International Integration, Risk and the Welfare State

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October 1999

How does international integration affect the welfare state? Does it call for a leaner welfare state to reap the benefits of integration or is it necessary to expand the welfare state to offset some negative consequences of international integration? This paper addresses these issues in a fully specified intertemporal two-country model focusing on the implications of product market integration reducing trade frictions across national product markets. Lower trade frictions may increase the marginal costs of public funds, which gives an argument for reducing (steady-state) public consumption. However, tighter integration of product markets unambiguously leads to more variability in private consumption, and this gives a case for expanding the social insurance provided by the public sector via state-contingent consumption (automatic stabilizers).

Jel Classification: E30, F10, H11

Keywords: Trade frictions, risk-sharing, optimal public consumption, social insurance

* Comments by Peter B. Sorensen and participants at the workshop ”Structural Change and European Economic Integration” and seminar participants in Bonn are gratefully acknowledged.
1 Introduction

The importance of international integration for the welfare state is a vividly debated topic\(^1\). To some international integration calls for a leaner welfare state partly to make the economy more competitive and therefore capable of reaping the benefits of international integration and partly because international integration expands the scope for risk diversification and therefore reduces the need for social insurance supplied by the welfare state. To others the gains from international integration come at the costs of more volatility due to enhanced dependence on foreign economic developments. Accordingly, proponents of the "globalization" view argue that the need for public sector activities in particular those related to the provision of social insurance is increased.

Considering the two divergent views on the relation between openness and welfare state activities from a theoretical perspective, we have to address two issues, namely, how international integration affects both the distortionary effects of public activities and the exposure to risk. The former effect arises because tighter international integration change market forms and behaviour and therefore the distortionary effects of public sector activities as well as the scope for taxation (tax mobility).

The major channels through which international integration affects the exposure to risk arise through mechanisms enhancing the exposure to foreign shocks as well as possible specialization of production tending to make the production structure less diversified\(^2\). On the other hand possibilities for risk diversification arise via both trade (especially in financial assets) and mobility of factors.

The present paper focuses on the fact that international integration enhances the possibilities for mutually advantageous trades while at the same time enlarging the exposure to risk (foreign shocks) as well as creating possibilities for risk diversification (domestic shocks). Risk diversification may go through many routes in financial, labour and product markets. The present paper focuses on the role of product markets for two reason. First, the most important changes induced by international integration are related to product markets making it easier for domestic firms to enter foreign markets and vice versa. Second, it is widely accepted that mobility of labour in a European context will play a modest role within a foreseeable future (Pedersen (1996)), and ample evidence indicates that financial markets do not - despite

\(^1\)For an introduction to the economics literature see eg Atkinson (1999) and Rodrik (1998), and to the political science literature see eg Pierson (1998).

\(^2\)Frankel and Rose (1998) find in a recent empirical study that tighter integration tends to make business cycles more alike contradicting the "specialization hypothesis".
being closely integrated - reach the ideal with respect to risk diversification (see e.g. Lewis (1999))\(^3\). Moreover, it is a central result in international trade theory that mobility of goods is sufficient to produce a situation equivalent to one where factors of production are mobile (the factor price equalization theorem).

To address the interrelationship between international integration and the exposure to risk involves aspects of both international trade and open macroeconomics. To this end a fully specified intertemporal two-country model with trade frictions is set-up. Trade frictions do in practice take many forms including transportation costs, tariffs, taxes, costs associated with border control, information costs about foreign markets and costs associated with product approval in foreign markets. While these frictions can have both fixed and variable components the present paper only focuses on the role of variable (proportional) costs of international trade. Increased international integration is assumed to lead to a reduction of these frictions. This allows us to interpret international integration as a process which works by lowering (explicit and implicit) trade costs. While capital markets are fully integrated, it is assumed that they are incomplete in the sense that it is not possible to fully diversify the risk of consumption changes induced by domestic or foreign changes in production.

The welfare state may provide social insurance both with respect to individual (idosyncratic) risks and aggregate risks and thereby mitigate the consequences of market failures. International integration is likely to have consequences for the distribution of both idiosyncratic and aggregate risks\(^4\). The present paper focuses on the latter for several reasons. Some aspects of international integration will affect all individuals in the same way, this applies in particular to changes originating in financial and goods markets. Since these are the areas in which international integration has proceeded the farthest, it is natural to start by analysing these. Secondly, concerning the overall size and structure of the welfare state aggregate shocks are likely to be most important, while a change in the distribution of idiosyncratic shocks across individuals may require restructuring, it is not necessarily going to affect the overall size of the public sector. Finally, international integration will unquestionably have aggregate effects, and it is therefore natural to ask how the welfare state can cope with these, without denying that there also will be important consequences for idiosyncratic risks. To this may be added the

\(^3\)While there is an increasing trend in the holdings of foreign assets, this is primarily for institutional investors, i.e. pension funds. Accordingly, this does not ensure full diversification of risks.

\(^4\)See e.g. Bec (1999) and Davis (1998) for analyses of how international integration affects different groups in the labour market.
more technical argument that a proper analysis of these problems requires an intertemporal general equilibrium model, and it is very difficult to construct a workable model with heterogeneous agents to study the role of idiosyncratic shocks.

Despite the political interest in the issue of how the welfare state is affected by international integration, the theoretical literature on this issue is very scant. Focussing on factor mobility Wildasin (1995) argues in a static partial model that international integration reduces the need for social insurance since international mobility of production factors tend to work as a shock absorber at the same time as it reduces the scope for taxation since the tax-base becomes more mobile. This model thus supports the view that international integration reduces the need and scope for welfare state activities. Rodrik (1998) takes international integration as something leading to increased volatility of the terms of trade and therefore enhancing the need for social insurance. This can be done by moving resources to sectors less exposed to market risks, ie shifting resources from the private sector to the public sector. Rodrik also presents some evidence indicating that more open economies tend to have a larger public sector which supports his conclusion that international integration calls for an expansion of public sector activities.

The present paper differs from the above-mentioned in two important respects. First the analysis is based on an explicit model of international trade in goods, and captures in this way basic gains from international trade as well as how volatility in the terms of trade can arise. Second, the model is a fully specified intertemporal two-country general equilibrium model.

The paper is organised as follows: Section 2 sets up the model, and the properties of the steady-state equilibrium are analysed in section 3. In section 4 the interrelationship between international integration and the exposure to risk is explored while section 5 considers the role for social insurance. Finally, section 6 offers some concluding remarks. All proofs are collected in the appendices.

2 An Intertemporal Two-Country Model with Trade Frictions

Consider a non-monetary two country model with specialized production, ie each country produces a commodity which can be traded internationally.

\footnote{In Rodrik (1997) mobility of capital is allowed for and it is found that tax policy can compensate for the consequences of increased terms of trade risks induced by international integration provided that capital is not too mobile}
Trade across borders are assumed to be associated with trade frictions implying that one has to buy $1 + \tau$ unit of the foreign good to attain one unit for final consumption, i.e. $\tau$ measures the friction associated with trade ("iceberg" costs). The process of international integration is taken to lower these trade frictions. To simplify, production is assumed exogenous.

**Households**

Households own the firms and are entitled to the flow of profits. The horizon is infinite and the representative household aims at maximizing expected utility given as

$$\Phi_t = E_t \left[ \sum_{j=0}^{\infty} (1 + \delta)^{-j} \left( U(b_{t+j}) + V(g_{t+j}) \right) \right]$$

where $\delta$ is the subjective rate of time preference, $U$ is the instantaneous utility function defined over private consumption in the following way:

$$U(b_{t+j}) = b_{t+j} - \frac{k}{2} (b_{t+j})^2 \quad k > 0, \quad b < k^{-1}$$

where $b$ is a composite index of consumption defined over consumption of home goods ($c$) and foreign goods ($c^*$), i.e.

$$b_{t+j} = \frac{1}{\alpha} \left( c_{t+j}^* \right)^{1-\alpha} \left( c_{t+j} \right)^{\alpha} \quad 0 < \alpha < 1 \quad a \equiv \alpha^\alpha (1 - \alpha)^{1-\alpha}$$

Note that the strict concavity of the utility function involves risk-aversion wrt. fluctuations in the consumption bundle. The utility of publicly provided services/goods is denoted $V(g_{t+j})$ possessing the properties $V' > 0, V'' < 0$.

International trade involves various frictions, hence to attain $c^*$ units of the foreign commodity for final consumption, the household has to acquire/pay for $c^* (1 + \tau)$ units ($\tau$ exogenous). The $\tau c^*$ units are absorbed by the trade frictions.

The optimal consumption decision can most easily be found by first considering how the household maximizes the value of the composite consumption bundle for given nominal expenditures $S_{t+j}$ in period $t + j$,

$$S_{t+j} = P_{t+j} c_{t+j} + P_{t+j}^* (1 + \tau) c_{t+j}^*$$

6This specification is also used in e.g. Glick and Rogoff (1995) and Razin (1995).

7This utility function is a convenient approximation allowing a mean-variance analysis in the case of uncertainty. The restriction on $b$ ensures that the marginal utility of consumption $b$ is positive.

8Note that $\tau$ is a pure friction and nobody derives any utility from this.
and $P$ is the price of home goods in the home market, and similarly $P^*$ is the price of the foreign product in the foreign market. With Cobb-Douglas preferences it follows straightforwardly that the optimal consumption implies

$$c_{t+j} = \alpha \frac{S_{t+j}}{P_{t+j}}$$

$$c^*_j = (1 - \alpha) \frac{S_{t+j}}{P^*_{t+j} (1 + \tau)}$$

The indirect utility implied by spending $S_{t+j}$ on the two commodities in period $t+j$ can be written

$$b_{t+j} = \frac{S_{t+j}}{Q_{t+j}}$$

where $Q$ is the consumer price index defined as

$$Q_{t+j} \equiv (P_{t+j})^\alpha \left( P^*_{t+j} (1 + \tau) \right)^{1-\alpha}$$

The intertemporal budget constraint reads

$$\sum_{j=0}^\infty \prod_{k=0}^{j-1} (1 + r_{t+k})^{-1} S_{t+j} \leq \sum_{j=0}^\infty \prod_{k=0}^{j-1} (1 + r_{t+k})^{-1} I_{t+j} + F_t$$

where $I$ is the after-tax nominal income ($\equiv P_1 y_t - T_t$), $r_{t+k}$ the nominal interest rate, $T_t$ a lump sum tax (see below), and $F$ is nominal non-human wealth at the start of period $t$.

In the international capital market risk-neutral institutions issue a bond offering a real rate of return specified in terms of the consumption bundle, i.e.

$$\frac{(1 + r_{t+1}) Q_t}{Q_{t+1}} = 1 + r$$

Equilibrium in the financial market ensures that $\delta = r^{910}$. Note that this assumption implies that the capital market offers possibilities for risk diversification. The arguments made below on social insurance does not, therefore, rely on highly inefficient capital markets.

The above assumptions imply that the real rate of return is constant; the real rate of return on the bond is therefore riskless. Capital market

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9In the symmetric case ($\alpha = \frac{1}{2}$) $Q = Q'$ and hence international trade in the asset ensures equal real returns in both countries.

10Ensuring that expected profits in the financial market is equal to zero. Note that the unit elastic demand implies that the trade balance is unaffected by relative prices changes.
imperfections arise because there is no trade in equities across countries, that is, domestic equities are held only by domestic residents (home bias in portfolio allocations). The risk associated with variation in production (income) can therefore not be fully diversified via the international capital market. While the specific assumptions on financial markets made here are highly stylized, they capture a basic capital market imperfection and maintains consistency with the observed ”home bias” in international portfolio allocations (see Lewis (1999)).

The budget constraint can now be written in real terms as

$$\sum_{j=0}^{\infty} (1 + \delta)^{-j} b_{t+j} \leq \sum_{j=0}^{\infty} (1 + \delta)^{-j} i_{t+j} + f_t$$

where $i_t \equiv I_t/Q_t$ and $f_t = F_t/Q_t$.

It is convenient to define

$$A_t \equiv \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t i_{t+j} + f_t$$

as the household’s total (human and non-human) wealth at time $t$. As is wellknown, the intertemporal utility maximization problem has the simple solution

$$b_t = \frac{\delta}{1 + \delta} A_t$$

with the associated no ponzi game condition

$$\lim_{T \to \infty} (1 + \delta)^{-T} f_{t+T} = 0$$

The household consumes the real return of its total wealth each year, and the martingale property holds for consumption, ie

$$E_t b_{t+1} = b_t$$

and likewise for wealth

$$E_t A_{t+1} = A_t$$

We find that the consumption of the home good can be written

$$c_t = \alpha \frac{\delta}{1 + \delta} \frac{A_t Q_t}{P_t}$$

(3)
and the foreign good

\[ c_t^* = (1 - \alpha) \frac{\delta}{1 + \delta} A_t \frac{Q_t}{P_t^* (1 + \tau)} \]  

(4)

Note that demand is equal consumption for the home good \((d_t = c_t)\), while demand for the foreign good is \(d_t^* = (1 + \tau) c_t^*\) due to the trade friction. For later reference note that risk associated with the consumption bundle \((b_t)\) arises from fluctuations in real wealth \((A_t)\).

**Government**

The government produces the public good by use of the home good as an input and finances these expenditures by a lump-sum tax. The model implies Ricardian Equivalence and hence we can without loss of generality, assume a balanced budget in each period, ie

\[ p_t g_t = \tau_t \]

The home bias in public consumption captures the observed tendency of the public sector to consume home produced goods (services).

The government is assumed to be utilitarian, ie the level of public consumption is set so as to maximize the utility function \((1)\).

**Equilibrium condition**

The equilibrium in the home market reads

\[ y_t = d_t + \overline{\tau}_t + g_t \]

where \(\overline{\tau}_t\) denotes foreign demand of the home product.

**3 Steady-State Equilibrium**

Let us start by analysing the properties of the steady-state equilibrium to this economy (see appendix A for details). The interesting question is how public consumption and thus the size of the public sector depends on trade frictions.

We find that the private consumption bundle in steady-state equilibrium can be written as a function of home goods less home government consumption, foreign goods less foreign government consumption, and the trade friction parameter \(\tau\), ie

\[ b = f (y - g, y^* - g^*, \tau) \]
First, note that trade frictions imply lower consumption, ie
\[ \frac{\partial b}{\partial \tau} < 0 \]
reflecting the standard argument that there are gains to be reaped by international integration.

Higher public consumption lowers private consumption
\[ \frac{\partial b}{\partial g} < 0 \]

However, the effect is numerically smaller the larger the trade friction, ie
\[ \frac{\partial}{\partial \tau} \left( \frac{\partial b}{\partial g} \right) > 0 \]

This can be interpreted in the following way, the larger the trade friction the less private consumption is generated out of a given income. Hence, increasing public consumption has a smaller effect on private consumption the larger the trade friction. This suggests that lower trade frictions lead to a lower level of public consumption.

The interrelationship between trade frictions and the optimal level of public consumption is, however, less straightforward as seen by the following result where \( g_0 \) denotes the optimal level of public consumption in the absence of trade frictions (\( \tau = 0 \)).

**Proposition 1** In a symmetric steady-state equilibrium the optimal level of public consumption is (i) increasing in the trade friction (\( \tau \)) if \( g_0 \geq g_0^* \), and (ii) decreasing in the trade friction (\( \tau \)) for \( \tau < \bar{\tau} \) and increasing for \( \tau > \bar{\tau} \) if \( g_0 < g_0^* \).

The reason why we don’t get an unambiguous result is that a change in the trade friction affects the optimal level of public consumption via two routes. First lower trade frictions imply that a change in public consumption has a larger effect on private consumption and this tends to lower the optimal level of public consumption. This effect runs via the effect public consumption has on relative prices\(^{11}\) and disposable income. Second lower trade frictions increase private consumption and thereby lowers the marginal utility of private consumption. This tends to increase the optimal level of public consumption.

\(^{11}\)It is well-known that a channel through which fiscal policy works in an open economy is via a change in relative prices (terms of trade) (see eg Andersen, Rasmussen and Sorensen (1996)).
One interpretation of proposition 1 is that if there is a strong preference for public goods \( (g_0 \geq g_0^*) \), then lower trade frictions unambiguously lead to a lower steady-state level of public consumption. However, with a less strong preference for public goods \( (g_0 < g_0^*) \), international integration may make room for a higher level of public consumption (for \( \tau \leq \tau^* \)).

Figure 1 displays the possible relationships between the optimal steady-state level of public consumption and trade frictions.

![Figure 1: Optimal steady-state public consumption and trade frictions](image)

Finally considering how steady-state private consumption depends on domestic and foreign output, we have

\[
\frac{\partial b}{\partial y} > 0 \quad \frac{\partial}{\partial \tau} \left( \frac{\partial b}{\partial y} \right) < 0
\]

\[
\frac{\partial b}{\partial y^*} > 0 \quad \frac{\partial}{\partial \tau} \left( \frac{\partial b}{\partial y^*} \right) < 0
\]

ie private consumption depends positively on the supply of both types of goods, but the sensitivity is lower, the higher the trade friction. The latter suggests that integration may affect the exposure to domestic and foreign shocks.
4 Risk

To analyse how international product market integration affects the exposures to risk, we now add uncertainty by assuming that production fluctuates. Specifically it is assumed that $\tilde{y}_k \equiv y_k - y$, i.e. the deviation of output from its mean (=steady-state value) has an expected value of zero and a variance $\sigma^2$, and similarly for the foreign country\(^{12}\). The contemporaneous correlation is denoted $\rho$, but the shocks are uncorrelated over time (transitory shocks)\(^{13}\). The information set in period $t$ is $\psi_t$.

The results reported below are derived for a version of the model linearized around the symmetric steady-state equilibrium (see appendix B) which can be solved analytically.

The first and obvious implication is that uncertainty wrt output creates uncertainty wrt consumption. We have that

$$\frac{\partial \tilde{b}_h}{\partial \tilde{y}_k} > 0$$

$$\frac{\partial \tilde{b}_b}{\partial \tilde{y}_k} > 0$$

and

$$\frac{\partial \tilde{b}_b}{\partial \tilde{y}_k} \leq \frac{\partial \tilde{b}_b}{\partial y_k} \quad \text{for} \quad \tau \leq \bar{\tau}$$

that is, higher output of both the home and the foreign product implies higher consumption. Private consumption is more sensitive to foreign output than domestic output for low levels of trade frictions. The reason is that a change in domestic output has both an income and a price effect going in opposite directions while a change in foreign output only releases the latter effect for the domestic consumers.

We also have that consumption becomes less sensitive to output changes if trade frictions are lower for fairly integrated economies ($\tau < \tau^*$) and vice versa.

$$\frac{\partial}{\partial \tau} \left( \frac{\partial \tilde{b}_b}{\partial \tilde{y}_k} \right) \geq 0 \quad \text{for} \quad \tau \leq \tau^*$$

\(^{12}\) Notice that the utility function implies that a mean-variance analysis is sufficient.

\(^{13}\) See eg Stockman and Tesar (1995) for some empirical regulations on business cycle movements and a two-country model with complete capital markets.
while lower trade frictions unambiguously increase the sensitivity to foreign output changes.

\[
\frac{\partial}{\partial \tau} \left( \frac{\partial \tilde{b}_h}{\partial y^*_t} \right) < 0
\]

which leads to

**Proposition 2** The higher the trade friction, the more (less) sensitive is consumption to changes in domestic output if \( \tau < \tau^* \) (\( \tau \geq \tau^* \)), while it is less sensitive to foreign output

The first part of this result reflects that trade frictions have two effects, namely, first to introduce a home-bias in consumption tilting consumption towards domestically produced goods\(^{14}\), and second to induce a loss through the frictions involved in trade, ie the larger the trade friction, the less the marginal gain in actual consumption of an increase in output. The former of these effects thus tends to make consumption more sensitive to domestic output and the latter to make it less sensitive the larger the trade friction. In general we have an ambiguous effects, but for fairly closely integrated economies (\( \tau < \tau^* \)) the former effect dominates. Trade frictions unambiguously reduce the sensitivity of private consumption to foreign output.

Given the opposite effects of trade frictions (for \( \tau < \tau^* \)) on the sensitivity of private consumption to domestic and foreign output, it might be expected that it is ambiguous whether lower trade frictions reduce or enhance the volatility of private consumption. This is not the case, however, since

**Proposition 3** Higher trade frictions (\( \tau \)) unambiguously lead to less consumption variability for \( \tau < \bar{\tau}, \) ie

\[
\frac{\partial \text{Var} \left( \tilde{b}_h | \psi_{t-1} \right)}{\partial \tau} < 0 \quad \text{for} \quad \tau < \bar{\tau}
\]

This is a surprising result since it might be expected that lower trade frictions would allow for risk-diversification between the two countries. While such risk diversification takes place, the effect found above is driven by the fact that the smaller the trade frictions, the more sensitive is final consumption to changes in output. Considering the consequences of trade frictions we thus have a trade-off between the mean value of consumption and its variance. Trade frictions lower both the mean and variance of consumption.

\(^{14}\)Note that \( c/c^* \) is increasing in \( \tau \). However, \( c/c^* (1+\tau) \) is independent of \( \tau \) due to the unit elastic demand function.
Is there any way by which the steady-state level of public consumption can affect the risk profile of private consumption and thus possibly be adjusted to compensate for the risk associated with tighter international integration? The answer is negative since we have\(^1\)

**Proposition 4** The steady-state level of public consumption does not affect the risk profile of private consumption

That is, the steady-state level of public consumption does affect the average level of private consumption but not its variability. This reflects that the (symmetric) level of public consumption does not affect the sensitivity of private consumption to output changes. Given that the size of the public sector does not in itself change the risk profile (compare to Rodrik (1996)) it becomes of interest to ask whether there are other ways of affecting the exposure to risk accompanying international integration.

5 Social Insurance

Since international integration leads to more risks, it is natural to question whether this leaves a greater role for (implicit) social insurance, or to put it differently, is there a case for an active management of public consumption so as to affect the risk-profile of private consumption by making public consumption state-contingent? If so, this is a way by which to provide social insurance to aggregate shocks and this has a positive effect on the welfare of private households since they are averse to risk. Note that since the present model displays Ricardian Equivalence and assumes non-distortionary taxes (exogenous production) the only possibility of introducing social insurance is via state contingent public consumption.

State contingencies can be made in several ways, and it is beyond the scope of the present paper to analyse the optimal way by which to design such contingencies. The aim is to show that such contingencies can lead to welfare improvements even if they take a very simple (ad hoc) form, and then to analyse how the need for social insurance is affected by international integration.

Assume for the sake of argument that public consumption is made contingent on income, i.e

\[ \tilde{y}_t = \kappa \tilde{\alpha}_t \]

\(^1\)This result obviously depends on the linearization, and it thus says that there are no first-order effects of variations in the steady-state level of public consumption on the volatility of private consumption.
where \( \kappa \) measures the degree of contingency in public consumption. This formulation is adapted since it is well-known that public sector activities (consumption, taxes) depend on the level of income in most countries (OECD (1993)).

It can be established that (see appendix C)

\[
\frac{\partial}{\partial \kappa} \left( \frac{\partial b_t}{\partial y_t} \right) < 0
\]

and

\[
\frac{\partial}{\partial \kappa} \left( \frac{\partial \tilde{b}_t}{\partial y_t} \right) < 0
\]

that is, more sensitivity of public consumption to the business cycle situation makes private consumption less sensitive to changes in both domestic and foreign output.

**Proposition 5** The variability of private consumption is decreasing in \( \kappa \) for \( \rho \geq \rho_0 < 0 \).

Proposition 5 implies that a policy aiming at stabilizing private consumption should choose \( \kappa > 0 \) (for \( \rho \geq \rho_0 \)), i.e. a pro-cyclical variation such that resources absorbed by the public sector should be higher when income is high and vice versa\(^{16}\)

The procyclical nature of the contingency may at first seem counter-intuitive. However, the way private consumption can be stabilized is by stabilizing disposable income. To stabilize disposable income requires that tax payments are high when income is high. The balanced budget constraint imposed enforces a one to one relation between changes in public consumption and taxes and therefore public consumption has to change pro-cyclically to stabilize disposable income. It is therefore perhaps best to think of the effect underlying proposition 5 as running primarily via the tax effect rather than the expenditure effect.

Taking this interpretation, it is interesting that a welfare improving contingency takes a ”procyclical form“, i.e. if private income is high, then taxes are high and vice versa. This corresponds fairly closely to the automatic stabilizers discussed at great length in the macroeconomic literature.

\(^{16}\) See Andersen and Holden (1999) for the optimal response of public consumption to supply and demand shocks in a small open economy.

\(^{17}\) Empirical evidence indicates that this is indeed the case since the public budget balance is strongly procyclical, see e.g. OECD (1993) and Braconier and Holden (1999).
Having established a case for contingencies built into public consumption as a way or providing social insurance raises the question how increased integration affects the need for social insurance. From the results reported in section 3 it may be inferred that it is difficult to obtain an unambiguous answer to this question. Tighter international integration (lower $\tau$) may reduce the exposure to domestic shocks ($\tau < \tau^*$) while increasing it to foreign shocks. It is, however, possible to establish the following result.

**Proposition 6** The larger the trade friction ($\tau$), the smaller the contingency in public consumption ($\kappa$) needed to attain a given stabilization of private consumption, i.e.

$$\frac{\partial \kappa}{\partial \tau} \bigg| \text{Var}(\delta_t | \psi_{t-1}) < 0 \quad \text{for} \quad \tau < \bar{\tau} \text{ and } \rho > \rho_0( < 0)$$

This says that for economies being fairly integrated we would expect a need for stronger contingencies in public consumption (social insurance) if they integrate further (provided the shocks are not too negatively correlated). This can be taken in support of the view that international integration may strengthen the need for implicit insurance provided by the public sector.

Private consumption smoothing is thus achieved via contingencies built into public consumption. However, such contingencies imply that public consumption becomes volatile. It is thus reasonable to ask whether contingencies in public consumption achieve anything but to substitute public consumption volatility for private consumption volatility. However, even if policy is set in accordance with an objective function equally averse to volatility in private and public consumption, we find that there are gains to be made by state-contingent public consumption since we have

**Proposition 7** If agents are equally averse to fluctuations in private and public consumption, overall variability can be reduced by state contingent public consumption ($\kappa > 0$).

This shows that tighter international integration in the form of lower trade friction makes a case for state-contingent public consumption or automatic stabilizers. While proposition 1 concerns the steady-state level of public consumption, proposition 6 implies that stronger contingencies or stronger elements of social insurance should be built into public sector activities when economies integrate.

Note that private agents do not have any possibilities of diversifying the increased consumption risk induced by lower trade frictions in the capital market. No assets exist which offer a return contingent on future domestic
and foreign output. Hence, even though agents are rational and forward-looking, there is a case for social insurance provided via state-contingent public consumption.

6 Concluding Remarks

The present analysis has shown that the two popular views on the relation between public sector activities and international integration both capture aspects which are supported by theoretical considerations. The supporters of the "Rolling back" view may be right that the distortionary costs of public sector activities may be larger the smaller the trade frictions and therefore integration calls for a smaller public sector. It is interesting that the response to international integration differs in the sense that countries biased towards a large public sector may reduce public consumption while countries biased towards a small public sector may increase public consumption, ie we may see a convergence to a more similar size of the public sector from both the bottom and the top. On the other hand supporters of the "globalization" view are right that tighter international integration may increase volatility and call for more social insurance achieved via state contingent public activities.

It follows that this analysis calls for reforms of the welfare state, since international integration has implications for both the size of the public sector and social insurance. The latter may be achieved by expanding so-called automatic stabilizers in the public budget running via both the revenue and the expenditure side. It is worth stressing that the results on social insurance found here are based on a model with forward looking agents and a perfect capital market, albeit the capital market is not complete in the sense of offering a set of assets contingent on all possible states of nature.

On the agenda for future research is extensions addressing two important shortcomings of the present model. First the unit elastic demand while a convenient and often made assumption is surely a special case which affects some of the results. Hence, it is of interest to see how the results are affected by a more general formulation. Second, while it would be fairly easy to endogenize production, it would be more difficult but also more relevant to do this in a way admitting unemployment as well as distortionary effects of taxation. In this way it would be possible to assess how unemployment is affected by international integration, and thereby integrate one of the major objectives underlying the welfare state, cf Agell (1999). This will also make it possible to capture that not all are affected equally by international integration, ie the distribution of idiosyncratic risk changes.
Appendix A: Steady State Equilibrium

The equilibrium condition for the home product reads

\[ y_k = \alpha \frac{\delta}{1+\delta} A_t \frac{Q_t}{P_t} + \alpha \frac{\delta}{1+\delta} A_t^* \frac{Q^*_t}{P_t} + g_k \]

Using that in steady state \( A = \frac{1+\delta}{\delta} P(y - g) \) where the time subscript is eliminated to indicate steady state and initial non-human wealth is assumed equal to zero, we get

\[ y - g = \alpha (y - g) + \alpha (y^* - g^*) \left( \frac{P^*}{P} \right) \]

implying

\[ \frac{P^*}{P} = \frac{(1 - \alpha) (y - g)}{\alpha (y^* - g^*)} \]

Inserting we find

\[ b = \frac{P}{Q} (y - g) \]

using

\[ \frac{P}{Q} = \left( \frac{P}{P^*} \frac{1}{1+\tau} \right)^{1-\alpha} = \left( \frac{1-\alpha}{\alpha} \right)^{\alpha-1} (y - g)^{\alpha-1} (y^* - g^*)^{1-\alpha} (1 + \tau)^{\alpha-1} \]

Hence,

\[ b = \bar{b} (y - g)^{\alpha} (y^* - g^*)^{1-\alpha} (1 + \tau)^{\alpha-1} \]

where

\[ \bar{b} \equiv \left( \frac{1-\alpha}{\alpha} \right)^{\alpha-1} \]

Note for later reference

\[ \frac{\partial b}{\partial g} = -\alpha \bar{b} (y - g)^{\alpha-1} (y^* - g^*)^{1-\alpha} (1 + \tau)^{\alpha-1} \]

Proof Proposition 1:

Optimal steady-state public consumption solves

\[ \max_g U(b) + V(g) \]
Hence, the first order condition reads

\[ V''(g) = -U'(b) \frac{\partial b}{\partial g} \equiv \phi(g, \tau) \]

Evaluating the marginal costs of public consumption in a symmetric steady state, we have

\[ \phi(g, \tau) \equiv \alpha \tilde{b} (1 - k \tilde{b}(y - g)(1 + \tau)^{\alpha-1})(1 + \tau)^{\alpha-1} \]

The second order condition reads

\[ V''(g) - \phi'(g) \leq 0 \]

We have that

\[ \frac{\partial g}{\partial \tau} = \frac{\phi_{\tau}}{V''(g) - \phi'(g)} \]

Hence,

\[ \text{sign} \left( \frac{\partial g}{\partial \tau} \right) = -\text{sign} \phi_{\tau} = \text{sign} \left( 1 - 2k \tilde{b}(y - g)(1 + \tau)^{\alpha-1} \right) \quad (5) \]

The expression on the right hand side is increasing in \( \tau \) and \( g \). Note that

\[ 1 - 2k \tilde{b}(y - g)(1 + \tau)^{\alpha-1} \rightarrow 1 \quad \text{for} \quad \tau \rightarrow \infty \]

Denote by \( g_0 \) the optimal level of public consumption in the absence of trade frictions \( (\tau = 0) \), i.e. \( g_0 \) solves

\[ V''(g_0) = \phi(g_0, 0) \]

If \( g_0 \geq g^* \equiv y - (2k \tilde{b})^{-1} \), it follows that \( 1 - 2k \tilde{b}(y - g) > 0 \) for \( \tau = 0 \) and therefore

\[ 1 - 2k \tilde{b}(y - g)(1 + \tau)^{\alpha-1} > 0 \]

for all values of \( \tau \), and hence \( \partial g / \partial \tau > 0 \). If \( g_0 < g^* \), it follows that \( \partial g / \partial \tau |_{\tau=0} < 0 \). However, since the expression in (5) is increasing in \( \tau \), it follows that there exists a \( \tau^* > 0 \) such that \( \partial g / \partial \tau < 0 \) for \( \tau < \tau^* \) and \( \partial g / \partial \tau < 0 \) for \( \tau > \tau^* \).
Appendix B: Stochastic Equilibrium

From (1) it follows straightforwardly that

$$\tilde{A}_t = \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \tilde{A}_{t+j} + \tilde{f}_t$$

where $\tilde{A}$ denotes that the variables are measured in deviations from steady-state equilibrium.

From (2) we have

$$\tilde{b}_t = \frac{\delta}{1 + \delta} \tilde{A}_t$$

The equilibrium condition for the product market reads ($\alpha = 1 - \alpha$)

$$y_t = \alpha \frac{\delta}{1 + \delta} Q_t A_t + \alpha \frac{\delta}{1 + \delta} Q_t^* A_t^* + \gamma_t$$

Making a first-order approximation around a steady-state equilibrium and assuming public consumption to be constant, we get

$$\tilde{y}_t = \eta_{d^A} (\tilde{A}_t + \tilde{A}_t^*) + \eta_{dp} \tilde{p}_t$$

where $\tilde{p}_t$ denotes deviations in the relative price ($P_t^* / P_t$) from its steady-state value and

$$\eta_{d^A} = \alpha \frac{\delta}{1 + \delta} \left( \frac{P_t^*}{P_t} \right)^{1-\alpha} (1 + \tau)^{\alpha} = \alpha \frac{\delta}{1 + \delta} (1 + \tau)^{\alpha}$$

$$\eta_{dp} = 2 \alpha^2 \frac{\delta}{1 + \delta} (1 + \tau)^{\alpha} \left( \frac{P_t^*}{P_t} \right)^{-\alpha} A = 2 \alpha^2 \frac{\delta}{1 + \delta} (1 + \tau)^{\alpha} A$$

Note for later reference that

$$\frac{\partial \eta_{d^A}}{\partial \tau} > 0 ; \quad \frac{\partial \eta_{dp}}{\partial \tau} > 0$$

Real income is defined as

$$I_t = P_t \frac{Q_t}{Q_t^*} (y_t - \gamma_t)$$

Hence, to a first order approximation we have

$$\tilde{i}_t = \eta_{Iy} \tilde{y}_t - \eta_{Ip} \tilde{p}_t$$
where
\[
\eta_{Iy} = \frac{P}{Q} = \left( \frac{P^*}{P} \right)^{-\alpha} (1 + \tau)^{-\alpha} = (1 + \tau)^{-\alpha}
\]
\[
\eta_{Ip} = \alpha (y - g) \left( \frac{P^*}{P} \right)^{-\alpha-1} (1 + \tau)^{-\alpha} = \alpha (y - g) (1 + \tau)^{-\alpha}
\]

Hence,
\[
\frac{\partial \eta_{Iy}}{\partial \tau} < 0 ; \quad \frac{\partial \eta_{Ip}}{\partial \tau} < 0
\]

Inserting from (6) we get
\[
\tilde{i}_t = \eta_{Iy} \tilde{y}_t - \eta_{Ip} \frac{1}{\eta_{dp}} \left( \tilde{y}_t - \eta_{dA} \left( \tilde{A}_t + \tilde{A}_t^* \right) \right)
\]
\[
= \left( \eta_{Iy} - \frac{\eta_{Ip}}{\eta_{dp}} \right) \tilde{y}_t + \eta_{Ip} \frac{\eta_{dA}}{\eta_{dp}} \left( \tilde{A}_t + \tilde{A}_t^* \right) \tag{8}
\]

Note that
\[
\eta_{Iy} - \frac{\eta_{Ip}}{\eta_{dp}} = (1 + \tau)^{-\alpha} - (1 + \tau)^{-2\alpha} > 0 \quad \text{for} \quad \tau \geq 0
\]
\[
\frac{\eta_{Ip} \eta_{dA}}{\eta_{dp}} = \frac{1}{2} \frac{\delta}{1 + \delta} (1 + \tau)^{-\alpha}
\]

Using that \(E_t A_{t+j} = A_t \forall j > 0\) and (8) we find
\[
\tilde{A}_t = \left( \eta_{Iy} - \frac{\eta_{Ip}}{\eta_{dp}} \right) \tilde{y}_t + \frac{1 + \delta}{\delta} \frac{\eta_{Ip} \eta_{dA}}{\eta_{dp}} \left( \tilde{A}_t + \tilde{A}_t^* \right) + \tilde{f}_t \tag{9}
\]

Considering the foreign country, we have
\[
\tilde{y}^*_t = \eta_{dA} \left( \tilde{A}_t + \tilde{A}_t^* \right) - \eta_{dp} \bar{p}_t
\]

Notice that the coefficients are not country specific since the approximation is made around a symmetric steady-state equilibrium.

Similarly, we have
\[
\tilde{i}^*_t = \eta_{Iy} \tilde{y}^*_t + \eta_{Ip} \bar{p}_t
\]
\[
= \left( \eta_{Iy} + \frac{\eta_{Ip}}{\eta_{dp}} \right) \tilde{y}^*_t - \frac{\eta_{Ip} \eta_{dA}}{\eta_{dp}} \left( \tilde{A}_t + \tilde{A}_t^* \right)
\]
It follows that
\[
\tilde{A}_t^* = \left( \eta_{I_y} + \frac{\eta_{I_p}}{\eta_{dp}} \right) \tilde{y}_t - \frac{1 + \delta \eta_{I_p} \eta_{dA}}{\eta_{dp}} \left( \tilde{A}_t + \tilde{A}_t^* \right) + f_t^* 
\] 
(10)

From (9) and (10) we get
\[
\tilde{A}_t + \tilde{A}_t^* = \left( \eta_{I_y} - \frac{\eta_{I_p}}{\eta_{dp}} \right) \tilde{y}_k + \left( \eta_{I_y} + \frac{\eta_{I_p}}{\eta_{dp}} \right) \tilde{y}_k^* + f_t + f_t^* 
\]
and therefore
\[
\tilde{A}_t = \eta_{Ay} \tilde{y}_k + \eta_{Ay} \tilde{y}_k^* + \eta_{Af} f_t + \eta_{Af} f_t^* 
\]

where
\[
\eta_{Ay} \equiv \left( \eta_{I_y} - \frac{\eta_{I_p}}{\eta_{dp}} \right) \left( 1 + \frac{1 + \delta \eta_{I_p} \eta_{dA}}{\eta_{dp}} \right) 
= \left( (1 + \tau)^{-\alpha} - (1 + \tau^{-2\alpha}) \right) \left( 1 + \frac{1}{2} (1 + \tau)^{-\alpha} \right) 
\]
\[
\eta_{Ay}^* \equiv \frac{1 + \delta \eta_{I_p} \eta_{dA}}{\eta_{dp}} \left( \eta_{I_y} + \frac{\eta_{I_p}}{\eta_{dp}} \right) = \frac{1}{2} \left( (1 + \tau)^{-\alpha} + (1 + \tau^{-2\alpha}) \right) (1 + \tau)^{-\alpha} 
\]
\[
\eta_{Af} \equiv 1 + \frac{1 + \delta \eta_{I_p} \eta_{dA}}{\eta_{dp}} = 1 + \frac{1}{2} (1 + \tau)^{-\alpha} 
\]
\[
\eta_{Af}^* \equiv \frac{1 + \delta \eta_{I_p} \eta_{dA}}{\eta_{dp}} = \frac{1}{2} (1 + \tau)^{-\alpha} 
\]

Proof Proposition 2:
It follows that
\[
\frac{\partial \tilde{b}}{\partial \tilde{y}_k} = \frac{\delta}{1 + \delta \eta_{Ay}} > 0 \quad \frac{\partial}{\partial \tau} \left( \frac{\partial \tilde{b}}{\partial \tilde{y}_k} \right) = \frac{\partial \eta_{Ay}}{\partial \tau} \geq 0 \quad \text{for} \quad \tau \leq \tau^* 
\]
where \( \tau^* : (1 + \tau^*)^{-\alpha} (1 + \frac{3}{2} (1 + \tau^*)^{-\alpha}) = 1 \)
\[
\frac{\partial \tilde{b}}{\partial \tilde{y}_k^*} = \frac{\delta}{1 + \delta \eta_{Ay}} > 0 \quad \frac{\partial}{\partial \tau} \left( \frac{\partial \tilde{b}}{\partial \tilde{y}_k^*} \right) = \frac{\partial \eta_{Ay}}{\partial \tau^*} < 0 
\]
Proof Proposition 3:
We have that

\[ \text{Var} \left( \tilde{A}_t \mid \psi_{t-1} \right) = \left( \eta_{Ay}^2 + \eta_{Ay} \eta_{Ay^*}^2 + 2 \rho \eta_{Ay} \eta_{Ay^*} \right) \sigma^2 \]

Hence,

\[ \frac{\partial \text{Var} \left( \tilde{A}_t \mid \psi_{t-1} \right)}{\partial \tau} = 2 \left( \eta_{Ay} + \rho \eta_{Ay^*} \right) \frac{\partial \eta_{Ay}}{\partial \tau} + \left( \eta_{Ay^*} + \rho \eta_{Ay} \right) \frac{\partial \eta_{Ay^*}}{\partial \tau} \] \sigma^2

By insertion it is found that

\[ \frac{\partial \text{Var} \left( \tilde{A}_t \mid \psi_{t-1} \right)}{\partial \tau} \bigg|_{\rho=1} = \left( \eta_{Ay^*} - \eta_{Ay} \right) \left( \frac{\partial \eta_{Ay}}{\partial \tau} - \frac{\partial \eta_{Ay^*}}{\partial \tau} \right) \sigma^2 < 0 \]

for \( \tau < \bar{\tau} < \tau^* \)

where \( \bar{\tau} : (1 + \tau)^{-\alpha} (1 + (1 + \tau)^{-\alpha}) = 1 \), and

\[ \frac{\partial \text{Var} \left( \tilde{A}_t \mid \psi_{t-1} \right)}{\partial \tau} \bigg|_{\rho=1} = \left( \eta_{Ay} + \eta_{Ay^*} \right) \left( \frac{\partial \eta_{Ay}}{\partial \tau} + \frac{\partial \eta_{Ay^*}}{\partial \tau} \right) \sigma^2 \]

\[ = \left( \eta_{Ay} + \eta_{Ay^*} \right) \left( -\alpha (1 + \tau)^{-\alpha-1} \right) \sigma^2 < 0 \]

Since

\[ \frac{\partial}{\partial \rho} \left( \frac{\partial \text{Var} \left( \tilde{A}_t \mid \psi_{t-1} \right)}{\partial \tau} \right) \bigg|_{\rho=1} = \left( \frac{\partial \eta_{Ay^*}}{\partial \tau} + \eta_{Ay} \frac{\partial \eta_{Ay^*}}{\partial \tau} \right) \sigma^2 \]

it follows that it is monotone in \( \tau \), and therefore it can be concluded that

\[ \frac{\partial \text{V} \left( \tilde{A}_t \mid \psi_{t-1} \right)}{\partial \tau} < 0 \]

for \( \tau < \bar{\tau} \)

Proof Proposition 4:
From the definition of the coefficient we have

\[ \frac{\partial}{\partial g} \left( \frac{\partial b_{y^*}}{\partial y} \right) = 0 \quad \frac{\partial}{\partial g} \left( \frac{\partial b_{y^*}}{\partial y^*} \right) = 0 \]
Appendix C: State Contingent Public Consumption

With state contingent public consumption we have that the product market equilibrium condition reads

\[ \tilde{y}_t = \eta_{dA} \left( \tilde{A}_t + \tilde{A}_t^* \right) + \eta_{dp} \tilde{P}_t + \tilde{g}_t \]

assuming \( \tilde{g}_t = \kappa \tilde{t}_t \) and using the approximation

\[ \tilde{t}_t = n_{ty} (\tilde{y}_t - \tilde{g}_t) - n_{tp} \tilde{P}_t \]

we get

\[ \tilde{t}_t = \phi \left[ \left( n_{ty} - \frac{n_{tp}}{n_{dp}} \right) \tilde{y}_t + \frac{n_{tp} \eta_{dA}}{n_{dp}} \left( \tilde{A}_t + \tilde{A}_t^* \right) \right] \]

where

\[ \phi \equiv \left[ 1 + \kappa \left( n_{ty} - \frac{n_{tp}}{n_{dp}} \right) \right]^{-1} \] (11)

Using the same procedure as in appendix B, it follows that

\[ \tilde{A}_t = \eta_{A_y}^p \tilde{y}_t + \eta_{A_y}^* \tilde{y}_t^* + \eta_{A_f}^p \tilde{f}_t + \eta_{A_f}^* \tilde{f}_t^* \]

where a p indicates that the parameters are policy dependent and

\[ \eta_{A_y}^p \equiv \phi (1 + \tau)^{-\alpha} \left( 1 - (1 + \tau)^{-\alpha} \right) \left( 1 + \frac{1}{2} \phi (1 + \tau)^{-\alpha} \right) \]

\[ \eta_{A_y}^* \equiv \frac{1}{2} \phi^2 (1 + \tau)^{-2\alpha} \left( 1 + (1 + \tau)^{-\alpha} \right) \]

\[ \eta_{A_f}^p \equiv \frac{1}{2} \phi (1 + \tau)^{-\alpha} \]

\[ \eta_{A_f}^* \equiv \frac{1}{2} \phi (1 + \tau)^{-\alpha} \]

Proof Proposition 5:

We have that

\[ \text{Var} \left( \tilde{b}_t \mid \psi_{t-1} \right) = \left( \frac{\delta}{1 + \delta} \right)^2 \text{Var} \left( \tilde{A}_t \mid I_{t-1} \right) \]

where

\[ \text{Var} \left( \tilde{A}_t \mid \psi_{t-1} \right) = \left( (\eta_{A_y}^p)^2 + (\eta_{A_y}^*)^2 + 2 \rho \eta_{A_y}^p \eta_{A_y}^* \right) \sigma^2 \] (12)
It follows that
\[
\frac{\partial \text{Var} \left( \tilde{A}_t \mid \psi_{t-1} \right)}{\partial \kappa} = 2 \left( (\eta^p_{A_y} + \rho \eta^p_{A_y^*}) \frac{\partial \eta^p_{A_y}}{\partial \kappa} + (\eta^p_{A_y^*} + \rho \eta^p_{A_y}) \frac{\partial \eta^p_{A_y^*}}{\partial \kappa} \right) \sigma^2
\]

where
\[
\frac{\partial \eta^p_{A_y}}{\partial \kappa} = \phi_\kappa \Gamma \left( 1 + \frac{1}{2} \phi (1 + \tau)^{-\alpha} \right) + \frac{1}{2} \phi \Gamma \phi_\kappa (1 + \tau)^{-\alpha} < 0
\]
\[
\frac{\partial \eta^p_{A_y^*}}{\partial \kappa} = \phi_\kappa \phi (1 + \tau)^{-2\alpha} (1 + (1 + \tau)^{-\alpha}) < 0
\]

and
\[
\phi_\kappa = -\frac{\Gamma}{(1 + \kappa \Gamma)^2} < 0
\]

where \( \Gamma \equiv (1 + \tau)^{-\alpha} - (1 + \tau)^{-2\alpha} \).

It follows that
\[
\frac{\partial \text{Var} \left( \tilde{b}_t \mid \psi_{t-1} \right)}{\partial \kappa} < 0 \quad \text{for} \quad \rho > -\min \left\{ \frac{\eta^p_{A_y}}{\eta^p_{A_y^*}}, \frac{\eta^p_{A_y^*}}{\eta^p_{A_y}} \right\} \equiv \rho
\]

Proof Proposition 6:

From (12) we have
\[
\frac{\partial \kappa}{\partial \tau} \text{Var}(\tilde{A}_t \mid \psi_{t-1}) = -\left( \eta^p_{A_y} + \rho \eta^p_{A_y^*} \right) \frac{\partial \eta^p_{A_y}}{\partial \tau} + \left( \eta^p_{A_y^*} + \rho \eta^p_{A_y} \right) \frac{\partial \eta^p_{A_y^*}}{\partial \tau}
\]

where the denominator is negative for \( \rho \geq \rho \), cf. proposition 5.

We have
\[
\frac{\partial \eta^p_{A_y}}{\partial \tau} = \left( \phi_\tau \Gamma + \phi_\tau \right) \left( 1 + \frac{1}{2} \phi (1 + \tau)^{-\alpha} \right)
\]
\[
\frac{1}{2} \phi \Gamma \left( \phi_\tau (1 + \tau)^{-\alpha} - \frac{1}{2} \alpha \phi (1 + \tau)^{-\alpha-1} \right) \geq 0
\]

\[
\frac{\partial \eta^p_{A_y^*}}{\partial \tau} = \phi_\tau (1 + \tau)^{-2\alpha} (1 + (1 + \tau)^{-\alpha})
\]
\[
- \alpha \phi^2 (1 + \tau)^{-2\alpha-1} (1 + (1 + \tau)^{-\alpha}) - \frac{1}{2} \phi^2 \alpha (1 + \tau)^{-3\alpha-1} < 0
\]
Since $\tau = 0$ implies $\Gamma = 0$ and $\phi = 1$, it is easily verified that
\[
\frac{\partial \kappa}{\partial \tau} \left| \text{Var}(\bar{b}_t|\psi_{t-1}) \right| < 0 \quad \text{for } \tau = 0 \text{ and } \rho > \rho
\]
By continuity it follows that there exists a $\tilde{\tau} > 0$ such that\[
\frac{\partial \kappa}{\partial \tau} \left| \text{Var}(\bar{b}_t|\psi_{t-1}) \right| < 0 \quad \text{for } \tau < \tilde{\tau} \text{ and } \rho > \rho
\]

Proof Proposition 7:
Consider next the problem
\[
\min_\kappa \text{Var} \left( \bar{b}_t \mid \psi_{t-1} \right) + \text{Var} \left( \bar{g}_t \mid \psi_{t-1} \right)
\]
It is easily verified that it is optimal to choose $\kappa \neq 0$. Note that for $\kappa > 0$ we have
\[
\text{Var} \left( \bar{b}_t \mid \psi_{t-1} \right) > 0 \quad ; \quad \frac{\partial \text{Var} \left( \bar{b}_t \mid \psi_{t-1} \right)}{\partial \kappa} < 0
\]
and
\[
\text{Var} \left( \bar{g}_t \mid \psi_{t-1} \right) = 0
\]

References


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