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The intraday liquidity management game

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The Intraday Liquidity Management Game*

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March 14, 2002

Abstract

We use a game theoretical framework to analyze the intraday behavior of banks with respect to settlement of interbank claims in a real time gross settlement setting. We find that the game played by banks depends upon the intraday credit policy of the central bank and that it encompasses two well-known game theoretical paradigms: the prisoner's dilemma and the stag hunt. The former arises in a collateralized credit regime where we confirm the result of earlier literature that banks have an incentive to postpone payments when daylight liquidity is costly and that this is socially inefficient. The latter arises in a priced credit regime where we show that the postponement of payments can be socially efficient. *Journal of Economic Literature* classifications numbers: C72, E58. Keywords: Payment systems, RTGS, prisoner's dilemma, stag hunt.

Resume

Bankers likviditetsstyring i et reeltids bruttoafviklingssystem analyseres ved hjælp af en spilteoretisk model. Spillet mellem banker vises at afhænge af centralbankens intradag kreditpolitik og at indbefatte to velkendte paradigmer inden for spilteori. Bankerne kan ende op i et fangernes dilemma, hvis centralbanken stiller intradag kredit til rådighed mod sikkerhed, da de har et incitament til at udskyde betalinger. Et koordinationspil inspireret af den franske filosof Jean-Jacques Rousseau er muligt, såfremt centralbanken opkræver et eksplicit gebyr for overtræk. I et regime med sikkerhedsstillelse er udskydelse af betalinger altid uhensigtsmæssig ud fra en samfundsmæssig synsvinkel, mens dette ikke nødvendigvis er tilfældet, hvis der opkræves et eksplicit gebyr.

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1 Introduction

Most transactions between agents are settled using payment instruments such as cash, check, or electronic money transfer. The system through which these payments flow is supplied in collaboration between the commercial banks and the central bank and is referred to as the payment system. A prerequisite for a well functioning economy is a well functioning payment system.

The *raison d'être* of central banks is partly the promotion of smooth operations of the payment system.¹ The extent to which the central bank is involved in the payment system varies however across countries.² Almost always, the central bank provides the medium to settle the smallest payments (cash) and the means to settle the largest payments, which typically are wholesale payments between banks. For the latter purpose the central bank usually operates a system through which banks can settle payments in central bank money. Besides a role in the operational part of the payment system the central bank often has a regulatory role as overseer of private payment system arrangements.³

The volume of interbank payments increased dramatically throughout the 1980s and 1990s as a result of rapid financial innovation and the integration and globalization of financial markets. Historically, interbank payments have been settled via (end-of-day) netting systems. As volume increased central banks became worried about the risks inherent in netting systems.⁴ Most central banks opted for the implementation of a Real Time Gross Settlement (RTGS) system.⁵ A RTGS system processes payments individually, immediately

¹See e.g. chapter 2, article 3 of the “Statute of the European System of Central Banks and of the European Central Bank.” For discussion of payment systems and the historical and current role of central banks see Pauli (2000).

²See Berg (1999) for a discussion of a minimalistic approach preferred in the Nordic countries.

³See Bank for International Settlements (2001).

⁴See Humphrey (1986) and Humphrey (1989) for some of the first discussion of these issues.

⁵An exception is Canada where the Large Value Transfer System (LVTS) is a net settlement system (see Dingle, 1998). See Kahn and Roberds (1998) for a discussion of the relative merits of net versus gross settlement of interbank payments.

and with finality throughout the day. The system avoids the situation where the failure of one participant may cause the failure of others due to the exposures that are accumulated over the day, as in a net settlement system with out proper risk controls. However, this elimination of risk comes at the cost of an increased need for intraday liquidity to smooth the non-synchronized payment flows. It has long been agreed among central banks that the provision of free intraday liquidity is not a viable option. It implies that the central bank (i.e., the tax payers) as guarantor of the finality of a payment assumes a credit risk and it creates an incentive for over use as is often the case when something of value is available for free.⁶ Today, central banks provide intraday liquidity for a fee or require shortfalls to be backed by collateral. Liquidity is thus costly either in form of an explicit fee or implicitly as the opportunity cost of the pledged collateral. Banks try to manage their liquidity throughout the day in order to minimize the cost of settling customer obligations and their own proprietary operations. Intraday liquidity management has become an important competitive parameter in commercial banking and a policy concern of central banks.⁷

Aspects of the alternative intraday credit policies of central banks are discussed in Furfine and Stehm (1998) and Zhou (2000). Incentives in RTGS systems are studied in Angelini (1998), Kobayakawa (1997) and McAndrews and Rajan (2000). Angelini (1998) and Kobayakawa (1997) use a setup derived from earlier literature on precautionary demand for reserves. Angelini (1998) shows that in a RTGS system, where banks are charged for intraday liquidity, payments will tend to be delayed and that the equilibrium outcome is not socially optimal. Kobayakawa (1997) models the intraday liquidity management process as a game of uncertainty, i.e., a game where *nature* moves after the players. Kobayakawa shows that both delaying and not delaying can be equilibrium outcomes when intraday overdrafts

⁶See Humphrey (1986) and Evanoff (1988).

⁷A recent discussion of the timing issues and risks associated with payment systems in connection with foreign exchange (FX) settlement is contained in an article entitled "The long, dark shadow of Herstatt," which appeared in the April 14th - 20th, 2001 issue of Economist magazine.

are priced. McAndrews and Rajan (2000) study the timing and funding of transfers in the Fedwire Funds Transfer system (Fedwire). They show that banks benefit from synchronizing their payment pattern over the course of the business day because it reduces the overdrafts. However, they also note that “the difficulty of achieving such a synchronized pattern is considerable because the timing of payments in some respects resembles a coordination game.” Based on the empirical work of McAndrews and Rajan (2000) and Richards (1995), Zhou (2000) states that in the Fedwire “there is evidence both of banks delaying sending outgoing payments and of banks cooperating in making payments”.

In this paper we analyze the incentives of commercial banks in a RTGS setting by specifying a Bayesian game, in which each bank has private knowledge about its own payment requests. The game-theoretic modelling allows us to understand the differences in incentives created by different intraday credit policies of the central bank and see the effect of these policies on equilibrium outcomes. We consider three credit regimes: free intraday credit, collateralized credit, and priced credit. Today almost no central bank provides intraday liquidity for free so this case is provided as a benchmark. Collateralized credit, in one form or another, is the prevalent option in Europe and elsewhere outside the United States.⁸ Priced credit is the system of choice in the United States.⁹ Quantitative limits or “caps,” are often used in combination with the different types of credit extensions.¹⁰ In what follows, however, we assume that quantitative limits are non binding.

We demonstrate that payment delays of the sort predicted by Angelini (1998) and Kobayakawa (1997) emerge in Bayesian equilibrium under various intraday credit policy regimes. However, we show that in some instances it is socially efficient for banks to delay

⁸Collateralized credit usually takes the form of pledging collateral to the central bank or entering into an intraday repurchase agreement with the central bank.

⁹In 1986 the Federal Reserve began implementing several different policies aimed at reducing the intraday overdrafts and in 1994 the Federal Reserve began to charge a fee for intraday overdraft.

¹⁰An often cited example of a quantity limit policy is the Swiss Interbank Clearing (SIC) system. In the United States depository institutions using the Fedwire are subject to capital based debit caps for overdrafts both at a daily and a biweekly frequency (Zhou 2000, pp. 33).

payments. Some intraday credit regimes produce a coordination game with two Bayesian equilibria, one that involves delaying payments and another that does not. We identify conditions under which coordination problems arise, and discuss a criterion for predicting which equilibria will be realized. The lessons learned from the analysis suggest some policy recommendations.

2 The Intraday Liquidity Management Game

Consider an economy with a generic RTGS system operated by the central bank and three possible intraday credit regimes as discussed above. Suppose that the system has two identical participants: Bank *A* and Bank *B*. The business day consists of three periods, which we shall refer to as morning, afternoon, and end-of-day. The sequence of events and the potential actions taken by the banks and the central bank are shown in Figure 1.

At the beginning of the morning period banks receive random payment requests from their customers and decide whether or not to process or delay them. Each bank is assumed to start the day with a zero balance on its settlement account. In order to overdraw its settlement account a bank might have to either post collateral or pay a fee depending on the intraday credit policy of the central bank. Morning payment requests arrive according to an exogenously specified probability distribution. Banks see their own requests, but not those of the other bank. Hence, a bank's decision to process a morning payment request is made without knowing whether or not the other bank received one, and if so, whether the other bank chooses to process it.

Banks also receive random payment requests from their customers at the beginning of the afternoon period. However, banks are assumed to process all requests on the day received, so all new requests and requests left over from the morning period must be processed in the afternoon period. Again intraday credit from the central bank, if needed, comes at a cost

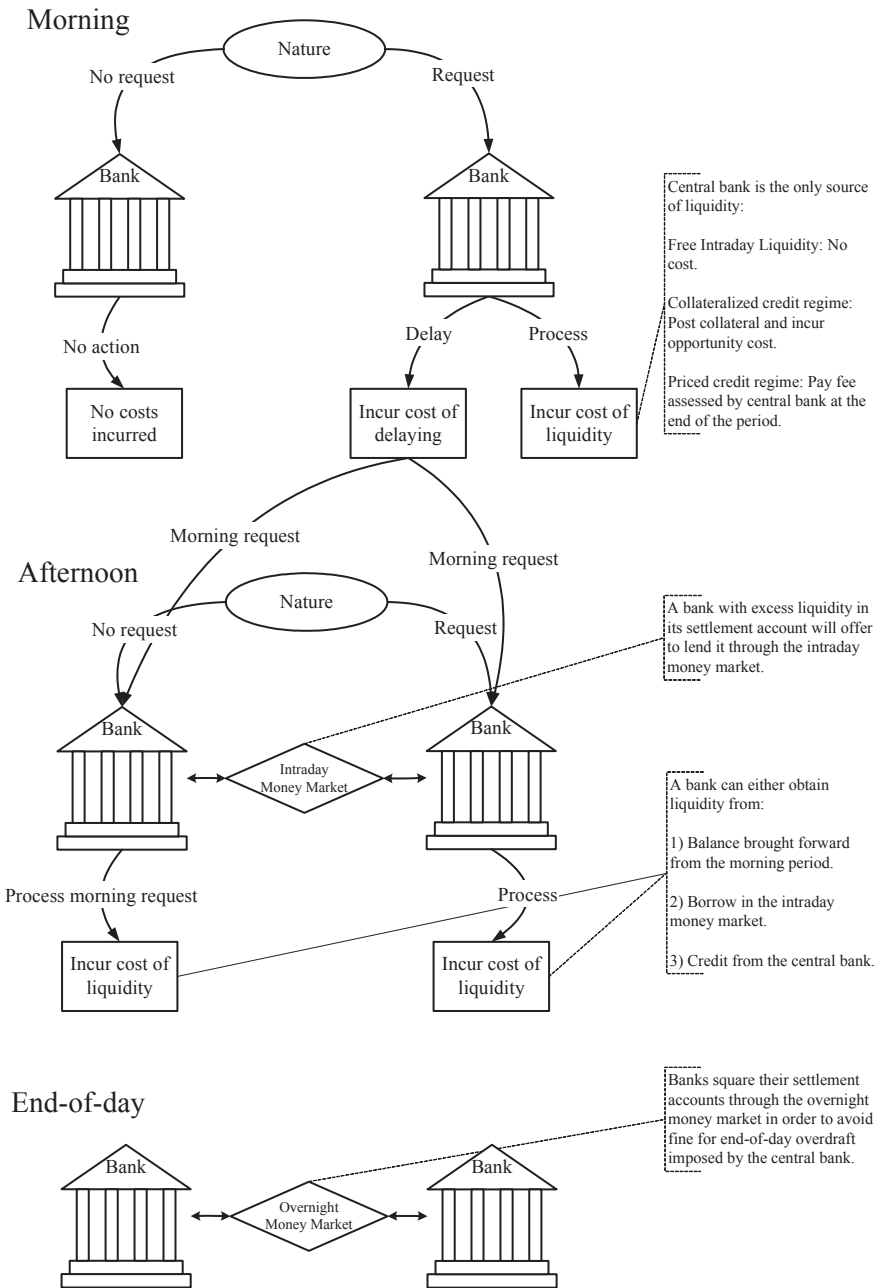


Figure 1: Sequence of events and actions for a bank

depending on the intraday credit regime.

In the afternoon period a bank with excess liquidity in its settlement account that it does not need will offer the liquidity to the other bank through an intraday money market.¹¹ The other bank will accept this offer as we assume the rate will be at least as good as the other alternatives. The central bank fines any roll over of intraday into overnight credit sufficiently in order to make this an unattractive option for banks.¹² The end-of-day period is thus used by the banks to clear imbalances on their settlement accounts via an overnight money market.

The fundamental problem from the viewpoint of the banks is whether to process a request received in the morning immediately, and bear the cost of obtaining intraday credit, or delay it until the afternoon. The decision is non trivial because delaying is costly. It damages the bank's reputation as an efficient payments processor and thus leads to a loss of goodwill and future business as discussed in Angelini (1998). Moreover, according to Furfine and Sterm (1998), delaying uses up computing resources and requires additional staff.

2.1 Notation and Definitions

The player set is $P = \{A, B\}$. Each bank $i \in P$ can be one of two possible types; $\theta_i \in \Theta_i = \{0, 1\}$, where 0 means Bank i receives no payment request in the morning and 1 means it does receive one. For simplicity we assume that all payment requests are one dollar (currency unit). Let $\theta = (\theta_A, \theta_B) \in \Theta = \times_{i \in P} \Theta_i$ denote a type profile. We assume that there is a given probability distribution over type profiles, $\mathbb{P} : \Theta \longrightarrow [0, 1]$, that is common knowledge.

¹¹An explicit money market for intraday liquidity does not seem to exist anywhere. However, the market implicitly exists in the form of a premium on overnight loans delivered earlier in the business day. For instance, Furfine (1999) finds evidence of overnight Federal funds loans being delivered early at a higher interest rate than equivalent loans delivered later during the business day. Angelini (2000) finds a modest deviation from the daily average in the morning for Italian data.

¹²The Swiss National Bank charges a penalty of 400 basis points above the overnight rate if the intraday credit is not paid at the end of the end day. This penalty is twice as high as that existing for a standard Lombard credit (Heller 2000).

For each bank $i \in P$, let $\mathcal{A}_i(\theta_i)$ denote the set of available actions given its type. The set of actions available to a bank of type $\theta_i = 0$ is $\mathcal{A}_i(0) = \{n\}$, where n indicates no action. The set of available actions to a bank of type $\theta_i = 1$ is $\mathcal{A}_i(1) = \{m, a\}$, where m and a indicate morning and afternoon, respectively. Let $\alpha = (\alpha_A, \alpha_B) \in \mathcal{A}(\theta) = \times_{i \in P} \mathcal{A}_i(\theta_i)$ denote an action profile.

Bank i 's situation in the afternoon is denoted by $\psi_i \in \Psi_i = \{0, 1\}$, where again 0 indicates no payment request and 1 indicates a payment request of one dollar. Let $\psi = (\psi_A, \psi_B) \in \Psi = \times_{i \in P} \Psi_i$ denote the state of world in the afternoon. There is a given probability distribution over the states of the world, $\mathbb{Q} : \Psi \rightarrow [0, 1]$ that is also common knowledge.

Let $\mathcal{I} \in \{\mathcal{F}, \mathcal{C}, \mathcal{P}\}$ denote the intraday credit policy chosen for the RTGS system by the central bank, where \mathcal{F} is free intraday liquidity, \mathcal{C} is collateralized credit, and \mathcal{P} is priced credit, respectively. Given the specification of the intraday credit policy, \mathcal{I} , each bank $i \in P$ has a payoff function $\pi_i^{\mathcal{I}} : \Theta \times \mathcal{A}(\theta) \times \Psi \rightarrow \mathbb{R}$ that gives the payoff under each type profile, $\theta \in \Theta$, action profile $\alpha \in \mathcal{A}(\theta)$, and state of the world profile, $\psi \in \Psi$. We assume that the price banks charge for processing payment requests is fixed, and for simplicity it is set to zero. The payoff function is thus equal to the negative of the settlement cost function, $c_i^{\mathcal{I}}(\cdot)$, that is,

$$\pi_i^{\mathcal{I}}(\alpha, \theta, \psi) = -c_i^{\mathcal{I}}(\alpha, \theta, \psi). \quad (1)$$

The settlement cost function depends on the intraday credit policy regime chosen by the central bank, as indicated by the superscript \mathcal{I} . The main difference of strategic interest is whether liquidity costs are incurred before or after payment requests are processed.

In a collateralized credit regime banks have to pledge collateral up front in order to obtain intraday liquidity. We assume that the act of pledging collateral entails an opportunity cost for the banks. This opportunity cost per period is denoted y and is incurred whenever a bank processes a payment request without having funds available in its settlement account to cover

the request.¹³ Since banks start the day with zero funds in their settlement accounts, any payment request processed in the morning incurs a cost of y . A payment request processed in the afternoon will also incur a cost of y unless the bank has funds available from the morning period.

Under a priced credit regime banks are charged a fee, x per dollar, whenever their settlement account is overdrawn at the end of either the morning or afternoon period. The fee is set by the central bank and can be thought of as an insurance premium reflecting the credit risk that the central bank assumes. The distinguishing feature of priced (versus collateralized) credit is that there is no cost of liquidity for the banks when off setting payment requests are processed in the same period.

The cost of delaying a request in either the collateralized credit or priced credit regime is denoted w and is assumed to reflect the true social cost of delaying. Afternoon money market loans between the two banks are at the rate e . Because interest paid in the overnight money market is unavoidable, regardless of the actions taken by banks, it is ignored in the analysis that follows. The settlement costs for every permutation of types, actions, states of the world and intraday credit policy regimes are shown in Tables 9 - 12 in appendix A.¹⁴ Some examples are discussed.

We restrict attention to pure strategies.

Definition 1 (Pure strategy) *A pure strategy for Bank $i \in P$ is a function $s_i : \theta_i \rightarrow \mathcal{A}_i(\theta_i)$ where for each type $\theta_i \in \{0, 1\}$, $s_i(\theta_i)$ gives the action, α_i , of bank i when it is of type θ_i .*

¹³We envisage an economy where banks have multiple uses for collateral within the day. We thus model the opportunity cost of collateral differently than Kobayakawa (1997), where the opportunity cost of collateral is described as a sunk cost and hence is ignored in the strategic analysis. The outcome of the collateralized credit regime in Kobayakawa (1997) resembles our free intraday credit regime.

¹⁴We ignore all other fixed or variable settlement costs, such as participation or transactions fees.

For each bank $i \in P$ the set of possible pure strategies is $\mathcal{S}_i = \{m(\cdot), a(\cdot)\}$ where

$$m(\theta_i) = \begin{cases} \text{morning} & \text{if } \theta_i = 1 \\ \text{no action} & \text{if } \theta_i = 0 \end{cases} \quad (2)$$

and

$$a(\theta_i) = \begin{cases} \text{afternoon} & \text{if } \theta_i = 1 \\ \text{no action} & \text{if } \theta_i = 0 \end{cases} \quad (3)$$

We shall refer to these strategies as the morning and afternoon strategies. Let $s(\theta) = (s_A(\theta_A), s_B(\theta_B)) \in \mathcal{S} = \times_{i \in P} \mathcal{S}_i$ denote a strategy profile.

We begin by assuming that banks are risk neutral and choose strategies in order to maximize expected payoffs (i.e., minimize expected settlement costs). Let $-i$ denote not i .

Definition 2 (Bayesian Equilibrium) *A strategy profile $s^*(\cdot) = (s_A^*(\cdot), s_B^*(\cdot))$ is a Bayesian equilibrium for the intraday liquidity management game given the intraday credit policy \mathcal{I} if and only if for each $i \in P$ and every $\theta_i \in \Theta_i$ occurring with positive probability*

$$s_i^*(\theta_i) \in \arg \max_{\alpha_i \in \mathcal{A}_i(\theta_i)} E_{\Theta_{-i}} [E_{\Psi} [\pi_i^{\mathcal{I}}(\alpha_i, s_{-i}(\theta_{-i}), \theta, \psi)] | \theta_i]. \quad (4)$$

2.2 Type 1 Game

Since banks only have one possible action when of type 0, a bank's strategy is determined solely by the action taken by the bank when of type 1. We can thus write the Bayesian game as a 2×2 normal-form game where the strategy sets of the players are the type 1 action sets $\mathcal{A}_i(1) = \{m, a\}$, $i \in P$, and the payoffs for player i from each action profile are given by the conditional expected payoffs $E_{\Theta_{-i}} [E_{\Psi} [-c_i^{\mathcal{I}}(\alpha_i, s_{-i}(\theta_{-i}), \theta, \psi)] | \theta_i = 1]$. By construction, a Nash equilibrium of what we shall refer to as the *type 1 game* is a Bayesian equilibrium of the underlying Bayesian game. The type 1 game is given in Table 1.

		B	
		m	a
A	m	$-E_{\Theta_A} [E_{\Psi} [c_B^{\mathcal{I}}(m(\theta_A), m, \cdot)] \theta_B = 1]$	$-E_{\Theta_A} [E_{\Psi} [c_B^{\mathcal{I}}(m(\theta_A), a, \cdot)] \theta_B = 1]$
	a	$-E_{\Theta_B} [E_{\Psi} [c_A^{\mathcal{I}}(m, m(\theta_B), \cdot)] \theta_A = 1]$	$-E_{\Theta_B} [E_{\Psi} [c_A^{\mathcal{I}}(m, a(\theta_B), \cdot)] \theta_A = 1]$
A	m	$-E_{\Theta_A} [E_{\Psi} [c_B^{\mathcal{I}}(a(\theta_A), m, \cdot)] \theta_B = 1]$	$-E_{\Theta_A} [E_{\Psi} [c_B^{\mathcal{I}}(a(\theta_A), a, \cdot)] \theta_B = 1]$
	a	$-E_{\Theta_B} [E_{\Psi} [c_A^{\mathcal{I}}(a, m(\theta_B), \cdot)] \theta_A = 1]$	$-E_{\Theta_B} [E_{\Psi} [c_A^{\mathcal{I}}(a, a(\theta_B), \cdot)] \theta_A = 1]$

Table 1: Type 1 Game

2.3 Efficiency

We view the central bank as a benevolent provider of the RTGS system and ask whether various intraday credit policies lead to equilibria that are efficient in the sense that aggregate expected settlement costs are minimized.

Definition 3 (Efficiency) *A strategy profile $s(\cdot) = (s_A(\cdot), s_B(\cdot))$ is efficient given the intraday credit policy \mathcal{I} if and only if*

$$\sum_{i \in P} E_{\Theta} [E_{\Psi} [c_i^{\mathcal{I}}(s(\theta), \theta, \psi)]] \leq \sum_{i \in P} E_{\Theta} [E_{\Psi} [c_i^{\mathcal{I}}(s'(\theta), \theta, \psi)]] \quad (5)$$

for all $s'(\cdot) \in \mathcal{S}$.

2.4 Further Assumptions

2.4.1 Intraday Money Market

As discussed above, if a bank finds itself to have excess liquidity in its settlement account after the morning period, and if these funds are not needed to cover a payment request received in the afternoon, then the bank will offer to lend the liquidity to the other bank at the rate e .¹⁵ We assume, that the bank will demand less than the cost of intraday liquidity

¹⁵The fee e can be thought of as a premium on overnight loans delivered earlier in the day.

provided by the central bank, that is

$$e = \begin{cases} x - \varepsilon & \text{if } \mathcal{I} = \mathcal{P} \\ y - \varepsilon & \text{if } \mathcal{I} = \mathcal{C} \end{cases} \quad (6)$$

where $\varepsilon > 0$. Consequently the implicit money market is the preferred option for afternoon credit. In the analysis below we shall equate the market interest rate for intraday credit with the cost of liquidity (i.e., set $\varepsilon = 0$).

2.4.2 Arrival Probabilities

We assume that, for each bank, the arrival of an afternoon payment request is independent of the arrival of a morning payment request. In addition, we assume that arrivals of payment requests occur independently across banks. Letting p denote the probability of a morning request and q denote the probability of an afternoon request we have $\mathbb{P}(1, 1) = p^2$, $\mathbb{P}(0, 1) = \mathbb{P}(1, 0) = p(1 - p)$, $\mathbb{P}(0, 0) = (1 - p)^2$, $\mathbb{Q}(1, 1) = q^2$, $\mathbb{Q}(1, 0) = \mathbb{Q}(0, 1) = q(1 - q)$, and $\mathbb{Q}(0, 0) = (1 - q)^2$.

3 Equilibrium Analysis

In this section we compute the Bayesian equilibria for each of the three credit regimes and identify efficient strategy profiles. To minimize notation it is useful to define the following expected cost functions. The function

$$\tilde{c}^{\mathcal{I}}(\alpha_i, s_{-i}(\cdot)) = E_{\Theta_{-i}} [E_{\Psi} [c_i^{\mathcal{I}}(\alpha_i, s_{-i}(\theta_{-i}), \theta, \psi) \mid \theta_i = 1]] \quad (7)$$

gives the expected settlement cost of a bank that receives a payment request in the morning and takes action α_i , and faces an opponent who plays the strategy $s_{-i}(\cdot)$. The function

$$\tilde{c}^{\mathcal{I}}(s_i(\cdot), s_{-i}(\cdot)) = E_{\Theta} [E_{\Psi} [c_i^{\mathcal{I}}(s_i(\theta_i), s_{-i}(\theta_{-i}), \theta, \psi)]] \quad (8)$$

gives the unconditional expected settlement cost of a bank that plays strategy $s_i(\cdot)$ and faces an opponent who plays the strategy $s_{-i}(\cdot)$. In what follows, these costs are computed under the assumptions on arrival probabilities outlined in section 2.4.2.

3.1 Free Intraday Liquidity

We shall use the free intraday liquidity regime as a benchmark. In the case of free intraday liquidity there is no incentive to delay any payments. The type 1 game under free intraday liquidity is shown in Table 2.

		Bank B	
		m	a
Bank A	m	0	$-w$
	a	$-w$	$-w$

Table 2: Type 1 Game - Free Intraday Liquidity

The action profile (m, m) is a Nash equilibrium in dominant strategies of this game and the Bayesian equilibrium of the intraday liquidity management game is thus for both banks to play the morning strategy i.e., $s^*(\cdot) = (m(\cdot), m(\cdot))$. Since the expected settlement cost is zero we have the following result.

Proposition 1 *Under free intraday liquidity the Bayesian equilibrium is efficient.*

As discussed in the introduction, free intraday liquidity transfers the settlement and credit risk to the central bank and this transfer of risk also creates a potential for moral hazard problems. Free intraday liquidity is thus not considered a viable intraday credit policy option by central banks.

3.2 Collateralized Credit

In a collateralized credit regime, the expected settlement costs of a bank conditional on having received a payment request in the morning are given by:

$$\tilde{\mathcal{C}}(m, m(\cdot)) = y + (1 - p)y + qy, \quad (9)$$

$$\tilde{\mathcal{C}}(m, a(\cdot)) = y + y + qy, \quad (10)$$

$$\tilde{\mathcal{C}}(a, m(\cdot)) = w + (1 - p)y + qy, \quad (11)$$

$$\tilde{\mathcal{C}}(a, a(\cdot)) = w + y + qy. \quad (12)$$

The settlement costs are divided into three components. The first component reflects the cost of the morning request in the morning period. Because collateral must be posted before processing a request, the morning cost depends solely on the action taken by the bank and not on the opponent's action. The cost is y if the bank processes the request and w if the bank decides to delay. The second component is the expected cost of the morning request in the afternoon period and it depends on the strategy played by the opponent and the probability that the opponent receives a morning payment request. If the other bank plays the morning strategy this cost is $(1 - p)y$ because with probability p the opponent receives a morning payment request and processes it, hence providing "free" afternoon liquidity. In cases where the opponent plays the afternoon strategy the bank has to post collateral in the afternoon period for the morning request in all circumstances and thus incurs the cost y . The last component qy is the same across all permutations of the possible actions and strategies. It reflects the fact that with probability q the bank will have to process an afternoon payment request.

The type 1 game for the collateralized credit regime is given in Table 3.¹⁶ If $w > y$, then

¹⁶The amount $y + qy$ is common to each of the expected settlement costs in the collateralized credit regime. Hence, this amount is excluded from each of the payoffs in the presentation of the normal form game. This simplifies the presentation of the game and makes it easier to see how strategic aspects of the game depend upon the parameter values.

the action profile (m, m) is the Nash equilibrium of the type 1 game while (a, a) is the Nash equilibrium when $w < y$.

		Bank B	
		m	a
Bank A	m	$-y(1 - p)$	$-w + py$
	a	$-y$	$-w$

Table 3: Type 1 Game - Collateralized Credit

The equilibrium of the intraday liquidity management game under a collateralized credit regime is given by lemmas 1 and 2.

Lemma 1 *If $w < y$, then the strategy profile $(a(\cdot), a(\cdot))$ is the unique Bayesian equilibrium.*

Lemma 2 *If $w > y$, then the strategy profile $(m(\cdot), m(\cdot))$ is the unique Bayesian equilibrium.*

The equilibrium outcome depends solely on the relative size of the opportunity cost of collateral and the cost of postponing a payment request. It does not depend on the arrival probabilities of payment requests in the morning and afternoon.

3.2.1 Prisoner's Dilemma

In the case where $y(1 - p) < w < y$, the type 1 game shown in Table 3 has the structure of the prisoner's dilemma game. Each player has a dominant strategy to play afternoon, yet they would both be better off if they both chose morning. The emergence of the prisoner's dilemma game in this case is noteworthy because much is known about how agents play the prisoner's dilemma game in both static and repeated settings. The prediction of how play might transpire in the repeated version of the intraday day liquidity management game is discussed in section 5.2 below. Note, that if $p = 1$ (i.e., certain arrival) the prisoner's dilemma emerges whenever $w < y$.

3.2.2 Efficiency

In order to evaluate the efficiency of an equilibrium strategy profile we must look at the unconditional expected settlement costs. The unconditional expected settlement costs incurred by a bank under each of the four possible strategy profiles are as follows:¹⁷

$$\hat{c}^{\mathcal{C}}(m(\cdot), m(\cdot)) = py + qy, \quad (13)$$

$$\hat{c}^{\mathcal{C}}(a(\cdot), m(\cdot)) = pw + qy, \quad (14)$$

$$\hat{c}^{\mathcal{C}}(m(\cdot), a(\cdot)) = 2py + qy, \quad (15)$$

$$\hat{c}^{\mathcal{C}}(a(\cdot), a(\cdot)) = p(w + y) + qy. \quad (16)$$

The efficient strategy profile is given by lemma 3.

Lemma 3 *The strategy profile $(m(\cdot), m(\cdot))$ is efficient.*

Proof. Observe that $2\hat{c}^{\mathcal{C}}(m(\cdot), m(\cdot)) = 2(py + qy)$, $\hat{c}^{\mathcal{C}}(a(\cdot), m(\cdot)) + \hat{c}^{\mathcal{C}}(m(\cdot), a(\cdot)) = pw + 2(py + qy)$, and $2\hat{c}^{\mathcal{C}}(a(\cdot), a(\cdot)) = 2w + 2(py + qy)$. The result follows from definition 3.

■

Based on lemmas 1, 2, and 3 we have the following result:

Proposition 2 *In the collateralized credit regime, the Bayesian equilibrium is efficient if the cost of delaying is greater than the opportunity cost of collateral. Otherwise it is inefficient.*

¹⁷Recall, that the cost is for the bank playing the strategy listed first facing an opponent who plays the strategy listed second.

3.3 Priced Credit

In the priced credit case, the expected settlement costs of a bank conditional on having received a payment request in the morning are given by:

$$\tilde{c}^{\mathcal{P}}(m, m(\cdot)) = (1-p)x + (1-p)(1-q(1-q))x + (1-q)qx, \quad (17)$$

$$\tilde{c}^{\mathcal{P}}(m, a(\cdot)) = x + (1-p)(1-q(1-q))x + (1-q)qx, \quad (18)$$

$$\tilde{c}^{\mathcal{P}}(a, m(\cdot)) = w + (1-p)(1-q(1-q))x + (1-q)qx, \quad (19)$$

$$\tilde{c}^{\mathcal{P}}(a, a(\cdot)) = w + (1-p)(1-q(1-q))x + (1-q)qx. \quad (20)$$

The settlement costs are again divided into three components. The first component reflects the expected cost of the morning request in the morning period. In the priced credit regime, no cost is incurred if payment requests are processed in the same period. Hence, the settlement cost in the morning depends upon both the strategy played by the opponent and the probability by which the opponent receives a payment request in the morning.

The first component is $(1-p)x$ if the bank takes the action morning and the opponent plays the morning strategy because with probability p the bank will receive a payment from the opponent that will offset the overdraft incurred by processing its own request. The first component is x if the bank processes the request and the opponent plays the afternoon strategy and it is w regardless of what the opponent does if the bank decides to delay. The last two components give the expected cost of the morning request in the afternoon period and the expected cost of an afternoon request, respectively. The expected cost of afternoon liquidity is the same across all action-strategy profiles.

The type 1 game for the priced credit regime is given in Table 4.¹⁸

¹⁸The amount $(1-p)(1-q(1-q))x + (1-q)qx$ is common to each of the expected settlement costs in the price credit regime. Hence, this amount is excluded from each of the payoffs in the presentation of the normal form game.

		Bank <i>B</i>	
		<i>m</i>	<i>a</i>
Bank <i>A</i>	<i>m</i>	$-(1-p)x$	$-w$
	<i>a</i>	$-x$	$-w$

Table 4: Type 1 Game - Price Credit

The Bayesian equilibrium of the intraday liquidity management game under the priced credit regime is characterized by lemmas 4 through 6.

Lemma 4 *If $w < (1 - p)x$, then the strategy profile $(a(\cdot), a(\cdot))$ is the unique Bayesian equilibrium.*

Lemma 5 *If $(1 - p)x < w < x$, then the strategy profiles $(a(\cdot), a(\cdot))$ and $(m(\cdot), m(\cdot))$ are Bayesian equilibria.*

Lemma 6 *If $w > x$, then the strategy profile $(m(\cdot), m(\cdot))$ is the unique Bayesian equilibrium.*

The equilibrium analysis of the intraday liquidity management game is somewhat more complicated than in the previously discussed regimes. If there is no uncertainty with respect to morning requests i.e., $p = 1$ then $(m(\cdot), m(\cdot))$ is the Bayesian equilibrium regardless of the relative magnitudes of x and w . Furthermore, as in the free and collateralized intraday credit regimes, $(m(\cdot), m(\cdot))$ is the Bayesian equilibrium when the cost of delay is greater than the cost of liquidity, i.e., $w > x$. On the contrary $(a(\cdot), a(\cdot))$ is the unique Bayesian equilibrium if $w < (1 - p)x$. This case is more likely when there is only a small chance of receiving an offsetting payment in the morning, i.e., for low values of p .

3.3.1 Stag hunt

In the intermediate case where $(1 - p)x < w < x$ the type 1 game has the structure of a well known *coordination* game, called the *stag hunt* game.¹⁹ The structure of the stag hunt game is shown in Table 5.

¹⁹The stag hunt game is based on the description of the stag hunt provided by the 18th century French philosopher Jean-Jacques Rousseau. See Fudenberg and Tirole (1993, pp. 3).

		Player <i>B</i>	
		Stag	Hare
Player <i>A</i>	Stag	<i>A</i>	<i>C</i>
	Hare	<i>B</i>	<i>D</i>

Table 5: Stag hunt game

The key features of the game are that $A > D > B$ and $A > C > B$. This implies that no player has a dominant strategy and there are two Nash equilibria, (Stag, Stag) and (Hare, Hare). The Nash equilibrium (Stag, Stag) yields the best outcome for both players but the other Nash equilibrium (Hare, Hare) is also plausible. The reason is that a player is assured a mediocre payoff by chasing the hare that, while not as good as the payoff he gets when both chase the stag, is better than what he gets from chasing the stag alone. In fact, (Hare, Hare) is the maximin solution. The type 1 game under a priced credit regime has the structure of the stag hunt game with $C = D$. Since we are assuming $(1 - p)x < w$ it is clear from Table 4 that action profile (m, m) is better than (a, a) for both banks. However, a cautious bank may choose to delay to ensure a payoff of $-w$, rather than risk a payoff of $-x$ for a chance of getting the preferred payoff of $-(1 - p)x$.

One criterion for evaluating these options is Harsanyi and Selten's (1988) notion of risk dominance. In symmetric 2×2 games, such as ours, the action profile (α, α') , $\alpha \in \{m, a\}$, is a risk dominant equilibrium if both players prefer the action α when their prediction is that the opponent randomizes $1/2, 1/2$ over the actions morning and afternoon. Depending on the parameters, risk dominance selects either the (morning, morning) or the (afternoon, afternoon) action profile. It is straight forward to show that the action profile (m, m) is the risk-dominant equilibrium if $w > x(1 - .5p)$ and the action profile (a, a) is the risk dominant equilibrium if $w < x(1 - .5p)$.

3.3.2 Efficiency

The unconditional expected settlement costs incurred by a bank under each of the four possible strategy profiles are as follows:

$$\hat{c}^{\mathcal{P}}(m(\cdot), m(\cdot)) = p(1-p)(1-q+2q^2)x + q(1-q)x, \quad (21)$$

$$\hat{c}^{\mathcal{P}}(m(\cdot), a(\cdot)) = (2p(1-q+q^2) + p^2(2q(1-q) - 1))x + q(1-q)x, \quad (22)$$

$$\hat{c}^{\mathcal{P}}(a(\cdot), m(\cdot)) = pw - pq(1-p-2q+2qp)x + q(1-q)x, \quad (23)$$

$$\hat{c}^{\mathcal{P}}(a(\cdot), a(\cdot)) = pw + p(1-p)(1-2q+2q^2)x + q(1-q)x. \quad (24)$$

Efficiency is characterized by two lemmas.

Lemma 7 *If $w < (1-p)qx$, then the strategy profile $(a(\cdot), a(\cdot))$ is efficient.*

Proof. Suppose $w < (1-p)qx$. Then, from equations (21) and (24),

$$\hat{c}^{\mathcal{P}}(a(\cdot), a(\cdot)) - \hat{c}^{\mathcal{P}}(m(\cdot), m(\cdot)) = p(w - (1-p)qx) < 0 \quad (25)$$

and from equations (22), (23) and (24)

$$2\hat{c}^{\mathcal{P}}(a(\cdot), a(\cdot)) - (\hat{c}^{\mathcal{P}}(m(\cdot), a(\cdot)) + \hat{c}^{\mathcal{P}}(a(\cdot), m(\cdot))) = p(w - (1-p)qx - px) < 0. \quad (26)$$

Hence, by definition 3, the strategy profile $(a(\cdot), a(\cdot))$ is efficient. ■

Lemma 8 *If $w > (1-p)qx$, then the strategy profile $(m(\cdot), m(\cdot))$ is efficient.*

Proof. Similar to the proof of Lemma 7. ■

Unlike in the collateralized credit regime it is now possible for the strategy profile $(a(\cdot), a(\cdot))$ to be efficient. This happens when the cost of delaying is sufficiently small and the underlying payment flow is skewed towards the afternoon period. If there is no payment in the afternoon, $q = 0$, then $(m(\cdot), m(\cdot))$ is always efficient.

We summarize the results from lemmas 4 through 8 in Table 6. BE stands for Bayesian equilibrium and RD stands for risk dominance.

BE	$a(\cdot), a(\cdot)$	$a(\cdot), a(\cdot) \& m(\cdot), m(\cdot)$	$m(\cdot), m(\cdot)$
RD	$a(\cdot), a(\cdot)$		$m(\cdot), m(\cdot)$
Efficient	$a(\cdot), a(\cdot)$	$m(\cdot), m(\cdot)$	

$\begin{array}{ccccccc} | & & | & & | & & | \\ 0 & & (1-p)qx & & (1-p)x & & (1-.5p)x & & x & & > w \end{array}$

Table 6: Equilibrium analysis - Priced Intraday Credit

Efficiency of the Bayesian equilibrium and risk dominant profiles is summarized in Table 7.

BE	Efficient	Inefficient	?	Efficient
RD	Efficient	Inefficient		Efficient

$\begin{array}{ccccccc} | & & | & & | & & | \\ 0 & & (1-p)qx & & (1-p)x & & (1-.5p)x & & x & & > w \end{array}$

Table 7: Efficiency - Priced Intraday Credit

4 Extensions of Model

In this section, we discuss two extensions of the model. First we consider the implication of introducing risk aversion, and second we consider repeated play of the intraday liquidity management game.

4.1 Risk Aversion

Above we assumed that the banks were risk neutral and thus that they only care about expected settlement costs when determining the best response to the strategy played by the other bank. However, one could argue that it is realistic to model banks as being risk averse. Banks might have an aversion to high levels of overdrafts because this exposes the bank to the risk of being cornered in the money market. Furthermore, banks might have an aversion

to highly variable intraday balances which induces extra liquidity management costs in order to insure a sufficient level of funding at the end of the day. We introduce risk aversion by assuming that the preferences of each bank have an expected utility representation. Let the expected utility of a bank that receives a payment request in the morning and takes action α and faces an opponent who plays the strategy $s(\cdot)$ be given by

$$u^{\mathcal{I}}(\alpha_i, s_{-i}(\cdot)) = E_{\Theta_{-i}} [E_{\Psi} [v(\pi_i^{\mathcal{I}}(\alpha_i, s_{-i}(\theta_{-i}), \theta, \psi))] | \theta_i = 1] \quad (27)$$

In the expected utility expression, $v(\cdot)$ denotes a preference-scaling function with $v'(\cdot) > 0$ and $v''(\cdot) < 0$; and $\pi_i^{\mathcal{I}}(\alpha, \theta, \psi)$ denotes the payoff to bank i as defined in equation (1).

Under risk neutrality a Bayesian equilibrium is determined by comparing the expected costs of the banks for the different actions chosen in the type 1 game. If risk aversion is introduced this expected value comparison is no longer sufficient to rank the different possible actions. However, it is well known that if the cumulative distribution function of the payoffs resulting from action α_i exhibits at least second-order stochastic dominance over the cumulative distribution function of the payoffs resulting from action α'_i then

$$u^{\mathcal{I}}(\alpha_i, s_{-i}(\cdot)) \geq u^{\mathcal{I}}(\alpha'_i, s_{-i}(\cdot)) \quad (28)$$

(e.g., Hirshleifer and Riley, 1992). It turns out that the following propositions hold:

Proposition 3 *Introducing risk aversion does not alter the conclusions of the intraday liquidity management game in a collateralized credit regime.*

Proposition 4 *Introducing risk aversion does not alter the conclusions of the intraday liquidity management game in a priced credit regime if there is a unique Bayesian equilibrium.*

The propositions are proven in appendix B. Together they show that introducing risk aversion does not alter the outcome of the intraday liquidity management game except (possibly) in instances of the priced credit regime where there are multiple equilibria (see Table 6). In such cases, risk aversion does not change the occurrence of the (afternoon, afternoon) equilibrium. However, depending on how risk aversion is introduced, i.e., the functional form of the preference scaling function $v(\cdot)$, it may eliminate the (morning, morning) equilibrium.

4.2 Repeated Game

In reality, the intraday day liquidity management game is played repeatedly, on a daily basis. A game played repeatedly might yield different equilibrium outcomes than when the same game is only played once. The key is that cooperation today can be rewarded by cooperation tomorrow and cheating can be punished by not cooperating tomorrow. It is thus not always optimal to pursue a short run gain in a repeated game. Different types of trigger strategies where cheating is punished can help sustain a cooperative equilibrium. This result hinges on the assumptions that the game is either played infinitely (or that the final period is unknown) and that the actions of the opponent are observable.

The first assumption is reasonable for the intraday liquidity management game but the second one may not be. The banks cannot observe the action of their opponent: They can observe the arrival time of payments but they are not able to decipher whether a payment received in the afternoon is a delayed payment or due to a request received in the afternoon. Over time, banks can compare the actual payment arrival frequencies with the underlying payment-request probability distributions (which are assumed to be common knowledge), and thus they can potentially detect cheating. Hence, it may be reasonable to assume that banks take advantage of opportunities for cooperation that arise in a repeated game setting.

In what follows, we consider the efficiency implications of repeated play in the collateralized credit regime. Recall, that in a collateralized credit regime the strategy profile $(m(\cdot), m(\cdot))$ is efficient for all parameter values. For $w > y$ the strategy profile $(m(\cdot), m(\cdot))$ is the unique Bayesian equilibrium (by Lemma 2) and is mutually preferred by both players to $(a(\cdot), a(\cdot))$, and hence we expect to see the morning, morning profile even in a repeated game. When $w < y$ the strategy profile $(a(\cdot), a(\cdot))$ is the unique Bayesian equilibrium (by Lemma 1). This is despite the fact that in some of the cases where $w < y$ the strategy profile $(m(\cdot), m(\cdot))$ is mutually preferred.

As mentioned in section 3.2.1, in a collateralized credit regime with $y(1 - p) < w < y$ the type 1 game has the structure of a prisoner’s dilemma game. In such cases, submitting morning payment requests in the afternoon is a dominant strategy for both banks even though they would both be better off if they submitted their requests in the morning. Based on well-known theory of repeated games, we expect the banks to coordinate on $(m(\cdot), m(\cdot))$ in a repeated version of the game, thus achieving the mutually preferred outcome.

In cases where $w < y(1 - p)$, the strategy profile $(a(\cdot), a(\cdot))$ is mutually preferred by both banks to the profile $(m(\cdot), m(\cdot))$, even though $(m(\cdot), m(\cdot))$ is efficient, as stated in definition 3. Banks playing a repeated version of this game will coordinate on the afternoon, afternoon strategy profile. Hence, in such cases, we do not expect efficiency even in the repeated game setting.

The efficiency implications of repeated game play in the collateralized credit regime are summarized in Table 8.

Static (BE)	Inefficient		Efficient
Repeated Game	Inefficient	Efficient	
	0	$y(1 - p)$	y

$> w$

Table 8: Equilibrium analysis - Repeated Game

As we see, moving to a repeated game setting improves the chances for efficiency over the one-shot game scenario.

5 Conclusion

In this paper we use a game theoretical model to analyze bank behavior under three intraday credit regimes. In our benchmark case of free intraday liquidity the unique equilibrium is for banks to settle early and this is efficient in terms of minimizing aggregate expected settlement

costs. However, the regime is not considered a viable option by central banks due to risk considerations and moral hazard issues.

In the case of collateralized credit, depending on magnitudes of the opportunity cost of collateral and the cost of delaying, both early settlement and delaying are possible equilibria. However, delaying payments is always inefficient. In some instances, the game played by banks has the structure of a prisoner's dilemma game. This means that in the static game, the inefficient outcome emerges as an equilibrium in dominant strategies. However, in a repeated game there is reason to expect banks will be able to coordinate on the mutually preferred, efficient outcome.

In the priced credit regime it is again the case that both early settlement and delaying are possible equilibria. However, with priced credit the equilibrium depends not only on the relative magnitude of the cost of liquidity and the cost of delaying, but also on the probability that the opponent receives a payment request. The opponent's payment probability matters because there are benefits to synchronizing payments under priced credit.

Equilibria with delaying are interesting as they match the empirical finding in Fedwire that both the number and value of payments peak in the late afternoon (see McAndrews and Rajan, 2000). In some instances of priced credit, delaying is efficient. This happens if the cost of liquidity is greater than the cost of delaying and if the underlying payment request flow is sufficiently skewed towards the afternoon, i.e., low p and high q .

In a priced credit regime the intraday liquidity management game can have the structure of a stag hunt game. Thus there are multiple equilibria. Here the policy issue is how to get banks to coordinate on the efficient equilibrium. Experimental evidence supports the notion that non-binding announcements, i.e., "cheap talk" can be useful to coordinate expectations on the efficient, and mutually preferred outcome (see Charness, 2000). This suggests that it might be in the best interest of banks to announce their processing policies even though

these announcements are non binding.

Efficiency is not guaranteed in the collateralized or priced credit regimes where intraday credit is costly. Since free credit is not a viable option the central bank must consider other options to promote efficiency. One approach to is to augment the rules of the game in order to tilt the incentive structure towards producing the desired outcome. For instance, the central bank or the commercial banks among themselves might require that banks submit a certain percentage of their payments before some specific time, i.e., noon in the context of this model.²⁰ However, monitoring is costly and removing the inefficiency requires that one can set the percentage(s) at the efficient level. Moreover, the analysis showed that in a priced credit regime this solution could even be counterproductive by requiring banks to settle in the morning when the efficient strategy profile involves delaying payment requests until the afternoon.

A second remedy is for the central bank to price settlement differently over the course of the business day in order to give the banks the incentive to settle early.²¹ However, if the central bank cannot observe when a payment request arrives at the commercial bank, it cannot price discriminate perfectly and thus payment requests that are received late will be overcharged.

The remedies just mentioned seek to improve efficiency by influencing the strategic behavior of the banks. A more direct approach is for the central bank to intervene in a way that eliminates strategic aspects altogether. This is done by inducing banks to provide their private information on delayed payments to the central bank. The central bank can use this information to find the largest possible subset of delayed payments that can be settled without requiring additional liquidity from the participants. The problem of choosing such

²⁰For example, in the United Kingdom members of the RTGS system (NewCHAPS) are required to manage their payment flows in such a way that on average 50% of the value throughput is sent by noon and 75% is sent by 2:30 pm.

²¹For instance, SIC applies a pricing schedule for sending banks that penalises late input (Bank for International Settlements, 1997, p. 19).

a subset of payments without breaching any liquidity limits is usually referred to as gridlock resolution and has recently been discussed in Bech and Soramäki (2001). RTGS systems operating under a collateral credit regime often offer a centralized queuing facility and the information is readily available, especially if banks do not utilize internal queues. Hybrid systems that actively combine gross settlement with liquidity savings features have been implemented in a number of countries, perhaps most notably in Germany with the RTGS^{plus} system.

6 Appendix

A Settlement Cost

In this appendix we provide the settlement costs used in the intraday liquidity management game. The settlement costs for the four possible type profiles in Θ are shown in Tables 9 - 12. If $\theta = (0, 0)$, for example, the cost of processing a payment request received in the afternoon is y when $\mathcal{I} = \mathcal{C}$ and either x or 0 when $\mathcal{I} = \mathcal{P}$ depending on whether or not the opponent also has a payment to process (see Table 9). Alternatively, suppose $\theta = (1, 0)$, $\psi = (0, 1)$ and $\alpha_i = m$ (i.e., the bank that receives the morning payment request chooses to process it immediately). The bank will have to post collateral for two periods at the cost $2y$ if $\mathcal{I} = \mathcal{C}$ and to overdraw its account for the morning period at the cost x when $\mathcal{I} = \mathcal{P}$ (See Table 10). The opponent will incur no cost since it can use the incoming liquidity to process its payment request in the afternoon. If the bank chooses to delay the payment its cost is $w + y$ if $\mathcal{I} = \mathcal{C}$ and w if $\mathcal{I} = \mathcal{P}$ (Table 10). Note, however, that when $\mathcal{I} = \mathcal{C}$ the decision to delay imposes the cost y on the opponent since it can no longer use incoming liquidity to process its own payments requests. The opponents costs are seen by examining row n , a , $(1, 0)$ in Table 11. In some cases, for example if $\theta = (1, 0)$, $\alpha_i = m$, $\alpha_{-i} = n$ and $\psi = (1, 0)$, the fee e appears. This represents a payment to the other bank for funds made available through the

intraday money market. Notice that the fee e shows up as a negative cost, i.e., as income for the bank which provides liquidity. See the row where $\alpha_i = n$, $\alpha_{-i} = m$ and $\psi = (0, 1)$ in Table 11.

α_i	α_{-i}	ψ	$c_i^{\mathcal{F}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{C}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{P}}(\alpha, \theta, \psi)$
n	n	$0, 0$	0	0	0
n	n	$1, 0$	0	y	x
n	n	$0, 1$	0	0	0
n	n	$1, 1$	0	y	0

Table 9: Settlement costs when taking action α_i (given opponent's action α_{-i}) for $\theta = (0, 0)$

α_i	α_{-i}	ψ	$c_i^{\mathcal{F}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{C}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{P}}(\alpha, \theta, \psi)$
m	n	$0, 0$	0	$y + e$	$x + e$
m	n	$1, 0$	0	$2y + e$	$2x + e$
m	n	$0, 1$	0	$2y$	x
m	n	$1, 1$	0	$3y$	$2x$
a	n	$0, 0$	w	$w + y$	$w + x$
a	n	$1, 0$	w	$w + 2y$	$w + 2x$
a	n	$0, 1$	w	$w + y$	w
a	n	$1, 1$	w	$w + 2y$	$w + x$

Table 10: Settlement costs when taking action α_i (given opponent's action α_{-i}) for $\theta = (1, 0)$

α_i	α_{-i}	ψ	$c_i^{\mathcal{F}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{C}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{P}}(\alpha, \theta, \psi)$
n	m	$0, 0$	0	$-e$	$-e$
n	m	$1, 0$	0	0	0
n	m	$0, 1$	0	$-e$	$-e$
n	m	$1, 1$	0	0	0
n	a	$0, 0$	0	0	0
n	a	$1, 0$	0	y	0
n	a	$0, 1$	0	0	0
n	a	$1, 1$	0	y	0

Table 11: Settlement costs when taking action α_i (given opponent's action α_{-i}) for $\theta = (0, 1)$

α_i	α_{-i}	ψ	$c_i^{\mathcal{F}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{C}}(\alpha, \theta, \psi)$	$c_i^{\mathcal{P}}(\alpha, \theta, \psi)$
m	m	0, 0	0	y	0
m	m	1, 0	0	$2y$	x
m	m	0, 1	0	y	0
m	m	1, 1	0	$2y$	0
m	a	0, 0	0	$2y$	x
m	a	1, 0	0	$3y$	$2x$
m	a	0, 1	0	$2y$	x
m	a	1, 1	0	$3y$	x
a	m	0, 0	w	w	w
a	m	1, 0	w	$w + y$	$w + x$
a	m	0, 1	w	w	w
a	m	1, 1	w	$w + y$	w
a	a	0, 0	w	$w + y$	w
a	a	1, 0	w	$w + 2y$	$w + x$
a	a	0, 1	w	$w + y$	w
a	a	1, 1	w	$w + 2y$	w

Table 12: Settlement costs when action α_i (given opponent's action α_{-i}) for $\theta = (1, 1)$

B Stochastic Dominance

In this appendix we utilize stochastic dominance relationships to prove propositions 3 and 4. The propositions relate to the effect of risk aversion on the type 1 game. Hence, we focus on the payoffs of a bank who receives a payment request in the morning and takes action α_{-i} , while its opponent plays strategy $s_{-i}(\cdot)$. The cumulative distribution functions for the payoffs corresponding to each action-strategy pair $\alpha, s(\cdot)$ are shown in Tables 13 - 16 and Tables 17 - 20 for the collateralized credit and priced credit regimes, respectively. Let $\pi^{\mathcal{I}}(\alpha_i, s_{-i}(\cdot))$ denote the random variable that gives the possible payoffs for a given action-strategy pair. In the tables, $P[\pi^{\mathcal{I}}(\alpha_i, s_{-i}(\cdot))]$ denotes the probability of each payoff; and $F_{\pi^{\mathcal{I}}(\alpha_i, s_{-i}(\cdot))}$ denotes the cumulative distribution function.

B.1 Collateralized Credit

The cumulative distribution functions for payoffs in the type 1 game under the collateralized credit regime are shown in Tables 13 - 16.

$\pi^c(m, m(\cdot))$	$P[\pi^c(m, m(\cdot))]$	$F_{\pi^c(m, m(\cdot))}$
$-3y$	$q - pq$	$q(1 - p)$
$-2y$	$2pq + 1 - q - p$	$pq + 1 - p$
$-y$	$p - pq$	1

Table 13: $m, m(\cdot)$

$\pi^c(a, m(\cdot))$	$P[\pi^c(a, m(\cdot))]$	$F_{\pi^c(a, m(\cdot))}$
$-(2y + w)$	$q - pq$	$q(1 - p)$
$-(y + w)$	$2pq + 1 - q - p$	$pq + 1 - q$
$-w$	$p - pq$	1

Table 14: $a, m(\cdot)$

$\pi^c(m, a(\cdot))$	$P[\pi^c(m, a(\cdot))]$	$F_{\pi^c(m, a(\cdot))}$
$-3y$	q	q
$-2y$	$1 - q$	1

Table 15: $m, a(\cdot)$

$\pi^c(a, a(\cdot))$	$P[\pi^c(a, a(\cdot))]$	$F_{\pi^c(a, a(\cdot))}$
$-(2y + w)$	q	q
$-(y + w)$	$1 - q$	1

Table 16: $a, a(\cdot)$

Let \succ_{FSD} and \succ_{SSD} denote first- and second-order stochastic dominance as defined in Hirshleifer and Riley (1992, pp. 106-108), respectively. We have the following lemma, that follows immediately from Tables 13 - 16.

Lemma 9 *In a collateralized credit regime*

$$\begin{aligned} F_{\pi^c(a,m(\cdot))} &\underset{FSD}{\succ} F_{\pi^c(m,m(\cdot))} && \text{if } w < y, \\ F_{\pi^c(m,m(\cdot))} &\underset{FSD}{\succ} F_{\pi^c(a,m(\cdot))} && \text{if } w > y. \end{aligned} \quad (29)$$

and

$$\begin{aligned} F_{\pi^c(a,a(\cdot))} &\underset{FSD}{\succ} F_{\pi^c(m,a(\cdot))} && \text{if } w < y, \\ F_{\pi^c(m,a(\cdot))} &\underset{FSD}{\succ} F_{\pi^c(a,a(\cdot))} && \text{if } w > y. \end{aligned} \quad (30)$$

B.2 Priced Credit

The cumulative distribution functions for payoffs in the type 1 game under the priced credit regime are shown in Tables 17 - 20.

$\pi^P(m, m(\cdot))$	$P[\pi^P(m, m(\cdot))]$	$F_{\pi^P(m, m(\cdot))}$
$-3x$	$(1-p)q(1-q)$	$(1-p)q(1-q)$
$-2x$	$2q^2 - 2pq^2 + 1 - 2q - p + 2pq$	$1 - p - q(1-q) + pq(1-q)$
$-x$	$q - q^2$	$1 - p + pq - pq^2$
0	$p - pq + pq^2$	1

Table 17: $m, m(\cdot)$

$\pi^P(a, m(\cdot))$	$P[\pi^P(a, m(\cdot))]$	$F_{\pi^P(a, m(\cdot))}$
$-(2x+w)$	$(1-p)q(1-q)$	$(1-p)q(1-q)$
$-(x+w)$	$1 - 2q + 2q^2 - p + 3pq - 3pq^2$	$1 - p - q(1-q) + 2pq(1-q)$
$-w$	$q - q^2 - 2pq + 2pq^2 + p$	1

Table 18: $a, m(\cdot)$

$\pi^P(m, a(\cdot))$	$P[\pi^P(m, a(\cdot))]$	$F_{\pi^P(m, a(\cdot))}$
$-3x$	$(1-p)q(1-q)$	$(1-p)q(1-q)$
$-2x$	$1 - 2q + 2q^2 - p + 3pq - 3pq^2$	$1 - p - q(1-q) + 2pq(1-q)$
$-x$	$q - q^2 - 2pq + 2pq^2 + p$	1

Table 19: $m, a(\cdot)$

$\pi^{\mathcal{P}}(a, a(\cdot))$	$P [\pi^{\mathcal{P}}(a, a(\cdot))]$	$F_{\pi^{\mathcal{P}}(a, a(\cdot))}$
$-(2x + w)$	$(1 - p)q(1 - q)$	$(1 - p)q(1 - q)$
$-(x + w)$	$1 - 2q + 2q^2 - p + 3pq - 3pq^2$	$1 - p - q(1 - q) + 2pq(1 - q)$
$-w$	$q - q^2 - 2pq + 2pq^2 + p$	1

Table 20: $a, a(\cdot)$

We have the following lemmas:

Lemma 10 *In a priced credit regime*

$$\begin{aligned}
F_{\pi^{\mathcal{P}}(a, a(\cdot))} &\underset{FSD}{\succ} F_{\pi^{\mathcal{P}}(m, a(\cdot))} \quad \text{if } w < x, \\
F_{\pi^{\mathcal{P}}(m, a(\cdot))} &\underset{FSD}{\succ} F_{\pi^{\mathcal{P}}(a, a(\cdot))} \quad \text{if } w > x.
\end{aligned} \tag{31}$$

Proof. Compare the cumulative distribution functions $F_{\pi^{\mathcal{P}}(\alpha, s(\cdot))}$ in Tables 19 and 20. ■

Lemma 11 *In a priced credit regime*

$$\begin{aligned}
F_{\pi^{\mathcal{P}}(a, m(\cdot))} &\underset{SSD}{\succ} F_{\pi^{\mathcal{P}}(m, m(\cdot))} \quad \text{if } w < (1 - p)x, \\
F_{\pi^{\mathcal{P}}(m, m(\cdot))} &\underset{FSD}{\succ} F_{\pi^{\mathcal{P}}(a, m(\cdot))} \quad \text{if } w > x.
\end{aligned} \tag{32}$$

Proof. The second part of the lemma follows immediately from the cumulative distribution functions in Tables 17 and 18. For the first part of the lemma note that $F_{\pi^{\mathcal{P}}(a, m(\cdot))}$ second-order stochastically dominates $F_{\pi^{\mathcal{P}}(m, m(\cdot))}$ if for $c \in \mathbb{R}$

$$H(c) = \int_{-\infty}^c (F_{\pi^{\mathcal{P}}(a, m(\cdot))}(r) - F_{\pi^{\mathcal{P}}(m, m(\cdot))}(r)) dr \leq 0 \tag{33}$$

with the inequality holding strict for some part of the range (e.g. Hirshleifer and Riley,

1992). Assuming $w < x$, it follows from Tables 17 and 18 that

$$H(c) = \begin{cases} 0 & \text{if } c < -3x \\ -(1-p)q(1-q)(3x+c) & \text{if } -3x \leq c < -(2x+w) \\ -(1-p)q(1-q)(x-w) & \text{if } -(2x+w) \leq c < -2x \\ -(1-p)q(1-q)(x-w) - (1-p-2q(1-q) \\ +2pq(1-q))(2x+c) & \text{if } -2x \leq c < -(x+w) \\ -(1-p-q(1-q)+pq(1-q))(x-w) \\ +pq(1-q)(x+w+c) & \text{if } -(x+w) \leq c < -x \\ -(1-p-q(1-q)+pq(1-q))(x-w) \\ +pq(1-q)w - (q(1-q)-pq(1-q))(x+c) & \text{if } -x \leq c < -w \\ -(1-p)(x-w) + pq(1-q)w \\ +(p-pq(1-q))(w+c) & \text{if } -w \leq c < 0 \\ w - (1-p)x & \text{if } c = 0 \end{cases} \quad (34)$$

Some algebraic manipulations show that $H(c) \leq 0$ for all c provided $w \leq (1-p)x$. ■

We are now ready to prove Propositions 5 and 6.

Proof of Proposition 5. From Lemma 2, if $w > y$, then the action profile (m, m) is the Nash equilibrium of the type 1 game. Suppose $w > y$. Then by Lemma 9, $F_{\pi^c(m, m(\cdot))} \succ_{FSD} F_{\pi^c(a, m(\cdot))}$ and $F_{\pi^c(m, a(\cdot))} \succ_{FSD} F_{\pi^c(a, a(\cdot))}$ and so, by Ranking Theorem I of Hirshleifer and Riley (1992, pp. 106), $u^c(m, m(\cdot)) > u^c(a, m(\cdot))$ and $u^c(m, a(\cdot)) > u^c(a, a(\cdot))$. Hence m is a dominant strategy for either player in the type 1 game, i.e., it is a best response to both $m(\cdot)$ and $a(\cdot)$.

From Lemma 1, if $w < y$, then the action profile (a, a) is the Nash equilibrium of the type

1 game. Suppose $w < y$. Then by Lemma 9, $F_{\pi^c(a,m(\cdot))} \succ_{FSD} F_{\pi^c(m,m(\cdot))}$ and $F_{\pi^c(a,a(\cdot))} \succ_{FSD} F_{\pi^c(m,a(\cdot))}$ and so, by Ranking Theorem I of Hirshleifer and Riley (1992), $u^c(a, m(\cdot)) > u^c(m, m(\cdot))$ and $u^c(a, a(\cdot)) > u^c(m, a(\cdot))$. Hence a is a dominant strategy for either player in the type 1 game, i.e., it is a best response to both $m(\cdot)$ and $a(\cdot)$. ■

Proof of Proposition 6. From Lemma 6, if $w > x$, then the action profile (m, m) is the Nash equilibrium of the type 1 game. Suppose $w > x$. Then by Lemmas 10 and 11, $F_{\pi^c(m,m(\cdot))} \succ_{FSD} F_{\pi^c(a,m(\cdot))}$ and $F_{\pi^c(m,a(\cdot))} \succ_{FSD} F_{\pi^c(a,a(\cdot))}$ and so, by Ranking Theorem I of Hirshleifer and Riley (1992), $u^p(m, m(\cdot)) > u^p(a, m(\cdot))$ and $u^p(m, a(\cdot)) > u^p(a, a(\cdot))$. Hence m is a dominant strategy for either player in the type 1 game, i.e., it is a best response to both $m(\cdot)$ and $a(\cdot)$.

From Lemma 4, if $w < (1-p)x$, then the action profile (a, a) is the Nash equilibrium of the type 1 game. Suppose $w < (1-p)x$. Then by Lemmas 10 and 11, $F_{\pi^p(a,a(\cdot))} \succ_{FSD} F_{\pi^p(m,a(\cdot))}$ and $F_{\pi^p(a,m(\cdot))} \succ_{SSD} F_{\pi^p(m,m(\cdot))}$, and so, by Ranking Theorems I and II of Hirshleifer and Riley (1992, pp. 106-108), $u^p(a, m(\cdot)) > u^p(m, m(\cdot))$ and $u^p(a, a(\cdot)) > u^p(m, a(\cdot))$. Hence a is a dominant strategy for either player in the type 1 game, i.e., it is a best response to both $m(\cdot)$ and $a(\cdot)$. ■

In section 4.1 the claim is made that under the priced credit regime, risk aversion does not change the occurrence of the (afternoon, afternoon) equilibrium, but may eliminate the (morning, morning) equilibrium. This is apparent from the fact that for $(1-p)x < w < x$, $F_{\pi^p(a,a(\cdot))} \succ_{FSD} F_{\pi^p(m,a(\cdot))}$, so the action a is a best response to $a(\cdot)$ for each player, even with risk aversion. In the same parameter range, $F_{\pi^p(m,m(\cdot))}$ does not stochastically dominate $F_{\pi^p(a,m(\cdot))}$. Hence, there is no guarantee that the action m is a best response to $m(\cdot)$ for each player when risk aversion is present.

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