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Endogenous Business Cycles and Systematic Stabilization Policy

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2002-03

ISSN 0908-7745

The activities of EPRU are financed by a grant from
The Danish National Research Foundation

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February 2002

*We thank anonymous referees for insightful comments. The paper has also benefited from comments and suggestions by Jean Pascal Benassy, Richard Disney, Jean Michel Grandmont, and Kala Krishna. Marta Aloi would like to thank the British Academy for financial support. Hans Jørgen Whitta-Jacobsen would like to thank Departament d'Economia i Empresa, Universitat Pompeu Fabra, Barcelona for hospitality during the period where the paper was finished. Teresa Lloyd-Braga and Marta Aloi would like to thank the Economic Policy Research Unit of Copenhagen University for support and hospitality.

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Abstract

We study the effects of stylized fiscal policy rules on the (global) determinacy of rational expectations equilibrium in a perfectly competitive monetary model with constant returns to scale and labor as the unique input. Government spending on transfers and/or demand implies a distortion of the monetary steady state due to the implied income or inflationary taxation. We show that policy rules for which the GNP share of government spending depends sufficiently negatively on increases in GNP can stabilize the economy with respect to endogenous fluctuations for an arbitrarily small level of distortion of the steady state at which stabilization occurs. These policy rules do not involve features such as positive lump sum taxation, negative income taxation, or exact knowledge of the economy's laissez faire steady state.

JEL Classification: E32 and E63.

Keywords: Endogenous business cycles, stabilization policy.

Running title: "Endogenous Fluctuations and Stabilization Policy"

1 Introduction

The present paper is concerned with fiscal policy in connection with stabilizing competitive *endogenous* fluctuations. Standard economic models based on intertemporal optimization and rational expectations may give rise to bounded equilibrium fluctuations driven by self-fulfilling volatile expectations without any intrinsic uncertainty being assumed. It is natural to ask how various principles for government stabilization policy, known by the economic agents, will affect the occurrence and properties of such endogenous fluctuations and to ask what are the most appropriate policies for stabilization. These are the kind of questions addressed in the present paper as well as in some related contributions to be referred to below.

A consideration that distinguishes the present paper is: Since stabilization policy is concerned with mitigating variations in economic activity *over time* it is a natural idea to assume that the government will let its stabilization effort depend on changes over time in economic activity. Therefore we consider policy rules which link the GNP share of government spending, and the implied taxation, to the rate of change in GNP (in contrast to the level of GNP).

We show, in a standard monetary model, that by applying a fiscal policy rule with this feature the government may eliminate endogenous fluctuations and stabilize the economy arbitrarily close to its laissez faire monetary steady state which is a Pareto optimum. The cyclical policy achieving this is that the GNP share of government spending should be relatively low in periods up to which GNP has increased by a relatively large amount. This may rationalize a tendency by policy makers to “punish” too rapid economic expansion.

We consider a simple monetary overlapping generations model with perfectly competitive markets where labor is the unique input in the production of a single

output, technology exhibits constant returns to scale (no externalities), and there are no fundamental, or intrinsic, uncertainties. There is a government that may pay lump sum transfers to the old consumers. (Alternatively real government spending can be interpreted as a demand for output if private and public consumption are assumed to be perfect substitutes). A policy rule is characterized by two parameters. One (called β) is the GNP share of government spending when the GNP is constant. The other ($-\alpha$) is the elasticity of the GNP share of government spending with respect to the growth factor of GNP from the preceding to the current period. Government spending is financed either by proportional income taxation or by seigniorage, the latter leading to inflationary taxation. The exact mix does not matter since direct proportional and inflationary taxation work in the same way in the model considered. Hence, fixing the GNP share of government spending is equivalent to fixing the tax rate and let spending be determined by budget balance, and consequently the policy rules studied can alternatively be interpreted as balanced-budget rules for the tax rate.

Among the model's rational expectations equilibria there is always a monetary steady state, but in case of indeterminacy of rational expectations equilibrium there will also be endogenous fluctuations, deterministic cycles and/or sunspot equilibria related to extrinsic uncertainty. According to such endogenous fluctuations, output fluctuates because of variations in labor supply driven by self-fulfilling volatile, deterministic or stochastic, expectations. Our main result can be stated shortly: For any $\beta > 0$, no matter how small, a sufficiently large α will imply global determinacy. In other words, by letting government spending depend sufficiently negatively on increases in GNP it is possible to stabilize the economy at a monetary steady state, while having to accept only an arbitrarily small distortion, implied by the rate of taxation β , of that steady state. Such stabilization has good welfare effects.

In our illustrative model the condition for indeterminacy under *laissez faire*, implying the existence of endogenous fluctuations, is the one well-known from, e.g. Grandmont (1985), that wealth effects dominate substitution effects in intertemporal choice to a degree so that labor supply has an elasticity with respect to the real wage less than minus one half. We have thus chosen to derive our results from a simplest possible model for which the condition for endogenous fluctuations under *laissez faire* may not be too realistic (we motivate this choice below). In Section 4, in connection with giving the intuition for our results, we explain that this condition is not important for our results. Basically two features, which are in particular features of our illustrative model, imply the output smoothing, or stabilizing, effect of a “growth punishing” policy: One is that labor supply is relatively sensitive to changes in lump sum transfers compared to changes in the net of tax real wage. The other is that current labor supply is substantially affected by future transfers, a feature linked to the monetary structure of the considered model. None of these features depend on the perverse slope of the labor supply curve.

The present paper is related to the (early) contributions of Grandmont (1986), Farmer (1986), Reichlin (1986), Goenka (1994), Sims (1994), and Woodford (1994), which also study the effects of various policies on endogenous fluctuations. The closest relationship is to Grandmont (1986): A particular case of the policy rules we consider, where government spending is always proportional to current GNP (corresponding to $\alpha = 0$), is equivalent to arranging spending such that, in the absence of direct income taxation, a constant money growth rate results. Grandmont studies such constant money growth rules. His main finding is that, under his assumptions on fundamentals, constant money growth rules will stabilize the economy at a monetary steady state if the money growth rate, or our β , is sufficiently large implying a correspondingly large distortion of the steady state. Our

results stand in some contrast since we emphasize stabilization for a small degree of distortion, which is a better stabilization in terms of welfare. We also show that constant money growth rules are only output stabilizing under some assumptions on fundamentals, under other they are destabilizing.

Our work is also related to the (recent) papers Schmitt-Grohe and Uribe (1997), Guo and Lansing (1997) and (1998), and Christiano and Harrison (1999).¹ These papers study *real* (non-monetary) models with capital accumulation. In the latter three papers productive externalities create the mechanism for endogenous fluctuations under *laissez faire* and the finding is that fiscal policy rules, according to which the tax rate, or the GNP share of government spending, depends *positively* on the level of GNP or employment, can create (sometimes local) determinacy. Schmitt-Grohe and Uribe (1997) find the related result that a countercyclical rule for the tax rate or spending share may itself create indeterminacy. Since the policy rules we find stabilizing condition the GNP share of government spending, or the tax rate, on the rate of *increase* in GNP, not on the *level* of GNP, their and our results are not directly comparable. Nevertheless, the fact that we find support for a negative dependence of the tax rate on the rate of growth in GNP, while they find support for a positive dependence of the tax rate on the level of GNP or employment suggests that the cyclicity of appropriate stabilization policies depends on the model considered and/or on the mechanism causing endogenous fluctuations.

The model we study is simple enough to give a *one*-dimensional, first order difference equation as the perfect foresight equilibrium dynamics. This makes it possible to establish enough global properties to be able to use *global* determinacy as criterion for stabilization. Global analysis is of importance for *excluding* the possibility

¹Other papers studying aspects of policy stabilization of endogenous fluctuations are Boldrin (1992), Evans and Honkapohja (1993), Chattopadhyay (1996) and (1999), Crés, Ghiglino, and Tvede (1997), Austin (1999), Guo and Harrison (2000).

of cycles and sunspot equilibria, since local determinacy is not a sufficient condition for the non-existence of endogenous fluctuations. With two-dimensional dynamic systems it is very difficult to provide more than local analysis. This motivates our choice of studying a simple model without capital.

In Section 2 we describe the basics of the economic model and the class of policy rules we consider. Section 3 derives the equilibrium dynamics, and Section 4 states the results on stabilization by the considered policy rules. Section 5 concludes. Proofs are given in Appendix A. Appendix B contains a technical result that is of importance for our purposes.

2 The Model

We consider an overlapping generations model with money as the only asset and no fundamental uncertainty. In each period the commodities are labor, output, and money. The money prices of labor and output in period t are $w_t > 0$ and $p_t > 0$ respectively, and labor and output markets are perfectly competitive.

2.1 Firms and Households

In each period a representative firm produces output $y_t \geq 0$ from labor input $l_t \geq 0$ under constant returns to scale, $y_t = l_t$. For $p_t < w_t$ the profit maximizing supply of output is zero, for $p_t > w_t$ it is “infinite”, and for $p_t = w_t$ any output is an optimal supply. As the demand for output is non-zero and finite competitive clearing of the output market implies $p_t = w_t$, and output y_t is given by demand.

The labor supply n_t in period t comes from an optimization by a representative young consumer, $\max_{n_t, c_{t+1}} E_t u(c_{t+1}) + v(1 - n_t)$ over the budget constraints $m_t = (1 - \tau_t)w_t n_t$ and $c_{t+1} = m_t/p_{t+1} + b_{t+1}$ (one for each value of p_{t+1} expected with positive probability), where c_{t+1} is (planned) consumption for the old age, m_t is

money holding at the end of period t , and τ_t and b_{t+1} respectively are the income tax rate in period t , and an expected lump sum transfer in real terms received during the old age in period $t + 1$. The demand by the old consumer in period t is accordingly $c_t = m_{t-1}/p_t + b_t = (1 - \tau_{t-1})w_{t-1}n_{t-1}/p_t + b_t$.

Following Woodford (1986), the description of the household sector is also compatible with a model of an infinitely lived consumer who, in each period t , maximizes the expectation of the utility $\sum_{s=t}^{\infty} \delta^{s-t} [u(c_s)/\delta + v(1 - n_s)]$, $\delta < 1$, subject (also) to cash-in-advance constraints stating that in any period t consumption cannot exceed the output value of money held at the beginning of period t plus the transfer received during period t , $c_t \leq m_{t-1}/p_t + b_t$.

If the cash-in-advance constraints are always binding ($u'(c_t) > (p_t/p_{t+1}) \delta u'(c_{t+1})$), then the consumer can do no better over the succession of any two periods t and $t+1$ than to perform exactly the former maximization described above. Since $\delta < 1$, for any dynamic trajectory that stays sufficiently close to a constant, or steady, state the cash-in-advance constraints will indeed be binding in all periods. Hence, in a neighborhood around a steady state, there is equivalence between the equilibria of the overlapping generations economy, as formulated above, and the equilibria of the corresponding cash-in-advance economy with infinitely lived consumers, and the “equivalence area” extends as the discount rate becomes smaller.

This equivalence is important for the interpretation of several features of the overlapping generations economy: One does not have to think of the implicit period length as literally a “working age”, but can think of a much shorter period, like a month. Further, the assumption that consumers only consume as old arises endogenously from the equivalence to the cash-in-advance economy. Finally, the assumption that transfers only go to the old is also motivated by the equivalence if, in the cash-in-advance interpretation of the model, it is assumed that a transfer can

be used in the same period it is received. We find this latter assumption natural and consider “transfers to the old” as the main case.²

We impose standard assumptions on u and v : Both are continuously differentiable several times, $u'(c)$ and $v'(e)$ are strictly positive and go to infinity as c and e respectively go to zero, and $u''(c)$ and $v''(e)$ are strictly negative. We denote the Arrow-Pratt measure of relative risk aversion in u by $R(c) := -u''(c)c/u'(c) > 0$, and also define $N(n) := -v''(1-n)n/v'(1-n) > 0$. We assume that $R(0) := \lim_{c \rightarrow 0} R(c)$, and $N(0)$ both exist (are $< \infty$).

The intertemporal budget constraint is,

$$c_{t+1} = (1 - \tau_t) \frac{w_t}{p_{t+1}} n_t + b_{t+1}, \quad (1)$$

so labor supply in period t comes from maximizing $E_t u \left((1 - \tau_t) \frac{w_t}{p_{t+1}} n_t + b_{t+1} \right) + v(1 - n_t)$ with respect to n_t . This has for any $(1 - \tau_t) \frac{w_t}{p_{t+1}} > 0$ and $b_{t+1} \geq 0$ a unique solution strictly between zero and one given by the first order condition,

$$v'(1 - n_t) = E_t u' \left((1 - \tau_t) \frac{w_t}{p_{t+1}} n_t + b_{t+1} \right) (1 - \tau_t) \frac{w_t}{p_{t+1}}. \quad (2)$$

Under our assumptions leisure and consumption are both normal goods.³

2.2 The Government and the Fiscal Policy Rules

In each period t , the government decides on the real lump sum transfer b_t and on the income tax rate τ_t , where $0 \leq \tau_t < 1$. Both b_t and τ_t are taken as parametric by

²However, the polar case where transfers are allocated entirely to the young is also of interest. In reality, transfers are assigned to agents of all ages. Further, for the alternative model interpretation, one could think of the cash-in-advance constraints as also involving transfers, so a received transfer can only be used in the next period, which would correspond to transfers going entirely to the young. Below we will therefore also report results for this alternative case. Since the equilibrium dynamic system will turn out to be two-dimensional, we will have to confine ourselves to local analysis.

³Labor supply $n(\omega, b)$ in case of a point expectation is given by $v'(1 - n) = \omega u'(\omega n + b)$, where $\omega = w_t(1 - \tau_t)/p_{t+1}$, and $b = b_{t+1}$. A larger b implies a lower right hand side, and to recreate equality n must fall since this both decreases the left hand, and increases the right hand, side, so $n'_b < 0$. A similar exercise on $v'(1 - (c - b)/\omega) = \omega u'(c)$ shows $c'_b > 0$.

the representative consumer. The real spending b_t can be interpreted alternatively as a government demand for output (or labor). It can easily be checked that if public and private consumption are perfect substitutes, so the utility maximized by the consumer over $(t, t + 1)$ according to the government demand interpretation of b_t is $v(1 - n_t) + u(c_{t+1} + b_{t+1})$, then the resulting dynamic model will be identical to the one where b_t is a real transfer going to the old. Note that this further justifies considering transfers to the old as the main case.

We assume that the government decides its level of spending in each period according to a fiscal policy rule of the form,

$$b_t = b(y_t, y_{t-1}) = \beta y_t^{1-\alpha} y_{t-1}^\alpha \quad \text{or} \quad \frac{b(y_t, y_{t-1})}{y_t} = \beta (y_t/y_{t-1})^{-\alpha}, \quad 0 \leq \beta < 1, \quad (3)$$

where the latter writing shows how government “intervention” can depend on the rate of change in GNP.

Government spending is financed either by proportional income taxation or by seigniorage, or by a mix of both, so that the government budget constraint $M_t - M_{t-1} = p_t b_t - \tau_t p_t y_t$ ($= p_t b_t - \tau_t w_t l_t$) is fulfilled, where M_t is the money stock at the end of period t . As shown below, the exact financing does not matter since in the model considered direct proportional taxation and inflationary taxation have the same effects. This means that a rule for *spending* $b_t = b(y_t, y_{t-1})$, which implies a period t rate of taxation $b(y_t, y_{t-1})/y_t$ - either as a direct tax in case of budget balance ($\tau_t y_t = b_t$) or as an inflationary tax in case of money printing - is equivalent to the rule $\tau_t(y_t, y_{t-1}) = b(y_t, y_{t-1})/y_t$ for the *tax rate*, if spending is determined by budget balance in each period.⁴

⁴This equivalence is important for comparison to related papers, e.g. Guo and Lansing (1998) and Christiano and Harrison (1999), which often proceed in terms of a rule for the tax rate combined with budget balance, and express their results in terms of the behavior of the tax rate over the cycle. Any cyclicity we arrive at for $b(y_t, y_{t-1})/y_t$ can alternatively be viewed as the cyclicity of τ_t in the equivalent balanced budget rule for the tax rate.

Our formulation implies that b_t is always (weakly) positive, so there is no lump sum taxation or proportional earnings subsidies. We think that this is a realistic feature, in particular in the context of stabilization policy. Further, the rules do not condition on exact knowledge of the economy's *laissez faire* steady state.

Each policy rule contains a level (or resting) component given by β , and a cyclical (or reactive) component given by α . From (3), β is the spending share in current output (or the tax rate) when output is constant so that β measures the “size of government”, while $(y_t/y_{t-1})^{-\alpha}$ is a cyclical component showing the responsiveness of the spending share in output (or of the tax rate) to changes in output, $-\alpha$ being the elasticity of the spending share with respect to the output growth factor. The larger α becomes, the more negative the reaction in the spending share to increases in output will be, that is, the more “countercyclical” will government spending be (we use quotation marks since the negative dependence is with respect to the increase in, not the level of, output).

The family of spending rules (3) includes as referential special cases *laissez faire*, $\beta = 0$, and a constant spending share, $\alpha = 0$, but not a constant absolute spending, $b(y_t, y_{t-1}) = \bar{b}$. This is an interesting bench mark case, and below we compare the performance of rules of form (3) to the performance of this particular rule.

In what follows it is assumed that the policy rule $b(y_t, y_{t-1})$ used by the government is known and believed in by the households who also have rational expectations with respect to next period's output price.

2.3 Equilibrium

The economy's basic equilibrium condition follows from competitive clearing of the output, labor, and money markets, that is, from $y_t = c_t$, $n_t = l_t$, and $M_t = m_t$ (by

Walras' Law one is superfluous). From the budget constraint (1) follows,

$$(1 - \tau_t) \frac{w_t}{p_{t+1}} = \frac{c_{t+1} - b_{t+1}}{n_t}.$$

Inserting this into the first order condition (2) gives $n_t v'(1 - n_t) = E_t(c_{t+1} - b_{t+1}) u'(c_{t+1})$. Using $y_{t+1} = c_{t+1}$, $n_t = l_t = y_t$, and $b_{t+1} = b(y_{t+1}, y_t)$ gives,

$$y_t v'(1 - y_t) = E_t[y_{t+1} - b(y_{t+1}, y_t)] u'(y_{t+1}). \quad (4)$$

This is the (temporary) equilibrium condition from which the economy's dynamic rational expectations equilibria are defined: A (deterministic or stochastic) intertemporal equilibrium with rational expectations is a sequence of positive random variables (y_t) satisfying (4).

The direct tax rate τ_t does not enter into (4), and hence rational expectations equilibrium does not depend on it. In the considered model it does not matter whether government spending is financed by proportional direct taxation or by seigniorage (inflationary taxation).⁵

3 Rational Expectations Dynamics

3.1 Steady State

A monetary steady state is a particular deterministic solution of (4), where $y_{t+1} = y_t = y > 0$. Since $b(y, y) = \beta y$ for the policy rules (3), such a y is given by,

$$\frac{v'(1 - y)}{u'(y)} = 1 - \beta. \quad (5)$$

Here the left hand side goes from zero to infinity as y goes from zero to one, so for any β , there is a unique monetary steady state $y(\beta)$, and $y(\beta) < 1$. In particular

⁵It follows, of course, from general market clearing etc. that the government's budget constraint is fulfilled (although τ_t does not enter directly into the equilibrium condition): Equilibrium in the money market implies $M_t - M_{t-1} = m_t - m_{t-1}$, where from the consumer's budget constraints $m_t = (1 - \tau_t)w_t n_t$ and $m_{t-1} = (c_t - b_t)p_t$. Inserting also $w_t = p_t$, $n_t = l_t = y_t$ and $c_t = y_t$ gives $M_t - M_{t-1} = p_t b_t - \tau_t p_t y_t$.

$y(0)$ is the monetary steady state under laissez faire. It follows directly that $y(\beta)$ is strictly decreasing in β , and that $y(\beta)$ goes to zero as β goes to one.

Welfare at the steady state should be defined as $W(\beta) := u(y(\beta))/\delta + v(1 - y(\beta))$, where $\delta \leq 1$ is a discount rate. Then $W'(\beta) = (u'/\delta - v')y'(\beta)$, and from $y'(\beta) < 0$ and (5), $W' < 0$ for all $\beta > 0$. This proves,

Proposition 1 *For all β , there is a unique monetary steady state involving production $y(\beta)$, with $0 < y(\beta) < 1$, and $y(\beta)$ is strictly decreasing in β and $y(\beta) \rightarrow 0$ as $\beta \rightarrow 1$. Welfare at the monetary steady state $W(\beta)$ is unambiguously decreasing in β , and optimal policy for steady state is $\beta = 0$.⁶*

Proposition 1 implies that government activity has to be motivated by the monetary steady state *not* being the appropriate descriptive equilibrium under laissez faire. Furthermore, should endogenous fluctuations prevail (under laissez faire) and should one, by use of a policy rule belonging to the class (3), manage to stabilize the economy at the monetary steady state, then it is unambiguously to be preferred that this is done at the lowest possible value of β .

For the particular bench mark rule, $b(y_t, y_{t-1}) = \bar{b}$, the steady state condition is $v'(1 - y)/u'(y) = 1 - \bar{b}/y$. It is easy to see that for a sufficiently large value of \bar{b} there is no steady state, while for all sufficiently low (and plausible) values of \bar{b} there are several steady states involving outputs larger than \bar{b} .

⁶If $\delta < 1$ and one allows $\beta < 0$ it would follow from $W'(0) < 0$ that optimal policy for steady state is some $\beta < 0$, which, in the absence of direct taxation, is equivalent to a constant negative money growth rate, a so-called Friedman rule. However, $\beta < 0$ means lump sum taxation combined with income subsidies coming either directly or through negative inflation. As said, we think it is realistic for the issue of stabilization policy not to consider lump sum taxation or earnings subsidies.

3.2 Perfect Foresight Dynamics

The economy's perfect foresight dynamics is obtained from (4) assuming that the next period's output is correctly foreseen from the current period in a deterministic sense. This gives a first order, one-dimensional difference equation in y_t and y_{t+1} ,

$$y_t v'(1 - y_t) = [y_{t+1} - b(y_{t+1}, y_t)] u'(y_{t+1}). \quad (6)$$

A dynamic perfect foresight equilibrium is a sequence (y_t) of production levels $0 \leq y_t < 1$, such that (6) is fulfilled for all t .

If, in (3), $\alpha \geq 0$, or $\beta = 0$, then $b(y_{t+1}, y_t)$ is (weakly) increasing in y_t , so the right hand side of (6) will, for any given $y_{t+1} > 0$, decrease weakly from a strictly positive value as y_t increases from zero. Since the left hand side increases from zero to infinity as y_t goes from zero to one, there is for every positive y_{t+1} a unique y_t between zero and one that solves (6), and (6) thus everywhere implicitly defines y_t as a function of y_{t+1} : $y_t = f(y_{t+1})$.

From the Implicit Function Theorem, f is continuously differentiable. So, for $\alpha \geq 0$, or $\beta = 0$, thus including laissez faire, the (backward) perfect foresight dynamic f is well-defined globally. For $\alpha < 0$ and $\beta > 0$ it is not. In that case there are for y_{t+1} small enough several solutions in y_t to (6), and for y_{t+1} large enough there are none. As just shown there is, however, a unique monetary steady state $y(\beta)$, and locally around $y(\beta)$ the perfect foresight dynamic f is well-defined and continuously differentiable.⁷

For the particular rule $b(y_t, y_{t-1}) = \bar{b}$, perfect foresight dynamics is given by $y_t v'(1 - y_t) = [y_{t+1} - \bar{b}] u'(y_{t+1})$, and for any $y_{t+1} \geq \bar{b}$, there is a unique solution

⁷From the Implicit Function Theorem, f is locally well-defined by (6) around steady state if the derivative of $y_t v'(1 - y_t) - [y_{t+1} - b(y_{t+1}, y_t)] u'(y_{t+1})$ wrt. y_t measured at steady state is not zero. This derivative is $v'(1 - y(\beta))(1 + N(y(\beta)) + \alpha\beta u'(y(\beta)))$, which, for any given β , is zero only for one particular (non-generic) negative value of α .

y_t , implying again a well-defined and differentiable function $y_t = f(y_{t+1})$. Since we know from above that (for \bar{b} not too large) there are at least two steady states with output larger than \bar{b} , it can be inferred that there is at least one steady state $y > \bar{b}$, where the slope of the perfect foresight dynamic fulfills $f'(y) > 1$.

3.3 Endogenous Fluctuations and Stabilization Criterion

Deterministic cycles and stationary (Markov) sunspot equilibria are our candidates for rational expectations dynamic equilibria exhibiting endogenous fluctuations. A deterministic r -cycle is a collection of r different production levels $0 < y_1, \dots, y_r < 1$ in the range where f is well-defined such that $y_1 = f(y_2), \dots, y_r = f(y_1)$. An r -state stationary (Markov) sunspot equilibrium, SSE, consists of r production levels $0 < y_1 \leq \dots \leq y_r < 1$, where $y_1 < y_r$, and r^2 transition probabilities q_{ij} , $\sum_{j=1}^r q_{ij} = 1$ for $i = 1, \dots, r$, where the matrix (q_{ij}) is irreducible, such that, whenever the consumer expects that the output level y_j will occur with probability q_{ij} next period, $j = 1, \dots, r$, then the current temporary equilibrium output level according to (4) is exactly y_i , that is,

$$y_i v'(1 - y_i) = \sum_{j=1}^r q_{ij} [y_j - b(y_j, y_i)] u'(y_j) \text{ for } i = 1, \dots, r. \quad (7)$$

The well-known idea is that one can imagine that an irreducible Markov chain (a sunspot) on states $1, \dots, r$, sending state i into state j with transition probability q_{ij} , though exogenous to the economic system, may govern its performance. If the agents know the transition probabilities and believe that in any period output must be y_i if the state is i , then output will indeed be governed by the sunspot and fluctuate accordingly, and the agents will have no reason to revise their beliefs since their expectations are probabilistically correct, i.e. rational. An r -cycle is a particular, non-stochastic r -state SSE.

It is well-known that if f is such that an r -cycle exists then there is also a truly stochastic r -state SSE close to the cycle, see Guesnerie and Woodford (1992). It is not generally true that the existence of a SSE implies the existence of deterministic cycles, or, equivalently, that non-existence of cycles implies non-existence of SSE. Since we are concerned with stabilization (establishing *non-existence* of endogenous fluctuations) it is important for our purposes to establish such a connection. In Appendix B we prove a proposition stating some general conditions under which the existence of a SSE implies the existence of a 2-period cycle. The proposition implies that a policy rule in the class (3), which eliminates all cycles by ensuring global stability according to f of the monetary steady, also eliminates all SSE.⁸ By virtue of these and some other well-known results it will suffice in what follows to study the perfect foresight dynamic f :

Indeterminacy. If f is locally well-defined around a monetary steady state and the slope of f at the steady state is smaller than minus one or larger than one, then the steady state is locally stable in the *forward* direction under perfect foresight, and one says there is indeterminacy. It is well known that indeterminacy implies the existence of SSE arbitrarily close to the steady state, see Guesnerie and Woodford (1992), and possibly also deterministic cycles (and SSE close to the cycles). In case of indeterminacy a cycle or a sunspot equilibrium is considered the relevant dynamic equilibrium (if it were nevertheless the steady state there would be no stabilization problem). It follows from above (last paragraph of Subsection 3.2), that the particular policy rule of constant government spending, $b(y_t, y_{t-1}) = \bar{b}$, will inevitably imply indeterminacy and the existence of SSE for all sufficiently small \bar{b} .

⁸The method used in Appendix B to establish conditions under which the existence of a SSE implies the existence of a 2-period cycle is similar to the one used by Grandmont (1986). However, the dynamics arising from our policy rules are not covered by the generality of Grandmont's result. Therefore the theorem in Appendix B generalizes Grandmont's result and it may be of independent interest.

Determinacy. Assume that by appropriate use of a policy rule in the considered class (3), it can be obtained that the steady state $y(\beta)$ becomes *globally* stable according to f (requiring that f is globally well-defined). Then there can be no deterministic cycles and, from Proposition B shown in Appendix B, there can be no SSE either. The steady state is then the only reasonable bounded and continuously well-defined rational expectations equilibrium, and one says that there is (global) determinacy. Determinacy will be considered a sufficient condition for stabilization.

4 Stabilization and Destabilization

Below we first state results on *stabilization*, that is, on how appropriate choices of α and β can create global determinacy of the steady state $y(\beta)$. For this type of results we assume that if the economy is left to itself it has a stabilization problem: Under *laissez faire*, $\beta = 0$, there is indeterminacy, $f'(y(0)) < -1$ (it follows from (10) below that > 1 is not possible), and both deterministic cycles and SSE exist. Then we state results on *destabilization*, that is, on how (in)appropriate choices of α and β can create (local) indeterminacy. For this second type of result one should have in mind that under *laissez faire* there is global determinacy, $y(0)$ is globally stable according to f , and cycles and SSE do not exist. In a final subsection results for the “transfers to the young” case are derived and compared to the main case.

Inserting the considered specific functional form of policy rules (3) into (6) gives,

$$y_t v'(1 - y_t) = (y_{t+1} - \beta y_{t+1}^{1-\alpha} y_t^\alpha) u'(y_{t+1}), \quad (8)$$

which defines $y_t = f(y_{t+1})$ at least locally around $y(\beta)$. For any $x > 0$, at which $f(x)$ is well-defined, the slope of f is obtained by implicit differentiation of (8)

written as $f(x)v'(1 - f(x)) = [x - \beta x^{1-\alpha} f(x)^\alpha] u'(x)$. This gives,

$$f'(x) = \frac{f(x)}{x} \frac{1 - \beta(1 - \alpha)\left(\frac{f(x)}{x}\right)^\alpha - \left(1 - \beta\left(\frac{f(x)}{x}\right)^\alpha\right) R(x)}{1 - \beta(1 - \alpha)\left(\frac{f(x)}{x}\right)^\alpha + \left(1 - \beta\left(\frac{f(x)}{x}\right)^\alpha\right) N(f(x))}. \quad (9)$$

Measuring f' at the monetary steady state where $x = f(x) = y(\beta)$ gives,

$$f'(y(\beta)) = \frac{1 - \beta(1 - \alpha) - (1 - \beta)R(y(\beta))}{1 - \beta(1 - \alpha) + (1 - \beta)N(y(\beta))}. \quad (10)$$

When $\alpha > 0$ one has that $b(y_{t+1}, y_t)$ is increasing in y_t so f is globally well-defined (as explained in Section 3.2), and for any $y_{t+1} > 0$, the y_t that solves (8) is below $y_{t+1}/\beta^{1/\alpha}$. Hence, as y_{t+1} goes to zero, so must this y_t , implying the property $f(0) := \lim_{x \rightarrow 0} f(x) = 0$ whenever $\alpha > 0$. This property hinges on the fact that the policy rules we consider may condition b_{t+1} on past GNP y_t , not only on current GNP y_{t+1} . Further, the property is important for our stabilization results: Whenever $\alpha > 0$, we know about f that 1) it is globally well-defined, 2) it starts at zero, $f(0) = 0$, and 3) it stays everywhere below one, $f(x) < 1$. It may have a number of “critical points” $(x^c, f(x^c))$ where $f'(x^c) = 0$. In any case, the properties 1) - 3) imply that if all critical points are below the 45°-line (including the case where there are no critical points), i.e. fulfill $f(x^c)/x^c < 1$, then $y(\beta)$ is globally stable according to f . This is used to establish our global stabilization results in Proposition 2. The fact that we consider policy rules that may condition real government spending on past as well as on current GNP (or condition the GNP share of spending on the growth rate) is thus of importance for our results.

4.1 Stabilization

Our main result is Proposition 2 below, which says that for any given positive value of β (no matter how small), one can create determinacy by choosing α sufficiently large. In other words, stabilization at the monetary steady state $y(\beta)$ can be obtained with arbitrarily little distortion by using a sufficiently “countercyclical” rule

for government spending. However, strong “countercyclicality” will be needed for little distortion. For completeness, the proposition also reports on stabilization by appropriate choice of β , given α .

Proposition 2 (*Creating determinacy*). Assume $f'(y(0)) < -1$.

(i) *Stabilization by appropriate choice of the cyclical component α :*

For any $\beta > 0$, there is an $\alpha^(\beta) > 0$, such that if a policy rule with $\alpha > \alpha^*(\beta)$ and β is used, then the steady state $y(\beta)$ is determinate and there are no cycles or stationary sunspot equilibria. Further, for all sufficiently small $\beta > 0$, it is necessary for determinacy that α is greater than or equal to a certain $\alpha^{**}(\beta) > 0$, and this $\alpha^{**}(\beta)$ goes to infinity as β goes to zero.*

(ii) *Stabilization by appropriate choice of the level component β :*

For any $\alpha > 0$, there is a $\beta^(\alpha)$ with $0 < \beta^*(\alpha) < 1$, such that if a policy rule with α and $\beta > \beta^*(\alpha)$ is used, then the steady state $y(\beta)$ is determinate and there are no cycles or stationary sunspot equilibria. If $R(0) < 1$ then also for $\alpha = 0$ there exists a $\beta^* < 1$, such that $\beta > \beta^*$ implies determinacy of the steady state.⁹*

Proposition 2, (i) is about stabilization of *output* by appropriate choice of α for a given fixed level of distortion β . This is of interest for *welfare*. Among monetary steady states, those with the lowest β are best welfarewise (Proposition 1), and indeed, the economy can be stabilized arbitrarily close to the laissez faire monetary steady state $y(0)$.¹⁰ On the other hand, Proposition 2 also says that little distortion requires much “countercyclicality”. The result thus points to a trade off between

⁹The result for $R(0) < 1$ and $\alpha = 0$ is equivalent to a main result of Grandmont (1986), since a constant spending share rule, $\alpha = 0$, can be shown to be equivalent to the kind of constant money growth rate rules studied by Grandmont, and Grandmont makes assumptions with the same effect as $R(0) < 1$.

¹⁰The stabilized situation does not necessarily Pareto dominate the fluctuating one. This cannot be achieved generally because some generations can benefit much from fluctuations (work little as young and consume much as old). However, in the stabilized situation the common utility of all generations is almost as high as it could be given that all generations should be equal.

“countercyclicality” and distortion.

A growth punishing or “countercyclical” spending rule is stabilizing because it tends to smooth output over time. Geometrically, by bringing the current equilibrium y_t close to the future y_{t+1} , the “transition curve” f is pulled towards the 45°-line (on the interval $[0, 1]$), which makes the monetary steady state stable according to f . Intuitively, the output smoothing effect is due to intertemporal incentive effects. Assume that GNP fluctuates, say according to a two-period cycle. The effects of a “countercyclical” policy rule can be partitioned into two effects, a “transfer effect”, and a “real wage effect”.

The transfer effect: In a period where output and labor supply are relatively low people will, since they anticipate an increase in output and know and believe in a “countercyclical” spending rule, expect relatively low transfers (or a low level of public consumption to substitute for private consumption) during the next period. Similarly, in a period where output and labor supply are relatively high, future transfers (or public consumption) will be expected to be high. Since leisure is a normal good this tends to mitigate the difference in labor supply between periods of low output and periods of high output and hence to smooth output over time.

The real wage effect: In a period of a relatively low GNP the transfer will be relatively high and hence the taxation (direct or inflationary) of the period’s income will be relatively high creating a relatively low intertemporal real wage (ceteris paribus). The effect on labor supply in the period of the relatively low real wage depends on a substitution effect that pulls in the direction of low labor supply and on a wealth effect that pulls in the direction of high labor supply. The sign of the real wage effect is therefore generally ambiguous. In the illustrative model we consider, the sufficient condition for fluctuations under laissez faire, $f'(y(0)) < -1$, implies that the wealth effect dominates the substitution effect (around the steady

state), so also the real wage effect tends to increase output in periods of low output, and vice versa, and hence to smooth output over time.

Therefore, in our illustrative model, both of the effects coming from a “countercyclical” spending rule may be stabilizing. However, for the output smoothing effect of a “countercyclical” policy rule it is not necessary that both of the above subeffects go in the “right” direction. It would suffice that the transfer effect dominates the real wage effect, should these go in opposite directions. Empirical investigations suggest that this is realistic. According to Pencavel (1986), estimates of the uncompensated real wage elasticity of labor supply are most often close to zero.

The general lesson from the above explanations seems to be that two features, which are in particular features of the model we have considered, tend to make “countercyclical” spending rules efficient in creating determinacy. 1) Labor supply is influenced relatively strongly by a received “exogenous income”, a lump sum transfer or a public consumption that substitutes for private consumption, compared to how it is influenced by the net of tax real wage. 2) Current labor supply is influenced (relatively strongly) by a future, compared to a current, “exogenous income”. The latter feature is linked to the monetary structure of our model: Out of the labor income earned in one period, people put money aside to have money holdings in the next period, and therefore the “exogenous income” received in the second of the periods substitutes for the labor income, and therefore substantially affects the labor supply, in the first of the periods. One reason why we arrive at a different cyclicity for most stabilizing policies than the related contributions mentioned in the Introduction is that we consider a monetary model (with the implication that current labor supply is affected by future government spending), while they consider real models.

It is an implication of Proposition 2, that a constant government spending \bar{b}

can be implemented without inducing the aggregate instability implied by the rule $b(y_t, y_{t-1}) = \bar{b}$, if a spending rule of the form $b_t = \beta y_t^{1-\alpha} y_{t-1}^\alpha$ is followed with β satisfying $\beta y(\beta) = \bar{b}$, and α chosen sufficiently large.

4.2 Destabilization

Proposition 2 leaves some open questions. It does not exclude that also for $\alpha < 0$ (or for $\alpha = 0$ and $R(0) \geq 1$), a sufficiently high β could imply determinacy, or that also large absolute values of a negative α could be stabilizing for certain values of β . The next Proposition rules out these possibilities. Note that in Proposition 3 we do not assume that $f'(y(0)) < -1$.

Proposition 3 (*Creating indeterminacy*).

(i) *If $\beta > 0$, then $f'(y(\beta)) > 1$ for all α , which are negative and sufficiently large in absolute value, and hence the steady state $y(\beta)$ is indeterminate and stationary sunspot equilibria exist.*

(ii) *If $\alpha < 0$, then $f'(y(\beta)) > 1$ for all sufficiently large β , and hence the steady state $y(\beta)$ is indeterminate and stationary sunspot equilibria exist.*

(iii) *If $\alpha = 0$ and $R(0) > 2 + N(0)$, then $f'(y(\beta)) < -1$ for all sufficiently large β , and hence the steady state $y(\beta)$ is indeterminate and both deterministic cycles and stationary sunspot equilibria exist.*

4.3 Transfers to the young

The intuitive explanation in Subsection 4.1 of why a “countercyclical” spending rule is (most) stabilizing emphasizes that current labor supply is influenced by future government spending. This suggests that the alternative case, where transfers are allocated to the young, would give different results with respect to (most) stabilizing policies. This is indeed true.

The alternative assumption gives rise to the following two-dimensional, perfect foresight dynamic system in output y_t and tax rate τ_t :¹¹

$$y_{t+1}u'(y_{t+1}) = \frac{1}{1-\tau_t}y_tv'(1-y_t),$$

$$\tau_{t+1} = \beta \left(\frac{y_{t+1}}{y_t} \right)^{-\alpha}.$$

This system has no predetermined variable (both y_t and τ_t will “jump” in reaction to a shift in the expectation y_{t+1}). A steady state is the same as before: output $y(\beta)$ is given by (5), the spending share is $\tau(\beta) = \beta$, and Proposition 1 holds. Linearizing the system around the steady state $(y(\beta), \beta)$, and isolating dy_{t+1} and $d\tau_{t+1}$ (the discrepancies from $y(\beta)$ and β) on the left hand side gives the system:

$$\begin{pmatrix} dy_{t+1} \\ d\tau_{t+1} \end{pmatrix} = \frac{1}{1-R(y(\beta))} \begin{bmatrix} 1+N(y(\beta)) & \frac{y(\beta)}{1-\beta} \\ -\frac{\alpha\beta}{y(\beta)}[R(y(\beta))+N(y(\beta))] & -\frac{\alpha\beta}{1-\beta} \end{bmatrix} \begin{pmatrix} dy_t \\ d\tau_t \end{pmatrix}.$$

We proceed by a standard analysis based on the linearized system.¹² The trace and determinant of the Jacobian are:

$$T = \frac{1}{1-R(y(\beta))} \left(1+N(y(\beta)) - \frac{\alpha\beta}{1-\beta} \right),$$

$$D = -\frac{1}{1-R(y(\beta))} \frac{\alpha\beta}{1-\beta},$$

and hence, for the characteristic polynomial $Q(\lambda) \equiv \lambda^2 - T\lambda + D$, one has:

$$Q(1) = -\frac{R(y(\beta))+N(y(\beta))}{1-R(y(\beta))},$$

¹¹With transfers going to the young, the intertemporal budget constraint of a consumer is $c_{t+1} = (1-\tau_t)(w_t/p_{t+1})n_t + (p_t/p_{t+1})b_t$, and the first order condition for the consumer's optimization is $v'(1-n_t) = E_t u'(c_{t+1})(1-\tau_t)(w_t/p_{t+1})$. Inserting the expression for $(1-\tau_t)(w_t/p_{t+1})$ isolated from the first, into the second, and using $n_t = y_t = c_t$ etc. gives: $y_tv'(1-y_t) = E_t[y_{t+1} - (p_t/p_{t+1})b_t(y_t, y_{t-1})]u'(y_{t+1})$. Under a balanced budget the stock of money is constant, so $y_t = M/p_t$ for all t , and then $p_t/p_{t+1} = y_{t+1}/y_t$. Inserting this, writing τ_t for $b_t(y_t, y_{t-1})/y_t$, and using the policy rule $\tau_t = b_t(y_t, y_{t-1})/y_t = \beta(y_t/y_{t-1})^{-\alpha}$ gives the stated system.

¹²For the mathematical results used see, e.g., Azariadis (1993) or Grandmont, Pintus, and de Vilder (1998).

$$Q(-1) = -\frac{1}{1 - R(y(\beta))} \left(R(y(\beta)) - 2 - N(y(\beta)) + \frac{2\alpha\beta}{1 - \beta} \right).$$

Assume, as in Proposition 2, that $f'(y(0)) < -1$. From (10) this implies $R(y(0)) > 2 + N(y(0))$, and then, for all sufficiently small β , $R(y(\beta)) - 2 - N(y(\beta)) > 0$, also implying $1 - R(y(\beta)) < 0$. It follows that for all sufficiently small β and positive α , one has that $Q(1)$, $Q(-1)$, and D are all strictly positive. Further, increasing α will eventually make D larger than one. Hence the steady state becomes a source (the Jacobian has two eigen values with moduli larger than one), so for any small β , a positive and sufficiently large α will imply local determinacy, a result seemingly similar to that in Proposition 2, (i).¹³

However, local determinacy does not suffice for the non-existence of endogenous fluctuations. In the two-dimensional case a deterministic cycle may arise by a so-called Hopf bifurcation. Still under the above assumptions, implying that $Q(1)$, $Q(-1)$, and D are strictly positive for α strictly positive, when α is exactly equal to $\hat{\alpha} \equiv -(1 - \beta)(1 - R(y(\beta)))/\beta > 0$, the determinant D is one (the Jacobian has two complex eigen values both with modulus one), and either for α (a little) smaller than or larger than $\hat{\alpha}$, quasi periodic or periodic deterministic orbits exist. Hence, with transfers allocated to the young it is no longer true that a sufficiently “countercyclical” rule for government spending will stabilize at steady state for a small degree of distortion of the steady state.

The results of this subsection highlights the importance of the global nature of the analysis for the main case (“transfers to the old”), and of the “timing” (whether current labor supply is most influenced by current or future government spending) for the appropriateness of a specific cyclicity in stabilization policy.

¹³Nothing similar can be obtained by a negative α . Indeed, under the same assumptions, a negative α will imply indeterminacy (real eigen values and at least one between zero and one, which suffices for indeterminacy since there is no predetermined variable).

5 Concluding Remarks

We have considered a monetary overlapping generations economy and fiscal policy rules where the share of government spending depends on the rate of change in GNP. We found that a fiscal policy rule according to which government spending is sufficiently “countercyclical” is able to stabilize economic activity with respect to endogenous fluctuations for an arbitrarily small degree of distortion. Further, a sufficiently “procyclical” spending rule will create indeterminacy.

The equilibrium dynamics of the considered model was summarized by a one-dimensional dynamic system. This allowed us to establish global results with respect to stabilization.

In our illustrative model the mechanism for the emergence of rational expectations endogenous fluctuations under *laissez faire* is the traditional one of strong wealth effects in intertemporal choice which ensure a sufficiently negatively sloped (individual) labor supply curve. However, this feature is not responsible for the stabilizing effect of “countercyclical” policy rules. The tendency of a “growth punishing” policy to smooth output over time is basically due to labor supply being more sensitive to changes in lump sum transfers (or public consumption) than to changes in the real wage rate and to the monetary structure of the considered model.

Our results are in contrast with results in several of the most recent papers on stabilization with respect to endogenous fluctuations where procyclicality (in the traditional sense) of government spending and implied taxation is usually desirable to achieve determinacy, or where countercyclicality may lead to indeterminacy. These other papers study real (business cycle) models and often rely on the existence of externalities in production, or similar mechanisms, as the source of indeterminacy under *laissez faire*.

The cyclicity of policy rules most appropriate for stabilization of endogenous fluctuations thus seems to depend on the specific model studied and, in particular, on the exact mechanism responsible for the emergence of endogenous fluctuations. On these grounds recommendations pointing towards *procyclical* (or “*countercyclical*”) policy rules for government spending and taxation should be taken with caution. A traditional (Keynesian) tendency of governments to react negatively to rapid economic expansion may be well motivated, even in the absence of price rigidities.

A Proofs

Proof of Proposition 2. First note that in Proposition 2, we consider only $\alpha \geq 0$. It therefore follows from Proposition B of Appendix B, that global stability of $y(\beta)$ according to f (global determinacy), which is established in this proof and which obviously eliminates all cycles, also eliminates all SSE.

(i) We are going to show that one can use $\alpha^*(\beta) = \frac{1-\beta}{\beta} \max_{x \in [0,1]} (R(x) - 1)$. Here $\alpha^*(\beta) > 0$, because it follows from $f'(y(0)) < -1$ and (10), that $R(y(0)) > 1$, and $0 < y(0) < 1$.

From (9), a critical point is given by $1 - \beta(1 - \alpha)(\frac{f(x)}{x})^\alpha - (1 - \beta(\frac{f(x)}{x})^\alpha) R(x) = 0$. This implies that at a critical point one must have $R(x) > 1$, whenever $\alpha > 0$, and,

$$\left(\frac{f(x)}{x}\right)^\alpha = \frac{1}{\beta} \frac{R(x) - 1}{R(x) - 1 + \alpha}. \quad (11)$$

A critical point $(x^c, f(x^c))$ is below the 45°-degree line if $f(x^c)/x^c < 1$, which has to be fulfilled if $x^c \geq 1$, since $f(x) < 1$ for all x . The denominator above is strictly positive at a critical point when $\alpha > 0$, so for $\alpha > 0$, $f(x^c)/x^c < 1$ is equivalent to,

$$\alpha > \frac{1 - \beta}{\beta} (R(x^c) - 1). \quad (12)$$

Now, if $\alpha > \alpha^*(\beta)$, then in particular (12) is fulfilled for any critical point $x^c < 1$, implying that $f(x^c)/x^c < 1$. This proves the first statement of Proposition 2, (i).

For the second statement we use that it is necessary for determinacy that $-1 \leq f'(y(\beta)) \leq 1$. From (10) one sees that if the denominator of $f'(y(\beta))$ is negative (which it can be for $\alpha < 0$), then $f'(y(\beta)) > 1$. So, to exclude $f'(y(\beta)) > 1$, one must set α such that the denominator is positive (for which $\alpha \geq 0$ suffices). On the other hand, for such an α , the necessary condition for avoiding indeterminacy, $f'(y(\beta)) \geq -1$, is equivalent to,

$$\alpha \geq \alpha^{**}(\beta) := \frac{1}{2} \frac{1 - \beta}{\beta} (R(y(\beta)) - N(y(\beta)) - 2).$$

From (10), $f'(y(0)) < -1$ implies $R(y(0)) - N(y(0)) - 2 > 0$, which means that for all small enough β , the parenthesis in the expression for $\alpha^{**}(\beta)$ is positive, so an α fulfilling the inequality also fulfils $\alpha \geq 0$. Finally, as β goes to zero, the required $\alpha^{**}(\beta)$ goes to infinity because the parenthesis goes to $R(y(0)) - N(y(0)) - 2 > 0$, and $(1 - \beta)/\beta$ goes to infinity.

(ii) The global stability of $y(\beta)$ is again established by sending all critical points below the 45°-line. For this to indeed imply global stability of $y(\beta)$, it is important that f is globally well-defined (which it is since $\alpha \geq 0$), and that $f(0) = 0$. For $\alpha > 0$ this follows as above. For $\alpha = 0$, (8) reads $y_t v'(1 - y_t) = (1 - \beta)y_{t+1} u'(y_{t+1})$. As y_{t+1} goes to zero, so will the right hand side if and only if $R(0) < 1$.¹⁴ Hence, if $R(0) < 1$, one still has $f(0) = 0$, whereas if $R(0) > 1$ one has $f(0) = 1$. The assumption $R(0) < 1$ thus implies $f(0) = 0$ also when $\alpha = 0$.

We will show that for $\alpha > 0$, one can use $\beta^*(\alpha) = \max_{x \in [0,1]} \frac{R(x)-1}{R(x)-1+\alpha}$, where one must have $0 < \beta^*(\alpha) < 1$, since, as above, $R(y(0)) > 1$, and $\alpha > 0$. From (11), since at any critical point $R(x^c) > 1$, and since $\alpha > 0$, $f(x^c)/x^c < 1$ is equivalent to,

$$\beta > \frac{R(x^c) - 1}{R(x^c) - 1 + \alpha}, \quad (13)$$

which is fulfilled for all critical points with $x^c \leq 1$ when $\beta > \beta^*(\alpha)$. When $x^c > 1$, one has $f(x^c)/x^c < 1$ from $f < 1$. This proves the first statement in (ii).

For the second statement simply note that for $\alpha = 0$, the perfect foresight dynamic (8) becomes $y_t v'(1 - y_t) = (1 - \beta)y_{t+1} u'(y_{t+1})$, so for β going to one the y_t that solves it must go to zero for any value of y_{t+1} . This means that $f(x)$ is pulled down arbitrarily close to the x -axis. Further, from (9), when $\alpha = 0$, a critical point is given by $R(x) = 1$ independently of β . So, as β is increased all critical

¹⁴Note that $R(0)$ is the elasticity measure of how fast $u'(y_{t+1})$ goes to infinity as y_{t+1} goes to zero. Hence, if $R(0) < 1$, the product $y_{t+1} u'(y_{t+1})$ goes to zero as y_{t+1} goes to zero etc.

points $(x^c, f(x^c))$ move downwards along the same value of x^c with $f(x^c)$ getting arbitrarily close to the x -axis, so eventually all critical points go below the 45° -line.

■

Proof of Proposition 3. (i) For $\beta > 0$, when α becomes negative and sufficiently large numerically, both the numerator and the denominator in (10) become negative with the numerator numerically the largest, so $f'(y(\beta)) > 1$.

(ii) When $\alpha < 0$, one sees from (10), that as β goes to one, $f'(y(\beta))$ goes to $\alpha/\alpha = 1$. B numerator and denominator become negative for a large enough β , but the numerator is numerically the largest, so $f'(y(\beta))$ goes to one from above. Hence, for all sufficiently large β , one has $f'(y(\beta)) > 1$, meaning that the steady state is indeterminate and an SSE exists.

(iii) Again from (10), if $\alpha = 0$, the slope of f at steady state is $f'(y(\beta)) = \frac{1-R(y(\beta))}{1+N(y(\beta))}$. As β goes to one, $y(\beta)$ goes to zero (Proposition 1), and hence $f'(y(\beta))$ goes to $\frac{1-R(0)}{1+N(0)}$, which is less than -1 exactly because $R(0) > 2+N(0)$. If $\lim_{\beta \rightarrow 1} f'(y(\beta)) < -1$, then from continuity also $f'(y(\beta)) < -1$ for all large enough β . Hence $y(\beta)$ is indeterminate, which suffices for the existence of SSE close to it. When f is globally well-defined and known to stay below a “ceiling”, $f(x) < 1$ for all x , then $f'(y(\beta)) < -1$ also suffices for the existence of deterministic cycles. ■

B Conditions for the Existence of SSE to Imply the Existence of Deterministic Cycles

Inserting the specific form (3) of policy rules into the equations (7) that a SSE must fulfill gives the equations,

$$y_i v'(1 - y_i) = \sum_{j=1}^r q_{ij} \left[y_j - \beta y_j^{1-\alpha} y_i^\alpha \right] u'(y_j) \text{ for } i = 1, \dots, r.$$

The left hand side can be called $v_1(y_i)$, and if one on the right hand side uses $v_2(y_i, y_j) := [y_j - \beta y_j^{1-\alpha} y_i^\alpha] u'(y_j)$, the equations become,

$$v_1(y_i) = \sum_{j=1}^r q_{ij} v_2(y_i, y_j) \text{ for } i = 1, \dots, r. \quad (14)$$

The backward perfect foresight dynamic $f(x)$ is then given implicitly (as the solution in z) by $v_1(z) = v_2(z, x)$. Under the assumptions made in this paper, $v_1(y_i)$ is strictly increasing, and when $\alpha \geq 0$, $v_2(y_i, y_j)$ is either independent of y_i (for $\alpha = 0$), or strictly decreasing in y_i (for $\alpha > 0$). Furthermore, still for $\alpha \geq 0$, the perfect foresight dynamic f is globally well-defined and continuous (and differentiable), stays below one, and with exactly one monetary steady state. This motivates,

Assumption 1. $v_1(y_i)$ is strictly increasing in y_i , and $v_2(y_i, y_j)$ is (weakly) decreasing in y_i .

Assumption 2. For every $x > 0$, there is a unique solution in z to $v_1(z) = v_2(z, x)$, and the backward perfect foresight dynamic $f(x) = z$ thus defined is continuous, $f(x) < 1$ for all x , and there is exactly one $y > 0$, that solves $f(y) = y$.

So, for all policy rules with $\alpha \geq 0$, these two assumptions are fulfilled for the model considered in the main text of this paper. They are also the assumptions underlying Proposition B below. This is why we have been able to conclude that for policy rules with $\alpha \geq 0$, if there are no deterministic cycles (as there cannot be if the steady state $y > 0$ is globally stable according to f), then there are no SSE either.

Proposition B. *Let v_1 and v_2 be such that Assumptions 1 and 2 are fulfilled. If there are $y_1 \leq \dots \leq y_r$, with $y_1 < y_r$, and an irreducible matrix (q_{ij}) of*

transition probabilities, such that (14) is fulfilled, then there are also y', y'' with $0 < y' < y'' < 1$, such that $y' = f(y'')$ and $y'' = f(y')$. That is, if there is a stationary Markov sunspot equilibrium SSE, then there is also a two-period cycle, or, if there is no two-period cycle, then there is no SSE either.

Proof.¹⁵ One can safely assume that all transition probabilities fulfill $q_{ij} > 0$.¹⁶ For each $i = 1, \dots, r$ define,

$$y_i^{\min} := \arg \min_{j \in \{1, \dots, r\}} v_2(y_i, y_j),$$

$$y_i^{\max} := \arg \max_{j \in \{1, \dots, r\}} v_2(y_i, y_j).$$

Since from (14), each $v_1(y_i)$ is an average of the r values of $v_2(y_i, y_j)$, $j = 1, \dots, r$, one must have $v_2(y_i, y_i^{\min}) \leq v_1(y_i) \leq v_2(y_i, y_i^{\max})$ for $i = 1, \dots, r$. In particular for $i = 1$ and r ,

$$v_2(y_1, y_1^{\min}) \leq v_1(y_1) \leq v_2(y_1, y_1^{\max}),$$

$$v_2(y_r, y_r^{\min}) \leq v_1(y_r) \leq v_2(y_r, y_r^{\max}).$$

Since v_2 is decreasing in its first argument we have: $v_2(y_r, y_r^{\max}) \leq v_2(y_1, y_1^{\max}) \leq v_2(y_1, y_1^{\min})$, and $v_2(y_1, y_1^{\min}) \geq v_2(y_r, y_1^{\min}) \geq v_2(y_r, y_r^{\min})$. So, now using that $v_1(y_i)$ is strictly increasing in y_i , we get,

$$v_2(y_r, y_r^{\min}) \leq v_2(y_1, y_1^{\min}) \leq v_1(y_1) < v_1(y_r) \leq v_2(y_r, y_r^{\max}) \leq v_2(y_1, y_1^{\max}).$$

Part of this is $v_2(y_1, y_1^{\min}) < v_2(y_1, y_1^{\max})$, and since all transition probabilities q_{1j} are strictly positive, one gets $v_1(y_1) > v_2(y_1, y_1^{\min})$. Similarly, $v_1(y_r) < v_2(y_r, y_r^{\max})$.

¹⁵This proof extends the result of Grandmont (1986) from the case where v_2 is independent of y_i , to the case where v_2 is weakly decreasing in y_i .

¹⁶We appeal here to known results. For dynamic systems as considered here, if there is a deterministic cycle, that is, a completely non-stochastic SSE where for each i , only one q_{ij} is greater than zero (equal to one), then there is also a fully stochastic SSE where all q_{ij} are strictly positive. By the same reasoning, if there is an SSE where for each i , some, but not all, q_{ij} are strictly positive, then there is also a fully stochastic SSE, cf. Guesnerie and Woodford (1992).

We have thus established,

$$v_2(y_1, y_1^{\min}) < v_1(y_1) \leq v_1(y_2) \leq \dots \leq v_1(y_r) < v_2(y_r, y_r^{\max}). \quad (15)$$

For one i , one has $y_i = y_1^{\min}$, and hence $v_1(y_1^{\min}) > v_2(y_1, y_1^{\min}) \geq v_2(y_1^{\min}, y_1^{\min})$, where the latter follows since v_2 is decreasing in its first argument. Hence, $v_1(y_1^{\min}) > v_2(y_1^{\min}, y_1^{\min})$, but this implies that $f(y_1^{\min}) < y_1^{\min}$. (Remember that $f(y_1^{\min})$ is the solution in z to $v_1(z) = v_2(z, y_1^{\min})$. For $z = y_1^{\min}$, one gets “strictly larger than”. The solution is then to be found strictly below y_1^{\min} , since v_1 is strictly increasing, and v_2 is decreasing, in z). Similarly, for one i , one must have $y_i = y_r^{\max}$, so $v_1(y_r^{\max}) < v_2(y_r, y_r^{\max}) \leq v_2(y_r^{\max}, y_r^{\max})$, implying $f(y_r^{\max}) > y_r^{\max}$. So, we have both $f(y_1^{\min}) < y_1^{\min}$ and $f(y_r^{\max}) > y_r^{\max}$. This implies, of course, that $y_1^{\min} \neq y_r^{\max}$, but also that,

$$y_r^{\max} < y_1^{\min}.$$

Otherwise one would have $f(y_1^{\min}) < y_1^{\min} < y_r^{\max} < f(y_r^{\max})$, which from the continuity and $f < 1$ parts of Assumption 2 would imply the existence of a monetary steady state strictly between y_1^{\min} and y_r^{\max} , and one strictly above y_r^{\max} , contradicting the uniqueness of monetary steady state part of Assumption 2.

Also from (15), one has directly that $v_1(y_1) > v_2(y_1, y_1^{\min})$, which implies $f(y_1^{\min}) < y_1$ (by the same reasoning as above), and similarly $v_1(y_r) < v_2(y_r, y_r^{\max})$, implying $f(y_r^{\max}) > y_r$. Since also $y_1 \leq y_r^{\max}$, and $y_1^{\min} \leq y_r$, one has,

$$f(y_1^{\min}) < y_r^{\max} \text{ and } y_1^{\min} < f(y_r^{\max}).$$

Combining the two last displayed inequalities gives,

$$f(y_1^{\min}) < y_r^{\max} < y_1^{\min} < f(y_r^{\max}).$$

Given that f is continuous and stays below the “ceiling” one, this suffices for the existence of a two period cycle: Note that the obtained inequality states that f

has a negative slope below minus one over an interval around the steady state, not necessarily infinitesimally close to it. However, the kind of non-local negative slope below minus one obtained suffices from a standard argument. If one constructs the mirror image of f around the 45° -line then this has, under the obtained condition and Assumption 2, to intersect f itself at two points y' and y'' different from the steady state. These y' and y'' define a two-period cycle. ■

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