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Equity Prices, Productivity Growth,
and the 'New Economy'

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EQUITY PRICES, PRODUCTIVITY GROWTH, AND ‘THE NEW ECONOMY’¹

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Abstract. The sharp increase in equity prices over the 1990s was widely attributed to permanently higher productivity growth derived from the New Economy. This paper establishes a rational expectations model of technology innovations and equity prices, which shows that under plausible assumptions, productivity advances can only have temporary effects on the fundamentals of equity prices. Using historical data on productivity of R&D capital, patent capital and fixed capital for 11 OECD countries, empirical evidence give strong support for the model by suggesting that technological innovations indeed have only temporary effects on equity returns.

JEL Classification: G120, G3, O4

Key words: New economy, productivity, economic growth, equity prices.

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Introduction

The worldwide increase in equity prices in the 1990s has been widely linked to permanent productivity-growth effects and the significant generation of intangible assets during the information and communication technology (ICT) revolution.² It has been extensively argued that the acceleration in productivity in the 1990s increased firms' current and expected real cash flows and therefore contributed to an increase in the value of firms.³ Hall (2000, 2001a) argued that the share market run-up in the 1990s was justified by the increasing value of intangible assets consisting of "e-capital" that has increased the expected cash flow of firms. Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) argue that the rise in the stock market from the 1980s onwards was linked to the rise of Information Technology (IT) based firms. However, the questions whether the increase in equity prices in the 1990s can be attributed to increasing growth in intangible and tangible capital productivity, and whether a sustainable higher capital productivity growth rate can be expected in the future, have gone almost unexplored.⁴

This paper introduces and tests a Tobin's q model of the interaction between capital productivity shocks and equity prices to gauge the short and long term effects of the ICT revolution on equity prices. Section 2 introduces some of the measurement issues and Section 3 develops a general equilibrium model to show that innovations have only temporary effects on capital productivity and hence on equity prices. In fact, changes in equity prices will precede the impact of the shock to productivity if equity markets react in a forward-looking way to news of innovations. Furthermore, productivity shocks lead to higher tangible and intangible capital stock in the long run, but equity prices revert back to a long-run equilibrium. It is suggested that the analysis is of considerable relevance given the growing prevalence of intangible as opposed to tangible capital in the New Economy. Using historical data for real equity returns, tangible and intangible capital stock for 11 OECD countries, we test the predictions of the model in Section 4, with considerable support being offered to its predictions.

² Other factors that have been suggested as important factors behind the increase in stock prices in the 1990s include a decrease in the risk premium, higher international liquidity, baby boomers, the disinflation, and irrational exuberance (IMF, 2000, Shiller, 2000).

³ See Business Week, 2003, Campbell and Shiller, 2001, Economist, 2001, Greenwood and Jovanovic, 1999, Hobijn and Jovanovic, 2001, IMF, 2000, Jovanovic and Rousseau, 2003, Keon, 1998, Laitner and Stolyarov, 2003.

⁴ An exception is the model of Datta and Dixon (2002) where it is shown that innovations increase profits of incumbents and share prices, but that entry of new firms drive profits back to zero. As discussed below the model of Datta and Dixon (2002) is quite different from the model presented in this paper.

2. Innovations, capital productivities and share returns

The New Economy brought with it expectations of productivity-induced increases in the growth in cash flow per share among share investors and several economists. Two closely related arguments have been used to account for the ICT-induced rise in equity prices in the second half of the 1990s which appears to have resumed in 2003. Some argue that the ICT revolution, or more generally the New Economy, has brought productivity growth rates up to a sustainable higher level, thus resulting in higher growth in expected earnings per share. Others have argued that the New Economy has created sufficient intangible wealth to merit the higher share prices in the late 1990s (Hall, 2000, 2001a, McGrattan and Prescott, 2001). These two arguments are closely related, because the value of the capital stock equals the discounted value of earnings in general equilibrium. This section examines these arguments and discusses the data issues relating to the measurement of intangibles.

2.1 The New Economy and productivity

Considering the productivity growth effects on share prices of the New Economy, the main international organisations and researchers have attributed a large part of the increase in equity prices in the 1990s to accelerations in actual and expected labour productivity and potential output.⁵ The problem with this line of reasoning is that labour productivity and potential output are severely biased proxies for firms' cash flow. The relevant productivity measure for firms' cash flow is the marginal productivity of capital, which has moved in a direction which was historically quite different from the growth in labour productivity and potential output. To see the consequences of using labour productivity and potential output as measures of earnings per unit of capital, consider the Cobb-Douglas production function, $Y = BL^\alpha K^{1-\alpha}$, where B represents total factor productivity (TFP), Y is aggregate value-added output, K is capital services and L is labour services. The growth in marginal productivities of labour and capital are given by:

$$\Delta \ln(Y/L) = (1-\alpha)\Delta \ln(K/L) + \Delta \ln B, \quad (1)$$

$$\Delta \ln(Y/K) = -\alpha\Delta \ln(K/L) + \Delta \ln B. \quad (2)$$

⁵ For example, in the IMF's *World Economic Outlook* (2000) and in Kennedy *et al* (1998) of the OECD, the growth in potential output is used as a proxy for expected dividend growth in a version of Gordon's growth model of equity valuation. Elsewhere, IMF (2000) suggests that labour productivity growth is the relevant measure of dividend growth. Similarly, a series of articles in the *Economist* and *Business Week* have argued that labour productivity is the relevant productivity measure for share prices (see for instance, *Business Week*, 2003, and *Economist*, 2001). Finally, Campbell and Shiller (2001) suggest that many analysts attribute the equity price boom in the 1990s partly to the accelerating labour productivity in the same period.

Comparing these equations, it is evident that TFP growth enhances growth in both capital and labour productivities. Capital deepening, however, increases the marginal productivity of labour but lowers the marginal productivity of capital and therefore explains why the real interest rate/return on equity tends towards a constant mean in the long run, while real wages show a continuous rise in the long run. Historically, capital deepening has counterbalanced total factor productivity growth to such an extent that tangible capital productivity has tended to decline only slightly in the OECD countries.

The K/L ratio has increased geometrically by 3.5% annually in the OECD countries used in this study over the period from 1960 to 2001 (see notes to Figure 1 below), whereas TFP has increased by 1.5% only on average, when α is set to 0.7; thus suggesting a strong growth in labour productivity but a slight decline in capital productivity. The bias from using the growth in potential output as a proxy for the growth in capital productivity is even larger than using labour productivity. The bias is given by $\Delta \ln Y - \Delta \ln(Y/K) = \Delta \ln(K)$. The bias was 34% over the period from 1980 to 1992 and 22% from 1993 to 2001 for the countries used in this study. From these numbers it is evident that share valuation based on growth in labour productivity or in potential output, severely overestimates the value of shares and is overly sensitive to fluctuations in labour productivity and potential output growth rates.

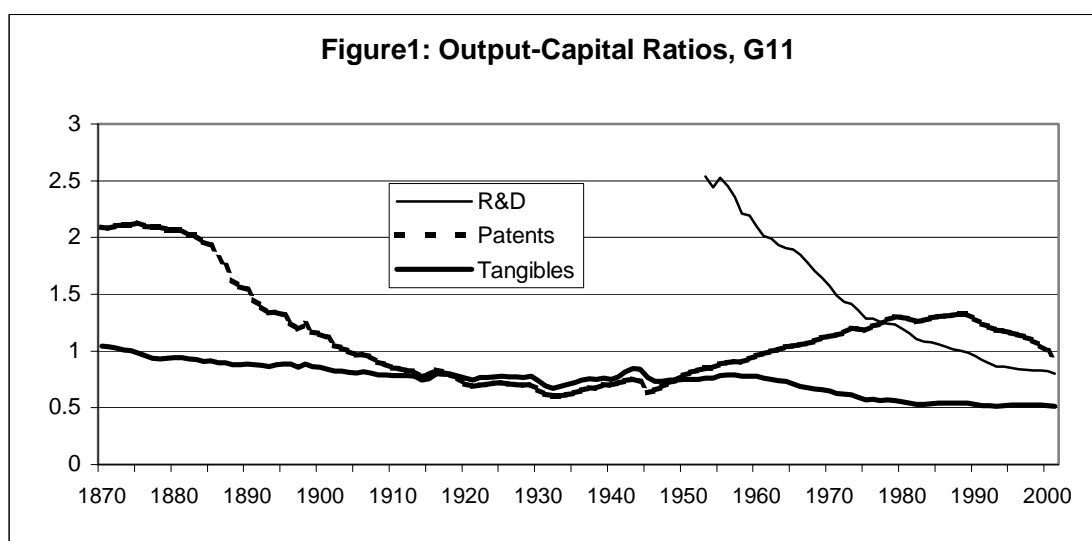
2.2 Some estimates of capital productivity and the New Economy

The estimates above are based on the tangible capital stock. However, several economists have argued that tangible capital stock is too narrow a concept of capital and that the creation of intangibles has been a vital part of the new economy (see for instance Brynjolfsson *et al*, 2002). Patent applications and R&D expenditures are probably the most accepted measures of the innovative activity, including the creation of intangibles during the ICT revolution.⁶ Hall (2001b) argues that the increase in the market value of firms in the 1990s are related to intellectual property and, to a much lesser extent, to advertising and R&D and writes that “much of the increase of firms in the past decades appears to be related to the development of successful differentiated products, protected to some extent from competition by intellectual property rights relating to technology and brand names” (p 1189). That the New Economy is well indicated by patent data is well documented. During the 1990s, for instance, ICT patent applications in the OECD countries grew at an annual rate

⁶ See for instance Griliches (1990) and Grupp (1998) for discussions of the merits in using patenting and R&D data as indicators of the innovative activity.

of 9% in the OECD countries, which is almost 50% higher than the growth rate of total patent applications (OECD, 2003). Furthermore, about a third of all OECD patent applications were ICT-related (OECD, 2003).

To get a picture of the historical paths of the marginal productivities of tangible and intangible capital stock, Figure 1 displays the unweighted average of the three Y/K ratios for the 11 countries that are used in this study. These 11 countries are listed in the notes to Figure 1 and are referred to the G11 countries as shorthand. The three ratios are the productivities of the tangible capital stock, R&D capital stock, and patent capital stock. We use the Cobb-Douglas productivity assumption under which the marginal productivity of capital type i is given by $(1-\alpha_i)Y/K_i$, which varies proportionally to Y/K_i , where $(1-\alpha_i)$ is the share of income going to capital type K_i , $\sum (1-\alpha_i) = 1-\alpha$. US data on R&D expenditure are used over the period from 1953 to 1965 since R&D data are not available for other countries before 1965. The patent capital stock is measured as patents applied for by residents and non-residents.⁷ The capital data are constructed using the perpetual-inventory method as detailed in the data appendix.



Notes. The figures are computed as an unweighted average for the following 11 countries: Australia, Canada, Denmark, France, Germany, Italy, Japan, the Netherlands, Sweden, the UK and the US. The output-R&D capital stock ratio is divided by five and is spliced to the US data before 1965. The output-patent capital stock ratio is measured in millions of USD in 1995 prices at 1996 purchasing power parity. See the data appendix for data sources.

⁷ Patent applications are almost always used in economic analysis as opposed to patents granted, because applications measure most precisely the timing of the innovation relevant for share price expectations and because the time lag between the lodgement of the application and the time at which the patent is granted, vary substantially over time. See Griliches (1994) for discussion of these issues.

Figure 1 shows that capital productivity measured by the ratio of output to the tangible capital stock has been declining over the past century. R&D capital productivity has been diminishing over the past 50 years, but at a declining rate, which, as argued in Section 3 below, is a result of a slow convergence towards the steady state. Patent capital stock productivity has also decreased over the past 130 years, but it appears that it has stabilised at the equivalent of 1 million USD at 1995 constant prices per patent.⁸ Common for all three indicators is the property that capital productivities have not been increasing in the long run.⁹ Coupled with the fact that labour productivities have been growing at a steady rate of 2-3% in the G11 countries over the past 130 years, this result underlines the point made in the previous sub-section, namely that growth in labour productivity is a directly misleading proxy for growth in returns to capital. This is particularly true over the past two decades where the returns to R&D effort and patenting have also been declining.

It can be argued that the productivity effects of the Second Industrial Revolution, which started around 1870, are quite instructive for projecting the earnings effects of the ICT revolution.¹⁰ The great inventions in the latter part of the 19th century such as the invention of electricity and the internal combustion machine, led in fact to declining and not increasing tangible and patent capital productivities as seen from Figure 1. A strong reduction in patent capital stock productivity can particularly be identified over the period from 1885 to 1913, which suggests diminishing returns to the patent capital stock. Thereafter patent capital stock and tangible capital stock productivities stabilised at a constant mean up to 1960, which covers a period in which the great inventions diffused (Gordon, 2000, Perez, 2002). The decrease in tangible capital stock productivities over the period from 1885 to 1913 is associated with a strong increase in the tangible and patent capital stock over the same period, and at least some of the tangible stock capital accumulation was associated with the high inventive activity in that period. Tangible capital accumulation is often associated with technological advances or embodied technological progress as shown by Hulten (1975) and advocated by Gordon (2003). Accordingly, the capital accumulation process during the Second Industrial Revolution was associated with declining tangible capital productivity and, therefore, diminishing returns to capital.

⁸ One potential problem associated with the patenting capital stock productivities for long run analysis is that the real value of patents may have changed over time. However, there is no clear evidence that the real value of patents has fluctuated in this manner (see Griliches, 1994).

⁹ Earlier data suggest that the patent capital stock and tangible capital stock productivities for the UK and the US were declining before 1870 and suggest that capital productivities have been declining over the past two centuries for these two countries. Very little data are available for other countries before 1870.

¹⁰ The exact dating of the Second Industrial Revolution differs among economists. Greenwood and Jovanovic (1999), for instance, date it to the period from 1890 to 1930, whereas Perez (2002) refers to the period after 1875 as the Third Industrial Revolution.

The lessons from the Second Industrial Revolution suggest that the declining capital productivities which are identified in Figure 1 over the past two decades, may well continue into the near future but at a declining rate as the diffusion process advances. The diffusion process is likely to be shorter than the experience from the first two industrial revolutions. Gordon (2000), for instance, argues that the reorganisation and the development of new systems as a consequence of the New Economy have been substantially easier than the implementation of the innovations which occurred in the latter part of the 19th century. Almost all workstations have computers today, whereas for example it took several decades to switch factories from centralised steam-driven power to decentralised electro motors in the last century. Hence, the delayed benefits of the New Economy may not be as large as thought by many investors.

2.3 The New Economy, share prices and intangibles

A number of economists have argued that the rising share prices in the 1990s in the OECD countries reflected increasing values of intangibles that have been created as a by-product of investment in ICT products, R&D, advertisement, and new brand names. Hall (2000, 2001a) and McGrattan and Prescott (2001) estimated the value of intangibles indirectly, whereas Nakamura (2001) provided a direct measure of the value of intangibles. A key question is whether these estimates can justify the value of shares in 2000 and at the end of 2003, and hence the expectations of higher earnings growth which has been induced by the ICT revolution.

Hall (2000, 2001a) defines intangibles as technical and organizational know-how that have been created by graduates using computers and software and names it *e-capital*. Since intangibles cannot be measured using this definition, Hall (2000) estimated the value of intangibles by subtracting the value of physical stock from the value of the stock market and found the value of *e-capital* to exceed the value of tangible capital stock for US corporations in 1999.¹¹ However, for Hall's measure of intangible to be correct, share prices should reflect earnings expectations in an efficient share market. Basing the fundamental value of shares on analysts' earnings forecasts, Bond and Cummins (2000) found that share prices increased substantially more than their value based on analysts' earnings forecasts during the 1990s. This suggests analysts' estimates of intangibles to be well below market's estimates but in line with managers' expectations since the estimates of Bond and Cummins (2000)

¹¹ Hall's method has been met with strong criticism (see for instance, Cummins, 2000, and Lamont, 2000).

also show a strong relationship between investment and the value of shares based on analysts earning forecasts. Coupled with the finding that Tobin's q does not provide incremental information on the investment function when analysts' earnings expectations are allowed for in their regression analysis, these results suggest that analysts and managers believed in a much lower increase in the value of intangibles during the 1990s than the share market.

McGrattan and Prescott (2001) estimate the value of intangibles in the US corporate sector to be 80% of the value of their tangible capital stock over the period from 1987 to 2000, which is up from 40% over the period from 1955 to 1962. Their method is based on the equilibrium conditions that equate the after-tax returns for all assets, that is, the after-tax profits per unit of intangible and tangible capital stock in the corporate sector equals profits per unit of tangible capital stock in the non-corporate sector (including imputed services to consumer durables and government capital). Thus, their method rests on the highly restrictive assumptions that the equity risk premium is the same in the two sectors and that the value of intangibles is zero in the non-corporate sector. Both restrictions bias the estimations of intangible capital upwards and McGrattan and Prescott (2001) also admit that the estimates of intangible capital stock are on the high side. Similarly Hansen *et al* (2004) argue that assumption of no intangible capital stock in the non-corporate sector is a "seemingly hard to defend restriction" (p 8).

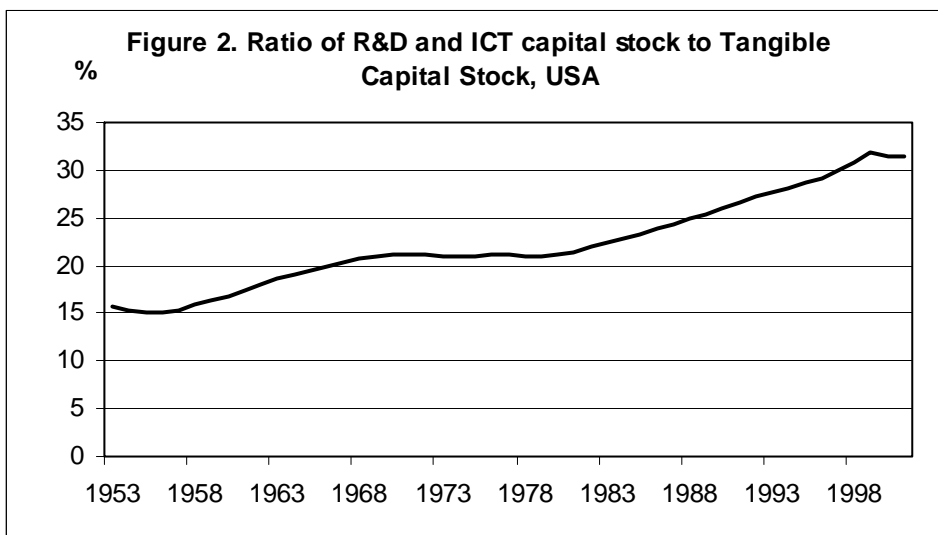
Based on the predictions of a standard growth model, Nakamura (2001) estimated the value of the intangible capital stock of US corporations to be USD 6.25 trillion in 2000, which exceeds the figure estimated by Hall (2000). Nakamura (2001) assumes that the steady state value of the intangible capital stock equals investment in intangibles divided by their depreciation rates under the assumption of no labour augmenting technological progress and no growth in employment. The following items were included in his estimates of intangible investment; 1) expenditures to R&D (\$181 bn in 2000); 2) software investment (\$183 bn); 3) expenditures on advertising (\$233 bn); 4) artistic expenditures (\$50 bn); 5) innovative expenditures by financial corporations (\$50 bn); and 6) items unaccounted for (\$303 bn).¹² Dividing this sum of \$1 trillion by a depreciation rate of 16% he finds the steady state stock of intangibles of \$6.25 trillion.

The estimates of Nakamura rest on the assumptions of zero Harrod-neutral technological progress, zero growth in the labour force, a depreciation rate of 16%, zero adjustment cost associated with

¹² Two other estimates are presented by Nakamura (2001). They are not discussed here to preserve space.

investment in intangibles, and the absence of externalities associated with the investment in intangibles. The zero-externality assumption has been questioned by Smithers and Wright (2000) and Gordon (2003), who argue that intangible investment that increases the value of an individual firm need not add to the aggregate value of firms because some of intangible investments are undertaken to gain market shares. Smithers and Wright (2000) argue that advertisement, for instance, is an intangible investment in customers by the individual firm, but, at the same time, lowers the customer capital of competing firms and is hence unlikely to significantly affect the aggregate value of the intangible capital stock. Similarly, Gordon (2000) argues that a large fraction of the ICT investment that has been generated by individual firms as a by-product of the New Economy has involved taking profits and customers away from other companies in a zero-sum game. R&D expenditures, however, have been found to add almost fully to the aggregate value of intangibles (Megna and Klock, 1993).

Since intangibles are by their very nature immeasurable but created from expenditures on factors of production, the growth in expenditures on items from which they are assumed to have been generated, will give an indication of the potential growth in their importance. Brynjolfsson *et al* (2002), for instance, argue that investment in computers and communication equipment lead to investment in unmeasured complementary intangibles such as organizational restructuring and business process design. Thus, the ratio of the capital stock of the factors of production that are assumed to generate the intangibles, and tangible capital stock will give an indication of the growth of the potential importance of intangible relative to tangible capital during the ICT revolution.



Notes: ICT capital stock is the sum of the stock of computer, software, and communication capital stock and tangibles is measured as total fixed capital stock excluding the land, consumer durable goods and inventories. See data appendix for data sources.

Figure 2 presents the ratio of the combined R&D and ICT capital stock to the tangible capital stock for the US economy since 1953, the first year for which data on R&D expenditures are available. The ICT capital stock is computed as the sum of computer, software, and communication capital stock using the data in Jorgenson (2001).¹³ The ratio of the R&D and ICT capital stock to the tangible capital stock has increased from approximately 15% in the beginning of the 1950s to 32% in 2001. More importantly, the *growth* in the ratio in the 1990s did *not* exceed the growth in the 1980s and particularly not the growth in the 1960s. Furthermore, and not shown in the graph, the real value of the R&D and ICT capital stock almost doubled in the 1960s, 1980s and 1990s, and increased by about 50% in the 1970s. Under the assumption that the shadow price of R&D and ICT capital stocks are constant over time, as maintained by Cummins (2003) and Brynjolfsson *et al* (2002), the real value of intangibles should have increased by the same rate. The growth path in Figure 2 is incompatible with the growth path in share prices and Hall's (2000) estimates that show that the nominal value of *e*-capital to have increased more than eight-fold in the 1990s.

Finally, provided that the share market run-up in the 1990s was associated with the creation of intangibles, intangibles should also have been produced at previous industrial revolutions and in Japan during the 1990s. Gordon (2003) argues that the organisational restructuring associated with the investment in the new capital during the Second Industrial Revolution, was much higher than the adjustment costs associated with the implementation of ICT products. Judging from Figure 1 the intangibles generated during the Second Industrial Revolution did not materialise in higher earnings per unit of capital. Similarly, the Japanese share market has been declining since 1990 although R&D and ICT capital services, based on the figures of Colecchia and Schreyer (2002), increased substantially more than in the US in both absolute values and relative to tangible capital stock.

For the US, Smithers and Wright (2000) find that Tobin's q based on tangibles has tended towards a constant mean over the past century. If previous clusters of innovations have been associated with significant investment in intangibles, then their estimates of Tobin's q would have decreased over time because they have omitted intangibles in the denominator. However, this has not been the case, which suggests that creation of intangibles cannot have been a significant part of past technology revolutions. The possibility that more intangibles have been created during the ICT revolution than previous industrial revolutions cannot be excluded. However, very little, if any, evidence to support this hypothesis has been produced thus far.

¹³ Note that computer hardware and communication equipment are also components of the tangible capital stock.

In summary, the evidence in this sub-section points towards several ambiguities that have been associated with the estimated value of intangibles. If the share market run-up in the 1990s should be justified by the growth in intangibles, we should have observed time-series and cross section evidence that was consistent with a positive relationship between share prices and various proxies of intangibles. Thus far, very little evidence on this account has been produced.

3 A model of equity prices and innovations

This section establishes a model to explain the main results in the previous section, namely that the capital productivities that drive returns to capital have not increased over the past 130 years and that previous technology epochs appear not to have had long term effects on earnings per unit of capital. Under plausible assumptions, the model in this section suggests that technological innovations have only temporary effects on real dividends and equity prices. The model allows for a two-way relationship between the equity return and the capital stock. The capital stock influences the returns to capital and therefore equity prices, whereas the real equity price determines the desired capital stock. Both exogenous and endogenous discount factors are considered. The model is based on the analytical framework developed by Abel (1982), Abel and Blanchard (1983), Hayashi (1982), Kiley (2000), Romer (2001), and Summers (1981), and allows for productivity-enhancing technological innovations in the investment-good producing sector.

3.1 Exogenous discount rate

The ICT revolution has two effects on real corporate cash flow. First, the real price of computers and ICT equipment decreases relative to economy-wide output prices and this hence enhances profits for any positive level of investment, because the effective acquisition price of fixed capital has declined. Relative prices of computers, for instance, have fallen substantially over the past two decades. Similar price developments have been observed in previous technological revolutions (see for instance Jovanovic and Rousseau, 2003). This will be referred to as embodied technological progress. Second, the ICT revolution increases the marginal productivity of the existing capital stock due to positive spill-over effects of the technological innovations, θ . These spill-over effects may not only affect the output of firms that have not undertaken the investment in computers and ICT technology, but also affect firms that have undertaken certain investment projects in the past, such as establishment of internet connections.

Incorporating spill-over and embodiment effects into the profit function of the representative firm, we arrive at the following equation of total profits:

$$\Pi_t = F(\theta_t, K_t^T, K_t^I, L_t^H, L_t^R) - I_t^I \cdot \phi_t^I - I_t^T \cdot \phi_t^T - w_t^H L_t^H - w_t^R L_t^R - C(I_t^T) - C(I_t^I), \quad (3)$$

where K^T and K^I is capital stock of tangibles and intangibles, respectively, I^T and I^I is investment in tangibles and intangibles, respectively, C is the adjustment cost of investment, which, for notional simplicity, is assumed to be the same for tangibles and intangibles, θ is the spill-over effect from investment in intangibles, ϕ^I and ϕ^T are real prices of investment in intangibles and tangibles, respectively, $\phi_t^T = P_t^T / P_t^O$, where P^T is prices of non-R&D investment and P^O is output prices, $\phi_t^I = P_t^I / P_t^O$, where P^I is the price of R&D equipment, or, more generally, intangibles, w^R is the real product wage of R&D workers, w^H is the real product wage of non-R&D workers, L^H is the employment of non-R&D workers, and L^R is the employment of R&D workers. Full employment is assumed to prevail so that $L = L^R + L^H$. Spill-over effects are assumed to enhance profits per unit of capital, $\partial MPK / \partial \theta > 0$, where MPK is the marginal productivity of capital. Taxes are assumed absent and the interest rate is assumed to be fixed and determined from abroad.¹⁴ The assumption of a fixed discount factor is relaxed below. R&D workers are defined broadly to encompass workers that are involved in R&D, implementation and usage of ICT products, and other activities that relate to investment in intangibles.

Embodied technological progress is represented by the (ϕ^I, ϕ^T) -terms that reflect the current state of the technology for producing tangible and intangible investment goods (Greenwood *et al*, 1997). The terms determine the amount of computers, machinery, software, information technology equipment, and communication equipment that can be purchased for one unit of output. They thus play an important role in capturing effects of technological revolutions that are usually characterised by reductions in prices of investment goods (Perez, 2002, Gordon, 2003, Jovanovic and Rousseau, 2003). Using hedonic pricing, Jorgenson (2001) shows that the real prices of computers and other ICT equipment have decreased substantially over the past few decades.

The firm's adjustment cost functions are assumed to have the usual convexity properties, but the interpretation of adjustment costs is broader than the conventional interpretation where adjustment

¹⁴ The effects of taxes on share prices are analysed in Summers (1981) and McGrattan and Prescott (2003).

costs consist of temporary production cut backs, waiting time and installation costs. We follow the interpretation of Cummins (2003) and Brynjolfsson *et al* (2002) where the adjustment costs associated with computer investment, for instance, include unmeasured intangibles such as training costs, costs associated with restructuring of the organisation, business process redesign and reallocation of decision rights. Based on the method of Wildasin (1984), Brynjolfsson *et al* (2002) estimate the intangibles that have been created jointly with investment in computers in the US are likely to be higher than the computer expenditures. Basing the value of firms on analysts' earnings expectations, Cummins (2003) obtained somewhat lower estimates but still found that investment in ICT hardware has created intangibles.¹⁵

The optimisation problem of the firm is to maximize its present value:

$$\max V = \int_{t=0}^{\infty} e^{-rt} \left[F(\theta_t, K_t^T, K_t^I, L_t - L_t^R, L_t^R) - I_t^T \cdot \phi_t^T - I_t^I \cdot \phi_t^I - w_t^H (L_t - L_t^R) - w_t^R L_t^R - C(I_t^T) - C(I_t^I) \right] dt \quad (4)$$

st.

$$\dot{K}_t^T = I_t^T / \phi_t^T - \delta^T K_t^T, \quad (5)$$

$$\dot{K}_t^I = I_t^I / \phi_t^I - \delta^I K_t^I, \quad (6)$$

where r is the real interest rate or the real required returns to equity, δ^T is the depreciation rate for the tangible capital stock, δ^I is the depreciation rate for the intangible capital stock, and a dot over a variable signifies the change in the variable.

The current-value Hamiltonian is given by:

$$J = F(\theta_t, K_t^T, K_t^I, L_t - L_t^R, L_t^R) - I_t^T \cdot \phi_t^T - I_t^I \cdot \phi_t^I - w_t^H (L_t - L_t^R) - w_t^R L_t^R - C(I_t^T) - C(I_t^I) + q_t^T \left[I_t^T / \phi_t^T - \delta^T K_t^T - \dot{K}_t^T \right] + q_t^I \left[I_t^I / \phi_t^I - \delta^I K_t^I - \dot{K}_t^I \right] \quad (7)$$

where q^T is the shadow price for the constraint given by Equation (5), and q^I is the shadow value for the constraint given by Equation (6).

The shadow prices of tangibles and intangibles are allowed to differ. This means that each dollar invested in tangibles and intangibles increases the value of the firm by q^T and q^I dollars, respectively. The question is whether the shadow prices of tangibles and intangibles differ in practice. Supposing

¹⁵ One problem associated with these studies based on firm data is that it is implicitly assumed that the firm effects equal the general equilibrium effects and, therefore, that share prices of individual firms reflect feedback effects from other

that ICT capital is complementary with unmeasured intangible assets such as organizational capital and other intangibles, the investment in ICT capital would increase the value of the firm by the value of the investment plus the increase in the value of organizational capital and business processes. In other words, by investing in another unit of ICT capital the firm can make better use of its organizational resources and the value of the firm will consequently increase more than the cost of the ICT investment. While shadow prices of different investment objects may differ in the short run they will not differ in a perfectly competitive steady state equilibrium because market forces will bring the value of various investments into equality.

Under the assumption of perfect competition in the goods market, equation (5) yields the first order conditions for optimality as follows:

$$\left(\phi_t^T\right)^2 + C'(I_t^T)\phi_t^T = q_t^T, \quad \left(\phi_t^I\right)^2 + C'(I_t^I)\phi_t^I = q_t^I, \quad (8)$$

$$MPK_T' = (r + \delta^T)q_t^T - \dot{q}_t^T, \quad MPK_I' = (r + \delta^I)q_t^I - \dot{q}_t^I, \quad (9)$$

$$\lim_{t \rightarrow \infty} e^{-rt} q_t^T K_t^T = 0, \quad \lim_{t \rightarrow \infty} e^{-rt} q_t^I K_t^I = 0,$$

where $MPK_T' = F'_{K^T}$ and $MPK_I' = F'_{K^I}$. Equation (8) is the investment function that links investment to the real shadow price of new capital goods. In equilibrium, the shadow price of additions to the capital stock equals the marginal cost of investment on the left hand side. Since the adjustment function is convex there is a positive relationship between investment and real share prices modified by relative prices of investment goods. Shadow prices of new capital goods and the real value of equity only differ to the extent that relative prices of investment goods differ from the numeraire of one. From the assumption that $C'(0) = 0$, it follows that investment is zero when $q_t^I = (\phi_t^I)^2$ and $q_t^T = (\phi_t^T)^2$.

Equations (8) and (9) form the simultaneous first-order differential equation system as follows:

$$\dot{K}^T = h(q^T, \phi^T) \quad (10)$$

$$\dot{q}^T = q^T \cdot (r + \delta^T) - MPK_T'(K^T, \theta), \quad (11)$$

$$\dot{K}^I = g(q^I, \phi^I) \quad (12)$$

$$\dot{q}^I = q^I \cdot (r + \delta^I) - MPK'_I(K^I, \theta), \quad (13)$$

where $h'_{q^T}, g'_{q^I} > 0$, $h'_{\phi^T}, g'_{\phi^I} < 0$, $MPK'_{K^T}, MPK'_{K^I} < 0$, $MPK'_\theta > 0$. Equations (10) and (12) are the investment functions and show the dynamic adjustment of capital stock to innovations in q and ϕ . Equations (11) and (13) show the dynamic adjustment of equity prices to innovations in the required return to equity and spill-over effects from innovations.

The long-run effects of technology innovations on share prices can be derived from the steady-state multipliers, which are given as follows:

$$\frac{dq^T}{d\theta} = 0, \quad \frac{dq^I}{d\theta} = 0, \quad (14)$$

$$\frac{dq^T}{d\phi^T} = -\frac{g'_{\phi^T}}{g'_{q^T}} > 0, \quad \frac{dq^I}{d\phi^I} = -\frac{g'_{\phi^I}}{g'_{q^I}} > 0. \quad (15)$$

The results given by (14) show that technological innovations, which are not embodied in new technology, do not have long-term effects on share prices. The results given by (15) show that the value of shares in the steady state is a negative function of embodied technological progress, which lowers ϕ , regardless of whether the companies predominantly employ intangible or tangible capital. This result applies only for incumbents who do not benefit from the additional investment. Companies that take advantage of the more advanced or cheaper equipment do not experience lower profits per unit of capital and hence lower share prices because the acquisition costs have counterbalanced the lower sales prices. The results that embodied technological progress lowers share prices of incumbents is consistent with the finding of Hobijn and Jovanovic (2001) and Greenwood and Jovanovic (1999) that new capital destroys old capital and that firms that do not implement the new technology, experience a reduction in their share prices.

The following results furthermore suggest that long-term spill-over effects of embodied technological progress between tangibles and intangibles are absent:

$$\frac{dK^T}{d\phi^I} = \frac{dK^I}{d\phi^T} = 0, \quad \frac{dq^I}{d\phi^T} = \frac{dq^T}{d\phi^I} = 0. \quad (16)$$

Finally, any technological innovation increases the steady state intangible or tangible capital stock:

$$\frac{dK^T}{d\theta} = -\frac{MPK'_{T,\theta}}{MPK'_T g'_{q^T}} > 0, \quad \frac{dK^I}{d\theta} = -\frac{MPK'_{I,\theta}}{MPK'_I g'_{q^I}} > 0, \quad (17)$$

$$\frac{dK^T}{d\phi^T} = -\frac{(r + \delta^T)h'_{\phi^T}}{MPK'_T h'_{q^T}} < 0, \quad \frac{dK^I}{d\phi^I} = -\frac{(r + \delta^I)g'_{\phi^I}}{MPK'_I g'_{q^I}} < 0. \quad (18)$$

These results are consistent with the fact that the ICT revolution has been associated with strong increases in investment, particularly investment in ICT equipment. The recent decline in the *growth* in ICT investment in the OECD countries (OECD, 2003) suggests that the pace of ICT-induced technological progress is declining.

Turning to the dynamic effects of technology shocks on share prices and the capital stock, the system (10)-(13) can be decomposed into two independent equation systems, that is, equations (10) and (11) can be treated separately from Equations (12) and (13). Since the results are the same we need not distinguish between tangibles and intangibles in the phase diagram exposition. Figure 3 shows the dynamics of the capital stock and equity values. The $\dot{q} = 0$ curve is negatively sloped because the marginal productivity of capital is a decreasing function of capital stock. The EE-line defines a stable manifold and the UU-line defines an explosive path. The explosive path is ruled out by the transversality condition.

The figure illustrates the short-run and long run effects of the ICT revolution on equity prices. Technological innovations lower the effective price of the capital stock and increase the marginal productivity of capital. The reduction in the effective price of capital shifts the $\dot{K}_t = 0$ schedule down by the reduction in the relative price of capital, because it reduces the effective acquisition price of new capital. The $\dot{q} = 0$ curve shifts to the right because the positive externalities that are associated with the new technology enhance the marginal productivity of the existing capital stock.¹⁶ The diagram shows that capital stock unambiguously increases whereas equity prices of firms that adopt the new technology are unaltered in the new long-run equilibrium, because the reduction in the relative price of capital has created a wedge between the shadow price of capital and equity prices. In other words, the lower acquisition price of capital stock for the firms that invest in the new capital counterbalances the lower shadow price of capital. Incumbents that do not adapt the new technology,

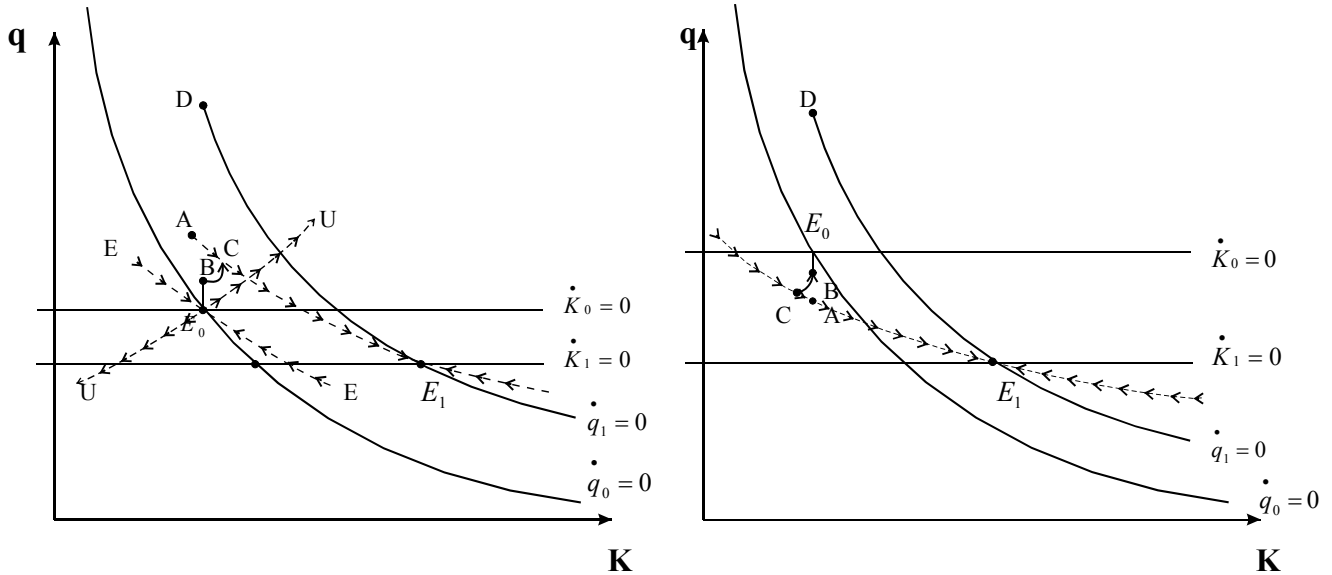
¹⁶ More correctly the slope of the $\dot{q} = 0$ curve becomes flatter by the technology shock. It is treated as a shift here for expositional simplicity.

however, will experience a fall in their share prices because share prices will follow share prices of companies that adapt the new technology.

Figure 3: The dynamics of share prices and investment

Figure 3.1

Figure 3.2



The dynamic path of the system following an unanticipated technology shock is as follows. On impact, a perfect foresight equity market jumps to the point A where it joins the stable manifold to capitalise on temporary higher earnings. Since Tobin's q exceeds one, investment will be positive and the capital stock starts increasing. The speed of adjustment towards the new equilibrium depends, among other things, on the shape of the adjustment cost function. Since the return to capital is constant, equity owners experience a capital loss along the path from A to E_1 to counterbalance the temporary higher return to capital. Equity prices stabilise in the new long-run equilibrium, E_1 . The increasing profit that follows from the lower cost of investment is counterbalanced by the lower marginal productivity of capital in the new equilibrium.

For a myopic share market that values shares based on current earnings, such as Gordon's growth model, share prices jump to the point D and moves along the $\dot{q}_1 = 0$ curve towards the new steady state equilibrium. If the technology shock is expected we get the same steady state outcome as in the unanticipated case, however, the dynamics becomes different. Share prices jump from E_0 to B in Figure 3.1 when news arrives about an anticipated technology innovation. The system then slowly

moves towards the point C and arrives at the point C when the technology innovation emerges. Thereafter the system moves along the stable saddle path towards the final equilibrium at E_1 .

Share prices need not jump on impact but may fall for both incumbents and new firms. If the embodiment effect is large and the spill-over effects small, then we get the dynamic path displayed in Figure 3.2. For the perfect foresight share market an unanticipated technology shock leads to a drop in share prices to the point A on impact where the economy joins the stable manifold. An anticipated technology shock leads to the result of Hobijn and Jovanovic (2001) and Greenwood and Jovanovic (1999), where it is argued that the forthcoming ICT revolution was already anticipated in the beginning of the 1970s and, consequently, put downward pressure on real share prices in the 1970s.¹⁷ In this case the share prices fall to the point B when news about the forthcoming ICT revolution arrives. The result that the ICT revolution has no lasting effects on share prices, however, is inconsistent with the hypotheses of Hobijn and Jovanovic (2001) and Greenwood and Jovanovic (1999).

The prediction of the model that productivity innovations have only temporary effects on share prices is consistent with the theoretical result reached by Datta and Dixon (2002). There is, however, one difference between their model and the model presented here. Their model predicts that incumbents gain from the innovations and not the new firms. In our model share prices of incumbents that do not adapt the new technology suffer from the technological innovations due to a creative destruction process. The evidence suggests that share prices of incumbents do not rise as much as share prices of new and innovative firms in periods of technological acceleration (Greenwood and Jovanovic, 1999).

3.2 Endogenous discount rate

The discount rate is made endogenous in this sub-section to allow for the possible impact of intertemporal utility maximization by rational consumers who perceive productivity shocks. Intuitively, it can be argued that consumers who expect higher future income move consumption forward, which puts upward pressure on the discount rate. Kiley (2000), for instance, shows that share prices drop on impact in response to positive productivity innovations because of the adverse interest rate effects. In terms of the phase diagram exposition above, the interest rate effect steepens the $\dot{q} = 0$ curve and it becomes ambiguous whether the productivity effect is sufficiently strong to

flatten the $\dot{q}_0 = 0$ curve on impact. The analytical framework with an endogenous discount factor is relegated to the appendix and is based on the Abel and Blanchard (1983) framework. A simple two-dimensional phase diagram cannot be used here. However, since this paper focuses on the long-run effects on share prices of the new economy it is sufficient to consider the steady-state multipliers.

The change in the steady state equilibrium share price of a technology innovation is given by:

$$\frac{dq}{d\phi} = \frac{i}{k} \cdot \phi^{-1} \cdot \left[2h'(i/k) + \frac{i}{k} h''(i/k) \right] > 0, \text{ and } \frac{dq}{d\theta} = 0, \quad (19)$$

where $h(i/k)$ is a convex adjustment cost function, $h' > 0$ and $h'' > 0$. These expressions shows that embodied technological progress unambiguously lowers share prices of incumbents and that the effect on share prices of spill-over effects of technological innovations in the steady state is zero. These results are similar to the results with exogenous discount rate, which is not surprising given that the discount rate only affects the slope of the $\dot{q} = 0$ curve. This in turn has no influence on share prices in steady state equilibrium as shown analytically in the previous sub-section.

The impact effects on share prices are also derived in the appendix. It is shown that the sign of $dq/d\theta$ is unambiguously positive whereas the sign of $dq/d\phi$ is ambiguous, which are similar to the result in the model with the exogenous discount rate. Coupled with the results that technology innovations have no long-term effects on share prices it can be concluded that the principal results obtained from the model with an exogenous discount rate continue to hold with an endogenous discount rate.

4 Empirical results

The implications of the model outlined above are tested in this section. Patent applications and R&D expenditures are used to measure the innovative activity as discussed in Section 2. The Cobb-Douglas technology assumption is adopted throughout the empirical section so that capital productivities, Y/K , vary proportionally with the marginal productivity of the capital stock, $(1-\alpha) \cdot Y/K$. Key aspects of the model in Section 3 are (1) that capital productivity shocks are only temporary and therefore have only temporary effects on equity prices; and (2) productivity shocks

¹⁷ One problem associated with their hypothesis is that there is scant evidence that a forthcoming ICT revolution was expected in early 1970s.

lead to higher tangible and intangible capital stocks in the long run, which drive capital productivity back to its base level. These allow us to derive the following testable hypotheses:

Hypothesis 1. Share markets predict intangible and tangible capital productivity. This follows from the fact that share markets react instantaneously to news of innovations, which owing to adjustment costs are only embodied later in capital.

Hypothesis 2. The response of intangible and tangible capital productivity to share prices is temporary and soon reversed, consistent with the dynamic path in share prices analysed in the phase diagrams in the previous section.

Our main focus is on R&D and patent capital, although we also present results relevant for assessing these hypotheses for tangible capital. The consistency of results across differing measures of capital is an important robustness check for our results. The hypotheses are tested using a combination of Granger causality, VAR and panel estimation methodologies. A preliminary to estimation is testing for unit roots, since variables entering a Granger Causality or VAR system should normally be stationary.

The results of Dickey-Fuller tests for the period 1965-99 over which we have data for R&D capital are shown in Table 1. They indicate that the second difference of the log of prices and the first difference of the log of productivity growth and real share price growth are stationary. Real long term interest rates and the dividend yield are borderline stationary. Share market volatility (the standard deviation of monthly share price changes, deflated by the CPI) and real equity returns are consistently stationary in levels. The deviation of GDP from a Hodrick-Prescott (HP) filter, justification for which is discussed below, is also stationary in levels by construction.

Whereas most of these results are as expected, note that real long term interest rates and dividend yields would generally be expected to be stationary in levels and the price level stationary in differences. The short sample may explain why these results are not obtained – we choose to retain the conventional variables – i.e. the level of the real long rate and dividend yields (where used) and the first difference of the log CPI - on the basis that the fact that these variables are difference stationary implies stationarity in variance. This is consistent with them being $I(0)$ about a trend or drifting $I(0)$ variables, which can still be bounded over a longer-term sample (as shown by long-sample unit root tests reported in Davis and Madsen (2001)).

Table 1: ADF Unit root tests (1965-99, annual data)

	US	DE	CA	UK	FR	IT	JP	DK	AU	NE	SE	Panel
<i>RLR</i>	-2.6	-2.3	-1.6	-2.4	-1.3	-2.1	-2.6	-1.9	-1.4	-1.9	-1.7	-2.0
Δ <i>RLR</i>	-4.6	-4.5	-4.2	-5.3	-4.0	-5.1	-6.4	-5.5	-5.5	-5.5	-6.1	-5.2
<i>VOL</i>	-5.2	-3.2	-4.1	-3.3	-3.1	-4.0	-2.7	-3.7	-3.9	-4.8	-3.5	-3.8
<i>EQR</i>	-4.5	-5.0	-5.6	-6.2	-5.1	-4.3	-5.2	-4.7	-5.8	-4.2	-3.9	-5.0
Δ <i>LRSP</i>	-4.4	-5.2	-5.2	-4.9	-4.2	-4.2	-5.0	-4.7	-4.9	-4.1	-4.1	-4.6
<i>LYD</i>	-4.6	-4.0	-3.6	-3.8	-3.4	-4.7	-3.1	-4.1	-3.3	-3.0	-4.2	-3.8
Δ <i>LCPI</i>	-2.6	-2.6	-1.8	-2.0	-1.4	-1.5	-2.0	-1.3	-1.4	-1.7	-1.5	-1.8
Δ Δ <i>LCPI</i>	-5.0	-3.5	-4.3	-4.8	-3.6	-4.2	-6.1	-5.3	-5.0	-4.6	-5.5	-4.7
Δ <i>LRDKP</i>	-4.2	-1.5	-2.5	-4.1	-2.0	-1.9	-3.5	-2.4	-3.0	-2.8	-2.8	-2.8
Δ <i>LTKP</i>	-5.3	-4.2	-4.0	-3.9	-3.8	-5.2	-3.7	-2.8	-4.1	-5.0	-5.4	-4.3
Δ <i>LTFPRD</i>	-3.7	-3.8	-3.3	-4.0	-2.0	-5.3	-2.7	-5.9	-4.4	-2.6	-3.2	-3.7
Δ <i>LPAKCP</i>	-4.2	-3.1	-2.8	-4.3	-1.7	-1.9	-3.5	-0.8	-3.2	-1.9	-2.1	-2.7
<i>DY</i>	-1.0	-1.8	-1.2	-3.6	-1.9	-2.7	-2.3	-2.9	-1.9	-2.6	-1.8	-2.2

Key: *RLR* = real long interest rate, *EQR* = real total return on equity, *CPI* = consumer price index, *VOL* = real share price volatility, *TFPRD* = total factor productivity including *R&D*, *TKP* = tangible capital productivity, *RDKP* = R&D capital productivity, *PATKP* = patent capital productivity, *YD* = deviation of GDP growth from the Hodrick-Prescott filter, *DY* = dividend yield, *KP* = tangible capital productivity. Δ Δ before the variable name indicates first difference, an *L* stands for log. Approximate critical values 10%: -2.6; 5%: -2.9; 1%: -3.6. Country code: DE = Germany, CA = Canada, FR = France, IT = Italy, JP = Japan, DK = Denmark, AU = Australia, NE = the Netherlands, and SE = Sweden. Panel: panel unit root following method of Im, Pesaran and Shin (1995).

4.1 Granger causality tests

For testing of Hypothesis 1, we initially undertook Granger causality tests on the relationship between the real return on equity and the marginal productivity of R&D and patent capital. Real returns to equity were computed as the proportional change in the real share index less inflation plus the dividend-price ratio. The Granger causality test assesses whether there is a consistent pattern of shifts in one variable preceding the other. Such tests do not give any proof on causality, but nevertheless where causal mechanisms based e.g. on expectations can be suggested, as outlined above, then a positive result gives grounds for further investigation.

Granger causality can only be a starting point in empirical investigation. Notably, there are a number of additional influences on real equity prices, so a multivariate regression approach needs to be adopted before reaching any conclusions. On the other hand VAR analysis as undertaken below has

some disadvantages, such as the problem of recursive ordering etc., that are not present in the Granger analysis and it is therefore an invaluable complement to the VAR analysis.

Following appropriate tests of lag length, tests were undertaken with two lags and data from 1965 to 1999, with the log of productivity differenced to ensure stationarity. As shown in the first two columns of Table 2 below, the broad conclusion is that we can reject the hypothesis that the equity return does *not* Granger-cause R&D productivity growth, for the vast majority of countries. On the other hand, realised R&D productivity growth does not precede equity returns. This is wholly in line with our theory as set out in the phase diagram (Figure 3). News of a technical innovation that increases the productivity of R&D capital gives rise to higher equity returns, which stimulate actual increases in R&D productivity via investment. This is consistent with the forward-looking nature of equity markets.

A similar test for patent capital productivity returned very similar results, as shown in the last two columns of Table 2, suggesting that equity returns also anticipate the outcome of R&D investment in terms of patents. All countries showed Granger causality from equity returns to patent capital productivity, while only Australia, Germany and the Netherlands showed a two-way Granger causation.

Table 2: Granger causality tests for equity returns and R&D and patent capital productivity growth (*F*-test and *P*-value) 1965-99.

	Equity return does not Granger cause Δ LRDKP	Δ LRDKP does not Granger cause equity return	Equity return does not Granger cause Δ LPATKP	Δ LPATKP does not Granger cause equity return
Australia	4.3 (0.02)**	0.82 (0.45)	3.7 (0.04)*	2.7 (0.09)*
Canada	6.6 (0.0)**	0.53 (0.6)	5.7 (0.01)**	0.3 (0.7)
Germany	4.8 (0.02)**	1.0 (0.4)	3.7 (0.04)**	4.7 (0.02)**
Denmark	3.0 (0.06)*	0.3 (0.7)	3.0 (0.06)*	1.2 (0.3)
France	2.1 (0.14)	0.3 (0.73)	3.1 (0.06)*	0.78 (0.46)
Italy	0.7 (0.5)	1.3 (0.3)	2.9 (0.07)*	0.01 (0.9)
Japan	5.2 (0.01)**	1.8 (0.19)	5.5 (0.01)**	1.8 (0.18)
Netherlands	8.2 (0.0)**	2.9 (0.07)	6.6 (0.0)**	9.2 (0.0)**
Sweden	4.7 (0.01)**	0.7 (0.52)	3.7 (0.04)**	1.4 (0.25)
UK	3.4 (0.05)**	1.4 (0.27)	2.8 (0.08)*	1.4 (0.27)
US	9.2 (0.0)**	0.5 (0.6)	7.8 (0.0)**	1.0 (0.4)

Key: See Table 1. ** indicates rejection of the hypothesis at 5% and * at 10% level.

A potential criticism of such results is that the Granger causality could simply be from equity returns to GDP growth which is a component of these measured productivity growth figures. This would be consistent with the leading indicator property of share prices which is often detected in empirical work. But as shown in Table 3 there is no Granger-causality relation between equity returns and labour productivity, over the same data period, which is inconsistent with this suggestion. The only case where Granger causality is accepted is for an inverse relation in Germany and the Netherlands. This set of results also lends support to the hypothesis that it is not labour productivity that drives equity returns, contrary to the suggestions of some of the works cited in Sections 1 and 2.

Table 3: Granger causality tests for equity returns and labour productivity (*F*-test and *P*-value) 1965-99

	Equity return does not Granger cause Δ LLP	Δ LLP does not Granger cause equity return
Australia	0.02 (0.98)	1.8 (0.19)
Canada	2.2 (0.12)	0.14 (0.87)
Germany	0.9 (0.4)	3.6 (0.04)**
Denmark	0.34 (0.7)	2.1 (0.15)
France	0.66 (0.52)	0.42 (0.66)
Italy	1.7 (0.2)	0.56 (0.58)
Japan	0.66 (0.52)	0.76 (0.47)
Netherlands	2.3 (0.12)	5.6 (0.0)**
Sweden	2.0 (0.14)	1.0 (0.37)
UK	1.5 (0.22)	0.4 (0.7)
US	1.1 (0.35)	0.09 (0.91)

Key: See Table 1, Δ LLP is difference of log of labour productivity. ** indicates rejection of the hypothesis at 5% and * at 10% level.

To compare and contrast with the results for patent and R&D productivity, we present a similar set of estimates featuring tangible capital productivity and the same additional variables. In this case we test the corresponding hypotheses to those set out above as drawn from Section 3. As was the case for R&D and patent capital productivity, the results for Granger causality in Table 4 are unequivocal in suggesting that equity returns Granger-cause capital productivity but the opposite is not the case. Only in Denmark and Italy are conventional significance levels not attained, and even there the result is far closer to rejection of the null than for a causal role for capital productivity.

Table 4: Granger causality tests for equity returns and tangible capital productivity (*F*-test and *P*-value) 1965-99

	EQR does not Granger cause Δ LKP	Δ LKP does not Granger cause EQR
Australia	3.39 (0.047)**	1.88 (0.17)
Canada	5.61 (0.0089)**	0.32 (0.73)
Germany	7.62 (0.002)**	0.01 (0.91)
Denmark	2.05 (0.15)	0.47 (0.63)
France	2.81 (0.08)*	0.63 (0.54)
Italy	1.69 (0.2)	0.44 (0.65)
Japan	4.03 (0.03)**	1.18 (0.32)
Netherlands	12.8 (0.0001)**	0.43 (0.66)
Sweden	9.49 (0.0006)**	0.70 (0.50)
UK	4.16 (0.03)**	1.19 (0.32)
US	9.06 (0.0008)**	0.65 (0.53)

** indicates rejection of the hypothesis at 5% and * at 10% level.

4.2 VAR estimates

To cast further light on Hypothesis 1 and also to address Hypothesis 2 we proceeded to wider estimation using multiple variables. The aim is to provide some quantitative estimates of the relationship between R&D or patent capital productivity and equity returns in the presence of related variables determining this nexus. This enables us to assess *inter alia* whether Granger causality results from omitted variables.

There is a voluminous literature on the determination of equity returns and their predictability which provides relevant background to our choice of variables. A helpful starting point for considering equity price determination is in terms of the Gordon valuation model, as employed recently for example by Harasy and Roulet (2000), Jagannathan *et al* (2001) and Nasseh and Strauss (2003). This highlights expected dividend growth, g , as well as real long term interest rates, rr , and the risk premium, pr , as key determinants of share valuations, V :

$$V_o = (D_0(1+g) + P_{t+1})/(1+(rr_{t+1}+pr_{t+1})) \quad (20)$$

$$V_o = D_{t+1}/(1+(rr_{t+1}+pr_{t+1})) + D_{t+2}/(1+(rr_{t+2}+pr_{t+2}))^2 + D_{t+3}/(1+(rr_{t+3}+pr_{t+3}))^3 \quad (21)$$

$$V_o = D_{t+1}/((rr + pr) - g) \quad (22)$$

Equation (20) shows that the value of a share depends on the dividend and the future price. The latter, as shown in equation (21) depends on future dividends suitably discounted. As shown in

equation (22), if dividend growth, the interest rate and the risk premium are expected to be constant, a series of discounted dividends can be simplified to an expression in dividend growth, the real interest rate, the risk premium and the level of dividends¹⁸. Equation (22) also follows from the theoretical framework in the previous section.

Equations (20) to (22) highlight that the real bond rate, expected dividends, and the equity risk premium are key determinants of share prices and will be included in the estimated models in this section. However, in contrast to the traditional valuation models based on the Gordon approach, it can be shown in the general equilibrium approach, as used in the previous section, that changes in these variables have only temporary effects on share prices.

We include capital productivity growth as a proxy for expected dividend growth.¹⁹ Meanwhile, real share price volatility will be employed as a proxy for investor uncertainty and the risk premium. In addition to the variables highlighted in the Gordon framework, measures of inflation are commonly considered to affect real equity returns. As noted by Fama (1981), expected inflation may be negatively correlated with shocks to future economic growth and thus affect share prices. Furthermore, finance theory suggests that in markets with risk averse agents, stock returns would vary with the state of the business cycle (Balvers *et al*, 1990).

The dividend yield is often thought to be a proxy for time variation in expected returns, see for example Campbell *et al* (1997). In this context, there is a very large literature which seeks to assess the forecasting power of the dividend yield over equity returns. Work by Fama (1990) and Schwert (1990) suggested that there was a strong and stable predictive power to the dividend yield, implying it should be included in regressions including the equity return (see also Campbell and Shiller, 1989, Fama and French, 1988, and the survey by Cochrane, 1997). On the other hand, in an international study, Canova and De Nicolo (1995) showed that the dividend yield's predictive power was limited to the UK and US, while more recent work by Goyal and Welch (2002) and Ang and Bekaert (2001) suggests that the predictive power of dividend yields in the US broke down in the 1990s. Robertson and Wright (2003) show that the predictability can be restored in the US by adjustments to dividends to include all cashflows to shareholders. Unfortunately, data are not available for such an exercise on

¹⁸ As noted by Harasy and Roulet (2000) the discount factor is in principle the weighted sum of future short rates, but following the expectations theory of the term structure it may justifiably be replaced by the long rate.

¹⁹ Dividend growth equals the returns on new investment multiplied by the retention ratio. Assuming that the retention ratio is approximately constant in the estimates it will be absorbed by the coefficient of capital productivity growth.

a multicountry basis. In a similar vein, McDonald and Power (1995) adopt the approach of replacing dividends with earnings, which are found to give superior predictors.

A general approach to equity return prediction is given by Pesaran and Timmermann (1995, 2000) who sought to simulate investors' search in real time for a model to forecast future stock returns in the UK. They found that key determinants include the dividend yield, the short rate, inflation, monetary growth, changes in oil prices and growth in industrial production as a cyclical variable.

In the context of the above discussion, and following the standard theory of equity price determination with Gordon's growth model, in our VAR we add to real equity returns and the log difference of R&D or patent capital productivity, the real long bond yield and real equity price volatility as a proxy for the equity risk premium, where real bond yield is estimated as the redemption yield on long government bonds minus the actual rate of consumer price inflation. These represent the discount factors applied to projected future dividends as proxied by the measure of capital productivity. Finally, we added the difference between the change in the log of GDP and the HP filtered GDP to allow for cyclical and effects on share prices, as well as the log difference of consumer prices given the observed sensitivity of real equity returns to inflation. We omit the dividend yield from our main approach given the breakdown of its predictive power in the 1990s and lack of power in countries other than the US and UK. However, we test whether its inclusion changes our results in a variant below.

A standard VAR system is the reduced form of a linear dynamic simultaneous equation model in which all variables are treated as endogenous. Each variable is regressed on lagged values of itself and on lagged values of all other variables in the information set. As noted by Canova and De Nicolo (1995), VARs can approximate arbitrarily well the joint unconditional distribution of variables of interest in the relation between stock returns and intangible capital productivity while standard OLS estimates cannot. To test our hypotheses we need to orthogonalise the estimated reduced form VAR model to identify the effect of shocks to the innovations of the variables in the VAR. The standard Choleski decomposition is used to identify the responses in VAR models. Identification then uses the Sims's triangular ordering. A well-known problem with the Sims triangular ordering is that it is arbitrary, and requires a justification for the ordering chosen. The presence of common shocks and co-movements among the variables makes the decision on ordering a crucial one.

As regards the recursive ordering, following Canova and De Nicolo (1995) and Nasseh and Strauss (2000), exogenous shocks that are largely technology driven will first affect R&D capital productivity, patent capital productivity and then output via investment. Shocks which are related to fiscal policy will affect output but not capital productivity. Hence for both kinds of shock the logic is for productivity to precede output in the recursive ordering. Stock returns, in line with the present value model, respond according to the effect of these shocks on expected future cash flow. As discussed, stock prices may also respond to changes in inflation, long-term real rates and real share price volatility, which may all also be affected by technological factors feeding through R&D capital productivity and other shocks. Hence, we order the variables with R&D or patent productivity first, followed by the output deviation from the HP filtered trend, the change in inflation, the change in long rates and real equity price volatility before real equity returns themselves. Real equity returns are thus constrained to only feed back on the other variables with a lag. Note that this need not exclude a marked leading indicator property of share prices and unpredictability of returns, if the data suggest it. We also tested for sensitivity by reversing the ordering for the US, as reported in the tables, which did not substantively change the results, and for patent capital productivity for a long data set since 1871 for the US only.

We began with tests for lag length, using the sequential modified LR test statistic, the final prediction error, the Akaike information criterion, the Schwarz information criterion and the Hannan-Quinn information criterion. In France, the Netherlands and the US the tests were unambiguous in selecting two as the appropriate lag length. In all other countries all but the Schwarz criterion lead to this conclusion. Accordingly, we selected two lags as appropriate in all cases.

Block exogeneity tests (Table 5) reveal that capital productivity measures are always endogenous to the VAR, while the other variables are less commonly endogenous across all the countries in our sample. Equity returns in particular are commonly exogenous, with the exceptions of the US (since 1965), UK, Netherlands, Italy (R&D), Germany (patent) and Sweden (patent). Interestingly, all variables are endogenous for the US and UK, with the exception of volatility for the US and equity returns since 1871. Exogeneity to domestic variables may indicate that there is a role for foreign variables in the smaller countries or where equity markets are less active. This is tested in the variants below.

Table 5: Block exogeneity tests using data over the period 1965 to 1999

Year	VAR with R and D capital productivity						VAR with patent capital productivity					
	Δ LR-DKP	LYD	Δ LCPI	VOL	RLR	EQR	Δ LPA-TKP	LYD	Δ LCPI	VOL	RLR	EQR
Australia		*		*		*		*		*		*
Canada						*						*
Germany		*		*		*		*		*	*	
Denmark			*	*	*	*		*	*	*	*	*
France		*	*	*		*		*	*	*	*	*
Italy		*			*				*		*	*
Japan				*		*		*		*		*
Netherlands				*	*					*		
Sweden		*				*						
UK												
US				*						*		
US since 1871	na	na	Na	na	na	na						*

Key: See Table 1. Exogenous at * 5% or ** 10%.

The key outputs of a VAR for the purposes of our current exercise are the variance decomposition and impulse responses. There may be effects in the whole system that are hidden from individual equations. With a model of this sort, there is a large amount of output generated by this exercise: six equations, subject to six different shocks, give 36 solutions. Therefore, only a few key results are presented. Given the focus of the work on real equity returns and R&D capital or patent capital productivity, we report only the variance decomposition of real equity returns to shocks in the innovations to productivity, and of productivity to real equity returns, together with the impulse response of productivity to equity returns because these are the focus variables of the study.

For R&D capital, the variance decompositions show the degree to which the variance of the “independent variables” explains the forecast variance of the target variable in the VAR system. Table 6 shows that equity returns help explain a significant proportion of R&D productivity in Canada, Italy, Japan and the US, suggesting forward looking behaviour by equity holders in response to expected increases in productivity. The US is similar with the ordering reversed. The opposite result is found for Sweden and Japan. This may of course relate to the fact that Sweden is a small country whose markets are subject to strong international influences. Also the Japanese market has been severely depressed for a decade in the 1990s despite a highly innovative environment.

Results for patents are very similar to those for R&D with significant proportion of patent capital productivity explained in Canada, Japan, the Netherlands and the US, and the opposite in Australia, Germany and the Netherlands. For the US when we extended the sample back to 1871 we found a very similar result for variance decomposition, with share prices predicting patent productivity also over this long sample, but not vice versa.

Table 6: Variance decompositions for real equity returns and R&D capital productivity (percent of forecast variance accounted for by variance in each variable) 1965-99.

	Δ LRDKP on EQR	EQR on Δ LRDKP	Δ LPATKP on EQR	EQR on Δ LPATKP
Years	4	4	4	4
Australia	15	10*	20**	8
Canada	8	20**	2	19**
Germany	16	2	16*	3
Denmark	22**	8	11	4
France	5	10	7	8
Italy	6	5	16	8
Japan	25*	25**	14	24**
Netherlands	17	5	19*	11**
Sweden	13	24**	9	7
UK	16	2	12	0
US	3	15**	4	14*
Memo: US with ordering reversed	3	26**	3	26**
US since 1871			3	20**

Key: See Tables 1 and 4. ** indicates rejection of the null hypothesis at 5% and * at 10% level.

Turning to impulse responses, as shown in Tables 7 and 8, a remarkable result emerges for effects of shocks to share prices on intangible capital productivity, in that a rise in real equity returns tends to raise capital productivity in year 2 but then depress it markedly in succeeding years. This is consistent with the valuation ratio effect as highlighted in the theory section, whereby high equity returns in response to a technical innovation prompt increasing R&D investment and patent applications, which given diminishing marginal productivity of intangible capital leads to lower capital productivity but permanently higher capital. For both R&D and patent capital productivity, the pattern is common to all countries and is significant at least in part of the cycle in Australia, Canada, Denmark (R&D), Japan, the Netherlands, Sweden, the UK (R&D) and the United States. Note that the long period sample for the US from 1871 gives significant responses throughout the five year period. In all cases the response returns to a level insignificantly different from zero in 5-10 years.

We repeated the impulse responses with the Pesaran and Shin (1998) generalised response approach. Technically, it constructs an orthogonal set of innovations that does not depend on the VAR ordering. The generalized impulse responses from an innovation to the j -th variable are derived by applying a variable specific Cholesky factor computed with the j -th variable at the top of the Cholesky ordering. This is not our preferred method as we consider that there are good economic reasons for the ordering we have chosen. However, the results are very similar in terms of the profile of the response, although the lags in years 3-5 are less commonly significant as seen from the lower part of Table 7. Again, the results are repeated for patent capital productivity (Table 8), with the long data sample for the US again having a significant response throughout.

Table 7: Impulse response functions for effect of change in real equity returns on change in R&D capital productivity (percent responses to 1 standard deviation shocks in real equity returns) 1965-99

Year	1	2	3	4	5
Australia	0	0.0003	-0.006**	0.002	0.001
Canada	0	0.005*	-0.007	-0.006	0.002
Germany	0	0.002	-0.002	0.000	0.003
Denmark	0	0.004	-0.005*	0.000	0.003
France	0	0.003	-0.003	-0.003	-0.001
Italy	0	0.005	-0.003	-0.004	0.005
Japan	0	0.007*	-0.006*	-0.009*	-0.003
Netherlands	0	0.004**	-0.002	-0.003	-0.001
Sweden	0	0.009**	0.0002	-0.008	-0.006
UK	0	0.002	-0.0003	-0.002	-0.004**
US	0	0.006**	-0.004*	-0.006*	-0.001
Unweighted average	0	0.0027	-0.0024	-0.0027	0.0002
Memo: with Pesaran – Shin Generalised IR					
Australia	0.002	0.001	-0.01**	0.002	0.001
Canada	-0.003	0.007*	-0.008	-0.006	0.002
Germany	-0.008*	0.003	-0.003	-0.003	-0.001
Denmark	0.006	0.006	-0.01**	-0.003	0.000
France	-0.002	0.004	-0.002	0.000	0.001
Italy	0.0002	0.004	-0.002	-0.003	0.005
Japan	0.001	0.006	-0.006	-0.008	-0.003
Netherlands	-0.005	0.01**	0.001	-0.003	-0.002
Sweden	-0.003	0.009**	0.003	-0.005	-0.005
UK	-0.002	0.005*	-0.0004	-0.002	-0.0006
US	-0.002	0.007**	-0.005	-0.008	0.002
Unweighted average	-0.0007	0.004	-0.0025	-0.0035	-0.00005

Table 8: Impulse response functions for effect of change in real equity returns on change in patent capital productivity with starting values (percent responses to 1 standard deviation shocks in real equity returns) 1965-99

Year	1	2	3	4	5
Australia	0	0	-0.007**	-0.001	-0.001
Canada	0	0.006**	-0.008*	-0.007*	-0.001
Germany	0	0.003	-0.002	-0.002	0.004
Denmark	0	0.002	-0.012	-0.004	-0.002
France	0	0.003	-0.003	-0.003	-0.000
Italy	0	0.002	-0.006	-0.005	0.000
Japan	0	0.01**	-0.004	-0.01	-0.005
Netherlands	0	0.003	-0.006*	-0.008*	-0.003
Sweden	0	0.003	0.001	-0.007**	-0.006**
UK	0	0.000	-0.000	-0.001	0.003
US	0	0.006**	-0.004*	-0.006*	-0.002
Unweighted average	0	0.002	-0.00371	-0.00371	-0.0007
US from 1871	0	0.016**	-0.011**	-0.011**	-0.007*
Memo: with Pesaran Generalised IR					
Australia	0.002	-0.001	-0.01**	-0.003	-0.002
Canada	0.001	0.006*	-0.009	-0.008	-0.001
Germany	-0.006	0.004	0.000	-0.000	0.003
Denmark	0.009	0.008	-0.009	-0.001	0.002
France	-0.003	0.002	-0.005	-0.003	-0.000
Italy	-0.004	0.0001	-0.009	-0.009	-0.004
Japan	0.004	0.011**	-0.005	-0.009	-0.005
Netherlands	-0.007	0.001	-0.012*	-0.014	-0.007
Sweden	0.001	0.003	-0.002	-0.010**	-0.009**
UK	-0.005	0.002	-0.002	-0.004	-0.003
US	-0.001	0.008**	-0.005	-0.008	-0.002
Unweighted average	-0.0008	0.0024	-0.0051	-0.006	-0.002
US from 1871	0.004	0.017**	-0.013**	-0.013**	-0.01**

Key: See Table 1.** significant at 5%, * significant at 10% level.

We ran three variants on the VAR to test for robustness in the case of alternative regressors and the results are shown in Table 9. One was to include the difference of log GDP instead of its detrended counterpart (marked Δ LGDP instead of LYD). The second was to split the equity return into the rise in share prices and the dividend yield, thus allowing the latter a separate effect. The third was to allow for foreign influence via the foreign equity yield. The experiments were performed for the US, Japan and the UK and for R&D capital productivity. In all cases the pattern of the impulse response of productivity to share returns or share prices is the same, with an initial rise followed by a reversal. In most cases, at least one of the annual responses is significantly different from zero.

Table 9: Variants on the basic VAR – impulse responses for R&D capital productivity (1965-99)

Years:		2	3	4	5
(1) Δ LGDP instead of LYD Response to equity returns	US	0.005**	-0.004**	-0.002	0.002
	UK	0.003	0.002	0.000	-0.003
	JP	0.008**	-0.003	-0.005*	0.003
(2) Split equity return Response to difference of log equity prices	US	0.002	0.001	-0.003**	
	UK	0.001	0.001	-0.002	-0.002
	JP	0.001	0.000	-0.005**	-0.003
(3) Including USEQR as exogenous (UKEQR for US) Response to equity returns	US	0.002	-0.001	-0.002	-0.001
	UK	0.002	0.000	-0.001	-0.004**
	JP	0.007**	-0.005	-0.008**	-0.005

Key: See Table 1

Following the same modelling strategy as above, Table 10 shows that the variance decomposition results for tangible capital productivity which suggest that equity returns explain the variance of capital productivity significantly in Canada, France, Italy, Japan, the Netherlands and the US. Again this is more countries than for R&D capital productivity. Only in Australia and the UK is the opposite the case.

Table 10: Variance decompositions and impulse response functions for effect of change in real share prices on change in tangible capital productivity (percentage responses to 1 standard deviation shocks in real share prices) 1965-99

	Variance decomposition (after 4 years)		Impulse response (years)				
	Δ LKP on EQR	EQR on Δ LKP	1	2	3	4	5
Australia	20**	6	0	0.0006	-0.0041*	0.0025	0.0026
Canada	5	23**	0	0.0051	-0.0099**	-0.0053	0.0019
Germany	10	2	0	0.0017	-0.0022	-0.0007	0.0033
Denmark	6	8	0	0.0013	-0.0058**	0.0004	0.0015
France	6	16**	0	0.0031**	-0.0030*	-0.0014	-0.0007
Italy	6	14**	0	0.0043	-0.0052	-0.0039	0.0035
Japan	16	21**	0	0.0044	-0.0082*	-0.0069	-0.0010
Netherlands	9	16**	0	0.0072**	-0.0016	-0.0041	0.0007
Sweden	9	22**	0	0.006**	-0.0013	-0.0073*	-0.0042
UK	22**	1	0	0.0018	-0.0003	-0.0013	-0.0024
US	6	20**	0	0.0045**	-0.0060**	-0.0051*	-0.0002
Memo: Unweighted average			0	0.0027	-0.0021	-0.0023	0.0005
Memo: US with ordering reversed	3	31**	-0.1	0.0069**	-0.0062*	-0.0063	0.0022

Key: See Table 1. ** indicates rejection of the hypothesis at 5% and * at 10% level.

A particularly relevant result for the model in Section 3 was again the impulse response from share prices to productivity, which for a number of countries shows a dynamic pattern as predicted by the phase diagrams, with an initial rise soon reversed, with a zero net effect. The results displayed in Table 10 give strong support for the predictions of the model that is outlined in Section 3, namely an increase in capital productivity growth following increasing share prices which is followed by a decline in capital productivity as the share-price-induced investment pushes capital marginal productivities down.

4.3 Cointegration tests

The VARs above by construction do not allow for a long run effect of equity returns on productivity growth, since we use stationary series. A simple assessment of whether there could be long run positive effects, as is implicit in the literature on the New Economy, is to test for cointegration between the underlying non stationary series, the log of the real accumulated share index (dividends reinvested) and the log of intangible capital productivity. A time-trend is included in the estimates to allow for constant required returns.

Table 11: Results of trace test for cointegration of log of productivity and log of accumulated real share prices (Trend included in cointegrating equation)

Number of cointegrating vectors

	1965-99		Long term data (from)
	Log R&D Productivity	Log Patent productivity	Log Patent productivity
Australia	None	None	2 (1885)
Canada	None	None	None (1918)
Germany	None	1	None (1886)
Denmark	None	None	1 (1918)**
France	None	None	None (1874)
Italy	None	None	None (1904)
Japan	None	None	None (1917)
Netherlands	1*	None	None (1923)
Sweden	1*	1*	2 (1904)
UK	None	None	None (1926)
US	None	1	None (1874)

Note: * only time trend is significant in cointegrating relationship, ** negative sign on share prices.

As shown in Table 11, the trace test indicates that in the vast majority of cases, there is no cointegration detectable between share prices and intangible capital productivity. This indicates that there is no long run impact of shocks to productivity growth on equity returns. It also means that the

stationary VAR is the appropriate modelling methodology. In the few cases where some evidence of cointegration was found, we tended to find the impulse response in the corresponding vector-error correction model to show that consistent with the model, productivity responds negatively to a shock to share prices in the long term, after an initial boost. We also assessed results with a wider range of cointegrating variables including the log of the CPI and GDP. Although there were more cases of cointegration, it still accounted for less than half of the countries.

A further cointegration test relevant for the model is to assess whether there is a long run relationship between capital productivity and real R&D expenditures and patents. This gives a view as to whether the trend in capital productivity is related with permanent growth in expenditures, which would be contrary to the predictions of the model. Again, as shown in Table 12, the majority of cases show either no cointegration or only the time trend as significant. These results reinforce the finding above that there are no long-run effects of productivity innovations on equity returns (indicating earnings per unit of patent or R&D capital).

Table 12: Results of trace test for cointegration of log of productivity and log of patent count and of R and D expenditures (Trend included in cointegrating equation)

Number of cointegrating vectors

	1965-99		Long term data (from)
	Log R&D Expenditures	Log Patent count	Log Patent count
Australia	None	None	1 (1878)
Canada	None	None	None (1918)
Germany	2*	1	None (1886)
Denmark	None	1	1 (1918)
France	2*	None	None (1921)
Italy	2	1	None (1909)
Japan	2	None	None (1916)
Netherlands	2	1	None (1922)
Sweden	1*	1	1 (1904)
UK	None	None	1 (1874)
US	None	None	None (1874)

Note: * only time trend is significant in cointegrating relationship.

4.4 Panel data estimates

As a further check on the model, panel data estimates are undertaken to gain efficiency by increasing the number of observations and by exploiting the contemporaneous correlation between the error terms. Furthermore, in addition to lagged variables contemporaneous regressors are included in the

estimates in this subsection, which enables us to trace fully the dynamic path of share prices and capital productivity as predicted by the model in Section 3. Again, we are interested in testing whether the result of short-term causality from equity to capital productivity is maintained in this framework. The seemingly unrelated regression method (SUR) is used, given possible simultaneous cross-country effects from international shocks such as wars, technology shocks, changing exchange rate regime, worldwide monetary and fiscal shocks, and commodity price shocks. Moreover, the estimates are weighted by cross-country variances to alleviate the potential effects of cross-country heteroscedasticity on the parameter estimates. All of the explanatory variables considered in the previous sub-sections are included in the estimates. Two lags of each variable are included in the estimates because further lags were insignificant. Country dummies were initially included in all estimates, but, except in one case, deleted because they were insignificant. Consumer price inflation was omitted from the patent and the R&D capital productivity equations, without affecting the results, because it created positive serial correlation in the residuals.

The model for patent capital productivity was estimated over the period from 1925 to 1999 and the model for R&D capital productivity was estimated over 1968-99 for all 11 countries considered in the paper. Instruments are used for all contemporaneous variables to deal with simultaneity. The current value of the dividend yield is not included in the share returns equations, and current GDP growth is not included in the capital productivity equations because of parts in common with the dependent variable. The instruments are listed in the notes to Table 12.

The estimation results are shown in Table 13. The diagnostic tests are based on “within” individual OLS residuals in order to remove the fixed country effects. The diagnostic tests do not indicate the presence of first-order serial correlation and the Breusch-Pagan LM tests for an off-diagonal covariance matrix strongly reject the null hypothesis of no cross-country correlation between the error terms, which suggests substantial efficiency gains from the SUR estimation method. Despite the potential efficiency gains from using the SUR method in the shorter period (1968-1999) the degrees of freedom to estimate the covariance terms are quite limited; thus potentially rendering the parameter estimates unreliable. However, since the SUR/IV estimates were almost identical to the OLS/IV method only the SUR/IV estimates are shown in the estimates below.

The null hypothesis of cross-country coefficient constancy cannot be rejected at any conventional significance level, as indicated by the F -tests. It follows that the coefficient estimates, which are restricted to be the same across countries, are unbiased. Leamer’s (1978, p. 114) formula is used to

calculate the critical F -values of diffuse priors, which takes into account that the likelihood of rejecting the null hypothesis grows with the sample size. The critical values are presented in Table 13.

Table 13: Pooled time series and cross section estimates

Estimation period	1925-1999		1968-1999	
	Patent productivity	Share returns	R&D Productivity	Share returns
EQR	0.17(3.87)		0.07(6.28)	
EQR(-1)	-0.02(1.83)	0.01(0.01)	0.04(5.20)	0.32(0.99)
EQR(-2)	-0.01(0.10)	-0.07(2.26)	0.00(0.65)	-0.32(3.08)
LYD		-4.63(3.66)		-11.4(2.06)
LYD(-1)	-1.62(7.03)	3.04(2.78)	-0.55(5.43)	0.05(5.48)
LYD(-2)	0.18(1.41)	0.41(0.65)	-0.12(0.64)	0.12(0.10)
DY(-1)	-0.00(1.76)	0.02(3.47)	-0.00(2.12)	0.05(5.49)
DY(-2)	0.00(1.20)	0.00(0.32)	0.00(1.99)	-0.02(2.08)
LR	-0.11(0.97)	-0.01(1.23)	-0.87(6.15)	-0.15(1.16)
LR(-1)	0.10(1.04)	0.02(1.81)	0.63(6.48)	0.12(1.30)
LR(-2)	-0.01(1.48)	-0.01(1.49)	-0.10(2.49)	-0.00(0.10)
Δ LCPR		0.43(1.06)		-0.17(2.93)
Δ LCPR(-1)		0.01(3.39)		0.22(2.23)
Δ LCPR(-2)		-0.03(1.82)		-0.24(2.26)
Δ LPATKP		2.22(5.32)		0.71(1.57)
Δ LPATKP(-1)	0.98(9.29)	-1.60(3.46)	0.87(8.91)	-0.54(0.53)
Δ LPATKP(-2)	-0.00(0.01)	-0.57(1.15)	0.02(0.25)	0.08(0.08)
VOL	-2.69(4.27)	0.67(0.18)	-0.02(0.04)	-1.26(1.85)
VOL(-1)	1.28(4.11)	-0.35(0.19)	0.00(0.01)	-0.45(1.10)
VOL(-2)	-0.01(1.99)	0.18(1.55)	0.00(0.02)	0.40(1.42)
D_{SWE}				0.11(3.64)
Wald(3)		0.04		0.21
R^2 (mom)	0.42	0.18	0.88	0.45
F	1.78(187,627)	2.05(209,605)	1.10(176,154)	1.33(209,121)
Leamer	12.70	13.73	19.34	23.13
$DW(M)$	2.18	2.03	2.27	2.09
$\chi^2(55)$	360.74	448.93	112.6	183.56

Notes: Absolute t -statistics are given in parentheses. Constants are included in the estimates and a 1946 dummy for Japan in the estimates in the second column, but not shown. R^2 = Buse's raw moment R -squared. $DW(M)$ = modified Durbin-Watson test for first order serial correlation in fixed effect panel data models (see Bhargava *et al.*, 1982). $F(i,j)$ = F -test for cross-country coefficient constancy, and is distributed as $F(i,j)$ under the null hypothesis of coefficient constancy. Leamer = Leamer's critical value for the F -test for coefficient constancy across countries. Wald(3) = Wald test for no sustained effect of the growth of patent capital productivity on share returns (differences sum to zero), and is distributed as $\chi^2(3)$ under the null hypothesis of no sustained effect. $\chi^2(55)$ = Breusch-Pagan test for off-diagonal covariance matrix, and is distributed as $\chi^2(55)$ under the null of zero off-diagonal coefficients. A 1946 dummy variable is used for Japan. A 1925 dummy variable is included in the auxiliary instrument variable regressions for Germany. D_{SWE} = constant dummy for Sweden. The following instruments are used for EQR, LYD, LCPR, LPATKP, and VOL: Two period logs of $M1$ deflated by consumer prices, real accumulated share index, real GDP and two lags of the dependent variable.

The estimates of the patent capital stock model are shown in the first column of Table 13. The results are consistent with the predictions of the model. Patent capital productivity increases in response to an increase in the equity return, as markets rise in anticipation, but the long effect is muted by a lagged negative response. Patent capital productivity is significantly negatively related to the one-period lagged value of GDP, which suggests that cyclical upturns are associated with declining patent capital productivity because of diminishing returns to investment in patents.

The results of regressing equity returns against patent capital productivity are presented in the second column of the table. Real share returns are shown to increase in response to an increase in patent capital productivity on impact but the increase is reversed after two years following the dynamic path in the phase diagram. This length of adjustment is consistent with economic intuition that most of the fixed capital stock is adjusted to its desired level within the first three years. Inclusion of the contemporaneous variable may help explain the contrast with the Granger causality result. We also find that the sum of difference terms on patent capital productivity sum to zero so there is no sustained effect on equity returns from a shock to patents. The null hypothesis of no long-run effects on share returns of changes in patent capital productivity cannot be rejected at any conventional significance level as indicated by the Wald test in the table. The dividend yield has a predictive power on equity returns, which is consistent with the results of Ang and Bekaert (2001) who found an effect emerged in cross-country pooled data.

Turning to the estimates of the models containing R&D capital productivity the principal results from the long historical estimates remain almost unaltered (columns 3 and 4 in Table 13). The estimation results in the third column in Table 13 show that share returns significantly predict R&D productivity, which is consistent with the predictions of our model. R&D capital productivity has a positive, although statistically insignificant, effect on share returns on impact but the null hypothesis of no sustained effect on share returns of R&D capital productivity growth can again not be rejected at any conventional significance level. The one-period-lag of dividend yield again has a significant positive effect on share returns.

Our cointegration shown in Table 11 above suggested that levels terms were not cointegrated and this is reflected in our main results cited in Table 13. As a variant, we did attempt to augment the panel models for equity returns with error correction terms to test for long run effects. For patent productivity, an error-correction term generated by regressing the log of the real accumulated share index on log of patent capital productivity, fixed effect dummies, and individual-country time trends,

was not significant with a coefficient of $-0.00(0.50)$. The coefficient of patent capital productivity was itself almost zero in the cointegration estimates. We consider this the more relevant result given the long time series. However, we note that for the error-correction term for equity returns using R&D capital productivity was significant at $-0.6(6.91)$, but on the other hand, the R&D capital productivity term was negative in the cointegration relationship. This indicates that the cointegration is not generated by a genuine long run relationship between the accumulated index and capital productivity, but that the accumulated share index gravitates towards a log time trend.

As a further variant, the models were augmented with time-dummies. Many of the estimated coefficients of the time-dummies were statistically significant, but none of the results above were overturned. Share returns were regressed on current and lagged values of growth in patent counts divided by either real GDP or population to investigate the short-run and the long-run reaction in the share returns to innovations. However, none of the estimated coefficients were significant at any conventional significance level. As a final sensitivity check on the models the models were re-estimated with the interest rate, dividend yield and share volatility measured in first differences and the log CPI in second differences, following the predictions of most models of share valuation that the level of, as opposed to the growth rate of, shares is negatively related to share volatility and the real bond rate. However, the estimates were little changed by this transformation and none of the central predictions of the model were overturned in these estimates. Hence the results are not reported in detail.

5 Conclusions

This paper has presented a model of technological innovations and share prices, which has the implication that technology-induced productivity advances will only have temporary effects on share prices, since an increased capital stock in the presence of diminishing returns drives capital productivity back to its original level. The results of an empirical investigation are strongly consistent with the model, using two different measures of intangible capital, namely capitalised R&D expenditures and capitalised patent applications. It is also striking that comparable results do not emerge for labour productivity, suggesting also that we are not merely capturing the tendency for equity markets to predict the cycle. It is worth noting that our dataset ends in 1999 and hence we are not taking into account recent falls in share prices in our estimation. On the other hand, those declines in share prices observed from the peak of the bull market in early 2000 to the end of 2002, are wholly consistent with the predictions of the model. Initial rises in share prices owing to the

innovations fell back once the capital stock had built up and the level of capital productivity returned to baseline.

Data Appendix

Intangibles. Intangibles are measured as R&D and patent capital stock. To construct R&D capital stock we deflate R&D expenditures by the economy-wide GDP deflator and use the perpetual-inventory method with a 5% depreciation rate. The initial R&D capital stock is set to R&D expenditure in 1965 divided by the depreciation rate plus the average geometric growth rate in R&D over the whole data period. The perpetual-inventory method applied to the number patent applications at the 5% depreciation rate for patent capital stock. The initial patent capital stock is set equal to the number of patent applications at the initial year divided by the depreciation rate plus the average geometric growth rate in number of patents over the whole data method. **R&D.** OECD, *Science and Technology Indicators*, Paris. For USA the R&D expenditures are from Department of Commerce, 1975, *Historical Statistics of the United States: Colonial Times to 1970*, Washington DC: Bureau of the Census. **GDP deflators.** OECD, *National Accounts, Vol. 2*, Paris, (NA). **Real GDP.** NA. **Consumer prices.** IMF, *International Financial Statistics (IFS)*; **Interest rates.** IFS. **Share returns.** See Madsen (2003). **Tangible capital stock.** See Madsen (2003). **Monthly share prices.** IFS. **Total factor productivity.** Real GDP divided by capital stock to the power of 0.33 times hours worked to the power of 0.66. **Hours worked.** Economy-wide employment times weekly hours worked. See Madsen (2004) for sources. **Dividends.** Madsen (2003). **Share prices.** Madsen (2003).

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Appendix: Endogenous discount rate

This section extends the model of Abel and Blanchard (1983) to allow for embodied technological progress and technological spill-over effects. Since the system presented in Section 3 is totally decomposable between intangibles and tangibles, tangibles and intangibles are merged in this appendix to simplify the exposition, without affecting the results. All non-pricing variables in the model are measured in per capita terms.

Firms

The optimization problem of the firm facing an interest rate that varies over time is:

$$\max \Pi = \int_{t=0}^{\infty} e^{-\int_{v=0}^{\infty} r_v dv} [F(\theta_t, k_t) - w_t - i_t / \phi_t - i_t \cdot h(i_t / k_t)] dt$$

st.

$$\dot{k}_t = i_t / \phi_t - \delta k_t,$$

where the number of workers are normalised to one following Abel and Blanchard. Here $h(i/k)$ shows convex adjustment costs of investment, k is the capital stock per worker, and i is investment per worker.

The optimality conditions are given by:

$$\dot{q}_t = (r_t + \delta) \cdot q_t - F'_k(\theta_t, k_t) - x_t^2 h'(x_t) \quad (\text{A1})$$

$$q_t = [\phi_t^{-1} + h(x_t) + x_t \cdot h'(x_t)] \quad (\text{A2})$$

$$\lim_{t \rightarrow \infty} e^{-\int_{v=0}^t r_v dv} q_t k_t = 0$$

where q is the shadow price of one extra unit of investment and $x = i/k$.

Consumers. The representative consumer has the preferences ordered by:

$$\max U = \int_{t=0}^{\infty} e^{-\rho t} U(c_t) dt$$

subject to the budget constraint

$$\dot{B}_t = \pi_t + w_t + r_t B_t - c_t,$$

where π is per capita dividends, B is the per capita value of bonds, and c is per capita consumption. Net investment is assumed to be financed out of bonds so that the change in bonds equals the change in net investment.

The optimality conditions for the representative consumer are given by:

$$U'(c_t) = \lambda_t \quad (\text{A3})$$

$$\dot{\lambda} = (\rho - r_t)\lambda_t \quad (\text{A4})$$

where λ is the shadow price of one extra unit of consumption.

General equilibrium

Defining $y_t = \lambda_t \cdot q_t$ it follows that

$$y_t = U'(c_t) \cdot [\phi_t^{-1} + h(x_t) + x_t \cdot h'(x_t)], \quad (\text{A5})$$

and

$$\dot{y}_t = (\rho + \delta) \cdot y_t - U'(c_t) [MPK(\theta_t, k_t) + x_t^2 h'(x_t)]. \quad (\text{A6})$$

Per capita output equals per capita consumption plus per capita investment in goods market equilibrium:

$$F(\theta_t, k_t) = c_t + i_t / \phi_t + i_t \cdot h(x_t). \quad (\text{A7})$$

Since investment adjustment costs are included as investment and not made an expense as in the traditional national accounting system, they increase income. Furthermore, embodied technological progress increases the real value of one unit of investment, which is consistent with the introduction of hedonic pricing of ICT products in the US national accounts.

Eliminating c_t in (A5) using (A7), differentiating (A5) with respect to time and eliminating \dot{y}_t using (A6) yields the following first-order differential equation:

$$\begin{aligned} & \left[2h'(x) + x \cdot h''(x) - \frac{U''(c)}{U'(c)} [\phi^{-1} + h(x) + x \cdot h'(x)]^2 k \right] \dot{x} = \\ & \left[(\rho + \delta) [\phi^{-1} + h(x) + x \cdot h'(x)] - x^2 h'(x) - MPK(k, \theta) \right] \\ & - \frac{U''(c)}{U'(c)} [\phi^{-1} + h(x) + x \cdot h'(x)] [MPK(k, \theta) - x \cdot (\phi^{-1} + h(x))] \dot{k}, \end{aligned} \quad (\text{A8})$$

where MPK is the marginal productivity of capital. Together with the following capital constraint Equation (8A) defines a simultaneous first-order differential equation system:

$$\dot{k} = k [x \cdot \phi^{-1} - \delta]. \quad (\text{A9})$$

This equation system have the two endogenous variables x and k . Share prices are determined in this system by x and are recovered below using (A2).

In the steady state ($\dot{k} = \dot{x} = 0$) the following system applies:

$$\begin{bmatrix} MPK'_K & x^2 h''(x) + 2x h'(x) - (\rho + \delta) [2h'(x) + x \cdot h''(x)] \\ 0 & \phi^{-1} \end{bmatrix} \begin{bmatrix} dK \\ dx \end{bmatrix} = \begin{bmatrix} -(\rho + \delta) \cdot \phi^{-2} & -MPK'_\theta \\ x \cdot \phi^{-2} & 0 \end{bmatrix} \begin{bmatrix} d\phi \\ d\theta \end{bmatrix}$$

From (2A) it follows that the change in the share price is a positive function of the change in x since the capital adjustment function is convex:

$$dx_t = \frac{dq_t}{2h'(x_t) + xh''(x)}$$

The change in the steady state equilibrium of share prices as a result of embodied technological change is given by:

$$\frac{dq}{d\phi} = \frac{dx}{d\phi} \frac{dq}{dx} = x \cdot \phi^{-1} \frac{dq}{dx} = x \cdot \phi^{-1} \cdot [2h'(x) + xh''(x)] > 0,$$

which means that embodied technological progress lowers the share prices of incumbents. This result is similar to the result with exogenous discount rate. The effect on share prices of spill-over-effects of technological innovations is:

$$\frac{dq}{d\theta} = \frac{dx}{d\theta} \frac{dq}{dx} = 0,$$

which is also consistent with the result with an exogenous discount factor. So share prices are unaffected by spill-over effects from technology innovations.

The effect on the capital stock in a steady state of the technology innovation is given by:

$$\frac{dk}{d\phi} = \frac{-(\rho + \delta) \cdot \phi^{-2} - x \cdot \phi^{-1} [x - (\rho + \delta)] [2h'(x) + x \cdot h''(x)]}{MPK'_k}$$

since $x - (\rho + \delta) = 0.10 - 0.10 - 0.08 < 0$, the second term in the numerator is most likely positive, which implies that the change in k is indeterminate as a result of embodied technological progress. This result is intuitive because the adverse interest rate effect pulls the $\dot{q}_0 = 0$ schedule to the left and, therefore, works in contractive direction for steady state capital stock.

Finally, the spill-over effects of technological innovations on the per capita capital stock are positive:

$$\frac{dk}{d\theta} = -\frac{MPK'_\theta}{MPK'_k} > 0.$$

Dynamics

Linearizing the system around their steady states [Equations (A8) and (A9)] yields:

$$\begin{bmatrix} \dot{x} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} (\rho + \delta - x)[2h'(x) + xh''(x)] & -MPK'_k \\ \phi^{-1} & 0 \end{bmatrix} \begin{bmatrix} x - \bar{x} \\ k - \bar{k} \end{bmatrix} \quad (A10)$$

The system is stable since the determinant is positive, $D = MPK'_k \phi^{-1} < 0$.

The equation system forms the solution as follows:

$$\begin{bmatrix} x - \bar{x} \\ k - \bar{k} \end{bmatrix} = c_1 e^{\mu t} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + c_2 e^{\xi t} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix},$$

where $\mu < 0$ (stable root) and $\xi > 0$ (unstable root). Ruling out the unstable root we get:

$$\begin{bmatrix} x - \bar{x} \\ k - \bar{k} \end{bmatrix} = c_1 e^{\mu t} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (\text{A11})$$

Given the eigenvalue of μ , the eigenvector x can be derived from the following system:

$$\begin{bmatrix} (\rho + \delta - x)[2h'(x) + xh''(x)] & -MPK' \\ \phi^{-1} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mu \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

A possible solution, which satisfies this equation system, is

$$\begin{aligned} x_1 &= \mu \\ x_2 &= \sigma a. \end{aligned}$$

Inserting this into (A11) yields:

$$\begin{bmatrix} x - \bar{x} \\ k - \bar{k} \end{bmatrix} = c_1 e^{\mu t} \begin{bmatrix} \mu \\ \phi^{-1} \end{bmatrix}.$$

This equation system yields the following linear relationship:

$$(x - \bar{x}) = \mu \phi (k - \bar{k}).$$

From this equation it follows that the impact on share prices of unanticipated embodied technological progress is given by:

$$\frac{dq}{d\phi} = \frac{dq}{dx} \frac{dx}{d\phi} = \left[\frac{d\bar{x}}{d\phi} - \mu \phi \frac{d\bar{k}}{d\phi} \right] \frac{dq}{dx} = \left[\frac{d\bar{x}}{d\phi} - \mu \phi \frac{d\bar{k}}{d\phi} \right] [2h'(x) + xh''(x)].$$

Given that the sign of $d\bar{k}/d\phi$ is indeterminate, share prices may jump or drop on impact. If $d\bar{k}/d\phi$ is positive share prices unambiguously jump in response to unanticipated technological progress.

The impact effect on share prices of spill-over effects of technological progress is unambiguously positive:

$$\frac{dq}{d\theta} = \frac{dq}{dx} \frac{dx}{d\theta} = \left[\frac{d\bar{x}}{d\theta} - \mu \phi \frac{d\bar{k}}{d\theta} \right] \frac{dq}{dx} = \left[\frac{d\bar{x}}{d\theta} - \mu \phi \frac{d\bar{k}}{d\theta} \right] [2h'(x) + xh''(x)] > 0.$$

These results are similar to the case of an exogenous discount factor.