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Working Paper Decomposing Firm-level Sales Variation

EPRU Working Paper Series, No. 2009-05

Provided in Cooperation with: Economic Policy Research Unit (EPRU), University of Copenhagen

Suggested Citation: Munch, Jakob R.; Nguyen, Daniel X. (2009) : Decomposing Firm-level Sales Variation, EPRU Working Paper Series, No. 2009-05, University of Copenhagen, Economic Policy Research Unit (EPRU), Copenhagen

This Version is available at: https://hdl.handle.net/10419/82106

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Decomposing Firm-level Sales Variation

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ISSN 0908-7745

Decomposing Firm-level Sales Variation^{*}

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> First version: August, 2008 This version: June 2009

Abstract

We measure the contribution of firm-specific effects to overall sales variation within a destination and find it remarkably low. Our empirical decomposition is structurally motivated by a heterogeneity model of exporting involving destinationspecific, firm-specific, and firm-destination-specific latent effects with incidental truncation. We use a highly detailed dataset with exports by products and destinations for all Danish manufacturing firms. We find the contribution of firm-specific heterogeneity to within-destination sales variation varies greatly across HS6 products, and that for the median product it drives 31% of the sales variation. When we remove first-time exports from our sample, the median value increases to 40%, implying that firm-destination-specific effects are most important the first year. We conclude that while firm-specific productivity can account for some of the variation, the majority is explained by firm-destination-specific heterogeneity sources such as firm-destination-specific demand.

Keywords: Firm heterogeneity, firm-level export data, truncation correction.

JEL Codes: F12, C24

^{*}We would like to thank participants at Purdue University, the University of Copenhagen, the Fourth Danish International Economics Workshop, Aarhus, the 2008 ETSG conference, Warsaw, the Fall 2008 Midwest Meetings International Economics Group Meetings, Ohio State University, 2008 EIIT, University of Colorado, the LMDG 2008 conference on 'Labor Economics and International Trade', Sandbjerg and NOITS 2009, Copenhagen for helpful suggestions and comments. In particular, we would like to acknowledge David Hummels, Chong Xiang, and Mette Ejrnæs for suggestions that have greatly improved the direction and exposition of this paper. Financial support from the Tuborg Foundation is gratefully acknowledged. All errors are our own.

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1 Introduction

There is substantial variation in firm-level export volumes. For example, year 2003 annual shipments for Danish exporters ranged from less than two hundred dollars to more than six-hundred million dollars. Recent theoretical works have attributed this variation to heterogeneity in firm productivities.¹ These theories were motivated by earlier empirical studies that identified differences between firms that do export and firms that do not²: on average, exporters produce more, hire more labor, pay higher wages, and exhibit higher productivities as measured by either total factor productivity or value added per worker. The contrasts between exporters and nonexporters supported the story that productivity and exporting status were linked.

This paper measures how much sales variation can be explained by productivity. We decompose firm sales variation within a destination into a firm-specific component and a firm-destination-specific component. The firm-specific component comprises all firm characteristics that would affect firm sales, such as productivity, quality, or economies of scale. Since productivity is thought of as anchored to the firm, the sales variation explained by the firm-specific effects is an upper bound of that explained by productivity heterogeneity.

This paper adds to a small but growing literature examining the destinations to which firms export. The lack of work in the area is due primarily to the dearth of firm-destination specific export observations. Eaton, Kortum, and Kramarz (2004) find that most French firms export to only one destination (the mode being Belgium), and that the entry of French firms into a market accounts for two-thirds of the growth of the French share of market sales. Newer studies show that firms supply domestically for several years before exporting, that they usually begin exporting to one destination country, and that many stop exporting activities soon after they begin³. Since productivity is realized before

¹Notable examples include Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), Melitz (2003), Bernard, Jensen, and Schott (2006), and Bernard, Jensen, Redding, and Schott (2007).

²Notable works include Aw, Chung, and Roberts (2000), Bernard and Jensen (1995, 1999) and Clerides, Lach, and Tybout (1998).

³For example, Eaton, Eslava, Kugler, and Tybout (2007), Damijan, Kostevc, and Polanec (2007),

supply to any destination and applies to all destinations, these studies present empirical patterns unreconciled by the productivity heterogeneity models. The current study adds to and extends this literature by utilizing a highly disaggregated and detailed dataset – we observe destination-specific shipment values for the universe of Danish exporters in 2001 to 2003. This disaggregation level allows us to identify the firm-specific component and a firm-destination-specific component of an export. We can estimate the contribution of each using our structural model. As it is the standard and flagship productivity heterogeneity model, Melitz (2003) forms the basis for our structural estimation. Since Melitz (2003) does not incorporate destination-specific effects, we incorporate demand heterogeneity à la Nguyen (2009) to account for destination-specific shocks. The resulting model is an amalgam of Melitz (2003) and Nguyen (2009).

Three contemporary studies have goals similar, but not identical, to our own. Eaton, Kortum, and Kramarz (2008) estimates the contribution of firm-specific productivity to both the probability of entering a destination and the variance of sales conditional on entry. They use a model incorporating firm-specific productivity shocks drawn from a Pareto distribution and firm-destination-specific taste and cost shocks drawn from lognormal distributions. By calibrating their model to exports of French firms, Eaton, Kortum and Kramarz (2008) estimate that the variance of firm-specific effects can account for 50% of the variation of entry into a destination and 25% of the variation of sales in a destination conditional upon entry into that destination. By contrast, our study estimates the contribution of firm-specific effects on the unconditional variation of potential sales within a destination. We do not separate the variation of entry from the variation of sales, since Melitz (2003), the basis for both studies, suggests that entry into a destination is determined entirely by potential sales.

Kee and Krishna (2008) examine Bangladeshi exports of textiles to the US and EU. They find that a textile firm's market share in EU cannot predict its market share in the US: the correlation between the two is not statistically different from zero.

Lawless and Whelan (2008) use firm-destination data from a survey of 676 Irish-owned Alvarez, Faruq, and Lopez (2007) exporters to explain to where and how much firms export. Using OLS regressions with fixed-effects, they find the variation in firm-year and country specific effects accounts for 57 percent of the total variation. By itself, the country specific effects explain 16 percent of the variation, leaving 41 percent of the variation explained by firm-year specific effects.⁴ Our results using OLS with fixed effects resemble those of Lawless and Whelan's. We show that truncation issues bias OLS results and must be accounted for. Honoré and Kyriazadou (2000) discuss that the Heckman (1979) two-step procedure cannot correct for this truncation bias when entry into a destination is related to the firm-destination-specific demand draws. Instead, this paper uses a monte carlo estimation maximization procedure to consistently account for truncation and the unobserved effects.

In addition to the differences outlined above, our study uses the most detailed dataset of the related studies. The data cover the universe of Danish firms and uniquely identifies exports by destinations at the eight digit product level. This level of disaggregation is not available in the three other studies. We pool observations at differing industry levels, allowing us to compare the contribution of productivity for both broadly defined and narrowly defined industries.

In our main results, we estimate the contribution of firm-specific heterogeneity to overall 2003 Danish export sales variance by HS6 product category. We find that the contribution varies greatly across products. For half of Danish exported products, the contribution is lower than 31%. The mean firm-specific contribution across our sample is only 33%, while firm-destination specific effects contribute 67%. Therefore, we conclude that firm-destination specific effects matter a great deal more than firm specific effects. As robustness checks, we look at different product aggregation levels and different years. We also remove small trade flows and new trade flows. Our results consistently point towards firm-destination-specific effects as the driver of sales variation.

In the next section, we present an illustrative example of how firm-specific effects may or may not drive firm sales across destinations. Section 3 descripes the Danish export

 $^{^{4}}$ In other specifications they estimate the explanatory power of observed firm characteristics such as value added per employee and sector dummies instead of firm fixed effects.

data. Next, we present a simple model, based on Melitz (2003) and Nguyen (2009), that shows how truncation biases standard estimation procedures. Section 5 outlines our strategy to overcome this bias. Results and conclusions follow.

2 An illustrative example

To aid the reader in understanding the goal of this paper, we begin with an illustrative example. Suppose Denmark exports to only two destinations: Sweden and Germany. Melitz (2003) predicts that firms that sell to both destinations should have relative revenues that are one-to-one correlated. If a Danish brick firm's sales to Germany are twice the average of all Danish brick firms selling to Germany, this firm must be twice as productive. The same firm's Swedish sales should therefore be twice the average. The one-to-one correlation predicts that the variation in German relative revenues should completely explain the variation in Swedish relative revenues.

We present the results of this test for building bricks, and for plastic boxes, in Figure 1. It depicts firm revenues to Sweden and Germany for the two products⁵. The revenues are relative to the mean Danish firm revenues of the respective product to the respective destination.

Insert Figure 1 here

The OLS results for boxes support a weaker interpretation of Melitz: that relative revenues are strongly and positively correlated, and close to one. The slope, although statistically different from one, is still high at 0.84. The variation in German relative revenues explains a little more than half of the variation in Swedish relative revenues.

In contrast, the OLS results for building bricks do not support the predictions of Melitz. The implied correlation is negative and not statistically different from zero. The $R^2 = 0.08$ suggests that little of the Swedish variation is explained by the German variation.

⁵The two products are more precisely "Boxes, cases, crates and similar articles for the conveyance or packaging of goods, of plastics" (CN8 product 39231000) and "Building bricks (excl. those of siliceous fossil meals or similar siliceous earths, and refractory bricks of heading 6902)" (CN8 product 69041000). Sweden and Germany are the two most popular destinations for Danish exporters.

Repeating this procedure for all Danish products exported to both Sweden and Germany, we can test this straight-forward prediction of Melitz (2003). We find a mean and median correlation of 0.18 and 0.12. The mean and median R^2 are both below 5 percent. These initial results suggest that firm-specific characteristics cannot explain much of the cross-destination variation.

For the remainder of the paper, we do not rely on the estimated slope as a measure of the contribution of productivity to relative revenues for several reasons. First, over one-third of our estimated slopes are negative, which would imply that productivity does not contribute to destination-specific revenues at all for these products. Second, the correlation does not tell us the contribution of productivity variation to the total variation. For example, consider Figure 2 below, which presents three graphs of simulated relative revenues in two destinations. All three scatters have fitted slopes of 1, but different R^2 values. If productivity heterogeneity is the sole source of the variation, we should expect to see scatters similar to that in the upper left panel of Figure 2. As the contribution of firm-destination specific heterogeneity rises, the scatters begin to look more like the ones in the upper right and bottom left of Figure 2. Therefore, estimated slopes close to unity are misleading confirmations of Melitz (2003). Instead we will focus on a variant of R^2 as our measure of the contribution of firm productivity to sales variation.

Insert Figure 2 here

3 Danish firm-level data

The Danish External Trade Statistics provides product-level destination-specific export data for the universe of Danish firms. Exports are recorded according to the eight-digit Combined Nomenclature (CN) product code which encompasses approximately 10,000 different product categories. While all trade flows with non-EU countries are recorded by customs authorities (and so the coverage rate in the data is close to complete), there is not a similar system in place for intra-EU trade. However, intra-EU trade is recorded through the Intrastat system, where firms are obliged to report trade data on a monthly basis. One source of inaccuracy in this system is that some firms appear not to report data to the system. Also, data on intra-EU trade is censored in a way such that only firms exporting goods with a total annual value exceeding a certain threshold⁶ are recorded in the files. No such data limitations exist for trade out of the EU. As a result the coverage rate in the Intrastat system is lower but still in the range 85-90 percent. See Statistics Denmark (2003) for further details.

This study examines Danish manufacturing exports in 2003, but for robustness checks we also use data from 2001 and 2002. We select all manufacturing firms with positive inputs of labor and capital and with positive export sales. Also, we consider only manufacturing products by selecting products in one-digit SITC categories 5, 6, 7 and 8. With these restrictions our 2003 dataset comprises 155,426 firm-destination-product sales observations by 4,304 firms in 5,339 eight-digit CN8 products to 223 destination countries, see Table 1. The aggregate value of all these trade flows totals 182 billion Danish kroner (DKK), which in 2003 roughly correspond to USD 28 billion.

Insert Table 1 here

Table 1 shows some similarities between exporters in Denmark and those in bigger economies. The median number of destinations for firm-product exports is 1, which is in line with the findings for the US (Bernard and Jensen, 1995) and France (Eaton, Kortum, Kramarz, 2004). Clearly, some firms ship their products to many destinations – the mean number of destinations is 3.4 and the maximum number is 138.

At the disaggregated eight-digit product level most destinations do not have many Danish firms present. The median number of firms is 1 and the mean is 2.2. At the slightly more aggregated six-digit Harmonized System (HS6) level the mean number of firms is 2.6, but still more than half of the product-destinations have only one firm. This presents a problem for our empirical strategy, because it cannot identify the destination-

 $^{^6{\}rm For}$ the years considered, this threshold was DKK 2.5 million corresponding to approximately USD 500,000.

specific effect with only a single firm. Therefore, in the following, we will only consider sufficiently important products by imposing some restrictions on the data. First, we disregard product-destinations with less than five firms and, second, products with less than 25 firm-destinations in total are deleted. Third, we also need cross destination variation to estimate the firm-specific effects, so products that are shipped to fewer than three destinations (by any firm) are omitted. With these restrictions we end up with just 491 CN8 products or 480 HS6 products. However, they constitute more than a third or a half of the overall trade volume respectively, see Table 1.

For the restricted samples there is not much difference between the HS6 and CN8 levels. In the following we focus on the HS6 level as it covers the largest fraction of the total Danish export volume, but we report results for the CN8 level as well.

4 Theory

Our model is based on Melitz (2003) and Nguyen (2009). We employ three types of heterogeneity (destination-specific, firm-specific and firm-destination-specific) to decompose sales variance. While Melitz (2003) and Nguyen (2009) work out the number of entering firms in a general equilibrium by clearing the labor market, our model exposition stops at the decomposition. Our model's predictions for the variation of revenues across destinations can be collapsed to those of either model. Table 2 in the appendix lists the notation for ease of reference.

4.1 A model of sales variation

The small open economy of Denmark exports goods produced by N products to foreign destinations $j \in J$. For each product $n \in N$, there are W_n Danish firms each producing a unique variety ω . A portion W_{nj} of these firms supply to destination j. For the rest of this section, we focus our attention on a single product and therefore drop the n without loss of generalization. The utility gained in destination j from consuming Danish varieties of this product is represented by u_j :

$$u_j = \sum_{\omega=1}^{W_j} \exp\left(\frac{x_{\omega j}}{\sigma}\right) \left(q_{\omega j}\right)^{\frac{\sigma-1}{\sigma}},\tag{1}$$

where $q_{\omega j}$ is consumption of variety ω in j and $\sigma > 1$ is a measure of the substitutability among the different varieties.

The utility function resembles a Dixit-Stiglitz utility function with a demand shifter. The demand shifter $x_{\omega j}$ represents destination j taste⁷ for variety ω . Higher $x_{\omega j}$ corresponds to greater demand for that variety relative to other varieties in the same destination. Destination j's demand for variety ω can be derived as:

$$q_{\omega j} = (p_{\omega j})^{-\sigma} \exp(x_{\omega j}) \frac{Y_j}{P_j}$$
(2)

$$P_j = \sum_{\omega=1}^{W_j} \exp\left(x_{\omega j}\right) \left(p_{\omega j}\right)^{1-\sigma},\tag{3}$$

where $p_{\omega j}$ is the price of ω and Y_j is j's total expenditure on Danish varieties. P_j is the corresponding Chamberlainian price index, which is unaffected by the actions of any single firm.

Firms share similar increasing returns to scale production technologies. Firm ω 's cost $c_{\omega j}$ of supplying $q_{\omega j}$ units of output to destination j is

$$c_{\omega j}(q_{\omega j}) = f + \exp\left(\frac{b_{\omega}}{1-\sigma}\right)\tau_j q_{\omega j},\tag{4}$$

where f and τ are fixed and variable costs identical to all firms supplying to j. The firm specific exp $\left(\frac{b_{\omega}}{1-\sigma}\right)$ is the firm's marginal cost of product that is constant across all destinations. The b_{ω} term is a normalized measure of ω 's productivity: a higher b translates to a lower marginal cost for the firm across all destinations.

Each firm $\omega \in \{1, ..., m_j\}$ draws its firm-specific productivity b_{ω} . In addition, each firm

⁷Nguyen (2009) defines this parameter as "perceived quality". We can also think of it as $\omega's$ popularity or appeal in j.

draws a firm-destination specific taste parameter $x_{\omega j}$. The two random variables b_{ω} and $x_{\omega j}$ determine firm ω 's potential sales $r_{\omega j}^*$ in destination j, which is presented in log form:

$$\ln r_{\omega j}^* = a_j + b_\omega + x_{\omega j} \tag{5a}$$

$$a_j = \ln\left(\frac{Y_j\tau_j^{1-\sigma}}{P_j}\right).$$
(5b)

The firm productivity draws, b_{ω} , are drawn from exogenous independent normal distributions with product specific mean \bar{b} and variance s_b^2 . Likewise, the firm-destination specific taste draws, $x_{\omega j}$, are drawn from exogenous independent normal distributions with product-specific mean \bar{x}_j variance s_x^2 . These normality assumptions are supported by the distribution of domestic revenues of Danish firms presented in Figure 3 and is consistent with previous studies of firm size distribution (Cabral and Mata, 2003) and export selection (Helpman, Melitz, Rubinstein, 2008). The productivity draw and taste draws are constructed to be uncorrelated with one another. If they were correlated, our empirical procedure would attribute all of the correlation to the firm-specific productivity draw. This would bias our estimation of firm-specific effects upwards.

Insert Figure 3

The variation in the potential sales of a firm to a destination is now decomposed into three latent effects: a destination-specific effect a_j , a firm-specific effect b_{ω} , and a firm-destination-specific effect $x_{\omega j}$. This study focuses on the contributions of the firmspecific effect and the firm-destination specific effect. We estimate the contribution of the the firm-specific effect to the variance of potential sales for firms within a destination, controlling for destination-specific effects. That is, this study estimates the statistic

$$Q^2 = \frac{s_b^2}{s_b^2 + s_x^2},\tag{6}$$

for each Danish product exported in 2003.

4.2 Truncation issues

If sales were observed for every firm-destination pair, a simple ANOVA of $\ln r_{\omega j}^*$ on destination and firm-specific effects would consistently decompose the variance, with the residual being attributed to $x_{\omega j}$. However, our dataset is an unbalanced panel where not every firm sells to every destination. The firm-destination sales $r_{\omega j}^*$ is truncated, with the truncation endogenously correlated with a_j, b_{ω} , and $x_{\omega j}$. In this section, we show how we correct for these sources of bias.

Melitz (2003) suggests that the presence of a firm in a destination is tied to its potential profit in that market. In the current model, firm ω 's profits π gained from supplying to j are

$$\pi_{\omega j} = \frac{r_{\omega j}^*}{\sigma} - f. \tag{7}$$

Profits are positive when $r_{\omega j}^* > c$, where $c = \sigma f$ and is unknown to the econometrician. Therefore, we cannot observe all $r_{\omega j}^*$. We only observe $r_{\omega j}$, where

$$r_{\omega j} = \begin{cases} r_{\omega j}^* & \text{for } r_{\omega j}^* \ge c \\ 0 & \text{for } r_{\omega j}^* < c. \end{cases}$$
(8)

Equation (8) given (5) is the standard Type 1 Tobit Model with latent effects described in Honoré and Kyriazidou (2000). In an earlier work, Honoré (1992) shows that if the latent effects⁸ are correlated with the probability of truncation, then the Heckman (1979) two-step procedure is biased. Honoré's solution to this problem treats the specific effects as nuisance variables and differences them out. This method renders the specific effects immeasurable. In our study, a_j is a nuisance variable, but b_{ω} is a parameter of interest, so we cannot use Honoré's approach. Instead, we treat a_j as a fixed effect, b_{ω} as a random effect, and $x_{\omega j}$ as a residual. We then estimate s_b^2 and s_x^2 using a Monte Carlo Expectation-Maximization Maximum Likelihood Estimation (MCEM) proposed by Walker (1996) and

⁸In our case the specific effects correspond to a_i, b_{ω} .

used widely in biometrics research. Kuhn and Lavielle (2005) show under very general conditions that the MCEM procedure obtains consistent estimates for nonlinear mixed-effects models. Using simulated datasets that resemble our actual dataset, we verify that our MCEM procedure estimates s_b^2 and s_x^2 consistently.

5 Estimation strategy

This paper estimates via MCEM the portion of sales variance contributed by firm-specific effects. Using our model given by equations (8) and (5), we can derive the distribution of $r_{\omega j}$ given a_j , b_{ω} , c, and s_x^2 :

$$\Pr\left(\ln r_{\omega j} = r | a_j, b_\omega, c, s_x^2, I = 1\right) = \Pr\left(a_j + b_\omega + x_{\omega j} = r\right)$$
$$= \frac{1}{s_x} \varphi\left(\frac{r - a_j - b_\omega}{s_x}\right)$$
(9)

$$\Pr\left(r_{\omega j} = 0|a_j, b_{\omega}, c, s_x^2\right) = \Pr\left(a_j + b_{\omega} + x_{\omega j} < c\right)$$
$$= \Phi\left(\frac{c - a_j - b_{\omega}}{s_x}\right).$$
(10)

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are the standard normal pdf and cdf, and where I denotes the indicator function that takes the value 0 if observed sales $r_{\omega j} = 0$ and 1 if $r_{\omega j} > 0$. Combining (9) and (10) with the indicator function I, we derive the conditional probability of $\ln r_{\omega j} = r$:

$$\Pr\left(\ln r_{\omega j} = r|a_j, b_{\omega}, c, s_x^2\right) = \frac{1}{s_x} \varphi\left(\frac{r - a_j - b_{\omega}}{s_x}\right) I + \Phi\left(\frac{c - a_j - b_{\omega}}{s_x}\right) (1 - I).$$
(11)

Following Wooldridge (2002), we find L_{ω} , the joint density of $\vec{r}_{\omega} = (r_{\omega 1}, r_{\omega 2}, ..., r_{\omega J})$ given b_{ω} , c, s_x^2 and the vector $\vec{a}_j = (a_1, a_2, ..., a_J)$:

$$L_{\omega}\left(\vec{r}_{\omega}|\vec{a}_{j},c,b_{\omega},s_{x}^{2}\right) = \prod_{j=1}^{J} \left(\frac{1}{s_{x}}\varphi\left(\frac{r_{\omega j}-a_{j}-b_{\omega}}{s_{x}}\right)I + \Phi\left(\frac{c-a_{j}-b_{\omega}}{s_{x}}\right)(1-I)\right).$$
(12)

We treat b_{ω} as a random effect drawn from a normal distribution with mean zero and

variance s_b^2 . Given s_b^2 , we can integrate out L_{ω} 's dependence on b_{ω} . Finally, we sum this integral over all m firms to arrive at our log-likelihood l of the unknown parameters given observations $\vec{r} = \{r_{\omega j} | \omega = 1, ..., W; j = 1, ..., J\}$:

$$l\left(\vec{a}_{j},c,s_{x}^{2},s_{b}^{2}|\vec{r}\right) = \sum_{\omega=1}^{W} \ln\left(\int_{-\infty}^{\infty} L_{\omega}\left(\vec{r}_{\omega}|\vec{a}_{j},b_{\omega},c,s_{x}^{2}\right)\frac{1}{s_{b}}\varphi\left(\frac{b}{s_{b}}\right)db\right).$$
 (13)

We treat the destination specific effect a_j as a fixed effect. We remain ambivalent as to the underlying distribution of a_j , since country-specific effects are not of interest. We obtain our estimates via MCEM. In each iteration, the b_{ω} 's are integrated out using a forty-point Gaussian Quadrature (the E-step). The parameters are then obtained by maximizing l (the M-step) using the MAXLIK procedure in Gauss. The steps are repeated until the squared sum of the gradient of estimated coefficients was less than 1e - 5.

The sample space of \vec{r}_{ω} is a function of the unknown parameter c. Zuehkle (2003) suggests estimating c with the minimum order statistic of the untruncated $r_{\omega j}$:

$$\hat{c} = \min\{r_{\omega j} | r_{\omega j} > 0; \omega = 1, ..., W; j = 1, ..., J\}.$$
(14)

Carson and Sun (2007) proves that \hat{c} converges to c at the rate of 1/W. They also show that MCEM estimates of the remaining coefficients are asymptotically normal with asymptotic variances identical to the case when c is known. We follow their lead and use $c = \hat{c}$. We then estimate the other parameters in (13) via MCEM, as previously described.

5.1 Monte Carlo simulation

We verify our procedure's ability to accurately estimate Q^2 under various conditions. We simulate 90 datasets of (W, J) = (100, 100) possible firms and destinations⁹ and compare the estimated \hat{Q}^2 with the known true $Q^2 = \tilde{Q}^2$. The simulation procedure is outlined in the appendix.

The results of our simulations are summarized in Figure 4 below. Our estimated \hat{Q}^2

⁹We also try (W, J) = (100, 50) wth similar results.

tracks the true value \tilde{Q}^2 well, with none of the median estimates more than one standard deviation from the true value. We use median values because there were a handful of outliers that resulted from the MCEM not converging.

Insert Figure 4 here

6 Estimation results

We use the MCEM procedure to obtain an estimate, Q^2_{MCEM} , for contribution of firmspecific effects for each Danish export in 2003. We do this at the HS6 product level. We also perform OLS dummy regressions of destination-mean-differenced observed revenues $\left(\ln r_{\omega j} - \sum_{\omega=1}^{W_j} \ln r_{\omega j}\right)$ on firm fixed-effects. From the OLS regressions, we retrieve the adjusted coefficient of determination \bar{R}^2 and estimate Q^2_{OLS} by¹⁰:

$$Q_{OLS}^2 = \max\left\{0, \bar{R}^2\right\}.$$
 (15)

 Q^2 is defined as a positive number, so we treat negative \bar{R}^2 values as estimates of 0 for Q^2 . In the following, we compare the MCEM estimates Q^2_{MCEM} to the OLS estimates Q^2_{OLS} . Our main results are derived at the detailed product level discussed in the next section, which is followed by a number of robustness checks.

6.1 Product level

As noted in section 3, there are many HS6 products that contain few firms selling to few destinations. We drop products containing fewer than 25 firm-destination observations, products that are exported to less than three destinations, and product-destinations categories containing fewer than five firms. A total 66,488 firm-destination-product observations remained, spanning 3,790 firms in 480 products to 84 countries and totalling

¹⁰We use the adjusted coefficient of determination to avoid small sample bias. Cramer (1987) shows that the unadjusted R^2 is heavily biased upwards for small samples.

DKK 91 billion. We estimate s_b^2, s_x^2 , and consequently Q^2 for each of these 480 products.

Our estimation procedure resulted in mean and median values of 33% and 31% for Q^2_{MCEM} across the 480 HS6 products. This is considerably lower than OLS estimates, which resulted in mean and median values of 43% and 46% for Q^2_{OLS} . For comparison, Lawless and Whelan (2008) obtain an \bar{R}^2 of 41% across their sample of Irish exporters. It should be noted that the product level dimension of the data is important, as the estimated Q^2_{MCEM} 's exhibit substantial variation across products. Histograms for the MCEM and OLS estimates are presented in Figure 5 below:

Insert Figure 5 here

The histogram of the MCEM estimates in Figure 5 is systematically to the left of that of the OLS estimates. To understand why, we compare the difference between Q_{OLS}^2 and Q_{MCEM}^2 for each product. Figure 6 shows that Q_{OLS}^2 generally overshoots Q_{MCEM}^2 . The overshooting is exacerbated at low values of Q_{MCEM}^2 . For products with Q_{MCEM}^2 between 10% and 20%, Q_{OLS}^2 averages 31%. For products with Q_{MCEM}^2 between 20% and 30%, Q_{OLS}^2 averages around 41%. This upwards bias shifts the histogram for Q_{OLS}^2 to the right. Q_{OLS}^2 actually undershoots Q_{MCEM}^2 at values of Q_{MCEM}^2 greater than 60%. Our simulations showed that Q_{MCEM}^2 slightly undershoots the true Q^2 at high values, so Q_{OLS}^2 's downward bias is even worse. That is, Q_{MCEM}^2 is a more accurate estimator for Q^2 than Q_{OLS}^2 across the entire range.

Insert Figure 6 here

To sum up we have found that OLS estimates of the contribution of firm-specific effects are generally biased. Our results show that the direction of bias is dependent on the degree to which firm-specific effects affect sales variation. In HS6 products where firm-specific effects do not contribute much to the overal sales variation, an OLS dummy regression overestimates the contribution of the firm-specific effect. In products where firm-specific effects play a large role, the OLS regression underestimates the true contribution. Since our Q^2_{MCEM} 's are product-specific, we investigate whether there are any patterns in the Q^2_{MCEM} 's across products. Our theoretical model is stylized and does not give us any predictions about how Q^2_{MCEM} varies with product characteristics, so a priori we do not have any expectations about any relationships. However, we regressed our estimates are several product-level characteristics to investigate any possible relationship.

We find no relationship between the contribution of firm-specific effects and a number of product-specific characteristics. We regressed Q^2_{MCEM} on the mean and variance of the capital labor ratio within the HS6 product code, the mean and variance of the value added per worker for firms within the HS6 product code, and the mean and variance of the total HS6 output. We found no significant correlation.

We also did not find any correlation between Q^2_{MCEM} and previously estimated measures of product differentiation. We regressed the Q^2_{MCEM} 's on import demand elasticities for the U.S. estimated by Broda and Weinstein (2006) and import demand elasticities for Denmark estimated by Broda, Greenfield and Weinstein (2006). Again we did not find any correlation. Finally, we partitioned our results by the Rauch (1999) classification of product differentiation. Approximately 90% of our products are classified as 'differentiated' while most of the remaining 10% are classified as 'reference priced.'¹¹. The 'differentiated' products had a median Q^2_{MCEM} of 33% while the 'reference priced' products had a median Q^2_{MCEM} of 20%. However, there were less than 50 estimated 'reference priced' products, so we refrain from speculating about any true differences.

6.2 Measurement error

Firm-destination specific effects contributes over two-thirds of the sales variation in a product-destination market for over half of Danish HS6 exports. Our theory suggests that this variation is due to firm-destination-specific demand variation. However, if the export sales data are riddled by measurement error, then that error could be a possible

¹¹Rauch (1999) also classifies products according to whether they are traded on organized exchanges, but we had only a handful of products of this type in our sample. This is because our dataset contains only manufacturing products.

source of variation that reduces the relative contribution from firm specific effects.

As a first robustness check we have calculated Q^2_{MCEM} for a similar sample but where small trade flows are excluded by deleting observations with a value less than DKK 1,000. This is to ensure economically unimportant and perhaps noisy observations do not affect our results. Those results are similar to the results presented above, with mean/median of 34%/33% for the MCEM procedure and 41%/38% for the OLS procedure.

Second, suppose that measurement error is the sole cause of the firm-destinationspecific variation. Our sample has a log sales mean of 10.6 and sample log sales variance of 8.4. If measurement error is the cause of two-thirds of that variance, that would imply that an average Danish export recorded at a value of DKK 40,000 has a 68% (1 standard deviation) confidence interval of DKK 4,000 to 420,000. Our data is customs trade data from which tariff revenues are calculated, and it does not seem plausible to have measurement errors that large.

6.3 Aggregation

We also estimate Q_{OLS}^2 and Q_{MCEM}^2 at the CN8 product level, the most disaggregated level available to us. The results are similar to estimates performed at the HS6 level. We obtain a mean and median of 31% for Q_{MCEM}^2 , and 46% and 44% for Q_{OLS}^2 . The histograms at the CN8 level are presented in Figure 7 below:

Insert Figure 7

The histograms in Figure 7 resembles those in Figure 5; OLS estimates are systematically higher than MCEM estimates.

As Table 1 shows, our data restrictions reduce the sample size to about a third of the trade volume at the eight digit level. This reduction did not result in a gain in the number of products: only 491 CN8 products passed the estimation restrictions, compared to 480 HS6 products. By disaggregating to CN8, we threw away observations without gaining much in return. We use the HS6 product classification for our robustness checks below,

as that sample comprises a higher total export volume.

As briefly mentioned in the introduction, our dataset contains product code information that are not contained in Eaton, Kortum and Kramarz (2008) or Lawless and Whelan (2008). To better compare our results to theirs, we aggregate our data to broader industries. We estimate Q^2_{MCEM} and Q^2_{OLS} at the HS2 industry level.¹² As before, we restrict our analysis to industries containing at least 25 firm-destination observations, industries that are exported to at least three destinations, and industry-destinations categories containing at least five firms. With these restrictions, we have 77,411 observations spanning 4,276 firms in 54 industries exporting to 161 countries, totalling DKK 178 billion.

For the 54 HS2 industries, we obtain median estimates of 32% for Q_{MCEM}^2 and 38% for Q_{OLS}^2 . This result is in line with our previous estimates at the HS6 product level. For over half of Danish exporting industries, firm-specific effects explain less than a third of total sales variation.

We obtain a mean of 43% for Q^2_{MCEM} , which is higher than the 35% mean obtained for Q^2_{OLS} . This is due to 16 industries having estimates of Q^2_{MCEM} greater than 80%. Figure 8 show this case.

Insert Figure 8 here

There was no obvious pattern why these industries exhibited higher contributions of firmspecific effects. These results suggests that, if anything, the estimated contribution of firm specific effects rises with the level of aggregation.

6.4 Consistency over time

To see if our results are consistent over time, we repeat the exercise for the year 2001, with similar estimates for Q^2 . For the 401 HS6 products that fit our restrictions in 2001, we obtain median estimates of $Q^2_{MCEM} = 34\%$ versus $Q^2_{OLS} = 43\%$.

The Q^2 estimates are not only correlated in the aggregate, but at the individual

¹²To compare more directly with existing studies we should estimate one Q^2_{MCEM} and one Q^2_{OLS} for firm level sales without any distinction between different products. However, that proved infeasible as Gauss was unable to handle the size of the dataset.

product level. There were 350 HS6 products that passed our estimation restrictions in both 2001 and 2003. We regressed Q^2_{MCEM} for 2003 on that for 2001 for these 350 products. Our estimated marginal effect was 0.76 with a standard error of 0.03. That is, a 10% increase in $Q^2_{MCEM,2001}$ corresponded to a 7.6% increase in $Q^2_{MCEM,2003}$.

The correlation is almost one-to-one when we restrict our regression constant to zero. That regression results in an estimated marginal effect of 0.92 with a standard error of 0.02. Figure 9 presents the point estimates for the two years:

Insert Figure 9 here

The strong correlation between $Q^2_{MCEM,2003}$ and $Q^2_{MCEM,2001}$ contrasts with the lack of correlation between OLS estimates $Q^2_{OLS,2003}$ and $Q^2_{OLS,2001}$. A regression of the two OLS estimates resulted in no significant correlation between the two. Figure 10 shows this lack of consistency across years:

Insert Figure 10 here

This exercise gives further evidence that our procedure accurately identifies the contribution of firm-specific effects, while OLS estimates do not.

6.5 Established exports

Nguyen (2009) suggests that much of the export sales variation is due to firms testing destinations in order to determine whether they can be successful exporting to that destination. Therefore, firm-destination-specific effects should play a larger role in the first year of exporting. To test that, we restrict our sample to only those firm-product-destination observations in 2003 that were also positive in 2002. That is, only 2003 exports by those firms that exported the same product to the same destination in both 2002 and 2003 were considered. This restriction leaves us with 31,242 observations spanning 303 HS6 products, 2,491 firms, and 64 countries and totalling DKK 69 billion.

The predictions from Nguyen (2009) are supported by the data. For the 303 established

exports in 2003, we obtain mean and median values of 39% and 40% for Q^2_{MCEM} and 49% and 50% for Q^2_{OLS} . These values are 20 – 25% higher than those estimates estimated for the sample which included first time exports. Therefore, firm-specific effects are more important for these established exports. Contrastly, firm-destination-specific effects are more important for the first year of exporting than for established exports. The histogram of results for the established exports is displayed in Figure 11.

Insert Figure 11 here

6.6 Core products

Firms typically export multiple products, and for such firms the within-firm output distribution across products is known to be highly skewed with typically one core product accounting for a major part of firm sales, see e.g. Bernard, Redding, and Schott (2009). Until now we have treated each firm-product combination as independent units of observations, but within-firm correlation across products in export markets may arise if non-core products are more likely to be sold in destinations where fixed costs related to sales of the core product already have been incurred.

Therefore, as a robustness check, we repeat our exercise for only the core product of each firm. We define firm ω 's core product as the HS6 category constituting the highest export sales for firm ω . We drop all other products exported by ω . With this and the forementioned restrictions, we are left with 6,686 observations spanning 73 HS6 products, 1,342 firms, and 61 countries, totalling DKK 3 billion.

The MCEM estimates for core products are similar to those for all products. We obtain a median $Q^2_{MCEM,CORE} = 40\%$ for the 73 HS6 categories comprising only core products. For these same 73 HS6 categories, we estimate a $Q^2_{MCEM,ALL} = 37\%$ when we include all products.

The OLS estimates, however, dropped significantly when we look only at core products. We obtain a median $Q_{OLS,CORE}^2 = 25\%$ for the core products compared to $Q_{MCEM,ALL}^2 =$ 43% for the sample with all products. Figure 12 compares the point estimates of Q^2 using both data restrictions and estimation techniques:

Insert Figure 12 here

 Q^2 estimates using just the core products can predict that using all products. A simple regression of $Q^2_{MCEM,ALL}$ on $Q^2_{MCEM,CORE}$ results in a positive and significant coefficient of 0.57 (standard error of 0.11). This estimate increases to 0.97 when we restrict the constant to zero. For Q^2_{OLS} , the same exercise results in a coefficient of 0.31 with a standard error of 0.08. We estimate a coefficient of 1.12 when we restrict the constant to zero.

This exercise shows that product scope does not adversely affect the measurement of the contribution of firm-specific effects to sales within a product category. Our estimates for Q^2_{MCEM} using just core products are on par with our results using all products. Coreproduct estimates can track all-product estimates well, and the marginal relationship is not significantly different from unity when we restrict the constant to zero.

7 Conclusion

We use a highly detailed dataset for Danish exporters to estimate the contribution of firm productivity to the variation of sales within a destination and find it to be remarkably low. When using firm-specific effects as the broadest interpretation of productivity, we find that the contribution of firm-specific effects varies greatly across products, and that it explain less than 31 percent of the variation for over half of Danish HS6 products. Our results suggest that firm-specific productivity is not capturing the majority of heterogeneity and is not the primary driver of variation in a market.

The Melitz (2003) model deftly explains variation between exporters and nonexporters. However, Melitz (2003) is limited to firm-specific differences, and our results suggest that the majority of variation is firm-destination specific. Nguyen (2009) shows how this variation can be generated with a single mechanism involving demand heterogeneity. In it, he presents a model in which firms test destinations and receive firm-destination-specific perceived quality draws. Higher perceived qualities result in higher sales. Since demands are firm-destination-specific, a firm can have high relative sales in one destination but low relative sales in another. Productivity heterogeneity models cannot generate this sales ranking inversion. Nguyen (2009) reconciles higher average domestic sales for exporters than for nonexporters by correlating a firm's perceived qualities with a firm-specific but unknown-to-the-firm latent quality.

Our results from restricting the dataset to established exporters also support Nguyen (2009). We find that firm-destination-specific effects are most important the first year of exporting.

We show that OLS estimates tend to overestimate low contributions and underestimate high contributions. Since the contribution of firm-specific effects are low in most products, OLS regressions tend to overestimate in general. To consistently estimate firm-specific effects, we employ a Monte Carlo Estimation-Maximization strategy used mainly in the Biometrics literature. We argue that this method can be employed fruitfully in studies of firm-level exporting with truncation issues.

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A Monte Carlo simulation

Our Monte Carlo simulation procedure consists of the following six steps:

- 1. Pick a $\tilde{Q}^2 \in \{0.1, 0.2, .., 0.8, 0.9\}$. Choose s_b^2 and s_x^2 such that $\frac{s_b^2}{s_b^2 + s_x^2} = \tilde{Q}^2$ and $s_b^2 + s_x^2 = 4$. Set *c* equal to 7.5.¹³
- 2. Draw a_j from a lognormal distribution for each of the *J* destinations. Draw b_{ω} from a $n(0, s_b^2)$ for each of the *W* firms. Draw $x_{\omega j}$ from a $n(0, s_x^2)$ for each of the $J \times W$ observations.
- 3. Generate $r_{\omega j}^*$ and $r_{\omega j}$ according to equations (5*a*) and (8).
- 4. Obtain parameter estimates $\hat{a}_j, \hat{c}, \hat{s}_x^2$ and \hat{s}_b^2 via the MCEM procedure described above. Calculate $\hat{Q}^2 = \frac{\hat{s}_b^2}{\hat{s}_b^2 + \hat{s}_x^2}$.
- 5. Repeat steps 2, 3, and 4 ten times.
- 6. Repeat steps 1-5 for all $\tilde{Q}^2 \in \{0.1, 0.2, .., 0.8, 0.9\}$.

 $^{^{13}}$ These values were chosen to so that between 30 and 70 percent of the observations would be truncated.

	CN8 product level		HS6 product level	
	Full	Restricted	Full	Restricted
	sample	sample	sample	sample
Number of				
Products	5339	491	3331	480
Firms	4304	3607	4304	3790
Destinations	223	79	223	84
Observations	155426	56297	145304	66488
Trade volume (billion DKK)	182	68	182	91
Destinations per firm-product				
Mean	3.4	2.8	3.5	3.0
Median	1.0	1.0	1.0	1.0
Min	1.0	1.0	1.0	1.0
Max	138.0	58.0	138.0	58.0
Firms per product-destination				
Mean	2.2	10.3	2.6	10.9
Median	1.0	7.0	1.0	8.0
Min	1.0	5.0	1.0	5.0
Max	373.0	373.0	412.0	412.0

Table 1: Descriptive statistics, 2003

Table	2:	Notation
Table	∠.	INDUATION

Notation	Description
j	A destination country. $j \in J$
n	An HS6 product
ω	The unique variety of a firm
σ	elasticity of substitution between varieties
$q_{\omega j}$	The quantity of variety ω supplied to j
$p_{\omega j}$	The price of variety ω supplied to j
$ au_{j}$	The iceberg trade cost
P_j	The price index in j
$r_{\omega j}$	The observed revenue of variety ω in j
$r^*_{\omega j}$	The theoretical latent revenue
Y_j	The total expenditure of j
b_{ω}	The firm-specific productivity of ω
~	The firm-destination-specific demand shock
$x_{j\omega}$	for variety ω in j
s_{b}^{2}, s_{x}^{2}	The variances of b_{ω} and $x_{j\omega}$, respectively
O^2	The theoretical proportion of total variance
Q	explained by firm specific effects



Figure 1: Sales relative to other Danish firms in Sweden and Germany for Danish Exporters of plastic boxes (left panel) and building bricks (right panel). Statistics for the lines with fitted values: Left panel: slope = 0.84, std.err. = 0.08, $R^2 = 0.54$. Right panel: slope = -0.22, std.err. = 0.12, $R^2 = 0.08$.



Figure 2: Simulated relative sales in two destinations for varying values of \mathbb{R}^2 .



Figure 3: The distribution of log domestic sales for Danish manufacturing firms, 2003.



Figure 4: Monte Carlo Simulation results for nine values of Q^2 with ten repetitions each. The MCEM-MLE estimates are compared to known true values. The circles indicate the median values of the estimates. Estimates lying on the 45° are exactly equal to the true value.



Figure 5: Estimated values for Q^2 , the contribution of firm-specific effects, for 2003 Danish exports at the HS6 level.



Figure 6: Comparison between MCEM and OLS estimates for the contribution of firm-specific effects.



Figure 7: Estimated values for Q^2 , the contribution of firm-specific effects, for 2003 Danish exports at the CN8 level.



Figure 8: Estimated values for Q^2 , the contribution of firm-specific effects, for 2003 Danish exports at the HS2 level.



Figure 9: Point estimates for Q^2_{MCEM} for the years 2001 and 2003 for HS6 Danish Exports.



Figure 10: Point estimates for Q_{OLS}^2 for the years 2001 and 2003 for HS6 Danish Exports.



Figure 11: Estimated values for Q^2 , the contribution of firm-specific effects, for 2003 Danish exports at the HS6 level. The sample includes only firms that also exported to the same destination in 2002.



Figure 12: Estimates of Q^2 using All and only Core products, using the MCEM and OLS techniques, for 2003 Danish HS6 Exports.