Kind, Hans Jarle; Koethenbuerger, Marko; Schjelderup, Guttorm

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Tax Responses in Platform Industries

Hans Jarle Kind, Marko Koethenbuerger, and Guttorm Schjelderup
Tax Responses in Platform Industries

By Hans Jarle Kind*, Marko Koethenbuerger†, and Guttorm Schjelderup‡

* Department of Economics, Norwegian School of Economics and Business Administration, Bergen; e-mail: hans.kind@nhh.no.
† Department of Economics, University of Copenhagen, Copenhagen; e-mail: marko.koethenbuerger@econ.ku.dk
‡ Department of Finance and Management Science, Norwegian School of Economics and Business Administration, Bergen; e-mail: guttorm.schjelderup@nhh.no.

Abstract

Two-sided platform firms serve distinct customer groups that are connected through interdependent demand, and include major businesses such as the media industry, banking, and the software industry. A well known result of tax incidence is that consumers of a more heavily taxed good pay a higher price and thus buy less of the good. The present paper shows that this result need not hold in a two-sided market. On the contrary, a higher ad valorem tax may lower end-user prices and spur sales. Thus, two-sided platform firms may not at all engage in tax shifting via price increases. We further show that a higher ad valorem tax may undermine a firm’s incentive to differentiate its product from that of its competitors. Finally, we demonstrate that the effects of increasing specific taxes may be the opposite of those of increasing value added taxes.
JEL classifications: D4; D43; H21; H22; L13
1 Introduction

Media is crucial to society both in terms of economic importance and its impact on information flows. The latter issue has recently been reconsidered in a variety of papers in order to shed more light on how the media industry works. A recent string of papers, for example, has looked at what determines the accuracy of reporting (Mullainathan and Shleifer, 2005, and Gentzkow and Shapiro, 2006, 2008) and how the media industry influences voting decisions (Strömberg, 2004, and Della Vigna and Kaplan, 2007). In this paper we turn to a different aspect of the media sector, namely how taxes influence market behavior of media firms. The media industry is subject to preferential tax treatment in many countries. Newspapers, for example, are typically taxed at a reduced rate or completely exempted from value-added taxation.\footnote{In Germany, newspapers are subject to a rate of 7\% (19\% is the regular rate) while in e.g. the UK, Denmark, and Norway they are exempted from value-added taxation all together (European Commission, 2004). Newspapers are also either fully or partially exempted from sales taxes in a number of U.S. states.} The reason for this is that governments consider newspapers to be an essential channel for disseminating vital information about e.g. culture, politics, and international affairs. Thus the public policy measures undertaken have aimed to stimulate high circulation and low prices.

In what we refer to as normal or one-sided markets, it is well known that reducing the \textit{ad valorem} tax, say, lowers the consumer price and increases output.\footnote{An overview of the tax incidence literature is given by Fullerton and Metcalf (2002).} However, we show that this need not apply for the newspaper industry and other platforms that operate in so-called two-sided markets.

Two-sided platform firms cater to two distinct groups of customers that are connected through quantity spillovers, and the firms maximize profit by facilitating value-creating interactions between these groups. Two-sided platforms operate in many economically significant industries, such as the media sector, the financial sector (payment card systems), real-estate brokerage, and the computing industry (computer operating systems, software,
game consoles etc.). The pricing strategies of a platform firm must account for interactions between the demands of different customer groups and the externalities that arise in these relationships.\(^3\) For instance, in the media industry, advertising may be perceived as a nuisance (a negative externality) or a benefit (a positive externality) by readers/viewers, while advertisers benefit from an increase in readers/viewers of the media outlet. In the credit card industry there are positive quantity spillovers between merchants and cardholders. Merchants who accept a credit card welcome an increase in the number of households joining the credit card system, and vice versa.\(^4\)

We show that the sign, size and direction of externalities in two-sided markets are decisive for the effects of changes in ad valorem tax rates. Specifically, an increase in the ad valorem tax in one side of the market affects the relative profitability between the two markets, such that a firm will want to shift its earnings to the side where the tax rate is unchanged. By doing so it reduces the burden of the tax increase. Contrary to what one might expect, this may involve increasing output and reducing prices on both sides of the market. The platform may thus decide not to shift taxes via price increases. Our analysis consequently has implications for the understanding of tax incidence in two-sided markets.

The behavior of the platform firm in response to a tax increase in one side of the market can be illustrated by a media firm. A media firm is a two-sided platform that derives income from selling a newspaper and advertisements, and where the income from advertisements depends positively on newspaper sales. An increase in the ad valorem tax rate on the newspaper may induce the media firm to rely more on income from advertisements. Thus, it may reduce the price of the newspaper in order to attract more readers. A larger readership means that the newspaper becomes more attractive for the advertisers, and the media firm may therefore end up selling more of both ads

\(^3\)Evans (2003a,b) provides examples and classifications of two-sided markets.

\(^4\)As will become clear in the discussion below, it is important to distinguish the concept of two-sided markets from that of complementarities. See also Rochet and Tirole (2003).
and newspapers following a tax increase.

In a setting with a multi-product monopoly Edgeworth (1925) showed that a higher specific tax on one of two substitutable goods may reduce the end-user price of both; this possibility has later been labelled Edgeworth’s Taxation Paradox.\(^5\) However, output of the more heavily taxed good will fall.\(^6\) In this sense the ‘paradox is somewhat less puzzling’, as stressed by Salinger (1991, p. 549).\(^7\) We have a similar ‘unsurprising’ result, as we show that a higher specific tax on a good reduces output of that good also in our setting. However, as explained above, we find that in a two-sided market higher *ad valorem* taxes may increase output. In this respect it should be noted that the externalities that arise in a setting with two substitutable goods are conceptually different from those arising in a two-sided market (Rochet and Tirole, 2003, 2006).

Our analysis is related to a growing literature on Industrial Organization that analyses the price-setting behavior of firms in two-sided markets. In this literature a key result is that two-sided platform firms may find it profitable to charge prices that are below marginal cost or even negative for one of its product (customer group).\(^8\) This is in contrast to conventional markets (one-sided) where marginal cost equal to marginal revenue pricing is well established as a guidance. In such markets the effects of taxation are well known both under perfect and imperfect competition. Under imperfect competition a tax can be overshifted onto the consumer side in certain circumstances,

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\(^5\) See also follow-up contributions by Hotelling (1932), Wicksell (1934) and Bailey (1954)

\(^6\) Concretely, Edgeworth considered demand for first-class and third-class railway tickets. His assessment was that a tax imposed on first-class tickets may give the railway company an incentive to reduce the price of the untaxed good - third-class tickets - in order to sell more of it. Indeed, under certain conditions the price of both types of tickets will fall subsequent to the tax increase. See Creedy (1988) for a good overview and discussion of the related literature.

\(^7\) Salinger (1991) uses the logic of the Edgeworth Taxation Paradox to show that mergers of successive monopolies in multiproduct industries may reduce welfare.

but, in general, the burden of the tax is shared between producers and consumers depending on elasticities of supply and demand.\textsuperscript{9} Except for Kind et al. (2008), who analyse tax policy in a monopoly market, the literature on two-sided platforms does not consider taxation issues. This paper differs from Kind et al. in various ways, however. For instance, while they look at the efficient choice of taxes, we focus on the issue of tax incidence in two-sided markets. More fundamentally, in the present paper we also consider duopolistic competition. This allows us to analyse how taxes affect media pluralism. Specifically, we show that increasing the \textit{ad valorem} tax may undermine a newspaper’s incentive to differentiate its content from that of its competitors. Interestingly, a higher specific tax may have the opposite effect. In contrast, it is well known that neither \textit{ad valorem} nor specific taxes tend to affect differentiation incentives in one-sided markets.

The rest of the paper is organized as follows: Section 2 sets up the basic model, while Section 3 analyses the effects of an \textit{ad valorem} tax on prices in monopoly. Section 4 carries out an analysis with respect to specific taxes. Section 5 analyses the effects of taxes in duopoly, and section 6 concludes.

\section{The model}

Consider a two-sided monopoly platform which sells good $N$ at price $p^N$ to one group of customers and good $A$ at price $p^A$ to another group of customers. Let $n$ and $a$ denote the respective quantities of the two goods.

We assume that both customer groups are price takers. The inverse demand function for each good is downward-sloping in own quantity; $p^N_n \equiv \frac{\partial p^N}{\partial n} < 0$; $p^A_a \equiv \frac{\partial p^A}{\partial a} < 0$ (subscripts henceforth denote partial derivatives). The willingness to pay for each good may also depend on how much is sold of the other good. The sale of good $A$ imposes a positive externality on buyers of good $N$ if the willingness to pay for $N$ is increasing in output of

\textsuperscript{9}See Keen and Delipalla (1992), Dierickx \textit{et al.} (1998) and Anderson \textit{et al.} (2001a,b), and Fullerton and Metcalf (2002) for a survey.
good $A$ ($p^N_A > 0$) and a negative externality if $p^N_A < 0$. In the same manner, good $N$ may impose a positive ($p^A_N > 0$) or negative ($p^A_N < 0$) externality on the demand for good $A$. The inverse demand functions can thus be written as $p^N = p^N(n, a)$ and $p^A = p^A(n, a)$. We resort to a partial equilibrium analysis by abstracting from other determinants of demand.

For the sake of convenience, and to emphasize the economic intuition and policy relevance of our results, we shall in what follows relate our model and results to a media firm (the platform). A newspaper is a typical example of a two-sided platform firm, which derives income from two distinct customer groups (newspaper readers and advertisers), and where there are externalities between the two groups (possibly positive from readers to advertisers, and negative from advertisers to readers). In such a setting we may interpret $n$ as sales of newspapers, and $a$ as sales of advertising space to firms.

An *ad valorem* tax ($t$) is levied on sales of newspapers (good $N$), which implies that the media firm receives the price $p^N/(1 + t)$ per copy it sells of the newspaper. The tax rate $t$ may deviate from the general VAT rate $\bar{t}$ which for simplicity is set to 0. Our focal point here is to examine the effects of a change in the tax rate $t$, holding $\bar{t}$ fixed.

The newspaper (the platform) has the following profit level:

$$\pi = \max_{n,a} \left[ ap^A(a, n) + \frac{np^N(n, a)}{1 + t} - k(n, a) \right], \tag{1}$$

where $k(n, a)$ is the cost function, with $k_i \geq 0$ ($i = a, n$) and $k_{na} \geq 0$.

The first-order condition for good $A$ ($\pi_a = 0$) implies

$$[p^A + ap^A_a] - k_a = -\frac{np^N_a}{1 + t}. \tag{2}$$

The squared bracket in eq. (2) measures marginal revenue on the advertising side of the market of selling more ads. In the profit maximizing optimum in a one-sided market this term is equal to marginal cost ($k_a$) so that the left-hand

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10 This is an externality since producers and consumers are price takers. Thus, they do not take into account the effect of their actions on the demand in either side of the market.
side would be zero. However, in a two-sided market there is an additional term (right-hand side) that captures the fact that the sales of advertising (good $A$) may influence the sales of newspapers (good $N$). This term is positive if the demand for newspapers is decreasing in the level of advertising (that is, $p_n^N < 0$), while it is negative if advertising imposes a positive externality on demand for newspapers. In the former case, the level of advertising should be set lower than the level that maximizes profit in the advertising market in isolation (i.e., in a one-sided market), while the opposite is true if a larger advertising volume increases the demand for newspapers.

From the first-order condition for good $N$ ($\pi_n = 0$), we likewise find that

$$\left[ \frac{p^N + np^N}{1 + t} \right] - k_n = -ap_n^A. \quad (3)$$

The squared bracket is marginal revenue from selling the newspaper (good $N$) to consumers, and would in optimum be equal to $k_n$ in a one-sided market (i.e., when $p_n^A = 0$). However, if demand for ads is higher the larger the number of readers ($p_n^A > 0$), profit is maximized by raising the sale of newspapers beyond the volume that maximizes profit on newspaper sales in isolation (and vice versa for $p_n^A < 0$).

From the first-order conditions we see that equilibrium prices and quantities on both sides of the market depend on the tax rate. Since $p^A = p^A(a, n)$ and $p^N = p^N(n, a)$, the price changes subsequent to a tax increase are given by

$$\frac{dp^A}{dt} = p_a^A \frac{da}{dt} + p_n^A \frac{dn}{dt}, \quad \text{and} \quad \frac{dp^N}{dt} = p_n^N \frac{dn}{dt} + p_n^A \frac{da}{dt} \quad (4)$$

We shall assume that the second-order conditions for profit maximization hold, which means that $\pi_{aa} < 0$, $\pi_{nn} < 0$, and $H \equiv \pi_{aa} \pi_{nn} - \pi_{an}^2 > 0$. In order to simplify the following discussion we further state:

**Assumption:** Let $p_n^A > 0$ and $\pi_{an} > 0$.

The assumption that $p_n^A > 0$ seems reasonable in our context, since it implies that the advertisers have a higher willingness to pay for ads the
larger is the readership of the newspaper. We might also have \( p^N_a > 0 \), in which case the willingness to pay for a newspaper is increasing in the ad volume. However, empirical evidence is inconclusive as to whether consumers consider advertising to be a good or a bad.\(^\text{11}\) We shall therefore not make any assumptions regarding the sign of \( p^N_a \).

The assumption \( \pi_{an} > 0 \) ensures that the marginal profitability for the media firm of selling advertising space is increasing in the newspaper circulation.

It should be emphasized that the model is applicable to two-sided markets in general, and that our mathematical derivations and results also hold for \( p^A_n \leq 0 \) (in which case two-sidedness requires \( p^N_a > 0 \)) and/or \( \pi_{an} \leq 0 \).\(^\text{12}\) In the Appendix we discuss how to interpret our results if \( \pi_{an} < 0 \).

### 3 Profit-maximizing platform responses to a tax increase

It is evident from our discussion above that the effect of a change in the *ad valorem* tax depends on assumptions linked to the externalities between the two customer groups. Our analysis should not be confused with the standard theory of complements. Complements are used to describe a situation where an increase in the price of one good causes a decline in consumption of both goods, measured by the change in the compensated demand by a single consumer (see e.g., Kreps 1990, p. 61). This is different from a two-sided market, where there are two distinct groups of customers that may respond differ-

\(^{11}\) Readers in European countries seem to be averse to advertising (see Ferguson 1983, p. 637; Blair and Romano 1993, and Sonnac 2000) For retail advertising there is some evidence showing that American readers like advertising.

\(^{12}\) Evans (2003b) defines a two-sided market as one where we have (a) two distinct groups of customers, (b) positive network externalities (at least from one of the customer groups to the other), and (c) an intermediary that internalizes the externalities between the groups. See Rochet and Tirole (2006) for a more formal definition.
ently to changes in prices (see Rochet and Tirole (2003, 2006) for a general discussion). Also, the main results of our analysis do not hinge on the goods being complementary in demand by the two groups of customers. In order to see this as simply as possible, we start out by considering a situation where newspaper readers are *indifferent* about the advertising level.

### 3.1 Consumers indifferent to the ad level \((p^N_a = 0)\)

There is no externality from good \(A\) to good \(N\) if newspaper readers are indifferent to the advertising level. Therefore the advertising level (i.e., output of good \(A\)) does not affect the willingness to pay for newspapers. In this case we have that \(p^N_a = 0\). The effect of a higher value-added tax can be found by using eq. (4) and totally differentiating first order conditions (2) and (3).

We then obtain\(^\text{13}\)

\[
\frac{dn}{dt}\bigg|_{p^N_a=0} = -\frac{\pi_{an} \left( ap^A_n - k_n \right)}{H (1 + t)}, \quad \frac{dp^N}{dt}\bigg|_{p^N_a=0} = p^N_n \frac{dn}{dt}\bigg|_{p^N_a=0}
\]

and

\[
\frac{da}{dt}\bigg|_{p^N_a=0} = \frac{\pi_{an} \left( ap^A_n - k_n \right)}{H (1 + t)}
\]

Equations (5) and (6) show that we may get the seemingly paradoxical result that a higher VAT on newspapers reduces the end-user price of that good and increases sales on both sides of the market. This happens if \((ap^A_n - k_n) > 0\). To see why, recall that the willingness to pay for advertising increases by \(p^A_n\) units if the newspaper attracts one more reader. With a total advertising volume equal to \(a\), the value for the newspaper of attracting one extra reader equals \(ap^A_n\). If the size of this indirect network effect is greater than the marginal cost \(k_n\) of serving one extra reader, it is profitable for the media firm to charge a lower price for the newspaper subsequent to the tax.

\(^{13}\text{The full derivation is stated in the Appendix.}\)
increase.\textsuperscript{14} Thereby the readership increases, allowing the media firm to sell more advertising and make a higher profit than if it increased the price and reduced the output of newspapers.\textsuperscript{15}

Whether \(ap_n^A - k_n > 0\) holds depends on the industry in question. In our media example there are high fixed cost of creating the first copy of a newspaper, but relatively low marginal cost of reproducing it (and on the internet \(k_n\) is approximately equal to zero even for pay-to-view sites). It should further be noted that advertising is the primary or only source of income for some media outlets, indicating that \(ap_n^A\) is relatively high.

The results in eqs. (5) and (6) are in stark contrast to benchmark results in one-sided markets, from which it is well known that (i) consumers buy less of a taxed good if marginal costs are positive \((k_n > 0)\), and that (ii) an \textit{ad valorem} tax is effectively a tax on pure profit with no effect on output if marginal costs are zero \((k_n = 0)\). Contrary to a firm operating in a one-sided market, a two-sided platform firm can reduce its tax burden by shifting revenue to the side of the market where the tax rate is unchanged. This is particularly profitable if the marginal costs of the more heavily taxed good are smaller than the size of the indirect network effect. In such a case our results demonstrate that consumers of the more heavily taxed good buy more of the good at a lower price. Thus, the platform does not shift even part of the burden onto the buyers.

The effect of the tax increase on the price of ads is from eq. (4) given by

\textsuperscript{14}Differentiating the equilibrium value of eq. (1) with respect to \(t\), and using the envelope theorem, we find \(d\pi/dt = -p^N(n, a)n(1 + t)^{-2} < 0\) so the profit level is strictly decreasing in the tax rate. However, the marginal change in profits earned in the ad market is \((p_n^A a + p_n^A) da/dt + p_n^A dn/dt\) which, by eq. (2) and \(p_n^A > 0\), is positive if quantity responses are positive (i.e., \(ap_n^A - k_n > 0\)).

\textsuperscript{15}To see the intuition for this result as clearly as possible, assume that \(t\) approaches infinity. Obviously, the newspaper would then have no reason to charge a positive consumer price. However, it can still raise revenue through the advertising market and give the newspaper away for free.
\[
\frac{dp^A}{dt}\bigg|_{p^a_N=0} = p^a_a \frac{da}{dt}\bigg|_{p^a_N=0} + p^a_n \frac{dn}{dt}\bigg|_{p^a_n=0} \geq 0.
\]

Since \(p^A(n, a)\) is downward-sloping in own quantity, an increase in the advertising volume tends to reduce \(p^A\) \((p^A_a < 0)\). At the same time, the firm can charge a higher advertising price if the size of the readership increases \((p^A_n > 0)\). Consequently, it is uncertain whether the price of advertising will go up or down.

### 3.2 Newspaper readers dislike ads \((p^N_a < 0)\)

When \(p^N_a < 0\), the demand for newspapers \((\text{good } N)\) depends negatively on the advertising level \((\text{good } A)\). One might think that higher value-added taxes are more likely to reduce the sales of newspapers the more consumers dislike ads \((\text{since tax-motivated increased sales of ads would reduce demand for newspapers})\). However, total differentiation of eqs. (2) and (3) makes it clear that the opposite is true:

\[
\frac{da}{dt}\bigg|_{p^N_a<0} = \frac{da}{dt}\bigg|_{p^N_a=0} + \left(\frac{1}{1+t}\right)^2 \pi_{na}^n p^N_a H
\]

\[
\frac{dn}{dt}\bigg|_{p^N_a<0} = \frac{dn}{dt}\bigg|_{p^N_a=0} + \left(\frac{1}{1+t}\right)^2 \frac{-\pi_{an}^n p^N_a}{H}.
\]

The first term in eqs. (7) and (8) shows how advertising and newspaper sales respond to a tax increase if consumers are indifferent about ads \((p^N_a = 0)\). As argued above, this term may be positive or negative. The second term, though, is unambiguously positive and increasing in the consumers’ disutility of ads. The reason is that if sales in the newspaper market are adversely affected by advertising \((p^N_a < 0)\) the media firm has incentives to set a smaller advertising level than the volume which maximizes profit in the advertising market \((\text{c.f. eq. (2)})\). However, this incentive becomes weaker with a heavier
taxation of newspaper sales, making it optimal to increase sales of ads. The media firm can achieve this by enlarging the size of the readership, which requires a reduction of the newspaper price. This implies that the tendency for the newspaper price to fall subsequent to a tax increase is even more pronounced when $p_{a}^{N} < 0$ than when $p_{a}^{N} = 0$.\(^{16}\) It should be noted, though, that we still cannot sign the change in the price of advertising if both the advertising level and the size of the readership increase. This opens up for the possibly surprising result that the price for both readers and advertisers fall subsequent to a tax rise, and that the platform bears the full tax burden.\(^{17}\)

Summing up the discussion so far, we can state:

**Proposition 1:** If $p_{a}^{N} \leq 0$, a sufficient condition for a higher value-added tax on good $N$ to increase equilibrium quantities of both goods is that $ap_{n}^{A} > k_{n}$. The price of good $N$ (inclusive of VAT) is lowered, while the sign of the change in the price of the untaxed good ($A$) is ambiguous.

Undoubtedly, the market price $p_{N}^{N}$ is only part of the total price readers pay when $p_{a}^{N} < 0$. The total, hedonic price includes the market price and the disutility readers incur from advertising exposure. Readers buy more of the more heavily taxed good when $ap_{n}^{A} > k_{n}$. Appealing to a revealed preference argument, the rise in advertising volume does not dominate the reduction of the market price. Hence, not only the market price $p_{N}^{N}$, but also the hedonic price falls subsequent to the tax rise.

\(^{16}\)With $p_{n}^{N} < 0$ and $p_{a}^{N} < 0$ it follows immediately from eq. (4) that $dp_{N}^{N}/dt < 0$ if $da_{a}/dt > 0$ and $dn_{a}/dt > 0$, and that the price reduction is larger the more consumers dislike ads.

\(^{17}\)Using Anderson and Coate’s (2005) well-established model of a monopoly newspaper it is straightforward to show that prices may indeed fall on both sides of the platform. The computations are available upon request.
3.3 Newspaper readers as ad-lovers \((p^N_a > 0)\)

Demand for newspapers is increasing in the advertising level if \(p^N_a > 0\), and reflects that readers have a positive attitude towards commercials (ad-lovers). This may be the case in for instance specialized magazines; car ads in automobile magazines and perfume ads in beauty magazines constitute examples where the ads seem to be appreciated by the readers (see Depken II and Wilson, 2004).\(^{18}\)

Equations (7) and (8) still hold when consumers are ad lovers, but with the potentially important difference that the last terms in both equations turn from positive to negative, that is,

\[
\frac{da}{dt} \bigg|_{p^N_a > 0} = \frac{da}{dt} \bigg|_{p^N_a = 0} + \left( \frac{1}{1 + t} \right)^2 \frac{\pi_{an} n p^N_a}{H}.
\]

\(9\)

\[
\frac{dn}{dt} \bigg|_{p^N_a > 0} = \frac{dn}{dt} \bigg|_{p^N_a = 0} + \left( \frac{1}{1 + t} \right)^2 \frac{-\pi_{an} n p^N_a}{H}.
\]

\(10\)

If \(p^N_a > 0\) is small, the last term is insignificant relative to the first term and our results in the previous sections are reproduced. If \(p^N_a\) is sufficiently high, it follows from eqs. (9) and (10) that the sales of newspapers and advertising are decreasing in taxes. To see why, notice that the newspaper has more ads than the quantity which maximizes profit on the advertising side when consumers are ad-lovers (c.f. eq. (2)). An increase in VAT, though, implies that it becomes less profitable for the media firm to attract readers by having many ads. Instead, the media firm will have incentives to reduce the level of advertising, and approach the volume that maximizes profit on the advertising side. If \(p^N_a\) is sufficiently high, both the level of advertising

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\(^{18}\)Another example is from the financial sector, where cardholders have a higher willingness to pay for holding a credit card the larger the number of merchants that accept it.
and newspaper sales will therefore fall, and the signs of \( dp^A/dt \) and \( dp^N/dt \) will be ambiguous (c.f. eq. 4).

To summarize:

**Proposition 2:** Suppose \( p^N_a > 0 \).

(a) If \( p^N_a \) is not too high, a higher value-added tax on good \( N \) increases sales on both sides of the market and lowers the price of good \( N \) if \( ap^A_n > k_n \).

(b) If \( p^N_a \) is sufficiently high, a higher tax on good \( N \) reduces sales on both sides of the market, while the effect on prices is ambiguous.

In the sections above we have shown that a higher *ad valorem* tax on newspapers may increase newspaper sales and reduce the newspaper price, particularly if consumers dislike ads. The purpose of the next section is to show that it may be a more robust policy recommendation to use negative specific taxes (unit subsidies) than to reduce the VAT rate if the aim is to increase newspaper circulation.

## 4 Specific taxation

Under a specific tax the profit of the platform is

\[
\pi = \max_{n, a} \left[ ap^A(n, a) + \left( \frac{p^N(n, a)}{1 + t} - \tau \right) n - k(n, a) \right],
\]

where \( \tau \) is the specific tax that falls on good \( N \) (newspapers). From the first order conditions \( \pi_a = 0 \) and \( \pi_n = 0 \), we can characterize the profit maximizing behavior of the platform as follows

\[
p^A + ap^A_n - k_n = -\frac{np^N_n}{1 + t} \quad (11)
\]

\[
\left[ \frac{p^N + np^N_n}{1 + t} \right] - k_n = -ap^A_n + \tau. \quad (12)
\]

The first-order conditions for the platform are the same as before (c.f. eqs. (2) and (3)), except that the specific tax imposes an additional cost on
the production of newspapers, as is evident from the right-hand side of eq. (12).

Totally differentiating eqs. (11) and (12), holding \( t \) fixed, we find

\[
\frac{dn}{d\tau} = \frac{\pi_{aa}}{H} < 0 \quad \text{and} \quad \frac{da}{d\tau} = -\frac{\pi_{na}}{H} < 0. \tag{13}
\]

Equation (13) makes it clear that specific taxes unambiguously have a negative impact on output in both markets, independently of consumer preferences for ads. The reason is that higher specific taxes are equivalent to increased unit costs, as shown by eq. (12). Since higher unit costs lower the marginal profitability for any given output, it is optimal to reduce sales of newspapers \((dn/d\tau < 0)\). As a result, the advertising level falls \((da/d\tau < 0)\). Note, however, that we would have \(da/d\tau > 0\) if \(\pi_{an} < 0\). The intuition for this is simple; if the marginal profit of advertising is decreasing in the newspaper circulation, a lower sale of newspapers will make it optimal for the media firm to sell more advertising space. In contrast, the equations in Section 3 make it clear that the sign of the change in sales of advertising do not depend critically on whether \(\pi_{an}\) is positive or negative under \textit{ad valorem} taxation (see also Appendix).

The change in the newspaper price is

\[
\frac{dp^N}{d\tau} = p^N_n \frac{dn}{d\tau} + p^N_a \frac{da}{d\tau}. \tag{14}
\]

Equation (14) is unambiguously positive if consumers dislike ads \((p^N_a < 0)\). However, with ad-lovers \((p^N_a > 0)\) the second term is negative, reflecting that the consumers’ willingness to pay for the newspaper falls when the level of advertising decreases. Unless this effect is sufficiently strong, we get the standard result from one-sided markets that the end-user price is increasing in the tax level \((dp^N/d\tau > 0)\).

For the advertising price we find

\[
\frac{dp^A}{d\tau} = p^A_a \frac{da}{d\tau} + p^A_n \frac{dn}{d\tau}. \tag{15}
\]
The fact that the advertising volume falls subsequent to a higher specific tax, tends to increase the advertising price. However, the smaller newspaper circulation \((dn/dt < 0)\) reduces the value of advertising. If this effect dominates (i.e., \(p_A^n\) is relatively large), the advertising price falls.

Our result above can be summarized as follows:

**Proposition 3:** A higher specific tax on good \(N\) reduces output of both goods. Unless \(p_A^n\) and \(p_a^N\) are positive and sufficiently large, end-user prices increase.

The analysis in Sections 3 and 4 makes it clear that raising *ad valorem* taxes and specific taxes may have opposite quantity effects. The reason for this is that with specific taxes, there is a one-to-one relationship between tax payments and quantity, while there is no direct link between output and the burden of taxation under *ad valorem* taxation. In fact, subsequent to a higher *ad valorem* tax the firm can in principle both reduce tax payments and increase the quantity by lowering the price.

The important insight from the discussion above, is that unit subsidies (a negative value of \(\tau\)) unquestionably increase newspaper circulation, and also reduces the newspaper price unless the readers are relatively strong ad-losers. A reduction of the VAT rate, on the other hand, has more ambiguous effects - in the worst case, such a policy may reduce newspaper circulation and increase newspaper prices.

## 5 Duopoly and newspaper differentiation

In this section we extend our analysis from monopoly to possible competition between two newspapers. The extension serves two purposes. First, we would like to examine the robustness of our results if there is competition. Second, given that the analysis pertains to the newspaper business, it is of interest to investigate whether changes in the *ad valorem* tax affect how newspapers differentiate themselves with respect to content such as the editorial stance.
Our analysis draws on the model in Gabszewicz et al. (2001, 2002) extended by taxation and the possibility of an ad-loving or ad-averse readership.\footnote{See also Peitz and Valetti (2008) on the possibility of ad-averse readers in a Hotelling model.} We assume that readers are uniformly distributed along a Hotelling line of unit size, and that they can choose between two newspapers. The locations of the newspapers are given by $\theta_1 = \alpha$ and $\theta_2 = 1 - \beta$, where $(1 - \beta) \geq \alpha$. The newspapers are perfect (horizontal) substitutes if $(1 - \beta) = \alpha$, while they are maximally (horizontally) differentiated if $\alpha = \beta = 0$.

Readers differ w.r.t. their preference for editorial stance as measured by $\theta$, which is uniformly distributed on the unit-interval. The utility of a $\theta$-type reader who consumes newspaper $i = 1, 2$ is given by

$$u_i = v - \lambda (\theta_i - \theta)^2 - (p_i^N - \gamma a_i), \quad (16)$$

where $p_i^N$ is the price that readers pay per copy of newspaper $i$ and $a_i$ is the advertising volume. The readers suffer a utility loss equal to $\lambda (\theta_i - \theta)^2$, $\lambda > 0$, when the newspaper’s editorial content $\theta_i$ is distinct from their most preferred one. Readers may (dis)like advertisements. They feel disturbed by advertisements when $\gamma < 0$, and appreciate them when $\gamma > 0$ (in terms of the analysis in Section 3 this means that $\gamma$ corresponds to $p_a^N$). As such, $p_i^N - \gamma a_i$ can be interpreted as the hedonic price readers pay per newspaper.\footnote{In Gabszewicz et al. (2001, 2002) $\gamma$ is set to zero.}

We denote the number of readers of newspaper $i$ by $n_i$, which is a non-increasing function of the hedonic price (such that $\partial n_i / \partial (p_i^N - \gamma a_i) \leq 0$).

Advertisers differ w.r.t. the benefit they derive from informing readers about the existence and characteristics of their product. The gross benefit of advertising in newspaper $i$ for an advertiser of type $\phi$ is equal to $\phi n_i$, and is thus proportional to the number of readers. Letting $p_i^A$ denote the advertising price in this newspaper, the advertiser’s net benefit of inserting an ad is $B_i = \phi n_i - p_i^A$. We follow Gabszewicz et al. (2001, 2002) in assuming that $\phi$ is distributed on $[0, 1]$ with density $4\kappa$ and that the advertisers are
price takers. The induced demand for advertising in newspaper \( i \) then reads\(^ {21} \)

\[
a_i = 4\kappa \left( 1 - \frac{p_i^A}{n_i} \right). \tag{17}
\]

The marginal cost for the newspaper of inserting an ad is set equal to zero, while the marginal cost of printing and distributing a newspaper copy is \( c \geq 0 \). An ad valorem tax \( t \) is levied on sales of newspapers (good \( N \)), which implies that the media firm receives the price \( p^N / (1 + t) \) per sold copy of the newspaper. This means that the profit level of newspaper \( i \) equals

\[
\pi_i = p_i^A a_i + \left( \frac{p_i^N}{1 + t} - c \right) n_i. \tag{18}
\]

We consider a three-stage game where the newspapers simultaneously and non-cooperatively choose their editorial stance at stage 1. At stage 2 each newspaper maximizes its profit with respect to the hedonic price, while they select advertising prices at stage 3.

We focus on subgame-perfect equilibria which exhibit positive newspaper prices (otherwise the tax would be neutral for firm behavior). Solving backwards, at stage 3 each newspaper maximizes profits with respect to \( p_i^A \) keeping the hedonic price \( p_i^N - \gamma a_i \) constant. At an interior solution, the first-order condition is

\[
\left( a_i + p_i^A \frac{\partial a_i}{\partial p_i^A} \right) + \frac{n_i}{1 + t} \left. \frac{\partial p_i^N}{\partial p_i^A} \right|_{d(\gamma a_i) = 0} = 0. \tag{19}
\]

Noting that \( \frac{\partial n_i}{\partial p_i^A} = -\frac{4\kappa}{n_i} \) and \( \left. \frac{\partial p_i^N}{\partial p_i^A} \right|_{d(\gamma a_i) = 0} = -\frac{\gamma a_i}{n_i} \) by eqs. (16) and (17), the third-stage equilibrium advertising price and the associated amount of advertising are

\[
p_i^A = \left( 1 - \frac{\gamma}{1 + t} \right) \frac{n_i}{2} \quad \text{and} \quad a_i = 2\kappa \left( 1 + \frac{\gamma}{1 + t} \right). \tag{20}
\]

\(^ {21} \)The platform has a monopoly over its readers as an advertiser can only contact a potential customer who reads newspaper \( i \) by placing an advert in that newspaper. Each newspaper firm is thus a competitive bottleneck; Armstrong (2006).
To ensure that the non-negativity constraints on the advertising price and quantity are not binding, we impose $|\gamma| < 1$ throughout. Total advertising revenue for newspaper $i$ is thus given by

$$p_i^A a_i = \tilde{\kappa} n_i,$$

where

$$\tilde{\kappa} := \kappa \left[ 1 - \left( \frac{\gamma}{1 + t} \right)^2 \right].$$

(21)

Hence, we find that per-reader advertising revenue $\tilde{\kappa}$ is increasing in $t$ provided $\gamma \neq 0$; i.e.

$$d\tilde{\kappa}/dt|_{\gamma \neq 0} > 0.$$  

(22)

Intuitively, if readers are indifferent to ads ($\gamma = 0$), the exposure to advertising does not affect revenues collected from readers and, thus, the newspaper tax neither influences advertising prices nor advertising revenues. If, in contrast, the audience is ad-averse ($\gamma < 0$), the newspaper incurs a cost of advertising. It recognizes the adverse effect of advertising on reader utility, and per-reader advertising revenues are set at a lower level than when $\gamma = 0$ (c.f. eq. (20)). Thus, a higher tax reduces the negative impact of advertising for newspaper revenues and, as a consequence, per-reader advertising revenues rise. An analogous type of reasoning applies when readers appreciate ads ($\gamma > 0$).

5.1 Local monopolies

To show that the results in Section 3 also turn up in the Hotelling model, it is useful to first consider a context where the market is uncovered and the newspapers maximize profits as local monopolies on the Hotelling line. This happens if the consumers’ willingness to pay for the newspapers, $v$, is sufficiently low. In this case it is not particularly interesting to analyse the newspapers’ localization decisions. For simplicity we therefore set $\alpha = \beta = 0$, such that they are located at each end of the Hotelling line. From the utility
function (16) we then find that demand for newspaper \( i \) equals
\[
n_i = \left( \frac{v + \gamma a_i - p_i^N}{\lambda} \right)^{\frac{1}{2}}.
\]
(23)

Inserting eqs. (20) and (23) into eq. (18) and solving \( p_i = \arg \max \pi_i \) we find
\[
p_i^N = \frac{c (1 + t)}{3} - \frac{(1 + t) (1 + t - 5\gamma) \bar{k} - 2v (1 + t - \gamma)}{3 (1 + t - \gamma)}.
\]
(24)

Using eqs. (20) and (24) and differentiating eqs. (23) and (24) yield
\[
\frac{dp_i^N}{dt} = \frac{c}{3} - \frac{(1 + t)^2 + 5\gamma^2}{3 (1 + t)^2} \kappa \quad \text{and}
\]
\[
\frac{dn_i}{dt} = \frac{1}{6\lambda n} \left[ -c + \frac{(1 + t)^2 - \gamma^2}{(1 + t)^2} \kappa \right].
\]
(25)

It follows immediately from eq. (25) that a higher tax reduces the price and increases sales of the newspaper if \( c \) is sufficiently small. If the marginal costs are ’high’, on the other hand, we get the standard result that a higher tax increases the price and reduces output. This, of course, is consistent with our general findings in Section 3.

Differentiation of eq. (20) implies
\[
\frac{da_i}{dt} = -\gamma \frac{2\kappa}{(1 + t)^2},
\]
so that a higher tax on newspapers increases the advertising level if the readers dislike ads \((\gamma < 0)\), and vice versa.\(^{22}\)

Finally, using eqs. (20), (23) and (24) we have
\[
\frac{dp_i^A}{dt} = \gamma \frac{kn_i (1 + \gamma + t)}{a_i (1 + t)^3} + \kappa \frac{dn_i}{a_i \; dt}.
\]
(26)

\(^{22}\)It is straightforward to show that \( \frac{\partial^2 \pi_i}{\partial a_i \partial p_i} = 2k \frac{\gamma}{1 + t} \). The first term on the right-hand side of eq. (7) in the general analysis is therefore equal to zero. This explains why the sign of the change in ad levels subsequent to a tax increase on newspaper sales depends solely on the sign of \( \gamma \).
If the consumers dislike ads ($\gamma < 0$), both the advertising level and newspaper sales might increase. The former tends to reduce the advertising price and the latter tends to increase it; these two effects are captured by the first and second term, respectively, on the right-hand side of eq. (26). The ambiguous net effect of a higher newspaper tax on the advertising price that we discussed in Section 3 thus also turns up in this Hotelling model.

5.2 Market coverage and duopolistic competition

Let us now turn to the case where the parameter $v > 0$ is sufficiently large to ensure market coverage; i.e. each consumer buys one newspaper both before and after a possible tax increase. To determine the size of the readership of newspaper $i$, $n_i(u_i)$, note that the willingness to pay for newspaper 1 is greater than for newspaper 2 for all consumers satisfying $u_1 > u_2$. Together with the previous finding $a_1 = a_2$ we thus find that demand for the two newspapers is given by

$$n_1 = \alpha + \frac{p_2^N - p_1^N}{2\lambda (1 - \alpha - \beta)} + \frac{1 - \alpha - \beta}{2}$$
$$n_2 = \beta + \frac{p_1^N - p_2^N}{2\lambda (1 - \alpha - \beta)} + \frac{1 - \alpha - \beta}{2}.$$  \hspace{1cm} (27)

Demand for newspaper $i$ is decreasing in its own price $p_i^N$ and increasing in the rival platform’s price $p_j^N$, $i \neq j$. More important for our purpose is the fact that solving $p_i = \arg \max \pi_i$ in eq. (18) subject to eq. (27) is equivalent to the optimization problem in Gabszewicz et al. (2001, 2002) even though they have set $\gamma = 0$ and $t = 0$. If $p_i > 0$, the second stage newspaper prices as a function of the editorial content choices $\alpha$ and $\beta$ are

$$p_1^N = p_2^N = (1 + t)(c - \bar{c}) + \lambda (1 - \alpha - \beta)(1 + \alpha - \beta).$$  \hspace{1cm} (28)

Following Gabszewicz et al. (2001, 2002), the first stage of the game - where the newspapers choose their location - yields an equilibrium with full
Full content differentiation and positive newspaper prices are inherently linked. With $p_i^N > 0$, advertising revenues are passed on to consumers in the form of reduced newspaper prices. In consequence, profits of the newspaper platform are independent of advertising receipts. As it only relies on newspaper receipts, the firm maximally differentiates editorial content in order to relax competition for newspaper readers (e.g., Shaked and Sutton, 1982).

Having solved for the equilibrium, we are equipped to analyse tax shifting incentives and the impact of taxes on the differentiation of newspapers. In equilibrium the reader market is shared between both platforms. Equation (28) yields

$$\frac{dp_i^N}{dt} = -\frac{(1 + t)^2 (\kappa - c) + \kappa \gamma^2}{(1 + t)^2}.$$  (30)

As in the monopoly case, we thus see that a higher tax reduces the newspaper price if $c$ is sufficiently small. However, with market coverage the size of the market is by assumption constant. In a symmetric equilibrium we therefore have $dn_i/dt = 0$.

From eq. (20) we further find

$$\frac{da_i}{dt} = -\gamma \frac{2\kappa}{(1 + t)^2}; \quad \frac{dp_i^A}{dt} = \gamma \frac{1}{2(1 + t)^2} \quad \text{and} \quad \frac{d(a_i p_i^A)}{dt} = \frac{2\kappa \gamma^2}{(1 + t)^3} > 0.$$  

As in the monopoly case, a higher newspaper tax increases the advertising volume if the readers dislike ads ($\gamma < 0$). Since the number of readers is unchanged, this unambiguously requires a lower advertising price. If the readers appreciate ads ($\gamma > 0$), we get the opposite result; advertising volumes fall and advertising prices increase. However, independent of whether $\gamma$ is positive or negative, total advertising revenue become higher if the tax rate on newspapers increases. As in the monopoly analyses above, the reason for this

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23We omit the details of the computations and refer the reader to Gabszewicz et al. (2001) and, in particular, to Gabszewicz et al. (2002).
is simply that the higher is \( t \), the more important it is for the newspapers to raise revenue from the advertising side of the market relative to the readers side of the market.

To examine how the tax affects the sustainability of a full-differentiation equilibrium, we analyse the propensity of taxation to render the non-negativity constraint on \( p_i^N \) binding. Using condition (29) and eq. (30) we can immediately state:

**Proposition 4:** The higher the ad valorem tax, the less likely it is that the two newspapers maximally differentiate editorial content.

As a final exercise we analyse firm responses to a specific tax on newspapers. Denoting the tax levied per newspaper sold by \( \tau \) profits are

\[
\pi_i = p_i^A a_i + (p_i^N - c - \tau) n_i,
\]

where, for simplicity, we have set the ad-valorem tax to 0. The specific tax works like an increase in the marginal cost \( c \). Hence, we may write \( \hat{c} = c + \tau \) as the effective marginal cost in what follows. It is straightforward to show that at stage 3 advertising revenues are independent of the newspaper tax and are given by

\[
p_i^A a_i = \hat{\kappa} n_i, \quad \text{where} \quad \hat{\kappa} := \kappa \left( 1 - \gamma^2 \right).
\]

Also, reiterating stage 1 and 2 of the game and keeping in mind that \( \hat{c} = c + \tau \) we find that the newspaper price becomes

\[
p_1^N = p_2^N = \hat{c} - \hat{\kappa} + \lambda (1 - \alpha - \beta) (1 + \alpha - \beta).
\]

Since \( dp_i^N/d\tau = d\hat{c}/d\tau = 1 \), we have the standard result that a higher specific tax on a good increases the consumer price.

The condition for a full differentiation equilibrium is now

\[
\hat{\kappa} < \hat{c} + \lambda/2 \iff p_i^N > 0.
\]
Thus we find:

Proposition 5: The higher the specific tax, the more likely it is that the two newspapers maximally differentiate editorial content.

To conclude, *ad valorem* and specific taxes thus have opposite effects on the newspapers’ differentiation incentives.

6 Conclusion

Traditional analysis of tax incidence has focused on conventional (one-sided) markets. In such markets a general insight is that indirect taxes are partly shifted (or even overshifted) onto consumers, resulting in lower sales of the taxed good. Our analysis has shown that this result is challenged in a two-sided market. If demand for the taxed good matters for the quantity sold to a different group of customers, the incidence of taxation changes. In a two-sided market an increase in an *ad valorem* tax may, under certain conditions, lead to lower prices for both goods as well as to higher sales. This is in sharp contrast to our findings under specific taxation.

We have also shown that taxation may affect media pluralism under duopoly. In particular we have seen that the higher the *ad valorem* tax is, the less likely it is that the two newspapers maximally differentiate editorial content. The conclusion is the opposite under specific taxation: the higher the specific tax, the more likely it is that the two newspapers maximally differentiate editorial content. Differently, neither *ad valorem* nor specific taxes tend to affect differentiation incentives in one-sided markets.

Even though our discussion is related to the media market, we believe to have used models sufficiently general in structure to highlight the most common mechanisms in two-sided markets. This said, we believe there is still a need for industry-specific analysis in both theoretical and empirical terms to identify peculiarities of the respective industries for tax policy design.

References


**Appendix**

1. Derivation of the relationship between quantities and ad valorem taxes

We assume that the second order conditions hold with non-negative prices and quantities, so that the equilibrium is characterized by first order conditions (2) and (3). To find how a higher value-added tax affects prices on the two sides of the market, we totally differentiate eqs. (2) and (3). This yields
\[
\frac{\pi_{aa}}{dt} \frac{da}{dt} + \frac{\pi_{an}}{dt} \frac{dn}{dt} = \left( \frac{1}{1+t} \right)^2 np_a^N
\]
\[
\frac{\pi_{an}}{dt} \frac{da}{dt} + \frac{\pi_{nn}}{dt} \frac{dn}{dt} = \left( \frac{1}{1+t} \right)^2 (p^N + np_n^N).
\]

Making use of the first-order condition (3), the effect of the tax on quantities is now given by

\[
\frac{da}{dt} = \left( \frac{1}{1+t} \right)^2 \frac{\pi_{an}}{H} \frac{(1 + t) (ap_a^A - k_n) + \pi_{nn}np_a^N}{H}
\]

and

\[
\frac{dn}{dt} = -\left( \frac{1}{1+t} \right)^2 \frac{\pi_{aa}}{H} \frac{(1 + t) (ap_a^A - k_n) + \pi_{an}np_a^N}{H}.
\]

2. Consequences of relaxing the assumption that \( \pi_{na} > 0 \)

Differentiating eq. (2) or eq. (3) we find

\[
\pi_{an} = \frac{p_a^N + np_a^N}{1 + t} + \frac{p_a^A + ap_a^A}{ap_a^A - k_n}.
\]

The cross derivative \( \pi_{an} \) measures how the marginal profitability of selling advertising space, \( \pi_a \), changes if the number of readers increases. In the main text we have assumed that \( \pi_{an} > 0 \), but from eq. (33) it is clear that \( \pi_{an} < 0 \) if for instance \( k_{an} \) is sufficiently large (such that a higher newspaper circulation significantly increases the marginal costs of selling and producing ads).

Suppose that \( \pi_{an} < 0 \) and \( p_a^N = 0 \). From eq. (5) we see that a higher ad valorem tax still increases sales of the newspaper and reduces the corresponding price if \( ap_a^A - k_n > 0 \) : thus the media firm’s incentive to sell a larger number of newspapers in order to shift revenue to the advertising side is unaltered. However, from eq. (6) we find that \( da/dt < 0 \) if \( \pi_{an} < 0 \).

If \( p_a^N < 0 \), we know that there will be less advertising than the volume which maximizes profit on the advertising side of the market. If the ad valorem tax rate on sales of newspapers increases, the media firm will care less
about the revenue it captures directly from the readers (independent of the sign of \( \pi_{an} \)). The second term in eq. (7) shows that the media firm thereby tends to sell more advertising space if \( t \) increases. The higher output of ads might in turn make it optimal for the media firm to reduce newspaper sales if \( \pi_{an} < 0 \), as shown by the second term in eq. (8).

The case where \( p^N_a > 0 \) has a similar interpretation. If consumers are ad lovers, the newspaper has more ads than the level that maximizes profit on the advertising side of the market. Independent of the sign on \( \pi_{an} \), the newspaper will therefore reduce the advertising level if \( t \) increases \( (da/dt < 0) \). However, a lower advertising level means that the marginal profit of selling newspapers increases if \( \pi_{an} < 0 \), which induces the newspaper to sell more newspapers \( (dn/dt > 0) \).

The effects of assuming \( \pi_{an} < 0 \) when we consider specific taxes are analogous, and seen from eqs. (13) - (15).