Abstract
Tax Policy, Venture Capital, and Entrepreneurship*

by

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Abstract

The paper studies the effects of tax policy on venture capital activity. Entrepreneurs pursue a single high risk project each but have no own resources. Financiers provide equity finance. They must structure the entrepreneur’s profit share and base salary to assure their incentives for full effort. In addition to providing equity finance, venture capitalists assist with valuable business advice to enhance survival rates. Within a general equilibrium framework with a traditional and an entrepreneurial sector, the paper investigates the effects of taxes on the equilibrium level of entrepreneurship and managerial advice. It considers differential wage and capital income taxes, a comprehensive income tax, incomplete loss offset, progressive taxation as well as investment and output subsidies to the entrepreneurial sector.

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1 Introduction

Financing early stage businesses involves special problems and is fundamentally different from financing investments by mature and well established companies. Because of lacking collateral and the absence of any past track record, and due to their informational advantages, pioneering entrepreneurs often face severe difficulties in convincing banks to finance projects with potentially high returns but high risks as well. Another problem that contains the roots of business failure, is the commercial inexperience of new entrepreneurs. They tend to be equipped with excellent technical science expertise but usually lack business experience and managerial training. Venture capital has come to specialize in financing early stage investment. Venture capitalists (VCs) not only supply equity finance but also provide valuable business advice to enhance survival chances of new start-ups. Viewing start-up investment as a key source of innovation, growth and employment, policy makers often emphasize the need to enhance entrepreneurship and venture capital activity.¹

The traditional literature on entrepreneurship, risk bearing and taxation ignores a distinct feature of venture capital finance, i.e. the productive contribution of financiers to the survival and success of start-up firms.² The tax literature on adverse selection in investment finance similarly excludes an active role of financiers.³ Despite the importance attached to venture capital by the policy community, a systematic analysis of public policy in this context has largely been neglected. The exceptions are Gordon (1998) and Poterba (1989a,b). Gordon points to the importance of tax avoidance through business ownership as a determinant of entrepreneurship. In this context, he briefly addresses the implications of asymmetric information for the availability of outside equity finance such as venture

¹ A recent OECD report on Austria, for example, includes a special feature on promoting entrepreneurship, see OECD (1999). Similar issues are also discussed in European Commission (1999).
capital and also studies the role of various tax instruments. Poterba investigates the effects of capital gains taxes on the supply of venture capital. Neither of them is very specific on the contractual problems in VC finance. Both abstract from managerial support and tax incentives on this margin. Many business failures, however, result from avoidable management mistakes that originate in the managerial incompetence of entrepreneurs in the early stages of their career. An active role of VCs in providing valuable business advice might be an important factor in raising survival chances of start-up businesses. It thus seems interesting to ask which factors determine the incentives to provide advice. Could taxes and other government activity improve upon such incentives and, thereby, boost survival rates by improving the ‘quality’ of venture capital finance?

This paper extends the existing literature on entrepreneurship and taxation in allowing for an active role of financiers and providing an analysis of various tax policy initiatives. We propose a stylized general equilibrium model of entrepreneurship and venture capital, featuring two sectors: one producing ‘traditional’ goods and another entrepreneurial sector where an ‘innovative’ good is produced with an inherently risky technology, and where informational problems loom large. The model conforms well with some important stylized facts of venture capital finance.\(^4\) Financiers provide start-up finance in exchange for an equity share. The typical arrangement consists of a low base salary combined with profit participation. In financing a portfolio of companies, venture capital funds are able to diversify risk and could, in principle, fully insure the entrepreneur. Risk diversification, however, is limited by the extent of moral hazard in the relation between entrepreneur and financier. The equity contract must thus be structured to retain the entrepreneur’s full commitment and effort in the face of a moral hazard problem that results from entrepreneurial effort being non-observable and non-verifiable. While the entrepreneur’s effort certainly is critical for the venture to have any positive survival chance at all, the financier also contributes with valuable business advice to further enhance survival rates.\(^5\)


\(^5\)In focusing on the advisory activity and tax incentives, the paper obviously neglects other important
Apart from solving incentive problems with respect to entrepreneurial effort, the venture capital contract must be sufficiently generous to attract entrepreneurs in the presence of alternative career opportunities. Agents may either go for a safe worker’s salary in the traditional sector or opt for an entrepreneurial career with potentially high rewards but high risk as well. The equilibrium solution with occupational choice splits the population into entrepreneurs, consultants and workers and endogenously determines the quality of venture capital finance (i.e. the extent of managerial advice). We then investigate the effects of a broad range of tax instruments such as differential wage and corporate income taxes, progressive taxation, incomplete loss offsets, an investment subsidy and an output subsidy to portfolio companies. The paper now proceeds with presenting the model in section 2. Section 3 discusses the effects of proportional taxes on the equilibrium level of managerial advice and venture capital backed start-up investment. Section 4 addresses the welfare implications of policy. Section 5 considers incomplete loss offset and progressive taxation. Section 6 summarizes and discusses future research.

2 The Model

2.1 Definitions

Overview: The economy consists of two sectors, producing ‘traditional’ and ‘innovative’ goods, respectively. A deterministic Ricardian technology is available for production of the traditional good with one unit of labor yielding one unit of output. Choosing the standard good as a *numeraire*, its price and the wage rate are both equal to one. Innovative goods result from an entrepreneurial activity which is inherently risky and requires a fixed start-aspects of venture capital finance such as two-sided moral hazard between entrepreneurs and venture capital firms [e.g. Repullo and Suarez (1998)], or stage financing and convertible debt [e.g. Cornelli and Yafeh (1997)]. Venture capital firms are also intensively screening projects. The effects of taxes in such a context are discussed in the above mentioned literature on investment with adverse selection, although the intensity of screening is usually not considered.
up investment on top of the entrepreneur’s input. Each entrepreneur pursues exactly one venture that yields one unit of output with probability $p$ and nothing with probability $1 - p$. Projects will fail with certainty, however, if entrepreneurs choose not to devote full effort and attention to their venture.

Households are risk averse and choose to become workers or entrepreneurs. Since entrepreneurs pursue only one project, they face an undiversifiable income risk. No income accrues if the venture fails. In face of this existential income risk, entrepreneurial activity can emerge only if financial intermediation provides sufficient insurance. Assuming project risks to be stochastically independent, financiers are able to partially insure entrepreneurs by financing a diversified portfolio of projects. By the law of large numbers, the aggregate economy is free of risk.

**Risk, Effort and Advice:** Survival probability $p$ is assumed to depend on effort $e$ which cannot be verified and contracted by an outside investor. A minimum amount $0 < \delta < 1$ of the entrepreneur’s time input is freely observable. Only the rest of time $1 - \delta$ is under discretion and is assumed not to be observable by outsiders. High effort means that, in addition to the basic activity $\delta$, the entrepreneur also devotes $1 - \delta$ of her time exclusively to the venture. Low effort or shirking means that it is directed to some lucrative outside activity. Only high effort implies a positive survival chance $p > 0$, while low effort results in business failure for sure, $p = 0$. We suppress the effort variable in the probability $p$, knowing that it is positive only if the entrepreneur supplies high effort. In addition to the entrepreneur’s effort, we also postulate a productive contribution of the VC consisting of some managerial services $a$:

$$
p = p(a), \quad p' > 0 > p'' , \quad p(0) = p_0 > 0, \quad \lim_{a \to \infty} p(a) < 1.
$$

**Taxation of Portfolio Company:** A corporate income tax (CIT) at rate $\tau$ and an output subsidy of $\sigma$ to innovative goods give an expected net income of the portfolio

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6 We use $p'$ as a short-hand for $dp/da$. 

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company equal to
\[(1 - \tau) (pQ(1 + \sigma) - b) - (1 - z) K,\]
where \(Q\) is the consumer price of innovative goods. Profits are reduced by the entrepreneur’s base salary \(b\). Setting up a business also requires a fixed start-up investment \(K\), part of which is subsidized with an investment tax credit at rate \(z\). Investment demand is for traditional goods. In case of business failure, the company runs up a loss equal to the base salary and the start-up investment cost net of taxes. At this stage, we assume that VCs can offset any losses against income from successful projects.

**Taxation of Venture Capitalist:** Entrepreneurs have no funds of their own. To get the firm started, the VC must thus inject equity in the amount of \(I = (1 - \tau) b + (1 - z) K\), which is in exchange for a share \(1 - s\) of the company’s cash flow. The VC earns gross revenues \((1 - s) Q(1 + \sigma)\) and pays CIT on her own expected operating profits equal to \(\tau [(1 - s) (1 + \sigma) pQ - a]\) where equity purchases equal to \(I\) are not deductible. VC firms hire a ‘consultants’ per project to supply business advice. Note that the VC calculates with expected profits because she is assumed to hold a diversified portfolio of start-up companies that eliminates all income risk on her part. For each project, her expected, net of tax profit is

\[
\Pi = (1 - \tau) [(1 - s) pQ (1 + \sigma) - b - a] - (1 - z) K. \tag{2}
\]

The entrepreneur’s income directly subtracts from the amount of income that may possibly be claimed by the VC. The expected cost of entrepreneurial compensation to the VC is

\[
c = (1 - \tau) [spQ (1 + \sigma) + b]. \tag{3}
\]

It will prove useful to write expected, net profits of the VC firm as

\[
\Pi = (1 - \tau) [pQ (1 + \sigma) - a] - (1 - z) K - c. \tag{4}
\]
Taxation of Entrepreneur: Apart from a base salary $b$, the entrepreneur receives income from her equity share $s$ in the company. Suppose, for simplicity, that the CIT rate $\tau$ is equal to the personal tax rate on capital income. There is no further tax burden at the individual level. The entrepreneur’s base salary, however, is subject to a wage tax at rate $t$. Expected entrepreneurial income net of taxes thus amounts to

$$c^N = s(1 - \tau) p Q (1 + \sigma) + (1 - t) b. \quad (5)$$

Demand: Agent $i$ with income $Y_i$ consumes quantities $C_i$ and $D_i$ of traditional and innovative goods, respectively. Demand derives from utility maximization subject to a budget constraint, $(C_i + Q D_i) (1 + v) \leq Y_i$, where $v$ is the rate of a uniform consumption tax. Being endowed with preferences $u = \ln (u_0 \cdot C^\alpha D^{1-\alpha})$, where $u_0 = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)}$ for convenience, agents spend

$$(1 + v) C_i = \alpha Y_i, \quad (1 + v) Q D_i = (1 - \alpha) Y_i, \quad V_i = \ln Y_i - \ln (1 + v) - (1 - \alpha) \ln Q. \quad (6)$$

Indirect utility $V_i$ is concave in disposable income $Y_i$. The logarithmic specification of utility implies constant relative risk aversion equal to unity.

Labor Allocation: Given our technological assumptions, and with $L$ denoting the number of workers and $E$ the number of entrepreneurs, the supply of traditional goods is $L$ and that of innovative goods $S = p E$. Apart from the entrepreneurial input, production of the innovative good is enhanced by managerial advice which requires $a E$ consultants in total. The production possibilities are thus traced out by an allocation of labor satisfying the resource constraint. Given a population of mass one, labor market clearing requires

$$1 = L + (1 + a) E. \quad (7)$$

Income: All agents potentially receive profits $\Pi_i$ from ownership of VC firms which will be zero, however, in equilibrium with free entry. Apart from this, individual disposable income depends on the agent’s occupation. A worker obtains a safe salary equal to the
wage rate net of the wage tax $t$, i.e. $Y_i = 1 - t + \Pi_i$.\footnote{Recall that the wage rate is unity by choice of the numeraire.} The entrepreneur’s income is risky and equal to $c^N + \Pi_i$ in expected value. Given symmetry within each occupational group, aggregate disposable income is\footnote{Profits from VC firms are $\int_0^1 \Pi_idi = \Pi E$ but will be zero in equilibrium with free entry.}

$$
Y = \int_0^1 Y_i di = c^N E + (1 - t) (L + aE) + \Pi E. \tag{8}
$$

The aggregate budget constraint reads $(C + QD)(1 + v) = Y$.

**Public Sector:** The government collects taxes and hands out subsidies. Any net revenue is rebated as a consumption subsidy. It will become apparent that a proportional consumption subsidy with a uniform rate is neutral and, thus, allows to isolate the allocative effects of other distortive taxes.\footnote{Lump-sum per capita transfers, in contrast, are not neutral because they affect the incentive compatible provisions of the equity contract. They are introduced in section 5.} The government budget constraint is

$$
\tau (pQ (1 + \sigma) - b - a) E + t (L + (b + a) E) + v (C + QD) = \sigma QpE + zKE. \tag{9}
$$

Apart from the salaries of $L$ workers, both the base salary $b$ of each entrepreneur and the wage income of $aE$ consultants are liable to the wage tax at rate $t$. While tax revenue from each project is risky, the government’s revenue is deterministic since the law of large numbers consolidates stochastically independent risks.

**Market Clearing:** Commodity market clearing requires

$$
C + KE = L, \quad D = pE. \tag{10}
$$

Using (2), (5), (9), and (10), disposable income in (8) is also written as

$$
Y = (1 + v) (QpE + L - KE). \tag{11}
$$

To verify Walras’ Law, substitute the budget constraint $(C + QD)(1 + v) = Y$ into (11) and get $(C + KE - L) + Q (D - pE) = 0$. The sum of valued excess demands is zero.
2.2 Venture Capital Activity

2.2.1 Incentive Contract

The following sequence of events determines individual decision making. Occupational choice comes first. Workers receive a safe wage, set their consumption levels and derive utility as in (6). If agents opt for an entrepreneurial career, they approach a VC to fund their project. An equity share and base salary is negotiated, and the VC promises to support the venture with a verifiable level of advice. Given the contractual arrangement, the entrepreneur chooses effort and the VC supplies managerial advice. Next, risk is resolved and state-dependent income determined. Knowing income, consumption and welfare of entrepreneurs is given by (6).

The VC buys an equity stake $1 - s$ at a cost $I$ that covers both the entrepreneur’s base salary $b$ and the fixed start-up investment $K$. The remuneration of entrepreneurs is optimally specified in a venture capital contract and must provide sufficient incentives for their participation and effort. To maximize profits in (4), the VC chooses $s$ and $b$ as well as a level of advice $a$. Her maximization problem is conveniently decomposed into two stages. For any given level of advice, she first minimizes the cost $c$ of obtaining the entrepreneur’s participation. Knowing how contract cost depends on advice, she then chooses $a$ to maximize profits. The second stage of profit maximization is taken up in the next subsection.

In contracting with the entrepreneur (agent), the VC (principal) structures the terms of the contract to solve the incentive problems arising from asymmetric information. Given the entrepreneur’s other job opportunities, the contract must be generous enough to secure her participation. For this reason, the contract cost in part reflects the foregone alternative income such as a worker’s safe salary equal to net wages,\[^10\] $Y_i = 1 - t$. To retain survival chances of start-ups, remuneration of entrepreneurs must also provide sufficient incentives for high effort. However, the contract cannot be made contingent on

\[^10\text{From now on, we set } \Pi_i = 0. \text{ Profits are zero in the competitive VC sector with free entry.}\]
non-verifiable effort, but only on freely observable outcome. If effort is high, the VC thus
concedes a gross income to the entrepreneur equal to $sQ(1 + \sigma) + b$ if the venture succeeds
but only $b$ if it fails, where $b$ is a safe, but moderate base salary. If the entrepreneur shirks,
the business always fails. In this case the entrepreneur is left with the base salary $b$ only,
but may reap some outside wage from tacitly working $1 - \delta$ of her time in manufacturing,
giving $b + 1 - \delta$ in total. With taxes, the entrepreneur’s net income is lower. A proportional
wage tax at rate $t$ is subtracted from all sources of wage income while the CIT cuts into
profit income. Defining

$$
\theta \equiv s(1 - \tau)Q(1 + \sigma), \quad \beta \equiv (1 - t)b,
$$

the entrepreneur receives an expected income of $c^N = p\theta + \beta$ net of CIT and personal
wage taxes if effort is high. The expected net cost to the VC is $c = p\theta + (1 - \tau)b$ and
may differ from $c^N$ because of taxes. Since the base salary is deductible from the CIT, the
effective cost to the VC is $(1 - \tau)b$ while the entrepreneur receives $(1 - t)b$. The problem
of the VC is now to obtain the venture at minimum cost,\footnote{Given indirect utility as in (6), the constraints should take into account the logarithm of the consumer price index, $-\ln(1 + v) - (1 - \alpha)\ln Q$. These terms, however, cancel out on each side. The consumption tax -- and indeed the CPI -- are neutral with respect to the contract! Only net income flows matter.}

$$
c = \min_{\theta, b} \quad p\theta + (1 - \tau)b \quad s.t.
\begin{align*}
PC : & \quad p\ln(\theta + \beta) + (1 - p)\ln(\beta) \geq \ln(1 - t), \\
IC : & \quad p\ln(\theta + \beta) + (1 - p)\ln(\beta) \geq \ln(\beta + (1 - \delta)(1 - t)).
\end{align*}
$$

The contract must specify profit participation and base salary such that both the participa-
tion (PC) and incentive compatibility (IC) constraints are satisfied. The PC compares
expected utility derived from entrepreneurship with utility from a safe worker’s salary.
The IC is fulfilled if expected utility from supplying high effort is no lower than utility
from shirking. Given that the VC cannot observe shirking, the base salary must be paid
in all cases. Income from shirking thus consists of the base salary plus any outside income
that the entrepreneur would derive from working $1 - \delta$ of her time somewhere else.
As a benchmark, we first consider the full information case. When the principal can costlessly verify effort, the contract may be conditioned on effort without any incentive problems. Since the principal holds a fully diversified portfolio of companies while the risk-averse agent pursues a single project only, it is efficient to provide insurance. The PC requires \( \theta = (1 - t) b \left( b^{-1/p} - 1 \right) \). Minimizing cost then gives an optimality condition 
\[ 1 - \tau = (1 - t) \left[ p + (1 - p) b^{-1/p} \right]. \]
The VC’s marginal cost of raising the base salary is \( 1 - \tau \) while the other side gives the savings from reducing the entrepreneur’s profit participation in return. Only if tax rates are equal do we have full insurance with a base salary equal to gross wages (\( b = 1 \)) and a zero profit share (\( \theta = 0 \)). However, if tax rates differ, entrepreneurs and VCs no longer agree on the value of safe income. If the wage tax rate exceeds the CIT rate, the VC will shift some risk to the entrepreneur even in the absence of incentive problems, because the VC’s effective cost of providing a safe salary exceeds the amount that the entrepreneur obtains net of taxes. Put differently, the entrepreneur prefers low taxed profit income over high taxed wage income and will accept some risk in exchange for the tax advantage. In Figure 1, the slopes of the PC and the cost line differ at the full insurance point.

With asymmetric information, both constraints are binding whence we obtain the solution by computing the intersection of them. Figure 1 illustrates. Moving away from full insurance along the PC, the reduction of the base salary must be compensated by ever higher profit shares such that expected income exceeds the safe alternative by a premium to reward for risk bearing. If \( \tau = t \), we have \( c = c^N \), and expected income \( c^N \) from the equity contract exceeds the net wage of a worker by the risk premium. For an analytical solution, substitute the definition of \( \beta \) and get \( \ln (1 - t) = \ln [\beta + (1 - \delta) (1 - t)] \) or \( b = \delta \). The PC then gives 
\[ 1 - t = (\theta + \beta)^p \beta^{1-p}, \]
or
\[ \theta = \beta \left( \delta^{-1/p} - 1 \right), \quad b = \delta, \quad \frac{d\theta}{dt} = \frac{\theta}{1 - t} < 0, \quad \frac{d\theta}{d\tau} = \frac{d\theta}{d\sigma} = 0. \quad (14) \]
We note some immediate implications for tax incidence. The base salary just compensates for the opportunity cost in terms of foregone wages of the entrepreneur’s basic time input \( \delta \) and is exogenous. For a given survival rate \( p \), the entrepreneur’s profit income \( \theta \) depends
only on the base salary net of the wage tax. Since $\theta = s (1 - \tau) Q(1 + \sigma)$, the CIT is thus fully shifted to the VC while the entrepreneur is compensated by an increase in the profit share to obtain the same overall income in case of success. Similarly, the output subsidy $\sigma$ fully accrues to the VC while the entrepreneur is able to capture not even part of it. In contrast, a higher wage tax reduces the alternative income and allows the VC, ceteris paribus, to cut the entrepreneur’s profit share and still retain her participation and high effort. The burden of the wage tax lies as much with entrepreneurs as with workers. In equilibrium, however, the survival rate is endogenously determined which will then affect the described pattern of tax shifting.

The overall contract cost that the VC must incur to attract the entrepreneur determines her residual expected income and willingness to invest. It depends on taxes. In raising survival chances $p(a)$ through more intensive managerial advice, the VC herself may control the cost. Appendix A proves the following properties:

**Proposition 1 (Cost of Contract)** The cost $c(p; \tau, t) = p\theta + (1 - \tau)b$ of incentive compatible compensation of the entrepreneur satisfies

$$
c' \equiv \frac{dc}{dp} < 0, \quad c'' \equiv \frac{d^2c}{dp^2} > 0; \quad \frac{dc}{dt} = -\frac{p\theta}{1 - t} < 0, \quad \frac{dc}{d\tau} = -b < 0.
$$

There are two offsetting influences of $p$ on cost. On the one hand, a higher survival rate raises cost since high income must be paid with higher probability. On the other hand, when project risk declines, the principal may ensure participation of the entrepreneur with a smaller risk premium. The VC is therefore able to squeeze the entrepreneur’s profit share in the successful state; $\frac{d\theta}{dp} < 0$. The second effect dominates and marginal cost falls. Furthermore, the cost function is unambiguously convex in the survival rate. The wage tax allows to cut the entrepreneur’s profit share and, thereby, reduces the VC’s contract cost since it makes the alternative career option less attractive. The CIT also squeezes cost. Since the base salary is tax deductible in the portfolio company, the government effectively pays for part of it. Note finally that contract cost is completely independent of
the output subsidy $\sigma$ to innovative goods. The subsidy boosts the company’s cash flow in case of success but the VC simply cuts the profit share and appropriates all of it.

2.2.2 Managerial Advice

Only successfully launched businesses eventually contribute to the VC’s revenues. According to (1), however, VCs may themselves contribute to higher survival chances of their portfolio companies and strengthen their revenues by giving business advice. In raising the survival rate, more advice also allows to squeeze the entrepreneur’s risk premium over safe wage income by making entrepreneurial income more certain, whence the venture may be obtained at a lower contract cost. Advice, however, is costly and results in operating costs gross of taxes equal to $a$ per project. The profit maximizing level of advice is most easily analyzed by rewriting (4) as

$$\Pi = \max_a p \left[ (1 - \tau) Q(1 + \sigma) - m \right], \quad m \equiv \frac{c(p; t, \tau) + (1 - \tau) a + (1 - z)K}{p}. \quad (15)$$

We refer to $m$ as cost to market which is the expected cost incurred in order to produce one unit of the innovative good. On average, one must start $1/p$ projects to accomplish this. Apart from savings in contract cost, an increase in the survival rate now reduces cost to market because a smaller number of projects need to be started for each successful one. The necessary and sufficient conditions for the VC’s advisory activity are, thus,$^{12}$

$$\Pi' = p' \left[ (1 - \tau) Q(1 + \sigma) - m \right] - pm'$$
$$= p' \left[ (1 - \tau) Q(1 + \sigma) - c' \right] - (1 - \tau) = 0,$$

$$\Pi'' = p'' \left[ (1 - \tau) Q(1 + \sigma) - m \right] - 2p' m' - pm''$$
$$= p'' \left[ (1 - \tau) Q(1 + \sigma) - c' \right] - p' p' c'' < 0. \quad (16)$$

The second order condition is fulfilled by the curvature properties of $p(a, g)$ and $c(p; t, \tau)$.

$^{12}$ $p'$ and $\Pi'$ denote derivatives with respect to $a$, while $c'$ denotes the derivative w.r.t. $p$. 

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2.3 Equilibrium

**Zero Profits and Managerial Advice:** As long as they make additional profits, VCs attract ever more entrepreneurs $E$ and generate more business start-ups. In equilibrium, the price of innovative goods must satisfy the zero profit condition relating to (15),

$$(1 - \tau) Q (1 + \sigma) = m. \quad (17)$$

With free entry and zero profits, the cost $m$ of bringing a venture to the market must be equal to the producer price net of the CIT. The intensity of managerial advice and the equilibrium market price are solved recursively. Imposing the zero profit condition (17) on the individual optimality condition of the VC in (16) gives

$$\Pi' = -pm' = p' (m - c') - (1 - \tau) = 0, \quad (18)$$

where $c$ depends on $a$ only via its effect on $p$. With taxes given, this equation autonomously fixes the level of managerial advice. In equilibrium, the VC’s marginal benefit of supplying more advice is $p' (m - c')$. More advice boosts survival rates which directly reduces cost to market, and does so indirectly as well, since a lower risk allows to cut the profit share of the entrepreneur. In providing more advice, the VC incurs a marginal cost equal to $1 - \tau$. Once the level of advice is known, $p$, $c$, and $m$ are determined which, in turn, fixes the demand price $Q$ according to (17).

**Number of Entrepreneurs:** We impose labor market clearing and budget constraints and solve for the number of entrepreneurs that equilibrate the market for innovative goods. Walras’ Law then implies market clearing for standard goods as well. Given neutrality of the consumption tax, equilibrium will be independent of its rate. We start with the observation in (6) that agents spend a fixed share of disposable income $(1 + v) (C + QD) = Y$ on innovative goods. Spending is thus $QD = \frac{(1-\alpha)Y}{1+v} = (1 - \alpha) Y^G$ where $Y^G \equiv C + QD = Y - v (C + QD)$ denotes gross factor income.\textsuperscript{13}

Before we equate\textsuperscript{13} To relate tax revenues, the government gives a consumption subsidy, i.e. $v < 0$. A consumption tax is charged only when an output or investment subsidy must be financed.

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demand and supply, we compute gross income by replacing the consumption subsidy from the government budget in (9). Using (8), (7), (5) and (12), disposable income is

\[ Y = [p\theta + (1 - t) b] E + (1 - t)(1 - E) \]

where the zero profit condition, by way of (15) and (13), implies \( p\theta E = [(1 - \tau) (pQ(1 + \sigma) - a - b) - (1 - z) K] E \). Taking \( vY^G \) from (9) and replacing \( L \) from (7), gross income emerges as

\[ Y^G = Y - vY^G = 1 + pQE - (1 + a + K) E. \]

Equate supply and demand in the entrepreneurial sector, \( pE = (1 - \alpha)Y^G /Q \). Note in particular that in zero profit equilibrium, managerial advice, survival probability, contract cost \( c \) and cost to market \( m \) are all autonomously determined by (18) independently from the rest of the model. Figure 2 illustrates the solution for the untaxed equilibrium with the number of entrepreneurs being the equilibrating variable. In this case, gross income is \( Y^G = 1 + (c - 1) E \). A part \((1 - \alpha)/Q\) of demand is autonomous but it increases with \( E \) for the simple reason that average income of entrepreneurs exceeds wages by a risk premium \( c - 1 \). The demand schedule is flatter than the supply curve such that the equilibrium number of entrepreneurs given by the intersection is smaller than unity. By the zero profit condition (17), the number of entrepreneurs is \( E = \frac{1}{1 - \alpha + a + K + \frac{\gamma}{\alpha c}} < 1 \). By the same steps, entrepreneurship in the taxed equilibrium is given by

\[ 1 - \alpha = E \cdot \Omega, \quad \Omega \equiv (1 - \alpha)(1 + a + K) + \frac{\alpha pm}{(1 - \tau) (1 + \sigma)} \quad (19) \]

### 3 Entrepreneurship

#### 3.1 Cost to Market

The recursive structure of the model greatly simplifies policy analysis. Cost to market and managerial advice may be solved independently of the rest of the model. Note that (18) is equivalent to \( m' = 0 \) which is also the condition for minimum cost to market

\[ m = \min_a \frac{d(p,t) + (1 - \tau) a + (1 - z) K}{p} \]

Profit maximization combined with free entry is, thus,
equivalent to cost minimization and yields the same level of advice. Applying the envelope theorem to the minimization problem and using proposition 1 gives
\[ \frac{dm}{dt} = -\frac{\theta}{1 - t} < 0, \quad \frac{dm}{d\tau} = -\frac{a + b}{p} < 0, \quad \frac{dm}{dz} = -\frac{K}{p} < 0, \quad \frac{dm}{d\sigma} = 0. \] (20)
Wage taxation allows to cut entrepreneurial compensation since it reduces alternative income. Wage taxes thus reduce cost to market and boost profits of VC firms. They start to attract more entrepreneurs and will bring more start-up companies to market. The market price falls until, in equilibrium, profits are squeezed to zero again and no more projects are funded. Because the base salary and advisory costs are tax deductible, the CIT effectively subsidizes these expenditures and reduces cost to market as well. A subsidy \( z \) to start-up investment similarly reduces cost to market. An output subsidy is unable to affect cost to market and, thereby, the zero profit producer price. It is completely passed on to consumers by reducing their demand price \( Q \).

### 3.2 Managerial Advice

VCs not only provide equity finance but also supply valuable business advice. Do taxes impair incentives to provide managerial advice? Condition (18) implicitly determines the extent of consulting when market entry is free and competition eliminates profits in VC finance. Taking the differential thereof shows how policy induces VCs to adjust advice in equilibrium. Using the partials listed in (B.2) yields
\[ \Pi''da = \frac{\mu b'}{1 - t} dt - \left[ 1 - \frac{(a + b) p'}{p} \right] d\tau + \frac{K p'}{p} dz, \]
where the elasticity \( \mu \) is defined in (A.2). Henceforth, we use \( r \) to denote a comprehensive income tax, i.e. \( r = t = \tau = z \). With signs determined in (B.2), the equilibrium effects of taxation on incentives to give advice are summarized as

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\[ ^{14} \text{Imposing zero profits on (16), the necessary and sufficient conditions of the two problems are related according to } \Pi' = -pm' = 0 \text{ and } \Pi'' = -pm'' < 0 \text{ whence the cost function } m \text{ is indeed convex.} \]
Proposition 2 (Managerial Advice) In equilibrium with free entry, the intensity of managerial advice increases with a higher corporate income tax, while a higher investment subsidy and higher wage taxes discourage advice. An output subsidy and a comprehensive income tax are neutral:

\[ \frac{da}{d\tau} > 0, \quad \frac{da}{dz} < 0, \quad \frac{da}{dt} < 0, \quad \frac{da}{d\tau} = \frac{da}{d\sigma} = 0. \]  \hspace{1cm} (21)

The CIT seemingly holds ambiguous incentives for advice. As indicated by (15) and (18), the marginal benefit \( p'(m - c') \) of giving more advice is a higher survival rate which saves costs, since fewer projects need to be started for each successful one. On the one hand, a higher CIT reduces the marginal benefit of business advice by \( p' \frac{\partial m}{\partial \tau} = -(a + b) p'/p \). When start-up cost becomes smaller due to the implicit tax subsidy associated with a higher CIT rate, then less is saved by raising the survival rate and the marginal benefit of advice declines accordingly. On the other hand, since advisory costs are tax deductible, a higher CIT rate reduces the marginal cost of advice by \(-d\tau\) and thereby encourages managerial support. The net effect is positive.\(^{15}\) Via the effect on managerial advice, the CIT tends to strengthen survival chances of new start-ups. A comprehensive income tax avoids interfering with VCs’ incentives since it affects marginal benefits and costs of advice proportionately.

### 3.3 Entrepreneurship

With an endogenous survival rate, the number of entrepreneurs \( E \) willing to start up new projects is no longer proportional to and must be distinguished from the number of successful projects \( pE \). How then is tax policy affecting entrepreneurship and supply in the innovative sector? The quality of VC finance, i.e. the intensity of consulting, determines entrepreneurial risk and thereby affects agents’ willingness to opt for an entrepreneurial career. Are tax incentives for advice in line with the tax effects on occupational choice? To

\(^{15}\)Since \( x \equiv 1 - (a + b) p'/p > 0 \) as shown in (B.2) of the appendix, the effect of the CIT is positive, \( \frac{da}{d\tau} = -x/\Pi'' > 0 \), since \( \Pi'' < 0 \).
build intuition, consider first the market for innovative goods in the untaxed equilibrium which is \( p(a) E = (1 - \alpha) [1 + (c - 1) E] / m \) with \( Q = m \) by the zero profit condition. In holding \( a \) constant, we identify some direct effects of taxes on the demand side. For a given number of entrepreneurs, taxes contribute to aggregate income and boost demand if they raise the risk premium, i.e. the income differential, of entrepreneurs. This income effect is enhanced by a price effect if taxes reduce cost to market and thus allow for a lower demand price. To eliminate excess demand, the number of entrepreneurs must increase. The indirect effect of taxes works through incentives for managerial advice. More intensive advice boosts survival chances and adds to aggregate supply when a larger fraction of start-up projects is successful. In reducing risk, more advice squeezes the income premium of entrepreneurs and erodes demand. Note also that a marginal increase in advice fails to affect the output price since \( m' = 0 \). If taxes encourage more intensive advice, they contribute to excess supply of innovative goods. The number of entrepreneurs must decline to restore equilibrium. With these transmission channels in mind, we now consider how various taxes affect entrepreneurship and industry supply.

**Wage Tax:** In the general case, we start from a taxed equilibrium. Take the differential of (19) and use the f.o.c. \( m' = 0 \). Defining \( \Phi \equiv 1 - \alpha + \frac{\alpha m'}{(1 - \tau)(1 + \sigma)} > 0 \), we have

\[
\frac{dE}{dt} = \frac{-E^2}{1 - \alpha} \frac{d\Omega}{dt} = \frac{E^2}{1 - \alpha} \left\{ \frac{\alpha \theta}{(1 - \tau)(1 + \sigma)(1 - t)} - \Phi \frac{da}{dt} \right\} > 0. \tag{22}
\]

As a direct effect, the wage tax reduces cost to market and thereby strengthens demand by means of a lower output price. On the other hand, lower net wages render the entrepreneur’s alternative career option less attractive. The VC is thus able to save on entrepreneurial compensation which reduces demand. The price effect is seen to dominate, and VCs will fund more projects and attract more entrepreneurs to restore equilibrium. The second term in the curly bracket is an indirect effect of the wage tax that stems from the diminished incentives for managerial advice, see (21). Less intensive advice contributes to excess demand and further stimulates entrepreneurship.

A higher wage tax entices more start-ups but each one receives less advice from VCs
and is, thus, less likely to succeed. What is then the net effect on \( pE \), the supply of innovative goods? Using (22) and introducing \( \Psi = 1 - \alpha - pE\Phi/p' \), we obtain
\[
\frac{d(pE)}{dt} = \frac{\alpha \theta (pE)^2}{(1-\alpha) (1-\tau) (1+\sigma) (1-t)} + \frac{p'\Psi \partial \alpha}{1-\alpha} \frac{\partial}{\partial t} > 0, \quad \Psi < 0. \tag{23}
\]
To sign \( \Psi \), we write \( \Psi = 1 - \alpha - \frac{pE(1-\alpha)}{p'} - \alpha pE \), where \( Q = \frac{m}{(1-\tau)(1+\sigma)} \) by (17). Replace \( p' \) in the second term by (18) and rearrange, \( \Psi = 1 - \alpha - pE - pE(1 - \alpha) (\sigma Q - \frac{\sigma}{1-\tau}) \).

Equation (19) helps to sign \( \Psi \). Expand \( \Omega \), such that \( \Omega = pQ - (1-\alpha) [pQ - (1 + a + K)] \).

Then, (19) implies \( 1 - \alpha < E pQ \Leftrightarrow [pQ - (1 + a + K)] > 0 \). The condition holds in the untaxed equilibrium where \( pQ = pm = c + a + K \). In this case, \( pQ - (1 + a + K) = c - 1 > 0 \) is equal to the risk premium. By continuity, the condition will be satisfied as long as tax rates are not too large. With a comprehensive income tax and a zero output subsidy, i.e. \( t = \tau = z > 0 \) and \( \sigma = 0 \), the condition is again related to the entrepreneur’s risk premium, \( [pQ - (1 + a + K)] = \frac{c}{e^z} - 1 = \frac{e^N - (1-z)}{1-z} > 0 \), and is therefore satisfied even for large taxes. With the inequality \( 1 - \alpha < E pQ \) thus established, the first two terms in \( \Psi \) are negative and the third one is negative anyway. With \( \Psi < 0 \), the wage tax is seen to boost output of innovative goods. The fact that the wage tax discourages consulting, reinforces the direct effects on entrepreneurship and aggregate supply.

**Capital Income Tax:** Similar calculations reveal the effects on entrepreneurship and industry supply that are induced by the CIT and the investment subsidy, respectively:
\[
\begin{align*}
\frac{\partial E}{\partial z} &= \frac{E^2}{1-\alpha} \left\{ \frac{\alpha K}{(1-\tau)(1+\sigma)} - \Phi \frac{\partial \alpha}{\partial z} \right\} > 0, \\
\frac{\partial E}{\partial \tau} &= \frac{E^2}{1-\alpha} \left\{ \frac{\alpha [\theta + (1-z)K]}{(1-\tau)^2(1+\sigma)} + \Phi \frac{\partial \alpha}{\partial \tau} \right\} < 0,
\end{align*}
\tag{24}
\]
An increase in the investment subsidy leads to a rise in both the number of entrepreneurs and of successful projects. It lowers cost to market of innovative goods which expands supply and attracts additional entrepreneurs. In discouraging VC advice, the subsidy reinforces both the number of start-ups and aggregate supply and thereby expands the entrepreneurial sector. The CIT has opposite effects. It lowers cost to market as well. However, since it hits revenues even more forcefully, the relative market price of innovative goods is higher which reduces output and demand for entrepreneurs.
Comprehensive Income Tax: A comprehensive income tax features common rates for the wage tax \( t \), the CIT \( \tau \), and the investment subsidy \( z \). A comprehensive income tax was shown to be neutral with respect to the level of managerial advice. It turns out that this broad-based tax with full loss offset neither affects entrepreneurship nor the supply of innovative goods. In (19), the direct effect of the tax would enter through \( m/(1 - r) \) but the tax factor cancels out since all terms in \( m \) including \( \theta \) as given in (14) are proportional to \( (1 - r) \). Consequently,

\[
\frac{\partial E}{\partial r} = \frac{\partial(pE)}{\partial r} = 0. \tag{25}
\]

Output Subsidy: In zero profit equilibrium with free entry of VCs, an output subsidy \( \sigma \) avoids to distort incentives for advice. It affects neither survival chances nor cost to market. In subsidizing consumer prices, however, the subsidy boosts demand for innovative goods and, thereby, encourages entrepreneurship and aggregate supply,

\[
\frac{\partial E}{\partial \sigma} = \frac{\alpha EY^G}{1 + \sigma} > 0, \quad \frac{\partial(pE)}{\partial \sigma} = \frac{p\alpha EY^G}{1 + \sigma} > 0, \tag{26}
\]

where we used \( \frac{m}{(1 - \tau)(1 + \sigma)} = Q \) and \( QpE = (1 - \alpha) Y^G \).

Proposition 3 (Entrepreneurship) A wage tax, an investment subsidy and an output subsidy raise the number of entrepreneurs and industry supply. The CIT works in the opposite direction. A comprehensive income tax with full loss offset is neutral.

4 Welfare

Tax policy affects the incentives of VCs to advise entrepreneurs and thereby influences the survival rates of start-up businesses. It also affects the propensity of households to opt for an entrepreneurial career. What are the normative implications of tax policy from a welfare theoretic point of view? The main complication compared to a standard competitive economy is the presence of asymmetric information in the relation between
entrepreneurs and financiers. To avoid moral hazard and to retain survival chances of start-up ventures, the equity contract must be arranged to provide entrepreneurs with powerful incentives for full effort. For this reason, entrepreneurs must bear risk via profit participation even though full risk diversification would be possible in principle. In the presence of unconsolidated risk, taxation could provide further insurance. It is expected, however, that further diversification is counterproductive since it conflicts with private arrangements to contain moral hazard. We now proceed with an explicit welfare analysis to check this conjecture.

The Welfare Measure: To investigate the welfare consequences of tax policy, we analyze (ex ante) welfare of agents prior to occupational choice.\textsuperscript{16} Given a price index $P = (1 + v) Q^{1-\alpha}$, an agent with real income $Y_i / P$ derives indirect utility $V_i = \ln (Y_i / P)$, see (6). In equilibrium, expected utility from entrepreneurship is exactly matched by utility from a safe job in industry. The participation constraint holds with equality. Therefore, utility $V_i$ of a worker which depends on the after tax real wage $(1 - t) / P$, is a complete welfare measure.\textsuperscript{17} The marginal welfare effect is, thus,

$$dV = -\frac{dt}{1 - t} - \frac{dv}{1 + v} - (1 - \alpha) \frac{dQ}{Q}.$$  \hspace{1cm} (27)

Welfare depends on the real wage net of taxes. Tax policy thus affects welfare via three channels: (i) the after-tax wage; (ii) the consumption tax; and (iii) the price of innovative goods. The wage tax determines (i) while (ii) and (iii) reflect the price index. As indicated in (20), tax policy affects the cost to market $m$ and, thereby, the price $Q = m / [(1 - \tau)(1 + \sigma)]$ which obtains under perfect competition and free entry of VCs. In particular,

\textsuperscript{16}Broadway et al. (1991) provide a welfare analysis in several models of occupational choice.

\textsuperscript{17}This ex ante welfare measure also corresponds to a social welfare function which adds up ex post utility levels of different agents. The population splits into $1 - E$ workers and $E$ entrepreneurs of whom $pE$ are successful and $(1 - p) E$ fail. Given net income $Y_i \in \{1 - t, \theta + \beta, \beta\}$, social welfare amounts to $SWF = (1 - E) \cdot V(\frac{1 - t}{1 + \beta}) + pE \cdot V(\frac{\theta + \beta}{1 + \beta}) + (1 - p) E \cdot V(\frac{\beta}{1 + \beta})$. Since the participation constraint binds with equality, $pV(\frac{\theta + \beta}{1 + \beta}) + (1 - p) V(\frac{\beta}{1 + \beta}) = V(\frac{1 - t}{1 + \beta})$, social welfare is again given by $V = \ln(\frac{1 - t}{1 + \beta})$.  

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the welfare evaluation of taxes must take account of the fact that revenues are rebated by means of a consumption subsidy.

As a first step in evaluating (27), we compute the differential of the government budget constraint to obtain the adjustment in the consumption subsidy. For the rest of this section, we start from an untaxed equilibrium position and derive the marginal welfare effects of introducing small taxes from zero. This way, we avoid complicated tax base effects that would identify the excess burden of taxes. With small taxes, the excess burden is zero to the first order. The remaining welfare effect must then be due to other distortions if there are any. The differential of the public budget constraint in (9) is

\[ Ydv = -[1 - (1 - b)E]dt - (p\theta + K)Ed\tau + KEdz + (1 - \alpha)Yd\sigma. \]  

In the untaxed equilibrium, the tax base of the consumption subsidy is equal to income \( Y = C + QD = 1 + (c - 1)E \). By way of (7), the wage tax base is \( L + (a + b)E = 1 - (1 - b)E \). Using the zero profit condition \( Q = m \) plus contract cost \( c = p\theta + b \), the corporate tax base is \( pQ - b - a = p\theta + K \).

**Marginal Taxes and Subsidies:** We start with the introduction of a small wage tax that generates revenues \( Ydv = -[1 - (1 - b)E]dt \). With free entry, \( Q = m \) and, by (20), \( dQ/Q = -\theta \cdot dt/m \). Substituting into the welfare differential in (27), and using \( Y = 1 + (c - 1)E \) and \( c = p\theta + b \), we find

\[ \frac{dV}{dt} = -1 + \frac{1 - (1 - b)E}{Y} + \frac{(1 - \alpha)\theta}{m} = \frac{(1 - \alpha)Y\theta - mpE\theta}{mY} = 0, \]  

where the last equality exploits the conditions for zero profits and equilibrium in the entrepreneurial sector, \( mpE = QD = (1 - \alpha)Y \). A small wage tax, with revenues rebated by means of a neutral consumption subsidy, boosts the number of entrepreneurs and supply of innovative goods, but it fails to raise welfare at the margin. There is no market distortion that would require a wage tax to correct private decisions. Starting again from the laissez-faire equilibrium, marginal changes in \( \tau \), \( z \), and \( \sigma \) yield the same result [use (20), (28), and the conditions for zero profits, \( Q = m/[(1 - \tau)(1 + \sigma)] \), and equilibrium in the innovative goods sector, \( mpE = (1 - \alpha)Y \)].
Proposition 4 (Welfare Effects of Taxes) Using a proportional consumption tax or subsidy to balance the budget, and starting from an untaxed equilibrium, the welfare effects from a small wage tax, CIT, and small output and investment subsidies are zero.

5 Extensions

Having discussed the major taxes being relevant for start-up investment by risk-bearing entrepreneurs, we now proceed with two further scenarios. In practice, tax systems are often restrictive to some extent in allowing VC funds to offset losses from failures against profits from successful start-ups. The first subsection thus addresses the effects on entrepreneurship, managerial advice, and welfare when the income tax allows for less than full loss offsets. The second subsection turns to a redistributive policy that levies a proportional consumption tax in order to finance a uniform per capita transfer. This scenario mimicks an indirectly progressive tax. Obviously, such transfers are relatively more important in case of failure when the entrepreneur is left with a modest base salary only, as compared to the exceptionally high income generated by a successful start-up. Such transfers provide welcome insurance to entrepreneurs. However, risk bearing and ex post income inequality is required to contain the moral hazard problem. It will be important to investigate how such a redistributive tax transfer scheme interacts with the financial arrangements of the VC contract.

Restricted Loss Offsets: Consider an initial situation where a comprehensive income tax at rate \( r \) with full loss offset is in place and revenues are rebated by means of a proportional consumption subsidy at rate \( v < 0 \). All other taxes and subsidies including lump-sum transfers are set to zero. The loss offset is relevant only for the VC since the portfolio company doesn’t come into existence when the project fails. While government fully taxes profits at rate \( r \), VCs are denied to deduct a fraction \( \varepsilon \) of losses from their tax liability with \( \varepsilon = 0 \) at the outset. We now consider the effects of restricting full loss
offset by increasing $\varepsilon$. The income tax rate is kept constant but the consumption subsidy adjusts to rebate any excess revenues.

The VC injects equity $I = b + K$ gross of taxes for a share $1 - s$ of prospective returns. The VC’s profit from a successful start-up is $\Pi^G = (1 - r) ((1 - s) Q - a - b - K)$ where $I$ is borrowed from other agents and must be subtracted. In case of failure, she incurs a loss $\Pi^B = -(1 - (1 - \varepsilon) r) (a + b + K)$ since only a fraction $1 - \varepsilon$ of losses can be offset against dividends from other projects. The VC’s expected, net of tax profit per project is thus $\Pi = p\Pi^G + (1 - p) \Pi^B$, or

$$\Pi = (1 - r) (1 - s) pQ - (1 - r \gamma) (a + b + K), \quad \gamma \equiv p + (1 - p) (1 - \varepsilon) \leq 1. \quad (2')$$

Full loss offset is given by $\varepsilon = 0$ and $\gamma = 1$ which results in (2) again when a comprehensive income tax with $\tau = t = z = r$ is in place and $\sigma = 0$. Appendix C now repeats in Table C.1 all those elements of the basic model that will change due to the presence of restricted loss offset and lump-sum transfers. The modified equations are primed.

In case of failure, the VC runs up losses of $a + b + K$ which include her own operating expenses for advice, the entrepreneur’s base salary, and the start-up investment cost. When restricting loss offset, the government fully taxes profits from a successful start-up but participates relatively less in the loss from project failure. Consequently, contract cost (13’) and cost to market (15’) both increase when a larger share $\varepsilon$ of losses are not tax deductible from other revenues. Profit maximization with free entry and minimization of cost to market are equivalent since they result in the same optimality condition (18’).

Applying the envelope theorem to $m = \min_u \frac{e^{(p_rT) + (1 - \gamma)(a + K)}}{p}$ shows that increasing the share $\varepsilon$ of non-deductible losses inflates cost to market according to $^{18}$

$$\frac{dm}{d\varepsilon} = r (1 - p) (b + a + K) > 0, \quad \frac{dm}{dT} = \frac{\partial \theta}{\partial T} < 0. \quad (20')$$

However, limiting the loss offset holds inherently ambiguous incentives for managerial

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$^{18}$For reasons of space, we also include the effects of transfers which are discussed below.
advice. Taking the differential of (18'), \( \Pi' da = -\frac{\partial \Pi}{\partial \varepsilon} d\varepsilon \), and using (C.5), gives

\[
\frac{da}{d\varepsilon} = \frac{r}{\Pi'} \left[ 1 - p - \frac{(b + a + K) p'}{p} \right] \geq 0, \quad \frac{da}{dT} = -p' \left[ \mu \frac{\partial \theta}{\partial T} + \theta \frac{\partial \mu}{\partial T} \right] < 0. \quad (21')
\]

Restricting loss offset \( \varepsilon \) raises marginal cost of advice by \( \frac{d(1-\gamma_0)}{d\varepsilon} = r \left( 1 - p \right) \). The square bracket in (18') identifies the marginal benefit which stems from an increase in the survival rate by \( p' \). The first part of the marginal benefit, \( p'm \), reflects the fact that a higher probability of success directly reduces cost to market. According to (20'), limiting loss offset raises the marginal benefit by \( p' \frac{\partial m}{\partial \varepsilon} = p' r \frac{(1-p)(b+a+K)}{p} \). The second component \( -p'c' = -p' \left[ \theta \left( 1 - \mu \right) - rb \varepsilon \right] \) captures the savings in entrepreneurial compensation. A higher survival chance not only squeezes the entrepreneur’s risk premium, but also reduces the VC’s effective contract cost of the base salary since a higher survival rate makes the tax disadvantage from limited loss offset hit less likely. If the loss offset is restricted further, the VC derives an even higher marginal benefit from more advice which helps to avoid a larger tax disadvantage. By (C.4), the marginal benefit increases by \( -p' \frac{\partial \beta}{\partial \varepsilon} = p' rb \).

Finally, part of marginal benefits stem from avoiding the tax disadvantage of limited loss offset for costs other than the base salary, \( p'r (a + K) \varepsilon \). By the same reasoning, if a larger share \( \varepsilon \) of losses is not deductible, the VC again faces higher incentives to avoid this tax disadvantage, and marginal benefit of advice increases by \( p'r (a + K) \). Adding up gives the second term in the square bracket of (21'). Restricting loss offset thus increases both marginal benefits and marginal costs of advice such that the net effect is ambiguous.

Restricting the loss offset affects entrepreneurship directly as well as indirectly by its implication for managerial advice. Using the optimality condition \( m' = 0 \) and defining \( \Phi \equiv 1 - \alpha + \frac{\alpha m'}{1-r} > 0 \), the differential of (19') yields

\[
\frac{dE}{d\varepsilon} = -\frac{E^2}{1-\alpha} \left\{ \frac{\alpha p \, dm}{1-r \, d\varepsilon} + \Phi \frac{\partial a}{\partial \varepsilon} \right\}, \quad \frac{dE}{dT} = -\frac{E^2}{1-\alpha} \left\{ \alpha p \frac{\partial m}{\partial T} + \Phi \frac{\partial a}{\partial T} \right\} > 0. \quad (30)
\]

By (20'), a restricted loss offset raises the tax cost of VC operations and inflates cost to market. By this direct effect, government discourages entrepreneurship and start-up investment activity. As with all other scenarios, the indirect effect on managerial advice, if it is positive, will reinforce the direct effect and further discourage start-ups. Restricting
loss offset, however, may as well diminish incentives for managerial support and thereby contribute to a larger number of start-ups, each one being more risky.

Output of the innovative sector is equal to the number of successful start-ups $pE$. Defining $\Psi = 1 - \alpha - pE\Phi/p' < 0$\(^{19}\), the net effect of the loss offset provision is

$$
\frac{d(pE)}{d\varepsilon} = -\frac{\alpha (pE)^2 \frac{\partial m}{\partial c} + p'E\Psi \frac{\partial \eta}{\partial c}}{1 - \alpha}, \quad \frac{d(pE)}{dT} = \frac{-\alpha (pE)^2 \frac{\partial m}{\partial T} + p'E\Psi \frac{\partial \eta}{\partial T}}{1 - \alpha} > 0. \quad (31)
$$

To summarize, the direct effect of restricting loss offset, i.e. increasing $\varepsilon$, is to discourage entrepreneurship and retard output of the entrepreneurial sector. This is reinforced if the limitation of loss offset induces more managerial support. Fewer entrepreneurs are needed if each project succeeds with higher probability. However, the relation between the generosity of loss offset and the extent of managerial advice is ambiguous.

Welfare depends on the disposable real wage. Keeping the income tax rate constant, real wages change along with transfers and the consumer price index as indicated in (C.6). By assumption, the government rebates tax revenues by means of a neutral consumption subsidy. The required change in the subsidy follows from the differential of the public budget (9') which we usefully rewrite as in (C.7). In analyzing the loss offset, we start from a position of $T = 0$ and $\gamma = 1$, implying $\frac{r}{\gamma r} = -\frac{n}{\gamma n}$ at the outset. Restricting loss offset generates higher revenues, allowing to raise the consumption subsidy by\(^{20}\)

$$
\frac{dv}{d\varepsilon} = -\frac{(1 + v)^2 r (1 - p) (a + b + K) E}{(1 - r) Y}, \quad \frac{dv}{dT} = \frac{1}{Y}. \quad (32)
$$

Substituting (20') and (32) into (C.6) yields a welfare effect of

$$
\frac{dV}{d\varepsilon} = -r (b + a + K) (1 - p) \left[ \frac{1 - \alpha}{mp} - \frac{(1 + v) E}{(1 - r) Y} \right] = 0. \quad (33)
$$

\(^{19}\)To sign $\Psi$, we write $\Psi = 1 - \alpha - \frac{pE(1 - \alpha)}{p} - \alpha pE Q$, where $Q = \frac{m}{1 - r}$ by (17). Replace $p'$ by (18'), evaluated at $\varepsilon = 0$, and rearrange: $\Psi = 1 - \alpha - pE Q + \frac{pE(1 - \alpha)c'}{1 - r}$. (19') helps to sign $\Psi$. Expand $\Omega$ such that $\Omega = pQ - (1 - \alpha)[pQ - (1 + a + K)]$. Then, (19') implies $1 - \alpha < E pQ \Leftrightarrow [pQ - (1 + a + K)] > 0$. The condition holds in the initial equilibrium where $pQ = p\frac{m}{1 - r} = \frac{c'}{1 - r} + (a + K)$. In this case, $[pQ - (1 + a + K)] = \frac{c'}{1 - r} - 1 > 0$ is positive due to the risk premium since $c = cN$ in the presence of a comprehensive income tax with full loss offset. With the inequality $1 - \alpha < E pQ$ thus established, the first two terms in $\Psi$ are negative which establishes $\Psi < 0$.

\(^{20}\)In the transfer scenario, we suppress the income tax ($r = 0$), giving a budget $T = vY/(1 + v)$. 

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The last equality follows from the fact that equilibrium in the E sector, \( D = pE \), together with expenditure \( QD = (1 - \alpha) \frac{Y}{1 + \tau} \) and zero profit \( Q = \frac{m}{1 + \tau} \) implies \( \frac{m}{1 + \tau} = QpE = \frac{mpE}{1 + \tau} \). When a comprehensive income tax with full loss offset is in place, the initial equilibrium is free of distortions. A small restriction of the loss offset provision entails negligible welfare effects that are zero to the first order.

**Proposition 5 (Restricted Loss Offset)** Starting with a comprehensive income tax with full loss offset, restricting the loss offset provision raises cost to market and, thereby, discourages entrepreneurship and industry output. However, it raises both marginal benefits and costs of managerial advice and creates ambiguous incentives for consulting and entrepreneurship. The welfare effect is zero to the first order.

**Lump-Sum Transfers:** When the government hands out uniform lump-sum transfers, cost to market falls according to (20'). The reasons are easily explained in terms of figure 1. Since the disposable wage from a safe job increases, transfers shift out the participation constraint while they do not directly enter the incentive constraint. It is easily shown that the slope of the incentive constraint is either negative or positive but less than unity. With uniform per capita transfers, disposable income \( \beta \) in the bad state increases one to one while income in the good state therefore rises less than one to one. While the VC’s cost of the base salary remains unaffected by transfers, she may save on the entrepreneur’s profit share \( \theta \). Alternatively, risk is reduced if income in all states is increased by the same absolute amount, and risk averse agents require a lower risk premium to compensate for entrepreneurial risk bearing. Consequently, the cost of the contract to the VC falls which translates into lower cost to market. For the same reasons, transfers reduce marginal benefits of advice, \( p' (m - c') \), while marginal costs remain the same. Consequently, transfers weaken the incentives to provide active consulting services, see (21'). With lower cost to market, VCs expand their operations and fund more projects until increased supply squeezes prices to restore zero profits in equilibrium. Entrepreneurship and industry output in (30) and (31) both increase. Since
equity finance comes with less intensive advice, however, business failure is more probable. More projects must be started to expand the industry. The negative effect on consulting reinforces the effects on entrepreneurship and industry supply.

By assumption, transfers are financed with a proportional consumption tax. Substituting (32) into (C.6), the welfare effect of a small transfer is \( \frac{dV}{dT} = 1 - \frac{1}{Y} - \frac{1 - \alpha}{Q} \partial Q \partial T \). The income definition \( Y = 1 + (c - 1) E \) thus yields

\[
\left. \frac{dV}{dT} \right|_{T=0} = \frac{E (c - 1)}{Y} - \frac{1 - \alpha}{Q} \partial Q \partial T > 0,
\]

since \( Q = m \) with free entry and \( \frac{\partial Q}{\partial T} < 0 \) by (20'). A proportional consumption tax combined with uniform per capita transfers in fact mimicks a progressive tax that redistributes from high to low incomes. The government is thereby able to provide further insurance of unconsolidated risk associated with entrepreneurial income. This yields a net welfare gain that is proportional to the risk premium \( c - 1 \). As taxation renders entrepreneurial income less risky, entrepreneurs accordingly require a lower premium for risk bearing. Financiers are thus able to attract entrepreneurs at a lower cost and at the same time retain their incentives for full effort. With free entry, lower cost to market is passed on to consumers by cutting the price \( Q = m \) where \( \frac{dm}{dT} = \frac{dc}{dT} = \frac{\partial}{\partial T} < 0 \), see (20').

**Proposition 6 (Progressive Taxation)** Starting from an untaxed position, a redistributive tax transfer scheme (uniform per capita transfers financed with a proportional consumption tax) retards managerial advice but boosts entrepreneurship and industry output. The tax transfer scheme reduces unconsolidated risk and raises welfare.

The welfare gains from the redistributive tax transfer scheme are in marked contrast to the proportional tax and subsidy scenarios of the preceding section. As Boadway et al. (1991) have demonstrated, welfare results in models of occupational choice should generally reflect three components: efficiency, equity and insurance. Given that the equity contract is optimal, and with no other obvious distortions, decentralized equilibrium is
efficient in our framework. Starting from an untaxed equilibrium, the first order efficiency gains of government policy are therefore zero. There cannot be any welfare gains on account of equity since all agents are identical ex ante. The only source of first order welfare gains is, thus, social insurance in the sense of Varian (1980). Broadway et al. (1991) further noted that the gains from insurance actually stem from two components which are also reflected in (34), namely a direct effect from the tax transfer scheme, and an indirect effect from equilibrium price adjustment. Apart from the direct insurance provided by the progressive consumption (income) tax, the induced price reduction compresses the return on start-up investment and diminishes the risk of entrepreneurial income. In the scenarios of the preceding sections, in contrast, the government balances the budget by means of a proportional consumption tax and, thus, fails to provide any insurance. High income people pay high taxes but also receive a high consumption subsidy because they have high consumption as well. For this reason, there cannot be any welfare gains on account of social insurance in the preceding section.

To a large part, the public finance literature has dealt with the benefits from social insurance by simply assuming the absence of private insurance markets. In this paper, unconsolidated risk is an endogenous outcome of the optimal decisions of financiers who could otherwise costlessly provide full insurance. Entrepreneurial risk bearing is an essential part of venture capital finance to retain incentives for high effort in the face of a moral hazard problem. It was not obvious a priori whether government should interfere with

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21 Given homothetic preferences, \( V_i = \ln (u_i) \) with \( u_i = u_0 C_i^\alpha D_i^{1-\alpha} \), we can interpret the subutility as consumption of a ‘final good’ in quantity \( u_i \). In allocating \( u_i \) to workers and entrepreneurs, the planner faces the same constraints as in (13), and will therefore allocate consumption of entrepreneurs to the two states as in figure 1. Private contracts just replicate this allocation by means of an equity share cum base salary and are, thus, optimal. Whenever a worker gets \( u \) units, expected consumption of an entrepreneur must exceed it by a factor \( c(p) \). The social optimum is attained by maximizing the utility of a worker, \( \max \ln (u) \), subject to four constraints. One cannot distribute more units of final consumption than what is obtained from demanding the two goods, i.e. \( u_0 C^\alpha D^{1-\alpha} \geq u (1 - E) + c(p(a))uE \). The other restrictions are the two commodity balances, \( p(a) E \geq D \) and \( L \geq C + KE \), and the labor constraint, \( 1 \geq L + (1 + a) E \). It can be shown that the planning solution replicates the market allocation.
private risk sharing arrangements by further consolidating risk. In the light of proposition 6 we conjecture that the redistributive scheme yields welfare gains from social insurance while the efficiency cost of doing so is zero to the first order. However, as the scheme is expanded, it starts to distort incentives for managerial advice ever more.

Proposition 6 also relates in a non-obvious way to existing studies on entrepreneurship and insurance. Assuming absence of private risk sharing, for reasons exogenous to his model, Kanbur (1980) finds that government can raise welfare by taxing the risky occupation and subsidizing the safe occupation which restricts entry into the risky activity. In the presence of costly state verification and partial private insurance, Black and de Meza (1997) find the opposite result that subsidizing entry into the risky occupation is welfare improving. In their case, government does not have superior information over outcomes but can create collective insurance more cheaply through its influence on equilibrium prices. Our analysis is in between. Since expected income of entrepreneurs is higher than a worker’s income, our tax subsidy scheme implies a net tax on risky activities, as in Kanbur but different from Black and de Meza. On the other hand, via its effect on the equilibrium price of the innovative good, the scheme encourages entry into the risky occupation, as in Black and de Meza but different from Kanbur.

6 Conclusions

Promoting entrepreneurship and business formation is widely recognized as an important policy objective. Among others, the OECD has recently concluded that entrepreneurial activity needs strengthening.\textsuperscript{22} The structure of taxes, the operation of financial markets, and the “entrepreneurial climate” are important policy areas. This paper proposed a model of entrepreneurship and start-up investment that emphasizes risk-bearing on the part of entrepreneurs and its implications for occupational choice as well as the \textit{quality of}

\textsuperscript{22}The recent OECD country report on Austria, for example, includes a special feature on promoting entrepreneurship and employment, see OECD (1999).
equity finance’. Even though financiers may, in principle, diversify project risk, some risk-bearing on the part of entrepreneurs is nevertheless required to contain a moral hazard problem in the relation between VCs and entrepreneurs. Apart from structuring equity contracts, VCs also supply valuable business advice to strengthen survival chances of their portfolio companies. We analyzed how tax policy might influence the propensity for entrepreneurship as well as the incentives for managerial support by financiers. We found that a capital income tax strengthens incentives for managerial advice but reduces the number of entrepreneurs while a wage tax holds precisely opposite incentives. A broad-based income tax is neutral on all margins, distorting neither occupational choice nor incentives for business advice. Output and investment subsidies to start-up firms both stimulate entrepreneurial activity. Unfortunately, our static framework is not well suited to analyze capital gains taxation. Since much of the income from venture capital investments actually accrues in the form of capital gains, it seems an important task for future research to address this issue in an enriched intertemporal model.

One might argue that the government could raise welfare by further consolidating uninsured risks. Because of a moral hazard problem, however, private financiers intentionally abstain from offering complete insurance even though they could in principle do so by means of diversification. The entrepreneur’s profit share is intended to preserve incentives for effort by making them bear part of the risk. Further consolidation by the government could thus be counterproductive. Since government has no informational advantage over private financiers, private risk sharing arrangements, and indeed the entire market allocation, are socially optimal in our framework. Efficiency losses of taxes and transfers are therefore zero to a first order. However, government may generate welfare gains from social insurance if it introduces a small tax transfer scheme. Further expansion of redistributive taxation would, however, increasingly impair efficiency by distorting equity contracts and incentives for managerial advice. Future research might investigate the role of taxes and subsidies in situations where private contracts are not necessarily optimal. Such cases might arise if there are more than two states of nature, or effort is continuous rather than discrete. Private agents might then not have enough instruments
available to achieve the optimal risk sharing arrangement. One could also contemplate more traditional market distortions such as imperfect competition among VCs, or learning and knowledge spillovers among them, to justify public policies other than redistributive taxation to promote entrepreneurship and venture capital finance.

Appendix

A Cost of Contract: To prove proposition 1, note that a higher survival chance affects the entrepreneur’s profit share according to

$$\frac{d\theta}{dp} = -\mu \frac{\theta}{p}, \quad \frac{d^2\theta}{dp^2} = \mu \frac{\theta}{p^2} \left\{ 2 + \frac{\theta + \beta}{\theta + \beta} \right\} > 0,$$

(A.1)

where the elasticity is defined as

$$\mu \equiv -\frac{\partial \theta}{\partial p} = \frac{\theta + \beta}{\theta + \beta} \ln \left( \frac{\theta + \beta}{\beta} \right) > 1,$$

$$\frac{d\mu}{dp} = \frac{\beta \mu}{(\theta + \beta)^2} - \frac{\mu}{p} = \frac{\mu}{p} \left( \frac{\beta}{\theta + \beta} - 1 \right).$$

(A.2)

The elasticity is positive and larger than unity. To see this, use (13) and write \(\theta + \beta = \frac{1}{x}\) where \(x \equiv \left( \frac{(1-r)\delta + T}{1-r} \right)^{1/p} < 1\) (transfers \(T\) and loss offset \(\varepsilon\) are introduced only in section 5 and are now set to zero). Therefore, \(\theta + \beta = 1/(1-x)\). With these transformations, \(\mu > 1\) is equivalent to \(-\ln x > 1-x\) which is fulfilled by concavity of the In-function.

With the base salary constant, contract cost depends on \(p\) according to

\[
\begin{align*}
(a) \quad & c' = \theta + \theta \frac{\varphi}{\varphi} - rb\varepsilon = \theta (1 - \mu) - rb\varepsilon < 0, \\
(b) \quad & c'' = (1 - \mu) \frac{\varphi}{\varphi} - \theta \frac{\varphi}{\varphi} = \frac{\varphi}{\theta + \beta} > 0.
\end{align*}
\]

(A.3)

Except for \(T\), \(x\) and therefore the elasticity \(\mu\) do not depend on policy parameters. To prepare the analysis of transfers \(T\) in section 5, we compute

$$\mu (x) = \frac{-\ln x}{1 - x} > 1, \quad \mu' (x) = \frac{-y (x)}{x (1 - x)^2} < 0, \quad y (x) = 1 - x + x \ln x > 0.$$  

(A.4)

To establish the sign of \(\mu' (x)\), we have to show \(y (x) > 0\). We have \(y (1) = 0\) and \(y' (x) = \ln x < 0\) for all \(0 < x < 1\). The graph of \(y (x)\) is negatively sloped over the unit
interval and is zero at \( x = 1 \). Hence, \( y(x) > 0 \) for all values strictly less than 1. Section 5.2 requires the sign of \( \frac{du}{\partial T} = \mu'(x) \frac{\partial u}{\partial T} \) at the position \( r = T = 0 \) and \( \varepsilon = 1 \). In this case, 
\[
x \equiv \left( \frac{x + T}{1 + T} \right)^{1/p} \text{ and } \frac{\partial x}{\partial T} \bigg|_{T = 0} = \frac{x(1-b)}{p\varepsilon} > 0.
\]
Therefore, \( \frac{du}{\partial T} \bigg|_{T = 0} = \mu'(x) \frac{x(1-b)}{p\varepsilon} < 0 \).

**B Managerial Advice:** To obtain the policy effects on managerial advice as stated in proposition 2, take the differential of condition \( \Pi' = 0 \) in (18) and use (20). As shown in (16), the second order condition is \( \Pi'' < 0 \). An investment tax credit, for example, is seen to discourage advice,

\[
\frac{da}{dz} = -\frac{1}{\Pi''} \frac{\partial \Pi'}{\partial z} = \frac{Kp'p}{p\Pi''} < 0.
\]

(B.1)

The equilibrium effects of other policy instruments are similarly derived, and the following partials are used in the main text,

\[
\frac{\partial \Pi'}{\partial p} = p' \left( \frac{\partial m}{\partial p} - \frac{\partial c'}{\partial p} \right) = \frac{p'(c' - \theta)}{1 - \tau} = -\frac{\mu p'p}{1 - \tau} < 0,
\]

\[
\frac{\partial \Pi'}{\partial \tau} = 1 - \frac{(a+b)p'}{p} > 0,
\]

\[
\frac{\partial \Pi'}{\partial z} = p' \frac{\partial m}{\partial z} = -\frac{Kp'}{p} < 0.
\]

(B.2)

In the first line, we have \( \frac{\partial p'}{\partial p} = \frac{c'}{1 - \tau} \) by (A.3), (14) and the fact that the elasticity \( \mu \) is independent of any tax rates as mentioned following (A.2). Proposition 1 then yields the effect of the wage tax. To verify that the sign of the second partial is positive, multiply \( x \equiv 1 - (a + b)p'/p \) by \((1 - \tau)\) and replace the tax factor on the r.h.s. by (18):

\((1 - \tau) x = p' \left( m - c' \right) - (1 - \tau) (a + b) p'/p \). Using (13) and (15) to replace \( m \) yields

\((1 - \tau) x = -p'c' + \left( p\theta + (1 - z) K \right) p'/p > 0 \) which is positive due to \( c' < 0 \). Finally, the effect of a comprehensive income tax with \( r = \tau = t = z \) is

\[
\frac{d\Pi'}{dr} = \frac{\partial \Pi'}{\partial t} + \frac{\partial \Pi'}{\partial \tau} + \frac{\partial \Pi'}{\partial z} = 1 + \frac{p'c'}{1 - r} - \frac{p'}{1 - r} \left[ \frac{p\theta + (1 - r) (b + a + K)}{p} \right].
\]

The square bracket is simply \( m \) under the income tax. Using the efficiency condition (18), it is seen that a comprehensive income tax with full loss offset does not interfere with the VC’s incentives to provide managerial advice,

\[
\frac{d\Pi'}{dr} = -\frac{p' \left( m - c' \right) - (1 - r)}{1 - r} = 0.
\]

(B.3)
Alternatively, one may verify that all terms in (18) turn out to be proportional to a common tax factor $1 - r$, which cancels. Note in particular that also $\theta$ is proportional to $1 - r$ which is obvious from (14).

**C Extensions:** Table C.1 repeats those equations that change as a result of the scenario analysed in section 5, and derives some intermediate results. The primed equations must be compared with the basic model when a comprehensive income tax is in place, $\tau = t = z = r$ with $\sigma = 0$. Restricted loss offsets inflate cost $c$ in (3') because government takes over a smaller part of losses. Lump-sum per capita transfers $T$ are part of the individual income and are thus included in disposable income in (8'). In computing the incentive compatible contract we obtain (14') which collapses again to the basic result in (14) for $T = 0$. Since $\varepsilon$ does not enter the constraints in (13'), $\theta$ is not affected. To prove the effect of $T$ on $\theta$, note that $\beta = (1 - r) b + T$ with $b = \delta$ fixed, and get the differential of (PC), $\frac{\beta}{\beta} (d\theta + dT) + \frac{1 - \beta}{\beta} dT = \frac{d\theta}{1 - r + T}$, which yields

$$\frac{d\theta}{dT} = \frac{(\theta + \beta) [X_1 - X_2]}{\beta} < 0, \quad X_1 \equiv \frac{\beta}{1 - r + T} + \frac{p - 1}{p}, \quad X_2 \equiv \frac{\beta}{\theta + \beta}. \quad (C.1)$$

To show $X_1 - X_2 < 0$, use (14'), $\theta + \beta = \beta \left(\frac{1 - r + T}{\beta}\right)^{1/p}$, and get $X_2 = \left(\frac{1 - r + T}{\beta}\right)^{1/p}$.

Consider $X_1 (\delta)$ and $X_2 (\delta)$ for $0 < \delta < 1$. We have $X_1 (\delta) = \frac{1 - r + T}{\delta}$ and $X_2 (\delta) = \left(\frac{1 - r + T}{\delta}\right)^{1/p} \cdot X_1 (\delta)$. Furthermore, $X_1 (1) = X_2 (1) = 1$ and $X_1 (1) = X_2 (1) > 0$. The slope $X_1 (\delta)$ is independent of $\delta$ while $X_2 (\delta) < X_1 (\delta)$ for $\delta < 1$. Plotting the $X$ schedules against $\delta$ proves that $X_1 - X_2 < 0$ for all $\delta < 1$. With these results, we may restate proposition 1 as

$$c' = \theta (1 - \mu) - rb \varepsilon < 0, \quad c'' > 0; \quad \frac{dc}{dT} = p \frac{d\theta}{dT} < 0, \quad \frac{dc}{d\varepsilon} = rb (1 - p) > 0. \quad (C.2)$$

The convexity of the cost function is proved exactly by the same arguments as in appendix A, except that variable $x$ introduced after (A.2) is now defined as $x \equiv \left(\frac{(1 - r) T}{1 - r + T}\right)^{1/p} < 1$. Note that the elasticity $\mu$ is independent of the loss offset provision $\varepsilon$ but it does depend via $x$ on transfers $T$. To obtain the sign of $\frac{dx}{dT} = \mu' (x) \frac{dx}{dT}$, we first compute, for $0 < x < 1$,

$$\mu (x) = \frac{- \ln x}{1 - x} > 1, \quad \mu' (x) = \frac{- y (x)}{x (1 - x)} < 0, \quad y (x) = 1 - x + x \ln x > 0. \quad (C.3)$$
If \( y(x) \) is positive, the sign of \( \mu'(x) \) is negative. Since \( y(1) = 0 \) and \( y'(x) = \ln x < 0 \) for all \( 0 < x < 1 \), the graph of \( y(x) \) is negatively sloped over the unit interval and is zero at \( x = 1 \). Hence, \( y(x) > 0 \) for all values strictly less than 1. At position \( r = T = 0 \) and \( \varepsilon = 0 \), \( x = \left( \frac{\varepsilon + T}{1 - r} \right)^{1/p} \) and \( \frac{\partial \mu}{\partial T} \bigg|_{T=0} = \frac{x(1-\varepsilon)}{\varepsilon^2} > 0 \). Therefore, \( \frac{\partial \mu}{\partial T} \bigg|_{T=0} = \mu'(x)\frac{x(1-\varepsilon)}{\varepsilon^2} < 0 \).

The program in (15') determines optimal managerial advice. In equilibrium with free entry, the necessary condition is given by (18'). The reader may easily verify that the sufficient condition (16') surely holds if evaluated at \( \varepsilon \) close to zero. To obtain the comparative statics in the level of advice, we first need the derivatives of marginal cost \( c' \). While \( \theta \) and \( \mu \) are independent of \( \varepsilon \), they do depend on \( T \) where \( \frac{\partial \mu}{\partial T} < 0 \) by the arguments following (C.3). Therefore,

\[
\frac{\partial c'}{\partial \varepsilon} = -rb < 0, \quad \frac{\partial c'}{\partial T} = (1 - \mu) \frac{\partial \theta}{\partial T} - \theta \frac{\partial \mu}{\partial T} > 0. \tag{C.4}
\]

(18') gives \( \Pi''d\theta a = -\frac{\partial \Pi'}{\partial \varepsilon} d\varepsilon \), and similarly for \( T \). Using (20') and (C.4), one obtains

\[
\frac{\partial \Pi'}{\partial \varepsilon} = r \left[ \frac{(b + a + K) p'}{p} - (1 - p) \right] \geq 0, \quad \frac{\partial \Pi'}{\partial T} = p' \left[ \frac{\mu}{\partial T} + \theta \frac{\partial \mu}{\partial T} \right] < 0. \tag{C.5}
\]

Evaluate the derivative w.r.t. \( \varepsilon \) at the initial position \( r > 0 \), \( \gamma = 1 \) and \( \varepsilon = 0 \) where \( m = \frac{\theta + (1 - r) \theta + a + K}{p} \) and \( p' (m - c') = 1 - r \) by (18'). Also, \( c' = \theta (1 - \mu) \) initially whence \( p' (m - c') = p' \theta \mu + (1 - r) \frac{a + b + K}{p} p' = 1 - r \). Substitute \( \frac{a + b + K}{p} p' = 1 - \frac{\varphi \theta m}{1 - r} \) into (C.5) and get \( \frac{\partial \Pi'}{\partial \varepsilon} = r \left[ p - \frac{\varphi \theta m}{1 - r} \right] \geq 0 \). We are unable to sign this derivative.

Welfare \( V = \ln ((1 - r + T) / P) \) depends on the net real wage where \( P = (1 + v) Q^{1-\alpha} \) is a price index. Keeping \( r \) constant, \( \frac{dQ}{Q} = \frac{dm}{m} \), and the marginal welfare effect is

\[
dV = \frac{dT}{1 - r + T} - \frac{dv}{1 + v} - (1 - \alpha) \frac{dm}{m}. \tag{C.6}
\]

It is useful to rewrite (9') by imposing zero profits, \( Q = \frac{m}{1 - r} \). Use (15'), (7), (13'), (5'), (12') and \( \frac{Y}{1 + v} = C + QD \) to obtain \( T = \frac{r}{1 - r} \left[ 1 - r + (cN - (1 - r)) E + (1 - \gamma) (a + b + K) E \right] + \frac{v}{1 + v} Y \). With \( \Pi = 0 \), (8') implies \( Y - T = 1 - r + (cN - (1 - r)) E \) whence

\[
T = \frac{r}{1 - r} \left[ Y - T + (1 - \gamma) (a + b + K) E \right] + \frac{v}{1 + v} Y. \tag{C.7}
\]
Table C.1: Restricted Loss Offset and Lump-Sum Transfers

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3')</td>
<td>$c = (1 - r) spQ + b (1 - \gamma r)$</td>
</tr>
<tr>
<td>(4')</td>
<td>$\Pi = (1 - r) pQ - (1 - \gamma r)(a + K) - c$</td>
</tr>
<tr>
<td>(5')</td>
<td>$c^N = (1 - r)(spQ + b)$</td>
</tr>
<tr>
<td>(8')</td>
<td>$Y = T + c^N E + (1 - r)(L + aE) + \Pi E.$</td>
</tr>
<tr>
<td>(9')</td>
<td>$T = r [pQ - \gamma (a + b + K)] E + r [L + (a + b) E] + v (C + QD)$</td>
</tr>
<tr>
<td>(12')</td>
<td>$\theta = s (1 - r)Q, \quad \beta = (1 - r)b + T$</td>
</tr>
<tr>
<td>(13')</td>
<td>$c = p\theta + (1 - \gamma r)b$</td>
</tr>
<tr>
<td>(PC')</td>
<td>$p \ln (\theta + \beta) + (1 - p) \ln (\beta) \geq \ln (1 - r + T)$</td>
</tr>
<tr>
<td>(IC')</td>
<td>$p \ln (\theta + \beta) + (1 - p) \ln (\beta) \geq \ln (\beta + (1 - \delta)(1 - r))$</td>
</tr>
<tr>
<td>(14')</td>
<td>$\theta = \beta \left[ \left( \frac{1 - \gamma r}{1 - \gamma r + T} \right)^{1/p} - 1 \right], \quad b = \delta, \quad \frac{d\theta}{dT} &lt; 0, \quad \frac{d\theta}{dc} = 0$</td>
</tr>
<tr>
<td>(15')</td>
<td>$\Pi = \max_a \ p [(1 - r)Q - m], \quad m \equiv \frac{c(p_a,T)+(1-\gamma r)(a+K)}{p}$</td>
</tr>
<tr>
<td>(18')</td>
<td>$\Pi' = -pm' = p'[m - c' + r(a + K) \varepsilon] - (1 - \gamma r) = 0$</td>
</tr>
<tr>
<td>(19')</td>
<td>$E = (1 - \alpha) / \Omega, \quad \Omega \equiv (1 - \alpha)(1 + a + K) + \alpha \frac{pm}{r}$</td>
</tr>
</tbody>
</table>
References


Figures

Figure 1: Equity Contract

Figure 2: Number of Entrepreneurs