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## Optimal Provision of Public Goods: A Synthesis

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# Optimal Provision of Public Goods: A Synthesis\*

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## Abstract

There currently exist two competing approaches in the literature on the optimal provision of public goods. The standard approach highlights the importance of distortionary taxation and distributional concerns. The new approach neutralizes distributional concerns by adjusting the non-linear income tax, and finds that this reinvigorates the simple Samuelson rule when preferences are separable in goods and leisure. We provide a synthesis by demonstrating that both approaches derive from the same basic formula. We further develop the new approach by deriving a general, intuitive formula for the optimal level of a public good without imposing any separability assumptions on preferences. This formula shows that distortionary taxation may have a role to play as in the standard approach. However, the main determinants of optimal provision are completely different and the traditional formula with its emphasis on MCF only obtains in a very special case. (JEL: H41, H23, H11)

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# 1 Introduction

Cost-benefit analysis is an important tool in everyday government decision making on public projects. When carried out in practice, the dominating view seems to be that the costs of a tax-funded project should be adjusted according to the marginal cost of funds (MCF), as a close reflection of the deadweight loss that will materialize if the project is added to the budget.<sup>1</sup> Today, the theoretical foundation for such a practice is less clear.

The simple view described above originates from the pioneering papers by Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974). They argued that the famous Samuelson rule — which equates the sum of the marginal willingness to pay for the public good of all citizens to the marginal rate of transformation (MRT) — relies on an unrealistic first-best setting where individual lump sum taxes are available. Instead, they base their analyses on distortionary taxation and arrive at a modified Samuelson rule where the effective cost of public goods is identified as MCF times MRT. This ‘standard approach’ has been very influential and also underlies the survey of Ballard and Fullerton (1992).

The standard approach has since been further developed by integrating the government spending side more thoroughly in the analysis and by allowing for heterogeneity in earnings abilities across households (Dahlby, 1998; Slemrod and Yitzhaki, 2001; Gahvari, 2006; Kleven and Kreiner, 2006). Two important conclusions emerge from these extensions. First, the evaluation of public projects should take account, not only of the distortionary effect of taxation as reflected by the MCF, but also of government revenue effects stemming from behavioral responses generated by the expenditure side of the projects. For example, a government investment in infrastructure or child care may increase working hours, and thereby tax revenue. Second, distributional concerns become important for the optimal level of public goods. It matters how benefits and costs are distributed across households.

In contrast, the ‘new approach’ to the optimal provision of public goods argues that distributional concerns are irrelevant to the evaluation of public projects. This line of research, initiated by Hylland and Zeckhauser (1979) and further pursued by Christiansen (1981) and

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<sup>1</sup>See, for example, Boardman et al. (2006) p. 104. Evaluation of tax-funded public projects in Denmark assumes that the cost of financing is 1.2 times the actual expenditures, corresponding to the official Danish marginal cost of funds (the Danish Ministry of Transportation and Energy, 2003).

Kaplow (1996), holds that unintended distributional effects can be undone by the income tax. Their analyses rely on the *benefit principle*, which, building on the flexibility of the non-linear income tax, argues that each individual should contribute to the financing of a public good corresponding to her own marginal willingness to pay. Formally, Christiansen (1981), in the context of the optimal non-linear income tax, and Kaplow (1996), for a general tax function, have shown that this principle restores the original Samuelson rule when preferences are separable in leisure and goods (including public goods). This somewhat surprising result arises because the effects on individual behavior from the benefit side and from the cost side of a government project cancel each other out, implying that a change in government consumption has no indirect effects on tax revenue.

The divergent results of the traditional approach and of the new approach have created a state of confusion as illustrated by the debate in the wake of Kaplow's (2004) survey (see Goulder et al., 2005, and the reply by Kaplow). One reason for this confusion may simply be that the underlying analyses appear to be very different. Another likely reason is that the new approach has been inextricably linked to a restrictive assumption on preferences, although the underlying benefit principle applies much more generally. The fundamental difference between the two approaches lies in the assumption made about the financing of the public good. Unlike the new approach, the standard approach imposes no restrictions on the way the project is financed. An argument in favor of this approach is that the income tax is not sufficiently flexible to exploit the information about the distribution of the benefits from the public good. However, the lack of restrictions on the financing scheme has the potential drawback of leading way to distributional concerns that are unrelated to the public goods problem itself. As a result, government consumption may become a means to compensate for a lack of appropriate tax instruments. In contrast, the new approach follows the tradition in analyses of optimal taxation by assuming away exogenous restrictions on the instruments available to the government, except the restriction that innate abilities cannot be observed and taxed directly. This eliminates any distributional concerns due to the specifics of the financing scheme. But, at its current state, the new approach suffers from the strong assumption of separable preferences.

This paper contributes in different ways to the literature on optimal provision of public goods. First, we generalize previous results in both the standard approach and the new approach

by considering a very general framework that accounts for heterogeneity in both earnings and preferences and allows for home production through Beckerian type household consumption technologies.

Second, we use the framework to reconcile the results of the two approaches. The traditional approach addresses the problem of optimal provision by examining whether a budget-neutral expansion of government consumption raises social welfare. The new approach, on the other hand, considers an expansion of government consumption together with an adjustment of the non-linear income tax that keeps everybody at the same utility level (the benefit principle). The optimality criterion then becomes whether government revenue increases or not. We demonstrate, using a simple duality property, that both approaches derive from the same basic formula, requiring that a public project is completed only when the social marginal benefit of the project (SMBP) exceeds the social marginal cost of public funds (SMCF).

Third, and most importantly, we contribute to the new approach by deriving a fully general, intuitive formula for the optimal level of public goods without imposing any separability assumptions on preferences. The formula shows that distortionary taxation may have a role to play as in the standard approach. However, the main determinants of optimal provision are very different and the traditional formula with its emphasis on MCF only obtains in a very special case where the willingness to pay for the public good is linear in ability.

Our general formula identifies the *partial* correlation between ability and the marginal willingness to pay for the public good as the driving force behind any deviations from the Samuelson rule. That is, public goods provision should only be less (more) than the Samuelson rule predicts if high ability individuals have a higher (lower) marginal willingness to pay for the public good — *when evaluated at a given earnings level*. We may observe that high earning, high ability individuals have a higher willingness to pay for the public good. However, if this correlation is driven entirely by the effect of income on the willingness to pay (as is the case with a standard normal good) the Samuelson rule still applies. Only a partial effect directly from ability to the willingness to pay leads to a departure from the Samuelson rule since any correlations with income can be made distributionally neutral through appropriate adjustments of the income tax.

The paper is organized as follows. Section 2 presents our model with a continuum of agents

and preference heterogeneity. Section 3 derives a general formula for the optimal level of a public good when there are no restrictions on the financing scheme as in the standard approach. Section 4 shows the relationship between the standard approach and the new approach, and derives a general, intuitive formula for the optimal level of a public good when marginal tax changes are governed by the benefit principle. In Section 5 we provide a special case where the two approaches lead to identical results, and where the simple, traditional formula with its emphasis on MCF applies. Finally, Section 6 offers a few concluding remarks.

## 2 The Framework

This section presents a general framework to analyze the optimal provision of public goods. The model has a continuum of agents, each characterized by an innate ability  $n$ , which is also our index of identification. The distribution of abilities across the population is given by the non-degenerate density function  $f(n)$ . Each agent derives utility from private consumption  $c$  and from public goods  $g$  provided by the public sector. Both  $c$  and  $g$  could be thought of as either a vector of consumption goods or a single composite good. Gross earnings or, more generally, taxable income is denoted  $z$ , and acquiring income imposes a utility loss on the agent. The utility of agent  $n$  equals

$$u(c, g, z, n), \tag{1}$$

where  $u_c \equiv \partial u / \partial c > 0$ ,  $u_g > 0$ ,  $u_z < 0$ , and  $u(\cdot)$  is quasiconcave. This utility specification embodies preference heterogeneity across individuals of different abilities. It also encompasses the traditional Mirrleesian specification,  $u(c, g, z/n)$ , as a special case. The term  $z/n$  builds on the notion that more able persons must exert less effort to attain a given income level. If this logic is extended to other domains of everyday life, as in Becker (1965), it seems natural that ability also has an impact on the utility of consuming, as long as the skills of home production are correlated with market productivity. The theory of household production views market goods as an input in a production process, which, along with individual skills, determines the output that ultimately enters individual utility. Thus, persons of different skills may benefit differently from a given input of  $c$  or  $g$ . For instance, an individual's ability to cook determines the utility derived from a basket of groceries. Similarly, the utility derived from public goods

such as the police or the judicial system depends on both the skill and the need to benefit from such institutions, which is likely influenced by individual ability. Thus, the formulation in (1) captures both innate preference differences between individuals of different abilities and preference differences due to the technology of home production.

Since the government cannot condition taxes on the unobservable ability, it is forced to operate a (possibly) non-linear income tax function  $T(z, \theta)$ , where  $\theta$  is a shift parameter used to capture the effects of changes to the tax function. Consumption equals  $c = z - T(z, \theta)$  which, together with the utility function (1), give

$$\text{MRS}_{cz}(z, n) \equiv -\frac{u'_z(z - T(z, \theta), g, z, n)}{u'_c(z - T(z, \theta), g, z, n)}, \quad (2)$$

$$\text{MRS}_{cg}(z, n) \equiv -\frac{u'_g(z - T(z, \theta), g, z, n)}{u'_c(z - T(z, \theta), g, z, n)}, \quad (3)$$

which measure the marginal rates of substitution between, respectively,  $c$  and  $z$  and  $c$  and  $g$  for a type  $n$  individual at the income level  $z$ . Notice that an increase in the ability level affects the MRS's both directly and indirectly through an impact on the earnings level  $z$ . The first-order conditions for the optimal choices of  $c$  and  $z$  imply

$$\text{MRS}_{cz}[z(n)] = 1 - m, \quad (4)$$

where  $z(n)$  denotes the optimal income level and  $m \equiv \partial T(z(n), \theta) / \partial z$  is the marginal tax rate at that income level. The indirect utility function is  $v(n) \equiv u[c(n), g, z(n), n]$  and gives the utility level of individual  $n$  when consumption and labor supply are chosen optimally. We follow the standard approach in optimal taxation and contract theory and assume (i) that utility is increasing in ability,  $\partial u / \partial n > 0$ , and (ii) that the Spence-Mirrlees single-crossing condition is satisfied (e.g., Salanié, 2003):

$$\partial \text{MRS}_{cz}(z, n) / \partial n < 0. \quad (5)$$

The first assumption along with the Envelope Theorem ensures that the indirect utility is increasing in ability,  $dv/dn = \partial u / \partial n > 0$ . The second assumption ensures that the tax system is implementable, i.e., that higher ability individuals always choose higher equilibrium earnings, implying that the government can use income as a signal of the underlying ability.

The government cares about redistribution as well as the provision of public goods. The preferences of the government are captured by a Bergson-Samuelson social welfare function of



the form

$$\Omega = \int_n \Psi [v(n)] f(n) dn, \quad (6)$$

where  $\Psi(\cdot)$  is a concave function reflecting the distributional concerns of the policymaker. The marginal rate of transformation between private goods and public goods ( $\text{MRT}_{cg}$ ) is normalized to one, without any loss of generality. The government budget constraint then becomes

$$R \equiv \int_n T(z, \theta) f(n) dn - g \geq 0,$$

where the public goods nature of  $g$  is seen from the fact that  $g$  enters only once in the government budget constraint but still appears in everyone's utility functions.

A reform is characterized by two parameters: the change in the supply of the public good  $dg$  and an associated adjustment of the tax function  $d\theta$ . Differentiating (6) and using the first-order condition (4) yields the effect of a marginal reform,  $(dg, d\theta)$ , on social welfare

$$\frac{d\Omega}{\lambda} = -d\theta \int_n \omega(n) \frac{\partial T(z, \theta)}{\partial \theta} f(n) dn + dg \int_n \omega(n) \frac{u'_g}{u'_c} f(n) dn, \quad (7)$$

where  $\lambda \equiv \int_n \Psi'(\cdot) u'_c(\cdot) f(n) dn$  is the average social marginal utility of income in society and  $\omega(n) \equiv \frac{\Psi'[u(\cdot)]u'_c(\cdot)}{\lambda}$  is the social marginal welfare weight of agent  $n$ . Similarly, the effect of a reform on government revenue is given by

$$dR = d\theta \int_n \frac{\partial T(z, \theta)}{\partial \theta} f(n) dn - dg + \int_n m \left( \frac{\partial z}{\partial \theta} d\theta + \frac{\partial z}{\partial g} dg \right) f(n) dn, \quad (8)$$

where the first two terms are the direct revenue effects while the last term captures the effect of behavioral responses on government revenue. These behavioral responses are driven both by changes to the tax schedule and by effects of government consumption on household utility.

### 3 The Standard Approach

The standard view of optimal public goods supply is due originally to Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974) and has exerted a tremendous influence on the practice of cost-benefit analysis (e.g., Ballard and Fullerton, 1992). This approach to deriving a formula for the optimal public goods supply does not impose any restrictions on the financing scheme other than the requirement that the reform is fully financed, i.e.,  $dR = 0$ . From eq. (8) this

yields

$$dg = \frac{d\theta \int_n \left[ \frac{\partial T(z, \theta)}{\partial \theta} + m \frac{\partial z}{\partial \theta} \right] f(n) dn}{1 - \int_n m \frac{\partial z}{\partial g} f(n) dn}.$$

A marginal expansion of  $g$  is desirable if it increases social welfare,  $d\Omega \geq 0$ . Insert the above expression in (7) and apply this test to get

$$\frac{\int_n \omega(n) \frac{u'_g}{u'_c} f(n) dn}{1 - \int_n m \frac{\partial z}{\partial g} f(n) dn} \geq \frac{\int_n \omega(n) \frac{\partial T}{\partial \theta} f(n) dn}{\int_n \left[ \frac{\partial T}{\partial \theta} + m \frac{\partial z}{\partial \theta} \right] f(n) dn}. \quad (9)$$

The earnings choice of the household, determined by eqs (2) and (4), may be written as a function  $\hat{z}((1-m), y, g, n)$ , where  $(1-m)$  is the marginal net-of-tax rate and  $y \equiv mz - T(z, \theta)$  is virtual income. The uncompensated elasticity of taxable income with respect to the net-of-tax rate may then be defined as  $\varepsilon \equiv \frac{1-m}{z} \frac{\partial \hat{z}}{\partial (1-m)}$ . From the Slutsky-equation, it may be decomposed into a compensated elasticity and an income effect, that is  $\varepsilon = \varepsilon^c - \eta$  where  $\varepsilon^c$  is the compensated elasticity and  $\eta \equiv -(1-m) \frac{\partial \hat{z}}{\partial y}$  is the income effect.<sup>2</sup> Further, let

$$\Phi \equiv \frac{\partial m / \partial \theta}{\partial a / \partial \theta}, \quad s(n) \equiv \frac{\partial T}{\partial \theta} / \left( \int_n \frac{\partial T}{\partial \theta} f(n) dn \right), \quad (10)$$

where  $a$  is the average tax rate. The parameter  $\Phi$  captures the progressivity of the implied tax reform, and  $s(n)$  is the share of the direct tax changes that is borne by agent  $n$ . Using this we can rewrite (9) in terms of behavioral elasticities to arrive at Proposition 1.<sup>3</sup>

**Proposition 1** *A marginal expansion of a public good is desirable iff*

$$\frac{\int_n \omega(n) \cdot \text{MRS}_{cg}(\cdot) f(n) dn}{1 - \int_n m \frac{\partial z}{\partial g} f(n) dn} \geq \frac{\int_n \omega(n) s(n) f(n) dn}{\int_n \left( 1 - \frac{m}{1-m} (\Phi \cdot \varepsilon^c - \eta) \right) s(n) f(n) dn}. \quad (11)$$

**Proof:** See Appendix A.  $\square$

<sup>2</sup>Previous contributions have defined hours-of-work elasticities. The elasticity of taxable income captures hours-of-work responses as well as all other behavioral responses that are relevant for total tax payments, and the empirical evidence indicates that this elasticity may be significantly larger than the hours-of-work elasticity (e.g. Gruber and Saez, 2002).

<sup>3</sup>When deriving the behavioral responses to the tax reform, we follow the standard approach and assume that the tax schedule is piece-wise linear. This ensures that there is no feed-back effect from the change in  $z$  to the marginal tax rate, and thus no additional earnings responses beyond those triggered directly by the tax reform. Mathematically, we avoid including second derivatives of the tax function ( $T''$ ) into the formula. The assumption of piece-wise linearity implies that there will be bunching at the various kinks in the tax schedule. This does not constitute a problem for our final results but may imply that taxable income elasticities are zero at a kink point because marginal changes are not sufficient to move the individual away from the kink point.

Expression (11) generalizes the result of Dahlby (1998), Gahvari (2006), and Kleven and Kreiner (2006) to a more general setting. Intuitively, a marginal expansion of the public good is desirable when the social marginal benefit of the project (SMBP, the left-hand side) exceeds the social marginal cost of public funds (SMCF, the right-hand side). The expression for SMCF is the continuous-setting equivalent to the social marginal cost of public funds derived in Dahlby (1998) with elasticities defined on taxable income rather than more narrowly on labor supply.<sup>4</sup>

Proposition 1 demonstrates the importance of tax distortions and distributional considerations for the optimal level of the public good. Without distributional weights,  $\omega(n) = \omega \forall n$ , and without initial tax distortions,  $m = 0 \forall z$ , the Samuelson rule applies (independently of how a marginal expansion of the public good is financed). Introducing positive marginal tax rates implies that the optimal  $g$  may be lower or higher than prescribed by the Samuelson rule, depending on the sizes of the behavioral effects stemming from changes to the tax schedule (the RHS denominator) and from changes to the public goods supply (the LHS denominator).

Distributional concerns affect the optimal level of public goods, even in the absence of any tax distortions. Consider, for example, the case where the aggregate willingness to pay for a public project exceeds the total costs of the project. Such a project should be implemented according to the original Samuelson rule but not necessarily according to the above modified rule which depends on the financing scheme. If, for example, high-income people receive most of the benefits and the public project is financed by a lump sum tax, the project might be discarded because the distribution of welfare is worsened. However, such a conclusion ignores the flexibility of the non-linear income tax, and thereby assigns a role to distributional considerations that are unrelated to the problem of public goods provision (see also Auerbach and Hines, 2002). This approach may have merit when there are exogenous constraints that limit the adjustment of the tax schedule as emphasized by Slemrod and Yitzhaki (2001) and Gahvari (2006). On the other hand, without any specific justification for constraining the tax function, it is natural to consider a financing scheme where those who benefit from the public good also pay the extra taxes, thereby neutralizing any distributional effects. This is the direction taken by the new approach.

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<sup>4</sup>Kleven and Kreiner (2006) include both intensive and extensive labor supply responses. We have chosen to follow the tradition in analyses of the optimal provision of public goods and MCF by focusing on intensive responses alone.

## 4 The New Approach

The new approach evaluates the benefits of an expansion of the public good by use of the *benefit principle*, introduced by Hylland and Zeckhauser (1979) and applied by Christiansen (1981) and Kaplow (1996, 2004). According to this principle, a (marginal) expansion of  $g$  should be financed by a benefit-offsetting, or distribution-neutral, change in the tax function. Since the reform keeps individual utilities unaffected the merits of a marginal expansion of  $g$  depend on the implied changes to government revenue, i.e., if  $dR \geq 0$  the expansion of  $g$  should be implemented. This is incompatible with the method used to derive the optimal level of  $g$  in the standard approach of the previous section. Indeed, condition (9) was derived by considering whether a budget-neutral reform,  $dR = 0$ , raised social welfare,  $d\Omega \geq 0$ . Instead, we use an alternative approach that keeps social welfare unaffected and determines the desirability of a marginal expansion of  $g$  by calculating the effect of the reform on government revenue. If the effect is positive, the reform is socially desirable. We show in Appendix B that the requirements  $d\Omega = 0$  and  $dR \geq 0$  are equivalent to

$$\frac{\int_n \omega(w) \frac{u'_g}{u'_c} f(n) dn}{1 - \int_n m \frac{\partial z}{\partial g} f(n) dn} \geq \frac{\int_n \omega(w) \frac{\partial T}{\partial \theta} f(n) dn}{\int_n \left[ \frac{\partial T}{\partial \theta} + m \frac{\partial z}{\partial \theta} \right] f(n) dn}, \quad (12)$$

which is the same as condition (9). The fact that we arrive at the same formula as in the standard approach is not surprising since we have merely applied a dual approach to determine the optimal level of  $g$ . Importantly, the equivalence of (9) and (12) provides a link between the two approaches. Indeed, they both derive from the same basic formula. The difference lies in the assumptions made regarding the associated tax reform.

The benefit principle makes the change to the entire tax schedule endogenous, i.e., at every income level both the direct change to the tax burden and the change in the marginal tax rate are determined endogenously by the requirement that the utility of all individuals is unchanged, implying that  $v(n)$  and  $v'(n)$  are fixed. Thus, we consider a reform,  $(dg, d\theta)$ , that affects  $g$  and the tax function  $T(\cdot)$  such that

$$dv(n) = u'_c(\cdot) dc + u'_g(\cdot) dg + u'_z(\cdot) dz = 0 \quad \text{for all } n, \quad (13)$$

$$dv'(n) = u''_{cn}(\cdot) dc + u''_{gn}(\cdot) dg + u''_{zn}(\cdot) dz = 0 \quad \text{for all } n, \quad (14)$$

where we have used that  $v'(n) = u'_n(\cdot)$  because of the Envelope Theorem. The benefit-offsetting expansion of  $g$  adjusts the tax function to capture the benefits of the additional  $g$  from each individual  $n$ . Since the tax function depends on income, not ability, the reform may have distortionary effects on the incentive to work.

Total differentiation of the relationship  $c = z - T(z, \theta)$  yields  $dc = (1 - m) dz - (\partial T / \partial \theta) d\theta$ . We can use this and the first order condition (4) to rewrite (13) as

$$\frac{\partial T(z, \theta)}{\partial \theta} d\theta = \frac{u'_g(\cdot)}{u'_c(\cdot)} \cdot dg = \text{MRS}_{cg}(z, n) \cdot dg. \quad (15)$$

This equation shows that the increase in the tax burden of an individual with earnings  $z$  is exactly equal to the extra benefit from the expansion of government consumption. After substituting for  $dc$  in condition (14), we obtain

$$dz = \frac{u''_{cn}(\cdot) \frac{\partial T(\cdot)}{\partial \theta} d\theta - u''_{gn}(\cdot) dg}{u''_{cn}(\cdot) (1 - m) + u''_{zn}(\cdot)}. \quad (16)$$

For any given individual  $n$ , this relationship displays the effect of the reform on the incentive to supply earnings. The first term in the numerator captures the effect on the marginal incentive to supply earnings from the increased tax burden, while the second term captures the effect from the expansion of  $g$ . Any discrepancy between the individual cost and the individual benefit of the tax-cum-public good reform functions just like a change in the effective marginal tax rate and thus affects earnings. Only when the two effects on the marginal incentive are exactly aligned is there no change in individual income. This is entirely consistent with the benefit principle, which cannot condition reform changes on the unobservable ability.

By differentiating the definitions in eqs (2) and (3) w.r.t.  $n$  and using eqs (4) and (15), we may write eq. (16) in the following way

$$dz = \frac{\partial \text{MRS}_{cg}(z, n) / \partial n}{\partial \text{MRS}_{cz}(z, n) / \partial n} \cdot dg, \quad (17)$$

where the single-crossing condition (5) implies that the denominator is negative. The partial derivatives in this expression measure the effect of ability on the marginal rates of substitution between, respectively,  $c$  and  $g$  in the numerator and  $c$  and  $z$  in the denominator.

The application of the benefit principle implies that the expansion of  $g$  and the accompanying change in the tax function keeps everyone's utility, and thus social welfare, unchanged. Now

eq. (15) gives  $\int_n \omega(w) \frac{u_g}{u_c} dg \cdot f(n) dn = \int_n \omega(w) \frac{\partial T}{\partial \theta} d\theta \cdot f(n) dn$  implying that condition (12) is equivalent to

$$\int_n \left[ \frac{\partial T}{\partial \theta} d\theta + m \left( \frac{\partial z}{\partial \theta} d\theta + \frac{\partial z}{\partial g} dg \right) \right] f(n) dn \geq dg. \quad (18)$$

From eqs (15), (17), and (18), it is now possible to establish our main result:

**Proposition 2** *A marginal expansion of a public good is desirable iff*

$$\int_n \left( \text{MRS}_{cg}(z, n) + m \cdot \frac{\partial \text{MRS}_{cg}(z, n) / \partial n}{\partial \text{MRS}_{cz}(z, n) / \partial n} \right) f(n) dn \geq \text{MRT}_{cg}. \quad (19)$$

**Proof:** This follows by inserting eqs (15) and (17) in condition (18).  $\square$

Proposition 2 shows that the Samuelson rule must be amended by a term that is affected by the *partial* correlation, i.e., *conditional on income*, between ability and the marginal willingness to pay for the public good. The additional term corrects for the revenue implications of the behavioral responses to the reform. The optimal level of  $g$  is affected by correlations with the unobservable  $n$  because the tax function is constrained to depend on the imperfect signal that is income. It is important to note that the partial effects on the MRS's in (19) are evaluated at a given income level. Thus, variations in MRS due entirely to variations in  $z$  do not affect the optimal public goods supply. The total effect of higher ability on the marginal willingness to pay for the public good is given by

$$\frac{d\text{MRS}_{cg}(z, n)}{dn} = \frac{\partial \text{MRS}_{cg}(z, n)}{\partial z} \frac{dz}{dn} + \frac{\partial \text{MRS}_{cg}(z, n)}{\partial n}.$$

This is illustrated on Figure 1, which displays indifference curves and the marginal rate of substitution between private consumption and public goods. A low-ability person who has low earnings/private consumption is at point  $L$ , while a high-ability person with high earnings/private consumption is at point  $H$ . Assume first that the preferences of both agents are given by the solid indifference curves  $i_1$  and  $i_2$ . In this case, the high-income person has a higher willingness to pay for the public good ( $\text{MRS}_{cg}$  is larger at  $H$  than at  $L$ ), which is only natural when  $g$  is a normal good because both agents receive the same level of public good consumption  $\bar{g}$ . This effect works entirely through earnings,  $dz/dn$ , and does not affect the optimal level of  $g$  since both types have the same willingness to pay when located at the same earnings/private consumption bundle. Rather, the crucial test is whether the slope of the indifference curves of

people of different ability differ *when evaluated at a given income/consumption level*. This situation arises if the preferences of the high ability person are instead represented by the dashed indifference curves  $i'_1$  and  $i'_2$ . In this case, the high-ability person has a higher willingness to pay at any given point, implying that the public good effectively redistributes based on the unobservable ability.

Intuitively, when marginal tax rates are positive, the supply of public goods is reduced relative to the first best if the marginal willingness to pay for the public good increases with ability. In this case, the benefit principle implies that higher incomes must contribute more to the financing of the public good. However, part (or all) of the additional benefit enjoyed by persons with higher incomes stems from their innate ability and is realized independently of the chosen income level. Thus, the additional taxes implied by the reform reduce the incentive to work. The size of the additional distortion depends on the responsiveness of earned income as captured by the denominator of the second term in (19). Also, the stronger is the influence of ability on the marginal willingness to pay, the more difficult it is for the government to finance  $g$  in a non-distortionary fashion. An alternative way to view this result focuses on how the concern for redistribution affects the optimal level of  $g$ . When persons of higher ability benefit relatively more from the presence of the public good, the supply of  $g$  adversely affects the government's scope for redistribution. Indeed, the public good effectively redistributes in favor of the rich. This point applies the same logic as do Nichols and Zeckhauser (1982) and Blackorby and Donaldson (1988) in the context of in kind transfers. Also, Kaplow (2008) provides a similar intuition for the case of public goods but does not arrive at our general formula (19). A reversal of this argument explains why the public goods supply should be *higher than advocated by the Samuelson rule* when there is a negative correlation between ability and the marginal willingness to pay for the public good. In this case, supplying  $g$  provides an additional means to redistribute in favor of the poor.

Education seems to be an example of a good that is valued higher by the more able, even conditional on income. Presumably, people of higher innate ability are better equipped to benefit from educational training. If so, the optimal public financial support for education is less than the Samuelson rule predicts because such support effectively redistributes income towards the

more able.<sup>5</sup> In contrast, public transportation is likely to benefit persons of lower ability more for a given income. Efficient public transportation reduces the travel time to and from the workplace, leaving more time for other activities. A low ability individual must work longer hours to uphold a given income and therefore, presumably, values her sparetime more. Thus, subsidies to public transportation effectively redistribute income towards the less able, over and above what is attainable through the income tax. Importantly, consumption patterns across incomes do not necessarily reveal the desirability of public transport subsidies. If low income individuals choose public transport because they cannot afford a car, not because they are of low skill, the Samuelson rule still applies.

Proposition 2 also clarifies when the original Samuelson rule obtains. The sufficient condition is that there is no *partial* effect from ability to the willingness to pay for the public good. Thus, the crucial question for the determination of the optimal  $g$  is whether the marginal willingness to pay is different for a person of high ability when she imitates the choices of a lower ability individual. If this is not the case, implying that people of different ability have the same  $MRS_{cg}$  for given  $z$ , the Samuelson rule applies and distributional considerations should not affect the level of the public good. This does not rule out that people of different ability, as they position themselves at different income levels, have different willingness to pay *in equilibrium*. In this case, the financing of the public good is not uniform under the benefit principle and, as a result, marginal tax rates are affected. But these tax variations are not distortionary as the marginal willingness to pay also varies with income. Differential financing is only distortionary when taxpayers can avoid the additional burden without reducing the benefit they enjoy from the public good. Thus, armed with Proposition 2 we can generalize the result of Kaplow (1996) to a more general class of utility functions:<sup>6</sup>

**Corollary 1** *Assume that individual utility satisfies the separability assumption:  $u(c, g, z, n) = \tilde{u}[w^1(c, g, z), w^2(z, n)]$ . Then an expansion of  $g$  is socially desirable whenever the Samuelson*

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<sup>5</sup>Education is, of course, not a public good but our argument also applies to externalities as discussed in the conclusion. Note also that redistribution policy may discourage private investments in education. This gives a second-best argument in favor of subsidizing education (Bovenberg and Jacobs, 2005).

<sup>6</sup>Note that  $u = \tilde{u}(w(c, g), l)$ , which is used in Kaplow (1996), is a special case of the utility function in Corollary 1, where  $w^1(c, g, z) = \tilde{w}^1(c, g)$  and  $w^2(z, n) = z/n$ .



condition holds, i.e.,

$$\int_n \text{MRS}_{cg} f(n) dn \geq \text{MRT}_{cg}.$$

**Proof:** The marginal willingness to pay for  $g$  is  $\text{MRS}_{cg} = \frac{u'_1(\cdot)w'_g(\cdot)}{u'_1(\cdot)w'_c(\cdot)} = \frac{w'_g(c,g,z)}{w'_c(c,g,z)}$ , which is independent of  $n$ . Thus,  $\partial \text{MRS}_{cg} / \partial n = 0$  implying that (19) reduces to the Samuelson rule.  $\square$

The above utility specification implies that variations in the marginal willingness to pay for the public good derive from income directly, not the underlying ability ( $\partial \text{MRS}_{cg} / \partial n = 0$ ). If the marginal willingness to pay increases with income, the benefit principle implies that marginal tax rates increase as a result of the reform but these changes are not distortionary as the individual benefit from the public good also increases with income (see Blomquist et al., 2008, for a similar point).

When utility is given by the standard Mirrleesian specification  $u(c, g, z/n)$  the formula for the optimal  $g$  can be written in terms of correlations between the marginal willingness to pay for  $g$  and labor supply,  $l = z/n$ . This is because with the standard utility function any positive correlation with  $n$  implies a negative correlation with  $l$ .

**Corollary 2** *With a Mirrleesian individual utility specification  $u(c, g, z, n) = \tilde{u}(c, g, z/n)$ , an expansion of  $g$  is socially desirable whenever*

$$\int_n \left( \text{MRS}_{cg}(z, l) + m \cdot \frac{\partial \text{MRS}_{cg}(z, l) / \partial l}{\partial \text{MRS}_{cz}(z, l) / \partial l} \right) f(n) dn \geq \text{MRT}_{cg}.$$

**Proof:** With the Mirrleesian utility function, we can use the relation  $z = n \cdot l$  to express the change in  $n$  as a function of the dependence of MRS on  $l$  instead. Indeed,

$$\frac{\partial \text{MRS}}{\partial n} = \frac{\partial \text{MRS}}{\partial l} \frac{\partial l}{\partial n} = -\frac{\partial \text{MRS}}{\partial l} \frac{z}{n^2} \Rightarrow \frac{\partial \text{MRS}_{cg}(z, n) / \partial n}{\partial \text{MRS}_{cz}(z, n) / \partial n} = \frac{\partial \text{MRS}_{cg}(z, l) / \partial l}{\partial \text{MRS}_{cz}(z, l) / \partial l}$$

Insert this in eq. (19) to arrive at the above result.  $\square$

When ability is restricted to affect utility only through  $l$ , the evaluation of a public project departs from the Samuelson rule if the marginal willingness to pay for the public good depends on individual labor supply. Thus, if  $\text{MRS}_{cg}$  displays a *negative partial correlation* with  $l$ , the optimal level of the public good is less than predicted by the Samuelson rule (notice that the denominator in the second term under the integral is now positive). In this case, the public good

is valued relatively more by those who must deliver fewer working hours to attain a given income, i.e., people of higher ability. Therefore, the public good impacts negatively on the government's ability to redistribute income. However, the opposite situation is equally plausible. When  $MRS_{cg}$  increases with  $l$  the optimal  $g$  is higher than the first best level. Finally, note that the correlation with working hours is only a sufficient statistic when the utility function has the shape considered in Corollary 2. It does not necessarily carry over to the general utility function (1).

## 5 Equivalence Between The Two Approaches: A Special Case

Generally, the formula for the optimal  $g$  deviates from Proposition 2 when the associated tax reform is not governed by the benefit principle. Thus, the standard approach generally leads to different results than those obtained in the previous section. However, in one special case the two approaches are equivalent and the simplest form of the standard formula obtains. The latter holds that public goods should be expanded if

$$\int_n MRS_{cg} \cdot f(n) dn \geq MCF \cdot MRT_{cg},$$

where MCF is the marginal cost of raising public funds. This simple representative agent version of the modified Samuelson rule focuses only on the distortionary effects of raising taxes and disregards distributional concerns. We now show that there is a special case where this simple formula obtains using the new approach.

Assume utility is given by

$$u = c + n \cdot w(g, z) - n \cdot h(z/n), \quad (20)$$

where the functional form of the disutility of labor is taken from Saez (2001) and implies that  $n$  reflects potential earnings. The above specification implies that utility from the public good is linear in ability. If we depart from this functional form, the simple standard formula does not obtain.

A marginal expansion of  $g$  is desirable iff (see Appendix C)

$$\int_n MRS_{cg} \left(1 - \frac{m}{1-m} \varepsilon\right) f(n) dn \geq MRT_{cg}, \quad (21)$$

where  $\varepsilon$  is the (compensated) elasticity of taxable income with respect to the net-of-tax rate. This formula identifies MCF as a central determinant of the optimal  $g$ . If, in addition, the income tax system is linear initially and the elasticity of taxable income is constant across individuals, the condition simplifies to

$$\int_n \text{MRS}_{cg} \cdot f(n) \, dn \geq \frac{1}{1 - \frac{m}{1-m}\varepsilon} \cdot \text{MRT}_{cg} = \text{MCF} \cdot \text{MRT}_{cg},$$

which is identical to the most simple version of the modified Samuelson rule (Browning, 1987, Dahlby, 1998, and Ballard and Fullerton, 1992). However, only when utility from the public good is *linear in ability* and the initial tax system is proportional is the traditional MCF correction valid.

## 6 Concluding Remarks

The central challenge involved in decisions on the optimal level of a public good is that correlations between the marginal willingness to pay and, respectively, ability and income are observationally equivalent but have vastly different policy implications as first noted by Hylland and Zeckhauser (1979). For instance, are wealthy people overrepresented among opera audiences because they are wealthy, or because they are of higher ability? For some purposes casual observation may be sufficient to decide on the desirability of a public project. When more detailed analyses are called for, one is left to search for observable characteristics that have a known (or estimable) relationship with ability. Indeed, if we have knowledge about the effect on the willingness to pay for the public good of some observable variable that is correlated with ability, this relationship should enter the determination of  $g$ . The observable characteristic could be education or even height. While there may be both ethical and practical concerns behind the absence of such variables in the tax function, such concerns have no bearing against their inclusion in the determination of  $g$  since the variables are not used to calculate individual tax burdens, only to identify the socially optimal  $g$ .

While the analysis in this paper has focused on public goods, the results may be directly applied to the correction of externalities. We may think of  $g$  as a global externality and  $\text{MRS}_{cg}$  as the willingness to pay for a marginal reduction of the externality. The cost of reducing  $g$  is then the costs of, e.g., abatement or alternative production methods. As argued by Kaplow and

Shavell (2002), the most efficient way to regulate externalities is through a price scheme that reflects marginal harm. When consumption patterns differ across individuals, the costs and benefits of such a scheme may be unevenly distributed. However, any distributional effects that are driven by preference variations due directly to income can be undone through adjustments of the income tax (see also Kaplow, 2006). Only when the willingness to pay for harm reduction is correlated with ability should the externality correction depart from first best rules.<sup>7</sup>

## A Proof of Proposition 1

The effect of the tax reform on government revenue is

$$\frac{\partial R}{\partial \theta} = \int_n \left[ \frac{\partial T}{\partial \theta} + m \left( \frac{\partial \hat{z}}{\partial y} \frac{\partial y}{\partial \theta} - \frac{\partial \hat{z}}{\partial (1-m)} \frac{\partial m}{\partial \theta} \right) \right] f(n) dn,$$

which is identical to the denominator on the right-hand side of (9), except that  $\partial z / \partial \theta$  has been decomposed into an income effect and an effect from the change in the marginal tax rate. The change in virtual income is

$$\frac{\partial y}{\partial \theta} = z \frac{\partial m}{\partial \theta} + m \frac{\partial z}{\partial \theta} - m \frac{\partial z}{\partial \theta} - \frac{\partial T(z, \theta)}{\partial \theta} = z \left( \frac{\partial m}{\partial \theta} - \frac{\partial a}{\partial \theta} \right),$$

where  $a \equiv T(z, \theta) / z$  is the average tax rate and  $\partial a / \partial \theta \equiv \frac{\partial T(z, \theta)}{\partial \theta} / z$ . This implies that

$$\frac{\partial z}{\partial \theta} = \left[ \frac{\partial \hat{z}}{\partial y} \left( \frac{\partial m}{\partial \theta} - \frac{\partial a}{\partial \theta} \right) - \frac{1}{1-m} \varepsilon \frac{\partial m}{\partial \theta} \right] z,$$

where  $\varepsilon \equiv \frac{1-m}{z} \frac{\partial \hat{z}}{\partial (1-m)}$  is the uncompensated elasticity of taxable income. We may rewrite this using the Slutsky equation

$$\frac{\partial z}{\partial \theta} = \left( \eta \frac{\partial a}{\partial \theta} - \varepsilon^c \frac{\partial m}{\partial \theta} \right) \frac{1}{1-m} z,$$

which implies

$$\frac{\partial R}{\partial \theta} = \int_n \left[ 1 + \frac{m}{1-m} \left( \eta - \frac{\partial m / \partial \theta}{\partial a / \partial \theta} \varepsilon^c \right) \right] \frac{\partial T}{\partial \theta} f(n) dn.$$

Insert this in the denominator on the right-hand side of (9) and use the definitions (10) in order to obtain formula (11) in Proposition 1.

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<sup>7</sup>If the externality is not global but affects only part of the population, it is necessary for the results that the income tax can follow the same demographic patterns. For instance, pollution in a major city mainly affects its citizens and compensation schemes must then be designed to affect only the citizens of that same city. This is possible if regional taxes are in place and can be adjusted freely. However, local tax functions are often subject to constitutional restrictions. In this case, and when the externality affects subsets of the population that cannot be explicitly targeted, the benefit principle can no longer be applied and alternative methods must be used.

## B Derivation of Equation (12)

From eq. (7) and the condition  $d\Omega = 0$  we get

$$dg = \frac{\int_n \omega(n) \frac{\partial T}{\partial \theta} d\theta f(n) dn}{\int_n \omega(n) \frac{u'_g}{u'_c} f(n) dn}.$$

We may rewrite eq. (8) as

$$dR = \int_n d\theta \left[ \frac{\partial T}{\partial \theta} + m \frac{\partial z}{\partial \theta} \right] f(n) dn - dg \left( 1 - \int_n m \frac{\partial z}{\partial g} f(n) dn \right).$$

Insert  $dg$  from above and apply the criterion  $dR \geq 0$  to get (12).

## C Derivation of Equation (21)

We start by deriving  $dz$  from eq. (16). With the utility function (20), we have  $u''_{cn} = 0$ ,  $u''_{gn} = w'_g(\cdot)$ , and the first-order condition for the choice of earnings (4) implies

$$h'(\cdot) = 1 - m \implies \frac{dz}{d(1-m)} = \frac{n}{h''(\cdot)},$$

which gives the (compensated) elasticity of earned income w.r.t. the take-home rate as

$$\varepsilon \equiv \frac{dz/z}{d(1-m)/(1-m)} = \frac{nh'(\cdot)}{zh''(\cdot)}.$$

The cross-derivative  $u''_{zn}$  then becomes

$$u''_{zn} = h''(\cdot) \frac{z}{n^2} = (1-m) \frac{1}{n} \frac{1}{\varepsilon}.$$

By inserting this relationship and  $u''_{cn} = 0$  into (16), we obtain

$$dz = -\frac{MRS_{cg}}{1-m} \varepsilon dg,$$

where we have used  $MRS_{cg} = nw'_g(\cdot)$ . By substituting the above expression and eq. (15) into condition (18), we obtain the inequality (21).

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Figure 1: High versus low ability

