

EPRU

Economic Policy Research Unit

Institute of Economics

University of Copenhagen

Studiestræde 6

DK-1455 Copenhagen K

DENMARK

Tel: (+45) 3532 4411

Fax: (+45) 3532 4444

Web: <http://www.econ.ku.dk/epru/>

**Improving the Equity-Efficiency Trade-Off:
Mandatory Savings Accounts for Social Insurance**

A. Lans Bovenberg
Peter Birch Sørensen

2003-07

ISSN 0908-7745

The activities of EPRU are financed by a grant from
The National Research Foundation

August 2003

Third draft

IMPROVING THE EQUITY-EFFICIENCY TRADE-OFF: MANDATORY SAVINGS ACCOUNTS FOR SOCIAL INSURANCE

A. Lans Bovenberg, Tilburg University, OCFEB, CEPR and CESifo*)

Peter Birch Sørensen, University of Copenhagen, EPRU and CESifo**)

Paper prepared for the 59th Annual Congress of the

International Institute of Public Finance

in Prague, August 25-28, 2003

Abstract

In the modern welfare state a substantial part of an individual's tax bill is transferred back to the same individual taxpayer in the form of social transfers. This provides a rationale for financing part of social insurance through mandatory savings accounts. We analyze the behavioral and welfare effects of compulsory savings accounts in an intertemporal model with uncertainty, endogenous involuntary unemployment and retirement decisions, credit constraints, and heterogeneous agents. We show that the introduction of (early) retirement and unemployment accounts generates a Pareto improvement by enabling the government to provide lifetime income insurance and liquidity insurance in a more efficient manner.

*) CentER, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands

E-Mail: A.L.Bovenberg@uvt.nl

***) Institute of Economics, University of Copenhagen, Studiestraede 6,
1455 Copenhagen K, Denmark. E-mail: Peter.Birch.Sorensen@econ.ku.dk

IMPROVING THE EQUITY-EFFICIENCY TRADE-OFF: MANDATORY SAVINGS ACCOUNTS FOR SOCIAL INSURANCE

A. Lans Bovenberg and Peter Birch Sørensen¹

1. Introduction

The prospect of population ageing in the OECD economies has generated an intense debate on the need for pension reform. Much of the academic controversy has focussed on the question whether moving from Pay-As-You-Go (PAYG) pensions to fully funded systems based on individual savings accounts can produce a Pareto improvement. The literature has concluded that the government cannot improve the welfare of all generations through such a switch (see e.g. Breyer (1989) and Sinn (2000)). If the current social security tax is replaced by public debt to finance the continued payment of pensions to the older generation during the transition to funding, the future taxes servicing the higher public debt exactly offset the gains of future generations from the higher return on pension saving offered by a funded system.

Homburg (1990) has argued, however, that funded individual accounts could make all generations better off if the initial transfer system distorts endogenous labor supply. The reason is that a closer individual link between contributions and benefits boosts labor supply, thereby alleviating labor-market distortions. The resulting efficiency gains can be distributed in such a way that all generations benefit. In the presence of intragenerational heterogeneity,² however, one has to ensure that not only all generations but also all households within a generation benefit (see also Fenge (1995), Brunner (1996) and Belan and Pestieau (1999)). Whereas a switch to a pension system in which all agents fund their own pensions eliminates the labor market distortion from a 'Beveridgean' PAYG system with a flat pension benefit, such a reform also does away with the intragenerational

¹In preparing this paper we have benefited from discussions with and criticisms by Henrik Kleven, Pierre Pestieau, Bo Sandemann Rasmussen and Dave Wildasin. Any remaining shortcomings are our own responsibility.

²Without intragenerational heterogeneity, the optimal tax-transfer system would not need to distort labor supply, as lump-sum taxes would be optimal.

redistribution achieved by such a PAYG system. The reform, therefore, is likely to hurt low-income households. In fact, if the initial Beveridgean pension benefit has been optimized by trading off the marginal equity gain against the marginal efficiency cost, a switch to individual funding cannot be Pareto-improving.

This analysis thus confirms the major lesson of the literature on the transition from funding to PAYG, namely that a Pareto improvement, which protects all agents, is feasible only if pension reform is accompanied by a reduction of a distortion somewhere in the economy. Examples of such possible distortions are the corporate income tax, inefficient redistribution, labor-market distortions due to incentives to retire early, inability of the political process to commit to promises (and the associated political risks), missing insurance markets (e.g. due to aggregate risks or the inability of agents to commit to insurance contracts before they are born), capital-market imperfections, and knowledge externalities. The associated efficiency gains should be ascribed to the cut in these distortions rather than the transition to funding. Indeed, the gains could be reaped also without changing to a funded system. Moreover, if the system has been optimized prior to the reform, one cannot obtain a Pareto improvement by moving to a funded system.

This paper shows that funding through compulsory savings accounts can be Pareto improving, even if (in contrast to most of the literature on the transition to funding) one allows for intragenerational heterogeneity. Hence, all agents gain from the reform, irrespective of the shocks they experience during their lifetimes. The key to this result is that the savings accounts protect households who suffer from low lifetime incomes. In particular, individual funding and the associated self insurance applies only to high-income earners and the middle class. Low-income earners still benefit from tax-financed transfers. As another extension of the voluminous literature on the role of savings accounts in old-age social security, we explore whether compulsory savings accounts can more efficiently finance social insurance for individuals of working age.³

The savings accounts considered in this paper are inspired by Fölster (1994, 1997) and work as follows. For each taxpayer an individual account is established. Part of the taxpayer's annual tax bill is replaced by a mandatory social security contribution, which

³Orszag and Snower (1997a, 1997b), Feldstein and Altman (1998), Orszag et al. (1999), and Fölster et al. (2002) propose savings accounts to finance social insurance for the working population.

is credited to his individual account. The contribution is computed as a percentage of the taxpayer's income. Whenever the taxpayer receives a social benefit payment from one of the transfer programs included in the individual account scheme (e.g. unemployment insurance, early retirement benefits), a fraction of this benefit is debited to the account. A (risk-free) market interest rate is added to or subtracted from the balance on the account each year. When one has reached the statutory retirement age, the government settles the account. In particular, part of the balance on the compulsory savings account is used to buy an annuity covering a fraction of the ordinary public pension. The balance that remains after buying this annuity can be used to supplement the public pension. In this way, the individual accounts are in fact integrated with the pension system. If the account balance at the statutory retirement age is not sufficient to buy an annuity for the part of the ordinary public pension benefit that should be financed out of individual accounts, the government supplements the funds in the savings account so that the individual receives the full ordinary public pension during old age. In this way, the government effectively bails out households with low lifetime incomes.

We distinguish three types of compulsory savings accounts depending on which transfer program is included in the accounts, namely unemployment accounts (UA), early retirement accounts (ERA) and retirement accounts (RA). These accounts provide *lifetime income insurance* by guaranteeing a minimum public pension benefit that does not depend on the funds in the account at statutory retirement. This provision ensures that unlucky individuals who draw large amounts from social insurance programs relative to their contributions have their benefits, just like today, financed out of general tax revenue contributed by all taxpayers without any negative consequences for their minimum old-age pensions. Moreover, the UA system also provides *liquidity insurance* by allowing the worker to collect unemployment benefits under eligibility rules identical to those existing at present, regardless of the size of the balance on his account. We show that, by adding unemployment accounts to the existing tax-financed system of social insurance, the government can provide liquidity insurance more efficiently.

The basic reason why these compulsory accounts can produce a Pareto improvement is that they add to the armory of fiscal instruments available to the government. By observing the balance in the compulsory savings accounts, the government in fact obtains

information about individual lifetime incomes. This additional information allows the government to offer lifetime income insurance and liquidity insurance in a more efficient manner.⁴ The key aspect of the savings accounts is thus not funding, but rather the additional information about individual lifetime incomes, which allows for more efficient insurance. Specifically, early retirement accounts combined with a minimum lifetime income guarantee improve the labor market incentives of higher- and middle-income workers (by creating an actuarial link between taxes and benefits) without cutting into the consumption of workers collecting only low lifetime incomes. Furthermore, in contrast to a general cut in social benefit rates, unemployment accounts enable the government to decrease the present value of public transfers without reducing the consumption possibilities of liquidity-constrained unemployed workers.

The analysis in this paper extends and generalizes the work of Sørensen (2003). In particular, we allow for a richer set of fiscal instruments and for more individual heterogeneity by introducing job search and uncertainty about involuntary unemployment as well as uncertainty about future wages. We show that even if the government has access to more fiscal instruments before the introduction of savings accounts and if additional shocks yield more ex-post heterogeneity in lifetime incomes, compulsory savings accounts can still make everybody better off. We also clearly separate the sources of the welfare gains of RAs, ERAs and UAs, respectively.

The recent contribution by Stiglitz and Yun (2002) explores the optimal design of unemployment accounts. These authors analyze how the optimal design of UAs is affected by the degree of risk aversion, the length of unemployment spells, and the sensitivity of job search intensity to economic incentives. The present paper does not seek to characterize the optimal UA providing lifetime income and liquidity insurance in the most efficient way. Instead, we demonstrate the possibility of a welfare-improving *reform* involving the use of a particular type of compulsory savings accounts.⁵ Stiglitz and Yun (2002) evaluate

⁴Setting up and enforcing compulsory individual accounts and registering the individual balances would obviously involve some costs. This paper does not analyze these costs. The benefits of improved lifetime income and liquidity insurance identified in this paper can be used to compute an upper bound for the costs of a welfare-improving individual savings system.

⁵Indeed, in addition to bailing out households with insufficient funds in their accounts, there may be other ways to offer lifetime income insurance, such as levying a linear tax on the accumulated account balances of all households.

the reforms on the basis of ex-ante utility (i.e. expected utility of individuals before shocks occur and individuals are still homogeneous). We, in contrast, explore whether the introduction of compulsory savings accounts can make everybody better off ex post (i.e. after the shocks have occurred) – even individuals who experienced adverse shocks. Compared to Stiglitz and Yun (2002), we also consider a richer set of shocks, including involuntary unemployment and wage shocks. In this way, we focus on achieving a Pareto improving reform in the presence of heterogeneous individuals rather than establishing the most efficient lifetime insurance system with ex-ante identical agents. Furthermore, we include incentive effects on labor supply of employed agents in addition to the moral hazard effects on job search considered by Stiglitz and Yun. Stiglitz and Yun also do not analyze the case for ERA and RA and the similarities and differences between UA, on the one hand, and ERA and RA, on the other hand. Rather, they take the existence of RA as exogenously given.

The rest of this paper is structured as follows. Section 2 describes the formal model underlying our analysis. Section 3 demonstrates that introducing (early) retirement accounts or unemployment accounts can yield a Pareto improvement. Section 4, finally, contains our main conclusions and suggestions for future research.

2. The model framework

2.1. The basic setting

Agents live for two periods. At the start of period 1, agents are young and have no prior labor market experience. As they are imperfectly informed about job opportunities, young workers face some risk of being unable to find employment during period 1. At the start of period 2, all workers have previous job search experience and are thus able to find a job. However, an exogenous fraction of the previously unemployed lose human capital as a result of being out of work. Due to this scarring effect of unemployment, these individuals earn a lower wage rate in period 2. This feature of the model allows us to explore the importance of the correlation between unemployment shocks and negative wage shocks during the life cycle (see section 3.3).

At some time before the end of period 2, workers decide (endogenously) to retire.

Subsequently, they collect public pensions and annuities from their mandatory savings accounts. Workers who are employed in both periods smooth their consumption through life-cycle saving. Unemployed workers, in contrast, face liquidity constraints and hence consume all their unemployment benefits during period 1. Consequently, they enter period 2 without any financial wealth.

Our framework includes both voluntary and involuntary unemployment. On the one hand, a young worker who fails to find a job in period 1 is involuntarily out of work. On the other hand, an old worker may voluntarily opt for non-employment by retiring early. The model also includes two sources of uncertainty. In particular, agents must decide on their job search without knowing whether that search will be successful and whether they will lose human capital if they would fail to find a job in the first period.

For the sake of simplicity, pre-tax factor prices are fixed. The exogeneity of factor prices may be rationalized by the assumption of a small open economy. Using a constant-returns technology and facing perfect capital mobility, such an economy produces and consumes a single good that is a perfect substitute for foreign goods. In this setting, the exogenous world real interest rate determines domestic capital intensity, which in turn fixes the domestic pre-tax real wage. We allow wages to differ across the two periods. This may reflect different period lengths (see also sub-section 3.3) or varying labor productivities across the two periods.

2.2. Policy instruments

Individuals become heterogeneous only after the completion of job search. Ex ante all individuals are identical and face the same risks. One may therefore ask why agents cannot write optimal private insurance contracts before the start of period 1, thereby eliminating the scope for Pareto-improving social insurance. To provide a rationale for social insurance in our model, we assume that private insurance companies, in contrast to the tax authorities, have difficulties in obtaining verifiable information about actual labor incomes. Private suppliers of income insurance thus face more serious moral-hazard problems than the government does. Another problem with private insurance is that private insurers fail to internalize the external effects of additional income insurance on the tax base and public benefits paid. Instead of leaving private insurers free to

offer these supplementary insurances, the government may therefore want to design and regulate social insurance.⁶

As far as liquidity insurance is concerned, the private market fails because of selection due to private information about a person's risk of human capital loss. Specifically, unemployed workers may well learn relatively early in period 1 whether they will lose human capital, while this information remains private until a person's wage rate in period 2 is revealed. Hence, banks do not know whether an unemployed worker asking for a loan in period 1 will earn a normal or a subnormal wage in period 2. They may therefore be unwilling to accommodate credit demand for fear that the value of the worker's human capital is not sufficient to prevent a default on the loan. The government can alleviate this capital-market imperfection⁷ by forcing everybody to save through compulsory savings accounts.

We assume that only linear taxes are available. Real-world tax systems are piece-wise linear, so our simplifying assumption of linear taxes is not necessarily less realistic than the alternative assumption of non-linear tax schedules. Indeed, in recent decades, many OECD countries have reduced the number of income tax brackets in order to simplify administration and make their tax systems more transparent. The assumption of linear taxes can also be rationalized by assuming that the government does not observe individual incomes and can thus levy only impersonal, proportional taxes on labor income.⁸ The proportional income tax rate therefore cannot depend on age.

We abstract from taxes on capital income because the government cannot observe individual savings.⁹ Otherwise, the model allows for a rich set of fiscal instruments, as the

⁶For the external effects between insurers in the presence of moral hazard, see Pauly (1974) and Greenwald and Stiglitz (1986).

⁷Our assumption that all unemployed workers are subject to liquidity constraints is made solely for expositional convenience. In any case, the case for UA as an efficient way to offer lifetime income insurance does not depend on the existence of liquidity constraints.

⁸The concluding section argues that the potential for a Pareto improvement via individual accounts does not rest on our assumption that non-linear taxes on static incomes are not available.

⁹In the face of this lack of information, the government cannot impose a residence-based tax on voluntary saving. Under the small-open-economy assumption, the government does not want to levy a source-based tax on capital. Indeed, with infinitely elastic capital supply from abroad, such a source-based tax would be shifted unto labor. It is thus more efficient to tax labor directly through a labor income tax rather than indirectly through a source-based capital tax, which distorts not only labor

government can observe both age and employment status (i.e. unemployed, employed, or retired). Hence, our model includes categorical social insurance benefits for employment, early retirement and ordinary old age. These benefits can be set independently of each other. The government can thus differentiate lump-sum transfers according to employment status and age. In practice, this might be achieved through instruments such as benefits to dependent children and education benefits.

The following sections will present the model and its assumptions in more detail.

2.3. Preferences

As noted, the economy includes three groups of individuals. Those who are fully employed in period 1 are termed *high-income earners* and are indicated by the superscript h . Those who are unemployed during period 1 but do not lose any human capital are called *medium-income earners* and are marked by the superscript m , while those who do lose human capital as a result of the scarring effect of unemployment are referred to as *low-income earners* and are denoted by superscript l . Furthermore, we employ the superscript u to refer to all workers who were unemployed in period 1.

Before the start of period 1, the expected lifetime utility U^e of an agent exerting job search effort a is

$$U^e = aV^h + (1 - a)V^u + g(G) - F(a), \quad (2.1)$$

$$g' > 0, \quad g'' \leq 0, \quad F' > 0, \quad F'' > 0, \quad 0 < a < 1,$$

where G is a public good that may be provided in either period of life, $F(a)$ represents disutility of job search effort, while V^h and V^u are, respectively, the expected lifetime utilities attainable by employed and unemployed workers, *excluding* the disutility of search effort and the utility from public consumption. Equation (2.1) assumes that a worker's probability of finding a job in period 1 simply equals his search intensity a . The disutility function $F(a)$ displays increasing marginal disutility of search effort.

supply but also the capital-labor ratio. Note that whereas the government cannot observe voluntary saving, it can observe the funds in the compulsory saving accounts.

Once the initial job search has been completed, the resulting disutility is a sunk cost. The private utility attainable by an employed worker during the remainder of his life (excluding the utility from public consumption) is given by

$$V^h = (f_1(C_1^h, 1 - e_1^h))^{1-\frac{1}{\sigma}} - \beta (f_2(C_2^h, 1 - e_2^h))^{1-\frac{1}{\sigma}}, \quad (2.2)$$

where $0 < \beta < 1$ represents a discount factor, σ is the intertemporal substitution elasticity,¹⁰ C_i^h stands for the high-income earner's consumption during period i ($i = 1, 2$) and e_i^h denotes the fraction of period i during which the high-income individual is working. The homothetic felicity function $f_i(., .)$ models intratemporal substitution between consumption C_i^h and leisure $1 - e_i^h$ in period i .¹¹

An unemployed worker faces an exogenous probability p of losing human capital as a result of joblessness in period 1. If he experiences such a scarring effect of joblessness, he has to accept a lower wage in period 2, in which case he attains private utility V^l during period 2. If he does not lose human capital, he earns the normal wage in period 2 and enjoys private utility V^m in that period. p measures the correlation between unemployment shocks in period 1 and adverse human capital shocks in period 2.

Right after the start of period 1, when the disutility of prior job search has been sunk but the unemployed agent does not yet know whether his human capital will be scarred as a result of this unemployment, the expected private utility of an unemployed worker over his remaining lifetime amounts to

$$V^u = (f_1(C_1^u, 1))^{1-\frac{1}{\sigma}} + \beta [pV^l + (1-p)V^m], \quad 0 < p < 1, \quad (2.3)$$

where C_i^u stands for an unemployed worker's consumption during period i . By analogy to the utility enjoyed by a high-income earner in period 2, the instantaneous private utility obtained by a previously unemployed worker (of both type m and type l) during period 2 amounts to

$$V^j = (f_2(C_2^j, 1 - e_2^j))^{1-\frac{1}{\sigma}}, \quad j = l, m. \quad (2.4)$$

¹⁰The reciprocal of the intertemporal substitution elasticity is the coefficient of relative risk aversion.

¹¹If $f_2(C_2^h, 1 - e_2^h)$ features a constant elasticity of substitution ϕ , we have $f_2(C_2^h, 1 - e_2^h) =$

$\left[\zeta_c (C_2^h)^{1-\frac{1}{\phi}} + \zeta_e (1 - e_2^h)^{1-\frac{1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$, where ζ_i , $i = c, e$, are constants.

2.4. Budget constraints

During period 1 an employed worker earns the standard wage rate w_1 , which is subject to the labor-income tax rate t . He also pays a mandatory social security contribution, which is levied at the rate s and is credited to his individual account. In addition to the mandatory saving in his individual account, the employed worker undertakes voluntary saving S^h . We can thus write the high-income earner's budget constraint for period 1 as

$$C_1^h = w_1 e_1^h (1 - t - s) - S^h + y_1, \quad (2.5)$$

where y_1 denotes transfers collected by employed, young individuals.¹² As a result of adverse selection in private capital markets, workers cannot borrow against their expected future labor and retirement income so that $S^h \geq 0$. This constraint is not assumed to be binding for high-income earners.

In the second period, the worker faces the same labor-income tax rate t and the same social security contribution rate s as in the first period. He also collects a lump-sum public transfer y_2 , which can be interpreted as an ordinary old-age pension granted from the date the worker reaches the exogenous statutory retirement age. In addition, the worker receives a benefit granted at the rate b_2 during that fraction $1 - e_2^h$ of the second period in which the worker is actually retired. At the margin, the transfer b_2 may be interpreted as an early retirement benefit,¹³ since it is paid out from the time the worker chooses to actually retire.¹⁴ Finally, the retired high-income earner may consume the positive balance A^h on his compulsory individual savings account plus the balance $(1 + r)S^h$ on his voluntary savings (where r is the real rate of interest). Thus, the high-income earner's budget constraint for period 2 becomes

$$C_2^h = w_2 (1 - t - s) e_2^h + b_2 (1 - e_2^h) + y_2 + (1 + r)S^h + A^h, \quad (2.6)$$

where w_2 denotes the wage in period 2. This wage may differ from the wage in period 1 because of different wage rates per hour or different period lengths.

¹²These transfers differ from transfers received by unemployed agents (see below). The government thus can observe employment status.

¹³ b_2 can alternatively be interpreted as an unemployment benefit with a lax work test, or as a disability benefit without strict medical tests.

¹⁴Actual retirement is assumed to occur above the age entitling a worker to early retirement benefits.

The balance on the high-income earner's individual account equals the contributions paid into the account during period 1 (with interest added) $(1+r)sw_1e_1^h$, plus the contributions during period 2, $sw_2e_2^h$, minus an exogenous fraction (α_2) α_y of the (early) retirement benefit received in the second period. We thus have

$$A^h = (1+r)sw_1e_1^h + sw_2e_2^h - \alpha_2b_2(1 - e_2^h) - \alpha_yy_2, \quad 0 \leq \alpha_2, \alpha_y \leq 1. \quad (2.7)$$

The parameter (α_2) α_y is a policy instrument reflecting the extent to which (early) retirement benefits must be financed by withdrawals from the recipient's individual account. Under a conventional tax-transfer system without mandatory individual savings accounts, we have $\alpha_y = \alpha_2 = s = 0$.

Consolidating (2.5), (2.6) and (2.7) to eliminate S^h and A^h , we obtain the lifetime budget constraint facing a high-income earner:

$$C_1^h + \frac{C_2^h}{1+r} = w_1e_1^h(1-t) + y_1 + \frac{w_2e_2^h(1-t) + b_2(1-\alpha_2)(1 - e_2^h) + (1-\alpha_y)y_2}{1+r}. \quad (2.8)$$

The social security contribution rate s has dropped out of (2.8). This contribution rate thus does not distort work effort. Indeed, marginal contributions to the individual account are in effect returned to the worker in the form of higher retirement benefits. (2.8) reveals that for the high-income worker the individual account system for retirement benefits in effect boils down to a cut in the effective rate of (early) retirement benefit $(b_2(1-\alpha_2))(1-\alpha_y)y_2$.

All unemployed workers collect an unemployment benefit b_1 during period 1. b_1 is so low that the credit constraint $S^j \geq 0$ ($j = l, m$) is binding for all unemployed workers during period 1, implying¹⁵

$$C_1^u = b_1. \quad (2.9)$$

Because a medium-income worker earns the standard wage w_2 in period 2, he is able to accumulate a positive balance A^m in his individual account. The surplus occurs even though a fraction α_1 of his unemployment benefit is debited to his account in period 1

¹⁵In Stiglitz and Yun (2002), workers in the first period anticipate liquidity constraints in the second period and thus engage in precautionary saving in the first period. Precautionary saving is absent in our model because workers are unemployed only in the first period and know the second-period wage shocks when they undertake first-period saving decisions.

and carried forward with interest. The budget constraints for a medium-income worker in period 2 are thus

$$C_2^m = w_2 e_2^m (1 - t - s) + b_2 (1 - e_2^m) + y_2 + A^m, \quad (2.10)$$

$$A^m = s w_2 e_2^m - \alpha_2 b_2 (1 - e_2^m) - \alpha_y y_2 - \alpha_1 b_1 (1 + r), \quad 0 \leq \alpha_2, \alpha_y, \alpha_1 \leq 1. \quad (2.11)$$

When the policy parameter α_1 is zero, we have a conventional tax-financed system of unemployment insurance. Substituting (2.11) into (2.10) to eliminate A^m , we obtain

$$C_2^m = w_2 e_2^m (1 - t) + b_2 (1 - \alpha_2) (1 - e_2^m) + (1 - \alpha_y) y_2 - \alpha_1 b_1 (1 + r). \quad (2.12)$$

Equation (2.12) reveals that, just as for the high-income earner, (early) retirement accounts imply a cut in the effective rate of (early) retirement benefit for the medium-income earner. Unemployment accounts in effect reduce the present value of unemployment benefits: collecting unemployment benefits in period 1 reduces the account balance by $\alpha_1 b_1 (1 + r)$ in period 2.

A low-income worker loses human capital as a result of first-period unemployment. Hence, his productivity in period 2 is only a fraction θ of the productivity of other workers, so he earns only a fraction θ of the standard wage w_2 . With this worker being hit by adverse shocks in both periods, social security contributions during period 2 are assumed not to be sufficient to cover the social security benefits that are to be financed from the accounts, i.e. $s w_2 \theta e_2^l - \alpha_2 b_2 (1 - e_2^l) - \alpha_y y_2 - \alpha_1 b_1 (1 + r) < 0$. The lifetime income insurance built into the individual account system ensures that the low-income earner still receives the full retirement benefits. The government bail-out in effect means that the government makes no deduction from the low-income earner's account for the retirement benefits but also does not return any of the previously paid social security contributions. The second-period budget constraint for a low-income earner thus amounts to

$$C_2^l = \theta w_2 e_2^l (1 - t - s) + b_2 (1 - e_2^l) + y_2, \quad 0 < \theta < 1. \quad (2.13)$$

Accordingly, for a low-income worker the social security contribution s works exactly the same way as the ordinary tax t . Indeed, this contribution s distorts second-period labor supply, as it is not returned to the low-income earner in the form of higher retirement benefits.

2.5. Generational accounts

To explore the impact of the reform on the public budget, we denote the *generational account* of a worker in group j by g^j , $j = h, m, l$. The generational account measures the present value of a worker's net payments to the public sector over his entire life cycle. Normalizing the total labor force at unity, the present value g of the total net payments from the private to the public sector can then be written as

$$g = ag^h + (1 - a) [pg^l + (1 - p)g^m], \quad (2.14)$$

where we recall that a is the employment rate of the young generation and that a fraction p of those who are unemployed in period 1 end up in the low-income category.

This specification of the government budget constraint implies that the government cannot raise the welfare of new entrants to the labor market by transferring resources away from older generations who are already on the labor market during the transition.¹⁶ Indeed, the government grandfatheres the initial arrangements for the older generations who already lived through the first period when the reform is announced and implemented. For these generations, the payments to the government are not put in individual accounts but remain regular tax finance. Also the collected social benefits are not debited to the individuals concerned. Hence, these generations continue to be treated in accordance with the old fiscal rules during the rest of their lives. The generational accounts of these generations are thus not affected by the reform. In this way, we ensure that a reform that is Pareto-improving for the newly entering generations does not come at the expense of older generations, so that the reform is truly Pareto improving for all income groups in all generations.

Treating the system of mandatory individual accounts as a part of the public sector,¹⁷ and using (2.7) to eliminate A^h , we find for the generational account of a high-income

¹⁶This contrasts with studies that employ a steady-state version of the government budget constraint.

¹⁷The generational accounts will be the same even if the individual accounts are administered by private financial institutions (assuming that administration costs are the same in the two sectors). For a demonstration of the equivalence between publicly and privately administered individual accounts, see Sørensen (2003).

earner

$$\begin{aligned}
g^h &= (t+s)w_1e_1^h - y_1 + \frac{(t+s)w_2e_2^h - b_2(1-e_2^h) - y_2 - A^h}{1+r} \\
&= tw_1e_1^h - y_1 + \frac{tw_2e_2^h - b_2(1-\alpha_2)(1-e_2^h) - (1-\alpha_y)y_2}{1+r}.
\end{aligned} \tag{2.15}$$

Remembering that a medium-income earner receives unemployment benefits during period 1, and using (2.11) to eliminate A^m , we write the generational account of a medium-income earner as

$$\begin{aligned}
g^m &= -b_1 + \frac{(t+s)w_2e_2^m - b_2(1-e_2^m) - y_2 - A^m}{1+r} \\
&= -b_1(1-\alpha_1) + \frac{tw_2e_2^m - b_2(1-\alpha_2)(1-e_2^m) - (1-\alpha_y)y_2}{1+r}.
\end{aligned} \tag{2.16}$$

Finally, since the low-income earner is bailed out at the end of his active life, his generational account is given by

$$g^l = -b_1 + \frac{(t+s)\theta w_2e_2^l - b_2(1-e_2^l) - y_2}{1+r}. \tag{2.17}$$

2.6. Individual behavior

After he has completed his initial job search and has received a job offer for the first period, the high-income earner maximizes his remaining lifetime utility (2.2) subject to the lifetime budget constraint (2.8). The main text assumes that agents fully participate in the first period (with full-time participation being normalized at unity so that $e_1^h = 1$), so that first-period working hours are exogenously given. The appendix considers the case with endogenous labor supply in the first period and shows that under weak conditions the major results in the main text continue to hold if first-period labor supply is endogenous. $1 - e_2^h$ is interpreted as the fraction of the second period spent in retirement.¹⁸ Endogenous

¹⁸The benchmark considered in the main text can thus be considered as the case in which labor supply is elastic on the extensive rather than the intensive margin. Indeed, recent empirical evidence suggests that labor supply is substantially more elastic on the extensive margin than the intensive margin. Heckman (1993, p. 118) writes "A revision is in order for George Stiger's dictum that all elasticities are 1 in absolute value. A dictum closer to the truth would be that elasticities are closer to 0 than 1 for hours-of-work equations (or weeks-of-work equations) estimated *for those who are working*. A major lesson of the past 20 years is that the strongest empirical effects of wages and nonlabor income on labor supply are to be found at the extensive margin – at the margin of entry and exit – where the elasticities are definitely not zero."

second-period labor supply thus reflects an endogenous retirement decision.

With $e_1^h = 1$, the outcome of this optimization is the following expression for second-period labor supply (see the appendix for a derivation)

$$\begin{aligned} \frac{de_2^h}{1 - e_2^h} = & \left(\frac{1}{Y^h} \right) \left[\left(w_1 e_1^h + \frac{w_2 e_2^h}{1+r} \right) dt - \frac{(1 - \alpha_2)(1 - e_2^h)}{1+r} db_2 \right] \\ & + \left(\frac{1}{Y^h} \right) \left[\frac{b_2(1 - e_2^h)}{1+r} d\alpha_2 - dy_1 - \frac{(1 - \alpha_y)}{1+r} dy_2 + \frac{y_2}{1+r} d\alpha_y \right] \\ & - [\delta\sigma\gamma_2^h + (1 - \gamma_2^h)\phi_2^h] \left[\frac{w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2}{w_2(1 - t) - b_2(1 - \alpha_2)} \right], \end{aligned} \quad (2.18)$$

$$Y^h \equiv y_1 + w_1(1 - t) + \frac{(1 - \alpha_y)y_2 + w_2(1 - t)}{1 + r},$$

$$0 < \gamma_2^h \equiv \frac{(1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]}{C_2^h + (1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]} < 1,$$

$$0 < \delta \equiv \frac{C_1^h + (1 - e_1^h)w_1(1 - t)}{C_1^h + (1 - e_1^h)w_1(1 - t) + \frac{C_2^h + (1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]}{1+r}} < 1,$$

$$\phi_2^h \equiv -d \left(\frac{C_2^h}{(1 - e_2^h)} \right) / d \left(\frac{\partial f_2}{\partial C_2^h} / \frac{\partial f_2}{\partial (1 - e_2^h)} \right) > 0,$$

where ϕ_2^h represents the (Allen) substitution elasticity between C_2^h and $1 - e_2^h$ in felicity $f_2(C_2^h, 1 - e_2^h)$. The first two terms at the right-hand side of (2.18) stands for the income effects on labor supply, whereas the last term represents substitution effects. In particular, a lower price of second-period leisure (i.e. $\frac{w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2}{w_2(1 - t) - b_2(1 - \alpha_2)} > 0$) induces both intratemporal substitution (away from C_2^h towards $(1 - e_2^h)$) and intertemporal substitution (away from C_1^h towards C_2^h and $(1 - e_2^h)$). Both effects reduce labor supply in the second period. A higher tax rate t thus exerts offsetting income and substitution effects on labor supply. A reduction in the effective early retirement benefit $b_2(1 - \alpha_2)$, in contrast, boosts second-period labor supply through both the income and the substitution effect.

At the start of period 2, the medium-income earner maximizes $f_2(C_2^m, 1 - e_2^m)$ with respect to C_2^m and e_2^m , subject to (2.12). As demonstrated in the appendix, this yields the following second-period labor-supply function

$$\begin{aligned}
\frac{de_2^m}{1-e_2^m} &= (1/Y^m) [w_2 e_2^m dt + (1+r) b_1 d\alpha_1 + (1+r) \alpha_1 db_1] \\
&+ (1/Y^m) [b_2 (1-e_2^m) d\alpha_2 - (1-\alpha_2) (1-e_2^m) db_2 - (1-\alpha_y) dy_2 + y_2 d\alpha_y] \\
&- (1-\gamma^m) \phi_2^m \left[\frac{w_2 dt + (1-\alpha_2) db_2 - b_2 d\alpha_2}{w_2 (1-t) - b_2 (1-\alpha_2)} \right], \tag{2.19}
\end{aligned}$$

$$\begin{aligned}
Y^m &\equiv w_2 (1-t) + (1-\alpha_y) y_2 - \alpha_1 b_1 (1+r), \\
0 < \gamma^m &\equiv \frac{(1-e_2^m) [w_2 (1-t) - b_2 (1-\alpha_2)]}{C_2^m + (1-e_2^m) [w_2 (1-t) - b_2 (1-\alpha_2)]} < 1,
\end{aligned}$$

where ϕ_2^m is the (Allen) substitution elasticity between C_2^m and $1-e_2^m$ in felicity $f_2(C_2^m, 1-e_2^m)$. The first two terms at the right-hand side of (2.19) represent income effects. To illustrate, an increase in α_1 reduces the net balance on the medium-earner's individual account and the associated negative income effect induces the medium-income earner to retire later. The third term on the right-hand side of (2.19) captures intratemporal substitution. As in the case of a high-income earner, a higher tax rate exerts offsetting substitution and income effects. A higher α_2 and a lower b_2 , in contrast, unambiguously boost labor supply through both income and substitution effects.

In period 2, a previously unemployed worker who has lost human capital maximizes $f_2(C_2^l, 1-e_2^l)$ subject to (2.13), resulting in the following labor-supply function (see the appendix for a derivation)

$$\frac{de_2^l}{1-e_2^l} = \frac{\theta w_2 e_2^l (dt + ds) - (1-e_2^l) db_2 - dy_2}{\theta w (1-t-s) + y_2} - (1-\gamma^l) \phi_2^l \left(\frac{dt + ds}{1-t-s} \right), \tag{2.20}$$

$$0 < \gamma^l \equiv \frac{[\theta w (1-t-s) - b_2] (1-e_2^l)}{C_2^l + [\theta w (1-t-s) - b_2] (1-e_2^l)} < 1,$$

where ϕ_2^l represents the (Allen) substitution elasticity between C_2^l and $1-e_2^l$ in felicity $f_2(C_2^l, 1-e_2^l)$.¹⁹ The two terms on the right-hand side of (2.20) represent, respectively, income effects and intratemporal substitution effects.

The optimal labor supplies e_2^m and e_2^l determine ex-ante expected utility V^u attainable by an unemployed worker (2.3), and the solution to the high-income worker's problem yields his maximum attainable lifetime utility V^h (see (2.2)). Having determined V^h and

¹⁹If felicity $f_2(\cdot, \cdot)$ is of the constant-elasticity-of-substitution (CES) shape, we have $\phi_2^h = \phi_2^m = \phi_2^l$.

V^u , we find optimal search effort by maximizing expected lifetime utility U^e (see (2.1)) with respect to a . The first-order condition implies

$$F'(a) = V^h - V^u. \quad (2.21)$$

Workers thus search up to the point where the expected marginal gain $V^h - V^u$ in lifetime utility from an additional unit of search effort exactly offsets the marginal disutility from search $F'(a)$. With $F''(a) > 0$, search effort (and hence the first-period employment rate) rises with the expected utility differential between fully employed and unemployed workers.

2.7. Consumer welfare

To analyze the welfare effects of individual accounts, we employ the agents' indirect 'private' utility functions. The high-income earner's indirect utility function has the properties²⁰

$$V^h = V^h \left(\bar{t}, b_2^+, \bar{\alpha}_2, y_1^+, y_2^+, \bar{\alpha}_y \right). \quad (2.22)$$

The appendix provides expressions for the partial derivatives. The signs of the income effects correspond to the income effects in the first term at the right-hand side of (2.18).

The *ex post* indirect lifetime utility function of a medium-income earner takes the form²¹

$$V^{um} = (f_1(b_1, 1))^{1-\frac{1}{\sigma}} + \beta V^m \left(\bar{\alpha}_1, \bar{b}_1, \bar{t}, \bar{\alpha}_2, b_2^+, y_2^+, \bar{\alpha}_y \right), \quad (2.23)$$

while the *ex post* indirect lifetime utility function of a low-income earner amounts to

$$V^{ul} = (f_1(b_1, 1))^{1-\frac{1}{\sigma}} + \beta V^l \left(\bar{t}, \bar{s}, b_2^+, y_2^+ \right). \quad (2.24)$$

Again, the partial derivatives of these indirect second-period utility functions V^l and V^m may be found in the appendix, while their signs correspond to the income effects in (2.19) and (2.20). The different arguments in the various indirect utility functions are explained by differences in the budget constraints of the three groups arising from varying labor market experiences.

²⁰These partial derivatives assume $\alpha_2, \alpha_y < 1$ and $b_2, y_2 > 0$.

²¹These partial derivatives assume $\alpha_1, b_1 > 0$, $\alpha_2, \alpha_y < 1$ and $b_2, y_2 > 0$.

3. Pareto-improving social insurance reform through individual accounts

This section starts out by demonstrating the potential for a Pareto improvement through the introduction of *retirement accounts* and *early retirement accounts*, relying on the more efficient *lifetime income insurance* offered by such accounts. It then illustrates the Pareto improvement made possible by the *lifetime income insurance* and *liquidity insurance* provided by *unemployment accounts*.

We employ a procedure inspired by Kaplow (1996). Under this procedure, we design a policy reform that keeps the (private) utility of all agents constant. We then analyze whether the reform raises (the present value of) net government revenue. If it does, the government can make everybody better off by spending the additional revenues on public goods, which (given the additive separability of utility functions) do not affect private sector behavior.

3.1. Retirement accounts

The economy starts out with a fiscal system without individual accounts, so that $s = \alpha_1 = \alpha_2 = \alpha_y = 0$ in the initial equilibrium. Suppose now that policy makers want to rely more on private saving for retirement. They might then decide to cut back on y_2 and at the same time reduce taxes on labor income to induce more life-cycle saving. However, such a reform is unlikely to be Pareto improving since it implies redistribution away from low-income households. Indeed, if the initial linear tax system is efficient, such a reform could not make everybody better off.

Whereas funding thus cannot accomplish a Pareto-improving reform, the introduction of individual accounts will. In particular, we will show that, starting from any initial equilibrium – including one in which the initial linear tax system is efficient – the government can obtain a Pareto improvement by introducing retirement accounts. We also analyze the features of the initial equilibrium, including behavioral parameters, that affect the magnitude of the marginal welfare gains from employing this new instrument.

Consider a fiscal reform involving a cut in the tax rate t along with changes in the instruments y_1 , s and α_y that are calibrated so as to keep the utilities of all agents

constant (the subscripts indicate partial derivatives of the indirect utility functions):

$$dV^l = V_t^l \cdot dt + V_s^l \cdot ds = 0,$$

$$dV^m = V_t^m \cdot dt + V_{\alpha_y}^m \cdot d\alpha_y = 0,$$

$$dV^h = V_t^h \cdot dt + V_{\alpha_y}^h \cdot d\alpha_y + V_{y_1}^h \cdot dy_1 = 0.$$

The partial derivatives of the indirect utility functions (provided in the appendix) imply that

$$ds = -dt, \tag{3.1}$$

$$d\alpha_y = -\frac{w_2 e_2^m}{y_2} dt, \tag{3.2}$$

$$dy_1 = \left[w_1 e_1^h + \frac{w_2}{1+r} (e_2^h - e_2^m) \right] dt. \tag{3.3}$$

Since $dt < 0$, we have $ds > 0$ and $d\alpha_y > 0$. Hence, starting from the initial equilibrium where $\alpha_y = s = 0$, the reform does indeed involve the introduction of retirement accounts.²²

Consider now the effects of the fiscal reform on the present value of net government revenue (2.14). Since the reform keeps private expected utilities of both high-income, employed workers and unemployed workers constant ($dC_1^u = db_1 = dV^l = dV^m = 0$ so that $dV^u = 0$ from (2.3)), the first-order condition for search (2.21) implies that the reform affects neither search nor the unemployment rate. The impact on the public budget therefore depends solely on the effects on the generational accounts of the three groups of workers. Using (3.1), (3.2), and (3.3) and recalling that $\alpha_2 = 0$ initially and that first-period labor supply is fixed at $e_1^h = 1$, we arrive at the following effect of the policy reform on the generational account of a high-income earner

$$dg^h = \left(\frac{tw_2 + b_2}{1+r} \right) \left(\frac{\partial e_2^h}{\partial t} \right)_c dt = \left(\frac{tw_2 + b_2}{1+r} \right) \left(\frac{w_2(1 - e_2^h)[\delta\sigma\gamma_2^h + (1 - \gamma_2^h)\phi_2^h]}{[w_2(1 - t) - b_2(1 - \alpha_2)]} \right) (-dt), \tag{3.4}$$

²²Given (3.1) and (3.2), the reader may verify that a low-income earner will end up with a negative IA balance if $\theta < \left(\frac{e_2^m}{e_2^l} \right) \left(\frac{1 - e_2^l}{1 - e_2^m} \right)$. A sufficient (but not necessary) condition for this, and hence for our assumption $A^l < 0$ to be valid, is that the uncompensated wage elasticity of labor supply is non-negative, since we then have $\left(\frac{e_2^m}{e_2^l} \right) \left(\frac{1 - e_2^l}{1 - e_2^m} \right) \geq 1$.

where the second equality follows from (2.18). $\left(\frac{\partial e_2^i}{\partial t}\right)_c$ $i = m, h$ denotes the *compensated* effects of the tax rate on second-period labor supply of household i . The compensated effects are relevant because the reform keeps the worker's lifetime utility constant. Since a rise in the tax rate t exerts a negative substitution effect on second-period labor supply, we have $\left(\frac{\partial e_2^h}{\partial t}\right)_c < 0$. The positive impact on net public revenue from the high-income earner measures the gain in welfare as a result of less distorted labor-supply behavior of the high-income earner. In particular, the non-distortionary social security contribution s replaces part of the distortionary tax rate t . Hence, the high-income household can raise his individual account balance by retiring later. The welfare gains depend on both the initial distortion of second-period labor supply, $tw_2 + b_2$, and the sensitivity of labor supply with respect to the marginal reward to labor.²³ The second factor depends on both intertemporal and intratemporal substitution (i.e. the elasticities σ and ϕ_2^h respectively). Intertemporal substitution becomes more important if leisure accounts for a large share in second-period felicity (so that $\gamma_2^h = \frac{(1-e_2^h)[w_2(1-t)-b_2(1-\alpha_2)]}{C_2^h+(1-e_2^h)[w_2(1-t)-b_2(1-\alpha_2)]}$ is large and e_2^h is only small). Intuitively, more expensive second-period leisure induces high-income earners to move aggregate consumption out of the second period into the first. With higher first-period consumption, saving declines in the anticipation of higher work effort in the second period. Interestingly enough, by delaying retirement, funding through individual accounts depresses national saving.²⁴

In a similar way, we find for the impact on the generational account of the medium-income earners (the second equality is found from (2.19))

$$dg^m = \left(\frac{tw_2 + b_2}{1+r}\right) \left(\frac{de^m}{dt}\right)_c dt = \left(\frac{tw_2 + b_2}{1+r}\right) \left(\frac{(1-\gamma^m)\phi_2^m w_2(1-e_2^m)}{w_2(1-t) - b_2(1-\alpha_2)}\right) (-dt) > 0. \quad (3.5)$$

This generational-account impact is quite similar to the corresponding impact for the high-income earner. Also the medium-income earner faces more incentives to delay his

²³Welfare gains are zero if an infinite degree of risk aversion implies $\sigma = 0$ and felicity $f_2(\cdot, \cdot)$ does not allow for substitution between consumption and leisure.

²⁴If labor supply is elastic also in the first period, this intertemporal substitution effect does not necessarily operate in the same direction. In particular, if first-period labor supply is more elastic than second-period labor supply, saving may actually rise as the rewards of higher first-period work effort are in part saved for additional second-period consumption. For a formal analysis of the case with endogenous first-period labor supply, see the appendix.

retirement because such a delay increases the balance in his RA.²⁵ The additional labor supply expands the tax base and thereby benefits the generational account. The main difference with the high-income earner is that intertemporal substitution effects on labor supply no longer apply; only intratemporal substitution away from second-period leisure to second-period consumption is relevant. The reason is that the medium-income household faces liquidity constraints and thus does not adjust its saving behavior.²⁶

For the low-income earners the reform affects neither incentives nor net incomes, so that g^l is unaffected. We therefore conclude that retirement accounts can be introduced so that private utility of all agents remains constant, while at the same time improving the public budget through an increase in the (present value of the) net tax payments of medium-income and high-income earners. The additional public resources enable the government to raise everybody's utility by offering more public goods. These additional public resources are especially substantial if second-period labor supply is taxed heavily at the margin (i.e. t and b_2 are large), the intratemporal substitution elasticities between second-period consumption and second-period leisure are substantial, and the intertemporal substitution elasticity σ is large. Intuitively, improved labor-supply incentives are especially important in the presence of substantial initial distortions and elastic behavior.

²⁵One can check from (2.11) that, starting from a situation without retirement accounts, the introduction of a marginal retirement account causes the medium-income household to accumulate exactly zero funds in its account if the household keeps second-period labor supply constant. By reducing second-period labor supply, the household could run a deficit in its account and draw on the government bail out. The household does not find this optimal, however, because doing so would reduce utility compared to the initial equilibrium. Indeed, at a negative account balance, households find it optimal not to change their behavior compared to the situation without retirement accounts (since nothing changes compared to the initial situation, see also the behavior of the low-income household below). By raising labor supply (compared to the initial equilibrium) in response to the improved incentives (at a non-negative account balance), the medium-income household accumulates a positive balance and enjoys a second-order gain in utility. The household finds it optimal to raise labor supply because in this way it can raise net retirement benefits. Intuitively, the individual account system implies a kink in the budget constraint at the initial equilibrium, with lower marginal tax rates for higher labor supplies. This induces households to move away from the kink by raising labor supply.

²⁶Without liquidity constraints (and with exogenous first-period labor supply), the reform would unambiguously depress saving because medium-run households would increase first-period consumption in anticipation of higher second-period labor income.

The efficiency gain associated with the Pareto improvement arises because the RAs establish an efficiency-enhancing actuarial link between contributions and resources in retirement for high-income and middle-income workers whose social benefits are currently paid by distortionary taxes. Improved incentives are obtained without cutting benefits paid to low-income workers. The reason is that the lifetime income guarantee protects these latter households, who continue to receive the same positive net transfers from those with higher lifetime incomes. The RA system effectively enables the government to implement a selective benefit cut for high-income and middle-income groups without having to cut net benefits at the bottom of the (lifetime) income ladder. In this way, the RAs improve the equity-efficiency trade-off.

3.2. Early retirement accounts

With the introduction of early retirement accounts, we use the same instruments as with the introduction of retirement accounts except that we employ α_2 rather than α_y to keep utilities of the medium-income and high-income households constant. Hence, (3.1) continues to hold, but (3.2) and (3.3) are replaced by, respectively,

$$d\alpha_2 = -\frac{w_2 e_2^m}{b_2(1 - e_2^m)} dt, \quad (3.6)$$

and

$$dy_1 = \left[w_1 e_1^h + \frac{w_2}{1+r} \left(\frac{e_2^h - e_2^m}{1 - e_2^m} \right) \right] dt.$$

The impacts on the generational accounts are given by the following expressions, where the second equalities are derived from (2.18) and (2.19), respectively:

$$\begin{aligned} dg^h &= \left(\frac{tw + b_2}{1+r} \right) \left[\left(\frac{\partial e_2^h}{\partial t} \right)_c - \left(\frac{we_2^m}{b_2(1 - e_2^m)} \right) \left(\frac{\partial e_2^h}{\partial \alpha_2} \right)_c \right] dt \\ &= \left(\frac{tw_2 + b_2}{1+r} \right) \left(\frac{w_2[\delta\sigma\gamma_2^h + (1 - \gamma_2^h)\phi_2^h]}{[w_2(1 - t) - b_2(1 - \alpha_2)]} \right) (-dt) > 0, \\ dg^m &= \left(\frac{tw + b_2}{1+r} \right) \left[\left(\frac{\partial e_2^m}{\partial t} \right)_c - \left(\frac{we_2^m}{b_2(1 - e_2^m)} \right) \left(\frac{\partial e_2^m}{\partial \alpha_2} \right)_c \right] dt \\ &= \left(\frac{tw_2 + b_2}{1+r} \right) \left(\frac{(1 - \gamma^m)\phi_2^m w_2}{[w_2(1 - t) - b_2(1 - \alpha_2)]} \right) (-dt) > 0, \end{aligned}$$

and

$$dg^l = 0,$$

where $\left(\frac{\partial e^i}{\partial \alpha_2}\right)_c$ represents the compensated effect of α_2 on second-period labor supply of household $i = m, h$. Compared to RAs, ERAs exert an additional positive welfare effect since they in effect reduce the effective rate of early retirement benefit $(1 - \alpha_2)b_2$ for medium-income and high-income households. Whereas a cut in the effective old-age retirement benefit $(1 - \alpha_y)y_2$ generates only income effects because households cannot affect their use of this benefit, the cut in the effective *early* retirement benefit exerts a positive substitution effect on second-period labor supplies of the medium-income and high-income households (i.e. $\left(\frac{\partial e^i}{\partial \alpha_2}\right)_c > 0, i = 1, 2$). Indeed, households can determine their own use of the early retirement benefit by selecting their date of retirement. Since medium-income and high-income households now pay part of their own early retirement benefit, they face an incentive to limit the use of this benefit by retiring later. The net contributions of the medium-income and high-income earners to the public budget thus increase through two channels: first, the lower marginal tax rate t expanding second-period labor supply (the so-called *tax base* effect) and, second, the lower effective rate of early retirement benefit $(1 - \alpha_2)b_2$ reducing the attractiveness of early retirement (the so-called *moral hazard* effect). Through these two channels, ERAs produce a double dividend for the government in the form of both higher labor income tax revenue and lower expenditure on early retirement benefits.

After the reform, low-income earners continue to receive their early retirement benefits from the government and thus do not face a direct link between their social security contribution s and retirement incomes. Hence, just as a RA, an ERA impacts neither incentives nor net incomes of these households, so that g^l is unaffected.

The Pareto improvement associated with the introduction of ERAs reflects more efficient lifetime income insurance.²⁷ This more efficient insurance produces not only a positive tax base effect on account of less distortionary finance (i.e. through the cut in the marginal tax rate t), but also less expenditure on insurance benefits as moral hazard

²⁷Just as with retirement benefits, cutting early retirement benefits b_2 and tax rates t at the same time is unlikely to produce a Pareto improvement (as it induces redistribution away from low-income households).

is reduced; agents face fewer incentives to draw on the social insurance benefits as they in fact finance part of these benefits themselves.

3.3. Unemployment accounts

Unemployment accounts provide both lifetime income insurance and liquidity insurance. We deal with both these aspects of unemployment accounts in turn.

3.3.1. Lifetime income insurance

The case for unemployment accounts as an efficient instrument for lifetime income insurance can be made by using the same instruments as with the introduction of retirement accounts, except that we employ α_1 rather than α_y to keep utilities constant. This implies that (3.2) and (3.3) are replaced by

$$d\alpha_1 = -\frac{w_2 e_2^m}{(1+r)b_1} dt,$$

$$dy_1 = \left[w_1 e_1^h + \frac{w_2 e_2^h}{1+r} \right] dt,$$

while (3.1) continues to hold. This experiment amounts to a cut in the effective unemployment benefit for medium-income households $(1 - \alpha_1)b_1$ and a cut in the first-period work benefit of the high-income household y_1 .

The effects on the generational accounts of the high-income and medium-income households are given by, respectively, (3.4) and (3.5), while the low-income households do not alter their behavior so that $dg^l = 0$. Medium-income households postpone their retirement because by working longer their social security contributions pay not only for the unemployment benefits they enjoyed during the first period of their life but also for higher incomes during retirement. Also high-income households expand their second-period labor supply, as part of the levies they pay on that labor income now result in higher retirement incomes for themselves.

3.3.2. Liquidity insurance

Unemployment benefits offer liquidity insurance by alleviating the liquidity constraints facing unemployed agents. We show that UA offer this liquidity insurance more efficiently than regular unemployment benefits do. In particular, we first look at the case in which

higher unemployment benefits offering liquidity insurance are tax financed. This reform is designed so that ex-post welfare of the low-income household remains constant²⁸

$$dV^{ul} = V_t^{ul} \cdot dt + V_{b_1}^{ul} \cdot db_1 = 0.$$

Using the definition of ex-post utility (2.24) and the partial derivatives of the indirect utility functions (see the appendix), this implies that (using $\alpha_1 = \alpha_2 = \alpha_y = s = 0$)

$$dt = \frac{\lambda^u}{\beta \lambda^l \theta w_2 e_2^l} db_1, \quad (3.7)$$

where λ^u stands for marginal utility of consumption for an unemployed agent in period 1, and λ^l denotes marginal utility of consumption for a low-income worker in period 2.

The impact on the ex-post welfare of the medium-income household amounts to (using the definition of ex-post utility (2.23), the partial derivatives of the indirect utility functions from the appendix, and (3.7))

$$dV^{um} = V_t^{um} \cdot dt + V_{b_1}^{um} \cdot db_1 = \left(1 - \frac{\lambda^m e_2^m}{\lambda^l \theta e_2^l}\right) \lambda^u db_1, \quad (3.8)$$

where λ^m denotes marginal utility of consumption for a medium-income worker in period 2.

The welfare of the high-income earner is kept constant by increasing y_1 :

$$dV^h = V_t^h \cdot dt + V_{y_1}^h \cdot dy_1 = 0.$$

These policy changes generate the following impact on the generational account of the low-income worker (with $s = 0$) :

$$dg^l = -db_1 + \frac{\theta w_2 e_2^l dt}{1+r} + \left(\frac{t\theta w_2 + b_2}{1+r}\right) \left[\left(\frac{\partial e_2^l}{\partial t}\right)\right] dt$$

²⁸The government could alleviate the liquidity constraints completely by raising b_1 and y_1 and simultaneously cutting y_2 . This could be done in such a way as to keep everybody's ex post utility unaffected. We assume, however, that the government wants to fight old-age poverty by keeping y_2 at a minimum level (which could in fact be zero) and cannot commit to reduce old-age pensions below this minimum level. Indeed, this lack of commitment can be one of the rationales behind the government guaranteeing a minimum pension level y_2 , even if agents do not have sufficient funds in their individual accounts to finance an annuity paying out this minimum pension. The higher is the minimum pension level y_2 , the more severe liquidity constraints are likely to be. The reason is that agents may then want to bring forward their higher second-period incomes to the first period. This holds true especially for medium-income households (as opposed to low-income households) who earn relatively high wages in the second period.

$$\begin{aligned}
&= \left[\frac{\lambda^u}{\beta(1+r)\lambda^l} - 1 \right] db_1 + \left(\frac{t\theta w_2 + b_2}{1+r} \right) \left(\frac{(1-e_2^l)\lambda^u db_1}{\beta\lambda^l[\theta w_2(1-t) + y_2]} \right) \\
&\quad - \left(\frac{t\theta w_2 + b_2}{1+r} \right) \left(\frac{(1-\gamma^l)\phi_2^l(1-e_2^l)}{1-t} \right) dt, \tag{3.9}
\end{aligned}$$

where dt is given by (3.7) and we have used (2.20). The first term at the far right-hand side of (3.9) represents the welfare gain from additional liquidity insurance. In particular, in a perfect capital market, an unemployed worker would borrow until the marginal utility gain of higher current consumption (λ^u) would equal the marginal utility loss from lower future consumption, $\beta(1+r)\lambda^l$. However, in the present setting with credit constraints we have $\lambda^u > \beta(1+r)\lambda^l$, so that the first term at the far right-hand side of (3.9) is positive. This reflects the pure *liquidity insurance effect* of tax-financed unemployment benefits. Since the reform shifts disposable income from a period with lower towards a period with higher marginal utility of consumption, it enables the government to extract more net revenue from unemployed workers over their lifetimes without harming their welfare.

The second term at the far right-hand side of (3.9) shows that the low-earner's generational account improves also due to a negative income effect in the second period. By shifting resources from the second to the first period of life, agents enter the second period with fewer resources. This stimulates agents to retire later. Indeed, the implicit loan the government provides to liquidity-constrained households in effect allows households to dissave more. These dissavings boost labor supply at the end of the working life.

In contrast to the other terms at the second terms at the right-hand side of (3.9), the final term is negative. It captures the negative substitution effect on labor supply that is associated with a higher tax rate t financing the unemployment benefits.

The generational account of the medium-income earner is affected in the following way (with $\alpha_1 = \alpha_2 = \alpha_y = 0$ and using (2.19)):

$$\begin{aligned}
dg^m &= \frac{e_2^m}{\lambda^l \theta e_2^l} \left[\frac{\lambda^u}{\beta(1+r)} - \lambda^m \right] db_1 + \left(\frac{tw_2 + b_2}{1+r} \right) \left(\frac{(1-e_2^m)w_2 e_2^m}{[w_2(1-t) + y_2]} \right) \left(\frac{\lambda^u db_1}{\beta\lambda^l \theta w_2 e_2^l} \right) \\
&\quad - \left(\frac{tw_2 + b_2}{1+r} \right) \left(\frac{(1-\gamma^m)\phi_2^m w_2 (1-e_2^m)}{[w_2(1-t) - b_2(1-\alpha_2)]} \right) dt - V^{um}/\lambda^u \tag{3.10}
\end{aligned}$$

The first two terms at the right-hand side of (3.10) are positive and correspond to the liquidity insurance effect (capital-market imperfections imply $\lambda^u > \beta(1+r)\lambda^m$) and

the second-period income effect in second-period labor supply, respectively. The negative substitution effect on second-period labor supply associated with distortionary tax finance is represented by the third term at the right-hand side of (3.10).

The policy changes impact the generational account for the high-income earners only through changes in the marginal tax rate t , and the overall impact on this generational account is given by expression (3.4) with dt given by (3.7). The compensated increase in the labor tax rate t thus reduces a high-income worker's net contribution to the public budget by motivating him to retire earlier.

We compare these effects on the generational accounts of tax-financed unemployment benefits with the corresponding impacts if the government provides liquidity insurance through an UA. In that case, ex post utility of the low-income earner is maintained by setting (using the derivatives of the indirect utility functions in the appendix)

$$ds = \frac{V_{b_1}^{ul}}{V_s^{ul}} db_1 = \frac{\lambda^u}{\beta \lambda^l \theta w_2 e_2^l} db_1.$$

This generates exactly the same impact on the low-earner's generational account as in (3.9). The positive first term versus the negative third term implies a trade-off between providing liquidity insurance and containing labor-supply distortions. Raising unemployment benefits through UAs does distort labor supply of the low-income households because the additional social security contributions are not returned to these agents in terms of higher retirement benefits. Hence, the financing of the additional unemployment benefits remains distortionary, even if provided through UA. Relieving capital-market distortions thus does not come free.

With the UA financing the additional unemployment benefits, the parameter α_1 is changed so that ex-post utility of the medium-income household is affected in exactly the same way as with a rise in regular tax-financed unemployment benefits²⁹

$$dV^{um} = V_{\alpha_1}^{um} \cdot d\alpha_1 + V_{b_1}^{um} \cdot db_1 = \left(1 - \frac{\lambda^m e_2^m}{\lambda^l \theta e_2^l}\right) \lambda^u db_1,$$

²⁹One can verify that the medium-income household accumulates exactly zero funds in its account. The household can slightly improve its welfare by reducing its labor supply so that the household is bailed out by the government. To prevent this from happening, the government has to marginally reduce the asset level below which the government tops up the individual accounts of agents at retirement. This reduces the scope for bailing out the low-income household. When UA are introduced, however, this effect is only second order.

which implies

$$d\alpha_1 = \frac{\lambda^u e_2^m}{\lambda^l \theta e_2^l} db_1 > 0.$$

The impact on the medium earner's generational account is the same as (3.10), except that the negative third term (with dt) drops out. The financing of unemployment benefits is no longer distortionary, as the social security contributions paid by the medium-income earner raise individual retirement incomes.

Since the employed policy instruments b_1 , α_1 , and s do not affect the welfare of the high-income earner, the government does not need to change any other policy instruments. Also, since the high earner is thus not affected at all, his generational account is not changed. Hence, just as the generational account of the medium-income earner, the generational account of the high-income earner improves compared to the case in which additional unemployment benefits are tax financed. Additional unemployment benefits provided through UA therefore alleviate capital-market distortions at lower efficiency costs in terms of harmed labor-market incentives than regular tax-financed unemployment benefits do.³⁰

Raising unemployment benefits paid out of UA thus increases the scope for intertemporal reallocation of disposable income towards the beginning of the working life. Essentially, unemployment accounts allow unemployed, medium-income workers to borrow against their own future labor income; part of the unemployment benefits collected in the first period is debited to the worker's individual account, thus reducing his consumption possibilities in the second period. In this way, UAs enable the government to increase an unemployed, medium-income worker's consumption possibilities during the period in which marginal utility of consumption is the highest, without increasing the present value of tax-financed unemployment benefits (discounted at the government's borrowing rate of interest) and without undermining incentives for these agents.

³⁰To establish this, one should note that both ways of raising unemployment benefits change V^l , V^m , V^h , and V^u in the same way and thus alter search effort a in the same fashion. Hence, as unemployment changes, both reforms modifies the composition of taxpayers in the same way. Through this channel, they thus generate the same impact on the government budget (2.14).

3.4. Correlation between adverse shocks

The parameter p , indicating the proportion of unemployed workers who lose human capital as a result of joblessness, measures the correlation between adverse shocks. The stronger this correlation, the smaller are the aggregate efficiency gains from the compulsory savings accounts. This is because a larger value of p implies that more households are unable to accumulate a positive balance on these accounts. The government thus needs to bail out more agents, who thus do not face improved labor-supply incentives and therefore do not enhance their generational accounts. Indeed, compulsory savings accounts providing more efficient lifetime income insurance benefit the generational accounts of high-income and medium-income earners only. Similarly, as far as liquidity insurance is concerned, compared to tax-financed benefits, funded unemployment benefits produce a stronger impact on the generational accounts of only these latter households. Hence, the larger the proportion of low-income households (which is determined by p and a), the less substantial the efficiency gains are.

Loosely speaking, higher values for p and lower values for a imply a smaller middle class and thus – in the context of a European welfare state – a smaller number of taxpayers whose taxes can serve to finance part of their own benefits. Within a more polarized society, the fiscal system to a larger degree redistributes resources from high lifetime-income earners to low lifetime-income earners, rather than reallocating resources over the life cycle of the same individuals. Indeed, more correlation between adverse shocks (a higher value of p) reduces the scope for improving incentives through self insurance in the basis of individual savings.

Another related factor in determining the scope for self insurance is the relative length of the two periods. A longer period of unemployment raises the likelihood that unemployment produces a scarring effect on human capital (thus raising p). In addition, it increases the number of individuals that need be bailed out through the public pension guarantee; the longer the period of unemployment, the larger the fraction of individuals who have not accumulated enough funds in their accounts at their statutory retirement age to be able to finance their own public pension. Moreover, with longer unemployment spells, liquidity constraints are also likely to become less serious as unemployed young individuals scale down their consumption in anticipation of low lifetime incomes. Hence,

long unemployment durations in slow-moving labor markets make individual accounts less attractive as an instrument to provide lifetime income insurance and liquidity insurance. Indeed, long unemployment spells in effect imply that adverse shocks are strongly correlated over time so that self insurance is less efficient.

To protect agents hit by correlated shocks against poverty, while at the same time enhancing their labor-supply incentives, the government must rely on other instruments than self insurance.³¹ In particular, the government may collect additional information by closely monitoring job search and imposing penalties on less active search. In this connection, workfare may play a useful role because the mere threat of being put on workfare is likely to boost job search.

4. Conclusions

This paper explored whether financing part of social insurance through mandatory contributions to individual savings accounts can produce a Pareto-improving welfare gain. Through these accounts, the middle class would engage in self insurance by saving for their own social insurance benefits. The system would continue to provide lifetime income insurance by offering a public pension guarantee to low-income workers with a deficit on their savings accounts at the time of statutory retirement. Moreover, the account system would allow unemployed workers to borrow against their future labor income by drawing unemployment benefits from their accounts, thereby alleviating credit constraints.

To investigate the incentive and welfare effects of such a fiscal reform, we set up a two-period model in which agents face an endogenous risk of involuntary unemployment and an exogenous risk (conditional on being unemployed) of losing future human capital as a result of the scarring effect of unemployment. The model includes endogenous job search, retirement decisions and life cycle saving by fully employed workers as well as credit constraints for unemployed workers. Within this framework, we demonstrate that the introduction of compulsory savings accounts would produce efficiency gains in credit markets (in the case of unemployment accounts) and labor markets. Paradoxically, these

³¹The model assumes that labor is homogeneous. If low-skilled labor is complementary to high-skilled labor in production, the improved labor supply incentives of high-skilled workers may raise the low-skilled wage. Through these indirect general equilibrium effects, also low-skilled labor supply may be boosted.

gains produced by individual savings accounts would not only postpone retirement but also raise first-period consumption, thereby depressing national saving. The efficiency gains associated with lower saving and higher labor supply allow the government to generate an ex-post Pareto improvement so that all agents benefit. These gains arise because the savings accounts establish an efficiency-enhancing actuarial link between taxes and benefits for high-income and middle-income workers – who currently pay distortionary taxes partly to finance distortionary social benefits to themselves – without reducing net transfers paid to the low-income workers who remain protected by the lifetime income guarantee. The savings accounts thus effectively enable the government to implement a selective benefit cut for high-income and middle-income groups without having to reduce benefits at the bottom of the income ladder. Savings accounts enrich the fiscal armory of the government by adding a non-linear element to an otherwise linear fiscal system. In this way, compulsory savings accounts improve the equity-efficiency trade-off, enabling the government to engineer a Pareto welfare improvement, even if it has optimized the pre-existing tax-transfer system.

The present paper is only a first step towards a full analysis of the economic costs and benefits of basing transfers on lifetime incomes. We see a number of related issues for future research.³² First, in future work, we plan to allow for more heterogeneity among agents – for example, by distinguishing a continuum of workers with different skills and by allowing for more periods with involuntary unemployment.

A second issue, closely related to the first, is to allow for non-linear income taxes. Such a rich tax schedule allows us to offset the redistributive effects of the compulsory saving scheme in order to generate a Pareto-improving reform for a continuum of agents.³³ Moreover, we can then investigate what the benefits would be of basing a per-

³²Another political-economy issue is how more individual funding accompanied by targeted redistribution affects the political economy of the welfare state and its associated political risks. In particular, individual funding may produce a stronger ownership of social insurance. This may make it more difficult for the government to change benefit rules, thereby reducing political risks but also reducing the flexibility to adjust the benefit rules in response to unanticipated shocks. Whereas individual funding may increase the political support of the middle class for social insurance, targeted redistribution to the underclass may undermine the support for this redistribution.

³³Kaplow (1996) employs a non-linear income tax to neutralize the income effects of a higher public good supply on a continuum of agents.

son's net fiscal contribution on his individual *lifetime* income, as reflected in the balance on his individual account at the time of retirement, as opposed to his individual *annual* income. In this connection, one may also want to explore the optimal mix between self insurance, compulsory insurance of verifiable events (e.g. disability), and active labor-market policies, including workfare, in enhancing labor-market incentives facing various skill levels.

Finally, whereas the present paper has considered only marginal reforms, which involve only one particular formulation of lifetime income insurance and which start from an initial situation without any compulsory savings accounts, we intend to characterize an optimal savings account system producing efficient lifetime income and liquidity insurance. In this context, we should explore how far the government can go in setting a non-linear tax schedule based on lifetime income and in offering liquidity insurance without violating self-selection constraints that must be respected in order to protect incentives for job search and work.

References

- Belan, P. and P. Pestieau (1999). "Privatizing Social Security: A Critical Assessment". *The Geneva Papers on Risk and Insurance* 24, 114-130.
- Breyer, F. (1989). "On the Intergenerational Pareto Efficiency of Pay-As-You-Go Financed Pension Schemes". *Journal of Institutional and Theoretical Economics* 145, 643-658.
- Brunner, J.K. (1996). "Transition from a Pay-As-You-Go to a Fully Funded System: The Case of Differing Individuals and Intragenerational Fairness". *Journal of Public Economics* 60, 131-146.
- Feldstein, M. and D. Altman (1998). "Unemployment Insurance Savings Accounts". *NBER Working Paper* 6860.
- Fenge, R. (1995). "Pareto-Efficiency of the Pay-As-You-Go Pension System with Intragenerational Fairness". *FinanzArchiv* 52, 357-364.
- Fölster, S. (1994). "Socialförsikring genom Medborgarkonto: Vilka Är Argumenterne?" (Social Insurance through a Citizen Account: What are the Issues?). *Ekonomisk Debatt* 22, 387-97.
- Fölster, S. (1997). "Social Insurance Based on Personal Savings Accounts: A Possible Reform for Overburdened Welfare States?". *European Economy* no. 4, 1997, 81-100.
- Fölster, S., R. Gidehag, M. Orszag, and D. Snower (2002). "Assessing Welfare Accounts". *CEPR Discussion Paper* no. 3479.
- Greenwald, B., and J.E. Stiglitz. (1986). "Externalities in Economies with Imperfect Information and Incomplete Markets," *Quarterly Journal of Economics*, 229-264.
- Heckman, J.J. (1993). "What Has Been Learned About Labor Supply in the Past Twenty Years?" *American Economic Review* 83, pp. 116-121.
- Homburg, S. (1990). "The Efficiency of Unfunded Pension Schemes". *Journal of Institutional and Theoretical Economics* 146, 640-47.
- Kaplow, L. (1996). "The Optimal Supply of Public Goods and the Distortionary Cost of Taxation". *National Tax Journal* 49, 513-533.
- Orszag, J. and D. Snower (1997a). "Expanding the Welfare System: A Proposal for Reform". *European Economy* no. 4, 1997, 101-118.

Orszag, J. and D. Snower (1997b). "From Unemployment Benefits to Unemployment Support Accounts". Mimeo, Birckbeck College, London.

Orszag, J., P. Orszag, D. Snower and J. Stiglitz (1999). "The Impact of Individual Accounts: Piecemeal versus Comprehensive Approaches". Paper presented at the Annual Bank Conference on Development Economics, World Bank, Washington D.C., 1999.

Pauly, M.V. (1974). "Over Insurance and Public Provision of Insurance. The Roles of Moral Hazard and Adverse Selection". *Quarterly Journal of Economics* 88, 44-62.

Sinn, H.-W. (2000). "Why a Funded Pension is Useful and Why it is not Useful". *International Tax and Public Finance* 7, 389-410.

Stiglitz, J. and J. Yun (2002). "Integration of Unemployment Insurance with Retirement Insurance". *NBER Working Paper* no. 9199.

Sørensen, P.B. (2003). "Social Insurance Based on Individual Savings Accounts". Forthcoming as Chapter 10 in S. Cnossen and H.-W. Sinn (eds.), *Public Finances and Public Policy in the New Century*, MIT Press.

Technical Appendix

1. Indirect utility functions

1.1 High-income earners

From (2.2) and (2.8), we construct the Lagrangian for a worker who has obtained a job at the start of period 1:

$$\mathcal{L}^h = (f_1(C_1^h, 1 - e_1^h))^{1-\frac{1}{\sigma}} + \beta (f_2(C_2^h, 1 - e_2^h))^{1-\frac{1}{\sigma}} - \lambda^h \left\{ C_1^h + \frac{C_2^h}{1+r} - w_1 e_1^h (1-t) - y_1 - \frac{[w_2 e_2^h (1-t) + b_2 (1-\alpha_2) (1-e_2^h) + (1-\alpha_y) y_2]}{1+r} \right\}. \quad (4.1)$$

Using the envelope theorem, and noting that the Lagrange multiplier λ^h measures the marginal utility of (exogenous) lifetime income, we find the properties of the high-income earner's indirect utility function by taking the partial derivatives of the Lagrangian (4.1):

$$\begin{aligned} V_t^h &\equiv \frac{\partial V^h}{\partial t} = \frac{\partial \mathcal{L}^h}{\partial t} = -\lambda^h [w_1 e_1^h + \frac{w_2 e_2^h}{1+r}], \\ V_{b_2}^h &\equiv \frac{\partial V^h}{\partial b_2} = \frac{\partial \mathcal{L}^h}{\partial b_2} = \frac{\lambda^h (1-\alpha_2) (1-e_2^h)}{1+r}, \\ V_{\alpha_2}^h &\equiv \frac{\partial V^h}{\partial \alpha_2} = \frac{\partial \mathcal{L}^h}{\partial \alpha_2} = -\frac{\lambda^h b_2 (1-e_2^h)}{1+r}, \\ V_{y_1}^h &\equiv \frac{\partial V^h}{\partial y_1} = \frac{\partial \mathcal{L}^h}{\partial y_1} = \lambda^h, \\ V_{y_2}^h &\equiv \frac{\partial V^h}{\partial y_2} = \frac{\partial \mathcal{L}^h}{\partial y_2} = \frac{\lambda^h (1-\alpha_y)}{1+r}, \\ V_{\alpha_y}^h &\equiv \frac{\partial V^h}{\partial \alpha_y} = \frac{\partial \mathcal{L}^h}{\partial \alpha_y} = -\frac{\lambda^h y_2}{1+r}. \end{aligned}$$

1.2 Medium-income earners

Using (2.4) and (2.12), we can write the Lagrangian for a medium-income worker in period 2 as

$$\mathcal{L}^m = (f_2(C_2^m, 1 - e_2^m))^{1-\frac{1}{\sigma}} - \lambda^m [C_2^m - w_2 e_2^m (1-t) - b_2 (1-\alpha_2) (1-e_2^m) - (1-\alpha_y) y_2 + \alpha_1 b_1 (1+r)]. \quad (4.2)$$

This yields the following derivatives of the medium-income worker's indirect utility function for period 2:

$$\begin{aligned}
V_{\alpha_1}^m &\equiv \frac{\partial V^m}{\partial \alpha_1} = \frac{\partial \mathcal{L}^m}{\partial \alpha_1} = -\lambda^m (1+r) b_1, \\
V_{b_1}^m &\equiv \frac{\partial V^m}{\partial b_1} = \frac{\partial \mathcal{L}^m}{\partial b_1} = -\lambda^m (1+r) \alpha_1, \\
V_t^m &\equiv \frac{\partial V^m}{\partial t} = \frac{\partial \mathcal{L}^m}{\partial t} = -\lambda^m w_2 e_2^m, \\
V_{\alpha_2}^m &\equiv \frac{\partial V^m}{\partial \alpha_2} = \frac{\partial \mathcal{L}^m}{\partial \alpha_2} = -\lambda^m b_2 (1 - e_2^m), \\
V_{b_2}^m &\equiv \frac{\partial V^m}{\partial b_2} = \frac{\partial \mathcal{L}^m}{\partial b_2} = \lambda^m (1 - \alpha_2) (1 - e_2^m), \\
V_{y_2}^m &\equiv \frac{\partial V^m}{\partial y_2} = \frac{\partial \mathcal{L}^m}{\partial y_2} = \lambda^m (1 - \alpha_y), \\
V_{\alpha_y}^m &\equiv \frac{\partial V^m}{\partial \alpha_y} = \frac{\partial \mathcal{L}^m}{\partial \alpha_y} = -\lambda^m y_2.
\end{aligned}$$

1.3 Low-income earners

In a similar way, we construct the period 2 Lagrangian for a low-income worker from (2.4) and (2.13) to obtain

$$\mathcal{L}^l = (f_2(C_2^l, 1 - e_2^l))^{1 - \frac{1}{\sigma}} - \lambda^l [C_2^l - \theta w_2 e_2^l (1 - t - s) - b_2 (1 - e_2^l) - y_2], \quad (4.3)$$

yielding

$$\begin{aligned}
V_t^l &\equiv \frac{\partial V^l}{\partial t} = \frac{\partial \mathcal{L}^l}{\partial t} = -\lambda^l \theta w_2 e_2^l, \\
V_s^l &\equiv \frac{\partial V^l}{\partial s} = \frac{\partial \mathcal{L}^l}{\partial s} = -\lambda^l \theta w_2 e_2^l, \\
V_{b_2}^l &\equiv \frac{\partial V^l}{\partial b_2} = \frac{\partial \mathcal{L}^l}{\partial b_2} = \lambda^l (1 - e_2^l), \\
V_{y_2}^l &\equiv \frac{\partial V^l}{\partial y_2} = \frac{\partial \mathcal{L}^l}{\partial y_2} = \lambda^l.
\end{aligned}$$

2. Labor supply

2.1 High-income earners

From the Lagrangian (4.1), a high-income earner's first-order conditions for optimal labor supply and savings are given by

$$\left(1 - \frac{1}{\sigma}\right) (f_1(C_1^h, 1 - e_1^h))^{-\frac{1}{\sigma}} \frac{\partial f_1(C_1^h, 1 - e_1^h)}{\partial C_1^h} - \lambda^h = 0, \quad (4.4)$$

$$\left(1 - \frac{1}{\sigma}\right) \left(f_1(C_1^h, 1 - e_1^h)\right)^{-\frac{1}{\sigma}} \frac{\partial f_1(C_1^h, 1 - e_1^h)}{\partial(1 - e_1^h)} - \lambda^h w_1(1 - t) = 0, \quad (4.5)$$

$$\beta \left(1 - \frac{1}{\sigma}\right) \left(f_2(C_2^h, 1 - e_2^h)\right)^{-\frac{1}{\sigma}} \frac{\partial f_2(C_2^h, 1 - e_2^h)}{\partial C_2^h} - \frac{\lambda^h}{1 + r} = 0, \quad (4.6)$$

$$\beta \left(1 - \frac{1}{\sigma}\right) \left(f_2(C_2^h, 1 - e_2^h)\right)^{-\frac{1}{\sigma}} \frac{\partial f_2(C_2^h, 1 - e_2^h)}{\partial(1 - e_2^h)} - \left(\frac{\lambda^h}{1 + r}\right) [w_2(1 - t) - b_2(1 - \alpha_2)] = 0, \quad (4.7)$$

$$C_1^h + (1 - e_1^h)w_1(1 - t) + \frac{C_2^h + (1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]}{1 + r} \quad (4.8)$$

$$= y_1 + w_1(1 - t) + \frac{(1 - \alpha_y)y_2 + w_2(1 - t)}{1 + r}. \quad (4.9)$$

Intertemporal behavior

We find intertemporal behavior by multiplying (4.4) and (4.5) by, respectively, C_1^h and $(1 - e_1^h)$ and adding the results to arrive at

$$\left(1 - \frac{1}{\sigma}\right) \left(f_1(C_1^h, 1 - e_1^h)\right)^{-\frac{1}{\sigma}} f_1(C_1^h, 1 - e_1^h) = \lambda^h [C_1^h + (1 - e_1^h)w_1(1 - t)], \quad (4.10)$$

where we have used the homotheticity of $f_1(C_1^h, 1 - e_1^h)$. We can manipulate (4.6) and (4.7) in a similar way to find

$$\left(1 - \frac{1}{\sigma}\right) \left(f_2(C_2^h, 1 - e_2^h)\right)^{-\frac{1}{\sigma}} f_2(C_2^h, 1 - e_2^h) = \lambda^h \{C_2^h + (1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]\}. \quad (4.11)$$

Dividing (4.10) by (4.11), we find

$$\left[\frac{f_1(C_1^h, 1 - e_1^h)}{f_2(C_2^h, 1 - e_2^h)}\right]^{-\frac{1}{\sigma}} = \frac{C_1^h + (1 - e_1^h)w_1(1 - t)}{f_1(C_1^h, 1 - e_1^h)} \frac{f_2(C_2^h, 1 - e_2^h)}{C_2^h + (1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]}.$$

Log-linearizing this expression, we arrive at

$$\tilde{u}_1^h - \tilde{u}_2^h = \sigma \gamma_1^h \tilde{t} - \sigma \gamma_2^h (w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2) / [w_2(1 - t) - b_2(1 - \alpha_2)], \quad (4.12)$$

where $u_i^h \equiv f_i(C_i^h, 1 - e_i^h)$, $\gamma_1^h \equiv \frac{(1 - e_1^h)w_1(1 - t)}{[C_1^h + (1 - e_1^h)w_1(1 - t)]}$, and $\gamma_2^h \equiv \frac{(1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]}{C_2^h + (1 - e_2^h)[w_2(1 - t) - b_2(1 - \alpha_2)]}$.

A tilde stands for a relative change (except for the tax rate, where $\tilde{t} \equiv dt/(1 - t)$).

Log-linearization of the household budget constraint (4.8) yields the overall income effect

$$\delta \tilde{u}_1^h + (1 - \delta) \tilde{u}_2^h = \tilde{z}^h \equiv \psi^h \left(V_t^h dt + V_{b_2}^h db_2 + V_{\alpha_2}^h d\alpha_2 + V_{y_1}^h dy_1 + V_{y_2}^h dy_2 + V_{\alpha_y}^m d\alpha_y \right), \quad (4.13)$$

where $\delta \equiv \frac{C_1^h + (1 - e_1^h)w_1(1-t)}{C_1^h + (1 - e_1^h)w_1(1-t) + \frac{C_2^h + (1 - e_2^h)[w_2(1-t) - b_2(1 - \alpha_2)]}{1+r}}$ and $\psi^h \equiv \frac{1}{\lambda^h \left(y_1 + w_1(1-t) + \frac{(1 - \alpha_y)y_2 + w_2(1-t)}{1+r} \right)}$.

Solving (4.12) and (4.13) for \tilde{u}_1 and \tilde{u}_2 , we find

$$\tilde{u}_1^h = \tilde{z}^h + (1 - \delta) \left\{ \sigma \gamma_1^h \tilde{t} - \sigma \gamma_2^h (w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2) / [w_2(1 - t) - b_2(1 - \alpha_2)] \right\}, \quad (4.14)$$

$$\tilde{u}_2^h = \tilde{z}^h - \delta \left\{ \sigma \gamma_1^h \tilde{t} - \sigma \gamma_2^h (w_2^h dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2) / [w_2(1 - t) - b_2(1 - \alpha_2)] \right\}. \quad (4.15)$$

Intratemporal behavior

Intratemporal behavior in period 2 is found by dividing (4.6) by (4.7) and log-linearizing the result to arrive at

$$\tilde{C}_2^h - \tilde{v}_2^h = -\phi_2^h [w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2] / [w_2(1 - t) - b_2(1 - \alpha_2)], \quad (4.16)$$

where $v_i^j \equiv 1 - e_i^j$; $j = h, m, l$; $i = 1, 2$. ϕ_i^h denotes the substitution elasticity C_i^h and v_i^h ($i = 1, 2$) in felicity $f_i(\cdot, \cdot)$. By using (4.4) and (4.5), we find in a similar way

$$\tilde{C}_1^h - \tilde{v}_1^h = -\phi_1^h \tilde{t}, \quad (4.17)$$

Log-linearization of $u_i^h \equiv f_i(C_i^h, 1 - e_i^h)$ yields

$$(1 - \gamma_i^h) \tilde{C}_i^h + \gamma_i^h \tilde{v}_i^h = \tilde{u}_i^h, \quad (4.18)$$

where we have used the definitions of γ_i^h and the first-order conditions (4.4), (4.5), (4.6), and (4.7).

Solving \tilde{u}_1^h and \tilde{v}_2^h from (4.16), (4.17), and (4.18), we arrive at

$$\tilde{v}_1^h = \tilde{u}_1^h + (1 - \gamma_1^h) \phi_1^h \tilde{t}, \quad (4.19)$$

$$\tilde{v}_2^h = \tilde{u}_2^h + (1 - \gamma_2^h) \phi_2^h [w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2] / [w_2(1 - t) - b_2(1 - \alpha_2)]. \quad (4.20)$$

Combining intratemporal and intertemporal substitution

Substituting (4.14) and (4.15) into (4.19) and (4.20) to eliminate \tilde{u}_i^h , we find for the changes in labor supply in both periods

$$de_1^h = -(1 - e_1^h) \left\{ \begin{aligned} &\tilde{z}^h + (1 - \delta) \left\{ \sigma \gamma_1^h \tilde{t} - \sigma \gamma_2^h [w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2] / [w_2(1 - t) - b_2(1 - \alpha_2)] \right\} \\ &\quad + (1 - \gamma_1^h) \phi_1^h \tilde{t} \end{aligned} \right\}, \quad (4.21)$$

$$de_2^h = -(1-e_2^h) \left\{ \begin{aligned} &\tilde{z}^h - \delta \{ \sigma \gamma_1^h \tilde{t} - \sigma \gamma_2^h [w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2] / [w_2 (1 - t) - b_2 (1 - \alpha_2)] \} \\ &+ (1 - \gamma_2^h) \phi_2^h [w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2] / [w_2 (1 - t) - b_2 (1 - \alpha_2)] \end{aligned} \right\}. \quad (4.22)$$

The case with exogenous labor supply in the first period is given by $e_1^h = 1$, so that $\gamma_1 = \eta_1^h = 0$. In that case, we find $de_1^h = 0$ and

$$de_2^h = -(1 - e_2^h) \left\{ \tilde{z}^h + [\delta \sigma \gamma_2^h + (1 - \gamma_2^h) \phi_2^h] \frac{[w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2]}{[w_2 (1 - t) - b_2 (1 - \alpha_2)]} \right\}.$$

With endogenous first-period labor supply, the introduction of retirement accounts affects the generational accounts of the high-income household as follows

$$dg^h = \left(\frac{tw_2 + b_2}{1 + r} \right) \left(\frac{\partial e_2^h}{\partial t} \right)_c dt + tw_1 \left(\frac{\partial e_1^h}{\partial t} \right)_c dt.$$

A lower tax rate (i.e. $dt < 0$) improves this generational account under weak conditions. A sufficient condition for this generational account to improve is that both $\left(\frac{\partial e_1^h}{\partial t} \right)_c$ and $\left(\frac{\partial e_2^h}{\partial t} \right)_c$ are negative.³⁴ These inequalities are met (see (4.21) and (4.22)) if intertemporal substitution is small compared to intratemporal substitution (i.e. σ is small compared to ϕ_i^h , $i = 1, 2$), or if the two periods are symmetric (i.e. $\gamma_2^h = \gamma_1^h$; $e_1^h = e_2^h$; $b_2 = \alpha_2 = 0$). With $b_2 > 0$ and $\gamma_2^h > \gamma_1^h$ (the latter inequality may be met because the second period includes retirement and is thus likely to feature relatively large leisure demand), a higher tax rate is likely to boost the generational accounts even if intertemporal substitution is so strong that $\left(\frac{\partial e_1^h}{\partial t} \right)_c < 0$ (and thus $\left(\frac{\partial e_2^h}{\partial t} \right)_c > 0$). The reason is that intertemporal substitution of leisure towards the first period benefits the public finances, as leisure is subsidized most in the second period on account of early retirement benefits (i.e. $b_2 > 0$). This holds also if (as in the case of the introduction of ERAs) the shadow cost of leisure in the second period is increased not only through a lower t but also a higher α_2 . Such a reform shifts labor supply from the first period to the more heavily taxed second period, thereby benefiting the public finances.³⁵

³⁴(4.21) and (4.22) (with $\tilde{z}^h = 0$) imply that $\left(\frac{\partial e_1^h}{\partial t} \right)_c$ and $\left(\frac{\partial e_2^h}{\partial t} \right)_c$ cannot both be positive at the same time

³⁵Moreover, if leisure demand is relatively large in the second period (i.e. $e_2^h < e_1^h$), labor supply is relatively elastic in the second period so that a cut in the marginal tax rate in the second period is especially effective in boosting the tax base. Indeed, in these circumstances, Ramsey considerations would dictate a lower optimal effective tax rate in period 2 than in period 1.

2.2 Medium-income earners

Maximizing the Lagrangian (4.2), we obtain the first-order conditions for the optimal second-period consumption and labor supply of a medium-income earner

$$\left(1 - \frac{1}{\sigma}\right) (f_2(C_2^m, 1 - e_2^m))^{-\frac{1}{\sigma}} \frac{\partial f_2(C_2^m, 1 - e_2^m)}{\partial C_2^m} = \lambda^m, \quad (4.23)$$

$$\left(1 - \frac{1}{\sigma}\right) (f_2(C_2^m, 1 - e_2^m))^{-\frac{1}{\sigma}} \frac{\partial f_2(C_2^m, 1 - e_2^m)}{\partial (1 - e_2^m)} = \lambda^m [w_2(1 - t) - b_2(1 - \alpha_2)], \quad (4.24)$$

$$C_2^m + [w_2(1 - t) - b_2(1 - \alpha_2)](1 - e_2^m) = w_2(1 - t) + (1 - \alpha_y)y_2 - \alpha_1 b_1(1 + r). \quad (4.25)$$

Dividing (4.23) by (4.24) and log-linearizing the result, we arrive at

$$\tilde{C}_2^m - \tilde{v}_2^m = -\phi_2^m [w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2] / [w_2(1 - t) - b_2(1 - \alpha_2)], \quad (4.26)$$

where ϕ_2^m denotes the substitution elasticity between C_2^m and v_2^m in felicity $f_2(C_2^m, 1 - e_2^m)$.

Log-linearization of $u_2^m \equiv f_2(C_2^m, 1 - e_2^m)$ yields

$$\tilde{u}_2^m = (1 - \gamma^m) \tilde{C}_2^m + \gamma^m \tilde{v}_2^m, \quad (4.27)$$

where $\gamma^m \equiv \frac{(1 - e_2^m)[w_2(1 - t) - b_2(1 - \alpha_2)]}{C_2^m + (1 - e_2^m)[w_2(1 - t) - b_2(1 - \alpha_2)]} = \frac{(1 - e_2^m)[w_2(1 - t) - b_2(1 - \alpha_2)]}{w_2(1 - t) + (1 - \alpha_y)y_2 - \alpha_1 b_1(1 + r)}$. Solving \tilde{v}_2^m from (4.26) and (4.27), we find for the change in labor supply

$$de_2^m = -(1 - e_2^m) \{ \tilde{u}_2^m + (1 - \gamma^m) \phi_2^m [w_2 dt + (1 - \alpha_2) db_2 - b_2 d\alpha_2] / [w_2(1 - t) - b_2(1 - \alpha_2)] \}.$$

Loglinearization of (4.25) yields

$$\tilde{u}_2^m = \psi^m \left(V_t^m dt + V_{\alpha_1}^m d\alpha_1 + V_{b_1}^m db_1 + V_{\alpha_2}^m d\alpha_2 + V_{b_2}^m db_2 + V_{y_2}^m dy_2 + V_{\alpha_y}^m d\alpha_y \right),$$

where $\psi^m \equiv \frac{1}{\lambda^m [C_2^m + (1 - e_2^m)[w_2(1 - t) - b_2(1 - \alpha_2)]}$.

2.3 Low-income earners

By a similar procedure as for the medium-income earners (see sub-section 2.2 of the appendix), the Lagrangian (4.3) yields the first-order conditions

$$\left(1 - \frac{1}{\sigma}\right) (f_2(C_2^l, 1 - e_2^l))^{-\frac{1}{\sigma}} \frac{\partial f_2(C_2^l, 1 - e_2^l)}{\partial C_2^l} = \lambda^l, \quad (4.28)$$

$$\left(1 - \frac{1}{\sigma}\right) \left(f_2(C_2^l, 1 - e_2^l)\right)^{-\frac{1}{\sigma}} \frac{\partial f_2(C_2^l, 1 - e_2^l)}{\partial (1 - e_2^l)} = \lambda^l [\theta w_2 (1 - t - s) - b_2], \quad (4.29)$$

$$C_2^l + [\theta w_2 (1 - t - s) - b_2](1 - e_2^l) = \theta w_2 (1 - t - s) + y_2 \quad (4.30)$$

We use (4.28) and (4.29) in a similar procedure as with the medium-income household above to arrive at

$$de_2^l = -\chi^l \left\{ \tilde{u}_2^l + (1 - \gamma^l) \phi_2^l [dt + ds] / [1 - t - s] \right\},$$

where $\gamma^l \equiv \frac{[\theta w_2 (1 - t - s) - b_2](1 - e_2^l)}{C_2^l + [\theta w_2 (1 - t - s) - b_2](1 - e_2^l)}$, ϕ_2^l is the substitution elasticity between C_2^l and $(1 - e_2^l)$ in felicity $f_2(C_2^l, (1 - e_2^l))$, and \tilde{u}_2^l is found from the log-linearized budget constraint (4.30) as

$$\tilde{u}_2^l = \psi^l (V_t^l dt + V_s^l ds + V_{b_2}^l db_2 + V_{y_2}^l dy_2),$$

where $\psi^l \equiv \frac{1}{\lambda^l [C_2^l + [\theta w_2 (1 - t - s) - b_2](1 - e_2^l)]}$.