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A Simple Explanation of Stock Price Behavior in the Long Run:

Evidence for Denmark *

by

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Abstract

Using Danish data for the post-World War II-period, we estimate a simple model for the long-run behavior of stock prices. We find a stable and strong cointegrating relation between stock prices and two macroeconomic “fundamentals” variables, firm profits and the nominal bond rate. Both “fundamentals” are highly significant. Growth in profits drives the long-run trend in stock prices while the bond rate explains the observed large deviations from trend growth. The behavior of the bond rate accounts for the evident split of the Danish stock market into a bearish period before the early 1980s and a subsequent bullish period. Likewise, a decline in the bond rate explains a major part of the large capital gains realized in recent years.

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1. Introduction

Like most stock markets in the OECD area, the Danish market experienced large capital gains in 1996 and 1997. In total, the increase in stock prices amounted to 84%. These bullish stock markets have stimulated interest in the behavior of stock prices both among academics and practitioners and the interest has, in particular, focused on the question whether stock markets are currently overvalued, see Campbell and Shiller (1998), Cochrane (1997), Kopcke (1997) and Cole *et. al* (1996) for just a few examples.

When examining the recent experience in a historical perspective, it is evident that the stock market developments in 1996 and 1997 are remarkable. However, they are not without precedents, see Figure 1 which shows the Danish All-Share Stock Price Index in the post-World War II-period. The Danish stock market has in three previous episodes experienced capital gains of a similar or even bigger magnitude, that is, in 1972 (89%), 1980-1983 (276%, with a capital gain of 114% in 1983 alone) and 1988-1989 (99%)¹. It is also interesting to note that the increases in 1996 and 1997 follow a period of several years where stock prices have effectively moved in a horizontal direction.

< Figure 1 >

Moreover, 1996 and 1997 may be viewed as merely the most recent part of a long-lasting bull market commencing in the early 1980s. Over the subperiod from 1980 to 1998, stock prices have increased at a trend growth rate of 10.3% per year. This is much faster than over the preceding years 1948-1980 where the trend in stock prices showed a growth rate of 4.0% per year, see Figure 1 where the straight lines indicate the trend growth rates. This evidence suggests that we can divide the post-World War II -period into a long-lasting bear market period before (around) 1980 where capital gains were below average and a subsequent long-lasting bull market period with capital gains above average².

¹ The stock market increases in 1972 and 1983 coincided with major changes in the Danish economy. In 1972, the Danish population decided by a referendum to join the EEC as of Jan 1 1973. In late 1982, a new conservative-liberal government came into office, marking the change to a new economic policy regime of tight fiscal policy, wage moderation and a fixed exchange rate policy. In 1983, a new tax on pension funds' bond returns was passed by the Parliament, favoring pension fund investments in stocks. The tax took effect as of Jan 1 1984.

² Our use of the terms 'bear market' and 'bull market' may differ from other definitions in the literature, but in this paper they are just short-hand expressions for periods with stock price increases below average and above average, respectively.

This picture is confirmed when relating the increase in stock prices to general inflation or the growth in economy-wide nominal income, cf. Figure 2. Real stock prices denote the level of stock prices deflated by the general price level (as measured by the factor price deflator for GDP in the private sector) whereas the growth-adjusted stock prices show the level of stock prices relative to nominal income (GDP in factor prices for the private sector). Stock prices have not been able to keep track with either the general price level or nominal income in the period before (around) 1980, the bear market period, but they have outpaced both the price level and nominal income in the subsequent period, the bull market period. The observation of a long-lasting bull market since the beginning of the 1980s has previously been made by Ritter and Warr (1999) for the US stock market³.

< Figure 2 >

The purpose of this paper is to explain the long-run behavior of the aggregate Danish stock market based on a few macroeconomic “fundamentals” variables. In particular, we want to address the question what can explain the apparent division into bearish and bullish subperiods of the stock market. We also provide some evidence on the question whether the large capital gains in 1996 and 1997 can be motivated by developments in fundamentals variables or whether they represent a non-fundamental innovation (a bubble, a fad or noise trading). We confine ourselves to the analysis of stock price behavior in the long run because this is the horizon where we expect fundamentals variables to have the most predominant effects. We focus on stock prices at the aggregate market level, that is, we formulate a model for the overall market price index of Danish stocks. By focusing on the pricing of the market rather than individual stocks, we reduce the importance of idiosyncratic shocks.

The main finding is that we are able to explain the long run behavior of stock prices by only two “fundamentals” variables, firm profits and the nominal bond rate. We establish a strong and stable cointegrating relation between stock prices and these fundamentals variables. The latter are highly

³ The observation that the Danish stock market can be divided into bearish and bullish subperiods is consistent with the findings in Nielsen and Olesen (1999a). They estimate a regime-switching model for the 1-year nominal stock return and

significant in both statistical and economic terms. We find that the increase in the bond rate until 1982 is a main explanation of the poor stock market performance in the bear market period and, likewise, that the decline in the bond rate following 1982 can account for the subsequent bull market. Furthermore, we conclude that a major part of the capital gains since 1995 can be motivated by the contemporaneous decline in the bond rate. However, the analysis also suggests a substantial degree of over-shooting in stock prices by late 1997. This overpricing of stocks was rectified during 1998.

Besides contributing to an understanding of the empirical issues raised above, this paper provides an empirical and macroeconomic alternative to more traditional valuation models for stocks, that is, we provide an alternative way to determine whether stocks are mispriced or fairly valued. Traditional models typically rely on financial or accounting ratios such as e.g. the price-earnings ratio, the dividend-price ratio or the book-to-market ratio. We take a different approach where we formulate and estimate an explicit empirical model for stock prices. We establish a well-determined and stable historical link between the long-run behavior of stock prices and a set of macroeconomic fundamentals variables. The level of stock prices predicted by this model (the fit) is interpreted as the fundamental level of stock prices. Thus, any deviations between actual stock prices and this prediction is taken as evidence of a mispricing of stocks in the market. Our approach is directed towards identifying an empirically valid structural model for stock prices which makes it possible to *explain* the behavior of stock prices either by contemporaneous fundamentals variables or by referring to a mispricing error. This distinguishes our approach from traditional models. These typically derive from predictability studies which have established that a certain (say) financial ratio has predictive power.

The outline of the paper is as follows. In section 2, we formulate an empirical long-run model for stock prices and take a look at the data. In section 3, we present the results from estimating the model. We use the VAR method of Johansen (1996) and include a check of the robustness of conclusions by using the two-step procedure of Engle and Granger (1987). Section 4 provides an interpretation and application of the results and section 5, finally, concludes with a summary and a discussion.

find a persistent regime shift in the return process in the early 1980s. This regime shift includes an increase in the mean

2. The Empirical Framework

The empirical long-run model for stock prices is based on a very simple theoretical framework. Let us assume that the fundamental value of stocks Q_t^* , defined as the expected discounted value of future profits, is determined as current profits per share divided by the discount rate used by investors to discount future profits:

$$(1) \quad Q_t^* = \frac{\Pi_t / N_t}{\gamma_t}$$

Π_t denotes the firm's total profits, N_t is the nominal value or number of all outstanding stocks in the firm and γ_t is the discount rate. (1) is basically an assumption about how investors value the uncertain stream of profits. As a special case, (1) comprises the textbook Gordon Growth Model for the price of stock with constant dividend growth and a constant discount rate if we assume that all profits are paid out as dividends, see Gordon (1962) or the outline in Campbell *et al.* (1997). In the Gordon case, the relevant discount rate is identical to the growth-adjusted real interest rate, defined as the excess of the nominal bond rate over inflation and real growth (in dividends). (1) can also be motivated as the relevant stock pricing formulae in the case where investors price stocks from a consideration of the price-earnings-ratio of the firm (the ratio of stock price to earnings) including a comparison with a relevant discount rate. This type of valuation is often used in practice.

In order to estimate the model, we need to identify the unobservable discount rate K_t . In this paper, we use the nominal bond rate as the discount rate. Theoretically, this is a controversial assumption because all rational asset pricing theories suggest that a real discount rate is the proper measure. This is for instance reflected in the use of a growth-adjusted real interest rate in the Gordon model. The main reason for this choice is that the nominal bond rate provides a good account of actual stock price behavior as the following results will show. On the other hand, the use of common proxies for a real discount rate fail to establish a cointegrating relation for stock prices, see section 4. The use of a nominal bond rate might also be motivated *apriori*. There are basically two possibilities. First of all, variations in the nominal bond rate may simply serve as a rough proxy for the variations in the 'true' but latent real discount rate used by a rational investor. To the extent that

return.

changes in the nominal bond rate are mainly driven by innovations in the underlying real interest rate, this proxy may do well.

The other possibility is that stock investors suffer from ‘money illusion’. In general, ‘money illusion’ is an irrational behavior where economic agents confuse nominal and real magnitudes or simply care more about the former. Hence, in the case of ‘money illusion’ stock investors may actually use the nominal bond rate as their discount rate even though a rational investor would do otherwise. This possibility has originally been advocated by Modigliani and Cohn (1979) for the US stock market. They argue that stocks were heavily undervalued by the late 1970s because investors irrationally used the high nominal bond rate in pricing stocks, thereby ignoring the fact that the high bond rate reflected a high level of inflation more than a high level of the real interest rate. In a rational world, an inflation-induced increase in the nominal bond rate will have no effect on stock prices because the increase in inflation eventually leads to a higher growth in future profits which compensates for the higher nominal bond rate. The result of ‘money illusion’ is that stock prices include an ‘irrational’ price component which reflects the gap between the nominal and the real interest rates, that is, the level of inflation. According to Modigliani and Cohn (1979), this lead to an irrational underpricing of stocks by the late 1970s. This argument has recently been resumed by Ritter and Warr (1999). They argue that the US bull market since 1982 can in part be explained by the decline in inflation which has reduced the irrational underpricing of stocks originating in money illusion (or ‘inflation illusion’, using their terminology)⁴. Moreover, they find that the level of inflation is a predictor of the undervaluation of stocks and interpret this as evidence that investors suffer from ‘money illusion’.

The possibility of ‘money illusion’ has also received attention in other parts of the academic literature. Recent studies that argue in favor of or incorporate the existence of ‘money illusion’ include Shiller (1999), Shafir *et al.* (1997), Shiller (1997) and Canner *et al.* (1997). Shafir *et al.* (1997) provide evidence from experimental studies suggesting that ‘money illusion’ is to be taken serious as a real world phenomenon⁵.

⁴ Notice that Ritter and Warr (1999)’s argument is not that stock investors have become more rational (less inflicted by ‘money illusion’) but that the source and hence impact of ‘money illusion’ (inflation) has been reduced.

⁵ Anecdotal support to the existence of ‘money illusion’ can be found in recent market studies by two of the largest Danish banks, see DDB (1999) and Valgreen (1999). DDB (1999) evaluates the prices of stocks in a sample of OECD markets by focusing on the so-called ‘earnings-yield-ratio’ which is a standard practical tool for pricing stocks. The

Based on (1), we formulate the empirical model for *actual* stock prices:

$$(2) \quad q_t = \beta_0 + \beta_1 \pi_t + \beta_2 b_t + \beta_3 n_t + u_t$$

(2) is obtained by taking the logarithm of (1) and using small letters to denote corresponding log-values. q_t is the log-value of *actual* stock prices (which we denote by Q_t) while b_t is the log-value of the nominal bond rate. The β_i 's are parameters to be estimated. We allow for a constant term because data for stock prices consists of an indexed series and not observations of stock prices *per se*. Thus, (2) is intended to model the behavior of stock prices over time rather than the actual levels. In the empirical specification, the fundamental value of stock prices is determined by the first part of (2), i.e., $\ln(Q_t^*) = \beta_0 + \beta_1 \pi_t + \beta_2 b_t + \beta_3 n_t$. The coefficients ($\beta_1, \beta_2, \beta_3$) denote partial elasticities of (fundamental) stock prices wrt. profits, the bond rate and the nominal value of outstanding stocks, respectively. According to (1), these should have a unit value in absolute terms but the empirical model allows for elasticities different from unity. u_t is the residual term of the equation and can be interpreted as the degree of mispricing of stocks. Hence, u_t is determined as the log-difference between actual and the fundamental level of stock prices, i.e., $u_t \equiv \ln\left(\frac{Q_t}{Q_t^*}\right)$.

< Figure 3 >

The data for stock prices, profits and the bond rate are shown in Figure 3. All observations are annual and cover the period 1948-1998. For stock prices, we use the official All-Share Stock Price Index by Statistics Denmark, comprising (today) all Danish stocks listed at the Copenhagen Stock Exchange (CSE). This market index captures the aggregate level of stock prices over a variety of firms in different branches. For profits, we rely on National Accounts. We use the measure of gross operating income, defined as value added at factor prices minus the labor costs or equivalently, as sales minus the indirect taxes, costs of material inputs (including raw materials) and labor costs.

latter is defined as the ratio of the earnings-yield (earnings per share as a percentage of the stock price) to the *nominal* bond rate. The use of a nominal bond rate represents 'money illusion' because it ignores the future growth (inflation and real growth) in earnings. Thus, the Gordon model suggests the use of a growth-adjusted real interest rate. Similarly,

This is not an exact measure of the profits or earnings of the firm because it ignores direct taxes, depreciation and interest payments on debt. However, it serves as a rough guide and captures what we believe to be the most important sources of variations in profits, namely, variations in sales and variable costs. For adequacy, we consider the gross operating income of the private sector⁶. For the bond rate, we use the yield-to-maturity on a 10-year government bond, as constructed by Nielsen, Olesen and Risager (1999).

In modeling stock prices, we have to take account of the fact that the nominal value or number of stocks listed at the CSE has increased substantially over time. Due to a lack of data for the entire sample period, we take a short-cut and use a deterministic trend to proxy the growth in the nominal value of outstanding stocks, that is, n_t in equation (2) is replaced by a simple time trend. The estimated coefficient, ϑ_3 , can then be interpreted as an estimate of the actual growth in the nominal value of stocks. Parum (1997) reports data for the total nominal value of Danish stocks listed at the CSE for selected years in our sample⁷. These suggest that the assumption of a trendwise growth is reasonable for a period of a couple of years (which is the relevant horizon as (2) provides a model for the long run, cf. below). Moreover, they suggest an estimate of ϑ_3 in the neighborhood of 8.0% per year which is the average (continuous) growth rate in the nominal value of stocks in the period from 1952 to 1996.

It is evident from Figure 3 that both stock prices and profits exhibit a strong time trend. Furthermore, unit root tests show that all three data series including the bond rate contain a unit root (and only one), that is, the series are integrated of order 1, see Appendix A. This implies that (2) has to be estimated by cointegration methods. If (2) is a valid model empirically, it forms a cointegrating regression defined by the statistical criterion that the residual term is stationary. The economic interpretation is that (2) forms a long-run equilibrium model for stock prices by determining an equilibrium level - the fundamental value - to which stock prices will be 'attracted'

Valgreen (1999) uses a nominal bond rate rather than a real discount rate in modeling stock prices. This is also an example of 'money illusion', see section 5 for details about this study.

⁶ New National Accounts figures, based on ENS95, are only available as of 1988. The data for profits before 1988 are therefore based on the old National Accounts figures (ENS79). A consistent series for profits is constructed by a multiplicative chaining backwards in time as of 1988. Note, that data for the years 1996-1998 are provisional. The data source is the official ADAM database by Statistics Denmark (the April 1997- and August 1999-versions).

⁷ The reported figures in mill DKK nominal value are (the year in parenthesis): 1420 (1952), 2439 (1962), 5869 (1972), 14123 (1982), 43360 (1992), 48764 (1996), cf. Parum (1997, Appendix Tables B1.5-B1.10).

in the long run. That is, in the short run stock prices may deviate from their fundamental value ($u_t \neq 0$) but in the long run any mispricing disappears because stock prices approach their fundamental value (u_t reverts to its unconditional mean of zero), see Engle and Granger (1991) for a discussion of the concept of cointegration.

3. Results⁸

We estimate (2) by the cointegrated Vector Autoregressive (VAR) method of Johansen (1996). This multivariate method has the advantages of allowing for more than one cointegrating relation in the data and of leading to consistent and asymptotically efficient estimates. To briefly sketch the procedure, we assume that stock prices, profits and the bond rate which all are modeled endogenously can be described by a VAR model of lag length k . Written in error-correction-form, this model is:

$$(3) \quad \Delta X_t = \Pi X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu_0 + \varepsilon_t$$

X_t denotes the (3×1) vector having the three endogenous series as elements and ΔX_t contains the corresponding first differences. We allow for an unrestricted constant term in each of the dynamic equations (captured by the vector of constants μ_0) and a deterministic time trend which is restricted to the cointegrating space. The latter restriction is enforced by including the trend in the cointegrating part of the VAR model, that is, as part of the augmented (4×1) vector $X_t^* \equiv (q_t, \pi_t, b_t, t)'$. The trend proxies the nominal value of listed stocks in the cointegrating regression, cf. section 2. By restricting the trend to the cointegrating space, we preclude the possibility of a quadratic trend in the endogenous variables, cf. Johansen (1996). This seems warranted by the data, see Figure 3. M_t is the vector of disturbance terms in the dynamic equations, assumed to fulfill the standard assumptions of being normally and independently distributed with mean zero and constant variance. The parameters to be estimated are $(\Pi, \Gamma_1, \dots, \Gamma_{k-1}, \mu_0)$.

⁸ The estimations are performed in PCFIML, see Doornik and Hendry (1997).

Our interest in (3) is attached to the first term on the right hand side which captures the cointegration features of the data. The number of cointegrating relations is determined by the rank of the (3×4) matrix of parameters, Π . If Π has zero rank, there will exist no cointegrating relation between the endogenous series (including the trend). If Π has a non-zero but reduced rank r ($0 < r < 3$), we will have r cointegrating relations⁹. In this case, the Π -matrix can be written as the product of two full column rank matrices I and ϑ^* with dimensions $3 \times r$ and $4 \times r$, respectively; $\Pi = I(\vartheta^*)'$. Each column vector of the ϑ^* -matrix is a cointegrating vector and contains the coefficients to stock prices, profits, the bond rate and the time trend in a cointegrating relation. The I -matrix contains the loadings by which the equilibrium errors from the cointegrating relations, i.e., the deviations from long-run equilibrium $(\vartheta^*)' X_{t-1}$, affect the short-run dynamics of the endogenous variables. Assuming that only one cointegrating relation exists ($r=1$), the ϑ^* -vector can in terms of (2) be formulated as the vector $(1, -\vartheta_1, -\vartheta_2, -\vartheta_3)$ where we have normalized on stock prices (the constant term ϑ_0 is estimated as part of the unrestricted constant terms in the μ_0 -vector). Both I and ϑ^* are estimated in the Johansen (1996) method.

The VAR model specification we use has a lag length of 3 ($k=3$) and includes an impulse dummy for the year 1960¹⁰. This specification is chosen by a general-to-specific procedure where we initially estimate a 'general' VAR model with a large number of lags (5 lags). In this 'general' version, specification tests indicate a significant heteroskedasticity (ARCH) in the residual term of the equation for the bond rate. This arises from an abrupt pattern in the bond rate in the period from 1957 to 1960 which ends with a sharp increase in 1960, cf. Figure 3. To eliminate ARCH, we include the dummy for 1960. The proper lag length is, subsequently, chosen by formal testing where we successively test for the significance of all terms at the highest lag order and eliminate any insignificant lags. Using conventional significance levels, we end up with the 'specific' VAR model with 3 lags¹¹.

⁹ In the case where Π has a full rank of 3, there will be three cointegrating relations. This is only possible if all the endogenous series are stationary. However, this is ruled out by the prior unit root testing, cf. above.

¹⁰ The impulse dummy has a value of one in 1960 and is zero otherwise. In terms of (3), the dummy gives rise to an additional (unrestricted) term $+\mu_{60}d60_t$, where $d60_t$ is the scalar dummy and μ_{60} denotes the (3×1) vector of coefficients to the dummy. The term enters in the same manner as the μ_0 -vector of constants.

¹¹ The simultaneous F-test of the null that all terms at lag 3 can be excluded from the VAR model with 3 lags leads to a clear rejection of the null at conventional significance levels (the critical significance level is 1.5%). Similarly, the null that the 'general' VAR model can be reduced from 4 to 3 lags is accepted as the critical significance level is 13.6%. The

< Table 1 >

< Figure 4 >

Specification tests and diagnostic graphics for the VAR model are shown in Table 1 and Figure 4. The specification tests test whether the residuals of the VAR model fulfill the requirements of being homoskedastic, serially uncorrelated and normally distributed. The model passes all tests for serial correlation and heteroskedasticity (including ARCH) at the standard 5% significance level and with the exception of the test for first order serial correlation in the equation for profits also at the 10% level. The univariate test for normally distributed residuals in the equation for stock prices leads to a clear rejection¹². However, for inference purposes the normality assumption can be relaxed in favor of the weaker assumption that the residuals are identically distributed over time, cf. Johansen (1996, Part II). Hence, the rejection is not critical. We conclude that the VAR model is well specified.

< Table 2 >

Table 2 reports the results from the cointegration analysis. We report two versions of the trace test on the cointegrating rank, the asymptotic version used in Johansen (1996) and the small-sample approximation suggested by Reimers (1992). Because our sample is relatively small, we rely on the latter version. We also report two sets of 95% critical test values, the standard values tabulated in Johansen (1996) and a set of critical values which have been simulated using the simulation program DisCo, cf. Johansen and Nielsen (1993), to take account of the inclusion of the impulse dummy for 1960. Using the small-sample trace test and the simulated critical values, we find strong evidence of cointegration. Hence, the null of no cointegration ($r=0$) is strongly rejected in favor of the alternative hypothesis that the rank is at least one. The critical significance level of the test is close to 1%. Testing the null that there is only one cointegrating relation, we accept the null by a clear margin using standard significance levels (the critical significance level is approximately 15%). The conclusion therefore is that there is one and only one cointegrating relation in the data.

use of information criteria in choosing the lag length gives ambiguous results: the Schwarz criterion supports the use of 1 lag, the Hannan-Quinn criterion 2 lags and the Akaike criterion 4 lags.

< Table 3 >

The estimates of the long-run parameters I and ϑ^* are shown in entry (I) of Table 3. Because we interpret the relation as an equilibrium relation for stock prices, we have normalized on this variable. The table also reports indicative standard errors of the parameter estimates (in parentheses) and for each parameter, the critical significance level of a Likelihood Ratio (LR) test of the null that the given parameter is zero (squared brackets). For instance, the LR test of the null that profits are weakly exogenous for the long-run parameters ($I_2=0$) has a critical significance level of 64.9%.

All ϑ^* -coefficients are highly significant. Among the I -loadings, only the loading in the direction of the bond rate (I_3) is significant when using conventional significance levels. In particular, the loading in the direction of stock prices (I_1) appears to be insignificant which suggests that a deviation from long-run equilibrium has no effect on stock prices. These results may actually be interpreted as an indication that the cointegrating relation identifies an equilibrium for the bond rate rather than stock prices. However, this would imply that the bond rate could be explained by the levels of stock prices, profits and a deterministic time trend which seems to lack any economic-theoretical or -intuitive foundation¹³. This is in particular true in the Danish case where the stock market has been small compared to the bond market historically and where the bond rate, furthermore, has been determined mainly by the foreign bond rate since the capital account liberalization in 1982. Moreover, it turns out that the lack of a significant error-correction in the direction of stock prices may be attributed to the small sample size. Thus, in the general version of the VAR model in (3) we have included several dynamic terms (lagged terms of ΔX_t) which turn out to be insignificant empirically. Excluding these terms, we end up with a more parsimonious VAR model where I_1 is significant at standard significance levels¹⁴. We therefore adhere to our interpretation that the cointegrating relation determines an equilibrium for stock prices.

¹² The rejection of normality is a result of right skewness which is driven mainly by the large capital gains in 1972 and 1983.

¹³ The implied relation for the bond rate would be (ignoring the constant term):

$$b_t = 2.15\pi_t - 1.43q_t - 0.076t$$

¹⁴ Estimating the VAR model by Full Information Maximum Likelihood and using consecutive LR tests to exclude individual terms in the stock price equation, we end up with a well specified and (more) parsimonious VAR model where the stock price equation only includes a constant, the second lag of $\Delta\pi_t$ and the equilibrium-error from the cointegrating relation. Performing a LR test of the null that I_1 is zero, we get a test statistic of 4.9, corresponding to a critical significance level of 3%. This leads to a rejection of the null at standard significance levels. The result can be

Entries (II) and (III) show the estimates of the (restricted) long-run parameters under two hypotheses on the I-loadings. In (II), we have assumed that profits are weakly exogenous for the long-run parameters, that is, that an equilibrium error has no effect on profits ($I_2=0$). This hypothesis is validated by the data as the relevant LR test on I_2 in the unrestricted specification (I) has a critical significance level of 64.9%. In (III), we have assumed that both profits and the bond rate are weakly exogenous ($I_2=I_3=0$). This hypothesis is rejected by the data as a LR test of the null that $I_2=I_3=0$ in (I) leads to a test statistic of 8.6 and a corresponding critical significance level of 1.4%.

By comparing entries (I) through (III), we find the ensuring result that the estimates of the coefficients in the cointegrating relation (the ϑ^* -vector) are rather robust to the assumptions on the I-loadings. We shall use the parsimonious specification in (II)¹⁵, leading to the following long-run equilibrium relation for stock prices (indicative standard errors in parentheses)¹⁶:

$$(4) \quad q_t = -15.1 + \underset{(0.17)}{1.55} \pi_t - \underset{(0.058)}{0.72} b_t - \underset{(0.015)}{0.057} t$$

All coefficients are strongly significant and have the expected signs. The point estimates of the coefficients of profits and the bond rate are significantly different from unity (which is the value expected from the theoretical framework of section 2)¹⁷ but the differences are not substantial in economic terms. According to the point estimates, a permanent increase in the level of profits by 1% (continuous rate) will *ceteris paribus* lead to a 1.55% increase in stock prices in the long-run, that is, when prices have adjusted to their new equilibrium or fundamental value. This somewhat stronger response than in the unit-elasticity case could be interpreted as an ‘overreaction’ in stock prices

confirmed by single-equation estimation of the dynamic stock price equation. Furthermore, using the two-step procedure of Engle and Granger (1987), we find that the (corresponding) error-correction term has a very significant effect on the short-run dynamics of stock prices, cf. below.

¹⁵ (II) is preferred to (I) because it is (more) parsimonious and hence leads to a more efficient inference. Moreover, recursive estimation suggests that the ϑ^* -vector of (II) is more stable over the sample period than that of (I). Recursive estimates for (II) are shown below.

¹⁶ The constant term is calculated to ensure that the deviations between actual stock prices and the model’s fitted stock prices have a sample mean of zero.

¹⁷ This can be shown by formal LR testing (not reported). Indicative 95% confidence intervals for the coefficients, constructed on the basis of the indicative standard errors, suggest that estimation uncertainty can sustain a coefficient to profits of 1.22 and a coefficient to the bond rate of -0.83 as the respective lower limits of the confidence intervals.

even in the long run, but could also merely reflect that the ‘true’ measure of profits of the firms considered is more volatile than the profit proxy we have used.

A permanent 1% *relative* increase in the bond rate (e.g. from 10.0% to 10.1%) will in the long run lead to a decline in the level of stock prices by 0.72%. This ‘underreaction’ compared to the unit-elasticity-case is consistent with other findings for the Danish stock market, see Nielsen and Olesen (1999b) and Olesen and Risager (1999). Nielsen and Olesen (1999b) find that the dividend-price ratio of stocks (the ratio of dividends to stock prices) responds less to innovations in the bond rate than one should expect in the unit-elasticity (Gordon Growth Model) case. Olesen and Risager (1999) find that stock returns tend to respond less than one-to-one to changes in bond yields at the 5- and 10-year investment horizons. A one-to-one relationship would be the outcome under a simple no-arbitrage relation, saying that the expected return on stocks equals the expected return on bonds plus a risk premium. Actually, our estimate of the bond rate elasticity comes close to the estimated long-run relationship between the 10-year stock return and the 10-year bond yield in Olesen and Risager (1999)¹⁸. Possible explanations of this ‘underreaction’ in stock prices are that investors may expect innovations in the bond rate to be partially transitory, cf. Nielsen and Olesen (1999b), or that bond returns are taxed at a higher rate than stock returns, cf. Olesen and Risager (1999).

The time trend is highly significant. This shows the importance of taking account of the growth in the nominal value of outstanding stocks. The estimated coefficient suggests that aggregate stock prices on average and in the long run tend to decline by a (continuous) rate of 5.7% per year due to new issuances that increase the number of stocks over which profits have to be distributed. The point estimate of 5.7% comes close to the average growth rate of 8.0% in the actual number of outstanding stocks on the CSE, see section 2. Performing a statistical test, we find that estimation uncertainty can account for the difference¹⁹.

Finally, we note from Table 3 that the sign and magnitude of the estimated loading in the direction of stock prices ($I_1 = -0.287$) is consistent with the interpretation that the cointegrating relation determines a long-run equilibrium (fundamental) level of stock prices. Thus, if stocks are currently

¹⁸ Olesen and Risager (1999) estimate a bond yield coefficient of 0.71 in a cointegrating relation for the 10-year stock return.

(say) overpriced, that is, if stock prices currently are above their fundamental value ($Q_t > Q_t^*$), prices will tend to decline in the next period by a fraction of the amount of mispricing. The estimate of I_3 suggests a simultaneous adjustment in the bond rate. Hence, the bond rate tends to decline whenever stocks are overpriced, and *vice versa*. This reinforces the adjustment to long-run equilibrium because a decline in the bond rate leads to an increase in the fundamental value of stocks, and *vice versa*.

< Figure 5 >

< Figure 6 >

We have performed two types of tests of the robustness of the estimated long-run model. First of all, we have tested whether results are stable over time by performing a recursive estimation of the key parameters. Figure 5 shows the recursively estimated eigenvalues of the (unrestricted) Π -matrix. All three eigenvalues seem fairly stable. This suggests that the conclusion that there is one and only one cointegrating relation in the data is robust over time. Figure 6 shows the recursive estimates of the ϑ^* -vector, having normalized on stock prices and assumed that profits are exogenous ($I_2=0$). The result is that the coefficient estimates are reasonably stable over time. Finally, it can be shown by recursive testing (not reported) that restricting profits to be exogenous is a valid hypothesis over the entire sample. We therefore conclude that results are stable over time.

Secondly, we have checked (indirectly) whether results are affected by the relatively small sample size of our study. Evidence suggests that the cointegrated VAR method of Johansen (1996) or more precisely, the embedded tests for cointegration and tests of hypotheses on the long-run coefficients, may suffer from a poor small sample performance (size distortions and low power of tests), see e.g. Gonzalo and Lee (1998), Juselius (1999) and Johansen (1999). To get an indication of whether results have been distorted by the small sample size, we have re-estimated the long-run model for stock prices using an alternative estimation method, that is, the single-equation two-step procedure of Engle and Granger (1987) (EG2) combined with methods suggested by Phillips and Loretan (1991) for testing hypotheses on the cointegrating vector. According to the Monte Carlo simulations

¹⁹ A LR test of the null that the (true) coefficient of the trend is 8.0% ($\vartheta_4^*=0.08$), gives a test statistic of 1.8. The critical significance level is 18.3%.

in Gonzalo and Lee (1998), the EG2 procedure appears more robust than the Johansen (1996) method when testing for cointegration in small samples.

< Table 4 >

Table 4 reports results of the alternative estimation. In entry a, we have listed the candidate cointegrating vector estimated in the first step of the EG2 procedure. This is obtained by regressing (OLS) stock prices on a constant term, profits, the bond rate and a time trend. Notice that the estimated coefficients come close to those estimated by the Johansen (1996) method. In the second step of the EG2 procedure, we test whether this regression represents a cointegrating relation by testing whether the residuals are stationary. Performing an augmented Dickey-Fuller test (we have included two significant lags of the first differences of the error-correction term which results in a well specified augmented Dickey-Fuller regression) of the null that the residuals are non-stationary, that is, that we do not have cointegration, we get a test statistic of -5.7. The critical small-sample test value at the 1% significance level is -5.1, cf. MacKinnon (1991). Hence, we clearly reject the null and conclude that there is strong evidence in favor of cointegration. Estimating an error-correction model, we can furthermore show that there is a highly significant error-correction in the direction of stock prices²⁰.

Having established cointegration, we can perform tests on the cointegrating vector. Because of a slow adjustment in stock prices or the regressors (profits and the bond rate) being endogenous, current, past or future innovations in the regressors may be correlated with the residual of the cointegrating relation. This leads to non-standard test distributions, cf. Hamilton (1994, Section 19.3). As a consequence, hypothesis tests on the cointegrating vector have to be conducted within an augmented regression where the tests have well-known distributions. In Table 4, we report the outcomes of estimating two types of augmented regressions, the Phillips and Loretan (1991) OLS (POLS) regression where the static cointegrating regression is augmented by current, lagged and leaded first differences of the regressors (results shown in entries b through d for three alternative lag- and lead-lengths) and the Phillips and Loretan (1991) Non-Linear Least Squares (PLNLS)

²⁰ The t-test statistic on the coefficient to the lagged error-correction term in a well specified parsimonious error-correction model for stock prices is -4.2, implying that the error-correction term is significant at any conventional

regression which also includes lagged error-correction terms (entries e through g for three alternative lag- and lead-lengths)²¹. In the latter case, the appropriate number of lags of the error-correction term is determined by a specific-to-general procedure, whereby lags are included until the residuals of the augmented regression fulfill the white noise requirements.

The results in Table 4 show that the augmented point estimates of the cointegrating vector are sensitive to the specification chosen. However, in most cases the differences are of a magnitude consistent with estimation uncertainty (which occasionally is large). Comparing with the estimates obtained by the Johansen (1996) method, we find that except for the large bond rate elasticities obtained in two cases (for $N_1=N_2=1$ for both the PLOLS and the PLNLS regression) the augmented coefficient estimates come reasonably close. In particular, the deviations usually (with the two exceptions mentioned) fall within one standard error of the augmented coefficient estimators. We conclude that the evidence provided by the single-equation analysis seems to support the earlier results.

4. Interpretation and Applications

The fit of the model in (4) is shown in Figure 7. The fitted value is interpreted as the estimated fundamental value of stock prices, defined as the equilibrium level which stock prices will approach in the long-run. The deviations between actual stock prices and their long-run levels show the degree of mispricing. These long-run residuals or mispricing errors are plotted in Figure 8 together with a correlogram and a density plot.

< Figure 7 >

< Figure 8 >

We see that the long-run model tracks actual stock prices remarkably well over the entire sample. This is reflected in the fact that any mispricing error disappears, that is, is eliminated or replaced by an error of the opposite sign within a few years. The observed maximum duration of a mispricing is seven years (corresponding to the underpricing of stocks over the period 1974-1980) but on average

significance levels. Results are very similar to those reported in Appendix B where we estimate an error-correction model for stock prices, based on the cointegrating relation obtained by the Johansen (1996) method.

²¹ See Phillips and Loretan (1991) or Hamilton (1994, Section 19.3) for an outline of the two methods.

an episode of mispricing lasts only 3 (2.7) years. This low persistence of the mispricing error which is also confirmed by its low degree of serial correlation reflects the strong cointegration in (4).

Even though a period of mispricing is usually short, the magnitude of a mispricing can be substantial. The largest overpricing error in the sample is 0.40 for the year 1983, corresponding to an overpricing of stocks by 49% relative to their fundamental value. The largest magnitude of an underpricing occurred in 1979 with a 27% undervaluation of stocks relative to fundamental value (a mispricing error of -0.32). As is evident from the density plot in Figure 8, the sample distribution of the mispricing error comes close to the normal distribution. In fact, according to the test of Doornik and Hansen (1994) we can not reject the hypothesis that the error distribution is normal²². The sample standard deviation of the mispricing error is 0.160 which implies that 95% of the errors in the unconditional distribution fall in the range between -0.31 and +0.31, i.e., in the range between an underpricing of stocks by 27% and an overpricing of stocks by 37% in relative terms.

< Figure 9 >

The contributions by the individual determinants of stock prices are illustrated in Figure 9. This figure shows actual stock prices, the fundamental level of stock prices implied by (4) and the contributions by the bond rate (the bond rate term in (4)) and ‘profits per share’ (the sum of the profits and the trend terms in (4)), respectively. The latter component is constructed as the sum of the contributions by total profits and the trend (the nominal value of outstanding stocks) and is, tentatively, interpreted as a proxy for the profits per share. In order to use a common scale, all series are normalized as deviations from their respective sample means.

As we might have expected, the strong trend in stock prices is (primarily) the result of the trend in profits per share. From 1951 to 1998 total profits have on average increased by 7.7% per year (continuous growth rate). Adjusting for the growth in the nominal value of outstanding stocks (5.7% per year), the trend in profits per share has according to (4) contributed by an annual growth in stock

²² The Doornik and Hansen (1994) test of the null that the mispricing error is normally distributed gives a test statistic of 1.1 which has to be compared to the $\chi^2(2)$ -distribution. The critical significance level is 57%, leading to an acceptance of the null at any conventional significance levels.

prices of 6.2% per year. For comparison, actual stock prices have increased by 7.4% per year over the sample.

Profits (per share) dominates the bond rate in modeling the long-run trend in stock prices. However, the bond rate is important in explaining the observed variations in stock prices around their long-run trend. In particular, the bond rate is the crucial variable in explaining the evident split of the Danish stock market into bearish and bullish subperiods. Thus, reflecting an almost continuous increase, the bond rate rose from 6.6% in 1951 to 21.4% in 1982. This represents more than a trebling or a relative increase by 3.8% per year (continuous growth rate). The rise in the bond rate increased the discount rate by which future profits were discounted and, hence, dampened the growth in stock prices. According to (4), the bond rate can account for a decline in the fundamental value of stocks by 2.7% per year over the period from 1951 to 1982. The latter roughly mirrors the difference between the average capital gain over the bearish period (4.0% per year) and the average capital gain over the entire sample (7.4% per year). Likewise, the trend in the bond rate following 1982 explains the subsequent bullish stock market. From its high in 1982, the bond rate declined to 4.2% by 1998, corresponding to only one fifth of the 1982-level or a relative decrease by 10.1% per year. According to (4), this enhanced the growth in stock prices by 7.3% per year. The latter is significantly larger than the difference between the average capital gain over the bullish period (10.1% per year) and the average capital gain over the entire sample (7.4% per year) which reflects the fact that growth in profits was below average in the bullish period, cf. Figure 9. In explaining the bear and bull market characteristics of the stock market, it is interesting to note that the behavior of profits actually works in the opposite direction. Hence, the fastest (above average) growth in profits occurred in the period from the late 1950s to the early 1980s (the bearish period) whereas growth in profits was relatively slow in the following period (the bullish period). The changes in the bond rate more than compensates for this behavior of profits²³.

The explanatory power of the bond rate is evident when examining the behavior of the growth-adjusted stock prices (stock prices relative to nominal income). These do not exhibit the same strong

²³ Even though an exact dating is difficult, the turn in the stock market seems to occur already in 1980, that is, two years before the decline in the bond rate. This earlier turn-around could, in principle, be explained by more temporary features such as the fact that stock prices according to (4) were significantly below their fundamental value in 1980. This underpricing disappeared from 1980 to 1982. Moreover, profits increased significantly in 1981 and 1982, leading to an increase in fundamental stock prices.

time trend as the level of stock prices. By re-formulating (4), we can immediately derive a model for growth-adjusted stock prices. Thus, subtracting (log-to) nominal income on both sides of (4), using the fact that profits can be written as the product of nominal income and the profit share (by the identity defining the profit share) and re-arranging terms, we get the following (implied) long-run model:

$$(5) \quad qy_t = -15.1 + 1.55\pi y_t - 0.72b_t + 0.55y_t - 0.057t$$

qy_t denotes growth-adjusted stock prices, y_t is the measure of nominal income and πy_t is the fraction of profits out of total income, all in logs. According to (5), the growth-adjusted stock prices are driven by four components. In economic terms, the most interesting are the first two, the profit share and the bond rate. Growth-adjusted stock prices will *ceteris paribus* increase, that is, stock prices will outpace growth in nominal income, whenever we see an increase in the profit share. This is because an increase in the profit share means that a larger fraction of total income in the economy accrues to stock holders. Because of the usual discount rate effect, an increase in the bond rate leads to a decline in the growth-adjusted stock prices, implying that stock prices will grow by less than nominal income. The trend term captures as before the growth in the nominal value of outstanding stocks. Finally, (5) includes an income term which reflects the result that innovations in profits and, hence, broad income y_t have a more than proportional effect on the level of stock prices. An increase in nominal income will therefore lead to an increase in the growth-adjusted stock prices²⁴.

< Figure 10 >

Panel a in Figure 10 shows the fit implied by (5). In calculating the growth-adjusted stock prices, we have used private sector GDP in current factor prices as the income measure. This corresponds to our measure of profits. Accordingly, the profit share is calculated as the ratio of gross operating income to GDP in factor prices, both for the private sector. This figure is basically a ‘mirror image’ of Figure 7 as the residuals are identical across the two figures. Panel b illustrates the contributions by the individual components of (5), showing the trend and the income terms together as a single

²⁴ Re-formulating (2), the implied model for growth-adjusted stock prices is in general (ignoring the error term):

$$(5') \quad qy_t = \beta_0 + \beta_1\pi y_t + \beta_2b_t + \beta_3n_t + (\beta_1 - 1)y_t$$

‘residual component’. In Panel c, we illustrate the explanatory power of the bond rate, having omitted the other components for expositional reasons. We observe that the by far most important component in explaining the long-run behavior of growth-adjusted stock prices is the bond rate. In particular, the bond rate almost by itself explains the continuous decline in growth-adjusted stock prices until the beginning of the 1980s (the bear market) and the subsequent continuous increase (the bull market), confirming the earlier insight obtained from the stock price model. The profit share has some explanatory power. Our measure of the profit share shows an almost steady declining trend from 1951 where the profit share was 49% until (around) 1980 where it reached a low of 38%. According to (5), this shift in the profit share can explain a decline in (growth-adjusted) stock prices by 1.4% per year (continuous rate) over the same period. Following a period of a steady increasing trend, the profit share was almost restored by 1998 (45%). This reversal increased the (growth-adjusted) stock prices by an equivalent 1.4% per year in the period from 1980 to 1998. Hence, the behavior of the profit share is consistent with the bear- and bull-market pattern of the stock market. However, the variations in the profit share have been rather modest and the explanatory power is therefore much lower than for the bond rate. Finally, the ‘residual component’ has some significance towards the beginning and towards the end of the sample, reflecting the fact that income growth deviated from its mean in these two periods. However, in most of the sample this component is fairly constant and can, hence, be ignored in explaining the behavior of growth-adjusted stock prices.

Our model also contributes to an understanding of the surge in the stock market in recent years. From late 1995 to late 1998, average capital gains amount to 18.5% per year (continuous rate), mainly reflecting the large capital gains realized in 1996 and 1997. The contemporaneous decline in the bond rate can to a large extent account for these stock price increases. Thus, by late 1995 stock prices were roughly in line with their fundamental value as implied by (4) (the mispricing error for 1995 is -0.037, indicating a slight underpricing of stocks by 4% in relative terms). The bond rate has fallen gradually from a level of 8.3% by late 1995 to 4.2% by late 1998 which according to (4) has increased fundamental stock prices by 16.0% per year over the same period, see also Figure 9. At the same time, profits per share have roughly been constant. Consequently, actual stock prices have only increased by slightly more than fundamental stock prices from 1995 to 1998. This suggests that

In the unit-elasticity case $\vartheta_1=1$, the income term disappears.

the recent surge in the stock market was to a large extent warranted by fundamental factors, that is, by the decline in the bond rate. We can also conclude that stock prices were close to their fundamental level by late 1998. According to (4), stocks were overpriced by 7.1% (the mispricing error for 1998 is 0.069). Given the usual magnitudes of a mispricing, we interpret this as evidence that stocks were roughly fairly valued. However, it is also interesting to note that the adjustment in stock prices differs significantly from that in their fundamental level, cf. Figure 7. Prices show a substantial overreaction in 1996 and 1997 with stocks being overpriced by 37% relative to fundamentals by late 1997. This mispricing was eliminated during 1998 as a result of a capital loss on stocks and the continued decline in the bond rate which led to a further increase in the fundamental level of stock prices.

< Figure 11 >

We have modeled the discount rate of investors by the nominal bond rate rather than a real discount rate measure. The reason is that the bond rate leads to a strong cointegrating relation for stock prices. As possible alternatives, we have experimented with a comprehensive set of common proxies for a real discount rate. However, none of these resulted in a cointegrating relation. To highlight this, we provide results for two possible real discount rate proxies and these are the proxies that came closest to sustaining cointegration. Both proxy the expected growth-adjusted real interest rate used in the Gordon Growth Model, see section 2. We take account of real growth and inflation by adjusting for the growth in nominal profits. Figure 11.a illustrates the proxies. The *ex post* real rate is a perfect foresight proxy for the growth-adjusted real interest rate. That is, the *ex post* real rate for e.g. year 1972 is constructed as the difference between the 10-year nominal bond rate in 1972 and the realized rate of growth in profits in the 10-year period from 1972 to 1982. This assumes that investors on average forecast growth in profits correctly. Because of the forward-looking nature of the proxy, data is only available until 1988. The *ex ante* real rate is backward-looking as it uses the historical growth in profits as a proxy for the investors' expectations about future profit growth. Thus, the *ex ante* real rate in (say) year 1972 is constructed as the difference between the 10-year nominal bond rate in 1972 and the average growth rate in profits in the historical 5-year period from 1967 to 1972. This proxy is available from 1953. In terms of long-run behavior, it is evident from Figure 11.a that the *ex ante* and the *ex post* real rates show a

resemblance to the nominal bond rate. In particular, the hike in interest rates in the late 1970s and the early 1980s is common to all three series. However, there are also differences. The most important difference is that the real discount rate proxies fail to track the increase in the nominal bond rate during the first two decades of the sample. From 1951 to the early 1970s the bond rate is roughly doubled which is a main contribution in explaining the large fall in the growth-adjusted stock prices over this period, see Figure 10.c. In comparison, the levels of the real rate proxies are stable or declining in the same period. We can also note that the real rates and, in particular, the *ex ante* real rate decline very rapidly from the high levels in the early 1980s.

Using the two-step procedure of Engle and Granger (1987), we have estimated a cointegrating relation for stock prices based on these alternative discount rate measures. The residuals obtained from regressing (OLS) stock prices on a constant term, profits, a deterministic trend and each proxy in turn (the first step of the procedure) are shown in Figures 11.b and 11.c²⁵. These also include the residuals from the cointegrating relation where the nominal bond rate is used as the discount rate measure. This is the relation outlined in the single-equation analysis of section 3, see Table 4 (entry a). For both real rates, we find that the residuals are very persistent, that is, there is a tendency for the residuals to be systematically negative or systematically positive several years in a row. In particular, the residuals are more persistent than when using the nominal bond rate. It is mainly in the bearish period until the early 1980s that the real rate relations suffer. Thus, from the mid-1960s to the early 1980s, actual stock prices are systematically overestimated when using the real rates. The bearish period includes the first two decades of the sample where the behavior of the real rates differs most evidently from that of the nominal bond rate. The residuals from the real rate relations also tend to be larger than the residuals from the bond rate relation. The residual has a standard error of 19.8% and 27.2%, respectively, for the *ex post* and *ex ante* real rate relations. The standard error is 16.2% for the relation based on the nominal bond rate.

²⁵ Because the real rate proxies are negative in some of the years, the regressions include the proxies in levels rather than log-levels. Notice that in the Gordon model the real discount rate is not allowed to be negative because this would imply that the investor's willingness to pay for stocks is infinitely large. However, in the empirical model we include a constant term, cf. (2), so what matters are the variations in the discount rate and not the levels.

We can test whether the regressions using the real rates form cointegrating relations by testing whether the residuals are stationary (the second step of the Engle and Granger (1987) procedure)²⁶. For the *ex ante* real rate, we get a Dickey-Fuller t-test statistic of -2.6 for the null hypothesis that the residuals are non-stationary. The critical test value at the 10% significance level is -4.0, cf. MacKinnon (1991). Hence, we accept the null by a clear margin and conclude that cointegration is strongly rejected. Likewise, for the *ex post* real rate, the Dickey-Fuller t-test statistic is -2.8 which should be compared to a critical test value of -4.1 at the 10% level. Again, we strongly reject that the regression forms a cointegrating relation. Finally, recall from section 3, that we find strong evidence of cointegration in the Engle and Granger (1987) analysis when using the nominal bond rate as the discount rate measure. From an empirical point of view, it therefore appears that the nominal bond rate performs significantly better than common proxies for a real discount rate.

We have focused exclusively on modeling the long-run behavior of stock prices. However, the model may also be used in explaining and possibly predicting short-run (1-year) movements in stock prices by formulating an error-correction model (ECM) which includes the adjustment to long-run equilibrium as part of the short-run dynamics. In Appendix B, we have estimated such an ECM, using a single-equation specification. The general insights are that the ECM has power in explaining short-run capital gains and that the ECM term, capturing the adjustment to the long-run fundamental level of stock prices, is an important explanatory variable. The most significant explanatory variable is (the change in) the contemporaneous bond rate.

²⁶ Unit root tests show that the *ex ante* and the *ex post* real rates are integrated of order 1 (tests not reported). For the *ex ante* real rate (denoted by K_t^A), the cointegrating regression is (indicative OLS standard errors in parentheses):

$$q_t = \underset{(5.2)}{0.41} + \underset{(0.40)}{0.27} \pi_t - \underset{(0.013)}{0.048} \gamma_t^A + \underset{(0.04)}{0.056} t + \hat{u}_t \quad (1953 - 1998)$$

For the *ex post* real rate (K_t^P), the regression is:

$$q_t = \underset{(5.3)}{-27.7} + \underset{(0.40)}{2.41} \pi_t - \underset{(0.013)}{0.065} \gamma_t^P - \underset{(0.03)}{0.135} t + \hat{u}_t \quad (1951 - 1988)$$

\hat{u}_t is the residual term. The test for cointegration in the second step of the Engle and Granger (1987) procedure is in both cases conducted within a simple Dickey-Fuller regression without augmenting terms. This lag structure is determined by a 'general-to-specific'-procedure where we initially estimate an augmented Dickey-Fuller regression with

5. Conclusion

We have formulated and estimated an empirical model which performs well in explaining the long-run behavior of stock prices. The model is simple as it only includes two economic explanatory variables, profits and the nominal bond rate. Both are highly significant in statistical and economic terms. Profits is the crucial variable when modeling the strong long-run trend in stock prices whereas the bond rate is important in explaining the large variations in stock prices around this trend, as reflected in the behavior of growth-adjusted stock prices. Despite its simplicity, the model gives rise to a strong cointegration as any mispricing (deviation between actual and fitted stock prices) disappears rather fast. All model coefficients are highly significant, have the right signs and plausible magnitudes and are stable over time. Furthermore, results appear to be robust to the use of a small sample.

We interpret the model as a model for the fundamental level of stock prices, defined as the equilibrium level which stock prices will revert to in the long run, based on the current levels of profits and the bond rate. Our definition of fundamental stock prices is empirically motivated as the point of reference is the way in which stocks have been priced over a historical sample. Hence, when determining whether stocks are fairly valued or not we examine whether the current formation of stock prices differs significantly from the way in which stocks have been priced historically. The fundamental stock price may therefore also be given the interpretation that it determines what the level of stock prices 'should be' when judged from the historical experience about the relationship between stock prices and the prevailing levels of the two fundamentals variables. We take results as evidence that we have established a good model for this fundamental level of stock prices.

As one application, the evident split of the Danish stock market into a bear market period before 1980 and a subsequent bull market period can be explained by the behavior of the bond rate. The decline in the bond rate from 1995 to 1998 also contributes to an understanding of the recent surge in the stock market. By late 1998, stock prices were roughly fairly valued, that is, in line with their fundamental levels. However, this only occurred after a substantial 'overshooting' in 1997. It is important to notice that a fair valuation of stocks by 1998 does not exclude the possibility that stock prices will decline in the future. The proper interpretation is that we do not find evidence of a

significant mispricing error in the current level of stock prices which *in itself* will trigger a decline in stock prices as they adjust to their long-run equilibrium level. However, as a hypothesis we might observe (say) an increase in the bond rate. By reducing the fundamental level of stock prices, this may lead to a decline in actual stock prices.

In spirit, our long-run model is closely related to a model presented in a recent market study by a major Danish bank, see Valgreen (1999). He estimates a long-run relation for stock prices using nominal GDP (GDP at market prices for the total economy) and a nominal bond rate (5-year maturity) as explanatory variables. This formulation comes close to ours. The main differences are that we take account of variations in the profit share and include a trend as a proxy for the growth in the nominal value of outstanding stocks. Valgreen (1999) estimates the relation for Denmark over the period from 1981 to (early) 1999 while imposing a coefficient of one to GDP. However, using the two-step procedure of Engle and Granger (1987), the estimated relation is rejected as a cointegrating relation at any conventional significance level. The lack of cointegration might possibly be explained by the short sample.

In modeling stock prices, we use the nominal bond rate rather than a real interest rate measure as the relevant discount rate of investors. This is motivated by the empirical finding that the bond rate performs well in explaining actual stock price behavior and, in particular, performs better than the set of common real interest rate proxies we have examined. The explanatory power of the nominal bond rate could be interpreted as evidence of ‘money illusion’ on behalf of stock investors, a possibility that has also been addressed for the US stock market by Modigliani and Cohn (1979) and Ritter and Warr (1999). The result that the increase in the bond rate can account for the bearish stock market period before the early 1980s is consistent with the findings of Modigliani and Cohn (1979). They refer to the increase in the nominal bond rate when explaining the poor performance of the US stock market during the 1970s. Ritter and Warr (1999) explain the US bull market over the last two decades by the decline in inflation. This parallels our conclusion on the Danish bull market because the decline in the nominal bond rate since the early 1980s mainly reflects the contemporaneous decline in inflation. Following Ritter and Warr (1999), the Danish bull market

could therefore be interpreted as the outcome of a reduced impact of ‘money illusion’ on stock prices.

In terms of the definition of fundamental stock prices, there is a difference to Modigliani and Cohn (1979) and Ritter and Warr (1999). In the latter studies, fundamental stock prices have a theoretical foundation as they are calculated on the basis of a stylized Gordon Growth type of model. Their definition of fundamental stock prices is intended to capture the valuation of stocks by a rational investor (within their specific theoretical framework). On the other hand, we have used an empirically motivated definition of fundamental stock prices. This implies that any ‘irrational’ components in stock prices, e.g. those arising from ‘money illusion’, are actually part of our measure of fundamental stock prices as long as they show up as empirically significant and regular components in actual stock price behavior. In particular, it means that the nominal bond rate and not a real interest rate measure is used as a fundamentals variable. This conceptual difference, for instance, explains why Ritter and Warr (1999) find that stocks are severely undervalued in the early 1980s whereas we find that stocks are close to their fundamental levels in (say) 1982²⁷. An advantage of using an empirical rather than a theoretical approach is that the valuation of stocks is not theory-dependent. It depends on a specific empirical model but a model that has proved useful in explaining actual stock price behavior over a historical sample.

It should be emphasized that even though results may suggest so, we have not provided strict evidence in favor of ‘money illusion’. Thus, we have not explicitly addressed the validity of this hypothesis. Moreover, the reason why the nominal bond rate performs so well in explaining actual stock price behavior may simply be that it provides a good proxy for the latent real discount rate used by a rational investor and, in particular, a better proxy than the real interest rate proxies we have examined. To quantify the discount rate of a rational investor is inherently difficult because we have to take account of his *rational expectations* about future inflation and real growth. In the case of a risk averse investor, we also have to include a risk premium on stocks, all of which are

²⁷ The (possible) presence of ‘irrational’ components in our measure of fundamental stock prices, of course, raises the subtle question whether a rational investor can beat the market, i.e., gain an abnormal return even in a situation where the market is deemed fairly valued. The answer is not obvious as indicated by the results of De Long *et al.* (1990). They show that ‘noise traders’ which by definition are irrational can actually survive in a market where rational, but risk averse investors take positions against them. Thus, in the end stock prices will persistently contain an irrational component (‘noise’) and, moreover, rational investors will not gain an abnormal return. An analysis of this question is outside the scope of the present paper.

unobservable. A further complication may arise from taxation. Hence, if investment income is taxed, a rational investor will use an after-tax discount rate. The bottom line is that the discount rate of a rational investor can be a highly complex after-tax and risk-adjusted *ex ante* real discount rate and it may simply be the case that the nominal bond rate shows a higher correlation with this 'true' discount rate than the real interest rate proxies we have examined. Therefore, the empirical significance of the nominal bond rate may not necessarily be inconsistent with rational asset pricing. Our study does not differentiate between the 'proxy' and the 'money illusion' explanations. In particular, results do not depend on which explanation is the relevant one. However, determining whether or not stock investors suffer from 'money illusion' is obviously an interesting and important issue for future research.

We have set up a model which has proved helpful in explaining certain historical features of the Danish stock market. A related question is to what extent the model is also useful for prediction purposes. Being able to predict future stock returns is, obviously, of interest to stock investors. It may also be important to policy makers if they are concerned that an overvalued stock market might collapse and that this will have a severe negative impact on the real economy and the financial sector. The policy relevance of valuing and predicting the stock market has become increasingly recognized in recent years, in particular, in the US and has, for instance, been advocated on several occasions by the Chairman of the US Federal Reserve System, see e.g. Sellon and Buskas (1999). This paper provides a simple empirical model for the valuation of the Danish stock market. The mere essence of cointegration implies some degree of predictability because we can predict that any mispricing error by definition will be eliminated over time. However, the issue of predictability needs a closer examination and needs, in particular, to be quantified.

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Appendix A: Unit Root Tests

The data series have been tested for unit roots, using both the Z_t -test by Phillips and Perron (1988) (PP), cf. Table A.1, and the unit root test by Kwiatkowski *et al.* (1992) (KPSS), cf. Table A.2. The former test tests the null hypothesis of a unit root against the stationary alternative while the latter tests the stationary specification as the null, treating the unit root specification as the alternative hypothesis. Both tests are performed for a prior specified lag length, see the notes to the two tables. Given the sample size, we report test statistics for up to six lags.

< Table A.1 >

< Table A.2 >

Using conventional significance levels, the PP test gives firm evidence that all series - stock prices, profits and the bond rate - are integrated of order 1, i.e., that the series are non-stationary in levels but stationary in first differences. The KPSS test confirms this result for stock prices while the evidence for profits depends on the lag length, as the KPSS test is indecisive between a unit root process and a stationary process with long lags. However, the KPSS test may suffer from a very low power for long lag lengths, cf. the simulations in Kwiatkowski *et al.* (1992), so we decide on the unit root alternative. For the bond rate, the KPSS test indicates that this series is integrated of order 2, i.e., that the series is also non-stationary in first differences, when not allowing for a deterministic trend. However, this test result is entirely driven by the apparent negative trend in Δb_t over the sample (cf. Figure 3) as the KPSS test leads to the unambiguous result that Δb_t is stationary when allowing for a trend in this series.

We, therefore, conclude the evidence that all series are integrated of order 1. This conclusion can be confirmed by the augmented tests of Dickey and Fuller (1979) (results available on request).

Appendix B: An ECM for Stock Prices

Based on the cointegrating relation identified by the Johansen (1996) analysis, we can estimate a (structural) Error-Correction-Model (ECM) for the short-run (1-year) changes in (log-to-)stock prices, i.e., for the short-run capital gains, using as explanatory variables the lagged and contemporaneous changes in the ‘fundamentals’ variables (profits and the bond rate) and an ECM-term which captures the adjustment in stock prices to long-run equilibrium. In this Appendix, we provide preliminary results, where we, for simplicity, have used a single-equation approach to estimate an ECM (rather than a more appropriate simultaneous approach, such as e.g. FIML estimation of a structural VAR model). We recognize that this approach may lead to inconsistent estimates of the dynamic coefficients because contemporaneous values of stock prices and the fundamentals variables may be determined simultaneously and, in this case, the regressors will be endogenous (correlated with the residual term in the stock price equation).

We use a ‘general-to-specific’ approach where we start out with a fairly general ECM for stock prices, having included up to five lags of the first differences of stock prices, profits and the bond rate. We estimate a structural ECM as we include the contemporaneous changes in the fundamentals variables. We also include two impulse dummies, one for 1972 and one for 1983, to exclude the extraordinary high capital gains observed in these two years. These extreme observations can be attributed to exceptional changes in the Danish economy, cf. Note 1 in the text, and we do not intend to model these changes. Having estimated the general model, we test for significance of the individual terms (t-test) and, consecutively, remove the insignificant terms. Using a 5% significance level, we end up with the following parsimonious ECM for stock prices, estimated by OLS over the period 1952 to 1998 (standard errors in parentheses):

$$(B1) \quad \Delta q_t = \underset{(0.046)}{0.097} - \underset{(0.12)}{0.668} \Delta b_t + \underset{(0.11)}{0.202} \Delta b_{t-1} + \underset{(0.44)}{1.04} \Delta \pi_t - \underset{(0.38)}{1.56} \Delta \pi_{t-2} \\ + \underset{(0.14)}{0.298} d72_t + \underset{(0.13)}{0.483} d83_t - \underset{(0.13)}{0.511} ecm_{t-1} + \varepsilon_t$$

ecm_{t-1} is the (lagged) error-correction-term, defined as the residual (in the previous period) from the cointegrating relation for stock prices in (4). $d72_t$ and $d83_t$ are the impulse dummies and M_t is the residual of this dynamic equation.

< Figure B.1 >

< Table B.1 >

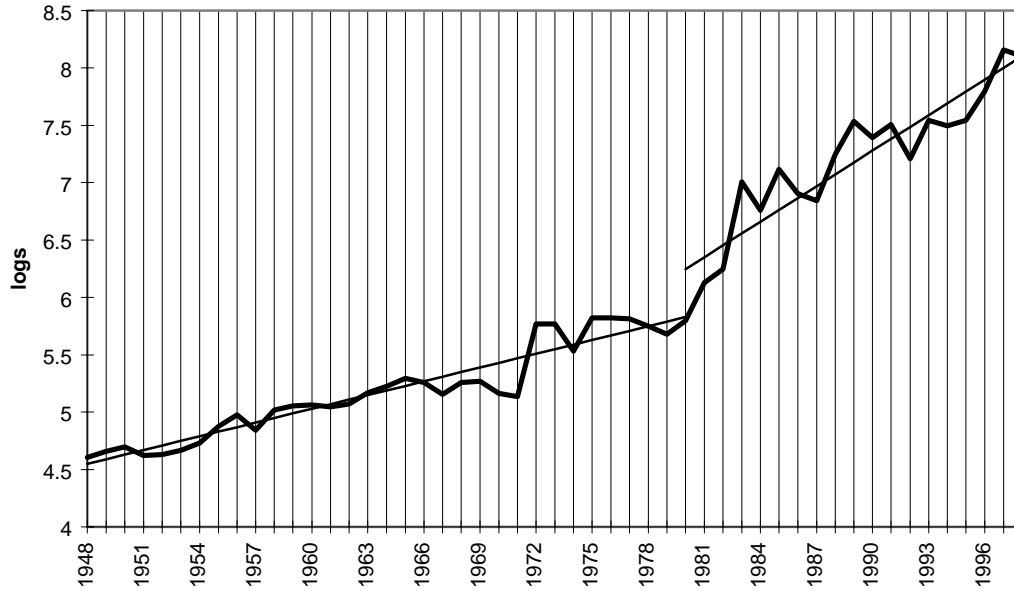
The fit of the ECM is depicted in Figure B.1 and the outcome of specification tests are reported in Table B.1. The specification tests show that the model is well specified as there is no evidence of serial correlation or heteroskedasticity in the residual term. Moreover, the model gives a good fit to the short-run fluctuations in stock prices. Excluding the two dummy years 1972 and 1983, the model explains 56% (= the coefficient of determination) of the sample variation in capital gains (when including 1972 and 1983, the coefficient of determination is 73%). This appears to be a satisfactory result, when recalling that the model explains the fluctuations in stock prices over a short (1-year) horizon.

With the exception of Δb_{t-1} (which is retained in the model to keep it well specified), all coefficients are significant at the 5% significance level. The most significant explanatory variable is the contemporaneous change in the bond rate, Δb_t (t-test statistic of 5.4). The bond rate exerts a strong negative influence on stock prices, with a 1% *relative* increase in the bond rate *ceteris paribus* leading to a 0.67% decline (continuous rate) in stock prices over the same 1-year period (and *vice versa*). The contemporaneous change in profits, $\Delta \pi_t$, has a significant positive effect on stock prices in the short run, with a change in profits inducing a roughly proportional change in stock prices over the same period (coefficient of 1.04). We find that the short-run effects from the fundamentals variables are of the same sign as the long-run effects, cf. (4), but according to the point estimates stock prices are (slightly) less sensitive to changes in the fundamentals variables in the short-run (in the contemporaneous period) than in the long-run, suggesting an element of (immediate) ‘underreaction’.

We observe that the error-correction-term is highly significant with a t-test statistic of 4.0. This shows the importance of incorporating the long-run equilibrium relation for stock prices when modeling the short-run capital gains. The estimated adjustment coefficient of -0.511 is consistent with a slow and smooth adjustment to the equilibrium or fundamental level of stock prices in the long-run. The estimate implies that roughly half of any mispricing error, relative to the cointegrating

relation, is eliminated, *ceteris paribus*, within one year which, at an informal level, seems consistent with the evidence on the duration of a mispricing provided in section 4.

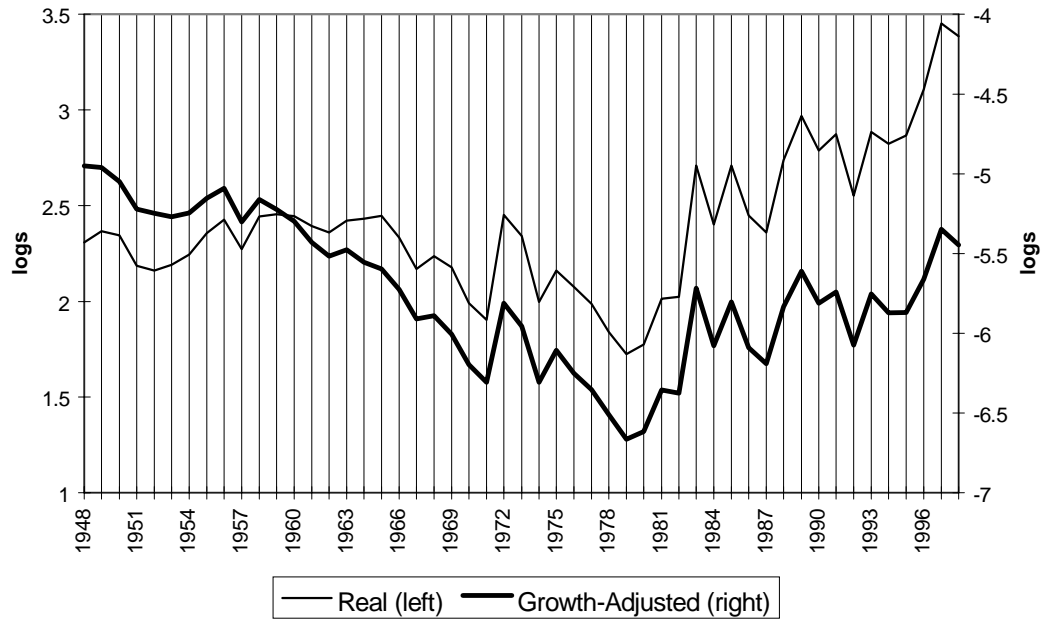
**Figure 1. Stock Prices
1948-1998**



Note: Straight lines indicate trend in stock prices over subsamples 1948-1980 and 1980-1998, respectively. The trend is estimated by a regression (OLS) of stock prices on a constant term and a time trend.

Source: All-Share Stock Price Index by Statistics Denmark.

**Figure 2. Real and Growth-Adjusted Stock Prices
1948-1998**



Note: Real and Growth-Adjusted stock prices show the level of stock prices deflated by the price level and nominal income, respectively.

general

Figure 3. The Data

Levels and first differences of (by row and from the top) stock prices, profits and the bond rate (all in logs), 1948-1998.

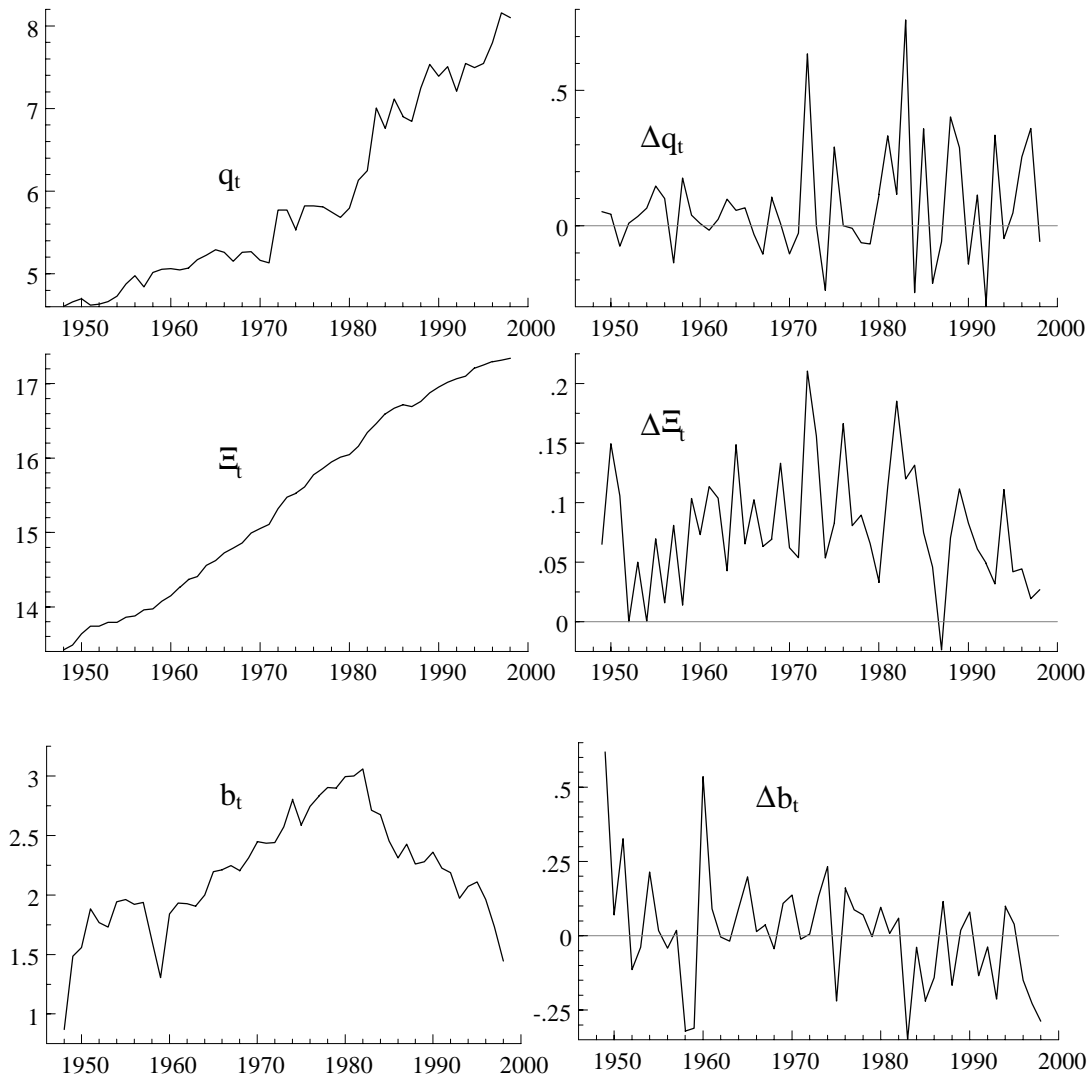


Figure 4. Diagnostic Graphics for the VAR Model

Actual and fitted values (in levels), residuals, residual correlogram and residual density for the equation for (by row and from the top) stock prices, profits and the bond rate.

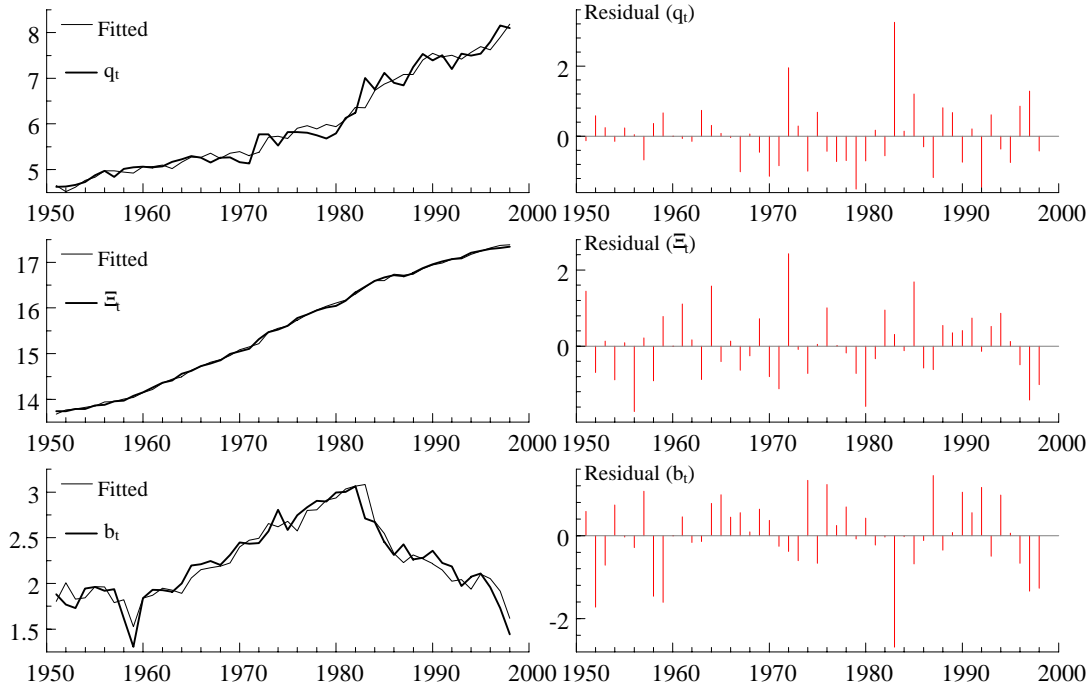
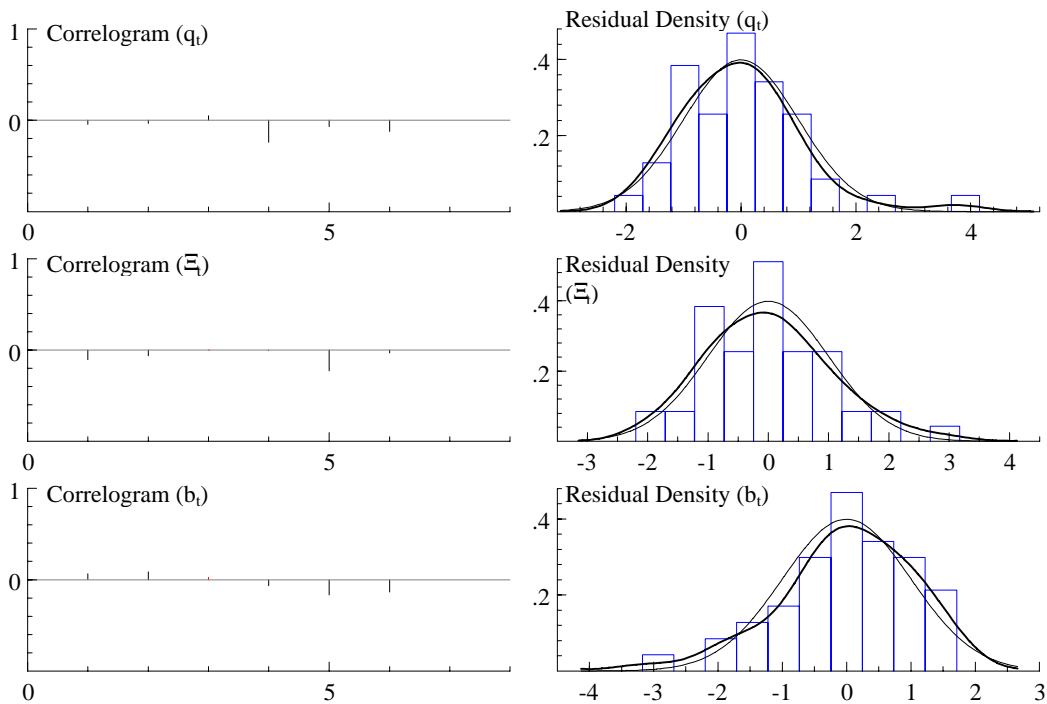


Figure 4, continued.



Note: Density plots include standard normal density for comparison (thin curve).

Figure 5. Recursive Estimates of the Eigenvalues
Eigenvalues labelled by numbers 1 to 3, 1970-1998.

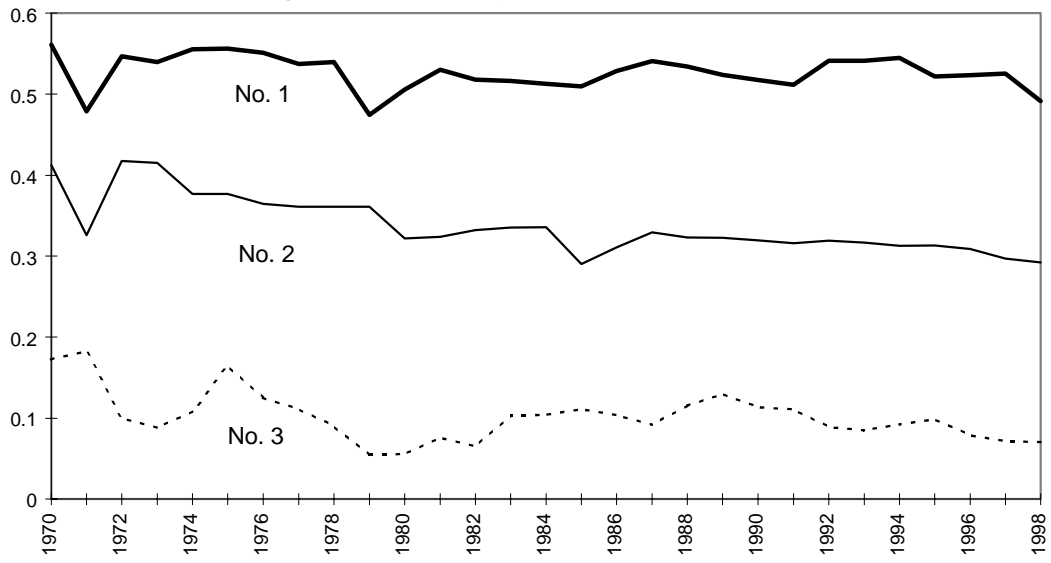
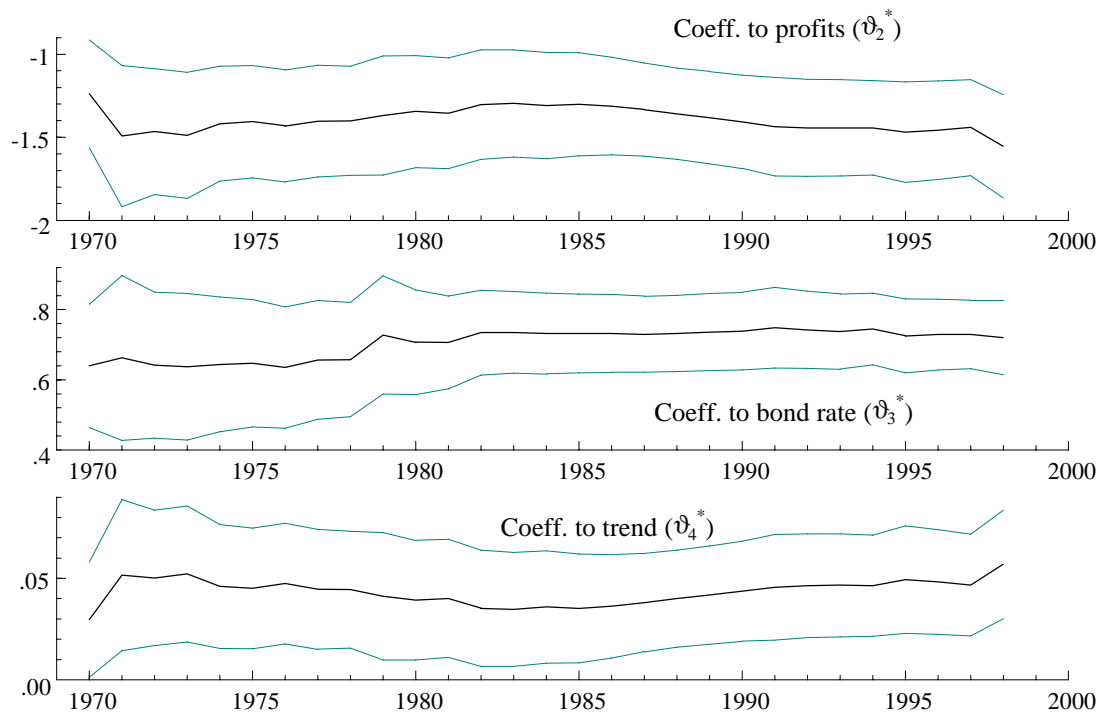


Figure 6. Recursive Estimates of the Cointegrating Vector

Recursive point estimates (solid line) and 95% confidence bands for the coefficient to, respectively, profits, the bond rate and the time trend, 1970-1998.

Cointegrating rank 1, cointegrating vector normalized on stock prices, and profits restricted to be exogenous for long-run parameters ($I_2=0$).



**Figure 7. Stock Prices: Actual and Long-Run Fit
1951-1998**

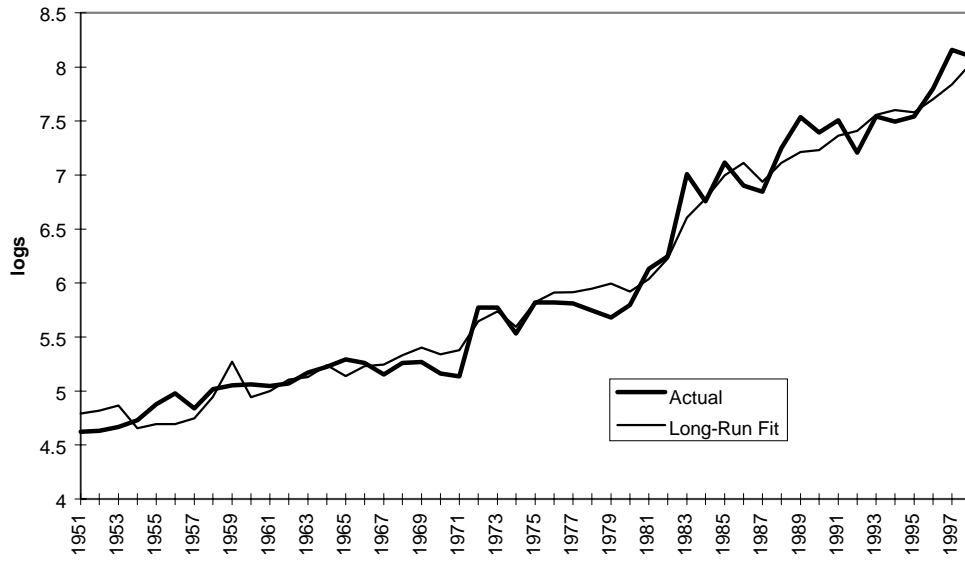
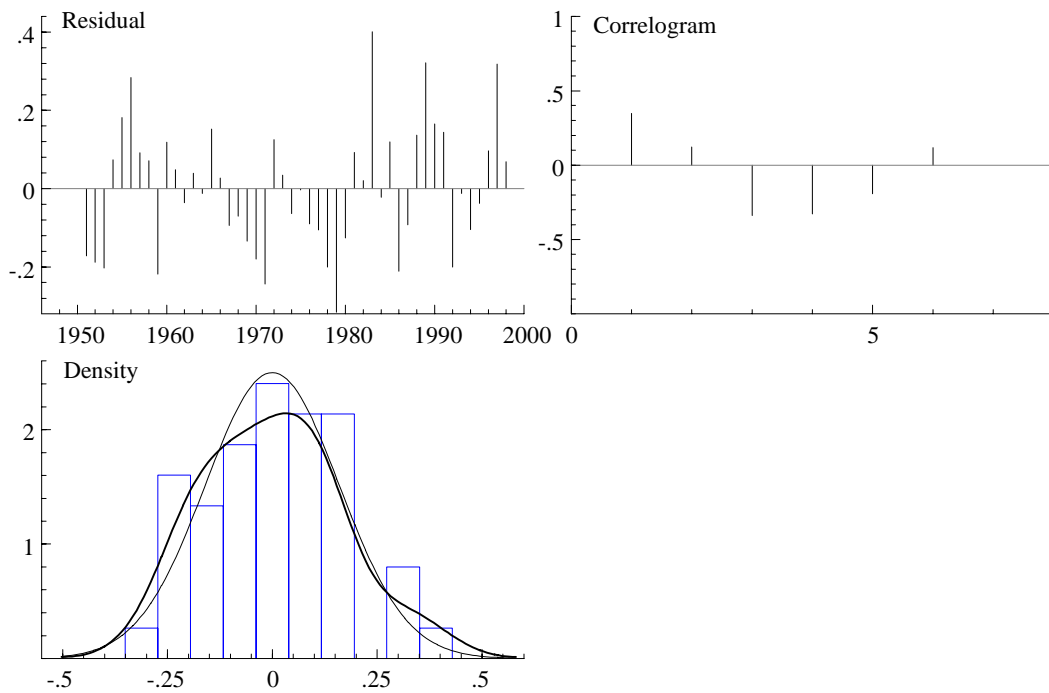


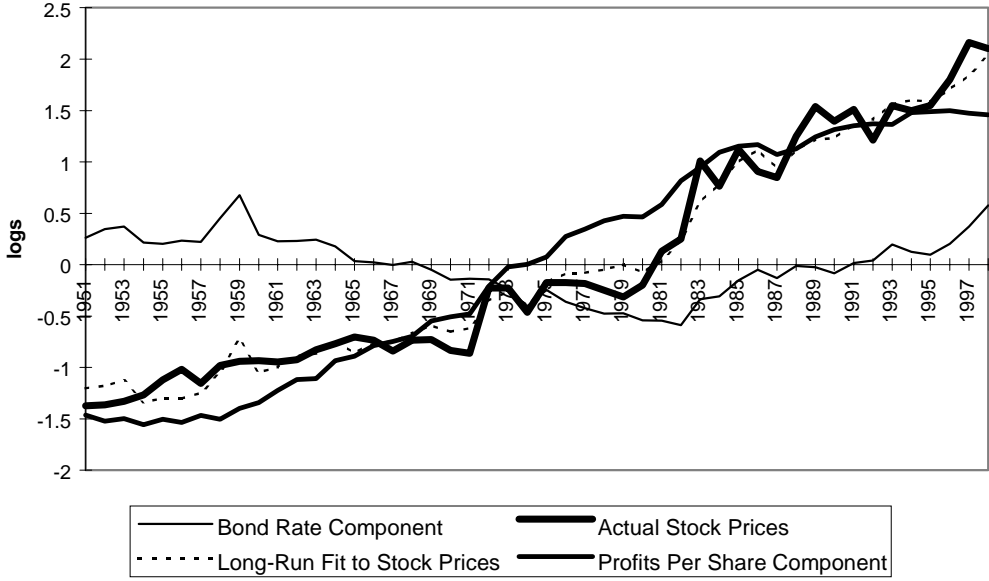
Figure 8. Mispricing Errors

Values, correlogram and density of residuals from cointegrating relation, 1951-1998.



Note: Density plot includes standard normal density for comparison (thin curve). The sample standard deviation of the residuals (using T-1 as degrees of freedom) is 0.160.

**Figure 9. Stock Prices and Contributions by Components
1951-1998**



Note: Series shown as deviations from respective sample means.

**Figure 10.a. Growth-Adjusted Stock Prices:
Actual and Long-Run Fit**

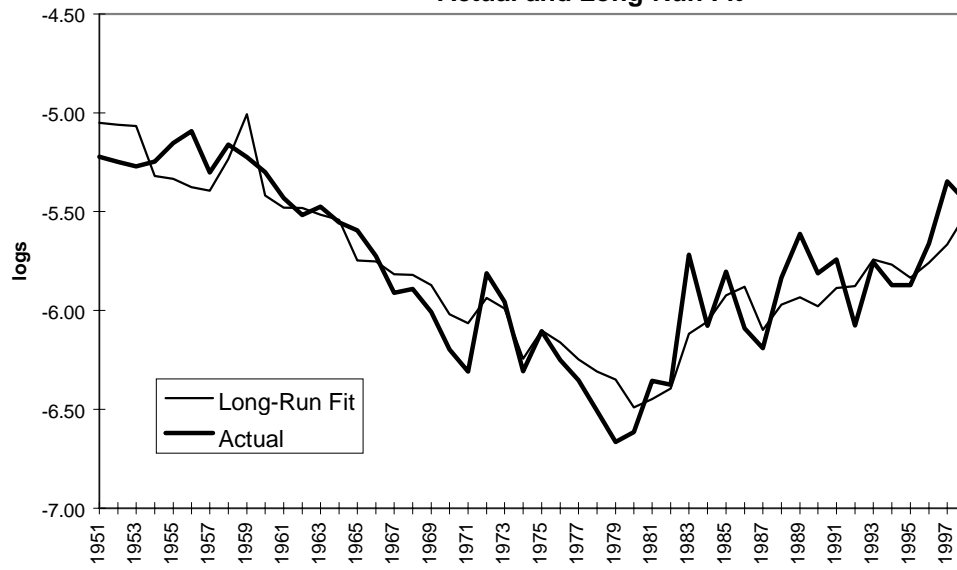


Figure 10.b. Contributions by Components

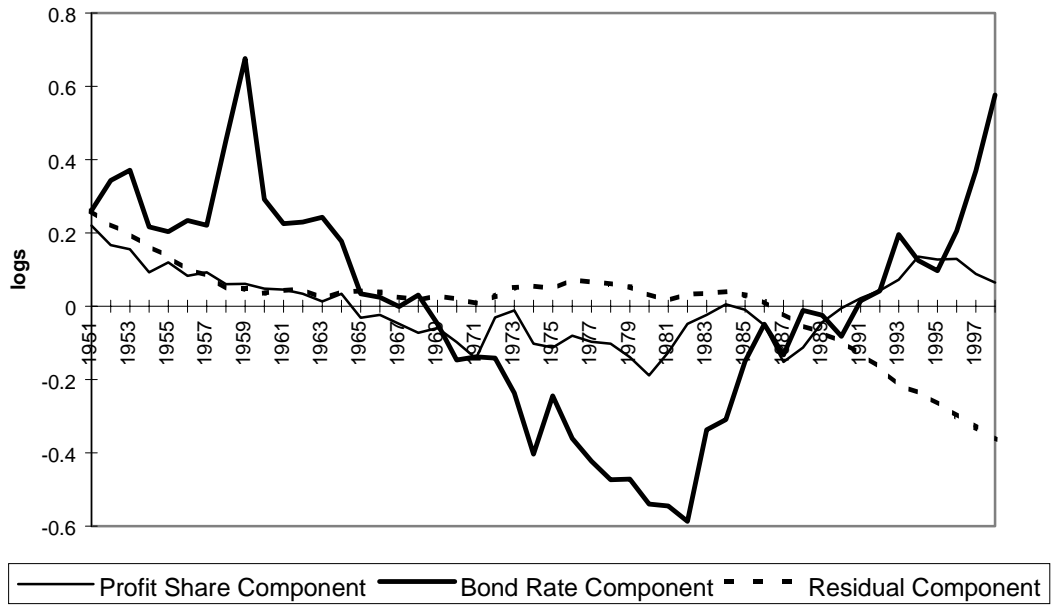
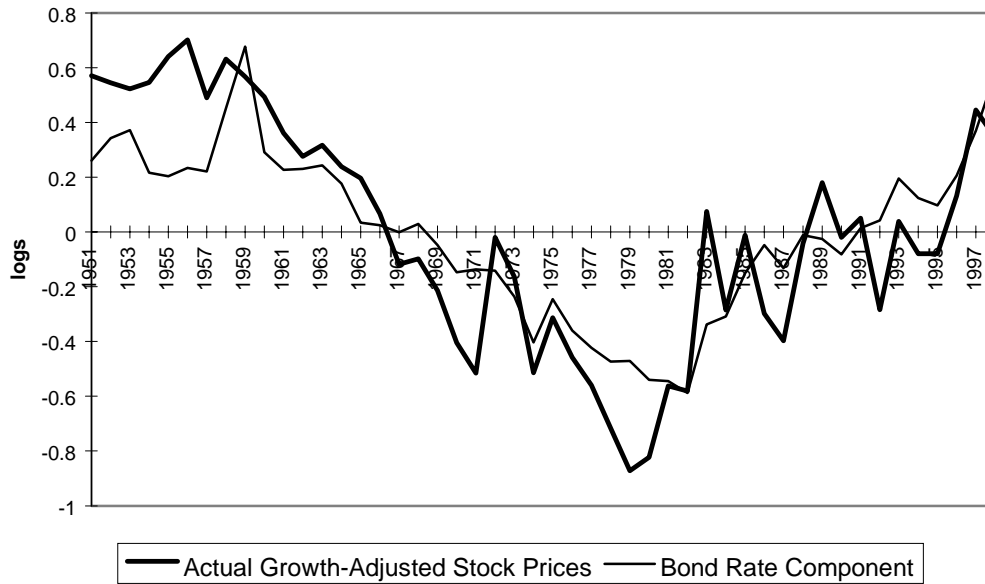
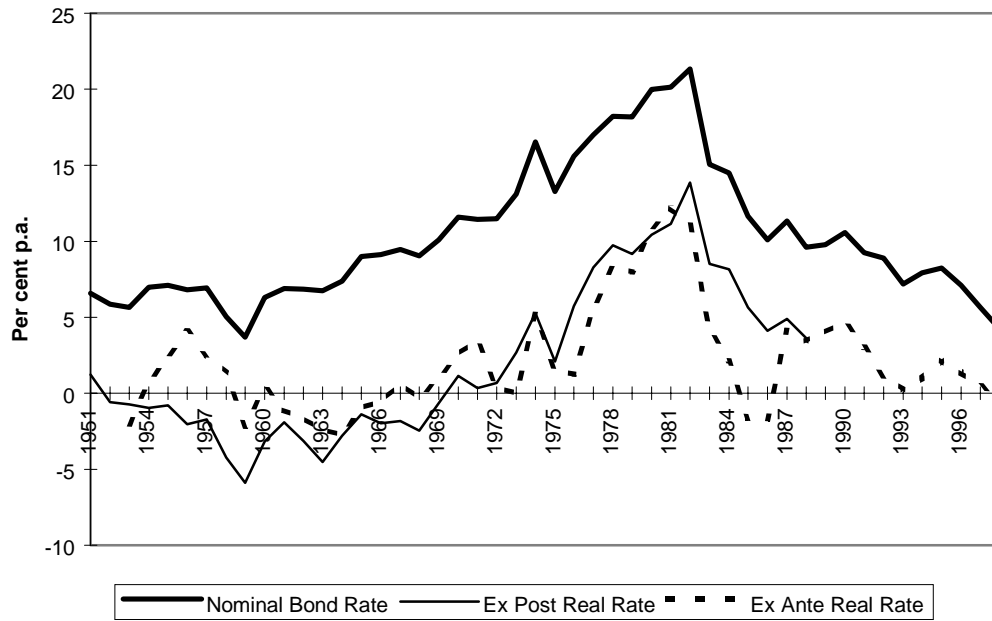


Figure 10.c. Growth-Adjusted Stock Prices and the Contribution by the Bond Rate

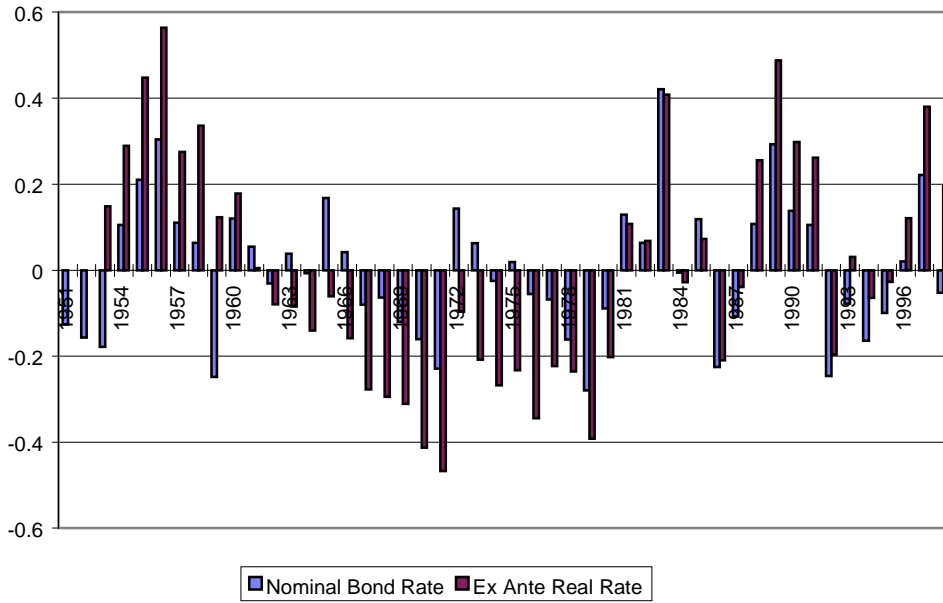


Note: In Panels b and c, the series are shown as deviations from respective sample means.

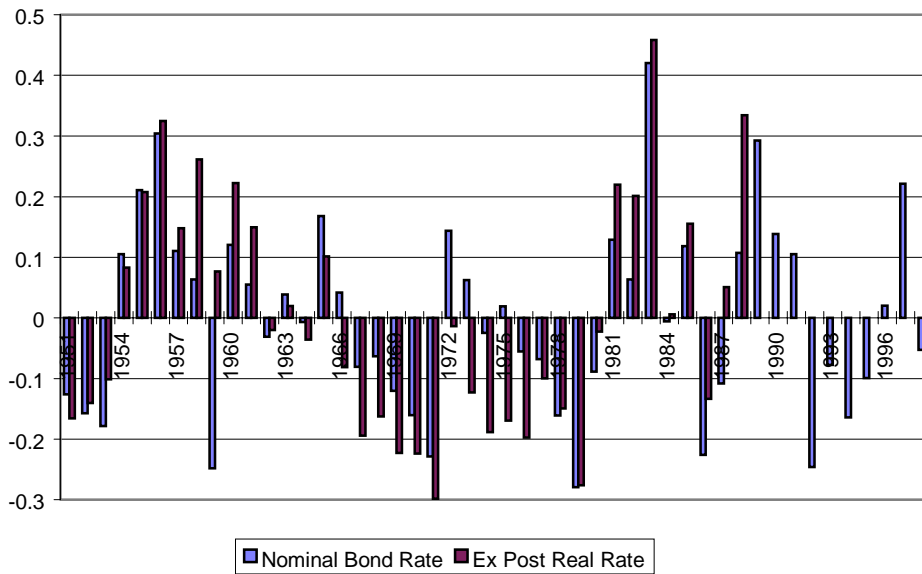
**Figure 11.a. Alternative Discount Rate Measures
1951-1998**



**Figure 11.b. Cointegration Residuals:
Ex Ante Real Rate vs. Nominal Bond Rate**



**Figure 11.c. Cointegration Residuals:
Ex Post Real Rate vs. Nominal Bond Rate**



Note: Ex ante real rate = difference between the 10-year nominal bond rate and the growth rate in profits in the preceding 5-year period. Ex post real rate = difference between the bond rate and the growth rate in profits for the same 10-year period.

Table 1. Specification Tests of the VAR Model
Estimation sample 1951-1998

<u>Multivariate tests:</u>			
Vector Autocorrelation order 1	F(9,75) =	0.81	[0.61]
Vector Autocorrelation order 2	F(18,79) =	1.20	[0.28]
Vector Autocorrelation order 4	F(36,65) =	1.42	[0.11]
Vector Autocorrelation order 6	F(54,48) =	1.32	[0.16]
Vector Heteroskedasticity (squares)	F(120,65) =	0.45	[1.00]
Normality	$\chi^2(6)$ =	8.65	[0.19]
<u>Univariate tests:</u>			
	Δq_t	$\Delta \Xi_t$	Δb_t
Autocorrelation order 1, F(1,35):	0.65 [0.43]	2.97 [0.09] *	0.98 [0.33]
Autocorrelation order 2, F(2,34):	0.37 [0.70]	1.56 [0.22]	0.69 [0.51]
Autocorrelation order 4, F(4,32):	1.24 [0.31]	0.74 [0.57]	0.37 [0.83]
Autocorrelation order 6, F(6,30):	1.33 [0.27]	0.78 [0.59]	0.43 [0.85]
ARCH (1), F(1,34):	0.31 [0.58]	0.48 [0.49]	0.33 [0.57]
Heteroskedast. (squares), F(20,15):	0.58 [0.87]	0.49 [0.93]	0.43 [0.96]
Normality, $\chi^2(2)$:	9.58 [0.01] ***	1.65 [0.44]	4.52 [0.10]
<u>Goodness-of-fit:</u>			
ρ	0.99	1.00	0.96
σ_ε	0.203	0.040	0.138

Note: The VAR model has a lag length of 3 and includes an impulse dummy for 1960. F-tests are small sample approximations to Lagrange Multiplier tests, being adjusted for degrees of freedom. Normality test by Doornik and Hansen (1994). For a description of the tests, see Doornik and Hendry (1997). Critical significance levels in brackets. *, ** and *** indicates misspecification at the 10%, 5% and 1% significance level, respectively. ρ denotes the correlation between actual and fitted values for each equation. σ_ε is the estimated standard deviation of the residuals.

Table 2. Cointegration Analysis in the VAR Model
 Estimation sample 1951-1998

<u>Cointegrating rank:</u>			
Rank(Π) ($r =$)	0	1	2
Eigenvalue	0.49	0.29	0.07
Trace test ¹⁾	52.5 ***	20.1 **	3.5
Trace test (adj. for df.) ²⁾	42.7 **	16.3	2.9
95% critical value (standard)	42.2	25.5	12.4
95% critical value (simulated)	36.5	19.9	NA
<u>Standardized eigenvectors β^*:</u>			
	β_1^*	β_2^*	β_3^*
q_t	1.000	-0.050	1.212
Ξ_t	-1.504	1.000	1.006
b_t	0.699	-0.435	1.000
t	0.053	-0.079	-0.133
<u>Standardized loadings α: ³⁾</u>			
	$\beta_1^* X_t^*$	$\beta_2^* X_t^*$	$\beta_3^* X_t^*$
Δq_t	-0.361	-0.438	0.062
$\Delta \Xi_t$	-0.035	-0.164	-0.004
Δb_t	-0.557	0.094	-0.044

Note: Maximum Likelihood Estimation by the procedure of Johansen (1996). The trace tests test for each value of r the null hypothesis $H_0: \text{rank}(\Pi) \leq r$ against the alternative $H_A: \text{rank}(\Pi) > r$. The null is rejected iff the trace statistic is larger than the critical value. The first (standard) critical values are from Johansen (1996, Table 15.4). The second (simulated) set of values are obtained using the simulation program DisCo, taking account of the dummy for 1960. *, ** and *** denote rejection of the null at the 10%, 5% and 1% significance level, respectively, using the simulated critical values.

The standardized eigenvectors are normalized on the diagonal wrt. the endogenous variables. Corresponding I-loadings.

NA: Not available in DisCo.

¹⁾ The asymptotic trace test of Johansen (1996).

²⁾ Small sample approximation to the asymptotic trace test, obtained by adjusting for degrees of freedom, cf. Reimers (1992).

³⁾ $X_t^* \equiv (q_t, \Xi_t, b_t, t)'$

Table 3. Hypothesis Testing

Parameter restriction	I-loadings			Cointegrating vector			
	I ₁	I ₂	I ₃	ϑ_1^*	ϑ_2^*	ϑ_3^*	ϑ_4^*
(I) Unrestricted	-0.361 (0.299) [0.241]	-0.035 (0.065) [0.649]	-0.557 (0.196) [0.004]	1 - [0.000]	-1.504 (0.172) [0.000]	0.699 (0.059) [0.000]	0.053 (0.015) [0.010]
(II) I ₂ =0	-0.287 (0.272) [0.270]	0 - -	-0.558 (0.192) [0.004]	1 - [0.000]	-1.554 (0.171) [0.000]	0.720 (0.058) [0.000]	0.057 (0.015) [0.000]
(III) I ₂ =I ₃ =0	-0.867 (0.160) -	0 - -	0 - -	1 - [0.000]	-1.556 (0.201) [0.000]	0.687 (0.068) [0.000]	0.056 (0.017) [0.004]

Note: All estimations are conducted with cointegrating rank restricted to 1. In (II), profits are assumed to be exogenous for the long-run parameters. In (III), both profits and the bond rate are restricted to be exogenous. Numbers in parentheses are indicative standard errors of the parameter estimates. Numbers in squared brackets give the critical significance level of a Likelihood Ratio test for significance, i.e., for testing the null that the specific parameter is zero. Cointegrating vector is normalized on stock prices. When testing for significance of stock prices, the cointegrating vector is normalized on the bond rate.

**Table 4. Estimates of the Cointegrating Relation:
Single-Equation-Analysis**

The regression is:

$$q_t = \beta_0 + \beta_1\pi_t + \beta_2b_t + \beta_3t + \sum_{i=-N_1}^{N_1} \gamma_{1i}\Delta\pi_{t-i} + \sum_{i=-N_2}^{N_2} \gamma_{2i}\Delta b_{t-i} + \sum_{i=1}^{N_3} \phi_i ecm_{t-i} + v_t, \text{ where } ecm_t \equiv q_t - \beta_0 - \beta_1\pi_t - \beta_2b_t - \beta_3t$$

The static cointegrating regression is augmented by current, leaded and lagged first differences of Ξ_t and b_t , and lagged equilibrium-correction-terms ecm_t . The augmenting terms, as indicated by N_1 , N_2 and N_3 , are shown in the first column.

Regression	Estimation sample	Coefficient Estimates (standard errors)			
		ϑ_0	ϑ_1	ϑ_2	ϑ_3
<i>a. Static</i>	1951-1998	-14.2 (3.1)	1.50 (0.24)	-0.786 (0.070)	-0.0502 (0.021)
<i><u>PLOLS:</u></i>					
<i>b. $N_1=N_2=0$</i>	1951-1998	-13.9 (2.5)	1.47 (0.19)	-0.793 (0.063)	-0.0478 (0.017)
<i>c. $N_1=N_2=1$</i>	1951-1997	-16.6 (2.9)	1.69 (0.23)	-0.936 (0.079)	-0.0661 (0.019)
<i>d. $N_1=N_2=2$</i>	1951-1996	-15.9 (2.1)	1.64 (0.16)	-0.803 (0.057)	-0.0659 (0.014)
<i><u>PLNLS:</u></i>					
<i>e. $N_1=N_2=0, N_3=3$</i>	1951-1998	-12.9 (3.2)	1.39 (0.25)	-0.752 (0.081)	-0.0410 (0.022)
<i>f. $N_1=N_2=1, N_3=3$</i>	1951-1997	-18.0 (5.2)	1.80 (0.41)	-0.926 (0.14)	-0.0727 (0.034)
<i>g. $N_1=N_2=2, N_3=4$</i>	1952-1996	-18.0 (6.4)	1.80 (0.50)	-0.830 (0.14)	-0.0770 (0.043)

Note: Entry a shows results from estimating (OLS) the static cointegrating regression, corresponding to the first step of the Engle and Granger (1987) two-step procedure (no augmentation). Standard errors of coefficient estimates are computed by OLS and are indicative only. Entries b through d give results from the Phillips and Loretan (1991) OLS procedure (PLOLS). $N_3=0$ in all three cases. The regressions are estimated by OLS. Standard errors take account of serial correlation in the disturbance term (v_t) by the method suggested by Hamilton (1994, p. 608f). An AR(3) (AR(4)) specification is used in cases b and c (case d). Entries e through g report results from the Phillips and Loretan (1991) NLS procedure (PLNLS). The regressions are estimated by the Non-Linear Least Squares method. Standard errors are calculated from numerical derivatives of the sum of squared residuals.

Table A.1. Phillips and Perron (1988) Z_t -Test for Unit Root
1948-1998

Series:	Lag length (l)						
	0	1	2	3	4	5	6
Levels:							
q_t (T)	-2.35	-2.18	-2.16	-2.15	-2.13	-2.12	-2.13
Ξ_t (T)	-1.02	-1.21	-1.28	-1.34	-1.38	-1.42	-1.46
b_t	-2.55	-2.56	-2.57	-2.58	-2.60*	-2.62*	-2.64*
First differences:							
Δq_t	-8.96***	-8.97***	-9.04***	-9.12***	-9.24***	-9.39***	-9.50***
$\Delta \Xi_t$	-5.62***	-5.62***	-5.62***	-5.62***	-5.63***	-5.64***	-5.65***
Δb_t	-7.22***	-7.23***	-7.27***	-7.26***	-7.25***	-7.22***	-7.20***
Critical test values:			10 %	5 %	2.5 %	1 %	
Without trend			-2.60	-2.93	-3.22	-3.58	
With trend			-3.18	-3.50	-3.80	-4.15	

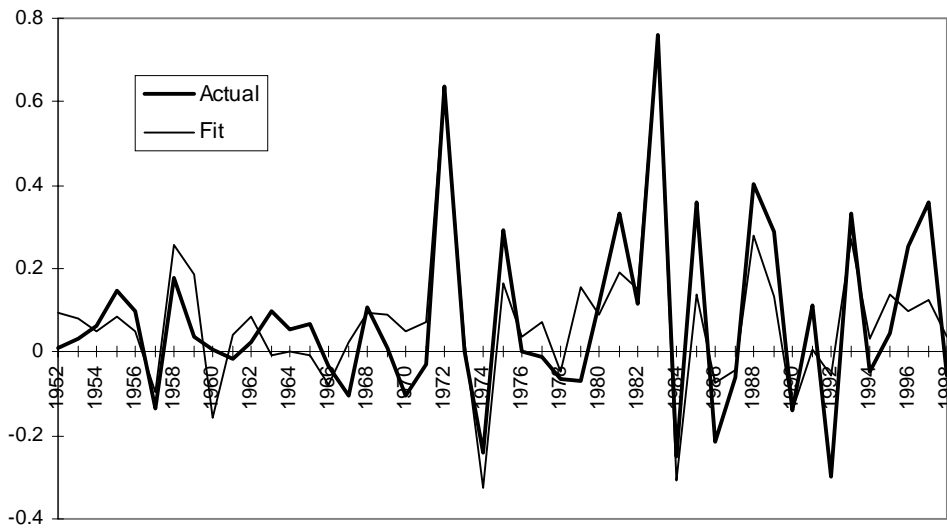
Note: The Phillips and Perron (1988) Z_t -test statistic is a modified t-statistic for the null hypothesis of a unit root ($\rho=1$) in the first order autoregression $x_t=\alpha+\rho x_{t-1}+u_t$ (without trend), respectively $x_t=\alpha+\rho x_{t-1}+\delta t+u_t$ (with trend), where the disturbance term u_t has mean zero but can otherwise be heterogeneously distributed and serially correlated up to lag l , see also Hamilton (1994). The null is rejected in favor of the stationary alternative ($\rho<1$) if Z_t is negative and sufficiently large in numerical value. Critical values are from Hamilton (1994, Table B.6) for a sample size of 50. *, ** and *** denote rejection of a unit root at 10%, 5% and 1% significance level, respectively. All regressions include a constant term. (T) indicates that a deterministic trend is included.

Table A.2. Kwiatkowski *et al.* (1992) Test for Unit Root
1948-1998

Series:	<i>Lag length (l)</i>						
	0	1	2	3	4	5	6
<u>Levels:</u>							
q_t (T)	0.90***	0.51***	0.37***	0.30***	0.25***	0.22***	0.20**
Ξ_t (T)	0.54***	0.29***	0.21**	0.16**	0.14*	0.12*	0.11
b_t	1.89***	1.04***	0.74***	0.58**	0.48**	0.42*	0.37*
b_t (T)	0.93***	0.52***	0.37***	0.30***	0.25***	0.22**	0.20**
<u>First differences:</u>							
Δq_t	0.14	0.19	0.22	0.24	0.26	0.30	0.31
$\Delta \Xi_t$	0.25	0.21	0.20	0.19	0.18	0.18	0.17
Δb_t	0.72**	0.69**	0.67**	0.65**	0.63**	0.59**	0.56**
Δb_t (T)	0.06	0.06	0.07	0.07	0.07	0.07	0.07
<u>Critical test values:</u>			10 %	5 %	1 %		
Without trend			0.35	0.46	0.74		
With trend			0.12	0.15	0.22		

Note: The Kwiatkowski *et al.* (1992) test for unit root is a Lagrange Multiplier test of the null hypothesis that the series can be described by a stationary process (possibly around a deterministic trend) of lag length l against the alternative that the process also includes a random walk component. The null of stationarity is rejected in favor of the unit root alternative if the test statistic is sufficiently large. Critical values are from Kwiatkowski *et al.* (1992). *,** and *** denote rejection of the null (unit root is present) at 10%, 5% and 1% significance level, respectively. (T) indicates that the null hypothesis is trend-stationarity. Otherwise, the null is mean-stationarity.

**Figure B.1. Annual Capital Gain on Stocks: Actual and Fit
1952-1998**



Note: Capital gain is calculated as the (backward looking) 1-year change in log-to-stock prices.

Table B.1. Specification Tests of the ECM
Estimation sample 1952-1998

Autocorrelation order 1	F(1,38)	=	0.07	[0.80]
Autocorrelation order 2	F(2,37)	=	0.79	[0.46]
Autocorrelation order 3	F(3,36)	=	0.54	[0.66]
Autocorrelation order 5	F(5,34)	=	1.07	[0.39]
ARCH (1)	F(1,37)	=	0.02	[0.89]
Heteroskedasticity (squares)	F(12,26)	=	0.99	[0.48]
Heteroskedasticity (cross-products)	F(22,16)	=	0.57	[0.89]
Normality	$\chi^2(2)$	=	0.04	[0.98]

Memo:

$$\sigma_{\varepsilon} = 0.119$$

Note: F-tests are small sample approximations to Lagrange Multiplier tests, being adjusted for degrees of freedom. Normality test by Doornik and Hansen (1994). Critical significance levels in brackets. σ_{ε} is the estimated standard deviation of the residuals.