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Wage Equality in a General Equilibrium Model with Indivisibilities

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Abstract

This paper looks at some consequences of a commitment to wage equality in some sectors of the economy which produce lumpy consumer goods. We show that in this setting, there are two equilibria, with high or low wages, incomes and output. In this closed economy, growth may be immiserizing. We also show that redistribution may eliminate the low wage equilibrium in productive economies but in unproductive ones it eliminates the market for indivisible good itself! This suggests why redistribution could be a “rich man’s game.”

1 Introduction

A feature of public sector enterprises in socialist economies is that workers are not paid solely on the basis of their productivity. Rather, wage equality is a primary concern. Moreover, in such economies, sectors which require significant capital investment are usually state owned. Lumpy or indivisible goods such as automobiles or refrigerators fall into this category. On the other hand, production which can be carried out on a small scale can be undertaken by individuals who then earn according to their productivity. In this paper, we explore the workings of such an economy. We develop a simple general equilibrium model where there are indivisibilities in consumption of a good which is produced in a sector where wages are independent of ability.¹

We model a zero-one kind of indivisibility in consumption: individuals buy one unit or none. This zero-one form is meant to reflect the idea that the good must be of a minimum size.² In practice, there are ways to make such goods divisible such as renting or sharing their services. However, to the extent that it is more costly to rent than buy and because of problems sharing, these are not perfect substitutes. For simplicity, we abstract from such alternatives.

Our assumption on the labor market is suitable for socialist economies as well as market economies with a large public sector. In these countries, state enterprises tend to offer wages independent of ability, and as a result they employ less able workers. Evidence in support of this can be found in Jefferson[8]. He shows that “the inability of state enterprises to monitor and reward high quality

¹Wage equality of this form could also arise because the production process is complex so that productivity cannot be verified in this sector and piece rates cannot be used.

²It is not possible to downsize endlessly as a way of reducing the extent of lumpiness: after all, a refrigerator has to be of a minimum size to be able to hold the needed perishables and a car can only be so small. At low income levels, even clothing is seen as indivisible good. One of the most successful projects undertaken by the World Bank involved subsidizing purchases of wood stoves. The initial cost of such stoves, around 10 to 25 dollars, prohibited their widespread usage although they are more efficient than native stoves made of mud.

labor is likely to create an adverse selection problem in which the most skilled and motivated workers exit from the state sector...”

Our model casts light on a number of issues. First, we show that in this setting, there are multiple equilibria. In one equilibrium, wages are high and so are incomes. As a result, market size is large as is output of the indivisible good. This, in turn, translates into strong demand for labor, and high wages. In the other equilibrium, such circular causation supports a low wage and output level. Second, in this economy, growth may be immiserizing.³ Third, we show that redistribution may have very different consequences in productive versus less productive economies. In the former, redistribution can eliminate the low wage equilibrium. However, in the latter, it eliminates the market for indivisible good itself! This suggests why redistribution could be a “rich man’s game.”

Our work fits squarely into the complementarities and cumulative processes literature. Greater output or effort by an agent results in conditions which elicit greater output or effort by other agents, which in turn is the basis for multiple equilibria.⁴

We proceed as follows. In Section 2 we develop the basic building blocks of the model. In Section 3 we solve for the equilibrium and show how and when multiple equilibria can exist. Then, we put the model through its paces and develop some applications. In Section 4, we look at the effects of technical change and the effect of redistribution. Section 5 looks at the dependence of the results on three main features of the model, namely distortions in the labor market, product market power and indivisibilities, while Section 6 contains some concluding remarks and directions for future work.

³Immiserizing growth exists for an open economy when terms of trade move against the country as a result of the technical progress as shown in Bhagwati[3].

⁴These complementarities arise in any number of settings. See, Cooper and John [4], Cooper [5] on macroeconomic applications, Murphy, Shleifer, and Vishny, see [14] and [15] on development and redistribution. See Matsuyama [13] for an elegant, though now slightly dated, survey of this literature.

2 The Model

There are two types of goods in the economy: One divisible good produced under perfect competition, and a continuum of indivisible goods, indexed by $\theta \in [0, 1]$, each produced by a profit maximizing monopolist.⁵ Individuals are heterogenous in two dimensions. Individual (θ, γ) has a potential demand for one unit of the durable good $\theta \in [0, 1]$ and his productivity is $\gamma \in [0, 1]$. A consumer is active in the indivisible good market with probability $\lambda < 1$.⁶ We assume that each consumer derives utility V each period from his indivisible good, and that V is large. Thus, he would purchase the indivisible good if he can afford it. This makes indirect utility discontinuous: consumers who are just able to buy the good are significantly better off than those with a slightly smaller income.

We assume that γ and θ are uniformly distributed on the unit square. This ensures that all monopolists face the same problem. We assume there is a unit mass of such firms while the mass of individuals is set at 2. Indivisible good firms are monopolists, but as the set of consumers who have a demand for the indivisible good θ is of measure zero relative to the set of all consumers, the pricing behavior of a monopolist does not have any general equilibrium effects. Similarly, due to their being a continuum of firms, no firm has factor market power, and takes wage as given.⁷ We take the divisible good to be the numeraire.

2.1 The Factor Market

Labor is the only factor of production in the economy. Workers with productivity γ make γ units of divisible good and they produce $\alpha\gamma$ units of output of indivisibles where $\alpha > 2$.⁸ However, workers in the indivisible goods sector are

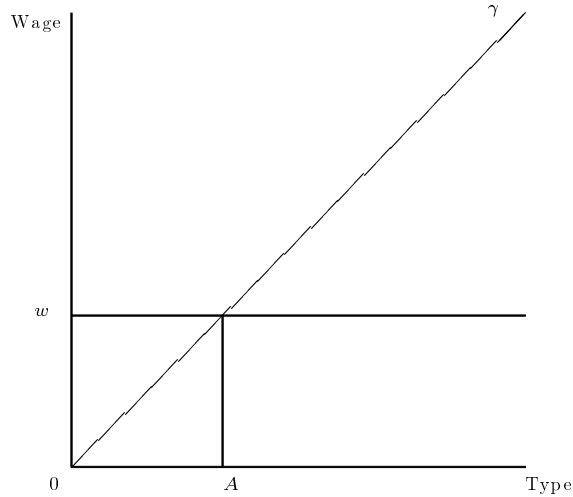
⁵It should be possible to generalize this to oligopoly at the cost of some complexity.

⁶It turns out that $\lambda < 1$ is needed for stability.

⁷This removes the familiar “Numeraire Problem” in the literature.

⁸This assumption is needed for the indivisible sector to be viable.

Figure 1: The Allocation of Labor Between Sectors



paid the going wage independent of their ability. Each agent has the choice of working in the divisible sector, and earning the value of his marginal product there, which acts as his reservation wage, or working in the indivisible goods sector, and earning the going wage. The marginal worker at a given wage is indifferent between the two. Workers with higher productivity prefer to make divisible goods while those with lower productivity work for a wage.

The allocation of labor is depicted in Figure 1. At wage w , OA workers choose to work in the indivisibles sector. These are agents who have productivity below w in the divisible good sector. The remaining workers choose to work in the divisible good sector. An increase in the wage rate attracts agents with a higher productivity level into the indivisibles labor pool and raises the average quality of labor there. All workers with $\gamma \geq w$ obtain their reservation wages while all workers with $\gamma < w$ earn the going wage which is higher than their reservation wage. The earnings of a worker with productivity γ is:

$$\omega(\gamma) = \max(w, \gamma). \tag{1}$$

The total indivisible good output when the wage is w is given by

$$Q^S(w) = 2\alpha \int_0^w \gamma d\gamma = \alpha w^2 \quad (2)$$

where the availability of labor, L , is normalized to be 2. As a share w of all workers work in this sector, the labor in this sector is $2w$, and output per worker is $\frac{\alpha w}{2}$. The unit labor requirement (the inverse of output per worker) times the wage, which equals average and marginal cost, is $\frac{2}{\alpha}$, which is independent of the wage! Marginal cost is just the wage times the unit labor requirement. While the former rises, the latter falls. In the uniform distribution case, the two effects exactly offset each other to leave marginal costs independent of the wage⁹. This gives the following.

Lemma 1 When the productivity of agents is uniformly distributed over the unit interval, marginal costs of production are constant at $c = \frac{2}{\alpha}$, and independent of the wage rate.

The equilibrium level of the wage in the economy is determined by the derived demand for labor from indivisibles. As the level of aggregate indivisible good production rises there will be a higher demand for labor in the indivisible good sector, and wage level will increase. From (2) we get the equilibrium wage as a function of the aggregate indivisible good production to be

$$w(Q) = \left(\frac{Q}{\alpha}\right)^{1/2}. \quad (3)$$

Now, we turn to the equilibrium level of indivisible good production.

2.2 Demand for Indivisibles

A consumer obtains one unit of utility per unit of the divisible good and V if he consumes the indivisible one. His income consists of his labor income, as given

⁹Note that at wage w , the total value of the product is $\frac{P\alpha w^2}{2}$ and total wage bill is w^2 . The value of the product will exceed total wage bill as long as $P\alpha > 2$ or $P > \frac{2}{\alpha} = c$. As a firm prices at or above cost, each firm covers its cost of production at any feasible wage rate.

by (1), and his profit share. We assume that profits made in the indivisible good sector are equally distributed across all individuals in the economy. Thus, each individual is entitled to $\frac{\Pi}{2}$ as the total population (labor force) in the economy is 2.

Let P be the price of the indivisible good. A consumer will purchase his variety of the indivisible good with probability λ if

$$P \leq \frac{\Pi}{2} + \gamma.$$

The balance of his income is spent on the divisible good. Assuming that V is high enough ensures that price is never the binding constraint in the choice of whether to buy or not¹⁰. Thus, all consumers will demand the indivisible good if they can afford it¹¹ and spend the balance of their income on the divisible good. Note that the lowest income level in the economy is $\frac{\Pi}{2} + w$ as all individuals with productivities lower than w will choose to work for the indivisible good sector and command a labor income equal to the market wage. The market demand for a typical indivisible good is¹²

$$Q^D(P) = 2\lambda \text{ if } P \leq \frac{\Pi}{2} + w \quad (4)$$

and

$$Q^D(P) = 2\lambda(1 - P + \frac{\Pi}{2}) \text{ if } P > \frac{\Pi}{2} + w \quad (5)$$

since $(P - \frac{\Pi}{2})$ gives the fraction of wage earners with incomes less than P .

¹⁰This assumption can be relaxed at the cost of some complexity.

¹¹Note that this makes the indirect utility as a function of income discontinuous and convex over a part of its support. Consumers who can just afford to buy the indivisible good are significantly better off than consumers with slightly lower incomes. As a result, consumers are willing to take fair gambles in some regions. This result is also derived in Ng (1965). As he assumes that V is low, consumers only buy the indivisible good after buying some of the divisible good which makes indirect utility continuous.

¹²Recall that the size of the population is 2, of which only a proportion, λ , demand the indivisible good.

3 Equilibrium

The endogenous variables in the system consist of the profits earned by firms, the wages prevailing in the factor market, the prices charged by profit maximizing firms and their sales. How can we solve the system? First we will show how to obtain the demand curve facing each firm for given aggregate profits and wages. This lets us derive the profit maximizing choice of the firm. Of course, once we know the prices chosen for given wages and profits, we know the quantity associated with it on the demand curve, profits, and from (3) the wages needed to elicit the required supply of labor. Finally, since there is a unit mass of firms, the profits of each firm exactly equal aggregate profits so that in equilibrium, the assumed level of aggregate profits and wages must equal the actual levels.

3.1 Profit Maximization

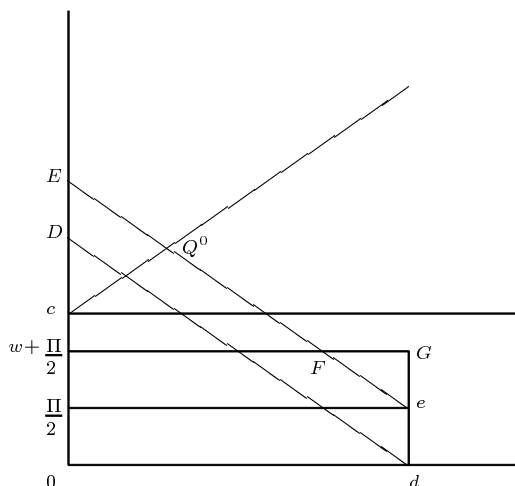
Aggregate profits and the wage level suffice to characterize the demand curve as consumers who can afford the indivisible good buy it. Let the inverse demand function be denoted by $P(Q, \frac{\Pi}{2} + w)$. The line Dd in Figure 2 depicts $P(Q, 0)$, the demand when aggregate profit level and wages are zero. In this event each worker earns the value of his product in divisibles, γ , and given our assumptions, is willing to pay all his earnings for the indivisible good. Thus demand at price P consists of the proportion of consumers with a productivity higher than the price, or $2\lambda(1 - P)$.¹³

If each agent gets profits of $\frac{\Pi}{2}$, he is willing to pay this much more for the indivisible good. Thus, the inverse demand curve is merely shifted up by $\frac{\Pi}{2}$. At a price of $\frac{\Pi}{2}$, even an agent with zero productivity is able to buy the good, but there are no further increases in demand as price falls below $\frac{\Pi}{2}$. Hence we get the vertical segment, ed , as depicted in the demand curve Eed .¹⁴

¹³ 2λ is the size of consumers with a demand indivisible goods.

¹⁴If profits are not evenly distributed the slope of the demand function after profits are added,

Figure 2: The Demand Curve



Finally consider the effects of a non zero wage of w . At this wage, some agents, namely those with a productivity below w , represented by the segment FG , will choose to work for the indivisible goods sector. All these agents will be willing to pay up to $\frac{\Pi}{2} + w$ for the indivisible good. This causes demand to jump to the right at this price as depicted in Figure 2 by the curve $EFGd$.¹⁵

This horizontal segment in the demand curve at price $w + \frac{\Pi}{2}$ causes a discontinuity in profits. Profits jump up at this price so that the use of calculus alone is suspect. We ask instead what is the highest iso profit contour attainable. Iso profit contours are rectangular hyperbolas emanating from c , the constant level of marginal costs. There are clearly only two candidates for profit maximization: to serve the whole market and price at $\frac{\Pi}{2} + w$ (a price lower than $\frac{\Pi}{2} + w$ will be sub optimal) or to choose a point along the downward sloping part of the demand curve. The maximum along the downward sloping part will occur along the ray cQ^0 . If the iso profit contour through Q^0 is higher than that through G ,

¹⁵Note that $D = 1$ and $E = 1 + \frac{\Pi}{2}$.

then Q^0 is chosen.

Let $\pi^*(\Pi)$ denote the profit level associated with the optimum choice on the downward sloping part of the demand curve¹⁶ and let $\bar{\pi}(\Pi, w)$ denote the profit level associated with serving the whole market. Firms will choose the higher of the two so that

$$\pi(\Pi, w) = \text{Max}\{\pi^*(\Pi), \bar{\pi}(\Pi, w)\}. \quad (6)$$

The maximized value of profits is continuous, though the profit function is not.

Next we show that $\pi^*(\Pi)$ and $\bar{\pi}(\Pi, w)$ are increasing in Π but flatter than the 45° line, and that $\pi^*(\Pi, w)$ is flatter than $\bar{\pi}(\Pi, w)$.

Lemma 1 $0 < \pi_{\Pi}^*(\Pi) < \bar{\pi}_{\Pi}(\Pi, w) < 1$.

Proof: Using the envelope theorem gives

$$\begin{aligned} \frac{d\pi(\Pi, w)}{d\Pi} &= \frac{\partial}{\partial \Pi} \left((P(Q, \frac{\Pi}{2} + w) - c)Q \right) \\ &= \frac{\partial P(Q, \frac{\Pi}{2} + w)}{\partial \Pi} Q \\ &= \frac{1}{2}Q(\cdot). \end{aligned}$$

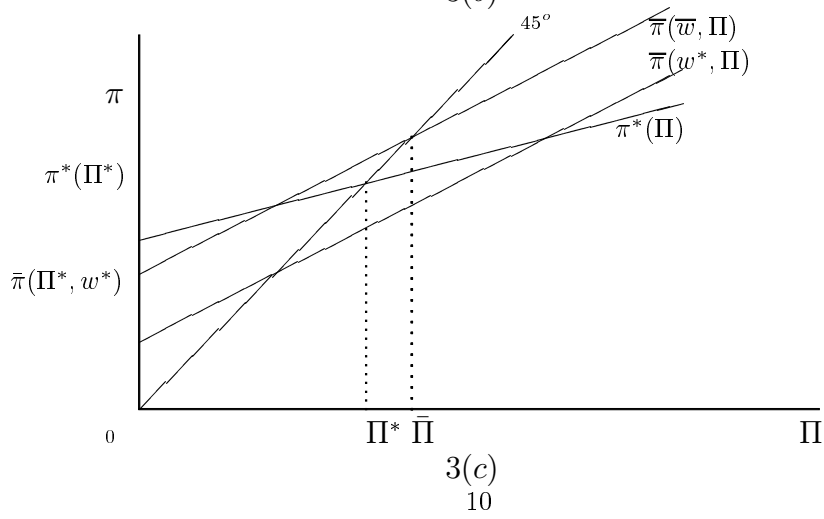
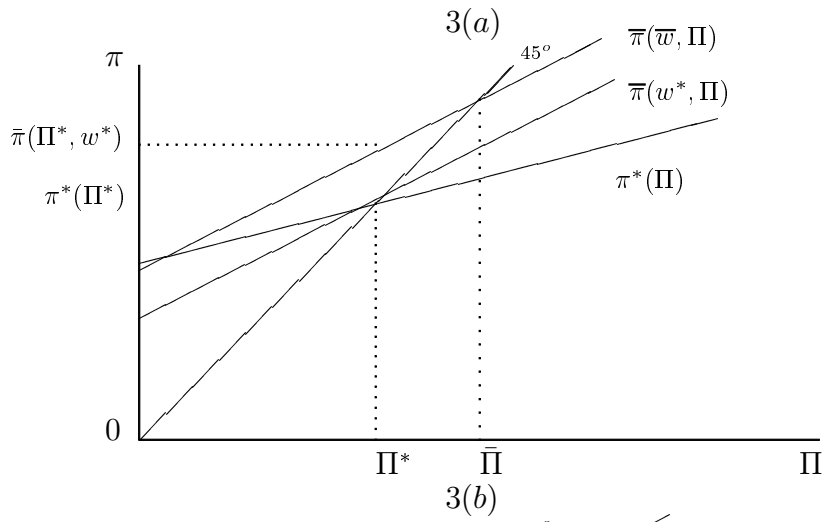
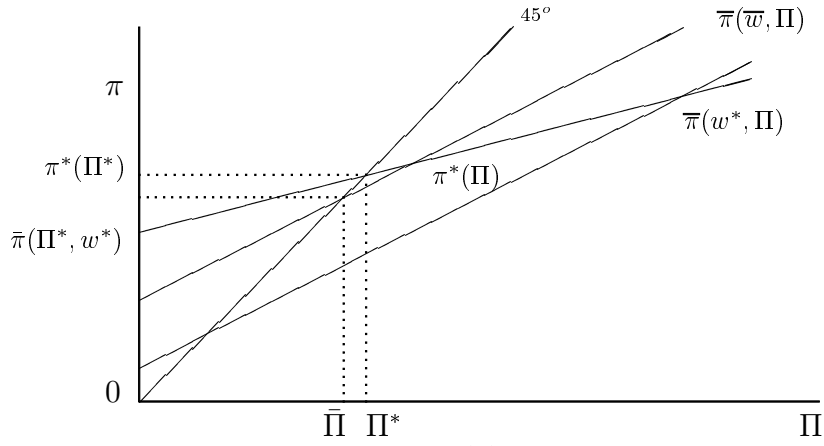
where $Q(\cdot)$ is the optimal choice of output¹⁷. Output is positive as cost is below the vertical intercept of demand for all profit levels which is ensured by our assumption that $\alpha > 2$. Output along $\pi^*(\Pi)$ is less than that along $\bar{\pi}(\Pi, w)$ so that $\pi_{\Pi}^*(\Pi) < \bar{\pi}_{\Pi}(\Pi, w)$. Also, since $Q(\cdot) < 2\lambda$ and $\lambda < 1$, it must be that $Q(\cdot) < 2$ so that $\frac{1}{2}Q(\cdot) < 1$.

Figure 3 depicts the level of individual profits as a function of aggregate profits. Note that $\pi^*(\Pi)$ does not depend on w while $\bar{\pi}(\Pi, w)$ is increasing in w . For $w = 0$, the intercept of the $\bar{\pi}(\Pi, w)$ line lies below that of $\pi^*(\Pi)$, that is,

¹⁶Of course, a firm cannot sell to more than 2λ so that the demand curve does not continue past this point.

¹⁷Although $\pi(\Pi, w)$ is differentiable except at $\pi^*(\Pi, w) = \bar{\pi}(\Pi, w)$.

Figure 3: Three Possibilities in Equilibrium



$\bar{\pi}(0, 0) < \pi^*(0)$. As w rises, $\bar{\pi}(\Pi, w)$ shifts up. Hence, for low w , the intercept of $\bar{\pi}(\cdot)$ lies below that of $\pi^*(\cdot)$ in Figure 3. Combined with the slope of $\bar{\pi}(\cdot)$ exceeding that of $\pi^*(\cdot)$, it follows that for low w , $\pi(\Pi, w)$ equals $\pi^*(\Pi)$ for low Π , and equals $\bar{\pi}(\Pi, w)$ for high Π . An increase in w reduced this switch-over point. For a high enough w , the intercept of $\bar{\pi}(\cdot)$ lies above that of $\pi^*(\cdot)$ as a result $\pi(\Pi, w)$ equals $\bar{\pi}(\Pi, w)$ for all Π .

As a result of Lemma 1, we get stability as well as a single intersection between $\pi(\Pi, w)$ and the 45 degree line. Both come from the slope of $\pi(\Pi, w)$ being less than unity. Note that $\pi(\Pi, w)$ cannot intersect the 45⁰ line more than once. If it did, it would mean that $Q(\cdot)$ exceeded 2, which would involve serving more than the entire market.

3.2 Equilibrium

Aggregate profits in equilibrium equal those of each firm as there is a unit mass of firms. Let \bar{w} denote the wage level needed to elicit the required labor when all firms serve their entire market and let w^* be the wage needed to elicit the lower level of labor needed when all firms serve only part of their market in equilibrium.

The candidates for an equilibrium are the level of profits at which $\pi^*(\Pi)$ and $\bar{\pi}(\Pi, \bar{w})$ cut the 45 degree line, i.e., Π^* and $\bar{\Pi}$ in Figure 3.¹⁸

Consider panel (a) where the only equilibrium is serving part of the market. If serving part of the market is an equilibrium, profits are Π^* . When profits are Π^* , then output must be Q^* if not all the market is served, and to generate the labor needed to produce Q^* , wages must be w^* . Finally, for this to be an equilibrium, deviation by a single firm should not be profitable so that profits

¹⁸We will draw these as straight lines for simplicity. While $\bar{\pi}(w, \Pi)$ is linear, $\pi^*(\Pi)$ is not.

along $\pi^*(\Pi)$ lie above those along $\bar{\pi}(\Pi, w^*)$ at $\Pi = \Pi^*$,

$$\pi^*(\Pi^*) = \Pi^* > \bar{\pi}(\Pi^*, w^*). \quad (7)$$

Similarly, serving the entire market is an equilibrium if

$$\bar{\pi}(\bar{\Pi}, \bar{w}) = \bar{\Pi} > \pi^*(\bar{\Pi}). \quad (8)$$

For this to be the only equilibrium, $\bar{\Pi} > \bar{\Pi}^* > \Pi^*$ as depicted in panel (b). Finally, both (7) and (8) could hold simultaneously as depicted in Figure 3(c) so that there are multiple equilibria. Each firm may choose to serve the whole market if every other firm is doing that, and they may choose to serve only a part of the market if all other firms are serving only a part of the market¹⁹. Note that in this case, $\bar{\Pi} > \Pi^* > \bar{\Pi}^*$.

Since monopolist are identical, their choice should be the same. There is one exception, which occurs when profits are such that $\pi^*(\Pi) = \bar{\pi}(\Pi, \tilde{w})$ for some \tilde{w} , i.e., we are at the switch-over point²⁰. In this event, firms are indifferent between their two relevant options and could choose either as depicted in Figure 4. In this case a proportion μ of the firms choose to serve their entire markets and μ is set so that the wage \tilde{w} prevails. There is only one such mix as any other mix would raise or lower $\bar{\pi}(\Pi, w)$ by changing the wage level. Note that this is not a stable situation²¹ and so we neglect it.

3.3 Determinants of Equilibrium Type

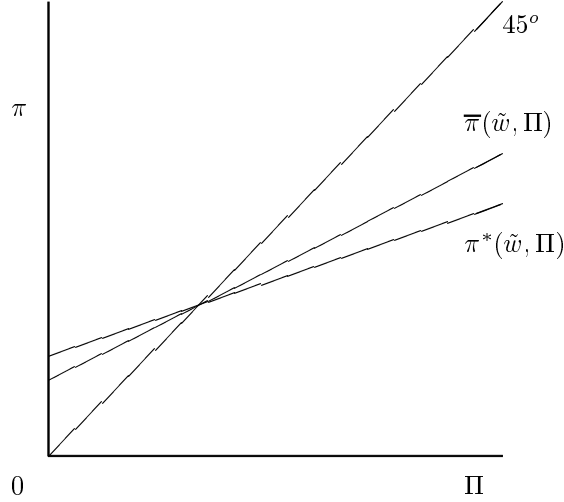
Recall that c must be less than unity (i.e., $\alpha > 2$) for profits to be positive, while $\lambda < 1$ is needed for stability, that is for profits to be flatter than the 45⁰

¹⁹What permits two maxima is the non concavity of the profit function due to profits jumping up at the point where all consumers are served.

²⁰In Figure 2, this would require the wage to be such that the same iso profits contour was tangent to the downward sloping part of demand and to the point where all consumers are just served. This is the wage \tilde{w} .

²¹If more firms choose to serve the whole market, the wage would rise as would $\bar{\pi}(\Pi, w)$ making it more profitable for a firm to serve the whole market. Similarly, if fewer firms choose to serve the whole market, the wage would fall as would $\bar{\pi}(\Pi, w)$ making it more profitable for a firm to serve only part of the market.

Figure 4: Asymmetric Equilibrium



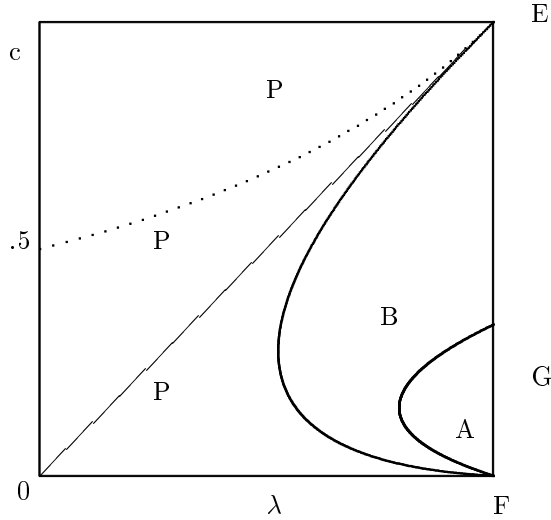
line as drawn in Figure 3. Hence, we need only consider the unit square in (c, λ) space. Our results are depicted in Figure 5.²²

Proposition 1 *Serving the entire market is an equilibrium in the region marked A in Figure 5, serving part of the market is an equilibrium in the region marked P , and in the region marked B both are possible.*

Proof: We sketch the proof here relegating the details to the Appendix. What gives the boundaries for the different regions in Figure 5? Recall that the regions are related to the ranking of $\bar{\Pi}$, Π^* and $\bar{\Pi}^*$. In region A , $\bar{\Pi} > \bar{\Pi}^* > \Pi^*$. In region B , $\bar{\Pi} > \Pi^* > \bar{\Pi}^*$. In region P , $\Pi^* > \bar{\Pi} > \bar{\Pi}^*$. Thus, the switch from A to B occurs when $\bar{\pi}(\Pi, w^*)$ goes through the intersection of the 45^0 line and $\pi^*(\Pi)$. In other words, where $\bar{\pi}(\Pi^*, w^*) = \Pi^*$. This is depicted by FG . The switch from B to P occurs when $\bar{\pi}(\Pi, \bar{w})$ goes through the intersection of the 45^0 line and $\pi^*(\Pi)$. In other words, $\Pi^* = \bar{\Pi}$ and this is depicted by EF . Thus, all we need to do is calculate these

²²Details of the simulations are in the Appendix.

Figure 5: Determination of Equilibrium Type



expressions in terms of (c, λ) and then set them equal to get the two boundaries. When this is done, we get Figure 5.

What is the economic intuition behind Figure 5? Serving all the market is the unique equilibrium only when c is small and λ is large. For a given c , a lower λ makes serving part of the market more attractive and only part of the market is served. A reduction in c , for a given λ , can result in moving from an equilibrium where everyone is served to one where only part of the market is served. As λ increases, market size increases, and serving the whole market becomes increasingly attractive. Thus, the A region is to the right in Figure 5 while the P region is to the left, and B is in between.

It is interesting to note that at moderate participation rates, serving the entire market is an equilibrium only for intermediate values of c ! Reducing c can move you through region P to B to A back to B to P ! At both high and low costs, only part of the market can be served. If c is very high, it is simply not feasible to serve the entire market. At the other extreme, if c is low, then

very little labor is needed to serve the whole market and wages are low. Due to this, the price that can be charged to the whole market is low. Profits from serving part of the market do not depend on wages, so that these exceed the profits from serving all the market at very low c , and serving part of the market is the only equilibrium.

In economics, there is a presumption that technical progress is welfare improving since it shifts the production possibility frontier outward and in the absence of distortions this is welfare improving. A famous exception, immiserizing growth, see [3], exists for an open economy. It occurs when terms of trade move against the country as a result of the technical progress. This could occur if productivity in the export sector grew and demand for exports was relatively inelastic. A weaker form of this, namely adverse, but not immiserizing, terms of trade effects is posited for primary product exporters by Prebisch and Singer²³.

Keynes voices similar concerns in *Essays in Persuasion* [10]. In the essay “*Economic Possibilities for our Grandchildren*” “technological unemployment” is seen as a consequence of productivity improvements. In his words “The increase of technical efficiency has been taking place faster than we can deal with the problem of labor absorption” and “We are being afflicted with a new disease of which some readers may not yet have heard the name, but of which they will hear a great deal in the years to come—namely, technological unemployment. This means unemployment due to our discovery of means of economizing the use of labor outrunning the pace at which we can find new uses for labor.” However, Keynes regarded such unemployment as a temporary phenomenon reflecting out of equilibrium behavior, while technological underemployment arises as an equilibrium phenomenon in our model.

²³A scenario in science fiction, as in Kurt Vonnegut’s “*Player Piano*”, is one where machines are so productive that only the gifted are employed and well off while the masses live with only basic comforts. The result is a kind of technological underemployment.

Our model provides such an example of immiserizing growth in a closed economy. Since agents who can obtain the indivisible good are strictly better off than those who cannot, total welfare will rise when all agents are served. However, as shown above, productivity improvements in indivisibles may result in fewer agents being served in equilibrium.

Proposition 2 *Technical improvements in indivisibles may result in a loss in total welfare.*

4 Effects of Redistribution

An interesting aspect of our model is that the distribution of income plays a very important role in shaping demand and the behavior of firms.²⁴ If income is relatively equal and the economy is quite productive, the demand curve is very elastic and serving the entire market becomes attractive. This would, of course, raise aggregate welfare as it is always optimal to serve all consumers given our assumptions. Thus, we get a novel rationale for redistribution, namely efficiency! In other cases, redistribution can actually hurt as it eliminates the market completely, despite there being no incentive distortion induced by redistribution. This occurs if the market is small and the economy is relatively unproductive²⁵.

The idea that the distribution of income affects the development of an economy is an old one. There is an old debate about inequality and development as well as inequality and growth. On the one hand, in poor countries inequality creates a market for certain kinds of goods. On the other hand, the very rich can

²⁴The importance of income distribution has been considered in the literature. For example, Murphy, Shleifer and Vishny [15] argue that this is a factor in the success of industrialization. In their model, the rich consume goods which do not have any spillovers on the economy and do not encourage industrialization while the consumption of the middle class does have such spillovers.

²⁵Of course, extreme inequality is very bad as well since the market size is small. Reducing such extreme inequality is likely to always be beneficial in our setup. Further reductions could be good or bad as described here.

only spend so much and tend to spend it on things that have few externalities. More recent modelling of such ideas includes [1], [15], [6] and [7].

In our model, labor supply is totally inelastic. Thus, taxing income will not have any incentive effects. We will assume thus, that the government implements a tax scheme which, for any combination of endogenous variables, gives all agents equal wage income. As profits are also equally distributed, all agents have the same incomes. We will argue that there are reasons to believe that complete redistribution is a “rich man’s game”! Poor (unproductive) economies are hurt by it, while rich (productive) ones gain.

Complete redistribution makes the firms’ choice a simple one of either serving all or none of the market. We will show that if the indivisible good sector is not very productive, that is c is high, then serving the whole market gives negative profits, and so nobody is served and the indivisible good industry vanishes. Comparing this outcome to that prior to redistribution where at least some consumers were served shows that welfare falls due to redistribution in this case. However, if c is low enough, redistribution must raise welfare by making it optimal for firms to serve their entire markets.

Average labor earnings in the economy are

$$w \int_0^w d\gamma + \int_w^1 \gamma d\gamma = \frac{1}{2} + \frac{w^2}{2}.$$

Recall that the wage level when all consumers are served is $\bar{w} = (\lambda c)^{1/2}$. Thus, if everyone is served as a result of complete redistribution, the average labor earning is $\frac{1}{2} + \frac{\lambda c}{2}$. Given this income level, firms will ask for a price of

$$P = \frac{1}{2} + \frac{\lambda c}{2} + \frac{\Pi}{2}$$

in equilibrium. The cost of production in the economy is constant at c . Since

all firms are serving their entire market, the profit level for each firm is:

$$\pi = 2\lambda\left[\frac{1}{2} + \frac{\lambda c}{2} + \frac{\Pi}{2} - c\right].$$

In equilibrium

$$\pi = \Pi = \frac{2\lambda}{1-\lambda}\left[\frac{1}{2} - c\left(1 - \frac{\lambda}{2}\right)\right].$$

Thus, the profit level is positive only if

$$c < \frac{1}{2-\lambda}.$$

This is depicted in Figure 5 by the dashed line. Above this dashed line, profits are negative when income is redistributed equally, so that the market is eliminated. Note that this region lies entirely in the P region and in it complete redistribution hurts the economy²⁶. On the other hand, in the remainder of region P , redistribution will ensure that everyone is served in equilibrium and improve welfare.

Proposition 3 *Although complete redistribution may improve welfare, it must reduce it if productivity is low, as is market size, that is, if $c > \frac{1}{2-\lambda}$.*

Clearly, those individuals whose productivity is higher than the average lose due to the transfer scheme. However, if complete redistribution results in the destruction of the indivisible good market, even individuals who received a positive wealth transfer may lose! This will be the case if the common income after redistribution²⁷, $1/2$, is lower than their income prior to redistribution, $w^* + \frac{\Pi^*}{2}$.

Although complete redistribution may fail, less severe forms which increase the proportion of consumers who are served may work. Note that transfers both from the very rich, *and the very poor*, to those on the verge of buying the

²⁶By reversing the experiment, this also provides conditions where creating greater inequality may be efficient.

²⁷This happens if for example $c = \frac{3}{4}$, $\lambda = \frac{1}{2}$.

good would tend to increase the size of the market. Of course, the regressive implications of such policies make them unlikely to be used. A possible example of the use of such policies might be found in Turkish History. It has been argued that after the Turkish Republic was founded in the early 20's, creating a new entrepreneurial class was seen as essential for the industrialization efforts in the country. For this purpose, heavy taxes were imposed on poor and middle income classes, and these tax revenues are used to subsidize the emerging entrepreneurial class as well as to boost the consumption of the bureaucrats by providing them a higher income level. (See ([9])).

5 Robustness

There are three distortions in our model. There is a product market distortion, a factor market distortion, and an indivisibility in consumption. As firms have monopoly power, too little of the indivisible is produced. Due to the factor market distortion, too few workers work in indivisibles. Finally, the indivisibility prevents poor consumers from buying some of the good so that in equilibrium too little of the good is consumed.

We have three main results so far and three distortions which could be responsible for them. Below we look at how the results change as we remove the distortions one by one to see which assumptions seem crucial. Table 1 provides an overview of the sensitivity of results to assumptions.

	No PMD	No FMD	No Indivisibility
Multiple Equilibria	YES	NO	YES
Immiserizing Growth	NO	NO	NO
Income Redistribution	YES	YES	NO

5.1 Product Market Distortion

Suppose that there is no product market distortion in the economy. In this case, prices will equal unit cost, $P = \frac{2}{\alpha} = c$, and the profit level will be zero for each firm. Then, income consists only of wage earnings, and all individuals whose wage income exceeds the price will purchase the indivisible good. The total quantity demanded will depend on whether the wage level weakly exceeds the price which equals cost, $w \geq c$. If this is so, then at a price of c everyone is in the market and in equilibrium the whole market can be served. If the wage does not exceed c , then only part of the market may be served.

When is part of the market served and when is everyone served? Can there be multiple equilibria? Consider Figure 6. The maximum level of demand is 2λ as the population is normalized to $L = 2$, and at a price of unity, nothing is demanded as the highest productivity worker makes exactly this. Thus, the line Dd is given by

$$Q = 2\lambda(1 - P). \quad (9)$$

Demand is Dd for prices above the wage in indivisibles and is 2λ for prices below the wage.

The curve $w(Q)$ depicts the wage level as a function of the aggregate quantity produced. \bar{w} gives the wage needed for the 2λ to be supplied so that from (3)

$$\bar{w} = \left(\frac{2\lambda}{\alpha}\right)^{1/2} = (c\lambda)^{1/2}. \quad (10)$$

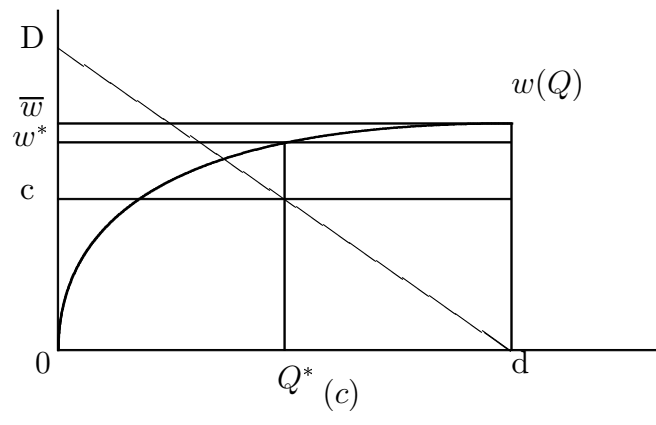
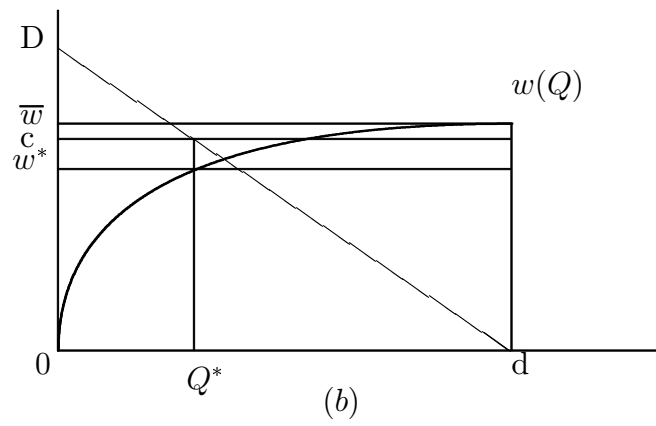
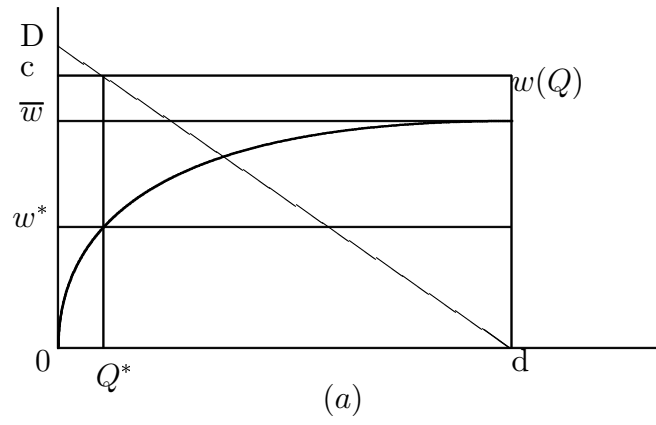
Look at the demand Dd when price is set at c . Using (9) this gives

$$Q^* = 2\lambda(1 - c). \quad (11)$$

Call the wage needed to elicit this quantity supplied, which of course is less than 2λ , to be w^* :

$$w^* = \left(\frac{2\lambda(1 - c)}{\alpha}\right)^{1/2} = (\lambda c(1 - c))^{1/2}. \quad (12)$$

Figure 6: Equilibria Without Market Power



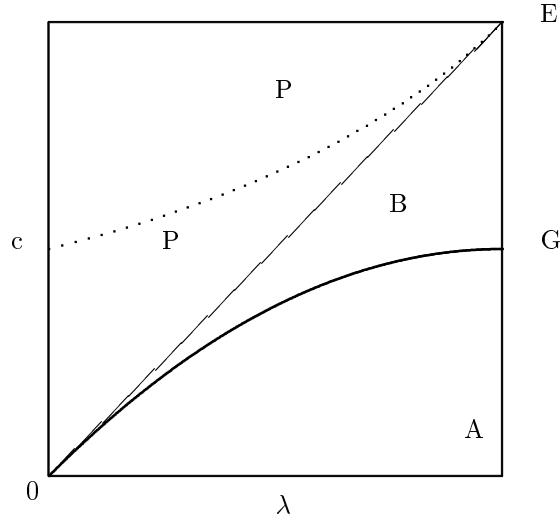
Note that $w^* < \bar{w}$ since $(1 - c) < 1$.

Hence, there are three possibilities. Either (a) $c > \bar{w} > w^*$, (b) $\bar{w} \geq c > w^*$, or (c) $\bar{w} > w^* \geq c$, or correspondingly, (a) $c > \lambda$, (b) $\frac{c}{(1-c)} > \lambda \geq c$, or (c) $\lambda \geq \frac{c}{(1-c)}$. These cases are depicted in Figure 6 panels (a) – (c). In case (a) serving part of the market, producing Q^* , and having a wage w^* is the unique equilibrium. Producing for the whole market and having a wage \bar{w} is not an equilibrium as demand is less than supply. In case (b), λ is high and serving the entire market is the unique equilibrium. This follows from noting that in this event when the wage is \bar{w} , demand at c is 2λ (since price equals c which is less than both \bar{w} and w^*) which equals supply at \bar{w} . Thus, a wage of \bar{w} and the entire market being served is an equilibrium. Serving part of the market is not an equilibrium as if part of the market was served, then at price c , the wage needed to elicit demand of Q^* namely w^* , exceeds price. Hence, at a price of c and wage of w^* , supply would fall short of demand and this could not be an equilibrium. In case (c) there are two equilibria: serving the whole market and having a wage of \bar{w} , or serving part of the market, producing Q^* , and having a wage w^* . The latter is an equilibrium as at a price of c and wage of w^* , demand is Q^* and this equals supply.

Note that the multiple equilibria result remains even without product market distortions. Also, that as c rises, we can move from case (c) to (b) to (a), that is, region A to B to P , but never from (a) to (b) to (c). Thus, an increase in c can make serving part of the market the equilibrium, or the only equilibrium but a reduction in c can never result in removing serving all the market as an equilibrium. Thus, immiserizing growth in a closed economy does not occur in the absence of monopoly power in the product market.

What about the effects of redistribution and technical change? Complete redistribution works under the same conditions as before: $c < \frac{1}{2-\lambda}$. There will

Figure 7: Equilibrium Type Without Market Power



be an improvement in the equilibrium outcome if the complete redistribution results in a switch from a part served equilibrium to an all served equilibrium. In Figure 7, the area above the dashed line is where the market vanishes with complete redistribution. In this region, redistribution hurts. On the other hand, outside this region, it can help as it can ensure that all the market is served²⁸.

5.2 No Factor Market Distortion

If there is no distortion in the factor market then piece rates can be paid to workers in all sectors. Let ω denote the piece rate in the indivisible sector (which of course then equals the marginal and average cost in the indivisible sector) while the piece rate in the divisible sector is unity. If both goods are produced, effective labor must earn the same in both sectors, that is

$$\alpha\gamma\omega = \gamma \tag{13}$$

²⁸It can be shown that with neutral technical progress immiserizing growth cannot occur. Furthermore, neutral technical progress is a Pareto improvement.

so $\omega = \frac{1}{\alpha}$. If ω lies above $\frac{1}{\alpha}$, then all labor will work in the indivisible good sector while if lies below $\frac{1}{\alpha}$, the opposite occurs. The analysis proceeds as before, but with costs of $c' = \frac{1}{\alpha}$. Producing indivisibles will be feasible if $c' < 1$, i.e., $\alpha > 1$. However, firms will never choose to sell to the whole market as it is not profitable to do so. If they do, they must price at $\frac{\Pi}{2}$ and incur costs of $\frac{1}{\alpha}$ per unit so that individual profits are given by:

$$\pi = \left[\frac{\Pi}{2} - c' \right] 2\lambda. \quad (14)$$

But as individual profits equal aggregate ones, this implies profits are negative! Thus, firms always serve only part of their market and it is impossible to obtain multiple equilibria.

Note that having a distortion in the factor market may improve the equilibrium outcome. Without the distortion, serving the entire market is never an equilibrium, while with the distortion, it is an equilibrium for some parameter values, and sometimes even the unique equilibrium. The factor market distortion may alleviate the distortion resulting from the indivisibility of one of the goods.

Is this assumption important for the other results? What are the implications of completely equal incomes? The average wage income in the economy is $1/2$. Thus, the profit from producing the indivisible good is

$$\pi = \left[\frac{1}{2} + \frac{\Pi}{2} - c' \right] 2\lambda. \quad (15)$$

In equilibrium $\pi = \Pi$, and $\pi = \frac{\lambda}{(1-\lambda)}(1-2c')$. This will be positive if $1 > 2c' = \frac{2}{\alpha}$ which implies that $\alpha > 2$. Recall that the condition for any production to take place is $\alpha > 1$. Thus, if $2 > \alpha > 1$, then redistribution will cause the indivisible good production to stop completely, and it will reduce welfare as in the case of no factor market distortion.

Finally, we look at the importance of the assumption of factor market distortions on the welfare effects of technical progress. When there is no factor market distortion, all individuals will earn the value of what they produce in the divisible goods sector. Thus, biased technical progress does not affect equilibrium wages in terms of divisibles, while it increases the profit level and the proportion of the market that is served in equilibrium. Hence, technical progress results in a Pareto improvement²⁹.

5.3 No Indivisibility

Indivisibility is critical to our central results. It is because of the indivisibility that there is a discontinuity in indirect utility as a function of income. Due to this indivisibility, consumers who cannot afford the good are much worse off than ones who can. If the indivisible good were divisible, this would not occur, as consumers would buy as much of the indivisible as they could afford equalizing marginal utility of the “indivisible” and the divisible.

If the “indivisible” good is divisible, then the price of the good will be equal to V , and all individuals will be indifferent as to how they allocate their income between the “indivisible” good and the divisible good. However, consumers’ decisions on how much to spend on the “indivisible” good will affect the income level. As more is spent on the indivisible the income level will increase and therefore there will be more to spend. In this case, it is possible that there will be a continuum of equilibria. Redistribution will no longer be desirable for efficiency reasons as individuals can always purchase part of the good.

²⁹Neutral technical progress will increase wages in terms of divisibles as well as indivisibles and will for similar reasons also result in a Pareto improvement.

6 Concluding Remarks

In this paper we showed that when coupled with labor and product market distortions, indivisibilities may lead to multiple equilibria in both product and factor markets, that biased technical progress could reduce welfare, and that redistribution toward equality could have destructive effects.

Variants of this model can explain several other phenomena. In Krishna and Yavas (2001a) assuming that the indivisible good is a consumer durable we show that endogenous business cycles are generically produced. In Krishna and Yavas (2001b), we argue that free trade may not be optimal because of general equilibrium “income shifting” in these models.

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7 Appendix: Calculations

When the whole market is served the output level is equal to 2λ , the monopolist serving its entire market asks a price of $p = w + \frac{\Pi}{2}$, and makes the profits of

$$\bar{\pi}(\Pi, w) = 2\lambda[w + \frac{\Pi}{2} - c]. \quad (16)$$

If all firms are serving their entire markets, by (3) the equilibrium wage is

$$\bar{w} = (\lambda c)^{1/2}. \quad (17)$$

Substituting (17) in (16) and setting $\bar{\pi}(\bar{\Pi}, \bar{w}) = \bar{\Pi}$ gives³⁰:

$$\bar{\Pi} = \frac{2\lambda c^{1/2}}{1-\lambda} [\lambda^{1/2} - c^{1/2}]. \quad (18)$$

Similarly,

$$\pi^*(.) = (1 + \frac{\Pi}{2} - \frac{q}{2\lambda} - c)q.$$

The maximized value of profits is obtained at

$$q^*(\Pi) = \lambda(1 + \frac{\Pi}{2} - c) \quad (19)$$

The value function for this problem is

$$\pi^*(\Pi) = \frac{\lambda}{2}(1 + \frac{\Pi}{2} - c)^2.$$

Setting individual profits equal to aggregate ones gives³¹

$$\Pi^* = \frac{4}{\lambda} - 2(1-c) - \frac{4}{\lambda}[1 - \lambda(1-c)]^{1/2}. \quad (20)$$

³⁰Note that for $c \geq \lambda$ this is non positive, so that the entire market can be served only below the 45⁰ line in Figure 5.

³¹Although there are two solutions to this quadratic equation, the positive root is infeasible as it implies output levels above 2λ .

Finally, to calculate $\bar{\pi}(\Pi^*, w^*)$ we first need to calculate w^* . By substituting (20) into (19) we get the total output level in a part served equilibrium as

$$Q^* = \lambda \left(\frac{2}{\lambda} - \frac{2}{\lambda} [1 - \lambda(1 - c)]^{1/2} \right) \quad (21)$$

Substituting the output level (21) into (3) and noting that $\alpha = \frac{2}{c}$, we get

$$w^* = \left(\frac{\lambda c}{2} \left(\frac{2}{\lambda} - \frac{2}{\lambda} [1 - \lambda(1 - c)]^{1/2} \right) \right)^{1/2}. \quad (22)$$

Substituting (22) into (16) and evaluating it at $w = w^*$ and $\Pi = \Pi^*$ gives:

$$\bar{\pi}(\Pi^*, w^*) = 2\lambda \left[(c(1 - 1[1 - \lambda(1 - c)]^{1/2}))^{1/2} + \frac{2}{\lambda} - 1 - \frac{2}{\lambda} [1 - \lambda(1 - c)]^{1/2} \right] \quad (23)$$

Next, we set $\Pi^* = \bar{\pi}(\Pi^*, w^*)$ and $\Pi^* = \bar{\Pi}$ in order to find the boundaries between regions. Using maple to plot these equalities,

$$\begin{aligned} & 2\lambda \left((c(1 - 1(1 - \lambda(1 - c))^{1/2}))^{1/2} + \frac{2}{\lambda} - 1 - \frac{2}{\lambda} (1 - \lambda(1 - c))^{1/2} \right) \\ &= \frac{4}{\lambda} - 2(1 - c) - \frac{4}{\lambda} (1 - \lambda(1 - c))^{1/2}, \end{aligned}$$

and

$$\frac{4}{\lambda} - 2(1 - c) - \frac{4}{\lambda} (1 - \lambda(1 - c))^{1/2} = \frac{2\lambda c^{1/2}}{1 - \lambda} (\lambda^{1/2} - c^{1/2})$$

we obtain the following:

