

Optimal Taxation in a Two Sector Model of Endogenous Growth

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Abstract

This paper examines the effects of capital and labor income taxation in a two sector model of endogenous growth. There is an education externality in the final goods sector and a public input in the education sector. The main result is that labor income taxation as well as subsidization of educational effort have positive effects on the balanced growth rate in the market economy, *ceteris paribus*. In addition, capital income taxation has no effect on the growth rate in the steady state. Furthermore, the optimal subsidy rate is unambiguously positive, when the public and private input factors in education are substitutes. Labor income should be taxed at a lower rate than capital income as long as there is a positive education externality in final goods production. The bigger the externality is, the bigger is the difference between the two tax rates. Finally, welfare maximization is not equivalent to growth maximization.

JEL classification:

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1. Introduction

This paper analyzes the effects of factor income taxation in a two sector model of endogenous growth with public investment in education and a positive education externality in final goods production.

The relevance of such an analysis is supported by two recent empirical papers. Firstly, Mendoza, Milesi-Ferretti and Asea (1995) show that there are significant negative investment effects from factor income taxation and that these are consistent with small negative growth rate effects. Secondly, Hansson and Henrekson (1994) show that educational expenditure by the government has a positive effect on productivity growth, while for instance government transfers and consumption have negative growth rate effects.

The present paper is inspired by four papers, namely Roubini and Milesi-Ferretti (1994a,b), Pecorino (1993) and Sørensen (1993). The first three papers present models that deal with optimal taxation in endogenous growth models, where the tax revenue is redistributed lump-sum to consumers. The last paper presents a model, which incorporates government spending on education and training. Thus in contrast to the first three papers, the Sørensen (1993) model uses the tax revenue for a productive purpose.

The first three papers reach different conclusions about the growth maximizing tax structure. The Roubini and Milesi-Ferretti models have three sectors; a final goods sector that produces both consumption goods and capital goods; an education sector; and a leisure sector. With respect to the latter sector, the authors consider different models of leisure, namely home production, leisure as quality time and no leisure at all. They assume that factor income only arises in the final goods sector, which implies that the optimal taxation rule is to set the capital income tax equal to the labor income tax in a version of their model with no leisure and a balanced government budget. The reason behind this result is that the sectoral allocation of factors is unaffected by taxation as long as both input factors are taxed at a common rate. The only implication of a comprehensive tax is that the interest rate is reduced by one minus the tax rate and that the growth rate is reduced by the fall in the interest rate multiplied by the intertemporal elasticity of substitution.

The Pecorino model also has three sectors; a consumption good sector; a physical capital sector; and an education sector. But in contrast to Roubini and Milesi-Ferretti he assumes that production in all three sectors give rise to factor income. As a consequence, the factor income from all three sectors is taxed. The implication of this assumption is that the optimal capital income tax is different

from the optimal labor income tax except, when the factor intensity in the two capital goods sectors as a whole equals the factor intensity in the consumption good sector. Thus, the optimal taxation rule given a balanced budget in the Pecorino (1993) model depends on the relative factor intensity in the three sectors. Thus, a critical assumption is whether factor income arises in all three sectors or just in the final goods sector.

Both the Roubini and Milesi-Ferretti (1994a,b) models and the Pecorino (1993) model are problematic in the sense that there is no market failure or externality to justify the use of factor income taxation. The only reason for introducing factor income taxation is that the government has to raise a revenue in order to finance a lump-sum transfer to consumers.

The Sørensen model has two sectors; a final goods sector and an education sector. As in the Roubini and Milesi-Ferretti models, factor income is assumed only to arise in the final goods sector. Compared with the other three models, the Sørensen model has the advantage that the raised tax revenue is used to finance a public input in education, which is a good reason for introducing factor income taxation. The optimal taxation of capital income equals the fraction of output which is used on public expenditure on education. Furthermore, the optimal taxation of labor income depends on the subsidy given to educational effort and is consequently different from the optimal taxation of capital income. And finally, the optimal subsidy to human capital investment in education can either be positive or negative. Unfortunately, these results depend to a large extent on the specification of the production function in the education sector, which is assumed to be of the Leontief form with complementarity between the public and the private input in education.

The model presented in this paper is an extended version of the Lucas (1988) model of a closed economy which allows for productive use of the collected tax revenue. Several assumptions are made. First, the representative household is assumed to allocate its entire time between work and education, which means that there is no labor-leisure choice in the model. Second, the government is assumed to be restricted in its ability to borrow and lend, which means that it balances its budget every period. Third, factor income is assumed only to arise in the final goods sector, because the human capital input in education usually is left untaxed. Fourth, a positive externality due to the level of education is assumed to be present in the final goods sector. Fifth, the tax revenue is used to finance a public input in education and a subsidy to human capital investment in education.

In a large part of the theoretical literature on optimal taxation in endogenous growth models, the tax revenue is redistributed lump-sum to consumers, see Roubini and Milesi-Ferretti (1994a,b) and Pecorino (1993). Thus, this paper

firstly contributes to the existing literature by allowing the obtained tax revenue to be used productively, namely to finance a public input in education and a subsidy to human capital investment in education. Secondly, the paper considers the case where government spending affects the productivity of the education sector, where previous models typically restrict government spending to be an input in the final goods sector.¹ Thirdly, the paper allows for a broader set of tax instruments, namely a subsidy to educational effort in addition to factor income taxation. Finally, the paper extends the Sørensen (1993) model by allowing for substitution possibilities between the public and private input in education.

The major conclusions drawn in this paper are firstly that the growth rate in the market economy is higher, the higher the subsidy rate to educational effort is and the higher the labor income tax rate is, *ceteris paribus*. In addition, capital income taxation has no effect on the growth rate in the steady state.

Secondly, the optimal subsidy rate is unambiguously positive, because the public and private input factors in education are substitutes. And finally, labor income should be taxed at a lower rate than capital income as long as there is a positive education externality in final goods production. Recall that the optimal taxation rule in the Roubini & Milesi-Ferretti models is to set the rate of capital income taxation equal to the rate of labor income taxation.

The basic model is presented in Section 2. Section 3 and 4 derive the balanced growth equilibrium in a socially planned economy and in a market economy, respectively. Section 5 finds the optimal tax-subsidy structure and Section 6 concludes the paper. In the Appendix to the paper, the effects of factor income taxation are analyzed in an equivalent model, where the human capital is assumed to give rise to income in the education sector.

2. The model

This section presents a two sector endogenous growth model of a closed economy. The production side of the economy consists of a large number of identical and perfectly competitive firms. A final goods sector produces consumption goods and physical capital, while an education sector produces human capital. Human capital is assumed to be embodied in people and is consequently a private good, which is both rival and excludable. The final goods sector uses physical capital and human capital as input factors, while the education sector uses human capital and a congested public good. Human capital is assumed to move freely between the two sectors within each period.

The government is assumed to tax factor income in order to finance the public

¹See for instance Barro and Sala-i-Martin (1995).

input in education and the subsidy to educational effort. Furthermore, the government is assumed to balance its budget every period such that total tax revenue equals total government expenditure.

The consumption side of the economy consists of a large number of identical infinitely lived households that own the input factors and rent them out to firms. Households are assumed to choose consumption and the allocation of human capital between sectors in order to maximize their lifetime utility.

In the following, time subscripts are left out for convenience.

2.1. The final goods sector

Firm i produces final goods Y_i by use of both physical and human capital according to the following Cobb-Douglas production function:

$$Y_i = A_c K_i^\alpha (u_c H_i)^{1-\alpha} H^\varepsilon \quad (2.1)$$

where $0 < \alpha < 1$ is the capital share, $\varepsilon \geq 0$ reflects a positive externality in production that arises from the work force's average level of education H , which firms do not take into account, A_c is a productivity parameter, K (H) is the stock of physical capital (human capital) and u_c is the fraction of human capital that is devoted to the production of final goods. Throughout the paper, depreciation of the physical capital stock is neglected for expositional convenience. Note that due to the presence of the positive education externality, there are constant returns to scale at the firm level, but increasing returns to scale à la Marshall at the aggregate level. Since all firms are identical, the subscripts i may be dropped in the following.

In order to produce final goods, firms rent physical capital from households at the interest rate r and hire human capital at the wage rate w . Firms are assumed to maximize their profits:

$$r = \alpha A_c \left(\frac{u_c H}{K} \right)^{1-\alpha} H^\varepsilon \quad (2.2)$$

$$w = (1 - \alpha) A_c \left(\frac{K}{u_c H} \right)^\alpha H^\varepsilon \quad (2.3)$$

According to equation (2.2) and (2.3) profits are maximized, when the marginal cost of each factor equals its marginal product. Both factor rewards are seen to increase with the size of the education externality. Note that the private return to human capital is lower than its social return. Thus, the smaller the labor share in final goods production and the greater the education externality, the bigger is the difference between the private and the social return to human capital.

2.2. The education sector

Production in the education sector is assumed to use two kinds of input factors. These factors are respectively human capital representing students' time² and a public good representing buildings and professors.³ The public good is congested in the sense that it has to increase relative to total output in order to raise the level productivity in education.

In this paper, the production technology in education is assumed to be of the Uzawa-Lucas type with constant returns to human capital. Furthermore, the production function assumes that there is a certain degree of substitutability between the effective labor input (uH) and the public input (g). This implies that a small effective labor input, e.g. less qualified students, and a large public input, e.g. a large number of professors lead to the same accumulation of human capital as would a large effective labor input, e.g. well qualified students, and a small public input.

The production technology in the education sector is given by the following function:⁴

$$\dot{H} = A_h u_h H g \left(\frac{G}{Y} \right) \quad (2.4)$$

where a dot above a variable indicates its derivative with respect to time, A_h is a productivity parameter, u_h is the fraction of human capital that is devoted to education, G is the public expenditure on education, $g(G/Y)$ is the amount of public input in education, where $g' > 0$ and $g'' < 0$. Depreciation of the human capital stock is neglected for expositional convenience.⁵

According to equation (2.4), there is a certain need for infrastructure such as school buildings in the education sector. In the present model, this input is assumed to be publicly financed, but it could just as well have been privately financed.

Equation (2.4) reveals that a constant growth rate of labor skills and thereby endogenous growth is obtained, when the time fraction spent in education u_h and the public input in education $g \left(\frac{G}{Y} \right)$ are constant.

Note that the following resource constraint applies to the use of human capital:

²Recall that human capital is embodied in human beings and therefore is both rival and excludable.

³The public sector is assumed to rent the services from school buildings, which implies that the public good in this paper is a flow variable. Some authors have looked at the role of a public good as a stock variable, see e.g. Turnovsky (1996).

⁴The Lucas (1988) model is obtained by setting $g \left(\frac{G}{Y} \right) = 1$.

⁵Since older generations are replaced by younger generations and since certain skills deteriorate, if they are not used on a regular basis, it is clearly more realistic to assume that the stock of human capital depreciates.

$$u_c + u_h = 1 \quad (2.5)$$

2.3. The government

The government taxes households' factor income and subsidizes human capital investment in education in order to finance both the public input in education and a lump-sum transfer to consumers. The government is assumed to be restricted in its ability to borrow and lend, which implies that it runs a balanced budget in every period. Thus, total government expenditure equals total tax revenue:

$$G + s_h w u_h H_a + T = \tau_k r K + \tau_h w u_c H \quad (2.6)$$

where G is public expenditure on education, s_h is the subsidy rate on the average level of labor income $w u_h H_a$ in the economy which is foregone when households invest in education, T is a lump-sum transfer to consumers and τ_k (τ_h) is the factor income tax rate on physical capital (human capital).⁶ The subsidy to educational effort is assumed to depend on the average level of labor income $w u_h H_a$, because it probably is the easiest policy to implement. When the subsidy to educational effort depends on the average wage level, the individual household does not take into account that its choice of time spent on education affects the subsidy rate it faces in the future. Alternatively, the subsidy is assumed to depend on the individual wage level $w u_h H$.

The resource constraint of the entire economy is given by:

$$Y - C - G - \dot{K} \geq 0 \quad (2.7)$$

which states that income should be equal to or exceed private and public consumption expenditures and investment in physical capital.

2.4. Households

Households choose consumption C and the allocation of human capital between the two sectors u_h in order to maximize their lifetime utility:

$$U = \int_0^{\infty} \left(\frac{C^{1-\theta}}{1-\theta} - 1 \right) e^{-\rho t} dt \quad (2.8)$$

⁶An implicit assumption in the human capital accumulation function is that taxable income earned by the public input is so small in comparison with the income foregone by students that it is safe to ignore. In appendix A, human capital is treated as a market good that give rise to income in the education sector.

where θ is the inverse of the intertemporal elasticity of substitution, and ρ is the rate of time preference. Note that the instantaneous utility function is assumed to take the Constant Intertemporal Elasticity of Substitution (CIES) form, and that there is no "leisure activity" in the model.⁷

Households maximize their lifetime utility (2.8) subject to the human capital accumulation function (2.4) and their instantaneous budget constraint:

$$(1 - \tau_k)rK + (1 - \tau_h)wu_cH + s_hwu_hH_a + T - C - \dot{K} \geq 0 \quad (2.9)$$

which says that consumption and investment in physical capital have to be financed by the net capital and labor income, the subsidy obtained by investing time in education and the lump-sum transfer.

This section has briefly described the model, which is used in the following analyses. Before the growth rate effects of taxation and subsidization are calculated, it proves convenient to solve both the social planner's and the representative agent's problem. This is done in section 3 and 4, respectively.

3. The planned economy

This section focuses on the social planner's problem and derives the first order conditions for an optimal growth path given the human capital accumulation function.

The social planner maximizes the lifetime utility of the representative household (2.8) subject to the constraint on human capital accumulation (2.4) and the resource constraint of the economy (2.7). The first order conditions with respect to C , K , H , G and u_h are given by:

$$C^{-\theta}e^{-\rho t} = \mu_k \quad (3.1)$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1 - \frac{G}{Y}\right) \alpha A_c \left(\frac{u_c H}{K}\right)^{1-\alpha} H^\varepsilon \quad (3.2)$$

$$-\frac{\dot{\mu}_h}{\mu_h} = \left(1 + \frac{\varepsilon}{1-\alpha} u_c\right) A_h g \quad (3.3)$$

$$\mu_h A_h u_h H g' \frac{1}{Y} = \mu_k \quad (3.4)$$

⁷In the Roubini and Milesi-Ferretti papers (1994a, 1994b and 1995) several different specifications of leisure activity are considered.

$$\mu_h A_h g = \mu_k \left(1 - \frac{G}{Y}\right) (1 - \alpha) A_c \left(\frac{K}{u_c H}\right)^\alpha H^\varepsilon \quad (3.5)$$

where μ_k (μ_h) is the shadow price of physical capital (human capital) in the social planning problem. Note that equations (3.1)-(3.5) describe the first best optimal growth path of the economy.

Equation (3.1) implies that the marginal utility of consumption in every period should equal the shadow price of consumption (physical capital). Equation (3.2) implies that the rate of change of the shadow price of physical capital should equal the marginal product of capital. Equation (3.3) implies that the rate of change of the shadow price of human capital should equal the marginal product of human capital in the education sector. Equation (3.4) describes the optimal allocation of resources between public expenditure on education and production of physical capital.⁸ Thus, if the value of the marginal product of public expenditure on education were higher than the shadow price of physical capital, then it would be optimal to reallocate resources towards the education sector until equality between the two is obtained. Equation (3.5) describes the optimal allocation of human capital between the education sector and the final goods sector. Thus, the value of the marginal product of human capital should be the same in the two sectors. If the marginal product of human capital were higher in the education sector than in the final goods sector, then it would be optimal to reallocate human capital to the education sector until the marginal products in the two sectors were equalized.

The transversality conditions to the maximization problem are:

$$\lim_{t \rightarrow \infty} \mu_k K = 0$$

$$\lim_{t \rightarrow \infty} \mu_h H = 0$$

These conditions rule out explosive paths by requiring that the present discounted value of each capital good equals zero in the long run. This is a reasonable requirement since optimizing agents do not want valuable assets at the end of their planning horizon. Consequently, the first transversality condition requires that the real interest rate should be positive.

This section solved the social planner's problem. Thus, the following section solves the representative agent's problem.

⁸The marginal cost of G in terms of foregone output is 1.

4. The market economy

This section focuses on the representative agent's problem. Firstly, the first order conditions for a balanced growth path in a market economy are derived. Secondly, the balanced growth rate is determined.

The representative household chooses its consumption C and the allocation of human capital u_h in order to maximize its lifetime utility (2.8) subject to the human capital accumulation function (2.4) and its instantaneous budget constraint (2.9) taking the paths of τ_k, τ_h, s_h and g as given.

The first order conditions with respect to C, K, H and u_h are given by:

$$C^{-\theta} e^{-\rho t} = \nu_k \quad (4.1)$$

$$-\frac{\dot{\nu}_k}{\nu_k} = (1 - \tau_k) r \quad (4.2)$$

$$-\frac{\dot{\nu}_h}{\nu_h} = \left(1 + \frac{s_h}{1 - \tau_h - s_h} u_c\right) A_h g \quad (4.3)$$

$$\nu_h A_h g = \nu_k (1 - \tau_h - s_h) w \quad (4.4)$$

where ν_k (ν_h) is the shadow price of physical capital (human capital) in the representative household problem.

Equation (4.1) is identical to equation (3.1). Equation (4.2) implies that the rate of change of the shadow price of physical capital should equal the *after-tax* marginal product of capital. Equation (4.3) implies that the rate of change of the shadow price of human capital should equal the marginal product of human capital in the education sector *adjusted* for the labor income tax and the education subsidy. Equation (4.4) describes the optimal allocation of human capital between the two sectors. Thus, human capital is optimally allocated between the two sectors, when the value of its marginal product equals its *tax and subsidy adjusted* rate of return.

In the following, the balanced growth rate in the market economy is calculated in three steps. Firstly, two steady state relationships are derived. Then, a semi-reduced expression of the balanced growth rate is determined in which the growth rate is a function of the fraction of human capital allocated to final goods production. And finally, the steady state fraction of human capital in final goods production and thereby the balanced growth rate is obtained.

The first step is to derive two steady state relationships, which are used in the derivation of the balanced growth rate. Note firstly that all endogenous variables

should grow at a constant rate in the steady state. According to equation (4.1) and (4.2), this implies that a constant steady state growth rate of consumption requires a constant interest rate.

Thus, logarithmic differentiation of equation (2.2) yields a relationship between the growth rate of human capital and physical capital, which holds in the steady state:

$$\frac{\dot{H}}{H} = \left(\frac{1 - \alpha}{1 - \alpha + \varepsilon} \right) \frac{\dot{K}}{K} \quad (4.5)$$

According to equation (4.5), human capital grows at a slower pace than physical capital as long as there is an education externality in the final goods sector. In absence of the externality, the two types of capital grow at the same rate. A further steady state relationship is obtained by logarithmic differentiation of (4.4) and use of (4.5):

$$-\frac{\dot{\nu}_k}{\nu_k} = -\frac{\dot{\nu}_h}{\nu_h} + \frac{\varepsilon}{1 - \alpha} \frac{\dot{H}}{H} \quad (4.6)$$

Equation (4.6) implies that the shadow price of physical capital must decline at the same rate as the shadow price of human capital plus the growth rate of the wage rate. Or equivalently, that the *after-tax* marginal product of capital should equal the marginal product of human capital in education *adjusted* for the labor income tax and the education subsidy plus the growth rate of wages at each skill level.

The second step is to derive a semi-reduced expression for the balanced growth rate in the market economy. This is done by noting that consumption and physical capital must grow at the same rate in the steady state, while human capital grows at a different rate due to the externality. A semi-reduced expression for the balanced growth rate of consumption and physical capital in the market economy is then obtained by logarithmic differentiation of equation (4.1):

$$\gamma_m \equiv \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{1}{\theta} \left[-\frac{\dot{\nu}_k}{\nu_k} - \rho \right] \quad (4.7)$$

and introduction of equation (4.6), (4.3) and (2.4):

$$\gamma_m = \frac{1}{\theta} \left[A_h g \left(1 + \frac{\varepsilon}{1 - \alpha} + \left(\frac{s_h}{1 - \tau_h - s_h} - \frac{\varepsilon}{1 - \alpha} \right) u_c^* \right) - \rho \right] \quad (4.8)$$

As can be seen from equation (4.8), the balanced growth rate depends on the fraction of human capital, which is allocated to the final goods sector in the steady state. It is immediately seen that an increase in the fraction of human capital allocated to education has a positive influence on the growth rate as long as

$\frac{s_h}{1-\tau_h-s_h} < \frac{\varepsilon}{1-\alpha_c}$, which happens to be the case whenever the subsidy to educational effort and the tax on labor income are smaller than their optimal levels.⁹ In this case, too much human capital is allocated to final goods production, which means that an increase in the time fraction spent in education is growth enhancing. Equivalently, an increase in u_h^* has a negative growth rate effect, whenever the subsidy to educational effort and the tax on labor income are larger than their optimal levels.

The third step is to derive u_c^* and thereby the balanced growth rate. In the following, the fraction of human capital allocated to the final goods sector is derived by use of three steady state properties. Firstly, the interest rate must be constant in the steady state, which implies that the growth rate of $\omega \equiv K^{1-\alpha}H^{-(1-\alpha+\varepsilon)}$ must be zero. Secondly, consumption and physical capital grow at the same rate in the steady state, which implies that the growth rate of $\chi \equiv C/K$ must be zero. Finally, the fraction of human capital used in the final goods sector must be constant in the steady state, since $0 < u_c < 1$. These steady state properties yield three equations in ω , χ and u_c , which determine the steady state fraction of human capital allocated to final goods production as:¹⁰

$$u_c^* = \frac{\frac{\theta-1}{\theta} + \frac{\varepsilon}{1-\alpha} \frac{\theta-1}{\theta} + \frac{\rho}{\theta A_h g}}{1 + \frac{\theta-1}{\theta} \frac{\varepsilon}{1-\alpha} + \frac{1}{\theta} \frac{s_h}{1-\tau_h-s_h}} \quad (4.9)$$

According to (4.9), the fraction of time spent at work in the steady state u_c^* depends positively on the education externality ε , the share of physical capital in final goods production α and the rate of time preference ρ , while it depends negatively on the intertemporal elasticity of substitution $\frac{1}{\theta}$, the level of productivity in education A_h , the public input in education g , the tax on labor income τ_h and the subsidy to educational effort s_h .

Consequently, human capital is reallocated from final goods production towards education, whenever the marginal product of human capital in education increases due to an increase in the level of productivity in education or in the public input in education. Equivalently, a reduction in the "tax and subsidy" adjusted rate of return to human capital in the final goods sector¹¹ due to an increase in the tax on labor income or in the subsidy to educational effort also result in a reallocation of human capital from final goods production towards education. Furthermore, a reduction in the education externality or an increase in the labor share in final goods production leads to a decline in the difference between the

⁹The optimal factor income taxes and the optimal subsidy to educational effort are obtained in the following section.

¹⁰See the appendix B for the derivation of u_c^* and the steady state values of χ and ω .

¹¹Or equivalently, the opportunity cost of education, namely the wage rate earned in the final goods sector reduced by the labor income tax and the subsidy which is foregone while working.

private and the social return to human capital in final goods production. The implication is that less human capital must be allocated to the final goods sector in order to exploit the positive externality stemming from education. Finally, the fraction of time spent in education is higher, the more willing households are to substitute present for future consumption and the more patient they are.

Proposition 4.1. *The fraction of human capital allocated to the final goods sector in the steady state is higher, the less willing agents are to substitute present for future consumption, the lower the level of productivity in education, the larger the education externality, the smaller the share of human capital in final goods production, the smaller the public input in education, the lower the labor income tax rate, the lower the subsidy to educational effort and the less patient agents are.*

Note that the tax on capital income τ_k and the level of productivity in the final goods sector A_c have no influence on the time fraction spent in final goods production in the steady state.

Now, the balanced growth rate in the market economy can be determined by introduction of (4.9) in (4.8):

$$\gamma_m = \frac{1}{\theta} [A_h g \left(1 + \frac{\varepsilon}{1-\alpha} + \left(\frac{s_h}{1-\tau_h-s_h} - \frac{\varepsilon}{1-\alpha} \right) \frac{\frac{\theta-1}{\theta} + \frac{\varepsilon}{1-\alpha} \frac{\theta-1}{\theta} + \frac{\rho}{\theta A_h g}}{1 + \frac{\theta-1}{\theta} \frac{\varepsilon}{1-\alpha} + \frac{1}{\theta} \frac{s_h}{1-\tau_h-s_h}} \right) - \rho] \quad (4.10)$$

Finally, the growth rate effects of changes in the tax rates, the subsidy and the parameters of the model are determined from equation (4.10). Inspection of the semi-reduced expression for the balanced growth rate in the market economy (4.8) reveals that there are two effects at work, namely a direct "growth rate effect" and an indirect "human capital allocation effect". These effects work in the same direction as long as the subsidy to educational effort and the tax on labor income are smaller than their optimal levels, while they work in opposite directions whenever s_h and τ_h are larger than their optimal levels.

In general, the balanced growth rate in the market economy depends positively on the education externality ε , the physical capital share in final goods production α , the labor income tax τ_h and the subsidy to educational effort s_h .¹²

Whenever the subsidy to educational effort and the tax on labor income are lower than their optimal levels, the two effects work in the same direction, which

¹²Thus in these cases, the direct "growth rate effect" outweighs the indirect "human capital allocation effect", whenever the subsidy and the labor income tax are greater than their optimal levels.

implies that the balanced growth rate in the market economy in addition depends positively on the intertemporal elasticity of substitution $\frac{1}{\theta}$, the level of productivity in education A_h and the public input in education g , while it depends negatively on rate of time preference ρ .

When the subsidy and the labor income tax are higher than their optimal levels, the "growth rate effect" and the "human capital allocation effect" work in opposite directions, which means that the growth rate effects of $\frac{1}{\theta}$, ρ , A_h and g become ambiguous.

Proposition 4.2. *The balanced growth rate in the market economy is higher, the larger the education externality is and the larger the physical capital share in final goods production is. In addition, the balanced growth rate is larger, the more willing agents are to substitute present for future consumption, the higher the productivity in the education sector is, the larger the public input in education is and the more patient agents are, whenever the subsidy to educational effort and the labor income tax rate are lower than their optimal levels.*

Returning to equation (4.10), it is immediately seen that the balanced growth rate effect of capital income taxation is zero, while the growth rate effects of labor income taxation and subsidization of educational effort are unambiguously positive for all values of τ_h and s_h , see equation (B.7) and (B.8) in Appendix B.2.

Proposition 4.3. *A change in the tax on capital income has no effect on the balanced growth rate in the market economy, ceteris paribus. However, both an increase in the tax on labor income and an increase in the subsidy to educational effort leads to an increase in the balanced growth rate in the market economy, ceteris paribus.*

The reason behind the zero balanced growth rate effect of capital income taxation is that physical capital is not used as an input factor in education. Thus, an increase in the capital income tax leads to an offsetting increase in the *pre-tax* interest rate such that the *after-tax* interest rate and thereby the growth rate is left unaltered. However, the physical capital intensity in final goods production is reduced by an increase in the capital income tax.

In addition, the balanced growth rate effect of labor income taxation becomes zero in the absence of a subsidy to educational effort ($s_h = 0$). The reason behind this result is that in this case the opportunity cost of education equals foregone wages. Thus, the opportunity cost of education and the after-tax wage rate are affected by a change in the labor income tax by the same amount. This implies that the allocation of human capital between the final goods sector and education is left unaltered by a change in the labor income tax, whereby the growth rate is left unaltered.

In the presence of a subsidy to educational effort ($s_h > 0$), the growth rate effect of labor income taxation becomes positive, because the opportunity cost of education is reduced less by an increase in the labor income tax, than the after-tax wage rate earned in the final goods sector. This leads to a reallocation of human capital towards education and thereby to an increase in the growth rate.

Note that changes in the tax and subsidy rates and in public spending on education result in changes in the tax revenue, which are captured by the lump-sum transfer to consumers, see equation (2.6). Section 5 therefore derives the optimal tax and subsidy rates.

This section has obtained the balanced growth rate in the market economy and analyzed the balanced growth rate effects of changes in the tax and subsidy rates and in the parameters of the model.

In the following section, the optimal tax-subsidy structure is derived.

5. Optimal tax and subsidy rates

This section derives the optimal tax and subsidy rates in the model. Throughout the section it is assumed that the government runs a balanced budget every period, where the tax revenue is spend solely on the subsidy to education and the public input in education, see equation (2.6).

The optimal tax and subsidy rates are derived by comparison of the first order conditions to the social planner's problem (3.1)-(3.5) and the representative household's problem (4.1)-(4.4):

$$\tau_k^* = \frac{G}{Y} \quad (5.1)$$

$$\tau_h^* = \frac{G}{Y} - s_h^* \quad (5.2)$$

$$s_h^* = \left(1 - \frac{G}{Y}\right) \frac{\varepsilon}{1 - \alpha} \quad (5.3)$$

$$g' \left(\frac{G}{Y} \right) \frac{u_h}{u_c} = \frac{g \left(\frac{G}{Y} \right)}{(1 - \alpha) \left(1 - \frac{G}{Y} \right)} \quad (5.4)$$

where the obtained optimal tax and subsidy rates are welfare maximizing. Note that the derived optimal tax-subsidy structure holds both in the steady state and outside the steady state.

It is immediately seen that $\tau_k^* = \tau_h^* + s_h^*$. This implies that it is optimal to reduce the social rate of return to physical and human capital by the same amount in the presence of a public input in education, namely by $1 - \frac{G}{Y}$. Thus, the optimal rate of capital income taxation τ_k^* equals the fraction of output, which is used as government spending on education. The optimal rate of labor income taxation τ_h^* equals public expenditure on education relative to total output less the optimal subsidy to education. The optimal subsidy rate on education s_h^* is seen to depend negatively on the public input in education G/Y , positively on the education externality ε and negatively on the human capital share in final goods production $(1 - \alpha)$. Finally, the optimal fraction of output, which is used as government spending on education G/Y is determined in equation (5.4). It is seen that the more human capital is allocated to the education sector u_h , the higher is the marginal product of public spending in education and thereby the bigger is the optimal fraction of output spent on public input in education.

When the public input in education increases it becomes possible to lower the subsidy to educational effort, because human capital investment and public investment in education are substitutes. The higher the human capital share in final goods production $(1 - \alpha)$ is and the lower the education externality ε is, the smaller is the difference between the private and the social marginal product of human capital in final goods. Consequently, the subsidy to educational effort does not need to be so high in order to exploit the positive externality stemming from the average level of human capital.

Proposition 5.1. *The optimal subsidy rate on education is higher, the smaller the public input in education is, the larger the human capital externality is and the smaller the human capital share in final goods production is.*

According to equation (5.3), the optimal subsidy to educational effort is unambiguously positive in a model with substitution possibilities between input factors in education. In contrast, the optimal subsidy can either be positive or negative in an equivalent model with no substitution possibilities between the public and the private input in education, see Sørensen (1993). Due to the complementarity between the public and the private input in education in the Sørensen model, a high requirement of the public input relative to human capital makes it optimal to charge a tuition fee in order to finance the public expenses associated with the input in education. In the presented model, a high input of the public good in education just leads to a low subsidy to educational effort due to the substitutability between the public and the private input in education. Consequently, it is never optimal to charge tuition fees as long as there are substitution possibilities between input factors in the education sector.

Implementation of the optimal policies (5.1)-(5.4) means that the government has to resort to lump-sum taxation in order to achieve the first best allocation of resources, since the optimal factor income taxes cannot fully cover the expenses on the optimal subsidy to educational effort and the public input in education. However, in order to achieve the first best optimum, there is still a need for factor income taxation and subsidization of educational effort even in the presence of a lump-sum tax instrument.

If the subsidy alternatively is assumed to depend on the individual wage level $wu_h H$, then the optimal subsidy would be given by s_h^* in equation (5.3) multiplied by the fraction of human capital devoted to final goods production u_c . Thus, the optimal subsidy rate is lower, when households take into account that its choice of time spent on education affects the subsidy rate it faces in the future. In appendix A, the optimal tax and subsidy rates are derived in an equivalent model, where human capital is treated as a market good that give rise to income in both sectors. In this case the optimal capital and labor income tax are unaltered, but the optimal subsidy to educational effort is negative. When agents earn the same wage rate both in final goods production and in education, they tend to allocate too small a fraction of their time to work in the final goods sector. Thus, in order to exploit the positive education externality it becomes optimal to charge a tuition fee, which induces agents to allocate more human capital to final goods production. However, the assumption that human capital is a market good is unrealistic, since the implicit labor income in the production of human capital is usually not taxed.

Proposition 5.2. *The optimal subsidy rate on human capital investment in education is unambiguously positive, when the public and the private input factor in education are substitutes.*

Proposition 5.3. *In the presence of a positive education externality in final goods production, labor income should be taxed at a lower rate than capital income. The bigger the externality is and the smaller the labor share in final goods production is, the bigger is the difference between the rate of capital and labor income taxation.*

In table 1 below, the optimal tax and subsidy rates are given for several special cases of the presented model. Recall that the Lucas (1988) model is obtained, when there is no public input in education $g = 1$. Firstly, table 1 reveals that it is optimal not to impose any taxes or subsidies in the Lucas model without an education externality in the final goods sector ($\varepsilon = 0$). Secondly, in the Lucas model with a positive education externality ($\varepsilon > 0$) it is optimal to subsidize human capital investment in final goods production and education at the same

rate, because it leads to an increase in the marginal product of human capital adjusted for the labor income tax and the subsidy, while leaving the opportunity cost of education unaffected. In this case, the first best optimum can only be achieved, if these subsidies are financed by lump-sum taxation $T < 0$. Thirdly, in a version of the presented model with a public input in education ($g \neq 1$) and no education externality in the final goods sector ($\varepsilon = 0$), the optimal tax structure is to set a comprehensive tax on factor income in order to finance public spending and a zero subsidy to educational effort. In this case, the first best optimum is obtained without the use of the lump-sum tax instrument. Recall fourthly that the first best optimum in the presented model only can be achieved, if a lump-sum tax is imposed in addition to the optimal factor income taxes.

Table 1	τ_k^*	τ_h^*	s_h^*	T
$g = 1, \varepsilon = 0$	0	0	0	0
$g = 1, \varepsilon > 0$	0	$-s_h^*$	$\frac{\varepsilon}{1-\alpha}$	—
$g \neq 1, \varepsilon = 0$	$\frac{G}{Y}$	$\frac{G}{Y}$	0	0
$g \neq 1, \varepsilon > 0$	$\frac{G}{Y}$	$\frac{G}{Y} - s_h^*$	$\left(1 - \frac{G}{Y}\right) \frac{\varepsilon}{1-\alpha}$	—

Thus, the presence of productive public spending suggests the efficiency of non-lump-sum taxes.

Now the steady state growth rate in the planned economy or equivalently the optimal growth rate is obtained by introduction of the optimal policies (5.1)-(5.4) in equation (4.10):

$$\gamma_p = \frac{1}{\theta} \left[A_h g \left(1 + \frac{\varepsilon}{1-\alpha} \right) - \rho \right] \quad (5.5)$$

where it is seen that the optimal growth rate depends positively on the intertemporal elasticity of substitution $\frac{1}{\theta}$, the education externality ε , the level of productivity in education A_h , the physical capital share in final goods production α and the public input in education g , while it depends negatively on the rate of time preference ρ .

Comparison of the balanced growth rate in the socially planned economy and the market economy reveals that:

$$\begin{aligned} \gamma_m &< \gamma_p, \text{ when } s_h < s_h^* \text{ and } \tau_h < \tau_h^* \\ \gamma_m &> \gamma_p, \text{ when } s_h > s_h^* \text{ and } \tau_h > \tau_h^* \end{aligned} \quad (5.6)$$

Since the social planner growth rate is welfare maximizing, welfare is rising (falling) for increasing growth rates below (above) the optimal growth rate (5.5).

Proposition 5.4. *The balanced growth rate in the market economy is lower (higher) than the optimal growth rate, whenever the subsidy to educational effort and the tax on labor income are smaller (bigger) than their optimal levels. Thus, growth maximization is not equivalent to welfare maximization except in cases where the balanced growth rate in the market economy equals the optimal growth rate.*

This section derived the optimal tax and subsidy rates, which should be imposed in a market economy in order to achieve the first best optimum. The following section summarizes and concludes the paper.

6. Summary and conclusion

This paper has examined the effects of capital and labor income taxation in a two sector model of endogenous growth, where the tax revenue is used to finance a public input in education and a subsidy to educational effort.

Households were assumed to allocated their time between work in the final goods sector and education. The analysis revealed that households allocate more time to education, the higher the marginal product of human capital in education is, the lower the opportunity cost of education is, the more willing households are to substitute present for future consumption and the more patient they are.

In addition, the growth rate effects of changes in the tax-subsidy structure were found. It was shown on the one hand that a change in the tax on capital income has no effect on the balanced growth rate in the market economy, eventhough it reduces the physical capital intensity of the final goods production. On the other hand, the growth rate effects of a labor income tax and a subsidy to educational effort were found to be unambiguously positive. In fact there were two effects at work behind the latter two results. The first effect is a direct "growth rate" effect, which is positive because both an increase in the labor income tax rate and the subsidy rate tend to increase the tax and subsidy adjusted marginal product of human capital in education. The second effect is a "human capital allocation" effect, which is positive whenever the labor income tax and the subsidy are below their optimal levels, but negative when they are above. In the latter case, the "growth rate" effect dominates the "human capital allocation" effect, whereby the overall effect of a labor income tax and a subsidy to educational effort on the balanced growth rate becomes positive.

Furthermore, the balanced growth rate was shown to increase with the growth rate of wages at each skill level that is with the education externality in the final goods sector and the physical capital share in production.

The paper shows that the optimal subsidy rate on education is unambiguously positive, when the public and private input factors in education are substitutes.

A bigger optimal subsidy to educational effort is required, the smaller the public input in education is, because more human capital is needed as an input in education in order to maintain a certain level of quality. In order to give households the right incentives to allocate enough human capital to education, a bigger optimal subsidy is also required, the higher the growth rate of wages at each skill level is.

The optimal capital income tax was shown to equal the fraction of output which the government spends on the public input in education, while the optimal labor income tax was shown to be lower than the optimal capital income tax due to the presence of the positive education externality in the final goods sector.

If the optimal policies were imposed, the government would have to resort to lump-sum taxation in order to achieve the first best allocation of resources, since the optimal factor income taxes cannot fully cover the expenses on the optimal subsidy to educational effort and the public input in education.

Finally, it was found that it is not necessarily welfare improving to achieve a higher balanced growth rate through an increase in the labor income tax and the education subsidy. In fact it is only welfare improving to increase the labor income tax and the subsidy as long as they are smaller than their optimal levels.

There are several ways in which the present paper can be extended. Firstly, the government could be assumed to have no borrowing constraints. This would probably lead to high taxation in the short run in order to build up sufficient assets to finance government spending in the long run. Secondly, the dynamic adjustment to the balanced growth path could be derived in cases, where the initial physical to human capital ratio in the final goods sector either is higher or lower than its steady state value.

A. Appendix

A.1. Human capital as a market good

This appendix considers the case, where human capital is a market good that gives rise to income both in the final goods sector and in the education sector. This assumption changes the household budget constraint (2.9) as follows:

$$(1 - \tau_k)rK + (1 - \tau_h)wH + s_h w u_h H + T - C - \dot{K} \geq 0 \quad (\text{A.1})$$

An implicit assumption in equation (A.1) is that agents take into account that their choice of time spent in education affects the subsidy rate they face in the future.

The representative household maximizes its lifetime utility (2.8) subject to the constraint on human capital accumulation (2.4) and its budget constraint (A.1).

The first order conditions with respect to C , K , H and u_h are given by:

$$C^{-\theta} e^{-\rho t} = \psi_k \quad (\text{A.2})$$

$$-\frac{\dot{\psi}_k}{\psi_k} = (1 - \tau_k) r \quad (\text{A.3})$$

$$-\frac{\dot{\psi}_h}{\psi_h} = \left(-\frac{1 - \tau_h}{s_h} \right) A_h g \quad (\text{A.4})$$

$$\psi_h A_h g = \psi_k (-s_h) w \quad (\text{A.5})$$

where ψ_k (ψ_h) is the shadow price of physical capital (human capital) in the representative household problem.

Now, the optimal tax and subsidy rates can be derived by comparison of the first order conditions to the social planner's problem (3.1)-(3.5) and the representative household's problem (A.2)-(A.5). The resulting optimal policies are:

$$\tau_k^* = \frac{G}{Y} \quad (\text{A.6})$$

$$\tau_h^* = \frac{G}{Y} - \left(1 - \frac{G}{Y} \right) \frac{\varepsilon}{1 - \alpha} u_c \quad (\text{A.7})$$

$$s_h^* = - \left(1 - \frac{G}{Y} \right) \quad (\text{A.8})$$

As before, the rate of capital income taxation equals the fraction of output, which is used as government spending on education and the optimal labor income tax is also unaltered.

However, the optimal subsidy to educational effort becomes negative, which means that it is optimal to charge a tuition fee.

This results hinges on the fact that households earn the same wage rate in both sectors. Thus, household maximization leads to a too small fraction of time spent at work in the steady state given the positive education externality in the final goods sector. Thus, in order to induce agents to allocate more human capital to final goods production it becomes optimal to charge a tuition fee.

B. Appendix

B.1. Derivation of the steady state values of u_c , χ and ω

In the steady state, the values of $\omega = K^{1-\alpha} H^{-(1-\alpha+\varepsilon)}$, $\chi = \frac{C}{K}$ and u_c are constant.

Firstly, the growth rate of ω is derived from the household budget constraint (2.9) and the human capital accumulation function (2.4):

$$\begin{aligned} \gamma_\omega = 0 = & (1 - \alpha) \left[\left(\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_h) + (1 - \alpha)s_h \frac{1 - u_c}{u_c} \right) \right. \\ & \left. A_c u_c^{1-\alpha} \omega^{-1} - \chi \right] - (1 - \alpha + \varepsilon) A_h g(1 - u_c) \end{aligned} \quad (\text{B.1})$$

Secondly, the growth rate of χ is derived from (4.7), (4.2), (2.2) and the household budget constraint (2.9):

$$\begin{aligned} \gamma_\chi = 0 = & \frac{\alpha(1 - \tau_k) - \theta \left(\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_h) + (1 - \alpha)s_h \frac{1 - u_c}{u_c} \right)}{\theta} \\ & A_c u_c^{1-\alpha} \omega^{-1} + \chi - \frac{\rho}{\theta} \end{aligned} \quad (\text{B.2})$$

Thirdly, the growth rate of u_c is derived by logarithmic differentiation of (4.4) and use of (4.2), (4.3), (2.9) and (2.4):

$$\begin{aligned} \gamma_u = 0 = & \frac{1}{\alpha} A_h g \left(1 + \frac{s_h}{1 - \tau_h - s_h} u_c \right) \\ & + (1 - \alpha) \left(-(1 - \tau_k) + (1 - \tau_h) + s_h \frac{1 - u_c}{u_c} \right) \\ & A_c u_c^{1-\alpha} \omega^{-1} - \chi - \frac{\alpha - \varepsilon}{\alpha} A_h g(1 - u_c) \end{aligned} \quad (\text{B.3})$$

Now, the steady state values of u_c , χ and ω are derived by solving the system of three equations (B.1)-(B.3) above:

$$u_c^* = \frac{\frac{\theta-1}{\theta} + \frac{\varepsilon}{1-\alpha} \frac{\theta-1}{\theta} + \frac{\rho}{\theta A_h g}}{1 + \frac{\theta-1}{\theta} \frac{\varepsilon}{1-\alpha} + \frac{1}{\theta} \frac{s_h}{1-\tau_h-s_h}} \quad (\text{B.4})$$

$$\begin{aligned} \chi^* = & \frac{\left(\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_h) + (1 - \alpha)s_h \frac{1 - u_c^*}{u_c^*} \right) \rho}{\alpha(1 - \tau_k)} \\ & - \frac{\alpha(1 - \tau_k) - \theta \left(\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_h) + (1 - \alpha)s_h \frac{1 - u_c^*}{u_c^*} \right)}{\alpha(1 - \tau_k)} \\ & \frac{1 - \alpha + \varepsilon}{1 - \alpha} A_h g(1 - u_c^*) \end{aligned} \quad (\text{B.5})$$

$$\omega^* = A_c u_c^{*(1-\alpha)} \left(\frac{\rho}{\alpha(1 - \tau_k)} + \frac{\theta}{\alpha(1 - \tau_k)} \frac{1 - \alpha + \varepsilon}{1 - \alpha} A_h g(1 - u_c^*) \right) \quad (\text{B.6})$$

where the calculations are available from the author upon request. Note that u_c^* is bounded between 0 and 1, which requires that $\rho < A_h g \left(1 - \frac{s_h}{1 - \tau_h - s_h} \right)$.

B.2. Balanced growth rate effects

The growth rate effect of labor income taxation is:

$$\frac{\partial \gamma_m^*}{\partial \tau_h} = \frac{1}{\theta} A_h g \frac{s_h}{(1 - \tau_h - s_h)^2} u_c^* \left(1 - \frac{\frac{1}{\theta} \left(\frac{s_h}{1 - \tau_h - s_h} - \frac{\varepsilon}{1 - \alpha} \right)}{1 + \frac{\theta - 1}{\theta} \frac{\varepsilon}{1 - \alpha} + \frac{1}{\theta} \frac{s_h}{1 - \tau_h - s_h}} \right) > 0 \quad (\text{B.7})$$

The growth rate effect of subsidizing educational effort is:

$$\frac{\partial \gamma_m^*}{\partial s_h} = \frac{1}{\theta} A_h g \left(\frac{1 - \tau_h}{(1 - \tau_h - s_h)^2} \right) u_c^* \left(1 - \frac{\frac{1}{\theta} \left(\frac{s_h}{1 - \tau_h - s_h} - \frac{\varepsilon}{1 - \alpha} \right)}{1 + \frac{\theta - 1}{\theta} \frac{\varepsilon}{1 - \alpha} + \frac{1}{\theta} \frac{s_h}{1 - \tau_h - s_h}} \right) > 0 \quad (\text{B.8})$$

The growth rate effect of the education externality is:

$$\frac{\partial \gamma_m^*}{\partial \varepsilon} = \frac{1}{\theta} A_h g \frac{1}{1 - \alpha} (1 - u_c^*) \left(1 + \frac{\theta - 1}{\theta} \frac{\left(\frac{s_h}{1 - \tau_h - s_h} - \frac{\varepsilon}{1 - \alpha} \right)}{1 + \frac{\theta - 1}{\theta} \frac{\varepsilon}{1 - \alpha} + \frac{1}{\theta} \frac{s_h}{1 - \tau_h - s_h}} \right) > 0 \quad (\text{B.9})$$

The growth rate effect of the physical capital share in final goods production is:

$$\frac{\partial \gamma_m^*}{\partial \alpha_c} = \frac{1}{\theta} A_h g \frac{\varepsilon}{(1 - \alpha)^2} (1 - u_c^*) \left(1 + \frac{\theta - 1}{\theta} \frac{\left(\frac{s_h}{1 - \tau_h - s_h} - \frac{\varepsilon}{1 - \alpha} \right)}{1 + \frac{\theta - 1}{\theta} \frac{\varepsilon}{1 - \alpha} + \frac{1}{\theta} \frac{s_h}{1 - \tau_h - s_h}} \right) > 0 \quad (\text{B.10})$$

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