holds the surplus that can arise due to a demographic bulge and is the recipient of the government’s payment of rights not financed directly by contributions from earnings, *e.g.*, contributions to accounts of the disabled from the government in lieu of actual individual contributions earnings, contributions to the accounts of parents in conjunction with child birth to compensate for earnings-based contributions in conjunction with childbirth, etc. These could in fact be invested in tradable financial market instruments, with an expected rate of return at least as high as the internal rate. However, it is more in keeping with the closed NDC system to issue these NDC bond shares comparable to the government commitment, with the rate of return  $(1+g_t)(1+\lambda_t)(1+\xi)$ . In other words, there is no reason for the government to attach a risk premium to this debt component and by not doing this the cost to the taxpayers is exactly on par with the value of the commitment behind the bond.

As we have claimed in introducing the concept of the NDC bond above, compared with just going through the accounting exercise of NDC, issuing and assigning NDC bond shares to the individual rights of participants strengthens the government commitment. NDC bond share values would appear in the NDC schemes double entry bookkeeping and account statements would be regularly sent to participants. In this way, the creation of NDC bonds strengthens the barrier against *ad hoc* political intervention. This means that in times of budget crisis, government cannot simply cut pensions to strengthen the national coffer. With this framework, instead of cutting pensions to cut back on government expenditures, which is a clear tax on pensioners, government could choose to increase the tax on both wages and pensions.

Finally, it is clear that there would be no marginal gain in marketing the NDC bond. Selling the NDC bond on the market would lead to marginal costs in the form of transaction costs and a risk premium to compensate the holder for holding the bond. This would however

create no welfare gain for the participants – this only shifts individuals' money from one pocket to the other; the taxpayers are in principle the same individuals as the workers and pensioners.

## **6. Other Alternatives - Setting Market Values on Liabilities, Creating a Measure of the Contribution Asset and Using a Solvency Ratio to Maintain Asset-Liability Balance**

The NDC bond creates a uniquely defined one-to-one relation between individual account values and bond shares. Together the bond shares constitute the NDC asset and liabilities are always covered by assets. This way of closing the system is attractive since it means that the system functions in accordance with equation (11), where the internal rate of return is that which growth of contributions. In principle, system revenues should cover expenditures with the main divergence being that created through imprecise life expectancy estimates. The government picks up the residual risk, created by uncertain longevity, the strength of which depends on life expectancy projection technology and the risk distributional model chosen by the policy maker.

Before we close the books, there are two alternatives that can be weighed against the NDC bond approach. The first would be to value or even go as far as selling the NDC debt on the market, which ostensibly would cover both the longevity risk and the risk of government default. The market would thus value the NDC debt and the taxpayers would service the market debt. The market will want a premium to cover the risk described above. The second is to estimate the contribution flow implicit in current and/or projected information on the population, work force participation, age earnings profiles and the age distribution of benefit payments. The asset can be defined in terms of the expected future flow of contributions, with the estimate used to construct a solvency ratio. These alternatives are considered here.

### **6.1 Should NDC liabilities be valued in the financial market?**

The recent literature on pay-as-you-go pensions contains several proposals to set market values on pay-as-you-go liabilities. One proposal is to sell a portion (not necessarily all) of the NDC debt on the market to establish a market value (Valdès-Prieto 2006; Robalino & Bodor 2008). Once the market value has been established, the argument goes, all liabilities can be valued taking the market valuation of the risk into account. Another possibility is to this is to set a price on liabilities using asset pricing methods.

Blocker *et al.* (2008) and Geankoplos & Zeldes (2009a; 2009b) and/or to market the whole NDC debt. In NDC, liabilities are always known, as opposed to what is often the case in non-financial DB pension schemes, so the risk sold to the market would be the cost to the provider (the government) of covering uncertain outcomes, for example the longevity risk to be discussed in the next section. The market would value this risk and set a price on the government commitment. The risk premium that emerges would become a cost to be deducted from the internal rate of return. In essence, participant workers and pensioners would pay the cost for covering the risk of government default on their future benefits.

The value added of these methods to NDC is not clear since they disregard the economic fundamentals of the NDC scheme in attempting to give liabilities a market value. Going back to the fundamentals, what is required to keep an equilibrium intergenerational contribution rate is that 1) the systematic longevity risk is covered and 2) that liabilities (pension rights of workers and pensions) grow with the economic internal rate of return. Marketing the debt creates transaction costs and introduces a new risk premium to be covered by taxpayers. The risk for the market is that the government will become bankrupt and renege on its payments. The higher the risk of the government the greater is the tax that is required to finance selling the risk. In practice, the government could afford to pay the internal rate of return, which in a Golden Rule economy, would be the same rate at which tax revenues increase, with a tax rate that is proportional to income, less the cost of selling the debt to the market. This round-about procedure leads to a deadweight loss for taxpayers, who are also workers and pensioners. The bottom line is that the NDC bond provides sufficient coverage of the real system risks and that there's nothing to be gained by marketing the debt.

## **6.2 Defining “the NDC asset” in terms of the average time a unit of money is in the system**

In principle, a solvency ratio of assets to liabilities can be constructed to determine the relationship between NDC liabilities and the expected flow of future contributions. As we have already established in Section 2, liabilities are completely known, as is any degree of funded there may be, but the future course of contributions is unknown at any accounting time  $t$ . Given this point of departure, one could use standard modelling techniques to estimate the future flow of assets. This approach would provide stochastic forecasts. The next step would be to determine statistical bounds beyond which liabilities would be adjusted, both upwards and downwards,

with an index based on the solvency ratio outcome. In this way participants would experience “additional” indexation beyond that necessitated by the internal rate of return. This is also a practice often availed in private mutual insurance schemes.

An alternative definition of assets is employed to create the solvency ratio in Sweden, defined very generally in Settergren and Mikula (2006) and described more detail in the appendix of the annual report of the Swedish pension system. This method defines the time a liability is in the system until a payment must be made, that is, the turnover duration (TD) of liabilities. The measure is based on *known outcomes* of the flow of contributions and payments at the time it is constructed.

We begin by showing the assumptions underlying this definition of assets, which have not previously appeared in the literature. Our point of departure is the definition of NDC liabilities, *i.e.* the balances on the individual accounts of workers and pensioners in expression (5) above. We begin with the fact that personal account values are based on contributions and are a product of a fixed contribution rate and individual earnings. Looking back from any “present” time T, the historical rate of growth of wages (W) for a given discrete time period is known. The sum of wages earned by an individual (or the average for a cohort of individuals) can be expressed as  $\sum_{\gamma}^T W_t (1+g)^{T-\gamma} = (T-\gamma) W_T$ , where g is the average rate of growth of the individual’s wages. Hence, an average cohort rate of growth,  $\bar{g}(a)_T$  can be calculated at time T, which yields the average cohort wage,  $\bar{W}(a)_T$  for an average number of years of work  $\overline{(T-\gamma)}(a)$  for the average cohort member up to time T. It follows then that the liability to workers, *i.e.* the value of accounts summed over all workers at time T, can be expressed as

$$(17) \quad \sum_a^m c \overline{(T-\gamma)}(a)_T \bar{W}(a)_T L_T.$$

The insurance system’s liability to pensioner cohort a is the average pension,  $\bar{P}(a)$  times the number of years, denoted by the life expectancy at time T for persons in birth cohort a,  $\bar{\rho}_T(a)$ , the pension is expected to be paid out from time T,

$$(18) \quad \sum_a^m \bar{\rho}_T(a)_T \bar{P}(a)_T R_T.$$

System liabilities to workers and pensioners are:

$$(19) \quad PV(\Lambda_T) = \sum_a^m c \overline{(T-\gamma)}(a)_T \overline{W}(a)_T L_T + \sum_a^m \overline{\rho}_T(a)_T \overline{P}(a)_T R_T .$$

Denoting the share of workers and pensioners in the population as  $L_T$  and  $R_T$  we rewrite (19) as

$$(20) \quad PV(\Lambda_T) = \sum_a^m c \overline{(T-\gamma)}(a)_T \overline{W}(a)_T L_T + \sum_a^m \overline{\rho}_T(a)_T \overline{P}(a)_T R_T$$

Multiply the first expression on the r.h.s. of (20) by  $\overline{W}_T / \overline{W}_T$  and the second expression by  $\overline{P}_T / \overline{P}_T$  and note that in equilibrium contributions equal pension payments, *i.e.*

$$(21) \quad \overline{P}_T R_T = c \overline{W}_T L_T .$$

To complete the derivation, substitute contributions (the r.h.s. of (21)) into the second expression on the r.h.s of (20) to get:

$$(22) \quad PV(\Lambda_T) = \sum_a^m \overline{(T-\gamma)}(a)_T \frac{\overline{W}(a)_T}{\overline{W}_T} c \overline{W}_T L_T + \sum_a^m \overline{\rho}_T(a)_T \frac{\overline{P}(a)_T}{\overline{P}_T} c \overline{W}_T L_T .$$

Rearranging terms, gives:

$$(23) \quad PV(\Lambda_T) = \left\{ \sum_a^m \overline{(T-\gamma)}(a)_T \frac{\overline{W}(a)_T}{\overline{W}_T} + \sum_a^m \overline{\rho}_T(a)_T \frac{\overline{P}(a)_T}{\overline{P}_T} \right\} \cdot c \overline{W}_T L_T .$$

The expression on the r.h.s. of (23) is a *description of liabilities*. The term within parentheses is the sum of the wage weighted average number of years of labor force participation of current cohorts and the pension weighted average of the length of payments to currently living cohorts of pensioners. It is a measure of the amount of time a unit of money will remain in the system from the average time the contribution is paid until it is paid out as a benefit given the pension system “statistics” on the age distribution of workers and pensioners and the age related average amounts of wages and benefits at any given time,  $t$ . This is the turnover duration, denoted TD.

In equilibrium assets equal liabilities, implying that the r.h.s of (23) is a possible definition of assets. Assume we adopt this view. Bringing a possible reserve fund (F) back into the picture, using expression (23), the present value of assets equals the right hand side of (23), with the addition of a possible fund, *i.e.*,  $PV(A_t) = TD_t(C_t) + F_t$ . This is the basic idea behind the Swedish balancing mechanism, employed in Sweden since 2001. Given this definition of assets, the solvency ratio implicit in (23) is

$$(24) \quad \Psi = \frac{PV(A_t)}{PV(\Lambda_t)}$$

This is the definition employed to maintain financial balance in Sweden. It is also the definition tested on a well-behaved set of US data projections in Auerbach and Lee (2009). We note that the philosophy behind the Swedish NDC design is to index accounts with  $1+g$  and to accumulate a reserve in times when  $\psi > 1$ , to provide a reserves that can then cover, if only partially, times when  $\psi < 1$ . Clearly, as Auerbach & Lee demonstrate, this approach creates a welfare loss, since reserves created with  $\psi > 1$  constitute foregone consumption opportunities – which can affect decades of workers and pensioners.

The problem with the NDC asset as defined in (22) is that it does not provide a *unique* definition of *the* asset. To see this we need only compare perpetual steady states where *total contributions are forever constant and the same in every steady state*, as are the mortality rates (longevity) at all ages, and the money-weighted distribution of annuities. With these limitations, the steady states differ only with respect to the age distribution workers, which in the real world could reflect the age patterns of entrance and exit over time. An easy example to understand is one in which younger birth cohorts enter the labor force at successively later ages. The decreases the time during which workers are in the labor force. The use of TD to measure assets yields a gradually declining value of assets, *even though total contributions are forever unchanged*. This property, which is clearly undesirable is illustrated with examples in Table 1. What may ameliorate this effect is that it occurs gradually over time. In addition, it could be offset in practice within the calculation by a simultaneous increase in the age of pensioners. However, this may or may not happen, and whether it happens does not change the fact that expression (23) does not yield a unique definition of assets.

The effect of an ageing labor force is clearly a liquidity effect, which would have relevance in determining the content of a *financial pension system* investment portfolio, but in the example presented here a liquidity effect is of no consequence. There are liquidity effects of consequence, however. For example, from the component  $\sum_a^m \bar{\rho}_T(a)_T \frac{\bar{P}(a)_T}{\bar{P}_T}$  in expression (23) it is evident that for any total amount of pension payments more liquidity will be needed to make payments if younger pensioners have relatively higher pensions.

Table 1. Changes in Turnover Duration with a Steady State Labor Force (LF). Labor force participation by age.

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	Scenario A	Scenario B	Scenario C	Scenario D
Age	Young LF participation PA=65	Same LF participation all ages PA=65	Middle aged LF participation PA=65	Old LF participation PA=65
16-20	0.8	0.8	0.5	0.3
21-25	0.9	0.8	0.7	0.5
26-30	0.95	0.8	0.8	0.7
31-35	0.95	0.8	0.9	0.95
36-40	0.95	0.8	0.95	0.95
41-45	0.95	0.8	0.95	0.95
46-50	0.8	0.8	0.9	0.95
51-55	0.8	0.8	0.8	0.95
56-60	0.5	0.8	0.8	0.95
61-64	0.4	0.8	0.7	0.8
Total LF Participation	0.80	0.80	0.80	0.80
TD of Workers	21.9	20	19.4	17.8
No. of workers and wage sum (unit wage)	40 million	40 million	40 million	40 million
Contributions (20%)	8 million	8 million	8 million	8 million
Assets (workers' TD + pensioners' TD times contributions)	279 million	264 million	259 million	246 million
Assets in the scenario as a % of assets in the "young" Scenario A	100.0	94.6	92.8	88.2

Assumptions. The total population in each age group is the same (1 million workers) and all workers receive a unit wage. In Scenarios A-D exactly 80 per cent of all persons age 16-64 work. In scenario E, 73 percent of all persons 16-69 work, giving exactly the same number of persons working as in Scenarios A-D. The contribution rate is 20 %. The life expectancy and age-distribution benefits is assumed to be the same in all scenarios. TD of pensioners is set to 13 years for pensioners from age 65 (Scenarios A-D) and 8 years for pensioners from age 70 (Scenario E).

With balancing based on the TD concept, accounts of workers and pensioners alike would be reduced as a result of the natural process of increasing pensions associated with continuous real growth. However, the resultant devaluation of accounts may not be enough or at the right time. This is a "negative" liquidity effect. The process of postponing retirement to higher ages creates liquidity, *i.e.*, constitutes a positive liquidity effect. One might argue, thus, that the real value of using a solvency ratio based on TD would be to provide a mechanism for



dealing with liquidity effects. For example, one might argue that the effect running through the aging labor counteracts the effect of higher benefits of new pensioner cohorts, creating a desirable net effect.

In sum, the drawback that the TD method does not yield a unique estimate of assets means that, *ceteris paribus*, postponed entrance into the labor force, *e.g.*, a more compact distribution of labor force participation to older ages, leads to a decline in the NDC asset, if defined in this manner, and consequently to a slow reduction in pension rights that is not consistent with the system's ability to pay. This drawback alone makes the NDC bond more attractive. In addition, both the TD-definitional approach to defining the asset and the NDC bond distribute residual risk to workers and pensioners. In fact the Swedish balancing mechanism works as a proportional tax on both groups, not discriminating with regard to any factor. By externalizing this process, when necessary, through the NDC bond, the risk and burden of payment is passed on to taxpayers in general. This means that the government budget performs the function of a reserve (buffer) fund, which means that pensions of pensioners are not automatically affected when solvency is temporarily less than unity, due to a recession, financial crisis, etc. If the situation giving rise to a deficit is of a chronic nature, for example a longer decline in the labor force, then the government has the option of tailoring tax or

## **7. Summary Remarks**

The paper derives the equilibrium properties of NDC and discusses these with respect to different scenarios for economic and demographic development. The paper shows how a criterion can be established for setting the desired level of the contribution rate, based on a target level of average per capita years of work and retirement. Once the contribution rate is fixed valuation of account balances with the internal rate of return steers the system towards financial equilibrium at this rate. This is an important insight in setting up an NDC scheme. Valuation of pension rights of workers and benefits of pensioners is driven by economic and demographic factors, *i.e.*, the dynamics of the average covered wage and the factors steering the growth of the covered labor force - births, mortality, net migration and labor supply decisions that determine work force participation.

Changes in cohort mortality after retirement are covered by the life expectancy factor used to compute annuities. However, many decades of experience of mortality forecasting suggest that both statistical models and expert judgments systematically underestimate improvements in mortality and, hence, longevity. This residual risk calls for a procedure to correct for deviations of the projected from actual outcomes. The paper discusses alternative possibilities for this and their consequences for the distribution of risk. One can argue for adjustable annuities for younger retirees in an age setting where either increased labor supply, including postponing retirement until a later age, is a reasonable alternative. Arguably, older pensioners with limited or no opportunity to adjust their labor supply should have a fixed life expectancy factor, *i.e.*, annuity, albeit with yearly indexation reflecting the internal rate of return. For example, for a country where it is not entirely uncommon to work up to the age of 67 the pension annuity factor could be variable for persons who choose to claim a part or all of a pension prior up until this age. In addition, in setting the value of the annuity, given uncertainty and the historical direction of the bias, it is possible to take an upper bound of the estimate.

If demographers and the pension scheme actuaries were to succeed in fulfilling the NDC condition of an expected deviation from the actual longevity outcomes of zero, then the government's role is that of a buffer fund that evens out the random fluctuations. In keeping with NDC accounting transparency principles, the NDC yearly accounts show this process. This procedure does not require an NDC bond. However, the device of an NDC bond makes an explicit declaration that the government (taxpayers in general) is covering this risk. This in turn means that instead of taxing pensions of pensioners the government would be spreading the burden of financing the residual longevity risk to *all* taxpayers, based on their *overall* personal income. This is a more appealing distributional result than distributing the burden explicitly, for example, through some form of variable annuity in the extreme a whole life or distribution through a solvency ratio approach, which puts more of a burden on pensioners, such as the Swedish solvency ratio approach.

With the NDC bond the internal rate of return is the growth of the covered contribution base, which reflects real per capita wage growth, labor force growth and inflation. This is done by using nominal contribution growth in practice. Under many circumstances this rate of growth will be close to the rate of growth of GDP. The *sine quo non* of NDC is the fixed contribution rate. This is what generic NDC, with a mechanism to cover residual risks, *i.e.*, the NDC bond or

a solvency ratio, delivers. This means that all generations pay the same portion of their earnings and, in principle, GDP, to the public pension system. Once again, this requires calibrating the NDC scheme from the beginning to achieve a level that delivers a satisfactory pension to the average worker, given the remainder of the overall pension landscape of a country. It is also shown in the paper that the solvency ratio approach requires a rule for measuring assets and at present there is no rule that does not have more or less serious drawbacks. Given this drawback in combination with the distributional advantages of the NDC bond, the NDC bond appears to be the prime candidate to close the NDC scheme financially.

Finally, the paper argues that NDC does not really lash policy to the mast, as some have claimed as for example Brooks and Weaver (2006) many others have claimed. The government's hands are free to enhance rights (accounts) for socially desirable goals, by creating rights and financing them with transfers (contributions) to the NDC scheme on behalf of the recipient individuals, for example, in conjunction with childbirth. Disability and other insured periods of absence from the labor force can be treated in a similar manner.

A remaining risk is the political risk. NDC accounting makes clear the commitments of the government to participants. Can shares in the NDC bond be viewed as a contract? It is argued in the paper that an NDC bond is comparable to traditional financial market instruments used to finance government debt, albeit it is not marketed, which it is argued in this paper has no economic logic. Marketing the bond enables the market to evaluate the sovereign risk. If the NDC bond is actually marketed, the cost of covering this risk will nevertheless be paid by the taxpayers as a financial transfer to holders of the government's paygo bonds, which in the end is a non-transparent way to distribute national income, which also brings deadweight through transaction costs. Since the taxpayers of today and the future are always the guarantors of last resort it seems logical that they are given the burden of covering the residual risk through the straight-forward procedure of the unmarketed NDC bond.

Of course, in the end there is always a political risk. Legislators can always change the rules for the game. What NDC brings to the table is a transparent rule system that clearly identifies the cost of renegeing on liabilities to today's and tomorrow's pensioners. This makes it much easier to compare alternative uses of tax revenues. Also, if a pressed government must cut pensions, then it is more reasonable to use the general income tax as the instrument to cover the

same reduction, since the income tax base is overall personal income. In fact, adopting this view, it is always possible for legislators to cut or enhance pension income after tax.

In sum, the generic NDC scheme is based on economic logic. The device of the NDC bond adds to this by creating a more appealing distributional mechanism for dealing with residual risks. In addition, it provides a better base and distributional outcome for adjusting – both up and down – the after-tax income of pensioners, should this become necessary.

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## Appendix 1. Demonstration of the Equivalence of the Macro Contribution Rate with the Individual Contribution Rate

Here we demonstrate the equivalence between the macro contribution rate in expression (11) in the text and expression (1).  $\bar{P}$  in (11) is the average of individual pensions, determined by individual capital balances and cohort life expectancy at retirement (3). Indexing accounts with  $(1 + g)(1 + \lambda)$ , the average individual in pensioner cohort  $a$  has an annuity,  $\bar{P}_a$ , which can be expressed in terms of the current average wage,  $\bar{W}_a$ , where  $\bar{\rho}(a)$  is the value of life expectancy used to calculate the benefit of pensioners in cohort  $a$ :

$$(A1.1) \quad \bar{P}_a = \frac{\bar{K}_a}{\bar{\rho}_a} = c \bar{W}_{a,t} + \bar{K}_{a,t-1} (1+g)(1+\lambda) \left( \frac{1}{\bar{\rho}_a} \right),$$

Assuming the average individual in the cohort works  $T$  years until retirement then (A1.1) becomes:

$$(A1.2) \quad \bar{P}_a = \left\{ c \bar{W}_{a,t} + c \bar{W}_{a,t-1} (1+g)(1+\lambda) + c \bar{W}_{a,t-2} [(1+g)(1+\lambda)]^2 + \dots + c \bar{W}_{a,T} [(1+g)(1+\lambda)]^{T-1} \right\} \left[ \frac{1}{\bar{\rho}_a} \right]$$

Similar reasoning can be applied to all cohorts of pensioners at any time  $t$ , the between cohorts being the life expectancy factor used in computing their annuities. The average pension for all pensioners together is the sum of pensions of all cohorts,  $a$ , weighted by the cohort's relative share in the entire population of pensioners, which is,  $\bar{P}$ , can thus be expressed in terms of the current average wage of all wage earners in  $t$ .  $\bar{W}$  is derived in a similar fashion. Expression (A1.2), for *all* pensioners at a specific time is, thus:

$$(A1.3) \quad \bar{P} = c \frac{\bar{T}}{\bar{\rho}} \bar{W}_t (1+\lambda)^{\bar{T}-1}$$

Without labor force growth (or with labor force growth, but with solely wage indexation) (A1.2) simplifies further to:

$$(A1.4) \quad \bar{P} = c \frac{\bar{T}}{\bar{\rho}} \bar{W}_t$$



Where  $L_t$  denotes all workers and  $R_t$  denotes retirees at any time,  $t$ , the discrete-time version of expression (11), the financial identity for the entire pension scheme, is:

$$(A1.5) \quad c = \frac{\bar{P}_t}{\bar{W}_t} \frac{R_t}{L_t}.$$

$R_t/L_t$  is simply the system dependency ratio. Without labor force growth, substituting (A1.4) into (A1.5) and making use of the fact that *ex post*,  $W_t = (1+g)W_{t-1}$ , gives:

$$(A1.6) \quad c = \frac{c \frac{\bar{T}}{\bar{W}_{t-1}} (1+g) R_t}{\frac{\rho}{\bar{W}_{t-1}} (1+g) L_t}$$

and with labor force growth  $L_t = L_{-T} (1+\lambda)^{\bar{T}-1}$  and indexation with both  $g$  and  $\lambda$ , seen *ex post* over  $T$  periods is:

$$(A1.7) \quad c = \frac{c \frac{\bar{T}}{\bar{W}_{t-1}} (1+g) (1-\lambda)^{\bar{T}-1} R_t}{\frac{\rho}{\bar{W}_{t-1}} (1+g) (1+\lambda)^{\bar{T}-1} L_{-T}}$$

*Ex post* it is always possible to calculate values for  $g$  and  $\lambda$  that have generated the development of average earnings and the labor force during the period in which rights have been acquired.

We can rewrite the expression for individual benefits, (A1.4) as:

$$(A1.8) \quad \frac{\bar{P}_t}{\bar{W}_t} = c \frac{\bar{T}}{\rho}.$$

And the macro financing condition for a pay-as-you-go scheme (A1.5) as:

$$(A1.9) \quad \frac{\bar{P}_t}{\bar{W}_t} = c \frac{L_t}{R_t}$$

Which together give:

$$(A1.10) \quad \frac{\bar{T}_t}{\rho_t} \frac{R_t}{L_t} = 1.$$

Substitution of this into expressions (A1.6) and (A1.7) shows there is exact equivalence in equilibrium between the macro contribution rate on the left hand side, which reflects the ratio of total system expenditures to total system revenues, and the contribution rate used in computing

individual pension accounts on the right hand side. Clearly, with non-zero labor force changes maintenance of equilibrium requires indexation with both  $g$  and  $\lambda$ , as is shown by (A1.7b). In (A1.7) both the labor force and the average pension are  $(1 + \lambda)^{\bar{T}-1}$  greater, if  $\lambda > 0$ , and  $(1 + \lambda)^{\bar{T}-1}$  smaller if  $\lambda < 0$ , than in (A1.6).

## **Appendix 2. Equilibrium Relationship between the Ratio of the Average Number of Years of Work and Retirement and the System Dependency Ratio**

There is a less obvious steady state equilibrium result inherent in (11), which we demonstrate here. The system dependency ratio in (11) has a counterpart expressed in years of work and years of retirement. In expression (A1.3) above  $\bar{T}(a)_t / \bar{\rho}(a)_t$  is the ratio of the average number of years of work to the average number of years of retirement for cohort  $a$ .  $\bar{W}(a)_t$  is the cohort's average wage and  $c$  is the fixed contribution rate. The average pension for all pensioners in period  $t$  is a weighted sum of the pensions of all cohorts in  $t$ , where the weights are the number of pensioners in cohort relative to the total pensioner population and the average wage is the weighted sum of the wages of all cohorts in period  $t$ , where the weights are the number of workers in a cohort relative to the total number of workers. This gives the macro replacement rate  $\bar{P}_t / \bar{W}_t$ . Likewise,  $\bar{T}_t / \bar{\rho}_t$  is the weighted macro ratio of years of work to years in retirement.

It follows then that in steady state the ratio of years of work to years of retirement for the whole (covered) population under consideration is the system replacement rate. Hence, in any period  $t$ , with macro replacement rate  $\bar{P}_t / \bar{W}_t$  (11) reduces to the equality

$$(A2.1) L_t / R_t = \bar{T}_t / \bar{\rho}_t$$

which is an equilibrium characteristic of NDC. If we were to simply “drop” an NDC pension scheme onto a steady state population we would observe then that (A2.1) holds.

A second characteristic of equilibrium is that for a given contribution rate  $c$  and wage  $\bar{W}$ , the country's average pension is determined by aggregate outcomes for years of work  $T$ , *i.e.* the average of all individuals' *actual lifetime work careers*, and years of average retirement,  $\rho$ .

Given expression (12), in contemplating a suitable magnitude in establishing the fixed contribution rate for an NDC scheme, the policy maker would consider the average benefit an actual or targeted number of years of work and retirement for the mean (or median) worker

would yield.<sup>10</sup> Note that the dependency of an NDC benefit on lifelong earnings (expression (1)) and life expectancy at retirement (expression (3)) are key characteristics of NDC that distinguish it from defined benefit schemes.<sup>11</sup> In comparison, in a defined benefit scheme, individual benefits are determined by a separate benefit rule, which does not necessarily fulfill either of these properties.

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<sup>10</sup> For example, a replacement rate of 50 % with an actual or targeted relation between the average number of years of retirement to years of work of 18 years to 45 years is consistent with a contribution rate of 20 %.

<sup>11</sup> Note that a polar opposite scheme to NDC would be a DB scheme that provides a demogrant, based neither on years of work or years of residence, but simply on the fact that one is alive at a given age from which the benefit is granted.

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