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PUBLIC PROVISION OF PRIVATE GOODS AND NONDISTORTIONARY MARGINAL TAX RATES: SOME FURTHER RESULTS

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Public Provision of Private Goods and Nondistortionary Marginal Tax Rates: Some Further Results*

by

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Abstract
The incidence and efficiency losses of taxes have usually been analyzed in isolation from public expenditures. This negligence of the expenditure side may imply a serious misperception of the effects of marginal tax rates. The reason is that part of the marginal tax may in fact be a payment for publicly provided goods and reflects a cost that the consumers should bear in order to face the proper incentives. Hence, part of the marginal tax may serve the same role as a market price in the sense that it conveys information about a real social cost of working more hours.

We develop this idea formally by studying an optimal income tax model in combination with a type of public provision scheme not analyzed before; the provision level is individualized and positively associated with the individual’s labor supply. As examples we discuss child care, elderly care, primary education and health care. We show that there is a potential gain in efficiency where public provision of such services replaces market purchases. We also show that it is necessary for efficiency that, other things equal, marginal income tax rates are higher than in economies where the services are purchased in the market. This is because the optimal tax should be designed so as to face the taxpayers with the real cost of providing the services. Hence, it might very well be that economies with higher marginal tax rates have less severe distortions than economies with lower marginal tax rates.

Keywords: Nonlinear income taxation; Marginal income tax rates; Public provision of private goods; In-kind transfers

JEL classification: H21, H42, I38

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1. Introduction

Why does the Bumble Bee fly? It is a common saying that according to the laws of aerodynamics the Bumble Bee can’t fly. The wings are too small in relation to the weight of the body. Still it does fly! The parallel in economics might be: Why does the Swedish economy function? It has the highest marginal tax rates in the world, and, according to standard economic theory, the distortions would severely hamper the economy. Some would say that the Swedish economy ought to collapse because of the high taxes. Still, the Swedish economy functions very well and outperforms many economies nurturing substantially lower marginal tax rates.  

Clearly, if a version of aero-dynamic theory predicts that the Bumble Bee can’t fly, something important is missing in that theory. Likewise, if a version of economic theory predicts that high marginal taxes necessarily imply damaging distortions and poor economic performance, something is missing in that theory. What we will argue in this article is that if there is public provision of private goods, then, for reasons explained below, a significant part of the marginal income tax might be nondistortionary.

There is a long standing interest in quantifying the deadweight losses of taxation. Harberger’s work in the sixties laid the foundations for a first generation of empirical studies. A second generation of empirical work was inspired by Feldstein in the mid nineties. More recently, Prescott (2002, 2004) has argued that high (marginal) taxes severely inhibit the performance of an economy. The common view seems to be that marginal income taxes are purely distortive. However, as shown below, under certain conditions, a significant portion of the marginal income tax faced by individuals is nondistortive. This part of the marginal income tax should not enter the calculations when computing the deadweight loss of a tax. We believe one important reason why the Swedish economy performs so well although it has very high marginal tax rates is that a significant portion of those tax rates is nondistortionary.

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4 The Bumble Bee image was originally used on March 10, 2000, by former Swedish Prime Minister Göran Persson in the Opening Address to the Extra Party Congress of the Social Democrat Party in Stockholm.
5 See for example Harberger (1962, 1964). In many cases the estimated welfare losses were surprisingly small.
6 See Feldstein (1995, 1999). Feldstein argued that previous studies had neglected many important margins that are distorted by taxes. By estimating how total taxable income reacts to changes in the marginal tax one would be able to capture distortions of all relevant margins. Feldstein’s own estimates indicated large welfare losses whereas many later studies arrived at estimates of the welfare loss that was larger than those obtained in pre-Feldstein studies, but considerably lower than the estimates obtained by Feldstein (Gruber and Saez, 2002, Saez, 2003, Kopczuk, 2005). See also Chetty (2008) for a recent re-assessment of the taxable income elasticity as the correct measure of excess burden in the presence of evasion and avoidance.
The reason why part of the marginal tax is nondistortive is that it is a payment for publicly provided goods/services and reflects a cost that the consumers should bear in order to face proper incentives. That is, marginal tax rates sometimes play the same role as prices in the sense that they convey information on resource costs. The part of the tax that reflects a real cost of working is nondistortionary.

Public provision of private goods is common in all developed countries and often is of the order of 20% of GDP. Previous contributions have usually considered public provision schemes that furnish each consumer with the same fixed quantity. In this paper we address another type of public provision scheme not analyzed before, although being empirically important. The provision level is individualized and positively associated with the individual’s hours of work. In section 6, where we discuss specific examples, we will argue that some important public provision schemes are of a form such that provision levels are individualized and positively related to hours of work.

There is a small, related literature addressing how taxes and public spending affect labor supply (Ragan, 2005, Rogerson, 2007). Rogerson uses a labor supply model with taxes and public expenditures to explain differences in market work across the US, Continental Europe and Scandinavia. He argues that differences in the spending patterns of governments can account for the large labor supply in Scandinavia in spite of high taxes. In Scandinavia a larger portion of public expenditures is devoted to provision of family services, child care, elderly care, or, in general, transfers that are conditional on working. While we share Rogerson’s emphasis on the need to consider how tax revenues are being spent, our concern is the extent to which marginal taxes are distortionary whereas Rogerson focuses on explaining labor supply. Ragan’s message is very similar to that of Rogerson but she uses more detailed data for a larger number of countries to show the combined effects of taxes and public spending on labor supply and welfare.

Inspired by the distinction originally made by Olson (1982) between “encompassing organizations” and “narrow distributional coalitions”, Summers et al. (1993) put forward a different explanation why labor taxes may be less distortionary and therefore higher in some countries, including Scandinavia. Their argument is that in these countries labor supply is to a larger extent determined collectively in settings where the decision makers internalize the

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8 We want to emphasize that what we study in this paper is public provision, i.e. publicly financed goods. Whether the goods are privately or publicly produced does not matter for our analysis.
labor supply effects on government revenue. While the argument may be of some interest, we believe that it tends to overstate the “corporatist” nature and underestimate the flexibility of the Scandinavian labor markets, but a further discussion is beyond the scope of the present paper.9

As a vehicle for our analysis we will use an extension of the Stern (1982) and Stiglitz (1982) two-type version of Mirrlees’ optimal income tax model (Mirrlees (1971)). A non-linear redistributive income tax is imposed under the assumption that knowledge of who is high-skilled and who is low-skilled is private information not available to the government. The tax schedule must then be designed subject to the self-selection constraint ensuring that a high-skilled person does not select an income point intended for a low-skilled person. If he were to, we would refer to his behavior as mimicking. If the high-skilled person were to mimic, he would obtain more leisure than the low-skilled person with the same income as, being more productive, the high-skilled person could earn the same income in less time. However, if some of the transfer is given in-kind, it will be of less value to the mimicker than to the genuine low-skilled type if the good being transferred is less beneficial to someone who has more leisure time. Shifting to a transfer in-kind may therefore make mimicking less appealing, and thus alleviate the self-selection constraint and enhance welfare. Given the particular type of provision scheme we study here, it will also be the case that the marginal tax should reflect the real social cost of additional hours of work. That is, part of the marginal tax serves the same role as a market price in the sense that it conveys information about a real social cost of working longer hours, but the tax is on balance more efficient as it also discourages mimicking.

Before introducing our main model we in section 2 present a simple, preliminary case without any heterogeneity in order to highlight the key role for taxes in our analysis. As our next step we set up the Mirrlees type tax model where, in order to obtain sharp results, we assume that the need for the publicly provided good is a strictly positive monotone function of hours of work. In section 3 we show how a strict Pareto improvement can be achieved by

9 The distinction between “encompassing organizations” and “narrow distributional coalitions” was used by Olson (1990) himself, together with that between “explicit” and “implicit” redistribution, to provide a key for the understanding of the success of the Swedish economy. According to him, by exploiting the rational ignorance of the typical citizen, narrow distributional coalitions, which usually represent well off people who would not have been able to persuade the electorate to give them a transfer on altruistic grounds, have an incentive to seek redistribution in implicit forms, namely in forms that bypass the public treasury (as for instance protectionist measures or restrictions on competition). For a variety of reasons it can be maintained that the distortions and social costs associated with implicit redistribution far exceed those associated with explicit redistribution and are especially detrimental for growth. Olson claimed that part of the success of the Swedish economy was due to the fact that, if compared with many other countries, the degree of implicit redistribution was relatively low.
supplementing the optimal tax with a publicly provided private good, and we characterize the optimal tax/public provision scheme, showing that the real social cost of providing the private good should be reflected in the individuals’ marginal tax rates. The model used in section 3 is purposely simple and highly stylized since it is meant to capture relevant common features characterizing important publicly provided services. For each such service one could build a more specific model, tailored to fit that particular service, that uses less restrictive assumptions than those made in section 3. To save space we only perform such an extension for one particular service, namely child care. This is done in section 4. Section 5 extends the model to deal with the fact that not all income responses are hours-related. In section six we discuss four services that we believe fit the assumptions of our model. Using Swedish data we also discuss the empirical importance of publicly provided private goods and nondistortionary taxes. Finally, section 7 concludes.

2. A Simple, Preliminary Case

The fundamental message of our paper is twofold. On one hand we claim that, in the presence of public provision of private goods, the distortionary part of a marginal tax rate does not necessarily coincide with its face value. On the other hand we also claim that economies with higher statutory marginal income tax rates might actually be less distortionary than economies with lower marginal tax rates. To illustrate the first claim, we will start by presenting the basics within a model which is stripped down to a bare minimum. There is a large population of identical individuals each of whom is a parent with a single child, and initially there is no public sector. Denote by \( w \) and \( h \) the wage rate and the working hours of the representative agent, respectively. We assume that the wage rate reflects the true productivity of the worker. Let \( p \) be the cost per hour of child care, and denote by \( C \) the consumption of the agent. The agent has preferences for consumption and labor expressed by the utility function \( u(C,h) \). According to the budget constraint of the agent \( C=wh-ph \). Along the budget line, \( dC/dh=w-p \). The net income obtained from an hour of work is the wage rate minus the cost of working, which is the price paid for child care. This is the net social income, and where the agent faces no taxes and buys child care in the market, the net private income is equal to the social one. There is no distortion. The agent will maximize utility by setting the marginal disbenefit from working equal to the net marginal income, and the demand for child care is determined by the hours of work.

Assume now that there is a government providing child care free of charge and satisfying any demand for child care required in order to work. The child care is financed by a
lump sum tax on each agent. The social gain from an hour of work is of course unaffected, as nothing happens to productivity, and the need for someone to look after the child while working remains the same. However, the parent will now behave as if child care is a free good. There is no child care fee, and, from the perspective of each single agent, the increase in the lump sum tax caused by that agent’s separate working decision is negligible, as all lump sum taxes will increase and each one only marginally. The private trade off will be based on \( \frac{dC}{dh} = w - w - p \), and there is a distortion. While as such the lump sum tax is nondistortionary, returning it as a subsidy will conceal the true cost of working and cause an upward distortion of labor supply.

Now suppose that rather than levying a lump sum tax, the government imposes an income tax. Denote by \( \tau \) the income tax rate. An individual’s budget constraint will be \( C = w(1 - \tau)h \) and the private trade off will be based on \( \frac{dC}{dh} = w(1 - \tau) \). There is a tax wedge between the social gain and the private gain equal to \( w - p - (w - w\tau) = w\tau - p \). This wedge will vanish when one sets \( \tau = p / w \), which also happens to be the tax rate required for fully funding the child care. Thus, funding the child care through an income tax is nondistortionary. In fact, it is a corrective tax that fully corrects for the distortion created by the free provision of child care. The income tax simply replaces the market price in facing the agent with the true social cost of working. Where a higher tax is imposed it is only the part of the tax exceeding the cost of child care which constitutes a tax wedge.

3. The Model – Social Efficiency and Implementation

We are now ready to set up the model we will use to illustrate both the desirability of public provision of private goods and the fact that taxes used to finance these goods are nondistortionary. We will build on the discrete type version of the Mirrlees model in the tradition of Stern (1982) and Stiglitz (1982).

Contrary to previous contributions considering public provision of private goods in an optimal taxation setting, the public provision scheme we address in this paper is a system where agents get as much as they want of the publicly-provided good. As we will notice later on, this feature of the provision system is of special importance when the economy is populated by more than two types of agents; then, the provision system that we consider here tends to outperform a provision system of the kind previously considered in the literature, namely a system where the public sector offers a minimum amount of the publicly provided good and allows people to top up with private purchases in the market. However, for the
The purpose of illustrating our results it is sufficient to consider here a model with just two types of individuals. Extension to any number of types is straightforward.

The two types of individuals have different skill levels reflected by exogenous productivities. In a market economy the productivities are interpreted as wage rates denoted $w^1$ and $w^2$, where $w^1 < w^2$. For simplicity we normalize the population size of each type to unity. We let $Y(=wh)$ denote the before tax labor income. Each agent chooses how much labor to supply and the corresponding consumption level, which also depends on the tax liability. There is a private commodity which is a candidate for public provision. The demand for this good, which we in the following will call the $x$-good, is strictly positively related to the hours of work, i.e. $x = f(h) = f(Y/w)$, $f'(h) > 0$. (The case considered in section 2 is the one where $f(h) = h = Y/w$.) An amount of $x$ has no value beyond $f(h)$. We will refer to this case as one of satiation or more accurately satiation conditional on labor supply.

The $x$-good does not enter the utility function directly. It is instead a commodity one must acquire in order to work. Hence, it entails a cost of working. The best example is probably child care as in the case considered in the previous section. We will discuss further examples in section 6.

All agents have identical preferences over hours of work and consumption; these are represented by the utility function $U(C,h)$, where $C$ is consumption net of expenditures on the $x$-good.

The labor supply of agents of type $i$ is expressed as $Y^i/w^i$. We denote the per unit resource cost of the $x$-good by $p$, which would be the price in a competitive market. The resource constraint of the economy is then $\sum_{i=1}^{2} \left( Y^i - pf(\frac{Y^i}{w^i}) - C^i \right) = 0$. We also make the usual assumption that the policy maker can observe $Y$ but not $w$ or $h$ separately, and we assume the standard single crossing property that, for any given point in $Y,C$-space, the indifference curve of a low ability type is steeper than that of a high ability type – a property usually referred to as agent monotonicity.

**Characterization of the social optimum**

We can now derive the socially efficient allocation subject to the social planner being information constrained. The important implication is that the allocation must be chosen

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10 The link $f(h)$ between $x$ and $h$ need not be a direct link. It may be that $x$ (say, health service) is determined by some characteristic $z$ (say, health) that in turn varies systematically with labor supply.
subject to the incentive compatibility constraint that a type 2 agent does not mimic a type 1 agent by choosing the $Y,C$-bundle intended for the latter. Denote by $U^{21}(C^i,Y^i)$ the utility of type 2 were he to mimic type 1. We adopt the standard procedure of maximizing the utility of type 1 subject to a minimum utility being assigned to type 2 and subject to the incentive compatibility and resource constraints. The Lagrange function of this optimization problem will take the form:

$$\Lambda = U^1(C^i,Y^i) + \lambda(U^2(C^2,Y^2) - \bar{U}^2) + \beta(U^2(C^2,Y^2) - U^{21}(C^i,Y^i))$$

$$+ \mu \sum_{i=1}^{2}(Y^i - pf(Y^i/w^i) - C^i).$$  \hspace{1cm} (1)$$

The first order conditions are derived in appendix 1. Invoking those results and denoting by $MRS$ the marginal rate of substitution $-U_y/U_C$, we obtain from (a10)

$$MRS^2 = 1 + \frac{p}{w^2} f'(\frac{Y^2}{w^2}),$$

whereas from (a9) we obtain

$$MRS^1 = \rho(MRS^{21} - MRS^1) + 1 + \frac{p}{w^1} f'(\frac{Y^1}{w^1}) \leq 1 + \frac{p}{w^1} f'(\frac{Y^1}{w^1}),$$

where $\rho = \beta U^{21}_C/\mu > 0$ and the inequality follows from the agent monotonicity assumption ($MRS^{21} < MRS^1$).

Considering an arbitrary individual and omitting superscripts, we can write the consumption generated by labor effort $h$ as $C = wh - pf(h)$. We can then interpret $dC/dh = -pf'(h)$ as the net marginal product of labor or the marginal rate of transformation. In $Y,C$-space it would read $dC/dY = 1 - (p/w)f'(Y/w)$. The expression $-U_y/U_C = 1 - (p/w)f'(Y/w)$ gives the condition that the marginal rate of substitution between income and consumption be equated to the corresponding marginal rate of transformation. We see that this first best efficiency condition holds for the high-skilled type but is violated for the low-skilled owing to the information constraint. However, if the incentive compatibility constraint does not bind, the efficiency condition for the low-skilled (eq. (3)) will reduce to the first best efficiency condition and have the same form as for the high-skilled.

We will proceed to show that the constrained social efficiency can be implemented by a tax-public provision scheme which Pareto dominates a regime where a nonlinear income tax
is deployed but where the work complement $x$ is acquired in a market free of any government intervention.

*The tax-public provision optimum*

The regime where the government designs a nonlinear income tax and provides the $x$-good free of charge can be modelled by assuming that the government offers a menu of bundles $Y^i, C^i$ where the income tax is implicitly defined as $T(Y^i) = Y^i - C^i$. The public provision implies that an agent always gets the amount $f(h)$ if supplying $h$ units of labor. Where satiation prevails the $x$-good can simply be allocated according to need as expressed by the agents themselves. The satiation case is a “pure” case which yields clear-cut results. In section 4 below we will also discuss cases where the demand for the $x$-good is less strictly related to labor supply.

The objective function, the minimum utility requirement for the high-skilled and the incentive compatibility constraint are the same as for the social efficiency problem. Assuming the income tax is purely redistributive and raises no revenue beyond the funding of the $x$-good, the government budget constraint is $\sum_{i=1}^{2} (Y^i - C^i - pf(Y^i / w^i)) = 0$, which is identical to the resource constraint in the social efficiency problem. Hence, the optimum tax–public provision problem is identical to the social efficiency problem. The same conditions must hold and can be further interpreted in terms of marginal tax rates.

However, before doing this, it is useful to consider public provision more closely. The general intuition underlying the welfare-enhancing effect of public provision is that it allows the policy maker to repackage the consumption bundle for the low-skilled in such a form that it leaves the utility of the low-skilled unaffected but it does hurt a high-skilled if he were to choose the income point intended for the low-skilled. With respect to our problem, let’s see then how a tax-transfer regime with income-tax-financed public provision of the $x$-good Pareto dominates the optimum that can be achieved by a tax-transfer scheme, without public provision, where agents privately purchase the work-complement in the market. Notice first that, since there is satiation (conditional on labor supply), the public sector can offer any amount free of charge. Conditional on his labor supply, each person will then choose the amount that he needs. The actual demand for the $x$-good is given by $x^i = f\left(\frac{Y^i}{w^i}\right)$, for $i=1,2$, whereas a mimicker would demand $f\left(\frac{Y^i}{w^2}\right)$. It is evident that $\frac{Y^i}{w^2} < \frac{Y^i}{w^1}$ as

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11 Without satiation, at a reasonable level, it will not be possible to offer any amount free of charge as each agent would then expand his consumption beyond any reasonable limit unless some private disutility (time cost etc.) is incurred in order to consume the publicly provided good.
the low-skilled person has a lower wage rate than a mimicker. This simply means that a mimicker, being more productive, would earn the same income in less time and hence demand less of the x-good. Thus, starting from an optimum with a binding self-selection constraint and without public provision, if we let the individuals get the amount of x they want and decrease their after-tax incomes by \( pf(Y^i / w^i), \ i=1,2 \), the situation for both types is unchanged. However, a mimicker would be forced to pay, via taxes, for more of the x-good than he needs (the extra expenditure being equal to \( p\left[ f(Y^1 / w^1) - f(Y^2 / w^2) \right] \) and hence would suffer a utility loss, implying that the self-selection constraint no longer binds. This means that we can offer the low-skilled individuals less distorted consumption-leisure bundles where they work more and enjoy larger consumption. Hence, we can improve welfare for the low-skilled persons without hurting the high-skilled ones. Thus, a strict Pareto improvement is achieved by supplementing the optimal tax scheme with public provision of the x-good.

To elaborate on the income tax – public provision scheme, it is helpful to distinguish between a gross and a net tax concept, where the latter is defined net of transfers to the consumers in terms of x-good provision. The rationale is that an in-kind transfer can be perceived as a negative tax. We interpret \( T(Y) \) as the gross tax function and let \( \tau(Y) \) denote the tax net of the public provision of the x-good so that \( \tau(Y) = T(Y) - pf(Y/w) \). The corresponding marginal tax rates are \( T'(Y) \) and \( \tau'(Y) \), where \( \tau'(Y) = T'(Y) - (p/w) f'(Y/w) \).

Employing the usual measure of marginal tax rates in the Mirrlees-Stern-Stiglitz tradition, we

\[ \tau'(Y) = T'(Y) - (p/w) f'(Y/w) \]

Notice that a nonlinear commodity tax on the purchase of child care services would represent an alternative mechanism to let the mimicker pay more for the x-good. Both mechanisms rely heavily on government intervention in combination with consumer choices. We find the public-provision regime, as opposed to the nonlinear commodity tax regime to be of particular interest for two reasons. First, there may be a case for avoiding nonlinear commodity taxes which are conceivably more informational-demanding and complicated to enforce as they require a certain amount of monitoring and control from the tax collector. In practice, it seems that nonlinear commodity taxes are quite rare. Secondly, the public provision regime is one which exists to various degrees in Sweden and other countries, and we are interested in assessing this regime.

In a finite-class economy, when the government wishes to redistribute from the higher ability types to the lower ability types, an optimal allocation results in a so-called simple monotonic chain to the left (see Guesnerie and Seade, 1982), meaning that only downward adjacent self-selection constraints will be binding. Thus, if we had considered a model with \( k\geq2 \) differently skilled types of agents, the number of binding self-selection constraints would have been \( k-1 \). Ordering agents according to their wage from the lowest skilled type (with unitary wage rate \( w^1 \)) to the highest skilled type (with unitary wage rate \( w^k \)), all the binding self-selection constraints would have involved an agent of wage type \( w^i \) being tempted to mimic the allocation intended for an agent of wage type \( w^{i-1} \). The public provision system that we have analyzed would have then allowed mitigating all the \( k-1 \) binding self-selection constraints, implying that the redistributive power of the instrument is increasing in the number of skill types. Notice that this would not have been the case with a provision system that only makes available to agents a minimum level of the publicly-provided good and allows people to top up with private purchases in the market.
can define $T'(Y)$ as $1+U_y/U_c=1-MRS$. As observed from the optimality condition (2), the marginal gross tax rate for the high-skilled becomes:

$$T'(Y) = (p/w^2)f'(Y^2/w^2) > 0,$$

(4)

whereas the marginal net tax rate becomes:

$$\tau'(Y) = T'(Y) - (p/w^2)f'(Y^2/w^2) = 0.$$

(5)

That is, the marginal income tax should not be zero but should be equal to the social marginal cost of providing the $x$-good when an additional unit of gross income is earned. The implication is that type 2 agents face the same marginal price as in a situation with no public provision of $x$. The rationale for this result is exactly the one presented in section 2 above. Even if true that the individual obtains the $x$-good “for free” from the public sector, it is still the case that the individual acts as if he were facing the real cost of purchasing the $x$-good. He simply pays for it via the tax bill. Hence, the optimal tax/public provision scheme faces the high-skilled individual, locally at the individual’s optimum point, with exactly the same budget constraint as in the system where the $x$-good is bought in the market.14

Turning our attention to the agents of type 1, we can see that the consumption-leisure bundle of the low-skilled agents must be distorted in order to prevent these agents from being mimicked. The marginal income tax of type 1 is:

$$T'(Y^1) = \rho(MRS^1 - MRS^{21}) + (p/w^1)f'(Y^1/w^1)$$

(6)

and the marginal tax net of the cost of the $x$-good is:

$$\tau'(Y^1) = T'(Y^1) - (p/w^1)f'(Y^1/w^1) = \rho(MRS^1 - MRS^{21}) > 0.$$

(7)

We note that the marginal gross tax for the low-skilled is made up of two terms – one reflecting the social marginal cost of the $x$-good and the other being a distortionary term required to deter mimicking. The part that reflects the social marginal cost is corrective and

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14 As it often happens in two-type optimal taxation models we get the result that the labor supply of the high-skilled agents is undistorted. In this respect, the high Scandinavian tax rates at the top of the income distribution may appear to be at odds with the result derived from our model where the tax rate on the top person is solely reflecting the cost of the work-related publicly provided good. Without necessary claiming that actual tax rates are set optimally in accordance with our or a similar model, a few remarks are in order. We would like to play down the significance of the specific top-person result which may easily be exaggerated because of the simplifications that we have made by considering only a small finite number of types. From the standard Mirrlees optimum tax theory with a continuum of individuals we know that normally (and abstracting from public provision) it is only at the very top of the distribution that the marginal tax is zero and that the marginal tax may indeed be quite large very close to the top (see e.g. Tuomala (1984, 2008), Saez (2001) and, albeit in a slightly different setting, Varian (1980)). The zero marginal tax result would then apply only to a tiny fraction of the population and its role would appear more modest that in the two type model where all high-skilled individuals are at the very top since by assumption there is only one high-skilled type of agent.
nondistortionary, and serves the same role as a market price as it conveys information about the cost of working an additional hour. This is a crucial insight. Just taking the marginal tax rates at face value, one is easily led to exaggerate the distortionary effect as one may easily overlook that part of the marginal tax is indeed a payment for a true social cost. Only the self-selection term, which appears on its own in the net marginal tax, is truly distortionary. It follows from the expression for the net tax rate that the labor supply of the low-skilled agents is distorted downwards. However, it is important to realize that the distortion is smaller than it would be in an optimal taxation setting without public provision, where individuals would buy the $x$-good in the market. The reason is that the public provision scheme, relaxing the binding self-selection constraint, opens the way for the government to achieve a Pareto improvement upon the optimum without public provision. This in turn allows the government to offer agents less distorted bundles. The introduction of a public provision scheme can then be interpreted as having a twofold effect on the equilibrium marginal tax rates. On one hand, as required by an efficiency argument, it will clearly tend to raise the marginal tax rates: this is the effect of the corrective, nondistortionary component which serves to induce agents to internalize the real resource cost of the publicly provided work-complement. On the other hand, due to the beneficial effect on the binding self-selection constraint, the introduction of a public provision scheme will tend to decrease the marginal tax rates since it allows lowering the distortionary component needed to deter mimicking behavior. In any case, and this is especially important when making cross-country comparisons of tax induced distortions, even if the net effect of the introduction of a public provision scheme will arguably be to raise the statutory marginal tax rates, it might well be the case that distortions are less severe.

4. Child Care for Work and Leisure Activities
To obtain stark results the model in section 3 was purposely simple and highly stylized. However, if we specialize the model to a particular kind of service it is easy to generalize the model in other respects. Here, we do so for child care, which represents one important application of our model. We generalize the model to include demand for child care for leisure activities besides work. As we will see, the major result that part of the marginal income tax reflects the social cost of providing the $x$-good, and is therefore nondistortionary, still goes through.

Let the utility function be $U(C,l_g,l_k)$, where $C$ denotes consumption (of market goods), and $l_k$ is time spent together with the kid. The remaining leisure time can be spent
doing various things. As a shorthand, we will in the following call it “golfing” and denote it by \( l_g \). Since the child must be looked after all the time either by the parent or by professional child carers the number of hours provided by the latter must equal the time that the parent devotes to work (\( h \)) or golfing (\( l_g \)): \( x = h + l_g \). We will assume all goods in the utility function to be normal.

Public provision and taxes

We can make two alternative informational assumptions. If we assume that child care centers can observe whether parents use the free child care for golfing or for work we can design the provision system such that free child care is provided exclusively for hours of work.\(^{15}\) Under such a system we obtain results very similar to those in section 3; the formulas would be slightly simpler as the \( f(\cdot) \) function has the form \( x = h \) and \( f' = 1 \). Also the distortion introduced by free child care for hours of work would be fully corrected by the term \( p / w \) in the expressions for the marginal income tax rates. Since the analysis and results are so close to the case covered in section 3 we do not give the details of this analysis here. Interested readers can find the analysis in Blomquist et al. (2008).

If golfing hours cannot be observed and the public provision scheme is designed so that also child care needed for golfing is provided for free, the analysis is a bit different. Treating \( C \) and \( Y \) as given and denoting the time endowment by \( \theta \), the first stage of the individual’s optimization problem can be written as \( \max_{l_g} U(C, l_g, \theta - l_g - Y / w) \). From the first order condition, \( U_{l_g} = U_{l_g} \), we can derive a conditional demand function \( l_g(C, Y, w) \).

Substituting it into the conditional direct utility function we get

\[
V^i(C, Y) = U(C, l_g(C, Y, w'), \theta - l_g(C, Y, w') - Y / w') .
\] (8)

In the second stage the individual maximizes utility subject to the constraint \( C = Y - T(Y) \) and we obtain, as usual, \( T'(Y) = 1 + V_{l_g} / V_{C} = 1 - MRS_i \).

The government’s optimal tax problem is the same as the one in eq. (1) above except that the resource constraint is different, and the problem can be expressed by means of the Lagrange function:

\(^{15}\) This is basically the type of system in force in Sweden. In Sweden parents get free child care for work and certain type of studies but they are not allowed to use it for leisure activities. Before getting access to free child care parents sign a contract where they promise to only use child care services for the stated purposes.
\[ V^i(C^i,Y^i) + \lambda \left( V^2(C^2,Y^2) - I^2 \right) + \beta \left( V^2(C^2,Y^2) - V^2(C^1,Y^1) \right) + \mu \left( Y^i - C^i + Y^2 - C^2 - p \left( \frac{Y^i}{w^i} + l^1_g + \frac{Y^2}{w^2} + l^2_g \right) \right). \]

\[ C^i, Y^i, C^2, Y^2 \] are chosen so as to maximize the utility of the low-skilled subject to a minimum utility being assigned to the high-skilled, and subject to the asymmetric information-induced self-selection constraint and resource constraint.\(^{16}\) To economize on analysis we here only study the extent to which the income tax distorts the allocation of the high-skilled agent. The conditions for the low-skilled (agent of type 1) would be similar but also include a self-selection term.

The first order conditions with respect to the relevant variables are

\[ (\lambda + \beta) V^2 + \mu \left[ 1 - p \left( \frac{1}{w^i} + \frac{\partial l^2_g}{\partial Y^2} \right) \right] = 0 \quad \text{and} \quad (\lambda + \beta) V^2 - \mu \left[ 1 + p \frac{\partial l^2_g}{\partial C^2} \right] = 0. \]

Introducing the notation \( \left( \frac{dl^2_g}{dy^2} \right)_{dy^2=0} = \frac{\partial l^2_g}{\partial Y^2} + MRS^2 \left( \frac{\partial l^2_g}{\partial C^2} \right) \), these conditions imply that the marginal income tax can be written as

\[ T(Y^2) = 1 + \frac{V^2}{V^2_c} \left[ 1 - p \left( \frac{1}{w^2} + \left( \frac{dl^2_g}{dy^2} \right)_{dy^2=0} \right) \right] \quad \text{and} \quad \left( \frac{dx^2}{dy^2} \right)_{dy^2=0} = p, \quad (9) \]

where we have made use of the identity \( x = h + l^1_g \).

Eq. (9) illustrates once again the general principle according to which pre-existing distortions should be taken into account when judging how distortive an income tax is.\(^{17}\) In this case a pre-existing distortion is associated with the free provision of child care to make time available for work and golfing. That in itself distorts the individual’s choice between work, golfing and consumption, so that the individual is over-incentivized to work and golf. If the marginal income tax were set to zero, the distortion stemming from the free provision of child care would prevail. However, if the marginal income tax is set according to eq. (9), the income tax partially corrects for the distortion caused by the free provision of child care. This

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\(^{16}\) A standard assumption of optimal tax theory is that the tax authority does not know individual wage rates (skill levels). Notice that in our model this asymmetric information problem could be overcome if child care centers reported individual information on hours spent by the children in day care to the tax authority. In practice, however, there is no such reporting. In Sweden, for instance, child care centers presently do not record this type of information. However, even if they were, a number of issues would be involved. One is whether the tax authorities would have, or should have, the legal right to access the information of publicly and conceivably privately-run kindergartens. Principles of privacy are obviously at stake. Another issue is whether the information, even if available, would be considered verifiable in court. A third issue is that once this information were available to the tax authority, parents would have an incentive to cut back their use of child care or resort to black market child care. Finally, information would, of course, only be available at a cost.

\(^{17}\) See Kaplow (1998).
is similar to the role of the marginal resource cost term in eqs. (4) and (6). However, there is an important difference between the case studied in section 3 and the one represented by eq. (9). In the circumstances studied in section 3 the income tax fully corrects for the distortion created by the free provision of child care. In the context of eq. (9), instead, the income tax only partially correct for the distortions caused by the free provision of child care. One way to understand this is to recognize that there are several margins that are affected by the free provision, but only one instrument, i.e. part of the marginal income tax, available for correcting the distortions. We elaborate on these features below.

Suppose that the increase in $h$ is matched by a reduction in $l_g$, leaving $l_k$ unaffected. Then, since there is no pre-existing distortion due to the public provision of child care in the household’s choice between $l_g$ and $h$, the marginal tax on labor income should be equal to zero in order to be nondistortionary. This is exactly what is prescribed by eq. (9) above, since in this case $(dl_g^2 / dY^2)_{dY^2=0} = -1 / w^2$ where $w^2$ is the opportunity cost of golfing.

Suppose instead that the increase in $h$ is realised through a reduction in $l_k$, leaving $l_g$ unaffected. In this case, since due to the public provision of child care there is a pre-existing distortion in the household’s choice between $l_k$ and $h$ inducing over-supply of labor, the marginal tax on labor income should equal $p / w^2$ in order to be nondistortionary. This would imply that the household’s after tax marginal rate of substitution between $l_k$ and $C$ is equated to the opportunity cost $w^2 - p$ as in the (no tax, no public provision) undistorted setting. Once again, this is exactly what is prescribed by eq. (9) above, since in this case $(dl_k^2 / dY^2)_{dY^2=0} = 0$.

More generally, if the increase in $h$ is accompanied by variation in both $l_g$ and $l_k$, the logic applied above requires that, to be nondistortionary, the marginal income tax should be equal to the resource cost of the child care services required by the way time for earning additional income is actually made available. This is precisely what is prescribed by eq. (9).

When child care is provided for free only for working hours the marginal income tax fully corrects for the distortion introduced by the free provision. Since the free provision helps mitigating the self-selection constraint, it is apparent that the introduction of a system of free child care provision financed by increased taxes yields a Pareto improvement upon the tax optimum without public provision. In the case where child care is for free also for golfing time, instead, the marginal income tax is not fully corrective. In that framework it is in the end
an empirical question whether the benefits from softening the self-selection constraints outweigh the cost in terms of remaining distortions.\textsuperscript{18} If use of the free child care for other purposes than work is considered to be a severe problem, alternative schemes might be preferable. One possibility would be to let agents pay a fee corresponding to a fraction $0 < \alpha < 1$ of the total cost of the service that they get. Another possibility would be to provide the service free of charge only up to a given fixed amount and then let agents top up the publicly provided ration in the market. This scheme could be implemented through a voucher system. Both these schemes have the disadvantage that they do not mitigate the self-selection constraints as well as the pure public provision system does. On the other hand they work to deter the abuse of child care for other uses than work. In the first case efficiency requires that in the expression for the marginal income tax rates faced by agents the nondistortionary term should be scaled down by $1 - \alpha$ to reflect the fact that a fraction $\alpha$ of the cost of the service is already paid by agents through a fee. In the second case only agents who in equilibrium are not topping up the publicly provided ration should have their marginal income tax rates raised to reflect the cost of the publicly provided good.\textsuperscript{19} It is worth pointing out, however, that our model with just a publicly provided good and one single marketed consumption good is likely to overstate the potential problem associated with the overconsumption of child care services. The reason is that in a model with a larger number of marketed goods it would be possible to exploit differentiated commodity taxation to counteract the tendency of agents to over-use child care services for purposes other than work. What would be required then is to tax relatively more (less) those goods which are Hicksian substitutes (complements) with uses of leisure time that do not involve the consumption of child care services.

We have seen that the result from section 3 that the marginal income tax for the high-skilled is nondistortive goes through also in the present more general framework. If we were

\textsuperscript{18} Notice that, when child care is provided for free also during golfing time, a necessary condition for public provision to be a welfare-enhancing policy instrument is $Y^i / w^i + I_g^i > Y^j / w^j + I_g^j$, namely that the sum of working time and golfing time is larger for the true low-skilled than for the mimicker. For any given allocation in the Y,B-space, a utility maximizing agent will satisfy the f.o.c. $U_i^l = U_i^h$; differentiating this condition gives $dl_i / dw = -Y(U_i^l - U_i^h) / \left( w_i \Delta \right)$, where $\Delta = U_i^l - 2U_i^h + U_i^h < 0$. Thus, a sufficient condition for $dl_i / dw > 0$ (implying $Y^i / w^i + I_g^i > Y^j / w^j + I_g^j$) is that $U_i^l \geq 0$. The available empirical literature seems to confirm that our necessary condition for the desirability of public provision is in fact satisfied. See Kimmel and Connelly (2007) and Guryan et al. (2008) for evidence about a positive wage elasticity for time spent with children and a negative wage elasticity for time spent on leisure.

\textsuperscript{19} We leave for future research the characterization of the optimal level of public provision when it is efficient to set a maximum level for the amount of the good that each agent can get. Here we limit ourselves to notice that, whenever it is optimal to set such a maximum level, it is also in general optimal, at least in models with several types of agents (and therefore several binding self-selection constraints), to fix this level beyond the amount demanded by some groups of agents.
to write out the corresponding equation for the low-skilled we would find as before that the marginal income tax consists of a distortive term, originating from the self-selection constraint, and a term that, as for the high-skilled, corrects (fully or partially) for the distortions associated with the free of charge provision of child care services.

To highlight the common structure of the various formulas that we have so far obtained, it is useful to define the function $R(x)$, which shows the resource cost of providing $x$. Earning additional income requires extra work effort and more of the work complement involving an increase in the resource cost $(dR/dx)(dx/dY)_{dV=0}$. What we in section 3 have labelled the net marginal tax rate, i.e. the marginal tax after taking into account the transfer in-kind, is of the same form as the marginal income tax in a pure income tax system. However, in the expression for the gross marginal income tax there is now the additional term $(dR/dx')(dx'/dY')_{dV=0}$. Here, as well as in the context of section 3, $dR/dx$ is simply equal to $p$. Using this notation and noticing that in the context of section 3 $(dx/dY)_{dV=0} = dx/dY = f'(Y/w)/w$, we can therefore rewrite eq. (4) as $T'(Y^2) = (dR/dx^2)(dx^2/dY^2)$ and eq. (6) as $T'(Y^1) = \text{self-selection term} + (dR/dx^1)(dx^1/dY^1)$.

*Quality matters*

So far we have neglected the quality dimension of child care services but, recognizing that quality is important, it is of interest to see what principles should govern the policy-maker’s choice where the quality of child care services is treated as endogenous. For this purpose we will consider a government being the sole provider of child care services and setting a uniform level of quality for the provided child care services.\textsuperscript{20} Allowing for quality changes, we write the agents’ utility function as $U(C,q,h)$, with $q$ denoting the quality of child care services.\textsuperscript{21} Assume that quality is a continuous variable and that the producer price of child care services depends on quality through the function $p(q)$, with $p'(q) > 0$. With the quality of child care services as an additional choice variable for the government, its optimization problem can be rewritten as follows:

$$\begin{align*}
\text{Max}_{C^1,Y^1,C^2,Y^2,q} & \quad V'(C^1, Y^1, q) \\
\text{s.t.} & \quad V^2(C^2, Y^2, q) \geq \bar{V}^2
\end{align*}$$

\textsuperscript{20} These features closely mirror the Swedish regime.

\textsuperscript{21} We neglect here for simplicity the possibility of different uses of leisure time. See footnote 22.
\[ V^2(C^2, Y^2, q) \geq V^{21}(C^i, Y^i, q) \]  
\[ \sum_{i=1}^{2} \left( Y^i - C^i - \left( Y' / w' \right) p(q) \right) \geq 0. \]

It is obvious that endogenous quality makes no difference to the characterization of the optimal income tax as it is immaterial for the tax structure whether we consider an optimal income tax for some exogenous quality (previously suppressed) or for the optimally chosen quality.

Being uniform, the quality may be considered as a public good the level of which should be set according to the optimality principles applying to public goods financed by a nonlinear income tax (see e.g. Boadway and Keen (1993)).

Manipulating the first order conditions of the government’s problem (see Appendix 2) and denoting by \( MRS_{qC}^i = (V_q^i / V_C^i) \) the marginal rate of substitution between quality and consumption for a type \( i \) agent, we obtain the following condition for the optimal level of \( q \):

\[ \sum_{i=1}^{2} MRS_{qC}^i = p'(q) \sum_{i=1}^{2} h_i + \frac{\beta V_{C}^{21}}{\mu} \left[ MRS_{qC}^{21} - MRS_{qC}^{1} \right]. \]

Social efficiency requires that the sum of agents’ marginal rates of substitution between quality of child care services and consumption be equated to the marginal aggregate resource cost of providing quality, corrected by the presence of a self-selection term. The latter term shows how the purpose of discouraging mimicking influences the choice of quality for publicly provided child care services. In particular, an upward (resp. downward) distortion on the quality level chosen by the policy maker will be warranted whenever the mimicker values child care quality less (resp. more) than the low-skilled type. From an overall efficiency perspective, setting a uniform \( q \) is on one hand efficiency-decreasing, since in general it prevents agents from equating their marginal rate of substitution between quality and consumption to the corresponding marginal rate of transformation, while on the other hand

\[ \text{If the agents’ utility function had instead been of the form } U(C, q, l, l_g) \text{ with the government unable to distinguish for public provision purposes between child care used during working hours and child care used during time spent golfing, the only difference for the formula characterizing the efficient level of } q \text{ would have been that the budget term on the right hand side became } p'(q) \sum_{i=1}^{2} \left( h_i + l_i \right) + p(q) \sum_{i=1}^{2} \left( \frac{\partial l_g^i}{\partial q} \right), \text{ with a “tilde” denoting compensated demand.} \]

\[ \text{If agents were free to optimize with respect to their preferred level of } q, \text{ a type } i \text{ agent would choose } q' \text{ such} \]
it is another instrument enabling the government to relax self-selection constraints and improve efficiency. From other perspectives, a uniform level of $q$ might also entail additional benefits. This would for instance happen if we introduce equality of opportunity for the children into the model and, in an interpretation of Tobin’s (1970) specific egalitarianism argument, assume that society exhibits aversion to inequality in the specific domain of quality level of child care services.

5. Sheltering

Traditionally hours of work has been the important margin studied in connection with income taxation. However, since the seminal work by Feldstein (1995, 1999) other margins like effort, occupational choice, sheltering etc. have come into focus. It can be of interest to see how our results are modified if we introduce one of these margins. An extension beyond the simplest model may have important implications for our results but adding several dimensions will presumably add more complexity than further insights. We have opted for an extension which recognizes that an important decision margin of an agent is how much income to shelter from taxation. Intuitively, it will still be true that the marginal resource cost of providing the $x$-good should be mirrored in the marginal income tax. This will raise the marginal tax and make it more profitable to shelter. Below we spell out the details of this. The technical details of the analysis are rather similar to what has been done in earlier sections and are therefore relegated to appendix 3.

As in section 3 the utility function is given by $U(C,h)$ and there is a need for a work-complement, the $x-$good, given by a function $f(h)$. Let $M$ denote taxable income which is equal to $Y-a$, where $a$ denotes the amount of income which is sheltered by the taxpayer. The cost of concealing income is modelled in a very simple way through the non-negative and strictly convex function $g(a)$, where $g(0)=0$ but where any deviation from zero will involve a cost.\footnote{The tax evasion literature has traditionally analyzed tax evasion as a decision under uncertainty where there is a certain probability that an evader may be detected and penalized. More recently a literature has emerged which addresses sheltering in a broader context capturing also legal avoidance, and where (privately) successful sheltering requires engaging in a costly activity; see for instance Mayshar (1991), Boadway et al. (1994), Slemrod (2001), Kopczuk (2001) and Chetty (2008). Our modelling choice mirrors this approach.} This means that $g'(a)<0$ for $a<0$ and $g'(a)>0$ for $a>0$ and the $g$-function has a kink at $a=0$ where it reaches its minimum. For any level of taxable income $M$ the

\[ MRS_{xc} = p'(q')h' \]

that the condition $MRS_{xc} = p'(q')h'$ is satisfied. The corresponding condition for the model with alternative uses of leisure time would be $MRS_{xL} = p'(q')(h'+l_e)$.\footnote{The tax evasion literature has traditionally analyzed tax evasion as a decision under uncertainty where there is a certain probability that an evader may be detected and penalized. More recently a literature has emerged which addresses sheltering in a broader context capturing also legal avoidance, and where (privately) successful sheltering requires engaging in a costly activity; see for instance Mayshar (1991), Boadway et al. (1994), Slemrod (2001), Kopczuk (2001) and Chetty (2008). Our modelling choice mirrors this approach.}
The corresponding amount of tax is denoted by $T(M)$. Define $B$ as $B = M - T(M)$. The usual asymmetric information assumption is that $w$ or $h$ cannot be observed separately. Further, in our model $Y = wh$ is not observable, but $M$ is. We model the income tax design as the choice of a menu of income points $B,M^i$ ($i=1,2$), where the taxes paid by the respective agents are given by $T^i = M^i - B^i$ and $T^2 = M^2 - B^2$.

From the discussion contained in section 3 we know that, in the context of our framework, a necessary condition for the desirability of public provision is that the labor supply of a low-skilled agent exceeds that of a high-skilled mimicker. In appendix 3 we give conditions under which the mimicker works less than the low-skilled, $h_{21} < h^1$. (These conditions depend on income and substitution effects and properties of the $g(\cdot)$ function.) The mimicker will then benefit less than the low-ability type from subsidized child care, and, as both incur the same tax burden to finance the subsidy, the mimicker is made worse off. The self-selection constraint is relaxed and there is scope for welfare improvement.

When there is no public provision we obtain the usual result that the high-skilled person is undistorted and faces a zero marginal income tax. We also find that the high-skilled do not shelter. The low-skilled faces a positive marginal tax and might or might not engage in sheltering.

With public provision the marginal tax for the high-skilled and low-skilled take on the same general form as in earlier sections. As before, let $R(x^i)$ be the resource cost of providing the $x^i$-good to individual $i$. Then the marginal tax rate for the high-skilled is $T'(M^2) = (dR/dx^2)(dx^2/dM^2)_{dV^1=0}$ and that for the low-skilled is $T'(M^1) = self\_selection\_term + (dR/dx^1)(dx^1/dM^1)_{dV^1=0}$. In the present context with a fixed price of $x$ the marginal resource cost can be written as $dR/dx^i = p_i$. The expression $(dx/dM)_{dV^1=0}$ can be written as $(dx/dM)_{dV^1=0} = f'(h)(dh/dM)_{dV^1=0}$ or alternatively, since $M = wh - a$, as $(dx/dM)_{dV^1=0} = f'(h)[1 + (da/dM)_{dV^1=0}]/w$.

For the high-skilled there is the possibility that the optimum is at the kink of the $g(\cdot)$ function and $(da/dM)_{dV^1=0} = 0$. Then the formula for the high-skilled reduces to $T'(M^2) = (dR/dx^2)(dx^2/dM^2)_{dV^2=0} = pf'(Y^2/w^2)/w^2$.

We see that the expression for the marginal tax has the same form as in the previous cases considered above. Adding sheltering to the model does not change the basic form of the
expression for the marginal income tax and it is still true that the marginal resource cost of providing the $x$-good should be mirrored in the marginal income tax. This part of the marginal income tax is not distortive, but corrective.

Marginal income taxes will be higher when there is public provision. This might increase the amount of sheltering taking place. Since sheltering involves a social cost, but as far as we see no social benefits, sheltering is a wasteful activity. Hence, the benefits of public provision are decreased. It is an open question whether the benefits of public provision, slackening self-selection constraints, outweigh the distortion that is introduced. As is often the case, there are pros and cons of the policy undertaken. It is the task for empirical research to establish whether the net benefits of public provision are positive or not.25

6. Further Examples and Empirics

What has been described above is a normative model. We do not claim that actual taxes, like those in Sweden, are set in accordance with this model. Hence, we do not believe it is fruitful to use the model to attempt to explain why tax structures differ across countries or why there are high marginal tax rates on high incomes in Sweden and some other countries.26 However, the results of our model are useful when trying to assess how distortive tax systems are. As discussed in the introduction, the mechanisms being highlighted may help explaining how high marginal taxes and a good economic performance can be reconciled and therefore may also help explaining how large marginal tax rates can be sustained. Two results of our analysis are of interest. First, if there is public provision of a work-related service/good, then this can help to mitigate self-selection constraints that hamper redistribution. This implies that redistribution can be accomplished with less distortive taxes than otherwise. Second, public funding of work-related costs is in and by itself a subsidy to labor causing an upward distortion of labor supply. This effect can be offset by an increase in the marginal income tax. This part of the income tax is not distortive, but corrective.

25 It might be reasonable to assume that of the private sheltering cost $g(a)$ only a fraction $0 < 1 - \alpha < 1$ is a true social cost, while the remaining part is not since it accrues to the government as some form of revenue. Conceivable examples are fines or taxes on the resources expended on sheltering activities. As the value of $\alpha$ increases, so does the likelihood that the net benefits of public provision are positive. The distinction between private and social cost of sheltering is immaterial for individual behavior but becomes relevant when writing the government’s budget constraint since on the revenue side we would then also have the term $\alpha \sum_{i=1}^{2} g(a')$.

26 Even assuming that taxes are indeed set in accordance with the Mirrlees model extended to accommodate public provision, the marginal tax rates will be governed both by public provision and a host of other circumstances including distributional preferences, the skill distribution, revenue requirement, labor supply elasticities, etc. Whether there is public provision or not may to some extent explain marginal tax differences between countries but cannot fully account for large discrepancies which would also have to be attributed to differences in distributional preferences or other country-specific factors.
In this section we will argue that child care, elderly care, health care and primary education, to various extents, fit the assumptions of our model and that in economies where these goods are publicly provided part of the marginal income taxes are nondistortive. Since Sweden probably has more public provision of private goods than any other country we present some statistics for the Swedish economy. In Sweden public expenditures on child care amount to 2.1% of GDP, public expenditures on elderly care and care of persons with functional impairments to 4.6%, public expenditures on health care to 6.2% and public expenditures on primary and secondary education (up to 9th grade) to 2.9%. Together these expenditures amount to close to 16% of GDP. For all these expenditures there is the potential that they are provided in such a form that they imply that part of the marginal taxes individuals face are nondistortionary.

6.1 Child care

Various demographic groups are affected in quite different ways by publicly provided services. The expenditures that are easiest to couple to a specific group of people are child care expenses. Largely it is the mothers of children in ages 1 to 6 that are affected. Even though this is not the quantitatively most important expenditure category, we still single out these expenditures for a detailed analysis since it is fairly easy to figure out the effects of child care. We focus on women in the age group 25-45. The vast majority of women with children in ages 1-6 are in this age group. Of women in this age group 35% have at least one child in child care ages and are hence potential users of child care. To be more specific: 24% have one child in child care ages, 9.7% have two children and 0.8% have three children. Very few have four or more. The labor force participation of women in this age group is around 82%.

Table 1 in the next page shows the wage distribution for women with one, two or three children.

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27 These numbers are based on statistics in “Public Finances in Sweden, 2007”, published by Statistics Sweden.
28 Table 1 builds on register data from Statistics Sweden. Information from three data sets, “flergenerationsregistret”, “Louise-databasen” and “lönestrukturstatistikken” has been combined. The original data covers all women working in the public sector and in large companies but not in small companies. The original data reports standard monthly salaries after pay roll taxes. To obtain the wage rates before payroll taxes we multiply by 1.3246. To get to hourly wage rates we divide by 175, which is the standard monthly hours of work. Since there are very few women with four or more children in ages 1-6 we do not include the wage distribution for this group.
We see that the median wage is fairly similar irrespective of the number of children in child care ages. However, the dispersion in wage rates is larger for those with three children than for the other two categories. Still the distributions are fairly similar across number of children.

To get at the nondistortionary part of the marginal tax we must divide the producer price of child care by the wage rates in the table. Our estimate of this price for year 2005 is SEK 70.2.\textsuperscript{29} To have a simple model we in section 4 only considered the case where each individual had just one child. However, the idea is easily extended to the case where an individual has \( n \) children. If a female has \( n \) children the real resource cost that should be mirrored in the marginal tax is \( np \) and \( np/w \) of that individual’s marginal tax rate is nondistortionary. In table 2 we show the distribution of \( np/w \) for the three categories of women.

\begin{table}[h]
\centering
\caption{Distribution of gross wage rates, 2005 SEK, women in ages 25–45.}
\begin{tabular}{|l|c|c|c|c|c|c|c|}
\hline
& \textbf{5\textsuperscript{th} per-centile} & \textbf{10\textsuperscript{th} per-centile} & \textbf{25\textsuperscript{th} per-centile} & \textbf{50\textsuperscript{th} per-centile} & \textbf{75\textsuperscript{th} per-centile} & \textbf{90\textsuperscript{th} per-centile} & \textbf{95\textsuperscript{th} per-centile} \\
\hline
One child & 118.94 & 125.05 & 138.23 & 155.37 & 184.92 & 239.94 & 287.63 \\
Two children & 119.37 & 126.47 & 140.20 & 158.95 & 188.47 & 247.89 & 292.35 \\
Three children & 116.43 & 123.69 & 137.20 & 156.23 & 190.36 & 262.37 & 302.77 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Distributions of \( np/w \), women in ages 25–45.}
\begin{tabular}{|l|c|c|c|c|c|c|c|}
\hline
& \textbf{5\textsuperscript{th} per-centile} & \textbf{10\textsuperscript{th} per-centile} & \textbf{25\textsuperscript{th} per-centile} & \textbf{50\textsuperscript{th} per-centile} & \textbf{75\textsuperscript{th} per-centile} & \textbf{90\textsuperscript{th} per-centile} & \textbf{95\textsuperscript{th} per-centile} \\
\hline
One child & 0.59 & 0.56 & 0.51 & 0.45 & 0.38 & 0.29 & 0.24 \\
Two children & 1.18 & 1.11 & 1.00 & 0.88 & 0.74 & 0.57 & 0.48 \\
Three children & 1.81 & 1.70 & 1.53 & 1.35 & 1.11 & 0.80 & 0.70 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{29} This figure is computed using information found on the homepage of “Skolverket”, the authority supervising the preschools in Sweden. The annual production cost per child, given a 40 hour week, was in 2005 SEK 141,500. Assuming a child on average spends 45 weeks per year in child care this gives an estimate of about SEK 78 per hour. However, parents pay about 10% of the child care costs themselves, which implies that the publicly paid part is about SEK 70.2.
To compute how large the distortionary marginal taxes are we also need to know the total marginal taxes the women are facing. We calculate these marginal taxes for women working full time. This should give an upward bias to the calculated distortionary marginal tax rates. For women working full time the marginal tax rate would be 49% for individuals at the 5th, 10th, 25th, 50th and 75th percentiles. For women with one child in child care ages and a wage at the 5th percentile this would imply that the distortionary part of the marginal tax rate is negative (0.49 – 0.59 = –0.10). It would be negative also for women at the 10th and 25th percentiles. At the median wage the distortionary part would be 4 percentage points and at the 75th percentile 11 percentage points. For women with a wage at the 90th or 95th percentiles the marginal tax is 59%. For a woman at the 90th percentile 29 percentage points are nondistortionary leaving the distortiory part at 30%. For a woman at the 95th percentile 24 percentage points are nondistortionary implying that the distortionary part is 35%.

For women with two children in child care ages the value of free child care is even larger. The combination of marginal tax rates and free child care implies that only at the 90th percentile is the net result of marginal tax rates and free child care a positive distortionary marginal tax rate amounting to 2%. At the 95th percentile the distortionary part of the marginal tax would be 11%. For those with three children (a very small group of women) the combined effect of marginal tax rates and free child care implies that not even at the 95th percentile is there a positive distortionary marginal tax.

For a substantial fraction of women in ages 25-45, those with children in child care ages, the fact that there is publicly provided child care implies that the distortionary part of the marginal tax on income in many cases is very low. A common view is that this is a group of the labor force that is particularly sensitive to economic incentives with a high wage elasticity. The fact that this group face low distortionary taxes should therefore be of large importance.

It can be of interest to compare the distortionary marginal tax rates for women with children in child care ages in Sweden with the marginal tax rate in an economy considered to have low distortionary taxes in comparison to Sweden. We take California as such an example. Our aim is to calculate the distortionary marginal tax rate for a median income woman among those with children in child care ages. However, it is hard to find data to accomplish this. As a proxy we use the median income for single parents in California who in 2004 filed as “head of household”. The median calculated in this way was $22,580. Using

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30 We calculate the marginal tax taking into account the local and “state” income tax, the pay-roll tax and commodity taxes. See appendix 4 for details.
31 The figure is calculated from data in the Internal Revenue Service 2004 Public Use Tax File. We are grateful
the NBER Taxsim calculator we find that the implied marginal income tax rate is around 43\%.

In California there are public subsidies to child care, and we next investigate if these subsidies are such that part of the marginal income tax is nondistortionary. In contrast to Sweden there is no universal public provision of child care in California, and the subsidies take a variety of forms. First, there are subsidies via the tax system. For privately bought child care one can obtain the Child and Dependent Care Tax Credit (CDCTC). This credit is not refundable, which means that it is not so beneficial for households with low income. The CDCTC is not used much. In California in 2004 only 13.7\% of households with children used it. In general the effect on the after tax price of child care is minor, and the subsidy does not significantly decrease the distortionary part of the income tax.

The Dependent Care Assistance Program (DCAP), which works as if the child care expenses were tax deductible, has a larger impact on the size of the distortionary marginal income tax. A maximum amount of $5,000 can be “deducted”. If actual expenses are larger the taxable income is decreased by $5,000. For a woman with the median income of $22,580, this would not change the marginal income tax. If instead the child care expenses were just below $5,000 the distortionary marginal income tax would decrease from 43\% to 35\% because of the DCAP. An individual can use CDCTC or DCAP, but not both. From the above we conclude that, although in some cases the subsidies to child care via the tax system lower the distortionary marginal income tax, the decrease is not large.

It is also of interest to study the marginal income tax for women filing jointly with a spouse. In 2004 the median wage income for those married, filing jointly and claiming at least one exemption for a child was $51,410. Assuming no capital income and deductions, this income implies a marginal income tax of 37\%. As for women filing as head of household, the effects of the CDCTC and the DCAP can be that part of this marginal tax is nondistortionary. However, the magnitudes of these effects are small.

Second, there is large variety of publicly financed programs providing child care targeted at low income families or children with special needs. In general, to be eligible for
child care from these programs a family’s adjusted monthly income must be at or below 75% of the state median income adjusted for family size. In 2004 around 20% of children in ages 1-5 obtained some form of such child care. Hence, for those women who earn low wages and have access to publicly financed child care it is possible that the combined effect of taxes and publicly financed child care imply that the distortionary part of the marginal income tax is quite small.

Our interpretation of the calculations above is that, because of the universal public provision of child care in Sweden, the distortionary marginal income taxes are in general lower for Swedish women with children in child care ages than what they are for Californian women with children in that age group.

6.2 Elderly care and care of functionally impaired

Elderly care and care of functionally impaired have strong similarities to child care. To make a concrete example, if a woman is responsible for the care of her elderly father and this care requires, say, 10 hours a day, free elderly care would affect this woman’s budget constraint in the same way as if she received free child care for a child. The implication would be that part of the marginal income tax she faces can be regarded as corrective and not distortionary.

In many countries it is the case that elderly persons are cared for by a near relative, like a daughter, daughter in law, son or a (younger) spouse. For example, in Sweden in the past, before the system of publicly provided elderly care was as common as it is nowadays, this was quite usual. Anecdotal evidence suggests that it has been fairly common in the US and that it still occurs. Bonsang (2007), using European data, studies the extent to which adult children spends time caring for their elderly parents. He finds that the time spent caring is

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36 The eligibility rules for programs overseen by the California Department of Education are given in the California Education Code. See the following California Department of Education home page: [http://www.cde.ca.gov/sp/cd/lr/documents/title5.doc](http://www.cde.ca.gov/sp/cd/lr/documents/title5.doc). Eligibility rules for programs overseen by the California Department of Social Services or the federal Department of Health and Human Services, Administration for Children & Families, are similar.

37 There are in California many different publicly financed programs that supply child care to low income families and it is hard to find information on the exact number of children that obtain child care via these programs. The figure we state in the text is based on information from three different sources. From California Department of Education, Child Development Division, CD-801A Monthly Child Care Report, January-December 2004 (archived data), we have the number of children in programs overseen by California Department of Education. The number of children in CALWORKS stage 1 programs are from a homepage operated by California Department of Social Services ([http://www.dss.ca.gov/research/res/pdf/CW%2020115/2004/CW115Jun04.pdf](http://www.dss.ca.gov/research/res/pdf/CW%2020115/2004/CW115Jun04.pdf)). Finally the number of children in the Head Start program were obtained from the California Head Start Association (Office of Head Start, Program Information Report for 2007-2008).The figure 20% is conceivably an overestimate as there might be some double counting because some child care centers get money both from California Department of Education and Head Start funds from the federal government.
much higher in countries with little publicly provided elderly care, like Spain, Greece and Italy as compared to countries with a system of public elderly care as in Sweden and Denmark. He also finds a strong negative correlation between market work and time spent caring for an elderly parent.\footnote{38}

Quantitatively, publicly provided elderly care and care of the functionally impaired is in Sweden much more important than child care; 4.6% of GDP versus 2.1%. However, when trying to identify a group which benefits from publicly provided elderly care in the sense that they are relieved from a caring duty, there is an important difference between child care and elderly care. All children in ages 1-6 need care. This means that it is easy to identify the group of persons (women) affected by free child care. However, not all elderly need care and some of those who need care have financial means to buy it.\footnote{39} Therefore, even if many persons (potential care givers) are affected by the free elderly care it is hard to empirically pin down this group, as can be done for those benefitting from child care. Presumably it would to a large extent be women in ages 50-65.

According to Statistics Sweden, in 2005 there were around 900,000 women in Sweden in ages 50-65. Statistics from “Socialstyrelsen” (Statistik, Socialtjänst 2006:3) show that there were around 235,000 persons 65 years or older that received some form of public elderly care; around 135,000 in their own homes and 100,000 in special homes for elderly in need of care. There were around 56,000 persons who received care because of functional impairment (Socialstyrelsen, Statistik, Socialtjänst 2006:2). That is, there were around 290,000 persons who received some form of public elderly care or care for the functionally impaired. If each one of these otherwise had been cared for by a close relative, like a daughter, it means that as much as around one third of the women in ages 50-65 might be affected by the publicly provided care. This is an upper estimate, and most likely unrealistically high. These figures are still an indication that public provision relieves quite a few individuals of the responsibility to care for an elderly relative or functionally impaired, and that part of the marginal taxes they face should be seen as nondistortionary. Public provision of elderly care and care of the functionally impaired in Sweden might well be one important reason why the labor force participation for women in ages 50-65, with the exception of Iceland, is the highest in the OECD area. According to OECD Employment Outlook 2005, Table C, in

\footnote{38 See Bonsang (2007), table 6.}
\footnote{39 However, it should be noted that some elderly who need care and have the financial means to buy care, still might be cared for by a daughter (or son). It might be financially more advantageous to care for the elderly parent than to let him/her buy care for himself/herself if this would increase the future inheritance (see e.g. Bernheim et al. (1985)).}
Sweden the labor force participation for women in ages 50-65 is around 70%. This makes Sweden a remarkable outlier. The average for OECD Europe is 33% and for US 56%.

6.3 Health care
Quantitatively, health expenditures are the most important category we discuss. It would therefore be of interest to try and couple these expenditures to individuals whose consumption of health care depends on their hours of work, or more generally labor income. If health insurance would be bought in the market the premium would presumably be related, among other things, both to hours of work, riskiness of work and health hazards like those facing miners. However, not only men in risky occupations would have part of their health care needs attributed to their work. Many cleaners, nurses and carers claim to get neck, back and arm problems because of their work. It is also fair to say that there is satiation in the sense that (most) individuals only want to be treated for their actual health problems. If a person has hurt his right knee, he wants that injury to be cured. He does not want his unhurt left knee or his eye to be operated, even if he would get it for free.40

We believe it is a daunting task, worthy of a separate paper, to empirically try and pin down the extent to which the marginal tax for some groups of men and women would be nondistortionary because of publicly provided health care.41 However, given the large quantitative importance of publicly provided health care, even if only a fraction, but presumably a significant fraction, is work-related, health care is a quantitatively important service that to some extent fits the assumptions of our model.42

6.4 Primary education
Primary education is yet another example that to some extent fits our model. Children in ages for primary education can get their education either as home schooling, education in a private school or in a public sector school. In the absence of publicly provided primary education parents would have to undertake home schooling or buy private education. In some countries like US and UK parents have a legal right to educate their children at home and this is a right

40 The aim of public health care is to furnish people with care and treatment according to need. This is obviously not a sharply defined concept. The waiting time for treatment may vary and one may choose quality levels with different probabilities of successful cure or prospects for speedy recovery. In practice it is the doctors that define what represents an adequate treatment, and satiation may be defined by the standards that are actually set.
41 It would be difficult to handle omitted variables bias. At a given point in time those with poor health status probably work less and consume more health care. What we would like to measure is how, for given health status, the consumption of health care varies with hours of work or labor income.
42 Gahvari (1995) made a similar point stressing the link between health care and labor supply.
that is used by some parents. Like for child care the demand for schooling for children would be an increasing function of the parents’ hours of work as parents would have less time for teaching their children when they work more. Also, most parents would still like that their children had time for activities such as playing, rest, and social activities besides being educated. Thus there would be satiation in the demand for hours of primary education.

Primary education has the necessary properties to make it suitable for the type of public provision studied in this paper. This means that one might conceivably use public provision of primary education as a screening device to mitigate the informational deficiencies encountered by the policy maker in the design of an optimal tax system and that the real resource cost of providing primary education should be mirrored in the marginal tax rates. However, in practice education policy is governed by other concerns. In most countries the provision system is not designed so that parents can choose how many hours per week they want to use the public school. Rather, there is a requirement that the children should go to either a private or a public sector school for a certain number of hours per week.

6.5 Other work related benefits

More generally, there will be beneficial effects on labor supply and less distortion where labor supply entitles the worker to benefits provided by the government. Examples are public pension schemes (old age pensions, disability pensions, etc.), unemployment benefits, subsidised maternity leave, and other welfare payments conditional on previous work-activity or earnings. Public pension schemes in particular are quantitatively important for instance in the Nordic countries and provide more or less perfect substitutes for private savings or insurance (against disability, longevity, etc.).

7. Conclusions and Discussion

It is well known since long that public provision of certain private goods can relax the self-selection constraints in force where an income tax redistributes income between high-ability and low-ability people subject to the asymmetric information assumption that the government does not observe individuals’ ability. The typical good suitable for this purpose is a work complement, like for instance child care. An important and novel insight from this paper is

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43 For US it is estimated that something like 2-4% of children in relevant ages are educated in home schooling and that the percentage is on the rise. For more information see http://nces.ed.gov/pubs2001/Homeschool/background.asp and http://www.reason.com/news/show/36591.html.

44 A caveat is that the benefits themselves may have moral hazard effects on the recipients, e.g. by discouraging unemployed people to search for jobs or upgrade their skills.
that there can be provision schemes where provision levels are individualized and tailored to each individual’s hours of work. It is essential for the social efficiency of such systems that the marginal social cost of providing the work complement is fully reflected in marginal income taxes. Otherwise the consumer would not allow for the full cost of working since the good is available free of charge.

To illustrate the basic message, we start by setting up a simple model where the only margin affected by the income tax is the labor-leisure choice. In this model we show that the introduction of a publicly provided private good, which is a work complement, unequivocally diminishes the distortions in the economy and that part of the marginal income tax is nondistortive. In fact, it is a corrective tax that fully amends the distortion caused by the free provision of the work-complement good. This corrective tax serves the same role as a market price in conveying information on the marginal resource cost of the publicly provided good. To add realism, we introduce other margins of choice. For example, child care can be used either for work or leisure activities, and taxable income can be determined both by hours of work and sheltering activities. For these extensions it is still true that the marginal resource cost of providing the work complement should be mirrored in the marginal income tax, and that this part of the tax is not distortionary but rather corrective. However, in contrast to the simplest case, the marginal tax is not fully corrective. One way to understand why is to recognize that there are several margins that are affected by the free provision of the private good, but only one instrument, i.e. part of the marginal income tax, available for correcting the distortions.

In the models with several margins of choice there are pros and cons of free provision. The free provision will mitigate the self-selection constraints. On the other hand the free provision also introduces distortions that cannot be fully corrected by the marginal income tax. Let’s consider free provision of child care, an example we discuss at length in the paper. If sheltering is a common phenomenon, and the free child care is widely used for leisure activities it is likely that the distortions outweigh the benefits from mitigating self-selection constraints. However, if sheltering is rare, and it is easy to control that the free child care is only used for work, benefits are likely to outweigh the distortions.

The fact that in our model all agents need the publicly provided good might raise the worry that, in a model with both users and non-users, the reduction in the distortions faced by users would be matched by a corresponding increase in the distortions faced by non-users. This objection is however less well-grounded than it might appear at first sight. On one hand, it would only be relevant if the policy maker could not achieve a separating equilibrium when
optimizing the shape of the income tax, meaning that some users and some non-users were pooled together at the same level of pre-tax income.\textsuperscript{45} Then, the distortion faced by the pooled non-users would likely be exacerbated since their marginal tax rate would also incorporate a term reflecting the marginal resource cost of a publicly provided service that they don’t use. On the other hand, from a social welfare point of view, whether the net effect of a combined increased distortion on the non-users and a decreased distortion on the users turns out being positive or negative will depend, among other things, on the labor supply elasticities of the pooled agents. Arguably, the net welfare effect will more likely be positive if the labor supply elasticity of the users is higher than that of the non-users since then we would be reallocating distortions in a more efficient way between the pooled agents.\textsuperscript{46}

In the last part of the paper we use Swedish data to illustrate the empirical importance of the phenomenon that we study. For some groups and some individuals the public provision implies that, even though the total marginal income tax is substantial, the distortionary part is very small. This is, for example, true for women with children in child care ages. Our calculations indicate that the distortionary part of the marginal income tax is substantially lower than, say, for Californian women with children in child care ages. Recognizing that this is a segment of the labor force that is believed to be particularly sensitive to economic incentives, facing this group with low distortionary taxes is potentially of large importance. Another major group affected by public provision is those who would be responsible for the care of close relatives were provided elderly care and care for functionally impaired not available. We believe this group would mainly be women in ages 50-65 – another group often judged to be quite responsive to economic incentives. Building on these examples, it seems as if an income-tax-financed public provision of some work-related goods contributes shifting distortions away from some of the agents with larger labor supply elasticity.

Let us end by emphasizing the major conclusions. Going from a situation where individuals buy the work-related service in the market to a regime where the good is publicly provided, total and marginal taxes will increase. However, part of the marginal taxes will be nondistortionary. Moreover, public provision is potentially welfare-enhancing since its screening power, softening the incentives to mimic, lessens the urgency of distorting the

\textsuperscript{45} Under a separating equilibrium, only the marginal tax rate on the allocations that are chosen by the users of publicly provided services would be incorporating a term reflecting the marginal resource cost of the publicly provided goods.

\textsuperscript{46} We explore in more details the issues raised by the presence of both users and non-users of the publicly provided good/service in a companion paper. See Bastani et al. (2009).
agents’ labor supply for self-selection purposes, and therefore allows decreasing the distortionary component of the marginal tax rates.

An implication is that if one compares the tax systems in two countries it may very well be that it is the country with higher marginal tax rates that has the less severe distortions. One cannot judge the distortions generated by a tax system in isolation but must also consider the expenditure side.

**Appendix 1**

From the Lagrange function (1) we can derive the following first order conditions:

\[
\begin{align*}
\Lambda_{c^1} &= U^1_c - \beta U^{21}_c - \mu = 0, \quad (a1) \\
\Lambda_{y^1} &= U^1_y - \beta U^{21}_y + \mu \left[ 1 - \frac{p}{w} f' \left( \frac{y^1}{w} \right) \right] = 0, \quad (a2) \\
\Lambda_{c^2} &= (\lambda + \beta) U^2_c - \mu = 0, \quad (a3) \\
\Lambda_{y^2} &= (\lambda + \beta) U^2_y + \mu \left[ 1 - \frac{p}{w^2} f' \left( \frac{y^2}{w^2} \right) \right] = 0. \quad (a4)
\end{align*}
\]

The first order conditions can be manipulated to obtain further economic insights. Solving (a1) and (a2) for \(-U^1_y\) and \(U^1_c\), and dividing, we obtain

\[
\frac{-U^1_y}{U^1_c} = \frac{-\beta U^{21}_y + \mu \left[ 1 - \frac{p}{w} f' \left( \frac{y^1}{w} \right) \right]}{\beta U^{21}_c + \mu}, \quad (a5)
\]

which can be reformulated as

\[
\frac{-U^1_y}{U^1_c} (\beta U^{21}_c + \mu) = -\beta U^{21}_y + \mu \left[ 1 - \frac{p}{w} f' \left( \frac{y^1}{w} \right) \right]. \quad (a6)
\]

Straightforward manipulations yield the expression

\[
\frac{-U^1_y}{U^1_c} \left( 1 + \frac{\mu}{\beta U^{21}_c} \right) = -\frac{\beta U^{21}_y}{\beta U^{21}_c} + \frac{\mu \left[ 1 - \frac{p}{w} f' \left( \frac{y^1}{w} \right) \right]}{\beta U^{21}_c}. \quad (a7)
\]

Or, equivalently

\[
\frac{-U^1_y}{U^1_c} \left( \frac{\mu}{\beta U^{21}_c} \right) = \frac{-U^{21}_y}{U^{21}_c} - \frac{U^1_y}{U^1_c} + \frac{\mu \left[ 1 - \frac{p}{w} f' \left( \frac{y^1}{w} \right) \right]}{\beta U^{21}_c}. \quad (a8)
\]

Multiplying on both sides, we get
\[ -\frac{U_Y^1}{U_C^1} = \beta U_C^{21} \left( -\frac{U_Y^{21}}{U_C^{21}} - \frac{U_Y^1}{U_C^1} \right) + 1 - \frac{p}{w'} f'\left( \frac{Y^1}{W} \right). \]  

(a9)

(a3) and (a4) readily imply that

\[ -\frac{U_Y^2}{U_C^2} = 1 - \frac{p}{w^2} f'\left( \frac{Y^2}{W^2} \right). \]  

(a10)

**Appendix 2**

The first order condition with respect to \( q \) is given by:

\[ V_q^1 + (\lambda + \beta) V_q^2 - \beta V_q^{21} - \mu p'(q) \sum_{i=1}^2 Y_q^i = 0. \]  

(a11)

The first order conditions for \( C^1 \) and \( C^2 \) are respectively given by:

\[ V_{C^1}^1 - \beta V_{C^1}^{21} = \mu, \]  

(a12)

\[ (\lambda + \beta) V_{C^2}^2 = \mu. \]  

(a13)

Adding and subtracting \( \beta V_{C^1}^{21} V_q^1 / V_C^1 \) to (a11) and rearranging terms gives:

\[ \left(V_C^1 - \beta V_{C^1}^{21}\right) V_{C^1}^1 + (\lambda + \beta) V_{C^1}^2 V_{C^1}^2 - \beta V_C^{21} \left( \frac{V_q^2}{V_C^2} - \frac{V_q^1}{V_C^1} \right) - \mu p'(q) \sum_{i=1}^2 Y_q^i = 0. \]  

(a14)

Substituting (a12) and (a13) into (a14) and dividing all terms by \( \mu \) give eq. (10).

**Appendix 3**

**Characterization of individuals’ behavior**

An agent is supposed to chose the utility-maximizing point in the \( M,B \)-space. With labor supply \( h \) and wage rate \( w \), we can express \( M \) as \( M=wh-a \) and therefore labor supply as \( h=(M+a)/w \). The direct utility function is \( U(C,h) \).

Without public provision we can express the consumption of an individual as

\[ C = B + a - pf\left( \frac{a+M}{w} \right) - g(a). \]  

A convenient approach is to model the consumer’s choice in two steps where for any \( M,B \)-combination he selects the optimal combination of \( a \) and \( h \), and then selects the preferred \( M,B \)-combination conditional on the \( a,h \)-combination being optimally adjusted. Substituting the budget constraint into the individual’s objective function and expressing labor supply by means of \( M \) and \( a \), the individual’s problem can be stated as follows:

\[ \max_a U \left( B + a - pf\left( \frac{a+M}{w} \right) - g(a), \frac{a+M}{w} \right). \]  

(a15)
Where sheltering occurs (the optimal \(a\) is different from zero), we get the first order condition
\[
U_c \left[ 1 - \frac{p}{w} f'(\frac{a+M}{w}) - g'(a) \right] + U_h \frac{1}{w} = 0, \quad (a16)
\]
equating at an optimum the marginal benefit from the net increase in consumption achieved by acquiring more sheltered income with the marginal cost of the additional effort needed to earn the extra income. Alternatively, there may be an optimum at \(a=0\) where \(g'\) is undefined due to the assumed kink, so (a16) will not apply. \(^{47}\)

With public provision we can express the consumption of an individual as \(C = B + a - g(a)\).

Defining the indirect utility function \(V(B,M) = \max_a U\left( B + a - g(a), \frac{a+M}{w} \right)\), the solution is either characterized by \(1 + \frac{1}{w} \frac{U_h}{U_c} = g'(a)\), where \(a \neq 0\), or else \(a = 0\).

**Mimicker’s behavior versus low-skilled’s behavior (in the absence of public provision)**

To assess the relative magnitude of \(h^2\) and \(h^1\), consider the f.o.c. satisfied by a low-skilled at a given \(M,B\)-allocation. Taking into account that \(C = B + a - pf\left(\frac{a+M}{w}\right) - g(a)\) = \(wh - T(M) - pf(h) - g(wh - M)\), and maximizing with respect to \(h^1\) \(u\left(w'h^1 - T(M^1) - pf(h^1) - g\left(w'h^1 - M^1\right), h^1\right)\), we get (for an interior solution)
\[
\left[\left[1 - g'(w'h^1 - M^1)\right]w^1 - pf'(h^1)\right]u_c + u_b = 0 \quad (a17)
\]

Implicit differentiation of (a17) gives:
\[
\frac{dh}{dw} = \frac{\left[1 - g'(\cdot) - whg''(\cdot)\right]u_c - \left\{u_{ch} + \left[\left(1 - g'(\cdot)\right)w - pf'(\cdot)\right]u_{cc}\right\}[1 - g'(\cdot)]h}{u_{bh} + \left\{\left[1 - g'(\cdot)\right]w - pf'(\cdot)\right\}\left[2u_{ch} + \left[\left(1 - g'(\cdot)\right)w - pf'(\cdot)\right]u_{cc}\right\} - \frac{pf''(\cdot) + (w)^2 g''(\cdot)}{u_c}}
\]

The denominator of the r.h.s of the expression above is negative from second order conditions. Thus, the sign of \(dh/dw\) is the opposite of the sign of the numerator. We can see that the income effect on labor supply pushes in the direction of \(dh/dw < 0\). Interpreting \((1 - g')w\) as the marginal wage rate, we can see that when \((1 - g')w < whg''(\cdot)\), namely when

\(^{47}\) When \(a = 0\), \(1 - \frac{p}{w} f'(h) + \frac{1}{w} \frac{U_h}{U_c} \leq g'\) and \(-1 - \frac{p}{w} f'(h) - \frac{1}{w} \frac{U_h}{U_c} \leq g'\), where \(g' > 0\) represents the marginal cost of letting \(a\) deviate from zero in either direction (assuming symmetry for simplicity). We shall assume that where the optimum is characterized by \(a=0\), strict inequalities apply so that infinitesimal changes in \(M\) and \(B\) will leave \(a\) unaffected.
the change in the marginal wage rate is negative, the substitution effect also pushes in the direction of \( \frac{dh}{dw} < 0 \).

**Characterization of optimal marginal tax rates without public provision**

Defining by \( V(B,M) \) the indirect utility function that solves problem (a15), the Lagrangian of the government’s problem can be written as

\[
V'(B^1,M^1) + \lambda [V^2(B^2,M^2) - \bar{V}^2] + \beta [V^2(B^2,M^2) - V^2(B^1,M^1)] + \mu \sum_{i=1}^{2} (M^i - B^i)
\]

The first order conditions with respect to the government’s policy variables are the following:

\[(M^2): \ (\lambda + \beta) V^2_{M} = -\mu \quad (a18)\]
\[(B^2): \ (\lambda + \beta) V^2_{B} = \mu \quad (a19)\]
\[(M^1): \ V^1_{M} = \beta V^{21}_{M} - \mu \quad (a20)\]
\[(B^1): \ V^1_{B} = \beta V^{21}_{B} + \mu \quad (a21)\]

Writing the second stage of an agent’s maximization problem as maximizing

\[V(M - T(M), M)\]

with respect to \( M \) and using the corresponding first order condition, we can express the marginal tax rate faced by an agent as \( T'(M) = 1 + V_{M} / V_{B} \). Dividing (a18) by (a19) gives \( 1 + V_{M}^2 / V_{B}^2 = 0 \) and therefore \( T'(M^2) = 0 \). Now assume that \( a \neq 0 \) for the high-skilled. The envelope theorem would then imply that \( V_{B} = u_{C} \) and \( V_{M} = 1 \frac{w}{w^2} u_{B} - \frac{p}{w} f'(\cdot) u_{C} \).

However, from (a16) we know that \( a \neq 0 \Rightarrow 1 + \frac{1}{w^2} u_{B}^2 - \frac{p}{w} f'(\cdot) = g'(a) \neq 0 \), which would contradict the optimal tax condition. Therefore, \( a=0 \) at an optimum for the high-skilled.

Dividing (a20) by (a21), multiplying the result by the r.h.s. of (a21) and rearranging, gives:

\[
1 + \frac{V_{M}^1}{V_{B}^1} = \frac{\beta}{\mu} V^{21}_{B} \left[ \frac{V^{21}_{M}}{V^{21}_{B}} - \frac{V^{1}_{M}}{V^{1}_{B}} \right] = \frac{\beta}{\mu} V^{21}_{B} \left[ MRS_{MB}^{1} - MRS_{MB}^{21} \right] > 0 ,
\]

where \( MRS = -V_{M} / V_{B} > 0 \), \( MRS_{MB}^{1} - MRS_{MB}^{21} > 0 \) by agent monotonicity, and we can interpret the r.h.s. of (a22) as the marginal tax rate faced at an optimum by a low-skilled agent. If the marginal tax rate implied by (a22) is larger than \( g'(0^+) \), it will be optimal for the low-skilled to shelter part of his earned income. Otherwise, no sheltering occurs.

**Characterization of optimal marginal tax rates with public provision**

The Lagrangian of the government’s problem can be written as
\[ V^i(B', M') + \lambda \left[ V^2(B', M') - V^2 \right] + \beta \left[ V^2(B^2, M^2) - V^2(B^1, M^1) \right] + \mu \sum_{i=1}^{2} \left[ M' - B' - p \left( h' \right) \right], \]

where \( h' = \left( a' + M' \right) / w' \). The associated first order conditions are:

\[(M^2): \quad (\lambda + \beta) V_{M}^2 = -\mu \left[ 1 - pf'(h^2) \frac{\partial h^2}{\partial M^2} \right] \tag{a23} \]

\[(B^2): \quad (\lambda + \beta) V_{B}^2 = \mu \left[ 1 + pf'(h^2) \frac{\partial h^2}{\partial B^2} \right] \tag{a24} \]

\[(M^1): \quad V_{M}^1 = \beta V_{M}^{21} - \mu \left[ 1 - pf'(h^1) \frac{\partial h^1}{\partial M^1} \right] \tag{a25} \]

\[(B^1): \quad V_{B}^1 = \beta V_{B}^{21} + \mu \left[ 1 + pf'(h^1) \frac{\partial h^1}{\partial B^1} \right] \tag{a26} \]

Dividing (a23) by (a24), multiplying the result by the r.h.s. of (a24) and rearranging, gives:

\[ 1 + \frac{V_{M}^2}{V_{B}^2} = pf'(h^2) \left( \frac{dh^2}{dM^2} \right)_{d^{1}y=0}, \tag{a27} \]

where \( \left( \frac{dh^2}{dM^2} \right)_{d^{1}y=0} \) has been defined as \( \left( \frac{dh^2}{dM^2} \right)_{d^{1}y=0} = \frac{\partial h^2}{\partial M^2} + MRS_{MB} \frac{\partial h^2}{\partial B^2} \) and we can interpret the r.h.s. of (a27) as the marginal tax rate faced at an optimum by a high-skilled agent. Differentiating the identity \( h^1 = \left( a' + M^1 \right) / w' \), this marginal tax rate can be equivalently rewritten as \( T'(M^2) = \frac{P}{w'} f'(h^2) \left[ 1 + \left( \frac{da^2}{dM^2} \right)_{d^{1}y=0} \right] \). If the marginal tax rate implied by (a27) is larger than \( g'(0') \), it will be optimal for a high-skilled agent to shelter part of his earned income. Otherwise, no sheltering occurs.

Dividing (a25) by (a26), multiplying the result by the r.h.s. of (a26) and rearranging, gives:

\[ 1 + \frac{V_{M}^1}{V_{B}^1} = \frac{\beta}{\mu} \left[ V_{B}^{21} \left[ MRS_{MB} - MRS_{MB}^{21} \right] + pf'(h^1) \left( \frac{dh^1}{dM^1} \right)_{d^{1}y=0} \right], \tag{a28} \]

where we can interpret the r.h.s. of (a28) as the marginal tax rate faced at an optimum by a low-skilled agent. If the \( T'(M^1) \) implied by (a28) is larger than \( g'(0') \), it will be optimal for a low-skilled agent to shelter part of his earned income; otherwise, no sheltering occurs.

**Appendix 4**

We first describe how we calculate the Swedish marginal income tax. Let \( Z \) denote gross income, namely the total amount the employer pays, and let \( Y \) denote the employee’s taxable
income (beskattningsbar inkomst). Let $\theta$ be the rate for the proportional payroll tax (löneavgifter) and $\tau$ the average tax on commodities. Since in Sweden all of the payroll tax is formally paid by the employer the relation between gross income and taxable income is given by $Z = (1 + \theta)Y$. We assume that the total income is consumed as the purpose of savings is ultimately consumption even though savings may in fact be taxed at a future rate different from the one faced today. Total taxes are then given by:

$$\text{Tax} = \frac{\theta Z}{1 + \theta} + T\left(\frac{Z}{1 + \theta}\right) + \tau \left(\frac{Z}{1 + \theta} - T\left(\frac{Z}{1 + \theta}\right)\right).$$

The marginal tax $\frac{\partial \text{Tax}}{\partial Z}$ is given by

$$\frac{\partial \text{Tax}}{\partial Z} = \frac{\theta}{1 + \theta} + T''\left(\frac{Z}{1 + \theta}\right)\frac{1}{1 + \theta} + \frac{\tau}{1 + \theta} - T''\left(\frac{Z}{1 + \theta}\right)\frac{\tau}{1 + \theta}.$$

We need information on $\theta$ and $\tau$ for the year 2005. According to information on the home page of The Swedish Tax Authority (Skatteverket) $\theta$ was 0.3246 in 2005. However, part of this is often regarded as an insurance premium rather than a tax. According to calculations performed by The National Institute of Economic Research (Konjunkturinstitutet) a share equal to 0.59 should be regarded as a tax. In our calculations we therefore use the formula

$$\frac{\partial \text{Tax}}{\partial Z} = \frac{0.59\theta}{1 + \theta} + T''\left(\frac{Z}{1 + \theta}\right)\frac{1}{1 + \theta} + \frac{\tau}{1 + \theta} - T''\left(\frac{Z}{1 + \theta}\right)\frac{\tau}{1 + \theta}.$$

Combining VAT and excise taxes, also taking account of excise taxes on firms, with the assumption that they are fully shifted to the consumers, the estimate of $\tau$ is 0.16.

To calculate $T'(Y)$ we have used the official tax calculation brochure (skatteuträkningsbroschyren) 2006 (SKV 425 utgåva 12). In 2005 the average local proportional tax rate was 0.316.

Since in US half the payroll tax is paid by the employer and half by the employee the formula for calculating the marginal income tax is slightly different for the Californian marginal income tax. The relation between gross income and taxable income would be given by $Z = (1 + 0.5\theta)Y$. We also want to take into account the Dependent Care Assistance Program (DCAP), which works as if part of the child care expenses is tax deductible and

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modifies the taxable income. We assume the hours of child care needed equal the hours of work. Child care expenses are therefore given by \( ph = pZ / w \) where \( w \) is the payroll-inclusive wage rate. Let \( \bar{D} \) be the maximum amount of child care expenses that can be deducted and define \( D \) as \( D = \min \{ pZ / w, \bar{D} \} \). The taxable income is then defined by

\[
M = (Z - D) / (1 + 0.5\theta) \quad \text{if the DCAP is used, otherwise the taxable income is simply} \quad M = Z / (1 + 0.5\theta).
\]

The expression for taxes becomes:

\[
TaN = \theta M + T(M) + \tau(M - T(M) - 0.5\theta M)
\]

and the expression for \( \partial Tax / \partial Z \) as

\[
\frac{\partial Tax}{\partial Z} = (1 - \tau)T'(M) \frac{dM}{dZ} + (\theta + \tau - 0.5\tau\theta) \frac{dM}{dZ}.
\]

If the DCAP is not used or if the deductions are inframarginal, i.e., the actual expenses are larger than the maximum one can deduct, \( dM / dZ = 1 / (1 + 0.5\theta) \) whereas if the expenses are marginal \( dM / dZ = (1 - p / w) / (1 + 0.5\theta) \). Note that if child care is bought for all hours of work, then \( p / w = ph / wh \) = ratio of child care expenses to income.

The payroll tax in US is 0.153. The state sales tax in California is 0.0725. Local sales taxes vary. In our calculations we have assumed that the local sales tax is 0.005, implying a total sales tax of 0.0775.

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