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Cross-border hybrid instruments

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Abstract

The rules demarcating debt and equity for tax purposes differ across countries, hence the possibility that a hybrid instrument is treated as equity in one country and debt in another and that a firm with foreign investment can combine tax favored dividend income in the home country and tax deductible interest expenses in the foreign country. This paper characterizes the scope for cross-border hybrid instruments as a function of the properties of demarcation rules. For any given pair of countries, firms in at least one country and sometimes in both can finance investment in the other country with a cross-border hybrid instrument. When hybrid instruments can be embedded in multi-layered finance structures, firms in both countries can always achieve equity treatment in the home country and debt treatment in the host country.

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1 Introduction

"In exchange for capital, corporations can offer investors any set of rights that can be described by words, subject to any conceivable set of qualifications, and in consideration of any conceivable set of offsetting obligations". Hariton (1994)

While the universe of financial intruments comprises a myriad of hybrid instruments that combine characteristics of debt and equity in different ways, tax systems typically categorize all financial instruments as either debt or equity. Thus, tax codes need demarcation rules that define the sets of debt and equity instruments for tax purposes. Just as tax systems vary in other respects, there is considerable international variation in demarcation rules. In the U.S., the demarcation rule takes into account a number of characteristics, for instance whether the instrument has a fixed maturity or the principal is reimbursed at the will of the issuer; whether the return on the instrument represents a legally enforceable claim; whether such a claim is subordinate to the claims of general creditors; and whether the holder of the instrument has voting rights. In other countries, demarcation rules are markedly different. In France, for instance, the classification of a financial instrument as debt or equity for tax purposes depends solely on whether voting rights are conferred on the holder or not (Connors and Woll, 2001).

International differences in demarcation rules introduces the possibility that the same financial instrument is categorized as debt in one country and equity in another country. We refer to such financial instruments as cross-border hybrid instruments. As pointed out by several legal scholars, cross-border hybrid instruments represent an important tax planning opportunity for multinational firms (Rosenbloom, 1999; Krahmal, 2005). If a foreign investment is financed with an instrument treated as debt in the host country and equity in the home country, payments on the instrument are treated as tax deductible interest expenses in the host country and tax favored dividends in the home country. Hence, cross-border hybrid instruments can potentially generate considerable tax savings compared to standard debt and equity instruments.

An example of a financial instrument that serves as a cross-border hybrid instrument is a perpetual loan (Krahmal, 2005). In the U.S., a perpetual loan is usually categorized as equity with reference to the equity-like characteristic that the principal is never reimbursed. In France, one the other hand, it is categorized as debt if it does not confer voting rights on its holder. A perpetual loan that does not carry voting rights thus allows U.S. firms to reduce the effective tax rate on investment in a French subsidiary by combining deductible interest payments in France and tax favored dividends in the U.S. Other financial instruments that are usually treated as equity in the U.S. but as debt in many other countries include profit sharing loans and convertible loans.

This paper studies the conditions under which tax planning with cross-border hybrid instruments is feasible. At the heart of the paper is a simple model of hybrid instruments and their classification for tax purposes. The model posits that demarcation rules have two components: an assessment function that translates the multiple characteristics of financial instruments into a one-dimensional measure of equityness and a threshold level of equityness. Hybrid instruments with a level of equityness above the threshold are treated as equity; those with a level of equityness below the threshold are treated as debt. To see how the model works, consider again a perpetual loan combining the equity-like characteristic that the principal is never reimbursed and the debt-like characteristic that the holder has no voting rights. If the assessment function has a large weight on the maturity-dimension of financial instruments, the perpetual loan is considered to be relatively equity-like; if the assessment function has a large weight on the voting-dimension, it is considered to be relatively debt-like. Whether the perpetual loan is ultimately treated as debt or equity depends both on the assessment function and the threshold.

Equipped with this model, we ask: Under what conditions can firms finance foreign investment with an instrument that is treated as debt in the host country and equity in the home country? We find that for a given pair of countries with different demarcation rules, this is always feasible for firms in one of the countries and, in certain cases, it is feasible for firms in both countries. To see the intuition for this finding, consider the following two examples:

First, suppose that two countries apply the same assessment function but different threshold levels. A given financial instrument is thus considered to have the same level of equityness in the two countries but may be categorized differently for tax purposes due to the difference in threshold levels. At sufficiently high levels of equityness, financial instruments are treated as equity in both countries and at sufficiently low levels of equityness they are treated as debt in both countries. There exists, however, a range of intermediate levels of equityness where hybrid instruments are treated as equity in the country with the lower threshold and as debt in the country with the higher threshold. International differences in the threshold level thus create asymmetric opportunities for cross-border hybrid instruments: they facilitate tax planning by multinational firms in countries with a low threshold level and impede tax planning by multinational firms in countries with a high threshold level.

Second, suppose that two countries apply the same threshold level but different assessment functions. A given financial instrument is thus considered to have a different level of equityness in the two countries. If a firm finances an investment in the other country with a financial instrument that is equity-like in the dimensions where the home country attaches more weight than the host country and debt-like in the dimensions where the host country attaches more weight than the home country, the instrument is considered more equity-like in the home country than in the host country. If the level of equityness is appropriately adjusted, the instrument is treated as equity in the home country and debt in the host country. International differences in assessment functions thus create symmetric opportunities for cross-border hybrid instruments: they facilitate tax planning by multinational firms in both countries.

Finally, we also consider a setting where hybrid instruments can be embedded in more complex multi-layered finance structures and ask under what conditions a firm can finance foreign investment so as to combine equity treatment in the home country and debt treatment in the host country. We find that the scope for tax planning with hybrid instruments is radically enlarged in this setting. Indeed, a firm investing in a foreign country can always set up a finance structure that achieves equity treatment in the home country and debt treatment in the host country regardless of the properties of the demarcation rules. Intuitively, the scope for cross-border hybrid instruments derives from differences in demarcation rules between a pair of countries. Multi-layered finance structures allow firms to integrate more countries in the finance structure and thus exploit differences in demarcation rules not only between the home country and the host country but between any pair of third countries.

While the present paper takes international differences in demarcation rules as given, a companion paper studies the policy implications of tax planning with cross-border hybrid instruments in a simplified theoretical framework (Johannesen, 2011). The main finding is that non-cooperating governments face an incentive to set demarcation rules so as to facilitate tax planning by domestic multinational firms, which erodes foreign taxation of domestic firms, and deter tax planning by foreign multinational firms, which enforces domestic taxation of foreign multinational firms.

References to cross-border hybrid instruments in the economics and finance literatures are surprisingly scarce. A few papers briefly discuss the role of hybrid instruments and the closely related notion of hybrid entities in reducing the effective taxation of multinational firms but no formal analysis is provided (Auerbach, 2006; Altshuler and Grubert, 2005). There exists a small literature on the use of hybrid instruments for other purposes than international tax planning. Two empirical papers study the use of hybrid instruments that combine equity treatment for financial reporting purposes and debt treatment for tax purposes (Engel et al., 1999; Mills and Newberry, 2005). Several papers discuss the use of hybrid instruments issued by banks to serve as debt for tax purposes and equity for regulatory purposes (e.g. Gergen and Schmitz, 1997).

The paper proceeds in the following way. Section 2 develops the basic framework for analyzing taxation of hybrid instruments. Section 3 derives results characterizing the scope for cross-border hybrid instruments under very general assumptions about demarcation rules. Section 4 presents a special case where demarcation rules are linear. Section 5 extends the general model to include multi-layered financial structures. Section 6 provides some concluding remarks.

2 Tax treatment of hybrid instruments

The aim of this section is to develop a framework for analyzing hybrid instruments and their classification for tax purposes. We assume that financial instruments may differ in N dimensions where each dimension is scaled to range the interval [0;1]. In any dimension n, values closer to one imply that the instrument is more equity-like and values closer to zero imply that the instrument is more debt-like in this dimension. A financial instrument is thus fully described by a vector $\mathbf{z} = (z_1, z_2, ..., z_N)$ where $z_n \in [0;1]$ for n = 1, ..., N. We define pure equity and pure debt as $\mathbf{z}^E = (1, 1, ..., 1)$ and $\mathbf{z}^D = (0, 0, ..., 0)$ respectively and let \mathbb{Z} denote the set of financial instruments.

A demarcation rule consists of two elements: a continuous and differentiable assessment function $F(\mathbf{z})$, which assigns a value to each vector \mathbf{z} that reflects the position of the instrument on the debt-equity continuum, and a threshold level of equityness y. Under the demarcation rule $\{F(\cdot); y\}$, an instrument \mathbf{z} is categorized as debt if $F(\mathbf{z}) < y$ and as equity if $F(\mathbf{z}) \ge y$. Mirroring real-world corporate tax systems, demarcation rules thus classify any financial instrument as either debt or equity.

We impose some structure on demarcation rules in the form of the following three relatively weak assumptions:

Assumptions Any demarcation rule $\{F(\cdot); y\}$ must satisfy:

- (1) $F_n(\mathbf{z}) \geq 0$ for any n and any $\mathbf{z} \in \mathbb{Z}$
- (2) $F_n(\mathbf{z}) > 0$ for at least one n and any $\mathbf{z} \in \mathbb{Z}$
- (3) $F(z^D) < y < F(z^E)$

where $F_n(\mathbf{z})$ denotes the partial derivative of $F(\mathbf{z})$ with respect to z_n . The first assumption requires that partial derivatives of $F(\cdot)$ are non-negative so that endowing a financial instrument with more equity-like characteristics in a single dimension does not move the global assessment of the instrument in direction of debt. The second assumption imposes that at least one partial derivative of $F(\cdot)$ is strictly positive so that endowing a financial instrument with more equitylike characteristics in all dimensions moves the global assessment of the instrument in direction of equity. The final assumption ensures that demarcation rules allow for both pure debt and pure equity instruments.

Figure 1a provides an illustration of a demarcation rule in the two-dimensional case (N=2). The shaded square represents the full set of financial instruments \mathbb{Z} . The demarcation line depicts the subset of financial instruments satisfying that $F(z_1, z_2) = y$. This is the set of marginal hybrid instruments with characteristics that are just sufficiently close to equity to be categorized as such by the demarcation rule. Financial instruments above the demarcation line satisfy $F(z_1, z_2) > y$ and qualify as equity whereas financial instruments below the demarcation line satisfy $F(z_1, z_2) < y$ and qualify as debt. The slope of the demarcation line is negative reflecting that an increase in the equityness of a financial instrument in one dimension requires a reduction in the equityness in the other dimension to keep the overall level of equityness of the instrument constant.¹

It is convenient to scale demarcation rules in a particular way. For the purposes of this scaling, we provide the following formal definition of equivalent demarcation rules:

¹By the implicit differentiation theorem, the slope of the line is $-(\partial F(\mathbf{z})/\partial z_1)/(\partial F(\mathbf{z})/\partial z_2)$ which is negative under Assumptions 1 and 2.

Definition 1 Two demarcation rules $\{F'(\cdot); y'\}$ and $\{F''(\cdot); y''\}$ are equivalent if any instrument categorized as equity under $\{F'(\cdot); y'\}$ is also categorized as equity under $\{F''(\cdot); y''\}$ and any instrument categorized as equity under $\{F''(\cdot); y''\}$ is also categorized as equity under $\{F''(\cdot); y''\}$.

Since demarcation rules, by construction, categorize any financial instruments as either debt or equity, the definition implies that any instrument categorized as debt under one demarcation rules is also categorized as debt under any equivalent demarcation rule. We immediately proceed to introduce the relevant scaling of demarcation rules:

Lemma 1 For any demarcation rule $\{F(\cdot); y\}$ there exists a unique equivalent demarcation rule $\{G(\cdot); x\}$ satisfying that

(a)
$$G(\mathbf{z}') \geq G(\mathbf{z}'')$$
 if and only if $F(\mathbf{z}') \geq F(\mathbf{z}'')$

(b)
$$G(\widetilde{\mathbf{z}}) = \widetilde{z} \text{ for any } \widetilde{\mathbf{z}} = (\widetilde{z}, \widetilde{z}, ..., \widetilde{z}).$$

Proof. See Appendix.

Intuitively, the assessment function $F(\cdot)$ produces ordinal measures of the distance between any financial instrument and standard debt. By performing monotonic transformations of $F(\cdot)$ that retain the ranking of instruments on the debt-equity continuum while appropriately adjusting the threshold distance y that triggers equity treatment rather than debt treatment, we may construct any number of demarcation rules that are equivalent to the original demarcation rule $\{F(\cdot); y\}$. From this set of equivalent demarcation rules, we pick out the single rule $\{G(\cdot); x\}$ which satisfies $G(\widetilde{\mathbf{z}}) = \widetilde{z}$ for any $\widetilde{\mathbf{z}} = (\widetilde{z}, \widetilde{z}, ..., \widetilde{z})$. We refer to $\{G(\cdot); x\}$ as the scaled transformation of $\{F(\cdot); y\}$ or simply the scaled demarcation rule. Finally, it is convenient to let $\nabla G(\mathbf{z}) = (G_1(\mathbf{z}), G_2(\mathbf{z}), ..., G_N(\mathbf{z}))$ denote the vector of first-derivatives of $G(\cdot)$.

3 Tax planning with hybrid instruments

With this basic framework we address the question concerning the scope for cross-border hybrid instruments. Specifically, considering two countries A and B with scaled demarcation rules $\{G^A(\cdot); x^A\}$ and $\{G^B(\cdot); x^B\}$ respectively, we seek to identify conditions under which it is possible to construct (i) a financial instrument classified as equity in A and debt in B and (ii) a financial instrument classified as debt in A and equity in B.

Before proceeding to the formal analysis, it is instructive to consider some graphical illustrations. Figure 1b provides an example of two scaled demarcation rules $\{G^A(\cdot); x^A\}$ and $\{G^B(\cdot);x^B\}$ that differ in two respects: First, A applies a lower threshold level of equityness than B. This is reflected in the property that the $z_1 = z_2$ line intersects the demarcation line of A to the left of its intersection with the demarcation line of B. Second, A assigns more weight to z_2 relative to z_1 than B. This is reflected in the property that the demarcation line of A is flatter than the demarcation line of B. The two demarcation lines divide the set of financial instruments \mathbb{Z} into four subsets labelled \mathbb{Z}^{DD} , \mathbb{Z}^{ED} , \mathbb{Z}^{EE} and \mathbb{Z}^{DE} where the first and the second superscript refer to the classification in A and B respectively and D and Edenote debt and equity. Hence, \mathbb{Z}^{ED} is the set of hybrid instruments classified as equity in A and debt in B whereas \mathbb{Z}^{DE} is the set of hybrid instruments classified as debt in A and equity in B. Figure 1c provides another example where the distance between the threshold levels of equityness in the two countries is larger than in Figure 1b. This causes \mathbb{Z}^{DE} to vanish. Figure 1d provides a final example where the weights applied by the two countries are more similar than in Figure 1b. This also causes \mathbb{Z}^{DE} to vanish. These examples suggest that it is generally possible to construct cross-border hybrid instruments in one direction whereas it may or may not be possible to construct cross-border hybrid instruments in both directions depending on the properties of the demarcation rules. Moreover, the examples point to particular features of demarcation rules that appear to facilitate cross-border hybrid instruments. Specifically, the

examples suggest that more similar threshold values and less similar weights make it more likely that cross-border hybrid instruments exist in both directions.

We are now prepared to characterize the scope for cross-border hybrid instruments.

Proposition 1 Consider two countries A and B with scaled demarcation rules $\{G^A(\cdot); x^A\}$ and $\{G^B(\cdot); x^B\}$ and assume without loss of generality that $x^A \ge x^B$.

When
$$x^A > x^B$$

(a) there exists a hybrid instrument categorized as equity in B and debt in A

If
$$\nabla G^A(\mathbf{z}) \neq \nabla G^B(\mathbf{z})$$
 at $\mathbf{z} = (x^A, x^A, ..., x^A)$ and $x^A - x^B$ is sufficiently small:

(b) there exists a hybrid instrument categorized as equity in A and debt in B

When
$$x^A = x^B$$

If
$$\nabla G^A(\mathbf{z}) \neq \nabla G^B(\mathbf{z})$$
 at $\mathbf{z} = (x^A, x^A, ..., x^A)$:

- (c) there exists a hybrid instrument categorized as equity in B and debt in A
- (d) there exists a hybrid instrument categorized as equity in A and debt in B

Proof. See Appendix.

When two countries employ different demarcation rules, it is generally possible for firms in at least one of the countries to implement financial instruments that are categorized as debt in the host country and equity in the home country and sometimes it is possible for firms in both countries to achieve this result. Specifically, when the scaled demarcation rules have different threshold values, firms in the country with the lower threshold value can always make use of cross-border instruments (part a) whereas firms in the country with the higher threshold value can make use of cross-border instruments provided that the assessment functions are different and that the threshold values are not too different (part b). Intuitively, when the two demarcation rules have different weights in the neighborhood of \mathbf{z} , it is always possible to construct two financial instruments in this neighborhood, of which one \mathbf{z}' is considered more equity-like in A than in B (satisfying $G^A(\mathbf{z}') > G^B(\mathbf{z}')$) and the other one \mathbf{z}'' is considered more

equity-like in B than in A (satisfying $G^B(\mathbf{z}'') > G^A(\mathbf{z}'')$). If moreover the threshold values of the two demarcation rules are not too different, it is possible to have the former instrument categorized as equity in A and debt in B and the latter instrument categorized as debt in A and equity in B. When the scaled demarcation rules have identical threshold values but different assessment functions, firms in both countries can make use of cross-border instruments (part c-d).

4 An application with linear demarcation rules

With the aim of further exploring the roles played by differences in assessment functions and thresholds values, this subsection imposes the following additional linearity assumption on demarcation rules:

Assumption A linear demarcation rule $\{F(\cdot); y\}$ satisfies:

(4)
$$F_n(\mathbf{z}) = \mu_n \text{ for all } n \text{ at any } \mathbf{z} \in \mathbb{Z}$$

The crucial difference to the analysis in the previous subsection is that weights are assumed to be constant over all financial instruments, which excludes the possibility of interactions between different dimensions of financial instruments in the assessment of their position on the debt-equity continuum. For instance, it is possible to imagine a tax code stating that financial instruments are treated as equity if either the maturity is longer than a threshold a or the variable element in the return is above a threshold b and as debt in all other cases. Intuitively, this type of mechanism cannot be represented by a linear demarcation rule since the weight on each of the two dimensions depends on the characteristics in the other dimension.²

As in the general case, it is useful to normalize demarcation rules. The following lemma

² Formally, we may represent the tax rule with the assessment function $F(\mathbf{z}) = \max\{z_m, (a/b)z_r\}$ and the threshold value y = a where m is the maturity dimension and r is the return dimension of financial instruments. It is easy to see that $\partial F(\mathbf{z})/\partial z_m = 0$ for $z_m < (a/b)z_r$ and $\partial F(\mathbf{z})/\partial z_m = 1$ for $z_m > (a/b)z_r$, which violates the linearity assumption.

shows that the scaled transformation of a *linear* demarcation rule has the convenient property that the assessed equityness of a given financial instrument is simply a weighted average of its equityness in the N dimensions.

Lemma 2 The scaled transformation of a linear demarcation rule has an assessment function of the form $G(\mathbf{z}) = \sum_{n} \lambda_{n} z_{n}$ where $\sum_{n} \lambda_{n} = 1$

Proof. See Appendix

Before proceeding to the analysis, we introduce some notation. To asses the scope for cross-border hybrid instruments treated as equity in A and debt in B, we identify the instrument that is considered most debt-like in B given that the same instrument is categorized as equity in A. Formally, we let $\underline{z} \equiv \underline{z}(\lambda^A; \lambda^B; x^A)$ denote the financial instrument that minimizes $G^B(\cdot)$ subject to the constraint that $G^A(\underline{z}) \geq x^A$. Moreover, we let $\underline{x} \equiv G^B(\underline{z})$ denote the lowest value of $G^B(\cdot)$ attainable subject to the same constraint. By construction, there exists an instrument treated as equity in A and debt in B if and only if $x^B \geq \underline{x}$. Symmetrically, to asses the scope for cross-border hybrid instruments treated as equity in B and debt in A, we identify the instrument that is considered most enquy-like in B given that the same instrument is categorized as debt in A. Formally, we let $\overline{z} = \overline{z}(\lambda^A; \lambda^B; x^A)$ denote the financial instrument that maximizes $G^B(\cdot)$ subject to the constraint that $G^A(\underline{z}) \leq x^A$ and let $\overline{x} \equiv G^B(\overline{z})$ denote the highest value of $G^B(\cdot)$ attainable subject to the same constraint. By construction, there exists a financial instrument treated as equity in B and debt in A if and only if $x^B \leq \overline{x}$.

In the first step of the analysis, we derive $\underline{\mathbf{z}}$ and $\overline{\mathbf{z}}$.

Lemma 3 Without loss of generality order the dimensions such that $\frac{\lambda_1^B}{\lambda_1^A} \leq \frac{\lambda_2^B}{\lambda_2^A} \leq ... \leq \frac{\lambda_N^B}{\lambda_N^A}$

(a) The financial instrument $\underline{\mathbf{z}}$ that minimizes $G^B(\cdot)$ while satisfying that $G^A(\underline{\mathbf{z}}) \geq x^A$ is characterized by:

$$\underline{z}_n = \begin{cases} 1 & n < q \\ \underline{u} \in [0,1] & for \quad n = q \\ 0 & n > q \end{cases}$$

where q is implicitly defined as the highest number for $\sum_{n=1}^{q-1} \lambda_n^A < x_A$.

(b) The financial instrument $\overline{\mathbf{z}}$ that maximizes $G^B(\cdot)$ while satisfying that $G^A(\underline{\mathbf{z}}) \leq x^A$ is characterized by:

$$\overline{z}_n = \begin{cases} 0 & n p \end{cases}$$

where p is implicitly defined as the lowest number for which $\sum_{n=p+1}^{N} \lambda_n^A < x_A$.

Proof. See Appendix

The financial instrument $\underline{\mathbf{z}}$, which minimizes equityness in B while still being categorized as equity in A, is completely equity-like in dimensions where B attaches the least weight relative to A and completely debt-like in dimensions where B attaches the most weight relative to A. Intuitively, λ_n^B/λ_n^A expresses the cost of increasing z_n in terms of increased equityness in B per benefit of increasing z_n in terms of increased equityness in A. The instrument $\underline{\mathbf{z}}$ thus achieves the level of equityness required in A by setting $z_n = 1$ in dimensions with low cost-benefit ratios and $z_n = 0$ in dimensions with high cost-benefit ratios. Conversely, the financial instrument $\overline{\mathbf{z}}$, which maximizes equityness in B while still being categorized as debt in A, is completely debt-like in dimensions where B attaches the least weight relative to A and completely equity-like in dimensions where B attaches the most weight relative to A. Now, λ_n^B/λ_n^A expresses the benefit of

increasing z_n in terms of increased equityness in B per cost of increasing z_n in terms of increased equityness in A and the instrument $\overline{\mathbf{z}}$ thus achieves the highest level of equityness allowed in A by setting $z_n = 1$ in dimensions with high benefit-cost ratios and $z_n = 0$ in dimensions with low benefit-cost ratios.

It is now straightforward to show that the lowest possible level of equityness in B for an instrument considered equity in A is given by:

$$\underline{x} = x^A - \left\{ \sum_{n=1}^q \underline{z}_n (\lambda_n^A - \lambda_n^B) \right\}$$
 (1)

where q is defined in Lemma 3. If the weights of the demarcation rules are the same in the two countries ($\lambda^A = \lambda^B$), the expression in curly brackets disappears so that $\underline{x} = x^A$. In that case, an instrument treated as equity in A and debt in B exists if and only if x^B is larger than x^A . If the weights differ ($\lambda^A \neq \lambda^B$), the expression in curly brackets is strictly positive because of the ordering of the N dimensions so that $\underline{x} < x^A$. Hence, a hybrid instrument treated as equity in A and debt in B exists also if x^B is smaller but not too much smaller than x^A . Similarly, it is easy to show that the highest possible level of equityness in B for an instrument considered debt in A is given by:

$$\overline{x} = x^A - \left\{ \sum_{n=p}^N \overline{z}_n (\lambda_n^A - \lambda_n^B) \right\}$$
 (2)

where p is defined in Lemma 3. Again, if the weights are the same, it holds that $\overline{x} = x^A$, which implies that a hybrid instrument treated as equity in B and debt in A exists if and only if x^B is smaller than x^A . If the weights differ, it holds that $\overline{x} > x^A$, which implies that a hybrid instrument treated as debt in A and equity in B is also feasible if x^B is larger but not too much larger than x^A .

The expressions for \underline{x} and \overline{x} highlight the role of weight differences in creating a scope for cross-border hybrid instruments. The instrument \mathbf{z} achieves a given level of equityness in A with

equity-characteristics in the dimensions where B attaches little weight and debt-characterics in the dimensions where B attaches large weight relative to A. This implies that $\underline{\mathbf{z}}$ is assessed as a more debt-like instrument in B than in A and the difference in the assessed equityness is proportional to the differences in the weights. Similarly, the difference in the perceived equityness of $\overline{\mathbf{z}}$ in A and B is proportional to the differences in the weights.

It follows from this discussion that if weights are different in the two countries $(\lambda^A \neq \lambda^B)$, it holds that $\overline{x} > x^A > \underline{x}$. To relate to the general result derived in Proposition 1, assume that $x^A > x^B$. This directly implies that $\overline{x} > x^B$, hence there exists a set of instruments treated as equity in B and debt in A. This is an application of part (a) of Proposition 1. Moreover, if x^B is sufficiently close to x^A , it also holds that $\underline{x} < x^B$, in which case there also exists a set of instruments treated as equity in A and debt in B. This is an application of part (b) of Proposition 1, however, under the linearity assumption, the condition that x^B is sufficiently close to x^A has the precise meaning that the distance $x^A - x^B$ should be smaller than the distance $x^A - \underline{x}$.

Figure 2 illustrates the results in a numerical example with five-dimensional financial instruments (N=5). The figure holds the weights λ^A and λ^B constant and depicts \underline{x} and \overline{x} as functions of x^A . Each of the lines \underline{x} and \overline{x} consist of five linear pieces corresponding to the five dimensions of financial instruments. Each piece has the slope λ_n^B/λ_n^A reflecting that a marginal change ε in the equity assessment in A can be achieved with a change $(1/\lambda_n^A)\varepsilon$ in z_n , which in turn causes a change in the equity assessment in B of $(\lambda_n^B/\lambda_n^A)\varepsilon$. Consider the line \underline{x} : the first piece has the slope λ_1^B/λ_1^A , the second piece the steeper slope λ_2^B/λ_2^A and so on. For a low required level of equityness in A, the instrument \underline{z} is completely debt-like except for some degree of equityness in the dimension n=1 where equityness in A transforms into equityness in B at the lowest rate λ_1^B/λ_1^A ; for a somewhat higher required level of equityness in A, it is completely equity-like in dimension n=1 and has some degree of equityness in dimension

n=2 where equityness in A transforms into equityness in B at the higher rate λ_2^B/λ_2^A ; for a very high required level of equityness in A, it is completely equity-like in all dimensions except for the dimension n=5 where equityness in A transforms into equityness in B at the highest rate λ_5^B/λ_5^A . Now consider \overline{x} : the first piece has the slope λ_5^B/λ_5^A , the second piece the flatter slope λ_4^B/λ_4^A and so on. For a low required level of equityness in A, the instrument \overline{z} only has equity-characteristics in the dimension n=5 where equityness in A transforms into equityness in B at the highest rate λ_5^B/λ_5^A ; for a somewhat higher required level of equityness in A, it is completely equity-like in dimension n=5 and partly equity-like in dimension n=4 where equityness in A transforms into equityness in B at the somewhat lower rate λ_4^B/λ_4^A ; and so on. For a given value of x^A (horizontal axis), a value of x^B above \underline{x} (vertical axis) permits a cross-border hybrid instrument treated as equity in A and debt in B and a value of x^B below \overline{x} (vertical axis) permits a cross-border hybrid instrument treated as debt in A and equity in B. Clearly, for any value of x^A , both types of cross-border hybrid instruments are feasible if x^B is sufficiently close to x^A as stated in Proposition 1.

In a final step, we derive two comparative static results.

Proposition 2 A policy change in A raising the threshold level of equityness causes an increase in \overline{x} and an increase in x

Proof. See Appendix A ■

This proposition states that a small increase in x^A raises both \overline{x} and \underline{x} . Recall that an instrument treated as debt in A and equity in B exists if $x^B \leq \overline{x}$. An increase in x^A relaxes this constraint and thus facilitates the use of cross-border instruments by firms in B investing in A. Conversely, an instrument treated as equity in A and debt in B exists if $x^B \geq \underline{x}$. An increase in x^A tightens this constraint and thus impedes the use of cross-border instruments by firms in A investing in B. In conclusion, differences in the threshold values of demarcation rules create asymmetric opportunities for international tax planning with hybrid instruments.

Proposition 3 A policy change in A moving a small amount of relative weight from a dimension m to another dimension $n \neq m$ implies that:

where q and p are defined in Lemma 3.

Proof. See Appendix A

Consider a small change in weights from a dimension m to another dimension n < m in A. By the ordering of the dimensions introduced in Lemma, this implies a larger difference in the weights employed by the demarcation rules of the two countries in the sense that A moves weight from one dimension to another dimension where it has less weight relative to B. The proposition states that such a change in weights either decreases \underline{x} or leaves it unchanged and, moreover, it either increases \overline{x} or leaves it unchanged. Recall once again that an instrument treated as debt in A and equity in B exists if $x^B \leq \overline{x}$ and an instrument treated as equity in A and debt in A exists if A and equity in A and debt in A and equity in A are a larger weight difference relaxes both of these constraints and thus facilitates the use of cross-border instruments by firms in both countries. On the other hand, the proposition states, a small change in weights from a dimension A to another dimension A in A either increases A or leaves it unchanged and either decreases A or leaves it unchanged. A smaller weight difference thus tightens both of the constraints defining the scope for cross-border hybrid instruments and thus impedes the use of cross-border instruments

by firms in both countries. In conclusion, international weight differences in demarcation rules create *symmetric* opportunities for international tax planning with hybrid instruments.

The comparative statics properties are also evident in Figure 2. First, it is immediately clear from the figure that a small increase in x^A is associated with an increase in both \overline{x} and \underline{x} and thus increases the scope for cross-border hybrid instruments for firms in B investing in A while reducing the scope for firms in A investing in B. Second, if A moves weight from a dimension m to another dimension n < m, the line \underline{x} has a flatter slope on piece n and a steeper slope on piece m, which is somewhere to the right of n. This implies that the new line \underline{x} is strictly below the old line at any point from the beginning of piece n to the end of piece m and that the scope for cross-border hybrid instruments for firms in n investing in n has been enlarged. Also the line n has a flatter slope on the piece n and a steeper slope on another piece n, which is here somewhere to the left of n. This implies that the new line n is strictly above the old line at any point from the beginning of piece n to the end of piece n and that the scope for for cross-border hybrid instruments for firms in n investing in n has been enlarged.

5 Tax planning with conduit finance structures

The previous section derived conditions under which a firm in one country can finance an investment in another country with a hybrid instrument treated as equity in the home country and debt in the host country. To contrast with the type of financial structures analyzed in this subsection, we label hybrid instruments issued directly by the foreign subsidiary to the parent company direct hybrid instruments.

It is well-known that the tax planning of multinational firms often involves multiple jurisdictions. An emblematic example is the use of finance entities in tax havens to reduce the effective corporate tax burden on foreign investment (Desai et al., 2006). Another example is the use of conduit entities in third countries to minimize withholding taxes on internal capital flows

(Johannesen, 2012).

This section explores the significance of multiple jurisdictions in the current context of hybrid financing. The question we ask essentially remains the same as in the previous section:

Under what conditions can a firm that undertakes foreign investment combine the advantages of equity financing in the home country and debt financing in the host country? The present analysis, however, allows firms to involve entities in third countries in the finance structure.

We first consider the simplest finance structure involving a third country. In this finance structure, a parent company capitalizes a conduit entity in a third country, which passes on the funds to a subsidiary in the host country. We refer to this finance structure as a *single conduit* hybrid structure and provide the following formal definition:

Definition 2 A single conduit hybrid structure is a vector $\mathbf{s} = \{C, \mathbf{z}^U, \mathbf{z}^L\}$ where C indicates the conduit country; \mathbf{z}^U is the financial instrument between the parent and the conduit entity; and \mathbf{z}^L is the financial instrument between the conduit entity and the subsidiary in the host country.

Figure 3 illustrates tax planning with a single conduit hybrid structure. There are two ways in which the firm can achieve the desired combination of equity treatment in the home country and debt treatment in the host country. Either the financial instrument between the parent and the conduit entity takes the form of standard equity whereas the instrument between the conduit entity and the operating subsidiary is a hybrid instrument treated as equity in the conduit country and debt in the host country (left side of the figure). Or the financial instrument between the parent and the conduit entity is a hybrid instrument treated as equity in the home country and debt in the conduit country whereas the instrument between the conduit entity and the operating subsidiary takes the form standard debt (right side of the figure). In both cases, the firm achieves equity treatment in the home country and debt treatment in the host country. Moreover, if the two financial instruments are structured such that their cash-flows

match, no tax burden arises in the conduit country.³

The following proposition provides a formal characterization of the scope for tax planning with single conduit hybrid finance structures:

Proposition 4 Consider two countries A and B with scaled demarcation rules $\{G^A(\cdot); x^A\}$ and $\{G^B(\cdot); x^B\}$ and assume that $x^A > x^B$.

- (a) There exists a single conduit hybrid structure $\mathbf{s} = \{C, \mathbf{z}^U, \mathbf{z}^L\}$ satisfying that
 - B categorizes \mathbf{z}^U as equity;
 - C categorizes both \mathbf{z}^U and \mathbf{z}^L as debt or categorizes both \mathbf{z}^U and \mathbf{z}^L as equity
 - A categorizes \mathbf{z}^L as debt;

If $x^i > x^A$ or $x^i < x^B$ for some country $i \neq A, B$:

- (b) There exists a single conduit hybrid structure $\mathbf{s} = \{C, \mathbf{z}^U, \mathbf{z}^L\}$ satisfying that
 - A categorizes \mathbf{z}^U as equity;
 - C categorizes both \mathbf{z}^U and \mathbf{z}^L as debt or categorizes both \mathbf{z}^U and \mathbf{z}^L as equity
 - B categorizes \mathbf{z}^L as debt;

Proof. See Appendix.

Proposition 4 highlights that involving third countries in more sophisticated finance structures increases the scope for tax planning with hybrid instruments. Part (a) states that when the home country has a lower threshold level of equityness than the host country, there exists a single conduit hybrid structure with equity treatment in the home country and debt treatment in the host country. This mirrors part (a) of Proposition 1 stating that it is possible to achieve

 $^{^{3}}$ In the case where the conduit entity is deemed to hold matching ingoing and outgoing loans (right side), this is relatively clear. Provided that the interest rate applying to the ingoing and outgoing loans is the same, the tax base of the conduit entity is zero under standard corporate tax rules. In practice, country C may invoke the arms-length principle to require that the interest rate applying to the outgoing loan is slightly higher than the interest rate applying to the ingoing loan in order for the conduit entity to earn a taxable profit on its financing activities. In the case where the conduit entity is deemed to hold an equity stake in the operating subsidiary (left side), the presumption that the conduit entity faces no tax liability requires that country C like most countries in the world applies a territorial tax system under which foreign source dividends are tax exempt in the hands of corporate investors.

an equivalent result with a direct hybrid instrument. Part (b) states that when the home country has a higher threshold level of equityness than the host country, there exists a single conduit hybrid structure with equity treatment in the home country and debt treatment in the host country except in the unique case where the home country has the largest scaled threshold value among all countries and the host country has the smallest scaled threshold value among all countries. This compares to part (b) of Proposition 1 where the possibility to achieve an equivalent result with a direct hybrid instrument hinges on the distance between the threshold values of the home and host countries. Hence, there is a class of policy environments where the distance between the threshold values of the home and host countries is too large to permit tax planning with a direct hybrid instrument but the existence of third countries with threshold values larger than that of the home country or smaller than that of the host country permits tax planning with a single conduit hybrid structure.

The intuition for this finding is straightforward. The main insight from the basic model was that the scope for tax planning with hybrid instruments derives from differences in demarcation rules between the residence country of the entity issuing the instrument and the residence country of the entity acquiring the instrument. Firms using direct hybrid instruments for tax planning rely entirely on differences in demarcation rules between the home country and the host country. Under a single conduit hybrid structure, firms can exploit bilateral differences between the home country and any third country and between the host country and any third country. It is thus not surprising that single conduit hybrid financing expands the scope for tax planning relative to direct hybrid financing.

It should be noted that the condition $x^i > x^A$ or $x^i < x^B$ for some country i is a sufficient but not necessary condition for part (b) of proposition 4 to hold. To see this, recall from the previous section that even when $x^A > x^C$, there may exist a hybrid instrument, which is treated as equity in A and as debt in C rendering feasible tax planning with the single conduit hybrid

structure illustrated in the right side of Figure 3. This is more likely the smaller the difference $x^A - x^C$ and the more different the assessment functions $G^A(\cdot)$ and $G^C(\cdot)$. Similarly, even when $x^C > x^B$, there may exist a hybrid instrument, which is treated as equity in C and as debt in B rendering feasible tax planning with the single conduit hybrid structure illustrated in the left side of Figure 3.

We now turn to a more sophisticated finance structure involving two conduit entities in two different third countries. In this structure, funds are passed from the parent company to the first conduit entity, from the first conduit entity to the second entity and finally from the second conduit entity to a subsidiary in the host country by means of three distinct financial instruments. We refer to this finance structure as double conduit hybrid financing and provide the following formal definition:

Definition 3 A double conduit hybrid structure is a vector $\mathbf{d} = \{C_1, C_2, \mathbf{z}^U, \mathbf{z}^M, \mathbf{z}^L\}$ where C_1 is the first conduit country; C_2 is the second conduit country; \mathbf{z}^U is the financial instrument between the parent and the first conduit entity; \mathbf{z}^M is the financial instrument between the first and the second conduit entity; and \mathbf{z}^L is the financial instrument between the second conduit entity and the subsidiary in the host country.

Figure 4 illustrates tax planning with a double conduit hybrid structure. The financial instrument between the two conduit entities is a hybrid instrument classified as equity in C_1 and debt in C_2 . The instrument $\mathbf{z}^{\mathbf{U}}$ may be standard equity or any other instrument treated as equity in both the home country and the first conduit country whereas $\mathbf{z}^{\mathbf{L}}$ may be standard debt or any other instrument treated as debt in both the second conduit country and the host country. A double conduit hybrid structure with these properties achieves the desired combination of equity treatment in the home country and debt treatment in the host country. Moreover, if the three financial instruments $\mathbf{z}^U, \mathbf{z}^M$ and \mathbf{z}^L are structured such that their cash-flows match, no tax burden arises in the two conduit countries.

The following proposition provides a formal characterization of the scope for tax planning with double conduit hybrid finance structures:

Proposition 5 Consider two countries A and B with scaled demarcation rules $\{G^A(\cdot); x^A\}$ and $\{G^B(\cdot); x^B\}$. Assume that $x^i \neq x^j$ for two countries $i \neq A, B$ and $j \neq A, B$

There exists a double conduit hybrid structure $\mathbf{d} = \{C_1, C_2, \mathbf{z}^U, \mathbf{z}^M, \mathbf{z}^L\}$ satisfying that:

- B categorizes \mathbf{z}^U as equity;
- C_1 categorizes \mathbf{z}^U and \mathbf{z}^M as equity;
- C_2 categorizes \mathbf{z}^M and \mathbf{z}^L as debt;
- A categorizes \mathbf{z}^L as debt.

And that

- A categorizes \mathbf{z}^U as equity;
- C_1 categorizes \mathbf{z}^U and \mathbf{z}^M as equity;
- C_2 categorizes \mathbf{z}^M and \mathbf{z}^L as debt;
- B categorizes \mathbf{z}^L as debt.

Proof. See Appendix.

Proposition 5 has the striking implication that firms in any country investing in any other country may combine the benefits of equity treatment in the home country and debt treatment in the host country by means of a double conduit hybrid structure. Double conduit hybrid structures thus increase the scope for tax planning as compared to single conduit hybrid structures. Intuitively, under double conduit hybrid structures firms exploit differences in demarcation rules between any pair of third countries, which implies that the particular features of demarcation rules in the host and home countries are completely irrelevant.

It should be noted that under less idealized assumptions than those embedded in our model, conduit hybrid financing have at least two advantages over direct hybrid financing. First, demarcation rules are generally not perfectly deterministic and there may be some degree of a

priori uncertainty about the tax treatment of a given hybrid instrument. Suppose that for an instrument with $G(\mathbf{z}) > x$, there is a probability of debt treatment, which is decreasing in the distance $G(\mathbf{z}) - x$ while for an instrument with $G(\mathbf{z}) < x$, there is a probability of equity treatment, which is decreasing in the distance $x - G(\mathbf{z})$. In this setting, conduit hybrid financing can reduce the risk of an undesired classification of a hybrid instrument. Most obviously, the firm may choose conduit countries with the largest possible difference in demarcation rules so as to maximize the "safety margins" $G^{C_1}(\mathbf{z}) - x^{C_1}$ in C_1 and $x^{C_2} - G^{C_2}(\mathbf{z})$ in C_2 . Alternatively, the firm may place the conduit entities in third countries that offer advance tax agreements, which settle the tax treatment of a given financial instrument prior to implementation and thus eliminates any a priori uncertainty. Second, a few countries have recently adopted anti-avoidance rules specifically targeted at cross-border hybrid instruments (OECD, 2012). Essentially, these rules make the deductibility of interest payments to foreign entities conditional upon taxation of the corresponding interest income in the counterpart country and, symmetrically, make the exemption of dividends from foreign entities conditional upon nondeductibility of the corresponding dividend payment in the counterpart country. While such anti-avoidance rules eliminate the scope for tax planning with direct hybrid instruments, they have no impact on the scope for tax planning with double conduit hybrid structures provided that there are at least two potential conduit countries that do not apply the anti-avoidance rules. In Figure 4, for instance, it is futile for A and B to adopt the anti-avoidance rules since the dividend income in A is effectively matched by a non-deductible dividend payment in C1and the deductible interest payment in B is matched by taxable interest income in C2.

There are, however, two important qualifications to the results presented in this section. First, it is conceivable that the tax savings generated by conduit hybrid structures in the home and host countries are at least partly offset by tax liabilities in the conduit country, which

⁴Advance tax agreements are unavailable in most OECD countries but are used in certain smaller countries, such as Luxembourg, presumably with a view to attracting business related to international tax planning.

reduces the attractiveness of such finance structures. We noted above that if the different financial instruments composing a conduit hybrid structure are designed so as to generate identical cash flows, no tax liabilities arise in the conduit countries. This argument, however, sidesteps the fact that matching cash flows may effectively constrain the choice of financial instruments. For instance, if one of the instruments composing a single conduit hybrid structure is a profit sharing loan with a return linked to the stochastic financial performance of the host country investment and the other instrument is a standard loan with a fixed return, then the ingoing and outgoing cash flows of the conduit entity generally do not match. It follows that the conduit entity earns profits in some states of the world and incurs losses in other states of the world. If the conduit country does not allow tax losses to be carried forward and backward, the conduit entity may be liable to taxes even when the net present value of its income stream is zero. Second, it should be noted that some countries have special anti-abuse rules applying to multi-layered financial structures. In the United States, for instance, anti-abuse regulation allows tax authorities to recharacterize finance transactions involving multiple related parties as transactions directly between two entities for tax purposes.⁵ Clearly, such anti-abuse rules may limit the scope for tax planning with conduit hybrid financing.

6 Concluding Remarks

This paper has developed a theoretical model of hybrid instruments and their characterization for tax purposes. We have used the model to characterize the scope for tax planning with cross-border hybrid instruments under different assumptions about the complexity of the financial structures that firms can use. We first considered the scope for tax planning with direct hybrid

⁵Section 7701(1) of the Internal Revenue Code provides that "...the Secretary may prescribe regulations recharacterizing any multiple-party financing transaction as a transaction directly among any two or more of such parties where the Secretary determines that such recharacterization is appropriate to prevent avoidance of any tax imposed".

instruments. We found that for any pair of countries, firms in at least one of the countries and possibly in both countries can finance investment in the other country with a hybrid instrument treated as equity in the home country and debt in the host country. We then considered the scope for tax planning with hybrid instruments embedded in conduit finance structures. We found that for any pair of countries firms in both countries can finance investment in the other country with a finance structure that achieves equity treatment in the home country and debt treatment in the host country. We also argued that compared to direct hybrid instruments, conduit hybrid finance structures may reduce the risk of undesired classification of the hybrid instrument in the presence of stochastic elements in the demarcation rule and may render the anti-abuse rules currently applied by certain countries ineffective.

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Appendix

Proof of Lemma 1

Define $\mathbb{Z}_a \equiv \{\mathbf{z} \in \mathbb{Z} : F(\mathbf{z}) = a\}$. Let $\widetilde{\mathbf{z}}$ denote a financial instrument with identical elements $(\widetilde{z},\widetilde{z}...\widetilde{z})$. Assumption 1 implies that $F(\cdot)$ takes no higher values than $F(\mathbf{z}^E)$ and no lower values than $F(\mathbf{z}^D)$. The continuousness of $F(\cdot)$ thus ensures that for each $a \in [F(\mathbf{z}^D); F(\mathbf{z}^E)]$, the set \mathbb{Z}_a contains at least one vector $\widetilde{\mathbf{z}}$ whereas Assumption 2 ensures that any set \mathbb{Z}_a contains no more than one vector $\widetilde{\mathbf{z}}$. Hence, for each $a \in [F(\mathbf{z}^D); F(\mathbf{z}^E)]$ there is one and only one number \widetilde{z} for which $F(\tilde{z}, \tilde{z}, ..., \tilde{z}) = a$. Denote this number by $\tilde{z}(a)$ and note that, by Assumption 2, it holds that $\widetilde{z}(a') \geq \widetilde{z}(a'')$ for any $a' \geq a''$. Define $G(\cdot)$ in the following way: for $\mathbf{z} \in \mathbb{Z}_a : G(\mathbf{z}) \equiv \widetilde{z}(a)$ for all $a \in [F(\mathbf{z}^D); F(\mathbf{z}^E)]$. This definition implies that $G(\cdot)$ ranks financial instruments \mathbf{z} in the same way as $F(\cdot)$ while satisfying $G(\tilde{\mathbf{z}}) \equiv \tilde{z}$. Define $x \equiv \tilde{z}(y)$. This definition implies that the same set of financial instruments attain the threshold level of equityness under the demarcation rules $\{F(\cdot);y\}$ and $\{G(\cdot);x\}$. Consider a vector \mathbf{z} with $F(\mathbf{z})=b$ where $b\geq y$. By construction $\widetilde{z}(b) \geq \widetilde{z}(y)$. This in turn implies that $G(\mathbf{z}) \geq x$. Hence, any instrument classified as equity under the rule $\{F(\cdot);y\}$ is also classified as equity under the rule $\{G(\cdot);x\}$. An identical argument applies to any instrument classified as debt under the rule $\{F(\cdot);y\}$. It follows that the two demarcation rules $\{F(\cdot);y\}$ and $\{G(\cdot);x\}$ are equivalent. As for uniqueness, it is easy to see that $G(\cdot)$ is the only monotonic transformation of $F(\cdot)$ that satisfies (b). Moreover, given the defintion of $G(\cdot)$ any other threshold than x would result in non-equivalence with $\{F(\cdot);y\}$.

Proof of Proposition 1

First, consider the case $x^A = x^B$. Define the instrument $\mathbf{z}^S = (x^A, x^A, ..., x^A)$. By the property of scaled assessment functions, it holds that $G^A(\mathbf{z}^S) = G^B(\mathbf{z}^S) = x^A = x^B$. The fact that $G(\widetilde{\mathbf{z}}) = \widetilde{z}$ for any $\widetilde{\mathbf{z}} = (\widetilde{z}, \widetilde{z}, ..., \widetilde{z})$ implies that $\sum_n G_n(\widetilde{\mathbf{z}}) = 1$. The assumption that $\nabla G^A(\mathbf{z}^S) \neq \nabla G^B(\mathbf{z}^S)$ therefore implies that $G_m^A(\mathbf{z}^S) > \partial G_m^B(\mathbf{z}^S)$ for at least one m and that

 $G_k^A(\mathbf{z}^S) < G_k^B(\mathbf{z}^S)$ for at least one k. Consider the instrument $\mathbf{z}^{S'}$ which is identical to \mathbf{z}^S except that it is marginally more debt-like in dimension m and marginally more equity-like in dimension k and where the marginal deviations from x^A in these two dimensions are scaled such that the overall assessment in B is unchanged, that is $G^B(\mathbf{z}^{S'}) = x^B$. By construction it holds that $G^A(\mathbf{z}^{S'}) < x^A$. Hence, $\mathbf{z}^{S'}$ is categorized as equity in B and debt in A. This proves part (c). Now consider the instrument $\mathbf{z}^{S''}$ which is identical to \mathbf{z}^S except that it is marginally more equity-like in dimension m and marginally more debt-like in dimension m and where the marginal deviations from m in these two dimensions are scaled such that the overall assessment in m is unchanged, that is m0 is m1. By construction it holds that m2 is categorized as equity in m3. Hence, m3 is categorized as equity in m4 and debt in m5. This proves part (d).

Second, consider the case $x^A > x^B$. By construction any vector $\tilde{\mathbf{z}} = (\tilde{z}, \tilde{z}, ..., \tilde{z})$ with $x^A \geq \tilde{z} > x^B$ is categorized as equity in B and debt in A. This proves part (a). Note that the instrument $\mathbf{z}^{S''}$ satisfies $G^B(\mathbf{z}^{S''}) < G^A(\mathbf{z}^{S''}) = x^A$. This implies that $\mathbf{z}^{S''}$ is categorized as equity in A and debt in B provided that x^B is sufficiently close to x^A . This proves part (b).

Proof of Lemma 2

Assumption (4) imposes that the assessment function of a linear demarcation rule $\{F(\cdot); y\}$ takes the form $F(\mathbf{z}) = \sum \mu_n z_n + C$ where C is a constant. Consider a financial instrument \mathbf{z} and define $a \equiv \sum \mu_n z_n + C$. Following Lemma 1, we define the unique number $\tilde{z}(a)$ for which $F(\tilde{z}, \tilde{z}...\tilde{z}) = a$:

$$\widetilde{z}(a) = \frac{\sum \mu_n z_n'}{\sum \mu_n}$$

The assessment function of the unique scaled transformation thus equals:

$$G(\mathbf{z}) = \widetilde{z}(a) = \sum \lambda_n z_n \tag{3}$$

where

$$\lambda_n \equiv \frac{\mu_n}{\sum \mu_n} \tag{4}$$

Again following Lemma 1, we identify the unique number $\widetilde{z}(y)$ for which $F(\widetilde{z}, \widetilde{z}...\widetilde{z}) = y$:

$$\widetilde{z}(y) = \frac{y - C}{\sum \mu_n}$$

The threshold of the unique scaled transformation thus equals:

$$x = \widetilde{z}(y) = \frac{y - C}{\sum \mu_n} \tag{5}$$

It is easy to see that the two demarcation rules $\{F(\cdot);y\}$ and $\{G(\cdot);x\}$ are indeed equivalent. Under $\{F(\cdot);y\}$, an instrument \mathbf{z} is treated as equity if and only if $F(\mathbf{z}) \geq y$ or, using the functional form, if and only if $\sum \mu_n z_n + C \geq y$. Inserting from μ_n from (4) and y from (5), this expression can be rewritten as $\sum \lambda_n z_n \geq x$. It follows that any instrument \mathbf{z} satisfying $F(\mathbf{z}) \geq y$ also satisfies $G(\mathbf{z}) \geq x$, hence the demarcation rules $\{F(\cdot);y\}$ and $\{G(\cdot);x\}$ are equivalent. It is readily verified that $\{G(\cdot);x\}$ satisfies conditions (a) and (b) of Lemma 1.

Proof of Lemma 3

We minimize $F^B(\mathbf{z})$ over the N variables z_n subject to the constraint that $F^A(\mathbf{z}) \geq x^A$ and the 2N constraints that $0 \leq z_n \leq 1$. Let μ denote the Lagrange multiplier associated with $F^A(\mathbf{z}) \geq x^A$ and let δ_n denote the multipliers associated with $z_n \geq 0$ and ρ_n denote the multipliers associated with $z_n \leq 1$. The first-order condition for z_n thus reads:

$$\lambda_n^B - \mu \lambda_n^A - \delta_n + \rho_n = 0 \tag{6}$$

Rewrite (6) in the following way:

$$\frac{\lambda_n^B}{\lambda_n^A} = \mu + \frac{(\delta_n - \rho_n)}{\lambda_n^A} \tag{7}$$

Note that if $\underline{z}_n < 1$, then $\rho_n = 0$, which implies that $\lambda_n^B/\lambda_n^A \ge \mu$. It follows that for any m where $\lambda_m^B/\lambda_m^A > \lambda_n^B/\lambda_n^A$, it holds that $\lambda_m^B/\lambda_m^A > \mu$ and therefore that $\delta_m > 0$, which in turn implies that $\underline{z}_m = 0$. Similarly, if $\underline{z}_n > 0$, then $\delta_n = 0$, which implies that $\lambda_n^B/\lambda_n^A \le \mu$. It follows that for any m where $\lambda_m^B/\lambda_m^A < \lambda_n^B/\lambda_n^A$, it holds that $\lambda_m^B/\lambda_m^A < \mu$ and therefore that $\rho_m > 0$, which in turn implies that $\underline{z}_m = 1$. In sum, if $\underline{z}_n < 1$ then $\underline{z}_m = 0$ for any m where $\lambda_m^B/\lambda_m^A > \lambda_n^B/\lambda_n^A$ and if $\underline{z}_n > 0$ then $\underline{z}_m = 1$ for any m where $\lambda_m^B/\lambda_m^A < \lambda_n^B/\lambda_n^A$

We proceed by deriving the unique solution under the following two assumptions: (i) $\lambda_n^B/\lambda_n^A \neq \lambda_m^B/\lambda_m^A$ for any $n \neq m$; (ii) $\lambda_n^B/\lambda_n^A > 0$ for any n. Assumption (i) implies that \mathbf{z} has only one element \underline{z}_q with an interior value while $\underline{z}_n = 0$ for any n > q and $\underline{z}_n = 1$ for any n < q. This follows from our discussion of the first-order conditions above. Assumption (ii) implies that $\mu > 0$. To see this, note that since $0 < x^A$ by definition, there must be at least one n for which $\underline{z}_n > 0$ in order for $F^A(\mathbf{z}) \geq x^A$. This, in turn, implies that $\delta_n = 0$ and, combining with (7), that $\mu > 0$.

The direct implication of $\mu > 0$ is that $F^A(\underline{\mathbf{z}}) = x^A$, or, using the linear form, that $\sum \underline{z}_n \lambda_n^A = x^A$. Using that $\underline{z}_n = 0$ for any n > q and $\underline{z}_n = 1$ for any n < q, it must hold that:

$$\underline{z}_q = \left\{ x^A - \sum_{n=1}^{q-1} \underline{z}_n \lambda_n^A \right\} \frac{1}{\lambda_q^A} \tag{8}$$

Finally, q is implicitly determined by (8) and the constraints $0 \le \underline{z}_q \le 1$.

Now drop Assumption (i). Clearly, the instrument $\underline{\mathbf{z}}$ defined above still satisfies the first-order conditions and therefore minimizes $F^B(\mathbf{z})$ subject to the constraint $F^A(\mathbf{z}) \geq x^A$. However, if it holds that $\lambda_{q-1}^B/\lambda_{q-1}^A = \lambda_q^B/\lambda_q^A$ or if $\lambda_q^B/\lambda_q^A = \lambda_{q+1}^B/\lambda_{q+1}^A$, then there exist other financial

instruments, which satisfy the 2N + 1 constraints and has the same level of equityness in B as $\underline{\mathbf{z}}$. In the former case, this is true for the instrument:

$$(\underline{z}_1, \underline{z}_2, ..., \underline{z}_{q-2}, \underline{z}_{q-1} - \varepsilon, \underline{z}_q + \varepsilon, \underline{z}_{q+1}, ..., \underline{z}_N)$$

In the latter case, it is true for the instrument:

$$(\underline{z}_1, \underline{z}_2, ..., \underline{z}_{q-1}, \underline{z}_q - \varepsilon, \underline{z}_{q+1} + \varepsilon, \underline{z}_{q+2}, ..., \underline{z}_N)$$

Thus, $\underline{\mathbf{z}}$ is still a solution to the problem but is no longer unique.

Now drop Assumption (ii). Instead of assuming that $\lambda_n^B/\lambda_n^A>0$ for any n, we assume that $\lambda_n^B/\lambda_n^A=0$ for any $n\leq r$ and $\lambda_n^B/\lambda_n^A>0$ for any n>r. This formulation is completely general and encompasses the previous assumption that $\lambda_n^B/\lambda_n^A>0$ for any n as a special case (r=0). There are now two distinct cases. First, assume that $\sum_{n=1}^r \lambda_n^A \geq x^A$. Under this assumption, there are a set of instruments satisfying $F^B(\mathbf{z})=0$ and $F^A(\mathbf{z})\geq x^A$, namely all instruments with $z_n=0$ in dimensions n>r and sufficiently high values of z_n in other dimensions to ensure that $\sum_{n=1}^r z_n \lambda_n^A \geq x^A$. Hence, $\underline{\mathbf{z}}$ solves the problem but is not the unique solution. Second, assume that $\sum_{n=1}^r \lambda_n^A < x^A$. This assumption brings us back to the situation $\mu>0$. To see this note that in order for $F^A(\mathbf{z})\geq x^A$, there must be at least one element n>r for which $z_n>0$. This, in turn, implies that $\delta_n=0$ and, combining with (7), that $\mu>0$. Hence, $\underline{\mathbf{z}}$ is the unique solution to the problem. A perfectly symmetric procedure can be used to derive $\overline{\mathbf{z}}$.

Proof of Proposition 2

Insert the vector $\underline{\mathbf{z}}$ derived in the proof of Lemma 3 into (1) and rearrange to obtain:

$$\underline{x} = x^A \frac{\lambda_q^B}{\lambda_q^A} - \sum_{n=1}^{q-1} \lambda_n^A \left(\frac{\lambda_q^B}{\lambda_q^A} - \frac{\lambda_n^B}{\lambda_n^A} \right) \tag{9}$$

Similarly, insert the vector $\overline{\mathbf{z}}$ derived in the proof of Lemma 3 into (2) and rearrange to obtain:

$$\overline{x} = x^A \frac{\lambda_p^B}{\lambda_p^A} + \sum_{n=p+1}^N \lambda_n^A \left(\frac{\lambda_n^B}{\lambda_n^A} - \frac{\lambda_p^B}{\lambda_p^A} \right)$$
 (10)

For small changes in policy parameters, we can take q and p as given. It is easy to see that $d\underline{x}/dx^A = \lambda_q^B/\lambda_q^A > 0 \text{ and that } d\overline{x}/dx^A = \lambda_p^B/\lambda_p^A > 0$

Proof of Proposition 3

Differentiate the expressions for \underline{x} and \overline{x} derived in the proof of Proposition 2 with respect to λ_n^A while using that q and p can be taken as given for small changes in policy parameters.

$$\frac{d\underline{x}}{d\lambda_n^A} = \begin{cases}
-\lambda_q^B/\lambda_q^A & n < q \\
-\underline{z}_q \lambda_q^B/\lambda_q^A & \text{if} & n = q \\
0 & n > q
\end{cases}$$
(11)

$$\frac{d\overline{x}}{d\lambda_n^A} = \begin{cases}
0 & n p
\end{cases}$$
(12)

Consider a small change in the assessment function combining a small decrease in λ_m^A with an identically sized increase in λ_n^A where $n \neq m$. Using (11), it is easy to see that such a policy change has the following impact on x:

If n < m:

$$\left\{ \frac{d\underline{x}}{d\lambda_n^A} - \frac{d\underline{x}}{d\lambda_m^A} \right\} = \begin{cases}
0 & n < m < q \\
-(1 - \underline{z}_q)\lambda_q^B/\lambda_q^A < 0 & n < q = m \\
-\lambda_q^B/\lambda_q^A < 0 & \text{if } n < q < m \\
-\underline{z}_q\lambda_q^B/\lambda_q^A < 0 & n = q < m \\
0 & q < n < m
\end{cases}$$

If n > m:

$$\left\{ \frac{d\underline{x}}{d\lambda_n^A} - \frac{d\underline{x}}{d\lambda_m^A} \right\} = \begin{cases}
0 & m < n < q \\
\lambda_q^B/\lambda_q^A > 0 & m < q < n \\
(1 - \underline{z}_q)\lambda_q^B/\lambda_q^A > 0 & \text{if} \quad m < q = n \\
\underline{z}_q\lambda_q^B/\lambda_q^A > 0 & m = q < n \\
0 & q < m < n
\end{cases}$$

Similarly, using (12), it is easy to see that such a policy change has the following impact on \bar{x} :

If n < m:

$$\left\{ \frac{d\overline{x}}{d\lambda_n^A} - \frac{d\overline{x}}{d\lambda_m^A} \right\} = \begin{cases}
0 & n < m < p \\
\overline{z}_p \lambda_p^B / \lambda_p^A > 0 & n 0 & if n 0 & n = p < m \\
0 & p < n < m
\end{cases}$$

If n > m:

$$\left\{ \frac{d\overline{x}}{d\lambda_n^A} - \frac{d\overline{x}}{d\lambda_m^A} \right\} = \begin{cases}
0 & m < n < p \\
-\overline{z}_p \lambda_p^B / \lambda_p^A < 0 & m < p = n \\
-\lambda_p^B / \lambda_p^A < 0 & \text{if} m < p < n \\
-(1 - \overline{z}_p) \lambda_p^B / \lambda_p^A < 0 & m = p < n \\
0 & p < m < n
\end{cases}$$

Proposition 3 provides a qualitative summary of these results

Proof of Proposition 4

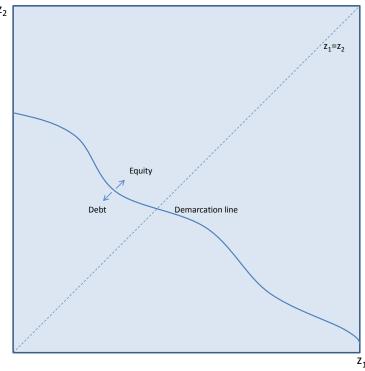
To prove part (a), we consider the following two cases in turn: (1) there exists a country $C \neq B$ for which $x^C < x^A$; (2) there exists no country $C \neq B$ for which $x^C < x^A$. First, consider case (1). Let $\mathbf{z}^U = \mathbf{z}^E$. This instrument, pure equity, is considered equity in both countries B and C. Let $\mathbf{z}^L = (\widetilde{z}, \widetilde{z}...\widetilde{z})$ for some $\widetilde{z} \in [x^C, x^A]$. By construction of the scaled demarcation rule, it holds that $G^C(\mathbf{z}^L) = G^A(\mathbf{z}^L) = \widetilde{z}$, hence this instrument is considered equity in Cand debt in A. In case (1), part (a) thus holds. Second, consider case (2) where all countries $i \neq A, B$ have $x^i \geq x^A$. Since $x^A > x^B$, this implies that there exists a country C for which $x^C > x^B$. Let $\mathbf{z}^U = (\widetilde{z}, \widetilde{z}...\widetilde{z})$ for some $\widetilde{z} \in [x^B, x^C]$. By construction of the scaled demarcation rule, it holds that $G^B(\mathbf{z}^L) = G^C(\mathbf{z}^L) = \widetilde{z}$, hence this instrument is considered equity in B and debt in C. Let $\mathbf{z}^U = \mathbf{z}^D$. This instrument, pure debt, is considered equity in both C and A. Also in case (2), part (a) thus holds. This proves part (a). To prove part (b), we consider the following two cases in turn: (3) there exists a country C for which $x^C > x^A$; (4) there exists a country C for which $x^C < x^B$. First, consider case (3). Let $\mathbf{z}^U = (\widetilde{z}, \widetilde{z}...\widetilde{z})$ for some $\widetilde{z} \in [x^A, x^C]$. By construction of the scaled demarcation rule, it holds that $G^A(\mathbf{z}^U) = G^C(\mathbf{z}^U) = \widetilde{z}$, hence this instrument is considered equity in A and debt in C. Let $\mathbf{z}^L = \mathbf{z}^D$. This instrument, pure debt, is considered debt in both C and B. In case (3), part (b) thus holds. Now turn to case (4). Let $\mathbf{z}^U = \mathbf{z}^E$. This instrument, pure equity, is considered equity in both A and C. Let $\mathbf{z}^L = (\widetilde{z}, \widetilde{z}...\widetilde{z})$ for some $\widetilde{z} \in [x^C, x^B]$. By construction of the scaled demarcation rule, it holds that $G^C(\mathbf{z}^L) = G^C(\mathbf{z}^L) = \widetilde{z}$, hence this instrument is considered equity in C and debt in B. Also in case (4), part (b) thus holds. This proves part (b).

Proof of Proposition 5

The assumption that $x^i \neq x^j$ for two countries $i \neq A, B$ and $j \neq A, B$ implies that there exist a pair of countries $C_1 \neq A, B$ and $C_2 \neq A, B$ for which $x^{C_1} < x^{C_2}$. Let $\mathbf{z}^U = \mathbf{z}^E$ and $\mathbf{z}^L = \mathbf{z}^D$. The former instrument, pure equity, is considered equity in all countries whereas the

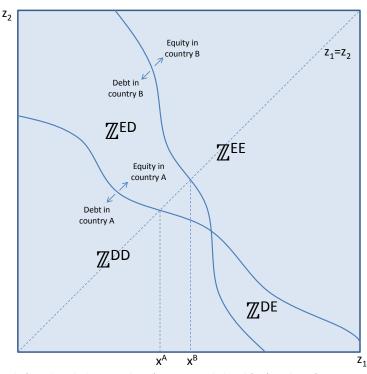
latter instrument, pure debt, is considered debt in all countries. Finally, let $\mathbf{z}^M = (\widetilde{z}, \widetilde{z}...\widetilde{z})$ for some $\widetilde{z} \in [x^{C_1}, x^{C_2}]$. By construction of the scaled demarcation rule, it holds that $G^{C_1}(\mathbf{z}^M) = G^{C_2}(\mathbf{z}^M) = \widetilde{z}$, hence this instrument is considered equity in C_1 and debt in C_2 . The double conduit hybrid structure $\mathbf{d} = \{C_1, C_2, \mathbf{z}^U, \mathbf{z}^M, \mathbf{z}^L\}$ thus satisfies the properties listed in Proposition 5.

Figure 1a: Two-dimensional example of a demarcation rule

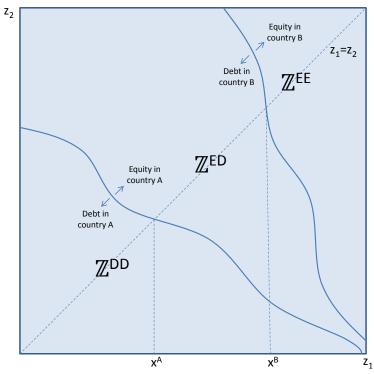


<u>Note</u>: The shaded square represents the set of financial instruments. The demarcation line defines the subset of instruments treated as debt and the subset treated as equity.

Figure 1b: Cross-border hybrid instruments in both directions

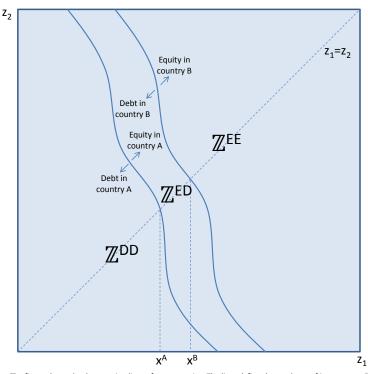


<u>Note</u>: The figure shows the demarcation lines of two countries. The lines define four subsets of instruments: Z^{DD} implies debt treatment in both countries, Z^{EE} implies equity treatment in both countries, Z^{DE} implies debt treatment in country A and equity treatment in country B, Z^{ED} implies equity treatment in country A and debt treatment in country B.



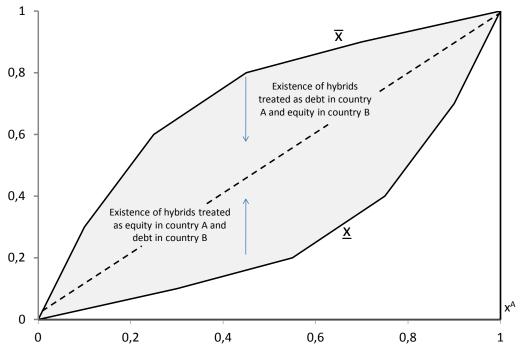
<u>Note</u>: The figure shows the demarcation lines of two countries. The lines define three subsets of instruments: Z^{DD} implies debt treatment in both countries, Z^{EE} implies equity treatment in both countries, Z^{ED} implies equity treatment in country A and debt treatment in country B. There are no instruments treated as debt in country A and equity in country B.

Figure 1d: Cross-border hybrid instruments in one direction



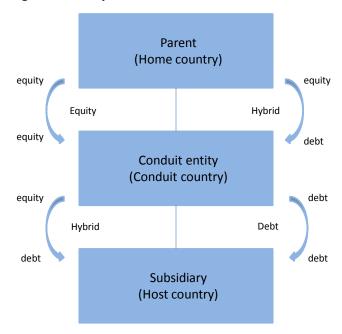
<u>Note</u>: The figure shows the demarcation lines of two countries. The lines define three subsets of instruments: Z^{DD} implies debt treatment in both countries, Z^{EE} implies equity treatment in both countries, Z^{ED} implies equity treatment in country A and debt treatment in country B. There are no instruments treated as debt in country A and equity in country B.

Figure 2: The scope for cross-border hybrid instruments – a linear example



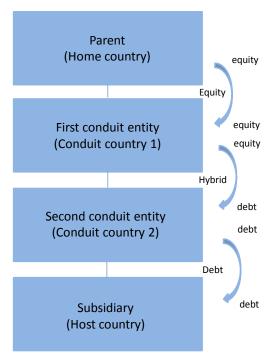
Note: The figure assumes that $\lambda^A=(0.3,0.3,0.2,0.1,0.1)$ and $\lambda^B=(0.1,0.15,0.2,0.25,0.3)$. These assumptions satisfy that $\Sigma \lambda_n=1$ for both countries and that λ^A_n/λ^B_n for any n<m. The lower bold line indicates the minimum level of equityness that can be achieved in B for an instrument treated as equity in A. The upper bold line indicates the maximum level of equityness that can be achieved in B for an instrument treated as debt in A. The figure thus depicts the scope for tax planning with cross-border hybrid instruments. Given a threshold value of equityness in A (on the x-axis), there exists a set of cross-border hybrid instruments treated as equity in A and debt in B if and only if the threshold value of equityness in B is higher than the lower bold line and there exists a set of cross-border hybrid instruments treated as debt in A and equity in B if and only if the threshold value of equityness in B is lower than the upper bold line.

Figure 3: The single conduit hybrid structure



Note: The figure illustrates the two ways in which a cross-border hybrid instrument can be embedded in a single conduit hybrid structure to create equity treatment in the home country and debt treatment in the host country. Either, the parent finances the conduit entity with a hybrid instrument, which is treated as equity in the Home country and as debt in the conduit country, and the conduit entity passes on the funds to the subsidiary as pure debt (right side). Or, the parent finances the conduit entity with pure equity and the conduit entity passes on the funds to the subsidiary in the form of a hybrid instrument, which is treated as equity in the conduit country and debt in the host country (left side).

Figure 4: The double conduit hybrid structure



Note: The figure illustrates how a cross-border hybrid instrument can be embedded in a double conduit hybrid structure to create equity treatment in the home country and debt treatment in the host country. The parent finances the first conduit entity with pure equity. The funds are passed on to the second conduit entity in the form of a a hybrid instrument, which is treated as equity in the first conduit country and as debt in the second conduit country. Finally, the second conduit entity passes on the funds to the subsidiary in the form of pure debt.