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in the Medium Term and in the Long Run

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Abstract. This paper develops a macro-model of share prices that predicts that the growth rates in real share prices and real dividends gravitate toward predictable constants in the long run, but fluctuate on approximately decennial frequencies due to movements in capital’s share in total income and the output-capital ratio. The model has important implications for medium-term and long-run movements in real share prices and dividends, required share returns, the effects of new technologies on share returns, share price valuation, and whether shares are less risky in the long run than in the short run. Using macroeconomic data over 130 years for 22 OECD countries, the data give support for the model.

JEL Classification: E44, G12.

Key words: Share prices and dividends in the long run, share valuation, required share returns, and macroeconomic factors.

1 Introduction
This paper develops a macroeconomic framework that is used to explain the medium-term and long-run movements in real share prices and dividends. It is shown that movements in labour’s share in total income and the capital output ratio dominate medium-term movements in real share prices and dividends. In the absence of retained earnings, real share prices and real dividends converge to constant means in the long run because of a Tobin’s $q$ effect. If a fraction of earnings is retained within the company then real share prices and dividends will, in the long run, grow at predictable constants independent of key macroeconomic aggregates such as growth in GDP and labour productivity.

The model seeks to fill the theoretical gap that prevails in the empirical literature on mean reversion in share prices, and to some extent, also share returns and earnings per unit of capital. Poterba and Summers (1988) and Fama and French (1988) have shown that real share prices revert toward constant means and Fama and French (2000) and, to some extent, also Chan et al. (2000) and Arnott and Asness (2001), demonstrate that earnings per unit of capital are mean reverting. Furthermore, Shiller (2001, p 253) and Siegel (2002), and implicitly also Smithers and Wright (2000), argue that earnings per unit of capital are mean reverting and that share prices converge to a

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mean that is trending upwards over time. In this literature, however, the forces that are responsible for the mean reversion, remain, to a large extent, unexplained. Furthermore, it appears that share prices overreact to movements in earnings since earnings per unit of capital revert toward a constant in the long run. This paper shows that the deviation of share prices from their long run equilibrium may be rational pricing behaviour.

The intuition behind the model in this paper is that earnings per unit of capital, and hence real share prices and dividends, for the full dividend payout firm converge toward a constant in the long run due to endogenous forces in the markets for labour and fixed investment. Suppose that share prices increase from a steady-state equilibrium because of a technology innovation that increases the returns to capital. The increase in Tobin’s $q$ leads to a higher capital stock, which in turn lowers the returns to capital due to diminishing returns to capital. The capital-induced reduction in earnings exactly counterbalances the technology-induced increase in earnings in the steady state equilibrium. Similarly, a reduction in the required share returns results in only temporarily higher share prices because the capital stock endogenously adjusts to the new level of required returns to such an extent that earnings per unit of capital matches the required returns. Finally, a supply shock such as a stronger union movement brings earnings per unit of capital down to a lower level. The effects on earnings and stock prices, however, are temporary because the wage pressure results in a reduction in the capital stock due to Tobin’s $q$. The capital stock is reduced until the pre-shock earning per unit of capital is established, which brings real share prices and dividends back to their initial equilibrium.

These considerations suggest that standard valuation models do not take account of the endogenous response in the market for fixed capital to changes in the variables that the valuation models consist of. Furthermore, traditional share valuation models have little to say about the expected returns to shares, whether shares are less risky in the long run than in the short run and the effects of shocks on share prices. The model in this paper seeks to fill this vacuum and shows that dividend discount models tend towards a constant in the long run. It is, furthermore, shown that the required or expected returns to shares can be read from the output-capital ratio or earnings per unit of capital. Hence, the serious empirical problems that are associated with the measurement of expected share returns can be easily overcome using data that are readily available or can easily be constructed.

Using data for 22 OECD countries over 130 years, the movements and the interaction between the output-capital ratio, earnings per unit of capital, real dividends and real share prices are examined using graphical illustrations and econometric analysis. It is shown that labour’s share in total income is a key determinant of the movements in real share prices and real dividends on decennial frequencies. Furthermore, \textit{ex post} real share returns are found to gravitate to a constant of

\footnote{The model of Campbell and Cochrane (1999) implies mean reversion of share returns on business cycle frequencies, but}
about 7% in the long run and that the growth in real share prices and real dividends converge to a constant of about 3% for given constant effective corporate tax rates and retention ratios.

2 Long run evidence on movements in corporate earnings

The basic idea in this paper is that earnings per unit of capital fluctuate on medium term frequencies due to fluctuations in income shares and capital stock, but converge to constant means in the long run under some regularity assumptions that are detailed below. Figure 1 displays before-tax corporate earnings per unit of capital in five of the G7 countries over the past 133 years (USA, UK, Japan, Italy, and Canada). All five countries are included in the data after 1929, but only the UK and Italy are included in the data before 1929. Since the statistics on capital stock have a broader coverage than earnings, the figures on returns to capital are on the lower side. The data sources and methods are listed in the data appendix.

The data in figure 1 indicate that earnings per unit of capital fluctuate about a constant mean of about 6.5% on medium term frequencies. It appears that earnings per unit of capital have been reduced slightly in the post-war period compared to the pre WWI period; an issue that is addressed later in the paper. Returns were reduced substantially during the interwar period because the strong labour movements increased labour’s share of total income and particularly because of the Great Depression. The surge in profits from 1940 to 1943 was a result of the increase in demand coupled by the sharp reduction in the investment activity in the 1930s, which reduced the capital stock and hence increased returns to capital at any level of demand. The reduction in earnings throughout the 1970s was caused by a strong labour movement that, in conjunction with adverse supply shocks, increased labours’ share in total income and hence squeezed profits. Labour’s income share has since been reduced substantially, which has paved the way for the profit recovery.

not on medium term and long term frequencies, which is the focus of this paper.
The path in earnings per unit of capital over the past 133 years has the following three implications for the medium term and the long term movements in real share prices and dividends. First, the tendency for earnings per unit of capital to converge to an almost constant mean in the long run implies that real share prices and dividends do not grow over time because of growth in GDP and labour productivity, as commonly assumed, but because earnings are retained within the company and, therefore, that the constant earnings per unit of capital gradually become concentrated on fewer shares over time. Second, medium-term movements in earnings per unit of capital are predominantly a result of changes in labour’s income share and the capital stock. Third, earnings are pro-cyclical due to asymmetrical adjustment costs in investment and, to some extent, also employment. These issues are addressed in the next section.

3 A macroeconomic model of share prices

The model in this section seeks to explain the observations from the previous section that earnings per unit of capital 1) fluctuate on medium term frequencies; but 2) converge to a constant or slow moving mean in the long run. Pro-cyclical earnings per unit of capital are also implied by the model; however, the demand side is not formally considered. The model consists of three markets, namely the share market, the market for fixed investment; and the labour market. To simplify the analysis, and without the loss of generality, the labour market is omitted from the analysis in the first part of this section. This can be done because the model is block recursive; there is no feedback effect from the markets for shares and fixed investment to the labour market. The model is augmented with the labour market in Sections 3.2 and 3.3.

Investment and share prices are determined jointly from the following optimization problem of the firm, which under the Cobb-Douglass technology assumption, is given by:

$$\max \Pi = \int_{t=0}^{\infty} e^{-\rho t} \left[ (1-\tau)\{P \cdot A L^\alpha K_i^{1-\alpha} - W I_t - P' C(I_t)\} - P' I_t (1-\Gamma) \right] dt,$$

st.

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where $\Pi$ is nominal profits, $P$ is the value-added price-deflator, $K$ is capital services, $I$ is net investment, $W$ is the wage rate, $L$ is labour services, $\rho$ is a constant required return to equity, $C(I)$ is the adjustment cost of investment, $C'(I) > 0$, $C''(I) > 0$, $\delta$ is the rate of capital depreciation, $\tau$ is the corporate tax rate, $P'$ is the investment deflator, $A$ and $\alpha$ are fixed parameters, and $\Gamma$ is the sum of investment tax credits as a percentage of acquisition costs and the present value of expected future depreciation allowances for tax purposes as a percentage of acquisition costs. The firm is an all equity firm and all earnings are paid out.

The current-value Hamiltonian of this optimisation problem is given by:
\[ H = (1 - \tau)[P_t AL_t^\alpha K_t^\gamma I(t) - W_t L_t - P_t I(t)] - P_t I(t)(1 - \Gamma_t) + Q[I_t - \Delta K_t - \delta K_t], \]  

where \( Q \) is the shadow price of capital or Tobin’s \( q \). The first order conditions for optimum under the assumption of imperfect competition are:

\[ (1 - \tau)(1 - \alpha)(Y_t / K_t)(1 + 1/\eta_t) = (\rho + \delta)Q_t - \dot{Q}_t, \]  

\[ \lim_{t \to \infty} e^{-\rho} Q_t K_t = 0 \]  

\[ 1 - \Gamma_t + C(I_t)(1 - \tau) = Q_t / P_t ^\gamma, \]

where \( \eta \) is the price elasticity of demand facing the firm and where a dot over a variable signifies first differences.

### 3.1 Equilibrium under perfect competition

Under the assumption of perfect competition in the goods and labour markets, returns to capital are independent of the labour market and share returns are determined by the interaction between the market for fixed investment and the share market. For the representative firm Equations (2)-(4) form the following simultaneous first-order differential system:

\[ \dot{Q}_t = Q_t \rho - (1 - \tau)(1 - \alpha)(Y_t / K_t), \]  

\[ \dot{K}_t = F[(Q_t / P_t ^\gamma + \Gamma_t - 1)/(1 - \tau)], \]

where the depreciation rate has been set to zero and \( \eta \to \infty \). Equation (5) is an ordinary equity market equilibrium condition saying that the required returns to equity are equal to expected capital gains plus dividends. Equation (6) is Tobin’s \( q \) model of investment where \( I = 0 \) for \( (Q_t / P_t ^\gamma + \Gamma_t - 1) = 0 \), \( I > 0 \) for \( (Q_t / P_t ^\gamma + \Gamma_t - 1) > 0 \), and \( I < 0 \) for \( (Q_t / P_t ^\gamma + \Gamma_t - 1) < 0 \). Tax credits and depreciations for tax purposes lower the effective acquisition cost of capital and therefore the benchmark level of \( Q \) at which investment is undertaken. Figure 2 displays the dynamics of the two equations, where the \( \dot{Q}_t = 0 \)-curve slopes downwards due to the assumption of diminishing returns to capital as implied by the homogeneity assumption in the Cobb-Douglas production function.

An important property of the model is that the share price converges to a constant mean in the long run as a result of an innovation in the marginal productivity of capital or a shift in the required share returns. Since \( Y/K \) is also earnings per unit of capital, the model implies that steady state earnings per unit of capital are constant provided that the required returns to equity remain constant.
This property is shown by considering two shocks to the system; an earnings shock and a shift in the required returns.

Suppose that the system is initially in its long run equilibrium and an unanticipated positive technology shock increases the marginal productivity of capital and therefore the \( \frac{Y}{K} \) ratio. Figure 2 shows the effects of an unexpected Hicks-neutral technological innovation that increases the marginal productivity of capital and therefore shifts the \( \dot{Q}=0 \)-curve to the right. The perfect foresight share market instantaneously jumps from point \( E_0 \) to the point \( A \) where it joins the stable manifold to capitalise on the higher earnings. The higher share prices at the point \( A \) trigger investment, which gradually lowers earnings per unit of capital due to diminishing returns to capital. Final equilibrium is reached at point \( E_1 \) where the capital stock has fully adjusted and share prices are back to their initial equilibrium, where the \( \frac{Y}{K} \) ratio equals its initial level.

**Figure 2. Dynamics of \( Q \) and \( K \) and a technology innovation**

The intuition behind this result is as follows. Hicks-neutral technological progress increases the marginal productivity of capital and therefore the value of equity per unit of capital and Tobin’s \( q \). Share prices jump because the share market capitalises on the temporarily higher earnings. The higher share prices, however, initiate a capital deepening process that gradually lowers the marginal productivity of capital and therefore earnings per unit of capital. The adjustment terminates when the initial \( \frac{Y}{K} \)-ratio and \( Q \) are reached. The speed of adjustment towards \( E_1 \) depends most importantly on the investment adjustment costs.

The model also implies that share prices are independent of changes in the required returns in the steady state. Shifts in the required returns shift the \( \dot{Q}=0 \)-curve along the horizontal \( \dot{K}=0 \)-curve and share prices remain unaffected in the steady state. Intuitively, the reduction in the required returns increases \( Q \) and, therefore, initiates a capital accumulation process, which, due to diminishing returns to capital, lowers returns to shares. The net investment terminates first when returns to shares equal the lower required returns. Consequently, share prices are only temporarily affected by the
shift in required returns. This result departs from the conventional analysis that is based on dividend
discount valuation models where share prices are inversely related to required returns. However,
standard valuation models do not account for the fact that capital investment endogenously responds
to changes in the required share returns to such an extent that the ratio between expected earnings
and required returns remain constant.

Taxes appear to permanently influence share prices in the model because they shift the
\( K = 0 \)-curve and therefore share prices. However, this effect does not transfer to shares based on
new capital stock. To appreciate this issue consider an introduction of a tax credit that lowers the
effective acquisition cost of capital and therefore increases the capital stock to a level where the
reduction in the discounted earnings per unit of capital equals the investment credit per unit of
capital. This implies that net earnings per unit of capital are unaltered for the companies that
undertake the investment, whereas the returns to the existing capital stock diminishes which
therefore adversely affects shares of established companies with low investment. In due course,
however, when all capital stock has been replaced, share prices revert to their initial level.

### 3.2 Imperfect competition

In the perfect competition case above, earnings are explained by the \( Y/K \) ratio. The \( Y/K \) ratio,
however, is far too stable over time to explain the large historical fluctuations in real share prices on
medium term frequencies.\(^3\) The medium term movements in earnings, however, are highly
correlated with factor shares, which exhibit large swings on medium term frequencies. The influence
of factor share movements on share prices and share returns is incorporated into the model by
relaxing the assumption of perfect competition and by introducing the labour market, which presents
the supply side of the economy.

Under imperfect competition the total pre-tax real profits in the economy are \( \Pi \equiv S^k Y \),
where \( S^k \) is capital’s income share. Thus the pre-tax earnings per unit of capital are equal to:

\[
\Pi / K \equiv (1 - S^L) Y / K ,
\]

where \( S^L = WL/YP \) is labour’s pre-tax share of income. Substituting this expression into Equation (2)
yields:

\[
\dot{Q}_t = \rho \dot{Q}_t - (1 - \tau) (Y_t / K_t) \left(1 - S^L_t \right) .
\] (8)

\(^3\) Under the CES technology assumption the \( Y/K \) ratio in the model above is replaced by \( (Y/K)^{\sigma} \), where \( \sigma \) is the elasticity
of substitution. In the short run \( \sigma \) is likely to be less than one and the CES technology will consequently give rise to
fluctuations in earnings that are somewhat larger than in the Cobb-Douglas case. However, the fluctuations in \( Y/K \), even
for very small \( \sigma \), are not sufficient to generate the observed earnings.
This equation says that expected share returns equal capital gains plus earnings per share. Earnings per share are a positive function of the marginal productivity of capital and monopoly profits, where monopoly profits are inversely related to the share of income going to labour. Factor shares are determined in the labour market.

3.3 Determination of factor shares
To show the factors that determine $S^L$, consider the following standard augmented Phillips curve, which summarises the supply side of the economy:

$$\Delta \ln W_t = \beta_1 \pi^w_t + \beta_2 \Delta \ln MP_L - \beta_3 (U_t - \phi) + \Delta \ln Z', \beta_4,$$  \hspace{1cm} (9)

where $\pi^w$ is the rate of inflation measured by the value-added price-deflator, $U$ is the rate of unemployment, $Z$ is a vector of wage push variables, $\phi$ is the equilibrium rate of unemployment, and $\beta_1-\beta_4$ are constants. The equilibrium rate of unemployment is probably time-varying, but is set to a constant here for expositional simplicity. The $Z$-variables consist of direct and indirect taxes, relative commodity prices, relative food prices, union wage pushiness, relative minimum wages, unemployment benefits, mismatch, and other wage push variables (see Madsen, 1998).

By imposing long-run price and productivity homogeneity, $\beta_1 = \beta_2 = 1$, Equation (9) reduces to:

$$\Delta \ln S^L_t = -\beta_3 (U_t - \phi) + \Delta \ln Z', \beta_4.$$  \hspace{1cm} (10)

This equation shows that labour’s share of income is intimately related to the deviation of unemployment from its equilibrium and supply shocks. Unemployment above its equilibrium reduces the growth in wages until the lower wage growth has brought unemployment back to its natural rate. Supply shocks change the wedge between labour costs and the value-added price-deflator and consequently influence firms’ earnings.

The rate of unemployment can be solved for from Equation (10) by letting it be a function of factor shares following mainstream theories of unemployment (see for instance Bruno and Sachs, 1985, and Madsen, 1998). Substituting into Equation (10) yields the following first-order difference equation:

4 The Phillips curve can easily be derived from an optimising framework (see for instance Mankiw and Reis, 2002). A complete model of the labour market is not presented here to make the presentation as straightforward as possible.
5 These assumptions are standard in the natural rate of unemployment framework. If they are not satisfied then a unique equilibrium unemployment rate does not exist.
6 Bruno and Sachs (1985) show that unemployment is a negative function of the wage gap, $W^*$, which in log-linear approximation is given by $\ln W^* = \ln (W/P) - \ln (W/P)^*$, where $(W/P)^*$ is the real wage, which is compatible with the natural rate of unemployment, $W/P = MP^*$. Hence, $\ln W^* = \ln S^L_t - \ln L$ under the Cobb-Douglas technology assumption, where $S^L_t$ is the share of labour in total income.
\[ \Delta \ln S_t^L = -\beta \left( a \ln S_t^L - a\alpha - \phi \right) + \Delta \ln Z_t^\prime \beta, \]  

(11)

where \( a \) is a constant. This first-order difference equation has a stable equilibrium since \( a \) and \( \beta \) are both positive constants.\(^7\)

The labour market feeds into share prices through the mark-up term in the share valuation equation. Suppose that the economy is hit by an unanticipated adverse supply shock that brings wages in excess of the full employment equilibrium. The resulting increase in \( S_t^L \) squeezes the monopoly profits and hence earnings per unit of capital. Share prices drop immediately to capitalise on the temporarily lower earnings. Since unemployment is in excess of its equilibrium, the wage growth rate, and hence \( \dot{s}_t^L \), will be negative until unemployment reverts to its equilibrium. Earnings per unit of capital and share prices are back to their initial level at the time at which the labour market returns to its equilibrium. This adjustment path, however, only applies to the case where the capital stock is exogenous and hence unaffected by share prices. When the capital stock is endogenous then the adjustment path is quite different as shown next.

### 3.4 General equilibrium

Equations (6), (8), and (11) define the following simplified simultaneous linearized first-order differential system:

\[ \dot{q}_t = q_t \rho - \alpha_t y(\omega)_t + \alpha_{s}k_t + \alpha_{s} \hat{s}_t^L, \]  

(12)

\[ \dot{k}_t = \beta (q_t + \Gamma_t - 1) \]  

(13)

\[ \dot{s}_t^L = -\theta \hat{s}_t^L + \varepsilon, \]  

(14)

where lowercase letters signify logs of capital letters, \( \alpha_1, \alpha_2, \) and \( \beta \) are positive constants, \( \omega \) is technology shocks, and \( \varepsilon \) is supply shocks.

Linearizing the equation system around its steady state yields:

\[
\begin{bmatrix}
\dot{q} \\
\dot{k} \\
\dot{s}_t^L
\end{bmatrix}
= \begin{bmatrix}
\bar{\rho} & \alpha_1(1-\partial y/\partial k) & \alpha_2 \\
\beta & 0 & 0 \\
0 & 0 & -\theta
\end{bmatrix}
\begin{bmatrix}
q - \bar{q} \\
k - \bar{k} \\
\hat{s}_t^L - \bar{s}_t^L
\end{bmatrix},
\]

(15)

\(^7\) Since \( \Delta \ln Z \) tends to zero in the long run, Equation (11) has the following solution:

\[ s^L(t) = (\alpha + \phi / a) + [s^L(0) - (1 + a\beta_t)(\alpha + \phi / a)(1 + a\beta_t)^{-1}]^t, \]

where \( s^L = \ln S_t^L \). This equation converges to a constant steady-state equilibrium as follows:

\[ \lim_{t \to \infty} s^L(t) = (1 + a\beta_t)(\alpha + \phi / a) = \bar{s}_t^L. \]
where \( \alpha_1, \alpha_2, \) and \( \beta \) are positive constants. This system has two stable roots \((-\theta, \mu)\) because of its block recursive nature and because the system given in Section 3.1, is stable.\(^8\)

As shown in the appendix the steady state multipliers for share prices are given by:

\[
\frac{\partial q}{\partial e} = \frac{\partial q}{\partial \omega} = \frac{\partial q}{\partial \rho} = 0 ,
\]

which shows that share prices converge to a constant in the long run following technology shocks, supply shocks, and shifts in the required returns. The result that \( \partial q / \partial \omega = 0 \) follows from the analysis in Section 3.1. An unexpected adverse supply shock leads to an instantaneous reduction in share prices, which in turn lowers the capital stock until the pre-shock earnings per unit of capital are established. A shift in the required returns leads to an endogenous response in the capital stock as analysed above.

4 Implications of the model
Beyond the dynamics of share returns following technology, supply and demand shocks, the model has implications for the movements of real share prices in the long run and on medium frequencies, the equity puzzle, the effects of the ‘New Economy’ on share prices, and the riskiness of share investment for short and long horizon investors. These issues are now considered.

4.1 Growth in share prices and dividends in the long run
The model predicts that real share prices in the long run are mean reverting provided that all earnings are paid out because earnings per unit of capital revert toward the required returns. Allowing for retained earnings it follows that earnings per share and, therefore, real share prices and dividends grow at the rate of \( r \kappa \), where \( \kappa \) is the retention ratio and \( r \) is the returns to new investment. Clearly, \( r \) may temporarily deviate from \( \rho \) following technology or supply shocks. However, \( r \) equals \( \rho \) in the steady state because of the endogenous response in the capital stock. Hence, the steady state growth in real share prices is \( \rho \kappa \), or about 3-4% annually depending on the exact magnitudes of \( \rho \) and \( \kappa \). From this it follows that earnings and dividend growth rates are unrelated to the growth in GDP and labour productivity because the retention ratio and earnings per unit of capital are both unrelated to growth in GDP and labour productivity.\(^9\)

\(^8\) Stability of the two-dimensional system follows automatically from Equation (15) by elimination of the labour market since \(-\alpha_1 \beta (1 - \partial y / \partial k) < 0\).
4.2 The required returns

A recent controversy in finance is whether the high *ex post* equity premium experienced in the 20th century will remain high into this century or whether it has been permanently reduced in the post-war period and, therefore, whether share holders can expect a substantial reduction in share returns relative to bond returns in the future. Measurement problems, however, have rendered it difficult to assess expected share returns and has led to results that are highly sensitive to the assumptions regarding the underlying process governing the expected growth in dividends (see, for different approaches and model assumptions, Arnott and Bernstein, 2002, Claus and Thomas, 2001, Fama and French, 2002, Harris and Marston, 2001). The model in this paper can be used to assess the required share returns. Due to the endogenous response of the capital stock to changes in the required returns the required returns can be directly read from the $Y/K$ ratio or earnings per unit of capital.

The key implication of the model in this paper is that the required returns vary proportionally to the $Y/K$ ratio and earnings per unit of capital due to the endogenous responses to changes in the required returns. A permanent shift in $\rho$ will permanently shift the $Q = 0$ curve and consequently alter the $Y/K$ ratio and earnings per unit of capital. All other earnings shocks will be temporary because capital will adjust endogenously until the initial $Y/K$ ratio and earnings per unit of capital are established. Changes in the tax structure can also change the $Y/K$ ratio but not after tax earnings per unit of capital. An investment credit, for instance, will increase the capital stock until the initial after-tax earnings per unit of capital are established.

More explicitly the steady state value of the required returns is given by the solution to Equations (6) and (8) in steady state and allowing the depreciation rate to be non-zero:

$$\rho = \left(\frac{1-\tau}{1-\Gamma}\right) \frac{Y}{K} \left(1-S^e\right) - \delta = \left(\frac{1-\tau}{1-\Gamma}\right) \frac{Y}{K} \left(1-(1+a\beta)(\alpha+\phi/\alpha)\right) - \delta.$$

This equation shows that the required returns are reflected in the steady state $Y/K$ ratio provided that the tax structure and the equilibrium rate of unemployment remain unaltered. To simplify the model consider the perfect competition counterpart of the required returns.

$$\rho = (1-\alpha) \left(\frac{1-\tau}{1-\Gamma}\right) \frac{Y}{K} - \delta. \quad (16)$$

For the US $(1-\tau)/(1-\Gamma)$ is close to one and shows little variation over time, as can be seen from the data that are provided by Summers (1981). The use of the $Y/K$ ratio as an approximation for

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9 Growth in real share prices and dividends are often related to growth in GDP and labour productivity. The problem associated with this approach is that economic growth is usually associated with capital deepening, which implies that earnings are dispersed over more units of capital. See Madsen (2003) for an analytical exposition.
\( \bar{Y} / \bar{K} \) hinges on an assumption of low adjustment costs of investment. Cummins et al. (1996), for example, find the investment adjustment costs to be well below 10% of the investment acquisition costs for most OECD countries, which suggests low costs of adjusting the capital stock to its desired level.

Note that Equation (16) does not indicate a causal relationship. For a given required share return, changes in the parameters on the right-hand-side will be counterbalanced by changes in the \( Y/K \) ratio. Changes in the \( Y/K \) ratio need, therefore, not reflect changes in \( \rho \) but could be an endogenous response to changes in the equilibrium unemployment rate or taxes. In the absence of changes in the tax structure and the equilibrium unemployment rate, however, changes in the \( Y/K \) ratio will be a good measure of changes in the required share returns.

![Figure 3. Y/K Ratio, OECD](image)

Source: See data appendix.

The path of the \( Y/K \) ratio over the past 100 years for 13 OECD countries is displayed in Figure 3, where \( K \) is based on the perpetual inventory method for total fixed investment.\(^{10}\) Since the curve exhibits low variability, the \( Y/K \) trend is easily identifiable. The \( Y/K \) ratio only deviates significantly from its long run trend in periods of major disruptions, such as the Great Depression and WWII. The \( Y/K \) ratio fluctuated around a constant slightly below 0.8 before 1950, declined thereafter and has stabilised slightly above 0.5 during the past three decades. The 2.5-percentage point decline corresponds to a 32% reduction in the required returns. This result is consistent with the result of Fama and French (2002) for the US and it is interesting to note that they use 1950 as the benchmark year of the decline in the required returns, which corresponds to the results obtained above.

\(^{10}\) Data from the following 13 countries are included in the figure: Canada, the US, Australia, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Spain, Sweden, and the UK. See Madsen (2003) for data sources.
As an alternative to the $Y/K$ ratio Figure 4 displays earnings per unit of capital adjusted for the influence of the business cycle and the deviation of factor shares from their long-term equilibrium for the same countries as those included in Figure 1. Like the $Y/K$ ratio, the figure indicates that earnings per unit of capital have stabilised at a lower level in the post-war period. In comparison with the pre WWI period earnings per unit of capital have been reduced by about 3 percentage points or 38%. These results are consistent with the results from the $Y/K$ ratio and suggest that the required share returns, and hence the expected returns to shares, have been permanently reduced to a lower level.

4.3 Share prices and technology innovations

The share market run-up in the 1990s has often been attributed to the technology innovations that were derived from the ‘New Economy’. The model in this paper implies that technological innovations only have short-term effects on real share prices and dividends. Consider again Figure 2. A technology shock increases earnings and therefore Tobin’s $q$, which will in turn induce investment until the earnings are back to their initial level. The share market will capitalise on the higher earnings and share prices and dividends will increase. A perfect foresight market will react less to a technology innovation than a myopic market, which values shares based on current earnings and therefore does not take into account that increasing investment drives the extraordinary profits down in the medium term. A myopic market will always be on the $\dot{Q} = 0$-curve, whereas the perfect foresight market will seek to move towards the stable manifold when it is out of equilibrium.

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11 The following procedure is used to cyclically adjust earnings per unit of capital. Earnings per unit of capital for each individual country are regressed on a constant and the deviation of the log of manufacturing factor shares and the deviation of the log of real GDP per capita from their time-trends. The estimated coefficients of factor shares and the business cycle times the cyclical factor shares and cyclical per capita income, respectively, are then subtracted from earnings per unit of capital.

12 The argument carries over to intangibles. See Madsen and Davis (2003) for theory and empirical evidence on this point.
To show this point more explicitly, the marginal productivity of capital under the Cobb-Douglas technology assumption can be written as:

$$\Delta \ln(Y/K) = \Delta \ln A - \alpha \Delta \ln(K/L),$$

which shows that the growth in the marginal productivity of capital depends on two counterbalancing forces, namely Hicks-neutral technological progress and capital deepening. From this equation it follows that Hicks-neutral technological progress increases the marginal productivity of capital. This leads to higher share prices and hence capital deepening until the marginal productivity of capital is back to its initial equilibrium. Historically, capital deepening multiplied by capital’s share of income has, in the long run, grown more than the rate as technological progress, as seen from Figure 3, which has therefore prevented the marginal productivity of capital from growing.

There are uncertainties regarding the time period that extraordinary high profits will last following a technology innovation. These uncertainties are due to the variations in the speed of adjustment of capital stock towards its long run equilibrium and variations in the time period in which firms can prevent imitation of their new technology. The patenting system gives the inventor the sole right to use the invention for up to 20 years depending on time and country. The average period in which the inventor, or the owner of the copyright, can benefit from the invention is in reality much shorter than 20 years because some innovations are not patented and competing firms are given incentives to innovate to remain competitive. Furthermore, Datta and Dixon (2002) show that technological innovations only have temporary effects on monopoly profits because entry of new firms drives the extraordinary profits down to zero in the medium term.

4.4 Shares and long-horizon investment

Traditional static financial theory of optimal portfolio allocation has concentrated on the mean and variance of asset returns without paying much attention to intertemporal decision-making. In the seminal paper of Samuelson (1969) it is shown that if asset returns are stochastic then the optimal portfolio of an investor with a power utility function is independent of the investment horizon. One important assumption underlying Samuelson’s model is that the accumulated share index is assumed to follow a random walk and therefore that share returns are independently distributed over time. Random walks have the important properties that the variance is independent of the data frequency and therefore, if risk is measured as the variance of the asset’s return or the covariance with consumption, that riskiness is independent of the investment horizon.

This paper has shown that the log of real share prices and dividends are mean reverting and, therefore, do not follow a random walk. The mean-reverting property implies that the riskiness of

---

13 See Campbell and Viceira (2002) for some recent attempts to model the portfolio choice in a multi-period framework.
investment is a declining function of the investment horizon and therefore that shares should have relatively higher weights in long horizon portfolios than in short-term portfolios.

4.5 Valuation models
The analysis in Section 3.1 suggests that share prices grow at the rate of $\rho \kappa$ in the steady state, which implies that static valuation models cannot be used as tools for share price valuation. A problem that is associated with the static valuation models is the factors that determine share prices are endogenous and, therefore, render many standard valuation models less useful tools for share price valuation.

To see the consequences of the endogeneity problem in valuation models consider Gordon’s growth model:

$$Q_t = \frac{D_{t+1}^e}{\rho - g} = \frac{D_{t+1}^e}{\rho - r \kappa},$$  

(17)

where $D_{t+1}^e$ is expected dividends per share at period $t+1$, conditional on all available information at period $t$, $r$ is the permanent returns on new investment and $g$ is the expected growth rate in the earnings per share.

Provided that the right-hand side variables in Gordon’s growth model are exogenous, the effects on share prices of shifts in $\rho$, $g$ and $D$ can be readily computed. However, in contrast to this common assumption, these variables are neither exogenous nor independent of each other. First, since earnings per unit of capital converge to a constant for a given $\rho$, as shown in the previous section, it follows that the returns on new investment, $r$, cannot permanently deviate from $\rho$. The denominator in Equation (17) consequently collapses to $\rho (1 - \kappa)$. Hence, $g$ will always be constant in Gordon’s model unless the retention ratio permanently changes. However, since changes in the numerator and denominator following a change in the retention ratio are the same, the valuation is independent of the retention ratio since taxes are absent from this valuation model.

Second, share prices are independent of $\rho$ as shown in the previous section. A shift in $\rho$ changes the steady-state dividends per share correspondingly. Consider a reduction in $\rho$. From a steady state equilibrium where the denominator in Equation (17) is given by $\rho (1 - \kappa)$, a reduction in $\rho$ to half its value, for instance, results in a 100% increase in share prices if dividends are assumed to be exogenous. The model in this paper suggests that dividends in steady state will be reduced by the same percentage as the reduction in $\rho$ and share prices will consequently only be temporarily affected by the change due to adjustment costs in investment and employment. Finally, innovations in dividends that are not caused by changes in $\rho$ or taxes, will not have permanent effects on dividends as shown above because the capital stock endogenously adjusts until dividends equal $\rho (1 - \kappa)$. 
It is, therefore, a straightforward task to find the fundamental value of shares in the steady state by using the recursive formula $Q_{t+1} = Q_t(1 + \rho \kappa)$, where $\rho$ is given by Equation (16) or $\rho \kappa$ is assumed to be constant or is allowed to evolve slowly. Some argue that the required stock returns are in part permanent (Heaton and Lucas, 1999). Others give evidence that suggests that expected stock returns are slowly mean reverting (Fama and French, 1989, and Cochrane, 1994, Siegel, 2002). However, since earnings are constantly changing due to shocks in demand, supply and technology, a more dynamic version of the Gordon growth model that takes these temporary shocks into account, such as the Campbell-Shiller model, will be needed. The initial jump in share prices following a technology shock, $\omega$, can then be computed from the following equation as derived in the appendix:

$$\dot{q} \Big|_{\omega=0} = \frac{\partial y / \partial \omega}{\beta(1-\partial y / \partial \kappa)} \mu^2 d\omega$$

where $\mu$ is a stable root.

However, this approach has practical problems in that it is too complex and cumbersome. A simple approximation is to use the following approximation equation:

$$Q_t \equiv \frac{D_{t+1}[1 + \kappa \cdot T (r - \rho)]}{\rho (1 - \kappa)}, \quad (18)$$

where $T$ is number of years for which the extraordinary profits last. Using the result that $Q_{t+1} = Q_t(1 + \rho \kappa)$ in steady state, Equation (18) collapses to:

$$Q_t \equiv [1 + \kappa \cdot T (r - \rho)][1 + \rho \kappa]Q_{t-1}, \quad (19)$$

where $r$ and $T$ are allowed to change to reflect the fact that earnings are constantly exposed to shocks. From Equation (19) we get the following approximation

$$\frac{dQ_t}{Q_t} \equiv 0.5T,$$

where $(1 + \rho \kappa)\kappa$ is set to 0.5, which is close to its historical average. This equation shows the percentage change in share prices in response to a one percentage point excess return that lasts for $T$ periods. From Figure 1 earnings per unit of capital are rarely more than two percentage points in excess of their long run equilibrium for more than five years. This translates to a share prices increase of 5%. Hence, share prices ought to be relatively insensitive to earnings shocks.

The steady-state P/E ratio is $\rho^{-1}$. If there is a temporary earning shocks the P/E ratio is given by:
where \( E \) is earnings per share or earnings per unit of capital under certain regularity conditions. In this paper it has been argued that trend earnings is the essential measure in the price-earnings ratio because real earnings converge towards a constant growth trend which is given by \( E_{t+1} = E_t(1 + \rho \kappa) \).

5 Empirical estimates

The most important empirical implications of the model of this paper are 1) that factor shares are important determinants of share prices; 2) that share prices are negatively affected by the output-capital ratio because the output-capital ratio echoes \( \rho \); and 3) that real share prices are trend stationary in the sense that they fluctuate about increasing trends provided that \( \rho \kappa \) is approximately constant. These implications are examined using pooled cross-section and time-series data for the OECD countries.

To investigate these issues, the following stochastic counterpart of Equation (8) is estimated for the 22 OECD countries that are listed in Table 1, using panel data over the period from 1953 to 2001:

\[
\Delta \ln Q_i = a_0 + a_1 \Delta \ln (Y/K)_i + a_2 \Delta \ln S_i^K + a_3 \Delta \ln S_i^{K_e} + a_4 \Delta \tau + a_5 \Delta \ln Y_i^{e} + a_6 \Delta \ln (S_i^{K}/S_i^{K_e}) + a_7 \Delta \ln (Q_{i,t-1}/\bar{Q}_i) + \varepsilon_i,
\]

where the subscript \( i \) signifies country \( i \), \( \varepsilon \) is a stochastic error term, \( Q \) is share prices deflated by consumer prices, \( \tau \) is corporate taxes divided by accounting profits before tax, \( Y/K \) is GNP divided by non-residential capital stock, the superscript \( e \) refers to expected value, and \( S^K \) is the share of capital in total income and is measured as net operating surplus divided by nominal GNP. Expected income growth is included in the estimation equation to allow for the fact that earnings are procyclical. Share price indices with the broadest sectoral coverage are used. Data sources are detailed in Madsen (2003). All the regressors are instrumented as detailed in the notes to Table 1.

The variable \((Q_{i,t-1}/\bar{Q}_i)\) is an error-correction term and is denoted \(ECT\) in Table 1 below, which ensures that real share prices converge toward their long-run equilibrium, which is defined here as a deterministic trend; thus implicitly imposing the restriction that \( \rho \kappa \) is constant for each individual country in the estimation period. More precisely, \( \ln(Q_{i,t-1}/\bar{Q}_i) \) is the lagged residuals from regressing the log of real share prices on a time trend for each individual country over the
period from 1953 to 2001. A similar method is used to compute \((S^k_{t-1} / S^k_t)\). This term is included in the model to allow for adjustment of real share prices to disequilibria in the factor market following the dynamic path in the phase diagram above. The coefficient of this term is negative in a perfect foresight share market but positive in a share market that extrapolates the trend in earnings. Such a market, therefore, ignores the mean-reverting nature of factor shares. Suppose that capital’s share in total income is above its steady state equilibrium. The perfect foresight share market is aware of the fact that earnings will subsequently be reduced to their steady state level and share prices will consequently adversely respond to the disequilibrium situation.

### Table 1. Parameter estimates of Equation (20).

<table>
<thead>
<tr>
<th>Term</th>
<th>Estimate (t-statistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \ln(Y/K)_t)</td>
<td>-2.93(4.56)</td>
</tr>
<tr>
<td>(\Delta \ln S_t^k)</td>
<td>0.70(2.18)</td>
</tr>
<tr>
<td>(\Delta \ln S^k_{t-1}/S^k_t)</td>
<td>1.06(7.97)</td>
</tr>
<tr>
<td>(\Delta \ln S^k_{t-1})</td>
<td>5.19(6.01)</td>
</tr>
<tr>
<td>(\Delta \tau_t)</td>
<td>-0.31(1.13)</td>
</tr>
<tr>
<td>(\Delta \ln Y^e_{t+1})</td>
<td>5.48(9.24)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.27</td>
</tr>
<tr>
<td>(DW)</td>
<td>1.84</td>
</tr>
<tr>
<td>(ECT)</td>
<td>-0.20(10.0)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.13(6.16)</td>
</tr>
</tbody>
</table>

**Notes:** Absolute \(t\)-statistics are given in the parentheses. \(DW\) = Durbin-Watson test for first order serial correlation in fixed effects panel data models. \(ECT\) = error-correction term. The following variables in first differences are used as instruments for the right-hand-side variables (except for \(ECT\) and \(\Delta \ln(S^k_{t-1}/S^k_t)\)):

- Lagged dependent variable
- One period lag of the real interest rate
- Unlagged values
- One period lag of the log of real GNP
- One period lag of the log of real share prices
- The log of consumer prices

The following 22 countries are included in the data sample: Canada, USA, Japan, Australia, New Zealand, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the UK. Estimation period: 1955 to 2000.

The results of estimating Equation (20) are presented in Table 1. The estimated coefficients of capital’s income share are economically and statistically highly significant. The sum of their estimated coefficients is 5.89. Since \(S^k\) typically fluctuates by 10-25% on medium term frequencies it is highly influential for the medium-term fluctuations in real share prices. For example, \(S^k\) increased by approximately 20% from the mid 1970s to 2000 and, therefore, accounts for an almost 120% increase in real share prices over the same period. This not only suggests that fluctuations in factors shares are very influential for movements in share prices. It also indicates that share markets overreact to changes in factor shares. The model in this paper predicts that the real share price elasticity of capital’s income share is significantly less than one. To appreciate this consider the Gordon growth model. If an increase in income shares is expected to be permanent without affecting the permanent growth in earnings, then the coefficient of capital’s income share is one. From this it follows that a large fraction of the innovation in factor shares is build into the expected growth term in the Gordon model. This result is consistent with the findings of Barsky and DeLong (1993) and is

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14 Corporate tax rates are calculated as total corporate taxes divided by net operating surplus. Net operating surplus is from OECD, *National Accounts*, and corporate taxes are from UN, *Yearbook of National Accounts Statistics* and OECD, *Revenue Statistics of OECD Member Countries*.
supported by the fact that the estimated coefficient of \( \left( \frac{S_{t-1}^K}{S_t^K} \right) \) is positive and statistically highly significant and, therefore, suggests that share markets extrapolate disequilibria in the labour market instead of taking into account that disequilibria are eliminated over time.

The estimated coefficient of \( Y/K \) is statistically and economically significant and has the expected negative sign. This result highlights the analytical findings in the previous section that the required returns move proportionally to the movements in \( Y/K \) and, therefore, that real share prices are inversely related to \( Y/K \). The estimated coefficient of expected income growth is economically and statistically highly significant, which suggests that share prices are highly sensitive to business cycle fluctuations. Finally, the estimated coefficient of the error-correction term has the expected negative sign and is statistically highly significant, which suggests that real share prices are mean-reverting around an upward trend.

Overall, the estimates are consistent with the predictions of the model presented in the previous section. First, factor shares are key determinants of share prices, which suggests that the labour market plays a central role for share prices and share returns. It is noteworthy that strikes per worker are increasing in periods of reduced corporate earnings, such as the mid 1970s, and are decreasing in periods of increasing earnings per unit of capital, such as the past two decades. Second, real share prices tend to gravitate towards a mean that is increasing over time because earnings are retained within the company. Third, share prices are negatively related to \( Y/K \) because \( Y/K \) mirrors the required returns. For example, an increase in the required returns, for instance, lowers share prices and initiates a capital reducing process which increases \( Y/K \) due to diminishing returns to capital.

6 Summary and conclusion

This paper has presented a theoretical framework that explains the long run behaviour of shares and dividends. The model predicts that earnings per unit of capital fluctuate on medium term frequencies due to innovations in earnings, in particular, and the required returns, but converge towards a constant mean in the long run by a Tobin’s \( q \) effect. This implies that real share prices and dividends are \textit{not} growing over time, in their steady state, due to increasing earnings per unit of capital but solely because earnings are retained within the company. Furthermore, shifts in earnings or required share returns have no permanent effects on real share prices and dividends. Hence, standard valuation model such as the Gordon growth model and other static valuation models, are rendered invalid for share valuation. Real share prices will only increase by the retention ratio multiplied by the required returns in the steady state equilibrium and more dynamic valuation models that allow for temporarily higher earnings are required for share valuation.
The model also allows the required share returns to be recovered from macroeconomic aggregates such as the output-capital ratio and earnings per unit of capital adjusted for the influence of the business cycle and the deviation of labour’s income share from its long run equilibrium. Graphical evidence suggests that the required share returns have decreased by about 30% over the past century. Despite this reduction, however, share returns can be expected to remain relatively high in the future and coupled with the possibility that the currently low real interest rates the equity risk premium will remain high and, perhaps, still be a puzzle.
APPENDIX

The equation system given by Equation (15) has two stable roots \((-\theta, \mu)\) and one unstable root \((\xi)\). Assuming that \(-\theta \neq \mu\) the general solution to the system is:

\[
\begin{bmatrix}
q - \bar{q} \\
k - \bar{k} \\
S^L - \bar{S}^L
\end{bmatrix} = c_1 e^{-\theta t} \begin{bmatrix} x_1^1 \\ x_2^1 \\ x_3^1 \end{bmatrix} + c_2 e^{\mu t} \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{bmatrix} + 0 \cdot e^{\xi t} \begin{bmatrix} x_1^3 \\ x_2^3 \\ x_3^3 \end{bmatrix},
\]

where \(x_j^i\) are eigenvalues corresponding to the \(x^i\) eigenvectors. Deriving the eigenvectors from this system we get, after a few manipulations:

\[
\begin{align*}
q_t - \bar{q} &= -c_1\theta e^{-\theta t} + c_2\mu e^{\mu t} \\
k_t - \bar{k} &= c_1\beta e^{-\theta t} + c_2\beta e^{\mu t} \\
S^L - \bar{S}^L &= c_1 \left( \frac{\mu + \xi + \theta}{\alpha_2} \right) e^{-\theta t}.
\end{align*}
\]

Since \(c_1\) and \(c_2\) are determined by the initial conditions for \(k\) and \(S^L\) we need to solve the system at \(t_0\).

**Steady state multipliers**

Total differentiating the system given by Equations (12)-(14), where \(\dot{q}_t = \dot{k}_t = \dot{S}_t^L = 0\), yields the following system:

\[
\begin{bmatrix}
\rho & \alpha_1(1 - \partial y / \partial k) & \alpha_2 \\
\beta & 0 & 0 \\
0 & 0 & -\theta
\end{bmatrix}
\begin{bmatrix}
d\bar{q} \\
d\bar{k} \\
d\bar{S}^L
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_1 \partial y / \partial \omega & 0 & 0 & q \\
0 & 0 & -\beta & 0 \\
0 & -1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
d\omega \\
d\epsilon \\
d\Gamma \\
d\rho
\end{bmatrix}.
\]

From this system the steady state share market multipliers are given by:

\[
\frac{\partial \bar{q}}{\partial \epsilon} = \frac{\partial \bar{q}}{\partial \omega} = \frac{\partial \bar{q}}{\partial \rho} = 0 \text{ and } \frac{\partial \bar{q}}{\partial \Gamma} = -\frac{1}{\alpha_1} < 0.
\]

From these multipliers it follows that share prices in their steady state are unaffected by supply shocks, technology shocks and changes in the required returns. An increase in investment tax credits reduces share prices by the inverse marginal productivity of capital because it lowers the effective acquisition cost of capital.

**Supply shock**

From the steady state multipliers the dynamic movements in the system following a technology shock are given by:

\[
\begin{align*}
q - \bar{q} &= -\frac{\partial y / \partial \omega}{\beta(1 - \partial y / \partial k)} e^{\mu} d\omega \\
k - \bar{k} &= -\frac{\partial y / \partial \omega}{(1 - \partial y / \partial k)} e^{\mu} d\omega
\end{align*}
\]
\[ S^L - \bar{S}^L = 0. \]

**Technology shock**

The dynamics of the system following a supply shock are given by:

\[ \begin{align*}
q - \bar{q} &= 0 \cdot d\varepsilon = 0 \\
k - k &= -\frac{\alpha_2}{\theta\alpha_1(1 - \partial y / \partial k)} d\varepsilon \\
S^L - \bar{S}^L &= \frac{1}{\theta} d\varepsilon .
\end{align*} \]

**DATA APPENDIX**

REFERENCES


