

EPRU

Economic Policy Research Unit

Institute of Economics

University of Copenhagen

Studiestræde 6

DK-1455 Copenhagen K

DENMARK

Tel: (+45) 3532 4411

Fax: (+45) 3532 4444

E-mail: Grethe.Mark@econ.ku.dk

Homepage: <http://www.econ.ku.dk/epru/>

Contests with Size Effects

Wolfgang Eggert and Martin Kolmar

2002-04

ISSN 0908-7745

**The activities of EPRU are financed by a grant from
The Danish National Research Foundation**

Contests with Size Effects*

Wolfgang Eggert[†] Martin Kolmar[‡]

February 28, 2002

Abstract

In this paper we analyze the structure of contest equilibria with a variable number of agents. First we analyze a situation where the total prize depends on the number of agents and where every single agent faces opportunity costs of investing in the contest. Second we analyze a situation where the agents face a trade-off between productive and appropriative investments. Here, the number of agents may also influence the productivity of productive investments. It turns out that both types of contests may lead to opposing results concerning the optimal number of contestants depending on the strength of size effects. Whereas in the former case individual utility is J-shaped when the number of agents increases, the opposite holds true for the latter case. We discuss the implications of our findings for the case of competition on markets and for the case of unstable property rights.

Keywords: Size Effects, Contests, Anarchy, Competition

JEL classification: D74, L10

*We thank Herschel Grossman, Søren Bo Nielsen, Peter Birch Sørensen, and seminar participants in Copenhagen for helpful comments. Corresponding author: Martin Kolmar, University of Konstanz, PO Box D 136, D-78457 Konstanz, Germany. Email: Martin.Kolmar@uni-konstanz.de

[†]University of Konstanz, Economic Policy Research Unit (EPRU) at the University of Copenhagen and CESifo. Eggert is part of a research network on ‘The Analysis of International Capital Markets: Understanding Europe’s role in the Global Economy’, funded by the European Commission under the Research Training Network Programme (Contract No. HPRN-CT-1999-00067). The activities of the EPRU are supported by a grant from the Danish National Research Foundation.

[‡]University of Konstanz. Kolmar acknowledges financial support by the German Research Foundation.

1 Introduction

In this paper we analyze economic environments that can be characterized as a contest, which implies that each agent faces a trade-off between productive and appropriative activities. Economic interactions in contests are coupled with the number of agents involved. A change in the size of the group has two effects on the individual perception of the economic environment. First, the individual influence on the outcome of the contest becomes smaller if the group size increases. Second, an increase in the group size may have an influence on the total amount of goods that can be distributed. We call any effect of the size of the group on the total amount of goods *size effect* and on the individual fraction of goods *slice effect*. Size effects are zero if the total amount of goods is independent of the size of the group. We are interested in the connection between group size and the structure of contest equilibria. Does an increase in the group size make the members of the group more or less aggressive? What group size maximizes individual, what group size aggregate utility? It turns out that the answers to these questions depend on the quantitative importance of size effects as well as on the effectiveness of appropriative activities.

In order to analyze the consequences of size effects on the outcome of the contest we distinguish between two generic forms of contests, both belonging to the class of ‘common-pool’ problems (Grossman 2000). In the first contest, the total prize that can be distributed among the agents is fixed and depends only on the number of agents in the economy. Investments in the appropriation of the prize, however, incur a cost for the agents that can be thought of as resulting from a labor–leisure choice, or, more generally, opportunity costs of alternative uses of time. In the second contest, the total prize depends on the amount of time invested in productive activities as well as on the number of agents. Then, the basic trade-off is between appropriative and productive activities.

Both contests differ with respect to the source of economic welfare as well as with respect to the magnitude of appropriable individual resources. In the first contest, every individual has a positive impact on the total amount of goods that can be distributed, and there exists an individual resource (‘leisure’) that is not due to appropriation by other individuals. In the second contest, the number of agents has an influence on the marginal productivity of production, however, there are no goods to be distributed without productive investments by the individuals. The total time endowment of an

agent has either to be devoted to productive activities, which can then be appropriated by other agents, or to appropriation.

We discuss our findings using two different applications. First, we apply the model to situations of incomplete or non-enforced property rights. During the last couple of years, situations of complete and partial anarchy have been fruitfully analyzed by the application of contest models (for example Bush and Mayer 1982, Hishleifer 1995, Skaperdas 1992, Grossman and Kim 1995, Grossman 2001 among others). We ask how conflict and production changes if the population increases. Size effects in anarchy can exist because of economies of scale or scope in the organization of economic activities.

Second, we apply the model to advertising activities of competitors on market places. Market places are often characterized by positive size effects. In developed economies there exists a complete and stable set of property rights on market places, however, the competition for customers creates the formal analogy to the appropriation of goods in anarchy.¹ If it is more attractive for customers to search on markets if the number of competitors supplying on this market is large, the total gross profit that can be earned on a market is increasing in the number of competitors supplying on the market. In order to sell goods on a market with potential competitors, however, each competitor has to promote the product, which requires investment in advertising. Hence, the market has the structure of a contest. The two contests we analyze correspond to different situations on a market. The first contest portrays a situation where competitors do not invest in the quality or attractiveness of the marketplace itself. Hence, the success of a competitor on the market depends only on its investment in individual advertising, where costs the costs of advertising are equal to the opportunity costs of investments. The second contest corresponds to a situation where the attractiveness of the marketplace can be influenced by the investments of a single competitor. Examples for this are investments in convenience and security by the creation of parking lots and the employment of private security guards in classical markets or the investment in hard- and software that simplifies access to internet platforms.

¹In a formal sense the contest-creating activities of competitors to attract customers can be interpreted as a result of imperfect property rights. All competitors would gain from a co-ordination of advertising activities on a minimum level. Hence, they could sign a contract implementing this strategy. In this paper we do not explain why those contracts are not signed. We take it as an empirical fact that competitors invest in advertising in order to attract customers. A reason for this may be that co-operation between competitors is explicitly forbidden by competition policy.

Both, the marketplace and the anarchy interpretation of our model allow to gain useful insights into the role of group size on the structure of conflict equilibria. We have decided to use the market-place interpretation throughout the presentation of the model. In a separate Section, we will then discuss how our results relate to the literature on anarchy and conflict.

This paper differs from the previous work on contests because it explicitly allows for size effects. Standard contest models either assume that the total prize is fixed and competitors compete for the slice they get (Huck, Konrad, and Müller 2000), or that competitors can either invest in production, which increases the size of the cake, or in appropriation, which increases its slice (Grossman and Kim 1995, Grossman 2001). With the exception of Grossman (2001) size effects of an increase in the number of participants in the contest have been neglected. A predecessor of our model is the paper by Hirshleifer (1995) who analyzes a variant of the second contest for the cases of no and constant size effects (both terms will be made rigorous throughout the text).

The paper proceeds as follows: Section 2 presents the model. We derive the basic results in Section 3 and discuss further applications of the model in Section 4. Section 5 concludes.

2 The model

Consider a market for similar goods where $n \geq 2$ producers indexed by i compete for customers. Assuming that goods are close substitutes, a change in producer's policy causes a change in demand which, however, is so small that competitors are not motivated to retaliate. Each producer chooses to invest $a_i \in \mathbb{R}^+$ units of money in advertising to appropriate part of the gross market value x . The fraction p_i of the gross value, for short prize, that accrues to producer i is given by the Tullock *contest-success function* (CSF) (Tullock 1980),

$$p_i(a_1, \dots, a_n) = \frac{a_i^b}{\sum_{j=1}^n a_j^b}, \quad (1)$$

where $b \in [0, 1]$ is the *decisiveness* parameter (Hirshleifer 1995) or the *discriminatory power* (Skaperdas 1995) of the CSF. From (1), investments do not change the fraction of the prize appropriated by competitors in the special case where $b \rightarrow 0$, and the marginal effectiveness of advertising has a maximum in cases where $b = 1$. Thus, other things equal, larger values of b tend to increase the level of investment in the contest.

Since products are close substitutes, the potential customers' evaluation of the market depends on the number of producers, n , and potentially also on the level of direct investment, l_i , chosen by each competitor to increase the overall attractiveness of the marketplace. The total value of sales on the market, f , the *market-value function*, is then given by $x = f(n; l_1, \dots, l_n)$. Introducing the parameter g to allow for a non-linear dependence between x and n , we use two different specifications of the market value function in the following:

1. $x = f(n; l_1, \dots, l_n) = n^g Z$, $g \in [0, 2]$, and the costs of appropriative investment are $Z = a_i$. This specification generalizes the contest analyzed in Grossman (2001).
2. $x = f(n; l_1, \dots, l_n) = n^g \sum_{j=1}^n l_j$, $g \in [0, 2]$, and the competitors face a trade-off between appropriative and productive investments, where $a_i + l_i = 1$.²

Both, the contest-success function as well as the market-value function have the same interpretation as a standard production function that abstracts from the exact technological and organizational process of the production process.

Let us assume that competitors are risk neutral. We consider Nash equilibria of the game. Under the first specification, competitor i chooses a_i to

$$\max_{a_i} u_i(a_1, \dots, a_n) = p_i(a_1, \dots, a_n) n^g Z - a_i, \quad (2)$$

and the first-order condition is:

$$\frac{du_i}{da_i} = \frac{\partial p_i}{\partial a_i} n^g Z - 1 = 0, \quad i = 1, \dots, n. \quad (3)$$

Hence, in the individual optimum, the marginal increase in the fraction of the market value accrued by competitor i is equal to the marginal costs caused by an increase in a_i .

Under the second specification, competitor i solves

$$\max_{a_i} u_i(a_1, \dots, a_n) = p_i(a_1, \dots, a_n) n^g \sum_{j=1}^n (1 - a_j), \quad (4)$$

which gives rise to the first-order condition:

$$\frac{du_i}{da_i} = \frac{\partial p_i}{\partial a_i} n^g \sum_{j=1}^n l_j - p_i n^g = 0, \quad i = 1, \dots, n. \quad (5)$$

²We use a market value function that is linear in investments for analytical convenience. Using a Cobb-Douglas specification would not change the qualitative results that follow. A proof is available from the authors upon request.

Let us denote by $a_i^*(a_{-i})$ the argument of maximization for both problems respectively, with a_{-i} denoting investments by all competitors except of i . A Nash equilibrium is a vector of investments $\{a_1^N, \dots, a_n^N\}$ such that $a_i^N = a_i^*(a_{-i}^N)$ for all $i = 1, \dots, n$. Under both specifications of the market value function, the simultaneous solution of the competitors' optimization problem gives rise to a unique symmetric Nash equilibrium.

With the first market-value function, the equilibrium investment in the contest a_i , the competitors' net equilibrium profit v_i , and the aggregate market surplus x , are

$$a_i = a_j = b(n-1)n^{g-2}Z, \quad (6a)$$

$$v_i = v_j = (n + (1-n)b)n^{g-2}Z, \quad (6b)$$

$$x = (n + (1-n)b)n^{g-1}Z, \quad (6c)$$

where x is the difference between total market value and aggregate costs of investments. Analogously we get for the second specification of the market-value function:

$$a_i = a_j = \frac{(n-1)b}{1+b(n-1)}, \quad (7a)$$

$$v_i = v_j = \frac{n^g}{1+b(n-1)}, \quad (7b)$$

$$x = \frac{n^{g+1}}{1+b(n-1)}. \quad (7c)$$

It proves useful throughout the paper to introduce the concept of dissipation.³ The rate of dissipation measures the fraction of the gross market value that is destroyed by investments in the contest. A contest implies under-dissipation if the sum of investments in the contest is smaller than the prize, $x > 0$. It implies over-dissipation if the sum of investments exceeds the prize, $x < 0$, and it implies complete dissipation if the sum of investments exactly balances the prize, $x = 0$.

Lemma 1. *There is under-dissipation in both contests.*

Proof: Market-value function 1: Total investments in the contest are given by na_i , and the total prize of the contest is n^gZ . Hence, $x > 0$ when $n^gZ > na_i$. Using (6c) we find $b(1-n) + n \lesseqgtr 0$, or $b \gtrless n/(n-1) > 1$ as required by the Lemma.

Market-value function 2: By the same token, under-dissipation implies $x > 0$. From (7c), $x > 0$ requires $1 + b(n-1) > 0$, or $(n-1) > -1/b$, which is always fulfilled. \square

³See Hillman and Samet (1987).

3 Analysis of size effects

3.1 Market-value function 1

3.1.1 Effects of changes in size effects, discriminatory power, and group size

We start the discussion of this section with an analysis of a market where competitors do not invest in its attractiveness. We first analyze the influence of the size parameter g on the equilibrium investment in the conflict and on the equilibrium value of competitors' net profit and the dissipation. Differentiation shows

$$\begin{aligned}\frac{\partial a_i}{\partial g} &= \ln(n) b (n - 1) n^{g-2} Z > 0 & \Leftrightarrow & n > 1, \\ \frac{\partial v_i}{\partial g} &= \ln(n) (n + b(n - 1)) n^{g-2} Z > 0 & \Leftrightarrow & \frac{n}{1 - n} > b, \\ \frac{\partial x}{\partial g} &= \ln(n) (n + b(n - 1)) n^{g-1} Z > 0 & \Leftrightarrow & \frac{n}{1 - n} > b.\end{aligned}$$

An increase in size effects has a positive impact on individual investments in the contest, individual net profits, and aggregate market surplus for all meaningful values of n . This result has an interesting interpretation. Other things equal, an increase in g has a positive effect on the total gross market value. Since marginal costs are fixed to be equal to zero, investments in the contest become more attractive. Moreover, the additional marginal prize is less than fully dissipated by the increase in investments, which implies that the aggregate market surplus also increases. The conclusion is that competitors become more aggressive in their attempt to attract potential consumers. However, the sharper competition does not result in a utility loss in the presence of size effects.

Next let us discuss the effects of an increase of the number of competitors on a_i , v_i , and x . We obtain:

$$\frac{\partial a_i}{\partial n} = (n + (n - 1)(g - 2)) n^{g-3} b Z, \quad (8a)$$

$$\frac{\partial v_i}{\partial n} = ((g - 1)n(1 - b) + b(g - 2)) n^{g-3} Z, \quad (8b)$$

$$\frac{\partial x}{\partial n} = (n(1 - b)g + b(g - 1)) n^{g-2} Z. \quad (8c)$$

To interpret (8) let us consider the benchmark case where size effects do not exist, $g = 0$. Inspection of (8) shows that the number of competitors has no influence on producers' policy. The argument is that none of the producers can profitably shift the

distribution of the aggregate market surplus. More formally, we find that $\partial a_i / \partial n \leq 0$ at $b = 0$. Moreover, an inspection of (8) shows that $\partial v_i / \partial n \leq 0$ and $\partial x / \partial n \leq 0$ at $b = 0$. Hence, the aggregate market surplus remains constant and the decrease of individual net profits is linear in the number of competitors.

If investments in the contest are productive, in the sense that competitors can shift some fraction of the total market surplus to their advantage by increasing a_i , then individual profits are no longer linearly decreasing in n . Increasing n reduces investments, individual net profits and the aggregate net surplus if $b > 0$. The reason is that the gross market value is fixed, which makes investments less attractive since the marginal profitability of investments decrease. Surprisingly, the positive effect of a reduction of wasteful investments in the contest does, however, not over-compensate the negative market-sharing effect resulting from intensified competition.

Let us allow for size effects now. An interesting scenario, which is based on previous work by Grossman (2001) for a an arbitrary but fixed number of competitors, is the case where the marginal size effect attributed to an additional competitor is constant. With $g = 1$, (8) simplify to

$$\begin{aligned}\frac{\partial a_i}{\partial n} &= b n^{-3} Z \geq 0, \\ \frac{\partial v_i}{\partial n} &= -b n^{-3} Z \leq 0, \\ \frac{\partial x}{\partial n} &= (1 - b) n^{-2} Z \geq 0.\end{aligned}$$

Assume that $b > 0$, to exclude the case discussed above. An increase in competition then increases the investment in the contest and competitors become more aggressive. The intuition is that the marginal return from investment increases when the aggregate net surplus is growing, causing a reduction of individual utility. Hence, taking the aggregate net market value as a proxy for welfare in this economy, there exists a discrepancy between the interest of the single competitor and the interest of a welfare-maximizing planner. To clarify the argument we make use of the Envelope theorem to derive the change in individual net profits as $dv_i/dn = -da_i/dn$, whereas the net market value changes by $dx/dn = (Z - a_i) - n da_i/dn$. With maximum discriminatory power, $b = 1$, the increase in the gross market size is exactly equalized by the increase in aggressiveness, $\partial x / \partial n = 0$. Hence, for a smaller discriminatory power, the positive market-value effect is less than offset by the increase in aggression.

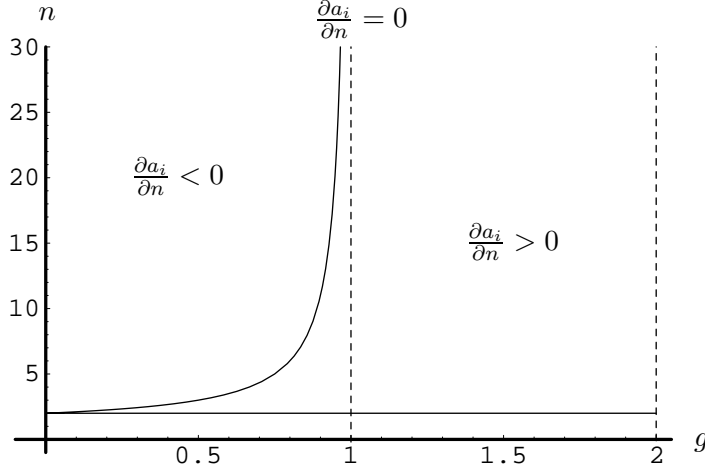


Figure 1: Effects of the size parameter and the number of competitors on appropriation.

This leads us to a discussion of the general case where $g \gtrless 1$. We start with a discussion of investment levels:

Result 1. *a. If $b = 0$, $\partial a_i / \partial n = 0$.*

b. If $b > 0$ and $g \geq 1$, $\partial a_i / \partial n > 0$.

c. If $b > 0$ and $g < 1$, $\partial a_i / \partial n \gtrless 0 \Leftrightarrow n \gtrless (g - 2)/(g - 1)$.

Proof: The second term in (8a) is equal to zero if $b = 0$ which proves part a. To prove part b and c, let $b > 0$. For $g = 1$, $\partial a_i / \partial n = bZn^{g-3} > 0$. Note that $\partial a_i / \partial n \gtrless 0$ if and only if $n(g - 1) \gtrless (g - 2)$. This implies $n \lesseqgtr (g - 2)/(g - 1)$ for $g < 1$ and $n \gtrless (g - 2)/(g - 1)$ for $g > 1$. For $g > 1$ the latter condition is always fulfilled because $(g - 2)(g - 1) \leq 0$. When $g < 1$ then $(g - 2)/(g - 1)$ approaches infinity if $b \rightarrow 1$ and it vanishes if $b \rightarrow 0$. Hence, $\partial a_i / \partial n$ depends on the relationship between n and g and the borderline is defined by $n = (g - 2)/(g - 1)$. \square

The intuition for this result is as follows. A larger number of competitors implies that the prize has to be shared among a larger group. Given constant marginal costs, investments in the contest are profitable when the slice effect is over-compensated by the size effect for each competitor. This implies that $b > 0$ and the number of competitors has to be finite. If the number of competitors becomes infinite, the marginal effect of a single competitor becomes negligible. However, if size effects are too small, an increase in competition tends to reduce appropriative investments in the contest exactly

because the slice effect dominates the size effect, the marginal return from investments decreases. *Result 1 therefore shows that it is not the effect of competition that makes competitors more aggressive, but the creation of size effects.*

Figure 1 gives a graphical representation of Result 1. The size parameter, g , is drawn on the horizontal axis and population size, n , is measured on the vertical axis. Investments in the contest are decreasing for all points above the dividing line $\partial a_i / \partial n = 0$, and increasing for all points below. Higher marginal size effects intensify the aggressiveness of competitors on the market. However, when marginal size effects are decreasing, then an increase in the number of competitors will finally temper the aggressiveness in the contest.

We now turn to the analysis of the effect of the number of competitors on the aggregate net value, x .

Result 2. *a. If $b = 0$, $\partial x / \partial n \geq 0$.*

b. If $b > 0$ and $g \geq 1$, $\partial x / \partial n \geq 0$.

c. If $b > 0$ and $g < 1$, $\partial x / \partial n \geq 0 \Leftrightarrow n \geq b(g - 1) / (g(b - 1))$.

Proof: Part a follows directly from (8c). Recall that $n \geq 2$. Let $b > 0$ and note that $\partial x / \partial n \geq 0$ if and only if $n(1 - b)g \geq b(g - 1)$, which is equivalent to $n \geq b(1 - g) / (g(1 - b))$. For $g = 1$ this boils down to $n \geq 0$. For $g > 1$ the limit of $b(1 - g) / (g(1 - b))$ for $b \rightarrow 0$ is equal to 0 (from below), and it is equal to $-\infty$ for $b \rightarrow 1$. For $g < 1$, the limit of $b(1 - g) / (g(1 - b))$ for $b \rightarrow 0$ is equal to 0 (from above), and approaches infinity for $b \rightarrow 1$. \square

Result 2 demonstrates that an increase in the number of competitors does increase aggregate net production if there are increasing marginal returns to size or if the number of competitors is sufficiently large. The size effect over-compensates every increase in the discriminatory power if n increases. The intuition for this finding is closely related to the intuition for the change in a : if $g > 1$ the prize increases over-proportionally with the number of competitors. This makes the competitors more aggressive but the increase in output is not fully destroyed by an increase in a because of under-dissipation. If there are decreasing marginal size effects, an additional competitor still adds to the aggregate value, however, this additional value decreases with an increase in n . Hence, the size effect decreases relatively to the slice effect, which is constant. This makes competitors less aggressive. However, this counterbalancing effect does not

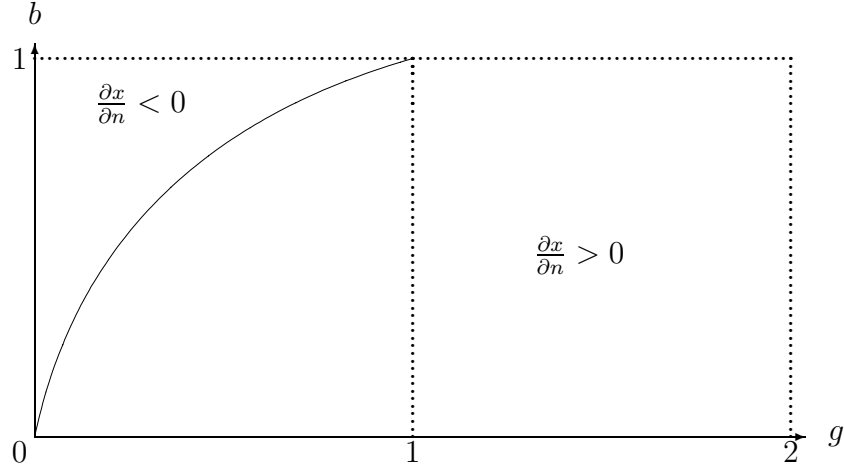


Figure 2: Change of net-aggregate wealth.

over-compensate the former effect because of under-dissipation. It should be pointed out that this result shows that there is a qualitative difference between models without size effects and models with decreasing or increasing marginal size effects: in a model without size effects the net aggregate value is linear in the number of competitors, $\partial x/\partial n = -bZ$, and the slope depends linearly on the discriminatory power of the contest. With size effects the slope can get positive. We will come back to this point when we discuss the optimal number of competitors in the contest.

Figures 2 and 3 illustrate Result 2. In Figure 2, the size parameter g is plotted along the horizontal axis and the decisiveness parameter b is plotted along the vertical axis for a given n .⁴ For large values of b and small values of g , the total net value is decreasing in the market size, whereas the opposite is true for low values of b and large values of g . In Figure 3, b is drawn along the abscissa and n is drawn along the ordinate at given $g < 1$. The Figure illustrates that an increase in n raises aggregate net value for low values of b . However, if investments in the contest are effective and b large, then an increase in group size leads to a reduction of aggregate net value.

Next we turn to an analysis of the change in individual net utility v_i .

⁴In order to determine the slope and curvature of the curve $\partial x/\partial n = 0$ use (8c) to get $g = b/((1-b)n+b)$. The curve of this function is increasing and convex in b .

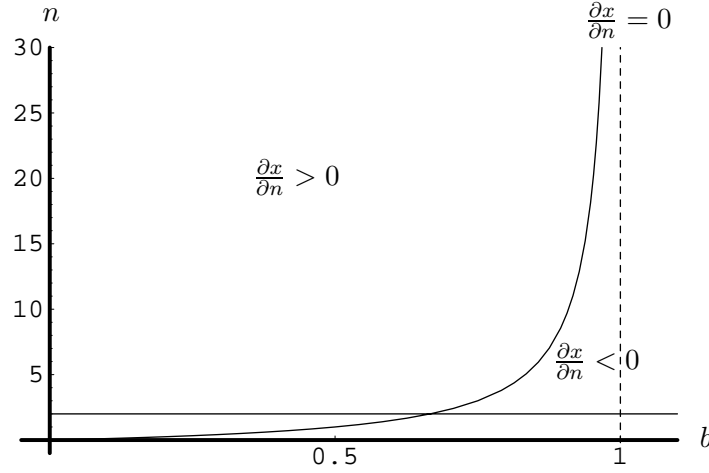


Figure 3: Effects of discriminatory power and the number of competitors on aggregate net value ($g \leq 1$).

Result 3. a. If $g < 1$, $\partial v_i / \partial n < 0$.

b. If $g \geq 1$, $\partial v_i / \partial n \gtrless 0 \Leftrightarrow n \gtrless b(g-2)/((g-1)(1-b))$.

Proof: Part a follows directly from (8b). Recall $n \geq 2$. Let $b > 0$ and note that $\partial v_i / \partial n \gtrless 0$ if and only if $n(g-1)(1-b) \gtrless b(g-2)$, which is equivalent to

i. $n \gtrless b(g-2)/((g-1)(1-b))$ for $g > 1$,

ii. $n \gtrless b(g-2)/((g-1)(1-b))$ for $g < 1$.

For $g = 1$ it follows immediately that $\partial v_i / \partial n = -b < 0$. In case i. the limit of the right-hand side for $b \rightarrow 0$ is 0 (from below) and $-\infty$ for $b \rightarrow 1$. Hence, the restriction $n \geq 2$ is binding, which implies that $\partial v_i / \partial n > 0$. In case ii. the limit of the right-hand side is 0 for $b \rightarrow 0$ (from above) and ∞ for $b \rightarrow 1$. \square

Result 3 implies that an increase in n increases the net utility of the competitors only if the size effects are large compared to the discriminatory power. The economic rationale of Result 3 is best understood when (8b) is evaluated for the boundary case where $g = 2$ and $b = 1$. Then, size and slice effects balance in a way that net utility is constant. As illustrated in Figure 4, the contest is less effective for low values of b , which implies that competitors lower their investments in aggression which leads to

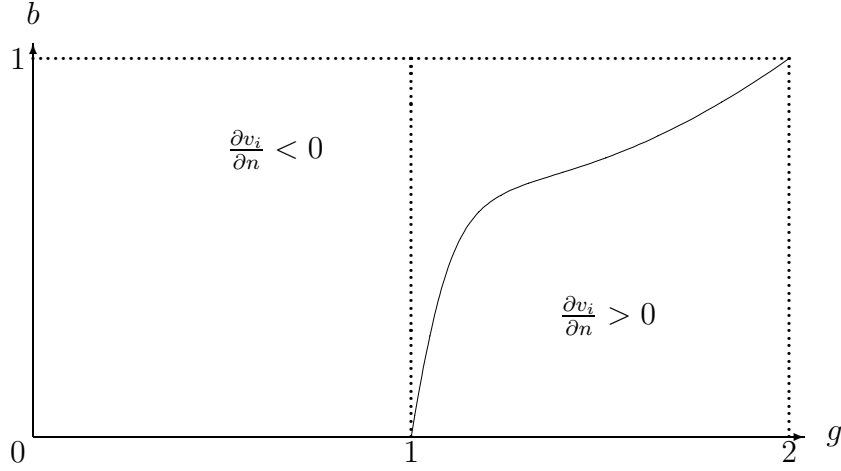


Figure 4: Change of individual utility.

an increase in individual utility. By the same token, individual utility decreases if size effects are less important.⁵

An interesting implication of Result 3 is that for sufficiently strong size effects ($g > 1$) individual utility is not monotonous in the number of competitors. It is *decreasing* up to a critical number of competitors and *increasing* thereafter. If n is relatively small, the negative slice effect of an increase in competition is relatively important and outweighs the size effect. However, increasing n implies that the slice effect is becoming less important. There exist pairs of (g, b) such that the slice effect is dominated by the size effect. This finding shows that the contest creates a J-curve effect that may hamper the development of markets if the market grows steadily.

A comparison of Result 2 and Result 3 demonstrates that there may exist a discrepancy between the change in individual utility and aggregate net value when the number of competitors varies. Figure 5 unifies both conditions. The change in individual utility and net aggregate value has the same sign for extreme values of b and g . However, there exists an interval of ‘intermediate’ values for b and g where aggregate net value increases whereas net utility decreases. The intuition for this discrepancy is as follows. Recall that net aggregate value, x , is $x = nv_i$. Thus, $\partial x / \partial n = v_i + n \partial v_i / \partial n$. The

⁵To determine the slope and curvature of the $\partial v_i / \partial n = 0$ -curve use (8b) to get $g = (n - (n + 2)b) / (n - (n + 1)b)$. The first and second derivatives of this function show that the curve is increasing. It is convex in b as long as $b \leq n / (n + 1)$ and concave thereafter.

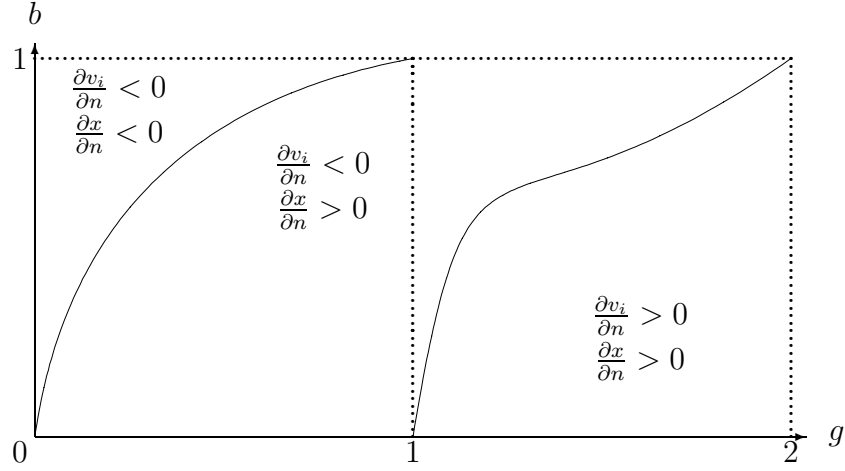


Figure 5: Comparison of changes in net-aggregate wealth and individual utility.

first term measures the effect of an additional competitors on aggregate utility, which is always positive. The second term measures the effect of an additional competitor on other competitors. This effect can be either positive or negative. Hence, if the individual utility is increasing, net aggregate value has to increase by definition. However, there is a region for which individual utility is decreasing but this decrease does not over-compensate the effect that a ‘new’ competitor adds to the aggregate. The effect on the ‘old’ competitors becomes dominant in cases where size effects are small and discriminatory power is large.

3.1.2 The optimal number of competitors in the contest

A straightforward question is about the optimal number of competitors in the market. There are two perspectives from which we can determine the optimal number of competitors: from the competitors’ perspective it is given by the number of competitors that maximizes net utility. From a social point of view the optimal number of competitors maximizes net aggregate value created on this market. Both measures do not necessarily lead to the same results as we demonstrate below. We start our discussion considering interior solutions.

Lemma 2. *For $b > 0$ and $\neg[g = 1 \wedge b = 1]$, if $\partial x / \partial n = 0$, then $\partial^2 x / \partial n^2 > 0$, hence, every interior solution constitutes a minimum.*

Proof: From Result 2 follows that an interior solution requires $g < 1$. In that case it is characterized by $n = b(g - 1)/(g(b - 1))$. This point is a maximum if the second-order condition is fulfilled at that point:

$$\begin{aligned}\frac{\partial^2 x}{\partial n^2} &= -\frac{b^{g-2}(g-1)^{g-2}}{(b-1)^{g-3}g^{g-3}}Z > 0 \\ &= n^{g-2}(1-b)gZ > 0,\end{aligned}$$

where the second line follows from part c of Result 2. \square

Lemma 3. *For $b > 0$ and $\neg[g = 1 \wedge b = 1]$, if $\partial v_i/\partial n = 0$, then $\partial^2 v_i/\partial n^2 > 0$, hence, every interior solution is a minimum.*

Proof: We know from Result 3 that an interior solution requires $g < 1$. In this case it is characterized by $n = b(g - 2)/((g - 1)(1 - b))$. It is a maximum if the second-order condition is fulfilled at that point:

$$\begin{aligned}\frac{\partial^2 v_i}{\partial n^2} &= \frac{b^{g-3}(2-g)^{g-3}}{(b-1)^{g-4}(g-1)^{g-4}}Z < 0 \\ &= n^{g-3}(b-1)(g-1)Z > 0.\end{aligned}$$

Form part c. of Result 3 both, $(b - 1)$ and $(g - 1)$ are smaller then zero in an interior extremum. \square

Lemma 2 and Lemma 3 together imply that the competitors' or public optimal number of competitors is either 2 or ∞ depending on the size effects and the discriminatory power. The next two results make this conjecture precise.

In order to determine the optimal size of the market we calculate the limit of x and v_i for $n \rightarrow \infty$. We get the following result for the net aggregate value.

Result 4. *a. If $b \in [0, 1)$, $\lim_{n \rightarrow \infty} x$*

$$\begin{aligned}&= Z \text{ if } g = 0, \\ &= \infty \text{ if } g \in (0, 2].\end{aligned}$$

b. If $b = 1$, $\lim_{n \rightarrow \infty} x$

$$\begin{aligned}&= 0 \text{ if } g \in [0, 1), \\ &= Z \text{ if } g = 1, \\ &= \infty \text{ if } g \in (1, 2].\end{aligned}$$

Proof: We have to take the limit of $x = (n + (1 - n)b)n^{g-1}Z$ for $n \rightarrow \infty$. It follows that

$$\begin{aligned} \text{at } b \in [0, 1), \quad \lim_{n \rightarrow \infty} x &= \begin{cases} Z, g = 0 \\ \infty, g > 0 \end{cases}, \\ \text{at } b = 1, \quad \lim_{n \rightarrow \infty} x &= \begin{cases} 0, g \in [0, 1) \\ Z, g = 1 \\ \infty, g \in (1, 2] \end{cases}. \end{aligned}$$

□

The above finding can be seen as a justification of the model of perfect competition: despite the fact that competitive pressure makes competitors more aggressive in fighting for customers, the net effect is positive if the size effect is large relative to the discriminatory power. The bang-bang character of the solution is a consequence of our assumption that size effect are either globally increasing, decreasing, or constant. This implies that the optimal market size is either minimal or maximal. A maximal market size is optimal if $b < 1$ and $g > 0$ or $b = 1$ and $g > 1$.

The effect on individual utility can be summarized as follows:

Result 5. *a. If $b \in [0, 1)$, $\lim_{n \rightarrow \infty} v_i$*

$$\begin{aligned} &= 0 \text{ if } g \in [0, 1), \\ &= Z \text{ if } g = 1, \\ &= \infty \text{ if } g \in (1, 2]. \end{aligned}$$

b. If $b = 1$, $\lim_{n \rightarrow \infty} v_i$

$$\begin{aligned} &= 0 \text{ if } g \in [0, 2), \\ &= Z \text{ if } g = 2. \end{aligned}$$

population size that, in a world with finite resources, sum-utilitarianism implies an infinite population with arbitrarily low individual utilities. This property of utilitarian welfare functions has been called the ‘repugnant conclusion’ by Parfitt (1984), see also Razin and Sadka (1995). On the other hand, average utilitarianism implies a minimal population with maximal individual utility.

Our results can be interpreted in the spirit of the repugnant conclusion. Individual utility, v_i , coincides with average utilitarianism and net aggregate value, x , with sum utilitarianism in our model. In our model, the repugnant conclusion does not occur as long as size effects are sufficiently weak ($g \in [0, 1)$). The repugnant conclusion holds when marginal size effects are moderate ($g \in (1, 2)$), since then individual utility and aggregate net value converge in opposite directions. However, if size effects are sufficiently strong ($g \geq 2$), individual utility and aggregate net value converge in the same direction again. In our model, the general logic of the repugnant conclusion is obtained as a special case: if the value of the market grows at a slower rate as the number of competitors the repugnant conclusion does not occur. It occurs, however, in cases where the growth of the market exceeds the growth of the competitors moderately. The reason is that dissipation of part of the potential value in the contest requires a minimum size effect in order to guarantee increasing aggregate or individual utilities.⁶

Let us summarize the basic results of this Section:

- First, if size effects exist but are moderately low ($g \in (0, 1)$) starting at $n = 2$ an increase in the number of competitors increases aggressiveness if the number of competitors is small and decreases aggressiveness if it is large. The larger g the larger becomes the critical number of competitors for which aggression is finally reduced. If $g \in [1, 2]$ competitors unambiguously increase aggression if the number of competitors increases.
- This implies that for an increase in the number of competitors and $g \in (0, 1)$, aggregate net value is first decreasing and then increasing. It is unambiguously increasing if size effects are sufficiently strong ($g \geq 1$).
- Individual utility is decreasing in the number of competitors if size effects are relatively small ($g \geq 1$). However, if they are strong enough ($g > 1$) there exists

⁶It is straightforward to show that, for $b = 0$ (there exists no conflict in the economy), $\lim_{n \rightarrow \infty} u_i = 0, g < 1; Z, g = 1, \infty, g > 1$ and $\lim_{n \rightarrow \infty} x = Z, g = 1, \infty, g > 1$, which replicates the repugnant conclusion in its standard formulation for $g \leq 1$.

a critical number of competitors below which individual utility is decreasing and from which on individual utility is increasing.

3.2 Market-value function 2

In this section we will analyze if the results change when competitors face a trade-off between investment in the contest success and investment in the market. Under this specification, an increase in the magnitude of the size effect, g , has the following influence on a_i , v_i , and x :

$$\frac{\partial a_i}{\partial g} = 0, \quad (9a)$$

$$\frac{\partial v_i}{\partial g} = \frac{\ln(n)n^g}{1 + b(n-1)} > 0, \quad (9b)$$

$$\frac{\partial x}{\partial g} = \frac{\ln(n)n^{g+1}}{1 + b(n-1)} > 0. \quad (9c)$$

In contrast to the previous case where competitors cannot directly invest in the attractiveness of the market, optimal investments in the contest are independent of the magnitude of the size effect as can be seen from (9a). The reason is that an increase in g increases the marginal revenue of investments. In addition the increase in marginal costs exactly offsets the effects of an increase in g , leaving the marginal rate of transformation between investments in the contest and investments in the market unaffected. Since individual investment decisions are independent of the magnitude of the size effect, the increase in the gross prize caused by the increase in g is equal to the increase in the net prize, which is equally divided among competitors. This latter property explains why the power of the numerator in (9c) exceeds the power of the numerator in (9b) by one. Hence, individual net utility as well as the aggregate net surplus are increasing in g .

We can analyze the consequences of an increase in the number of competitors next:

$$\frac{\partial a_i}{\partial n} = \frac{b}{(1 + b(n-1))^2}, \quad (10a)$$

$$\frac{\partial v_i}{\partial n} = \frac{(g(1 - b(n-1)) - bn)n^{g-1}}{(1 + b(n-1))^2}, \quad (10b)$$

$$\frac{\partial x}{\partial n} = \frac{(1 + g + b(g(n-1) - 1))n^g}{(1 + b(n-1))^2}. \quad (10c)$$

It follows from (10a) follows that an increase in the number of competitors intensifies their aggressiveness.⁷ The economic intuition is that investments in the attractiveness of the market create a positive externality for other competitors. Clearly, the associated free-rider problem is the more severe the larger is the number of competitors. In other words, the marginal rate of transformation between investments in the contest and investments in the market is a function of the number of competitors, which implies that aggressiveness intensifies if the number of competitors increases.

Following the route of the previous section, we will next analyze the benchmark case $g = 0$ before we turn to the analysis of the general case. For $g = 0$, equations (10b)-(10c) read

$$\begin{aligned}\frac{\partial v_i}{\partial n} &= -\frac{bn}{(1+b(n-1))^2} \leq 0, \\ \frac{\partial x}{\partial n} &= \frac{1-b}{(1+b(n-1))^2} \geq 0.\end{aligned}$$

The net utility of the competitors is weakly decreasing in n . It remains constant if $b = 0$ because the market technology has constant returns to scale, which implies that the individual net utility is independent of the number of competitors. However, if $b > 0$ it is decreasing in n : if $b = 1$, aggregate net value is constant, hence, individual net utility has to decrease. By the same token, as long as net aggregate wealth increases by less than one, individual net utility has to decrease. For all $b > 0$ the increase in net aggregate wealth is smaller than one because the market technology has constant returns to scale and there is a reallocation of investments in the direction of the contest.

Net aggregate value is weakly increasing in n because of under-dissipation: the potential increase in additional wealth is not completely offset by a reallocation of investments. Only in the case $b = 1$, aggregate wealth does not increase if the number of competitors increases.

Let us turn to the general case now. An inspection of (10c) reveals that $\partial x/\partial n \geq 0$ for all values of b and g . Increasing the number of competitors increases aggregate net wealth. An increase of competitors intensifies aggressive behavior, but the waste caused does not over-compensate the size effect. We may summarize with:

Result 6. $\partial x/\partial n \geq 0 \ \forall \ b \in [0, 1], g \in [0, 2]$.

The general effect on v_i of a change in n is more involved and can be stated as follows:

⁷Note that $\partial a_i/\partial n$ is not a function of g since a is independent of g .

Result 7. *a. If $g \geq 1$ or $g < 1$ and $g < 2b/(1+b)$ then there exists no interior maximum.*

b. If $g < 1$ and $g \geq 2b/(1+b)$ then there exists an interior maximum with $n = \frac{(b-1)g}{b(g-1)} \geq 2$.

Proof: (10b) implies that the value of n fulfilling the first-order conditions is given by $n = \frac{(b-1)g}{b(g-1)}$. Consistency requires that $n = (b-1)g/(b(g-1)) \geq 2$, which implies that $g \geq 2b/(1+b) \Leftrightarrow g \leq 1$. $g > 1$ and $g < 2b/(1+b)$ would imply $b > 1$, a contradiction. If $g < 1$ the condition $g > 2b/(1+b)$ can be fulfilled: for $b \rightarrow 0$ the right-hand side converges to 0. However, if $b \rightarrow 1$ the term converges to 1, which implies that it is fulfilled if b is small relative to g .

Assume that the necessary condition for a maximum is fulfilled for $n \geq 2$. An evaluation of the second-order condition at that point shows

$$\frac{\partial^2 v_i}{\partial n^2} = b \left(\frac{g(b-1)}{b(g-1)} \right)^{g-1} \frac{(g-1)^3}{(b-1)^2}.$$

The term in large brackets is equal to n , which implies that the product of the first two terms is larger than zero for $b > 0$. The third term is smaller than zero because $b < 0$ and $g < 0$ by assumption. \square

We will clarify the implications of Result 7 after the derivation of the optimal number of competitors in the contest. Notice that Result 6 implies that x is maximized if $n \rightarrow \infty$ because of the monotonicity in n . For v_i we have to compare the corner solutions $n = 2$ and $n \rightarrow \infty$. This leads to the following Result:

Result 8. *a. If $g > 1$, $n \rightarrow \infty$.*

b. If $g = 1$

i. and $b = 1$, $n \in [2, \infty)$,

ii. and $b < 1$, $n = 2$.

c. If $g < 1$

i. and $g > 2b/(1+b)$, $n = (b-1)g/(b(g-1))$,

ii. and $g \leq 2b/(1+b)$, $n = 2$.

Proof: Denote by n^* the optimal n . By the use of the rule of l'Hôpital, the limit of v_i with respect to n is equal to

$$\lim_{n \rightarrow \infty} Z \frac{n^g}{1 + b(n-1)} = \frac{gZ}{b} \lim_{n \rightarrow \infty} n^{g-1} = \begin{cases} \infty, g > 1 \\ \frac{gZ}{b}, g = 1 \\ 0, g < 1 \end{cases} . \quad (11)$$

On the other hand, for $n = 2$, we get $v_i(2) = 2^g Z / (1 + b)$. A comparison of this value with (11) yields for $g > 0$: $n^* \rightarrow \infty$, for $g = 1$, and (i) $b = 1$: $n^* \in [0, \infty)$, or (ii) $b < 1$: $n^* = 2$. For $g < 1$ we get: (i) if $g > 2b/(1 + b)$: $n^* = (b - 1)g/b(g - 1)$, and (ii) if $g \leq 2b/(1 + b)$: $n^* = 2$, as required by Result 8. \square

Results 7 and 8 show that there exists no conflict of interest between the individual and the aggregate perspective for $g > 1$: both, individual utility and aggregate net value increase in the number of competitors in the market. However, this is not true if $g < 1$. In this case, the optimal number of competitors from the perspective of a single competitor depends on both, the discriminatory power in the contest, b , and the size effect, g . If the size effect is relatively small compared to the discriminatory power, an increase in competition unambiguously reduces individual utility. In contrast, if the size effect is relatively large, a critical number of competitors exists which constitutes an upper bound. Below this bound an increase in competition increases individual utility and above this bound individual utility is reduced. We can summarize our findings with the conclusion that a finite interior number of competitors exists at which individual utility is maximized.

The economic intuition for the result is related to the public-goods character of productive investments. Starting at $n = 2$ with $a = b/(1 + b)$, an increase in the number of competitors intensifies aggressive behavior. Since a converges to 1 for $n \rightarrow \infty$, the total of the time endowment is invested in aggression. If size effects are relatively strong compared to discriminatory power, tougher competition has a positive effect on individual utility. The reason is that the reduction of investments in the attractiveness of the market is over-compensated by the increase in marginal productivity of these investments. If the discriminatory power, however, is strong, the reduction in productive investments immediately over-compensates the increase in marginal productivity causing a net utility loss. If, on the other hand, the discriminatory power is moderate but the number of competitors high, the increase in marginal productivity cannot over-compensate the decrease in productive investments since $g < 1$ for a large num-

ber of competitors. The implication is that there exists an interior optimal number of competitors from the point of view of the single competitor.

This finding extends the first contest model in two respects. First, without productive investments by the competitors individual utility is decreasing if $g < 1$ and may be either increasing or decreasing if $g > 1$. Extending the model by incorporating productive investments reduces the extend to which size effects are necessary in order to create positive effects of competition for the individual competitor. Second, we have seen that there exists a J-curve effect of increasing competitiveness if size effects are sufficiently strong, whereas this effect is inverted in the presence of productive investments.

4 Further applications

A re-interpretation of the model in the context of unstable property rights is straightforward:⁸ in a situation of unstable property rights the production of private goods essentially creates a common pool from which each individual obtains a share according to its appropriative investments. Size effects exist in this contexts because of economies of scale and scope that result from a better organization of individuals. As has become clear in the present paper, better organization of individuals does, however, not imply that appropriation becomes less important.

Our results suggest that the production technology matters for the implications of size effects in anarchy. If size effects merely increase the stock of goods that can be distributed, our model implies that for large size effects individuals either organize in very small or very large groups (families and nations, respectively). If the initial group is small and the group size has to be extended continuously, a growth trap may exist because of the J-curved shape of individual utility in population size. If size effects are small, however, it is optimal from an individual point of view to organize in small units.

In the extended model with productive investments, size effects may merely increase the return from productive investments. If size effects are sufficiently small but still large enough compared to the discriminatory power of the contest, our model implies

⁸See Grossman and Kim (1995), Grossman (2001) and Skaperdas (1992) for similar models with an exogenous population size. Hirshleifer (1995) discusses a model where the population size is endogenized without fully exploiting the consequences of this extension

that there exists a finite group size larger than 2 that maximizes individual utility. This finding can be interpreted as the formation of a tribe or a village. The mechanisms that are responsible for welfare-improvements if the group size increases are comparable to those discussed in the literature on agglomeration (see for example Krugman 1995), namely returns to scale. However, the counterbalancing effect that explains a finite group size differs in our approach. The literature on agglomeration focuses on transportation costs and crowding. In contrast, our focus is on the absence or instability of property rights. As we have demonstrated, appropriative activities are an alternative explanation for a finite group size even in the presence of globally increasing returns to scale.

5 Conclusions

In this paper we have analyzed the impact of size effects on contest equilibria. A primary focus of the analysis has been to explore the consequences of an increase in the number of competitors involved in the contest on the aggressiveness of the equilibria and the individual and aggregate utilities of the competitors. Our results show that the structure of contest equilibria crucially depends on the structure of the size effect. We have analyzed a situation where the number of agents increases the size of the prize to be distributed and a situation where the number of agents increases the marginal effectiveness of productive investments.

Size effects have a natural interpretation in the analysis of competitive forces on markets where competitors are confronted with two opposing effects. First, the number of competitors increases the attractiveness of the market from the point of view of the customers, which, in turn, increases aggregate profits that can be earned on this market. On the other hand, an increase in competition reduces the slice of the market a single competitor attracts. Hence, he will react by changing his advertising efforts to attract customers. Our model shows that an increase in competition may have positive as well as negative effects on the single competitors, and that the structure of these effects depends on the structure of the size effects.

6 References

- Bush, W.C., Mayer, L.S., 1974. Some Implications of Anarchy for the Distribution of Property. *Journal of Economic Theory* 8 (4), 401-412.
- Grossman, H., 2001. The Creation of Effective Property Rights. *American Economic Review* 91, 347-352.
- Grossman, H., Kim, M., 1995. Swords or Plowshares? A Theory of the Security of Claims to Property. *Journal of Political Economy* 103, 1273-1288.
- Hillman, A., Samet, D., 1987. Dissipation of Contestable Rents by a Small Number of Contenders, *Public Choice* 63, 63-82.
- Hirshleifer, J., 1995. Anarchy and its Breakdown. *Journal of Political Economy* 103 (1), 26-52.
- Huck, S., Konrad, K. and Müller, W., 2000. Divisionalization in Contests. Forthcoming: *Economics Letters*.
- Krugman, P., 1995. Development, Geography, and Economic Theory. *Ohlin Lectures*, vol. 6. Cambridge and London: MIT Press.
- Parfitt, D., 1984. *Reasons and Persons*. Oxford University Press.
- Razin, A. and Sadka, E., 1995. *Population Economics*. Cambridge and London: MIT Press.
- Skaperdas, S., 1992. Cooperation, Conflict, and Power in the Absence of Property Rights. *American Economic Review* 82 (4), 720-739.
- Tullock, G., 1980. Efficient Rent Seeking, in: J. Buchanan, R. Tollison, and G. Tullock: *Toward a Theory of the Rent-Seeking Society*. College Station: Texas A&M University Press, 97-112.

Titles published in EPRU's Working Paper Series, beginning in November 1993:

2002

- 02-01 Wolfgang Eggert and Martin Kolmar: *Information Sharing, Multiple Nash Equilibria, and Asymmetric Capital-Tax Competition*
- 02-02 Carl-Johan Dalgaard and Claus Thustrup Kreiner: *Endogenous Growth: A Knife-Edge or the Razor's Edge?*
- 02-03 Marta Loi, Teresa Lloyd-Braga and Hans Jørgen Whitta-Jacobsen: *Endogenous Business Cycles and Systematic Stabilization Policy*
- 02-04: Wolfgang Eggert and Martin Kolmar: *Contests with Size Effects.*

2001

- 01-01 Henrik Jensen: *Optimal Degrees of Transparency in Monetary Policymaking*
- 01-02 Thorvaldur Gylfason and Gylfi Zoega: *Natural Resources and Economic Growth: The Role of Investment.*
- 01-03 Wilhelm Kohler: *International Fragmentation of Value-added Chains: How Does it Affect Domestic Factor Prices?*
- 01-04 Mark A. Roberts and Eric O'N. Fisher: *Funded Pensions, Labor Market Participation, and Economic Growth.*
- 01-05 Søren Bo Nielsen, Pascal Raimondos-Møller and Guttorm Schjelderup: *Formula Apportionment and Transfer Pricing under Oligopolistic Competition.*
- 01-06 Christian Keuschnigg and Søren Bo Nielsen: *Public Policy for Venture Capital.*
- 01-07 Søren Bo Nielsen, Pascal Raimondos-Møller, and Guttorm Schjelderup: *Tax Spillovers under Separate Accounting and Formula Apportionment*
- 01-08 Peter Birch Sørensen: *International Tax Coordination: Regionalism versus Globalism.*
- 01-09 Michael M. Hutchison: *A Cure Worse Than the Disease? Currency Crises and the Output Costs of IMF-Supported Stabilization Programs.*
- 01-10 Rasmus Lenz and Torben Tranæs: *Job Search and Savings: Wealth Effects and Duration Dependence.*
- 01-11 Dieter Bös: *Bureaucrats and Public Procurement.*
- 01-12 Kala Krishna and Cemile Yavas: *Wage Equality in a General Equilibrium Model with Indivisibilities.*

- 01-13 Mark A. Roberts: *Funding the Transition from Pay-As-You-Go Pensions by Taxing Capital Gains on Land.*
- 01-14 Wilfred J. Ethier: *Punishments and Dispute Settlement in Trade Agreements.*
- 01-15 Peter Christoffersen, Eric Ghysels, and Norman R. Swanson: *Let's Get "Real" about Using Economic Data.*
- 01-16 James E. Alt, David Dreyer Lassen, and David Skilling: *Fiscal Transparency, Gubernatorial Popularity, and the Scale of Government: Evidence from the States*

2000

- 00-01 Andrew Hughes Hallett and Maria Demertzis: *When Can An Independent Central Bank Offer Lower Inflation at No Cost? A Political Economy Analysis.*
- 00-02 Torben M. Andersen: *International Integration, Risk and the Welfare State.*
- 00-03 F. Gulzin Ozkan, Anne Sibert and Alan Sutherland: *Monetary Union, Entry Conditions and Economic Reform.*
- 00-04 Michael M. Hutchison: *European Banking Distress and EMU: Institutional and Macroeconomic Risks.*
- 00-05 Bertil Holmlund: *Labor Taxation in Search Equilibrium with Home Production.*
- 00-06 Knud Jørgen Munk: *Administrative Costs and the "Double Dividend".*
- 00-07 Carl-Johan Dalgaard and Claus Thustrup Hansen: *Scale-Invariant Endogenous Growth.*
- 00-08 A. Lans Bovenberg and Ben J. Heijdra: *Environmental Abatement and Intergenerational Distribution.*
- 00-09 Jan Overgaard Olesen: *A Simple Explanation of Stock Price Behaviour in the Long Run: Evidence for Denmark.*
- 00-10 John E. Roemer, Rolf Aaberge, Ugo Colombino, Johan Fritzell, Stephen P. Jenkins, Ive Marx, Marianne Page, Evert Pommer, Javier Ruiz-Castillo, Maria Jesus San Segundo, Torben Tranæs, Gert G. Wagner, and Ignacio Zubiri: *To What Extent Do Fiscal Regimes Equalize Opportunities for Income Acquisition among Citizens?*
- 00-11 Christian Schultz and Tomas Sjöström: *Local Public Goods, Debt and Migration.*
- 00-12 Francesco Daveri: *Is Growth an Information Technology Story in Europe Too?*
- 00-13 Peter F. Christoffersen: *Dating the Turning Points of Nordic Business Cycles.*

- 00-14 Reuven Glick and Michael M. Hutchison: *Stopping “Hot Money” or Signalling Bad Policy? Capital Controls and the Onset of Currency Crises.*
- 00-15 Dan Anderberg and Carlo Perroni: *Renegotiation of Social Contracts by Majority Rule.*
- 00-16 Henrik Jacobsen Kleven: *Optimum Taxation and the Allocation of Time.*
- 00-17 Clemens Fuest, Bernd Huber and Søren Bo Nielsen: *Why Is the Corporate Tax Rate Lower than the Personal Tax Rate?*
- 00-18 Christian Keuschnigg and Søren Bo Nielsen: *Tax Policy, Venture Capital, and Entrepreneurship.*
- 00-19 Harry Huizinga and Søren Bo Nielsen: *Withholding Taxes or Information Exchange: The Taxation of International Interest Flows.*
- 00-20 David Dreyer Lassen: *Political Accountability and the Size of Government: Theory and Cross-Country Evidence.*

1999

- 99-01 Henrik Jacobsen and Peter Birch Sørensen: *Labour Tax Reform, The Good Jobs and the Bad Jobs.*
- 99-02 Ignacio Ortuno-Ortin and Christian Schultz: *Divide the Dollar, A Model of Interregional Redistributive Politics.*
- 99-03 Pekka Ilmakunnas, Vesa Kanninen, and Uki Lammi: *Entrepreneurship, Economic Risks, and Risk-Insurance in the Welfare State.*
- 99-04 Niels Thygesen: *Evolving Ambitions in Europe’s Monetary Unification.*
- 99-05 Sajal Lahiri and Pascalis Raimondos-Møller: *Lobbying by Ethnic Groups and Aid Allocation.*
- 99-06 Roel M.W.J. Beetsma and Henrik Jensen: *Structural Convergence under Reversible and Irreversible Monetary Unification.*
- 99-07 Wolfgang Mayer and Pascalis Raimondos-Møller: *The Politics of Foreign Aid.*
- 99-08 Dermot Leahy and Catia Montagna: *Temporary Social Dumping, Union Legalisation and FDI: A Note on the Strategic Use of Standards.*
- 99-09 Rasmus Fatum and Michael M. Hutchison: *Is Sterilized Foreign Exchange Intervention Effective After All? An Event Study Approach.*
- 99-10 Christian Schultz and Tomas Sjöström: *Public Debt, Property Values and Migration.*

- 99-11 Roel M.W.J. Beetsma and Henrik Jensen: *Risk Sharing and Moral Hazard with a Stability Pact.*
- 99-12 Henrik Jacobsen Kleven, Wolfram F. Richter and Peter Birch Sørensen: *Optimal Taxation with Household Production.*
- 99-13 Huw Dixon, Claus Thustrup Hansen and Henrik Jacobsen Kleven: *Dual Labour Markets and Menu Costs: Explaining the Cyclicalities of Productivity and Wage Differentials.*
- 99-14 Jim Malley and Hassan Molana: *Fiscal Policy and the Composition of Private Consumption: Some Evidence from the U.S. and Canada.*
- 99-15 Jonas Agell and Per Lundborg: *Survey Evidence on Wage Rigidity and Unemployment: Sweden in the 1990s.*
- 99-16 Satya P. Das: *North-South Trade, Capital Accumulation and Personal Distribution of Wealth and Income.*
- 99-17 Holger Bonin, Bernd Raffelhüschen and Jan Walliser: *Can Immigration Alleviate the Demographic Burden?*
- 99-18 Morten Hvidt and Søren Bo Nielsen: *Noncooperative vs. Minimum-Rate Commodity Taxation.*
- 99-19 Lisandro Abrego and Carlo Perroni: *Investment Subsidies and Time-Consistent Environmental Policy.*
- 99-20 Reuven Glick and Michael M. Hutchison: *Banking and Currency Crises: How Common Are Twins?*
- 99-21 Mark Gradstein and Moshe Justman: *Public Schooling, Social Capital and Growth.*
- 99-22 Jeremy S.S. Edwards and Alfons J. Weichenrieder: *Ownership Concentration and Share Valuation: Evidence from Germany.*
- 99-23 Henrik Jensen: *Targeting Nominal Income Growth or Inflation?*
- 99-24 Svend E. Hougaard Jensen and Thomas F. Rutherford: *Distributional Effects of Fiscal Consolidation.*

1998

- 98-01 Pascal Raimondos-Møller and Kimberley A. Scharf: *The Optimal Design of Transfer Pricing Rules: A Non-Cooperative Analysis.*
- 98-02 Michael Keen, Sajal Lahiri and Pascal Raimondos-Møller: *When Is Policy Harmonisation Desirable?*

- 98-03 Clemens Fuest and Bernd Huber: *Tax Progression and Human Capital in Imperfect Labour Markets.*
- 98-04 Frank Hettich and Minna Selene Svane: *Environmental Policy in a Two Sector Endogenous Growth Model.*
- 98-05 Harry Huizinga and Søren Bo Nielsen: *Is Coordination of Fiscal Deficits Necessary?*
- 98-06 Claus Thustrup Hansen: *A Note on Blanchard & Kiyotaki (1987).*
- 98-07 Claus Thustrup Hansen: *Long Run Impact of Increased Wage Pressure.*
- 98-08 Erkki Koskela and Ronnie Schöb: *Why Governments Should Tax Mobile Capital in the Presence of Unemployment.*
- 98-09 Mark A. Roberts: *Unfunded Social Security in the OLG Model with an Imperfectly Competitive Finance Market.*
- 98-10 Peter Birch Sørensen: *Tax Policy, the Good Jobs and the Bad Jobs.*
- 98-11 Roel M.W.J. Beetsma and Henrik Jensen: *Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts: Comment.*
- 98-12 Ole Risager: *Random Walk or Mean Reversion: The Danish Stock Market Since World War I.*
- 98-13 João Ejarque and Torben Tranæs: *Skill-Neutral Shocks and Institutional Changes: Implications for Productivity Growth and Wage Dispersion.*
- 98-14 Minna Selene Svane: *Growth, Training Leave and Unemployment.*
- 98-15 Svend E. Hougaard Jensen: *Nominal Stability, Real Convergence, and Fiscal Transfers in a Monetary Union.*
- 98-16 U. Michael Bergman and Michael Hutchison: *The Costs of EMU and Economic Convergence.*
- 98-17 Niels Thygesen: *Fiscal Institutions in EMU and the Stability Pact.*
- 98-18 Søren Bo Nielsen: *A Simple Model of Commodity Taxation and Cross-Border Shopping.*
- 98-19 Christian Schultz: *Monetary Policy, Delegation and Polarization.*
- 98-20 Knud Jørgen Munk: *Should Governments Create Production Inefficiency?*
- 98-21 Syed M. Ahsan and Panagiotis Tsigaris: *The Public Discount Rate and the Uncertain Budgetary Flows.*
- 98-22 Minna Selene Svane: *Emission Standards and Growth.*

1997

- 97-01 Harry Huizinga and Søren Bo Nielsen: *The Political Economy of Capital Income and Profit Taxation in a Small Open Economy.*
- 97-02 Torsten Sløk and Jens Peter Sørensen: *How Small Shocks and Heterogeneous Expectations Can Create Swings in the Exchange Rate.*
- 97-03 Thórarinn G. Pétursson and Torsten Sløk: *Wage Formation in a Cointegrated VAR Model: A Demand and Supply Approach.*
- 97-04 Jeffrey H. Nilsen: *Borrowed Reserves, Fed Funds Rate Targets, and the Term Structure.*
- 97-05 Carlo Perroni and Kimberley A. Scharf: *Tiebout with Politics: Capital Tax Competition and Constitutional Choices.*
- 97-06 Sajal Lahiri, Pascal Raimondos-Møller, Kar-yiu Wong, and Alan D. Woodland: *Optimal Income Transfers and Tariffs.*
- 97-07 Claus Thustrup Hansen and Hans Jørgen Jacobsen: *Rebalancing Unemployment Benefits in a Unionized Labour Market.*
- 97-08 Sören Blomquist and Vidar Christiansen: *Price Subsidies versus Public Provision.*
- 97-09 Amrita Dhillon, Carlo Perroni and Kimberley A. Scharf: *Implementing Tax Coordination.*
- 97-10 Peter Birch Sørensen: *Optimal Tax Progressivity in Imperfect Labour Markets.*
- 97-11 Syed M. Ahsan and Peter Tsigaris: *The Design of a Consumption Tax under Capital Risk.*
- 97-12 Claus Thustrup Hansen and Søren Kyhl: *Pay-per-view Television: Consequences of a Ban.*
- 97-13 Harry Huizinga and Søren Bo Nielsen: *The Taxation of Interest in Europe: A Minimum Withholding Tax?*
- 97-14 Harry Huizinga and Søren Bo Nielsen: *A Welfare Comparison of International Tax Regimes with Cross-Ownership of Firms.*
- 97-15 Pascal Raimondos-Møller and Alan D. Woodland: *Tariff Strategies and Small Open Economies.*
- 97-16 Ritva Tarkiainen and Matti Tuomala: *On Optimal Income Taxation with Heterogenous Work Preferences.*
- 97-17 Minna Selene Svane: *Optimal Taxation in a Two Sector Model of Endogenous Growth.*

- 97-18 Frank Hettich: *Growth Effects of a Revenue Neutral Environmental Tax Reform.*
- 97-19 Erling Steigum, Jr.: *Fiscal Deficits, Asset Prices and Intergenerational Distribution in an Open Unionized Economy.*
- 97-20 Rod Falvey and Geoff Reed: *Rules of Origin as Commercial Policy Instruments.*
- 97-21 U. Michael Bergman, Michael M. Hutchison and Yin-Wong Cheung: *Should the Nordic Countries Join A European Monetary Union? An Empirical Analysis.*
- 97-22 Kenneth M. Kletzer: *Macroeconomic Stabilization with a Common Currency: Does European Monetary Unification Create a Need for Fiscal Insurance or Federalism?*
- 97-23 Martin Richardson: *Trade Policy and Access to Retail Distribution.*
- 97-24 Sugata Marjit and Hamid Beladi: *Protection, Underemployment and Welfare.*
- 97-25 Bernd Huber: *Tax Competition and Tax Coordination in an Optimum Income Tax Model.*
- 97-26 Clemens Fuest and Bernd Huber: *Tax Coordination and Unemployment.*
- 97-27 Assaf Razin, Efraim Sadka, and Chi-Wa Yuen: *Quantitative Implications of the Home Bias: Foreign Underinvestment, Domestic Oversaving, and Corrective Taxation.*
- 97-28 Mark A. Roberts, Karsten Stæhr, and Torben Tranæs: *Two-Stage Bargaining with Coverage Extension in a Dual Labour Market.*

1996

- 96-01 Torben Tranæs: *A Simple Model of Raiding Opportunities and Unemployment.*
- 96-02 Kala Krishna and Ling Hui Tan: *Transferable Licenses vs. Nontransferable Licenses: What is the Difference?*
- 96-03 Jiandong Ju and Kala Krishna: *Market Access and Welfare Effects of Free Trade. Areas without Rules of Origin.*
- 96-04 Anders Sørensen: *Growth Enhancing Policies in a Small Open Economy.*
- 96-05 Anders Sørensen: *Industrialization and Factor Accumulation.*
- 96-06 Christian Schultz: *Announcements and Credibility of Monetary Policy.*
- 96-07 Christian Schultz: *Political Competition and Polarization.*
- 96-08 Ole Risager and William G. Tyler: *Macroeconomic Policy and Exchange Rate Policy Management in a Small Dependent Economy: Estimating the Effects of Currency Devaluation in Jordan.*

- 96-09 Neil Rankin: *How Does Uncertainty About Future Fiscal Policy Affect Current Macroeconomic Variables?*
- 96-10 U. Michael Bergman and Michael M. Hutchison: *The 'German View', Fiscal Consolidation and Consumption Booms: Empirical Evidence from Denmark.*
- 96-11 Eric Hansen and Michael M. Hutchison: *Exchange Rates, Non-traded Goods and the Terms-of-Trade: An Empirical Application for New Zealand.*
- 96-12 Michael M. Hutchison and Carl E. Walsh: *Central Bank Institutional Design and the Output Cost of Disinflation: Did the 1989 New Zealand Reserve Bank Act Affect the Inflation-Output Tradeoff?*
- 96-13 Rasmus Fatum and Michael M. Hutchison: *Is Intervention a Signal of Future Monetary Policy? Evidence from the Federal Funds Futures Market.*
- 96-14 Tryggvi Thor Herbertsson and Anders Sørensen: *Policy Rules for Exploitation of Renewable Resources: A Macroeconomic Perspective.*
- 96-15 Anders Sørensen: *International Welfare Effects from Country-Specific R&D Subsidies.*
- 96-16 Andreas Haufler and Søren Bo Nielsen: *Dynamic Effects of an Anticipated Switch from Destination- to Origin-Based Commodity Taxation.*
- 96-17 Harry Huizinga and Søren Bo Nielsen: *The Coordination of Capital Income and Profit Taxation with Cross-Ownership of Firms.*
- 96-18 Svend Erik Hougaard Jensen: *Wage Rigidity, Monetary Integration and Fiscal Stabilisation in Europe.*
- 96-19 Ole Risager and Jan Rose Sørensen: *Job Security Policies and Trade Union Behaviour in an Open Economy.*
- 96-20 Slobodan Djajic, Sajal Lahiri and Pascal Raimondos-Møller: *Transfer and the Intertemporal Terms of Trade.*
- 96-21 Slobodan Djajic, Sajal Lahiri and Pascal Raimondos-Møller: *Logic of Aid in an Intertemporal Setting.*
- 96-22 Svend E. Hougaard Jensen and Bernd Raffelhüschen: *Public Debt, Welfare Reforms, and Intergenerational Distribution of Tax Burdens in Denmark.*

1995

- 95-01 Vesa Kannianen and Jan Södersten: *On Financial Adjustment and Investment Booms: Lessons from Tax Reforms.*
- 95-02 Søren Bo Nielsen: *Withholding Taxes and Country-Specific Shocks.*

- 95-03 Vesa Kanninen and Rune Stenbacka: *Towards a Theory of Socially Valuable Imitation with Implications for Technology Policy.*
- 95-04 Bent E. Sørensen, Pierfederico Asdrubali, and Oved Yosha: *Channels of Interstate Risksharing: US 1963-1990.*
- 95-05 Peter Birch Sørensen: *Changing Views of the Corporate Income Tax.*
- 95-06 Robin Boadway: *The Role of Second-Best Theory in Public Economics.*
- 95-07 Sjak Smulders: *Environmental Policy and Sustainable Economic Growth - an endogenous growth perspective.*
- 95-08 Bernd Genser: *Patterns of Tax Arbitrage and Decentralized Tax Autonomy.*
- 95-09 Harry Huizinga and Søren Bo Nielsen: *Capital Income and Profits Taxation with Foreign Ownership of Firms.*
- 95-10 Ben Lockwood: *Commodity Tax Harmonisation with Public Goods - an Alternative Perspective.*
- 95-11 Saqib Jafarey, Yannis Kaskarelis, and Apostolis Philippopoulos: *Private Investment and Endogenous Fiscal Policy. Theory and Evidence from UK and USA.*
- 95-12 Svend Erik Hougaard Jensen: *Debt Reduction, Wage Formation and Intergenerational Welfare.*
- 95-13 Sajal Lahiri and Pascal Raimondos: *Public Good Provision and the Welfare Effects of Indirect Tax Harmonisation.*
- 95-14 Ruud A. de Mooij and A. Lans Bovenberg: *Environmental Taxes, International Capital Mobility and Inefficient Tax Systems: Tax Burden vs. Tax Shifting.*
- 95-15 David F. Bradford: *Consumption Taxes: Some Fundamental Transition Issues.*
- 95-16 Ole Risager: *On the Effects of Trade Policy Reform: The Case of Jordan.*
- 95-17 Niels Thygesen: *The Prospects for EMU by 1999 - and Reflections on Arrangements for the Outsiders.*
- 95-18 Christian Keuschnigg and Søren Bo Nielsen: *Housing Markets and Vacant Land.*
- 95-19 Hans Fehr: *Welfare Effects of Investment Incentive Policies: A Quantitative Assessment.*
- 95-20 Ben Lockwood, Torsten Sløk, and Torben Tranæs: *Progressive Taxation and Wage Setting: Some Evidence for Denmark.*
- 95-21 Claus Thustrup Hansen, Lars Haagen Pedersen, and Torsten Sløk: *Progressive Taxation, Wages and Activity in a Small Open Economy.*

- 95-22 Svend Erik Hougaard Jensen and Bernd Raffelhüschen: *Intertemporal Aspects of Fiscal Policy in Denmark*.

1994

- 94-01 Niels Thygesen: *Reinforcing Stage Two in the EMU Process*.
- 94-02 Kåre P. Hagen and Vesa Kanninen: *Optimal Taxation of Intangible Capital*.
- 94-03 Ed W.M.T. Westerhout: *The Economic and Welfare Effects of Taxing Foreign Assets*.
- 94-04 Slobodan Djajic: *Illegal Immigration and Resource Allocation*.
- 94-05 Sajal Lahiri and Pascal Raimondos: *Is There Anything Wrong with Tied-Aid?*
- 94-06 Ben Lockwood, Apostolis Philippopoulos, and Andy Snell: *Fiscal Policy, Public Debt Stabilization and Politics: Theory and evidence from the US and UK*.
- 94-07 Partha Sen: *Welfare-Improving Debt Policy under Monopolistic Competition*.
- 94-08 Sajal Lahiri and Pascal Raimondos: *Tying of Aid to Trade Policy Reform and Welfare*.
- 94-09 Mark Gradstein and Moshe Justman: *Public Choice of an Education System and its Implications for Growth and Income Distribution*.
- 94-10 Peter Birch Sørensen, Lars Haagen Pedersen, and Søren Bo Nielsen: *Taxation, Pollution, Unemployment and Growth: Could there be a "Triple Dividend" from a Green Tax Reform?*
- 94-11 Peter Birch Sørensen and Søren Bo Nielsen: *On the Optimality of the Nordic System of Dual Income Taxation*.
- 94-12 Sajal Lahiri and Pascal Raimondos: *Competition for Aid and Trade Policy*.
- 94-13 Niels Kleis Frederiksen, Peter Reinhard Hansen, Henrik Jacobsen, and Peter Birch Sørensen: *Consumer Services, Employment and the Informal Economy*.
- 94-14 Yoshiyasu Ono: *Market Segmentation and Effective Demand Shortage in a World with Dynamic Optimization*.

1993

- 93-01 Svend Erik Hougaard Jensen, Søren Bo Nielsen, Lars Haagen Petersen and Peter Birch Sørensen: *Tax Reform, Welfare, and Intergenerational Redistribution - An Intertemporal Simulation Approach*.
- 93-02 Bernd Genser, Andreas Haufler and Peter Birch Sørensen: *Indirect Taxation in an Integrated Europe. Is there a Way of Avoiding Trade Distortions Without Sacrificing National Tax Autonomy?*

- 93-03 Svend Erik Hougaard Jensen and Lars Grue Jensen: *Debt, Deficits and Transition to EMU: A Small Country Analysis.*
- 93-04 Torsten Persson and Guido Tabellini: *Federal Fiscal Constitutions. Part I: Risk Sharing and Moral Hazard.*
- 93-05 Martin Paldam: *The Political Economy of Stopping High Inflation.*
- 93-06 Roger H. Gordon and Jeffrey K. Mackie-Mason: *Why is There Corporate Taxation in a Small Open Economy? The Role of Transfer Pricing and Income Shifting.*
- 93-07 Peter Birch Sørensen: *From the Global Income Tax To the Dual Income Tax: Recent Tax Reforms in The Nordic Countries.*
- 93-08 Wilhelm Kohler: *Strategic Trade Policy and Integration.*
- 93-09 F. Gulcin Ozkan and Alan Sutherland: *A Model of the ERM Crisis.*